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Method of Packaging and Deployment of Large Membranes in Space

By

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Summary: The purpose of this paper is to present a new concept of packaging and deployment of large membranes in space. The problem of biaxially folding of a plane is transferred to the elastic problem of a biaxially compressed infinite plate. After solving the problem, the plate thickness is reduced infinitesimally small, and thus the result represents the isometric transfer of an infinite plane subject to biaxial shortening. As a result, the concave polyhedral surface is discovered, which is composed of a repetition of a fundamental region, which is further composed of four congruent parallelograms. It is shown that the packaging and deployment by this surface geometry satisfies various requirements as to operations in space.

Key words: Deployable structure; large space structure; packaging; membrane structure; solar power satellite; antenna; polyhedron.

1. INTRODUCTION

A number of future space missions will require ultra-low-mass, large space platforms or structures (LSS). As the representatives, we can exemplify the solar power satellite (SPS), the large antenna, etc. It is no doubt that whether such a kind of LSS project comes true depends upon how plane structures composing the LSS are made lighter and to what extent their cost is reduced. Thus, thickness of reflector surface, solar cell pannel, etc. is desired to be more and more thinner. As the result, it seems likely that the membrane structure which does not depend upon its bending stiffness, but depends upon in-plane stiffness comes to be greatly realized.

The study of membrane structures have been making considerable progress in its both theoretical and practical fields through various kind of structures on the earth. However, as for space applications, some problems, which have never been in serious consideration on the earth and therefore never been given investigation to them, become very important. They are the problems concerning packaging and deployment. Membrane elements of huge amount of square, regardless of their types such as deployable structure or erectable structure, are to be packaged, transported by shuttles, and deployed in

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space. Such a technique is beyond by far our imagination accustomed to familiar earth bound problems. For increase of the performance and reliability of LSS, it is conceived that the fundamental research should further be promoted in this field. The purpose of this paper is to present a new concept of packaging and deployment of large membranes in space.

2. Packaging Of Membrane Elements And Its Problems

Notwithstanding that membrane elements can naturally be admitted in-plane deformation, it can be dealt with as an inextensional object when packaging is studied macroscopically. To grasp a concrete image, one can take up a sheet of paper in imagination.

Packaging of membrane elements is tentatively classified as follows:

One-dimensional folding;

A. roll type

B. accordion-door or fan type

Two-dimensional folding;

C. orthogonal type (the operation B is repeated in two orthogonal directions)

D. other type

Among these A and B can be called one-dimensional folding owing to the fact that just one dimension of two-dimensional expansion of a surface is to be folded. Therefore one length is resticted to be not longer than the cargo hold. Theoretically C and D types can be, on the other hand, folded into optionally small size.

When one imagines a deployment operation in space, it is easily seen that the automatization of it cannot be attained unless the deploying process itself is simple and reliable to a considerable degree of extent. Thus simplification of the process is the utmost important requirement.

Since folding and deployment can generally be considered as two phases of a reversible process, the study of both may be performed by just conceiving one phase, say, the folding. The difference between them is that a complicated machine can be used for the folding process, while it is not so for the deployment in space.

Another important factor which must be considered is stress and fracture caused by folding. A thin sheet which is usually called a film has, in general, ductility endurable enough for the stress and deformation produced by giving a single fold. The stress concentration, however, occurs at the node when more than two folds cross each other. Illustrated in Fig. 1 is the deformation around a node with regard to a plastic sheet folded orthogonally. Hereafter, we will call this stress at the vicinity of a node the nodal stress.

Figure 2 is an enlarged sketch showing deformation of a sheet in the vicinity of a node. Qualitatively, breaking of membrane can be explained as follows.

