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Arm Length Matching in 10m Delay-Line Interferometer

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Summary: The noise due to laser frequency fluctuation in a Michelson interferometer can be suppressed by making the arm length difference equal to zero. But it is difficult to match the arm lengths in the interferometers with a large number of reflections in delay-lines. In this paper we show result that it is possible to achieve CMRR (Common Mode Rejection Ratio) of 119dB for the noise due to laser frequency fluctuation, by using the mirrors with small relative difference to radii of curvature and most tiltless surfaces.

1. INTRODUCTION

The 10m prototype for the laser interferometric gravitational wave antenna (TENKO-10 [*1*] has been operated to investigate its noise behavior. The schematic diagram of the 10m prototype is shown in Fig. 1. This interferometer, which is based on a Michelson interferometer formed of two opposite mirrors in each arm to achieve long path length, is illuminated by an argon-ion laser (GLG3304, NEC). The laser beam divided at a beam splitter (B.S.) enters the interferometer arms through the entrance holes cut in the near mirrors, which are spherical mirrors nearest the beam splitter. In the arms, the laser beam is reflected by near and end mirrors with a beam number N of 102. This multi-pass geometry is called optical delay-line [*2*]. Finally, each beam goes out the optical delay-line through the same hole it entered. One of the interfering beam, which reached the photo detector P.D.1, is held to the minimum of intensity by a lock-in system, whose feed-back signal would contain the gravitational wave signal.

The fundamental limit of the sensitivity in an interferometer is given by the inevitable shot noise of the photons. For a light power of 1W (at wavelength λ=515nm) this corresponds to an equivalent path difference noise of about 10^{-16}m/$\sqrt{\text{Hz}}$. Moreover, these are many kinds of noise sources which make the total sensitivity worse. The noise due to laser frequency fluctuation is one of them. If the total path lengths of two arms are different by a static offset ΔL_{ST}, this directly translates the frequency fluctuation $\delta\nu$ into apparent signal δL_{FR} in the interferometer output as follows.

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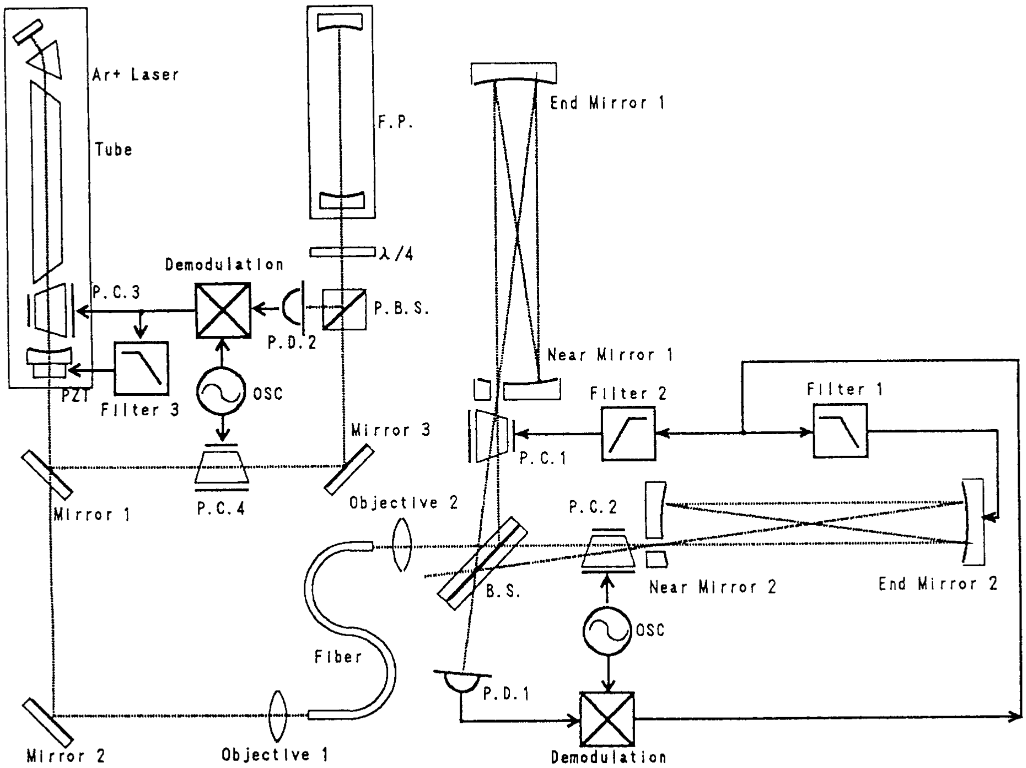