Consider a sheet of membrane having a thickness t, which is folded at fold a. At the fold the membrane is curved, with a radius of curvature r_1. A maximum tensile stress will be produced on the outer surface of the fold, in the direction perpendicular to the fold. This stress is proportional to t/r_1. Thus, it is seen that the tensile stress will be greater the larger the membrane thickness, and the smaller the radius of curvature. When

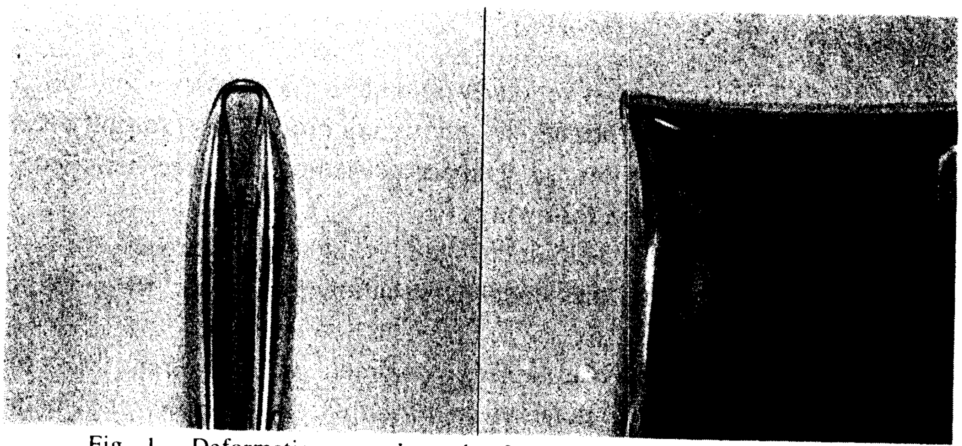

Fig. 1. Deformation around a node of plastic sheet folded orthogonally.

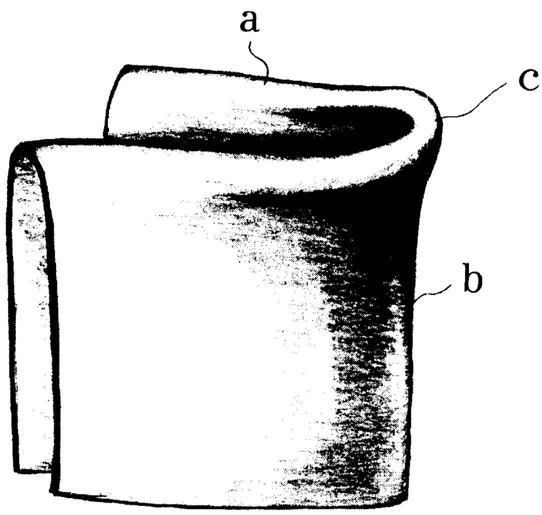

Fig. 2. Enlarged sketch showing deformation of a sheet at a node.

a second fold b (having a radius of curvature r_2) is made perpendicular to fold a, an additional tensile stress due to fold b is produced on the outer surface of the point of intersection of fold a and fold b, that is, the node c. Furthermore, since many more nodes on other sheet of membrane will usually come just inside the node c, a very large tensile stress will be exerted perpendicular to fold a. The outer edge of node c, where several tensile stress are superimposed, will be the starting point of breaking, from which a crack will be developed along the fold a. When one thinks of folding a Sunday edition of The New York Times, one will fully aware of these characteristics inherent in orthogonal folding.

It is thought that, heretofore, the orthogonal folding is a only known principal method of two-dimensional folding. It is the method already known in the era of ancient Egypt. In the era of human activities in space, other prossibilities of two-dimensional folding may be sought by the use of modern analytical procedures.

3. An Analytical Method For Studying Way Of Folding

The process in which a plane is regularly folded into a smaller plane belongs to a process called isometric transformation, whose conditions are severely restricted. Furthermore, particular conditions as a space structure must be satisfied. These conditions are summarized in the following.

1) Isometric condition must be held unchanged throughout the process.
2) Fold line is to be a two-dimensional tessellation of a plane by repetition of a fundamental region.
3) Folding (deploying) process itself must be complete within the fundamental region.
4) Deploying process is to be done through simple, continuous, and monotonous movement.

These conditions are so severe that it seems almost impossible to purposefully find a transformation that fulfils all of the conditions laid down and still attains the required function. Heretofore we have no analytical means to solve such a problem. If the situation is unchanged, are we obliged to depend upon such a classical method of Euclidean geometry, in which we are compelled to make use of our intuition and a trial-and-error method?

In reply to such a question, the author is going to explain that a problem of plane folding can be to some extent converted to an analytical problem described by partial differential equations.

Take an infinitely large elastic plate. Since this is, so to speak, an ordinary extensible plate, von Kármán's equations of large displacement configuration can be applied. Let us consider the case when the plate is uniformly contracted, on an average, in two orthogonal directions in its plane. The word 'infinitely large' cited here has some significant meanings.

First, it means that the thickness of the plate is infinitesimally small in relative sense. With reference to such a thin plate, the effect of decreasing thickness is that the strain energy of bending deformation decreases much faster than that of in-plane deformation. Since deformation certainly tends towards the one with minimum strain energy, if the thickness is reduced to zero, it should result in the pure bending deformation without any in-plane stretching and contraction. In short, both thickness and elasticity are fading away in accordance with this way, and the geometric solution of isometric transformation of a plane will finally remain.

Secondly, 'infinite' means that there is no terminal, therefore, a solution is assuredly periodic in both orthogonal directions. Thus we will be able to obtain at least an 'initial' solution of folding a plane by solving von Kármán's equations of plate and by getting to infinity. The term, 'initial' means that the deformation will be restricted within a domain to which von Kármán's equations are to be applied. Deformation to be followed to finality, required by present study, should be assured with the aid of other implements.

The computation was carried out by TANIZAWA and MIURA (1978) [1], and some of the results are shown in Fig. 3.

Figure 3 shows the normalized deformations of typical solutions by means of contour

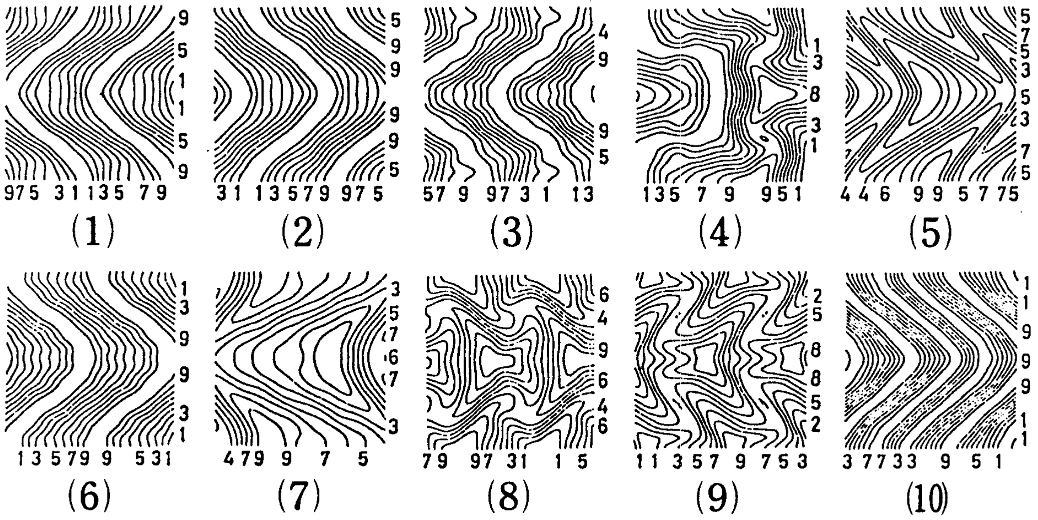

Fig. 3. Various solutions as to deformation of bi-axially shortened infinite plate (contour map).