Fig. 1. Schematic diagram of the 10 m prototype of a laser interferometric graviational wave antenna. The frequency stabilizing argon-ion laser, the optical fiber, the two arms of the interferometer with 4 beam delay-line, and fringe lock-in system are shown.

$$\delta L_{FR} = \Delta L_{ST} \frac{\delta \nu}{\nu} \tag{1}$$

or

$$\frac{\delta L_{FR}}{L} = \frac{\Delta L_{ST}}{L} \frac{\delta \nu}{\nu}, \tag{2}$$

where L is total path length of each arm and ν is laser frequency ($=5.8\times10^{14}$Hz). Provided l is separation between a near mirror and an end mirror, *i.e.* arm length, L is defined as follows.

$$L = l \times N. \tag{3}$$

Although simple Michelson interferometers without delay-lines can be easily adjusted so that the path length difference ΔL_{ST} of two arms may disappear, it is difficult to satisfy the condition in the interferometers with a large number of reflections in delay-lines. On the ohter hand, frequency fluctuation $\delta\nu$ of our free-run laser is about $10^3\mathrm{Hz}/\sqrt{\mathrm{Hz}}$ at 1kHz. In order to acquire a sensitivity equal to the level of photon shot noise, we need a path length difference ΔL_{ST} smaller than 10^{-4}m. To avoid this critical requirement, we have made the laser stabilized by RF reflection locking (Pound-Drever) method [3]. A frequency stability acquired in this stabilized laser is

$\sim$ 1Hz/$\sqrt{\text{Hz}}$ at 1kHz. In this case a required path length difference becomes approximately 0.1m.

2. Mirrors for Delay-Line Interferometer

Main mirrors, introduced recently, used in the delay-lines have diameters of 160mm, thicknesses of 25mm, radii of curvature of 10.115±0.005m (10.18±0.015m), entrance hole diameters of 6mm (near mirrors only), and reflectivities of 99.95% (99.5%). The arm lengths are adjusted to 9.803m (9.86m) to realize a beam number of 102 in the delay-lines. Values in parentheses show the specifications of old mirrors used previously. The new mirrors have better performance than the old mirrors on relative difference to radii of curvature.

In the re-entrance condition, where the beam goes out of the optical delay-line through the entrance hole, the arm length l is decided by beam number N in delay-line and radius R of curvature.

$$l \approx \left(1 - \frac{\pi}{N}\right) R. \tag{4}$$

That means the re-entrance condition with desired beam number N can be achived by adjusting the arm length l to eq. (4). From eq. (4),

$$dl \approx \frac{\pi R}{N^2}\, dN. \tag{5}$$

A change of beam number N occurs on a step of $dN = 4$ by every arm length change dl of about 12mm at $N = 102$. On the other hand, arm length change dl leads to a lateral translation of the outgoing beam by dy.

$$dy \approx \frac{N x_0}{R}\, dl. \tag{6}$$

where x_0 is distance of the entrance hole from the center of the near mirror. Only 1mm change in arm length causes lateral translations of the outgoing beam by approximately 0.6mm against the 6mm diameter of the entrance hole. In the case of our old mirrors, each radius of curvature has difference of 15mm. If one arm length is changed by 15mm to match arm lengths, the position of the outgoing beam on the near mirror shifts by 9mm. In this cace, the re-entrance condition in this arm is not fulfilled. Finally, arm length difference of 10 $\sim$ 15mm is residual in the re-entrance condition. In the Garching 30m prototype a total path length difference of about 2m is observed when both 90 beam delay-lines are re-entrant [*4*]. Even if the difference is smaller than 5mm as our new mirrors, the position of outgoing beam on the near mirror shifts from the center of the entrance hole. Fig. 2 shows an example of the contrast dependence of interference fringe upon the outgoing beam position on the near mirror with the same entrance position of beam. If we need a contrast better than 90%, the allowed region of the beam position change is about 0.9mm. This change corresponds to an arm length change of 1.5mm.