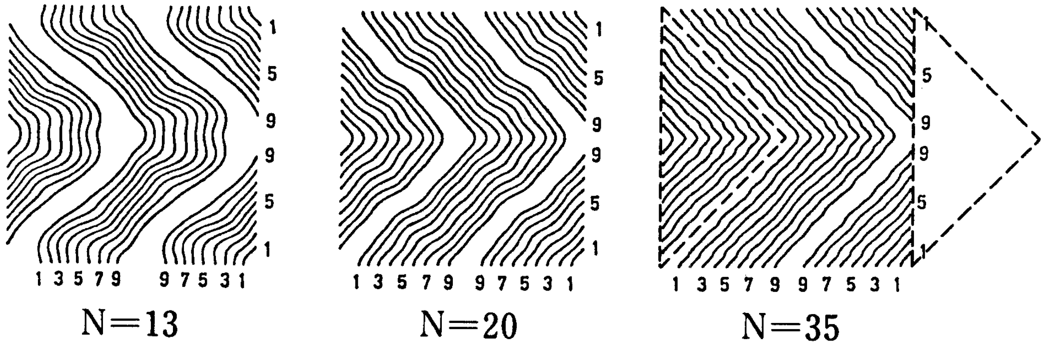

Fig. 4. Convergence to DDC surface (N: number of terms in Fourier series expression).

maps. The numbers at the border indicate the elevation and the solutions are shown within their square fundamental region. These solutions correspond to extremes of the strain energy. It is to be noted that the deformations of case number (1), (2), (6) and (10) resemble each other. Moreover, according to the calculation, the value of the strain energy of such a kind of solution is found to be, without exception, remarkably smaller than those of the other solutions.

4. Generation Of Developable Double Corrugation Surface

It is found that this configuration bears a close resemblance to the surface which has been predicted by MIURA (1970) at IASS Symposium, Wien [2], and it was later named the developable double corrugation surface (DDC surface). Further computation also revealed that, by increasing the number of terms in Fourier series expression of the solution, the solution converges unlimitedly to the idealized DDC surface, as shown in Fig. 4.

In principle, a fundamental region of a tessellation can be converted to another fundamental region by transferring some portion of it. Thus, the square fundamental region shown in the right figure of Fig. 4, can be modified to a herringbone shaped

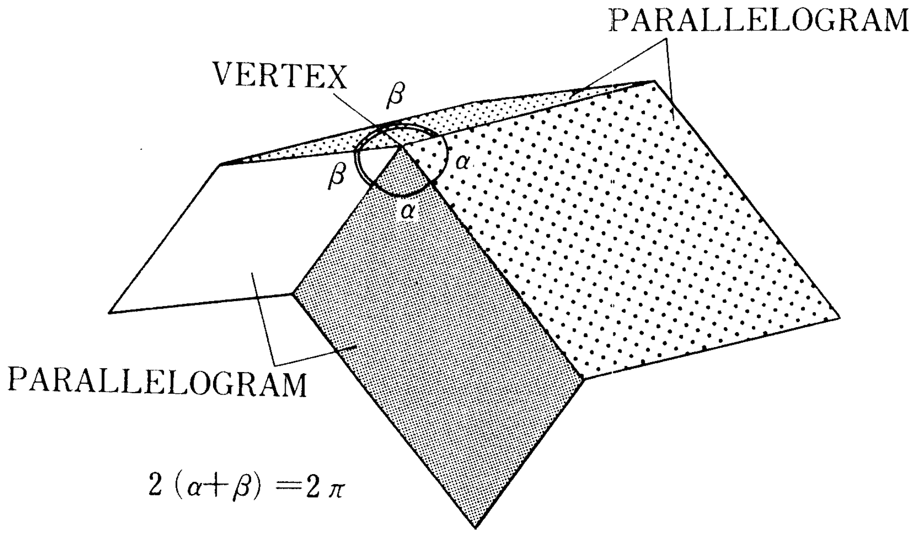

Fig. 5. Fundamental region of developable double corrugation (DDC) surface.

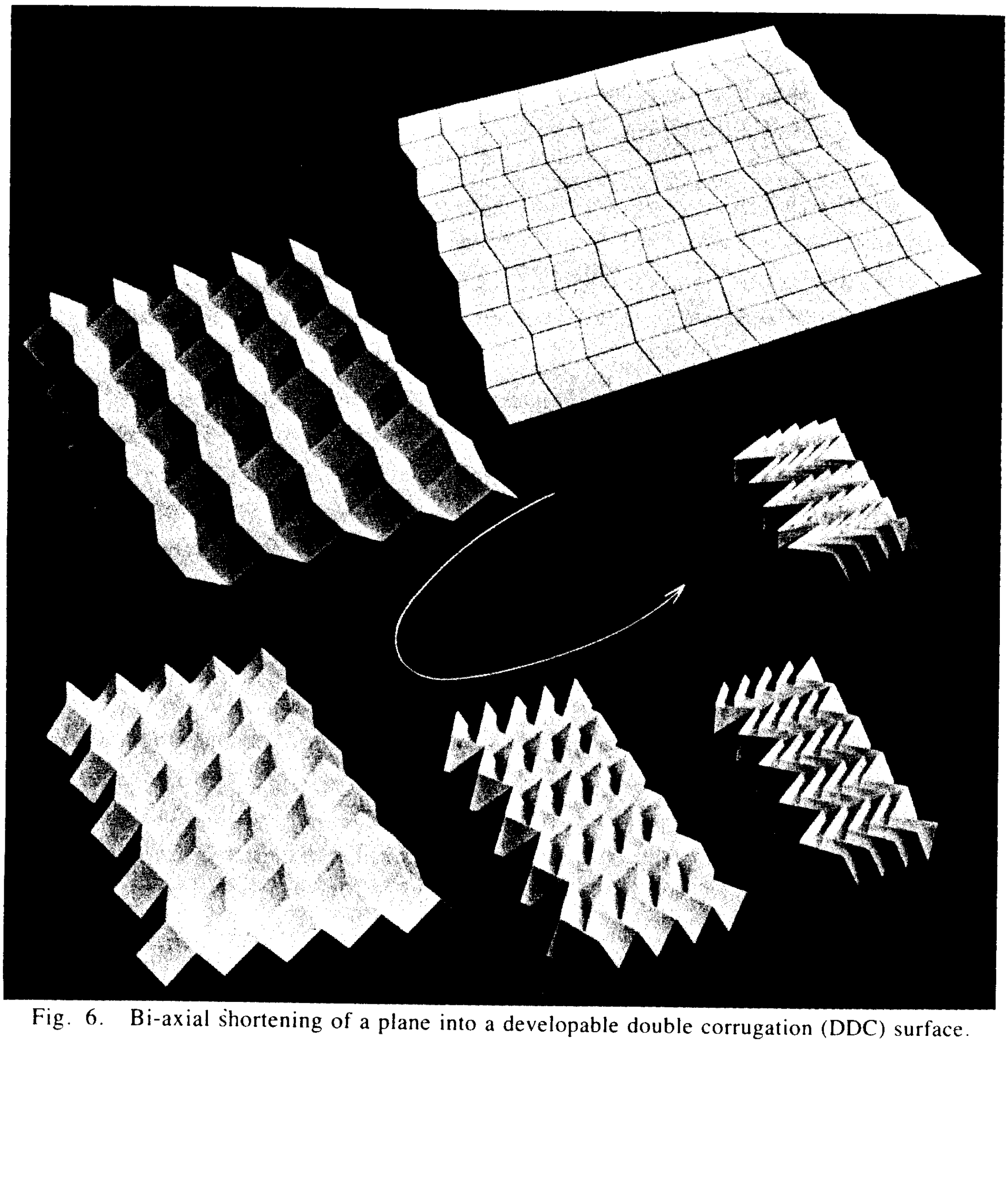

Fig. 6. Bi-axial shortening of a plane into a developable double corrugation (DDC) surface.

fundamental region. It is a polyhedral surface composed of four congruent parallelograms as shown in Fig. 5.

A whole configuration of the surface is obtained by two translations of this fundamental region in orthogonal directions X and Y, as shown in the upper right figure of Fig. 6.

In order to study the characteristics of the surface, particular attention must be given to the vicinity of vertex or, in terms of geometry, the polyhedral angle. It is observed that, if the difference between the right and the back side is discarded within the problem of chief interest, every polyhedral angle of it is congruent with each other. The polyhedral angle, shown in Fig. 5 is composed of four angles α, α, β, and β, where α and β are internal angles of a parallelogram, respectively, and thus they are supplemental with each other. Therefore, the sum of four angles of the polyhedral angle is always 2π. Furthermore, it goes without saying that at every edge of the polyhedral surface, the sum of internal angles is 2π. It can be concluded that the sum of internal angles is 2π everywhere on the surface. This is the proof that the surface is isometric with a flat surface or, in other words, the surface is developable.