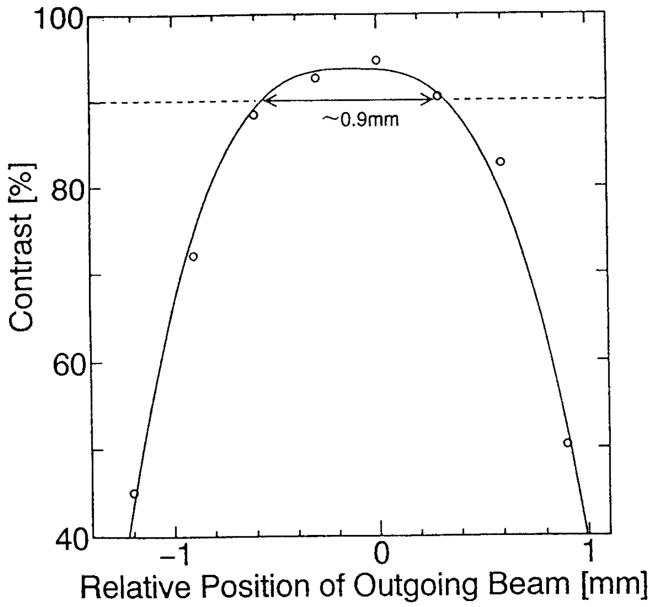

Fig. 2. An example of the contrast dependence of interference fringe upon the outgoing beam position on the near mirror with the same entrance position of beam. If we need a contrast better than 90%, the allowed region of the position change is about 0.9 mm.

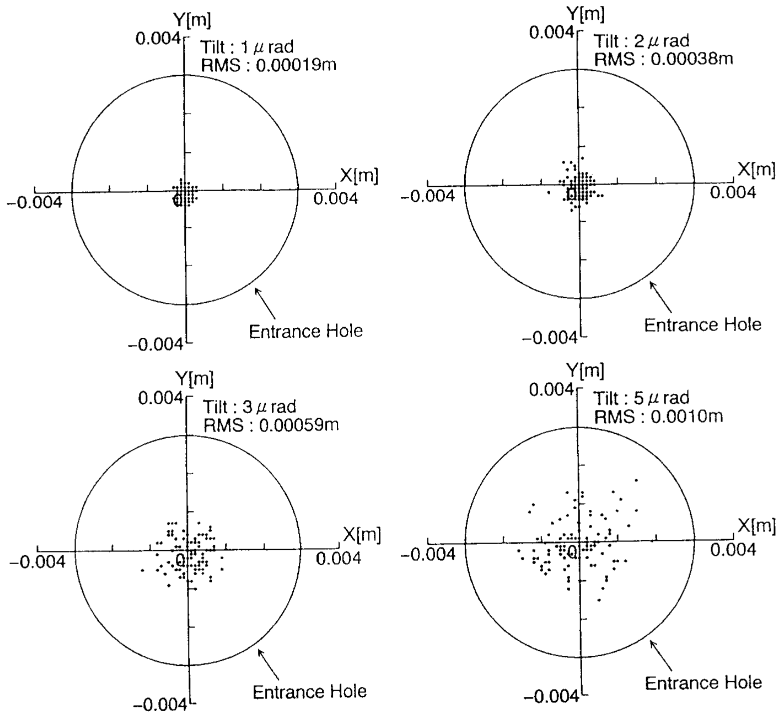

Fig. 3. The position of the outgoing beam from the delay-line consist of two mirrors with the local tilt. The cases of mirrors with the tilt of 1~5 μrad are shown. In the case of the mirrors with the tilt of 1 μrad, the beam position has a deviation of 0.2 mm_{rms} from the center of entrance hole.

There is another problem. A local tilt of mirror surface on the scale of beam spot size affects the direction of reflected laser beam. The position of outgoing beam on the near mirror shifts from expected position by this effect. For example, as shown in Fig. 3, the position of the outgoing beam from the delay-line consist of two mirrors with the local tilt of 1μrad has a deviation of 0.2mm_{rms} from the center of entrance hole. In order to achieve the deviation smaller than the allowed region of 0.45mm radius without adjusting arm length, we need mirrors with local tilt less than 3μrad. This tilt coresponds to mirror surface accuracy of 1/100λ.