At the beginning of this analysis, we define the analytical solution within the domain to which von Kármán's equations are to be applied. However, the above reasoning on the developability of the surface clearly does not depend on the finiteness of deformation. Thus, the resulting DDC surface may be considered as the solution of the problem up to where the shortening approaches the finality.

5. Two Dimensional Folding By DDC Surface

Is there any way in which a plane will be folded in two mutually perpendicular directions at the same time, and in a uniform way? At first thought one can hardly believe such a possibility, but the developable double corrugation (DDC) surface does give the solution. Remember that the DDC surface is obtained by the contraction of a plane in two orthogonal directions. In Fig. 6 a series of pictures of a paper model shows how a plane is folded up and contracts itself lengthwise and broadwise simultaneously. The intermediate product of this process is a DDC surface consisting of a number of congruent parallelograms. If an ideal paper of vanishing thickness is assumed, and it is folded up infinitesimally closely, then it will be folded up into a point. In other words, folding of this sort corresponds to transformation of a plane into a point.

A remarkable fact to be noted here is that the foldings or contractions in two mutually perpendicular directions are not independent with each other. The contraction in the X direction should always be accompanied by the contraction in the Y direction, and vice versa. In contrast, the foldings in the X and Y directios are completely independent in the case of the so-called orthogonal folding. This may help one understand the singular feature of the proposed method of folding.

Consider a sheet of papaer so folded that it forms a DDC surface, and see the motion of an arbitrary fundamental region on the plane. When the fundamental region is given some deformation, say, by making the fold angle sharper, the adjacent fundamental region will undergo exactly the same deformation, which, in turn, causes further

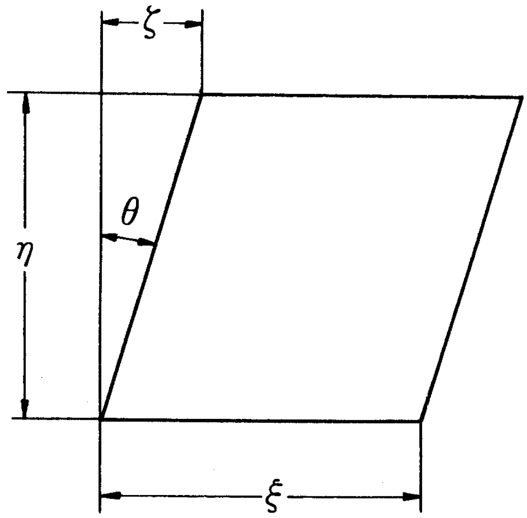

Fig. 7. Basic parallelogram element whose tessellation composes fold lines of two-dimensional folding by DDC surface.

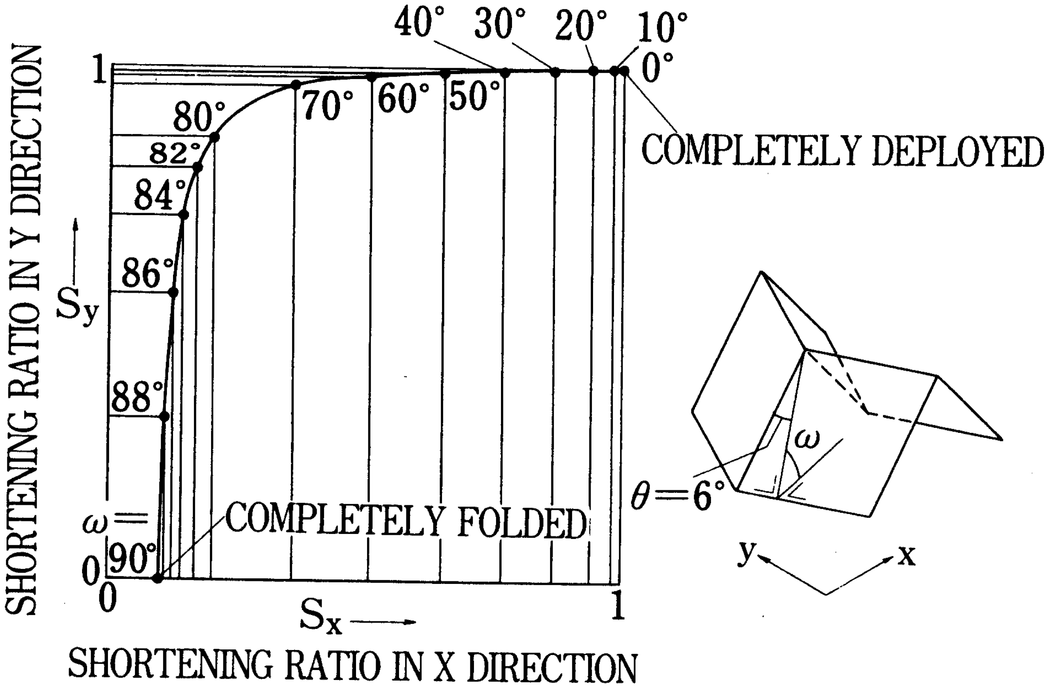

Fig. 8. Locus of folding and deployment of two-dimensional folding by DDC surface (relation between shortening ratios S_x and S_y).

deformation to the fundamental regions adjacent to each of the deformed regions. Thus, it is seen that the deformation given to a particular fundamental region will propagate over the entire surface in an instant. The paper, when so folded, behaves as if it has a built in linkage mechanism to fold or deploy. To deform, that is, to fold the whole paper, it is necessary only to fold any one of the fundamental regions.

Figure 7 shows a basic parallelogram element which determines fold lines of two-dimensional folding by a DDC surface. It is seen that there are only three independant parameters which completely determine the fold line geometry. In addition, to determine a DDC surface in three dimensional space, a parameter such as the angle ω between the parallelogram element and the original plane is necessary as shown in Fig. 8.

If S_x and S_y represents the shortening ratio in X and Y direction, respectively, these are expressed as follows:

$$S_x = \cos\left[\sin^{-1}(\cos\theta\cdot\sin\omega)\right] \tag{1}$$

$$S_x = \cos\omega/\cos\left[\sin^{-1}(\cos\theta\cdot\sin\omega)\right] \tag{2}$$

where the definition of θ and ω are described in Fig. 8. The relation between S_x and S_y, and ω is plotted graphically in Fig. 8 ($\theta=6°$). The solid curve in this figure indicates the locus of (S_x, S_y) along decreasing of ω. When $\omega=90°$, we obtain the completely folded position, while when $\omega=0°$, we have the completely deployed position, both are indicated so in the figure.

It is observed that there are two distinct phases in the deployment process. The first phase of a deployment occurs primarily along the Y direction up to 80% of full expansion with only 20% deployment in the X direction. On the contrary, the second phase followed is carried out primarily in the X direction. Nevertheless, on the whole, the deployment process is perfected through a single, simple, continuous, and monotonous movement.

In a previous section, we have enumerated four required conditions which must be fulfiled by any way of folding. It is seen that the characteristics of the method of folding by the DDC surface studied above are exceptionally favorable for all of the requirements.

Next, with regard to the strength of folds, the folding method under consideration has an essentially advantageous feature. In the case of the orthogonal folding, that portion of the sheet which thrusts itself deeply inside the second fold causes a large tensile stress at the node, as has been explained earlier, whereas, in the case of present folding, only a single sheet of membrane will come beneath the second fold, and that with less depth. This helps reduce the above-mentioned stress by a large margin. As a result, the folds in our case is much stronger than those in the case of the orthogonal folding.

6. Concluding Remarks

In summary, a new concept of packaging and deployment of large membranes is presented. The concept is founded on the characteristics of a developable double corrugation (DDC) surface. It is shown that the packaging and deployment by this surface geometry satisfies the necessary requirements. Further engineering study will be needed for developing the concept.

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