3. Arm Length Matching

The method of matching arm lengths is simple. By adding monochromatic frequency modulation to the incident laser beam, a signal δL_{FR} proportional to the statical path difference ΔL_{ST} appears in the interferometer output as shown in eq. (1). In principle, when ΔL_{ST} is equal to zero, this signal δL_{FR} is bolted out. Practically, by vibration of mirrors and scattered light effect [5], there are residual noises depending on the additional modulation.

The monochromatic frequency modulation is generated by changing the cavity length of the laser. In the laser cavity with length of 1.4m, there is a Pockel's cell (PM25, Gsänger), *i.e.* P.C.3 in Fig. 1. Efficiency of this Pockel's cell is 2.65×10^{-10}m/V. We applied a sinusoidal signal E to P.C.3 at 4kHz, with the amplitude of 1.4~7.1V_{rms}. From these values,

$$\frac{\delta\nu}{\nu} = 1.9 \times 10^{-10} \times \left(\frac{E}{1\text{V}}\right). \tag{7}$$

The arm length can be changed by moving a suspension stand of the end mirror in the direction of beam axis. The stand is controlled by a stepping motor remotely. One count of the stepping motor corresponds to the position change of 0.5μm.

Fig. 4 shows relations between noise due to artificial laser frequency fluctuation and statical path difference. Four kinds of data show measured values for additional signals of 1.4, 2.8, 5.3, and 7.1V to P.C.3, respectively. Each data is fitted to linear function by method of least squares. We can recognize linear dependence as expected in eq. (1). All of lines cross approximately at the same point where the noise due to the frequency fluctuation disappear. At this point, statical path difference is near zero. Further, $\delta\nu/\nu$s evaluated from the slopes of fitting lines are consistent with that calculated from eq. (7) by the factor of 2.

In order to investigate measurement error of statical path difference, data were taken in detail to arm length change. Fig. 5 shows relation between noise due to the frequency fluctuation and statical path difference. By using method of least squares, standard deviation of measured point in direction of X-axis is decided to 1.1×10^{-3}m. The data less than -0.03m or near 0m in path difference are not used for this caluclation. The ratio of total path length L and residual path difference ΔL_{ST} (eq. (2)) is called Common Mode Rejection Ratio (CMRR). In the case of our measurement, since L is 1km from eq. (3), calculated CMRR is 119dB.

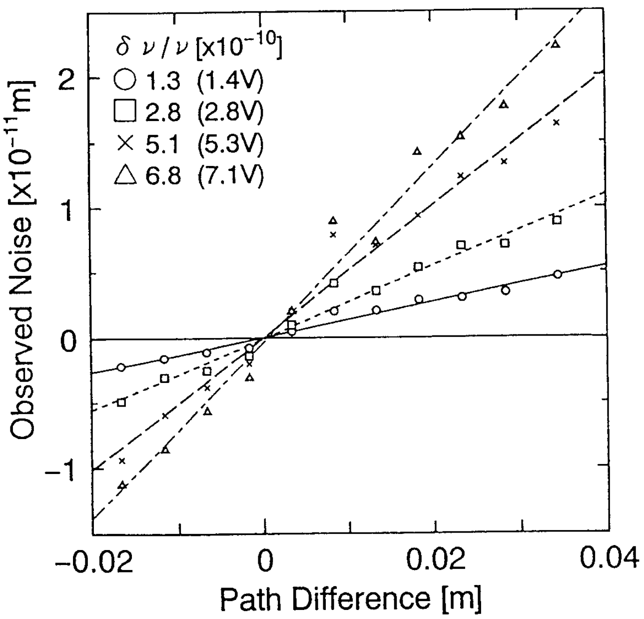

Fig. 4. Relations between noise due to artificial laser frequency fluctuation and relative statical path difference. Four kinds of data show measured values for additional signals of 1.4, 2.8, 5.3, and 7.1 V_{rms} to P.C. 3 (Fig. 1), respectively. Each data is fitted to linear function by method of least squares. All of lines cross approximately at the same point where the path difference is zero. $\delta\nu/\nu s$ evaluated from the slopes of fitting lines are written.

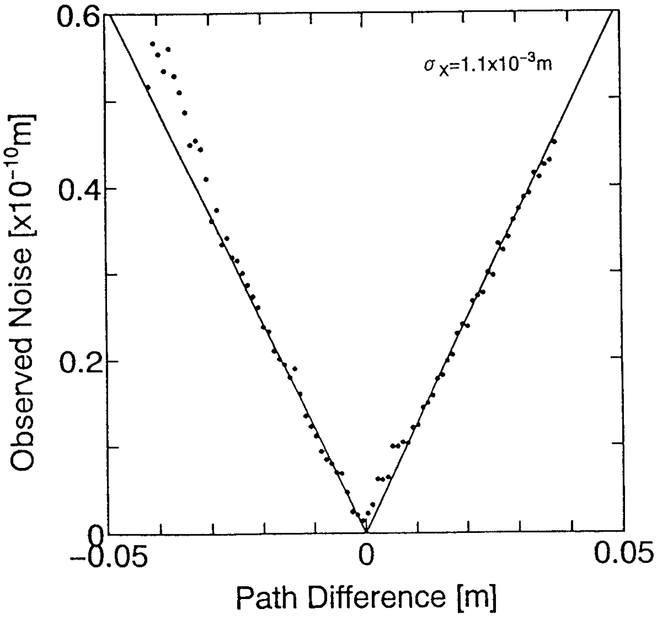

Fig. 5. The relation between noise due to the frequency fluctuation and statical path difference. In order to investigate measurement error of statical path difference, data were taken in detail to arm length change. By using method of least squares, standard deviation of measured point in direction of X-axis is decided to 1.1×10^{-3}m. The data less than -0.03 m or near 0 m in path difference are not used for this calculation.

4. Discussion

When we made matching of arm length, contrast of interefering light better than 90% was always able to be achieved. Therefore it is estimated that the difference of each mirror radius of curvature is less than 1.5mm, and local tilt of mirror surface is less than 3μrad.

If we acquire possible statical path difference ΔL_{ST} = 1.1mm, the contribution from laser frequency fluctuation is $\delta L_{FR} = 2 \times 10^{-18}$m/$\sqrt{\text{Hz}}$ with stabilizing laser, and $\delta L_{FR} = 2 \times 10^{-15}$m/$\sqrt{\text{Hz}}$ with free-run laser from eq. (1). In the case of using the stabilizing laser, this noise is much less than photon shot noise of 10^{-16}m/$\sqrt{\text{Hz}}$ for light power of 1W, and comparable with the shot noise level for 10kW light power. First in near future, scattered light effect will become critical when its relative field strength σ is larger than 10^{-3} because of its large path difference to the main beam. The contribution from scattered light is $\delta L_{FR} = 2 \times 10^{-15}$m/$\sqrt{\text{Hz}}$ at maximum with stabilizing laser supposing $\sigma = 10^{-3}$.

Fig. 6 shows sensitivities of our interferometer without frequency stabilizing system. In the case of using old mirrors, since the residual difference of arm length is approximately 10mm at the same number of reflection, noise due to frequency fluctuation is large (upper curve). On the other hand, the sensitivity a little larger than expected noise $\delta L_{FR} = 2 \times 10^{-15}$m/$\sqrt{\text{Hz}}$ at statical path difference ΔL_{ST} = 1.1mm was acquired with new mirrors (lower curve). An origin of this little larger measured noise could be mismatching of the arm lengths caused by drift of the mirrors. The mirrors' motion are dumped at low frequencies to prevent large motion due to the

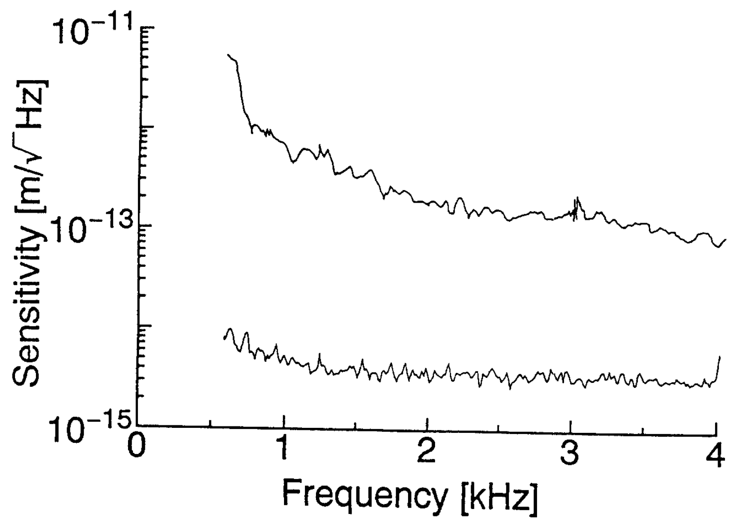

Fig. 6. Sensitivities of our interferometer without frequency stabilizing system. In the case of using old mirrors, since the residual difference of arm length is approximately 10 mm at the same number of reflection, noise due to frequency fluctuation is large (upper curve). On the other hand, the sensitivity a little larger than expected noise $\delta L_{FR} = 2.9 \times 10^{-15}$m/$\sqrt{\text{Hz}}$ was acquired with new mirrors (lower curve).

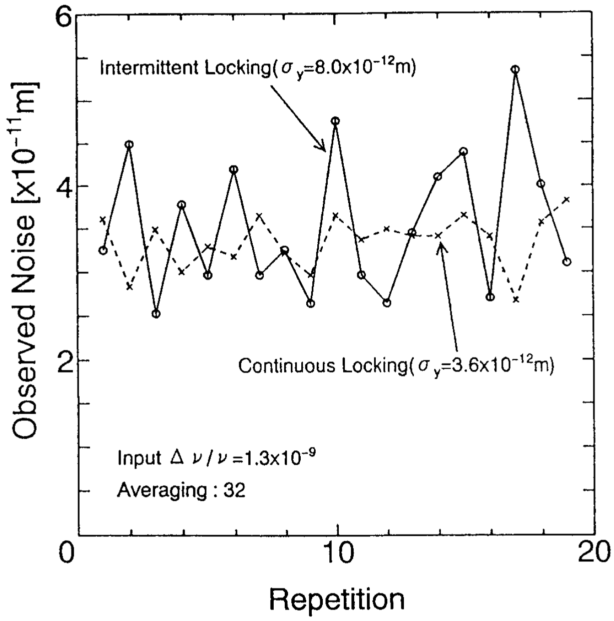

Fig. 7. Comparision between changes of observed noises due to additional frequency fluctuation in two conditions of locking the interference fringe by the main feedback system. Change of the observed noise in intermittent locking is larger than that in continuous locking.

ground vibration at the resonance frequency (near 1Hz). So that the influence of ground motion is relatively small and the fluctuation in the difference between the arm lengths is 5×10^{-7}m around 1Hz [*1*]. However, the drift of mirror in the dumping system is not small. This drift may be caused by thermal expansion and contraction of the vacuum chamber where mirrors are suspended. Even if mirrors are not moved artificially, the level of observed noise due to additional frequency fluctuation changes when the interference frings is locked by the main feedback system intermittently (Fig. 7). Therefore, without continuous locking after arm length matching, it is difficult to keep CMRR the level of 119dB at all times.

A plan of constructing a 100m laser interferometric gravitational wave antenna (TENKO-100) [*6*] based on the 10m interferometer is now going on. In this plan large size mirrors, which have diameters of 350mm, radii of curvature of ~ 100m, and entrance hole diameters of ~ 18mm, will be used. Provided a reflection number in the delay-line is 102 and a relation of contrast in Fig. 2 is scaled, accuracy of ~ 20mm to radius of curvature is required. In the same way, accuracy of 1μrad is necessary because of long base line for a local tilt of mirror surface on a scale of beam spot size. It will be not easy to achieve these specifications. If we give up matching the arm lengths, the re-entrance condition will be satisfied by adjustment of each arm length and by making use of oval entrance hole long in direction of mirror's radius. Our near mirrors for 100m interferometer have such an oval entrance hole.

5. Conclusion

We have developed the 10m prototype for the laser interferometric gravitational wave antenna (TENKO-10). Good CMRR of 119dB for the noise due to laser frequency fluctuation was achieved by making use of the mirrors with small relative difference to rudii of curvature and most tiltless surfaces, although the number of reflections in optical delay-lines is so large as 100. From these results, it is estimated that the difference of each mirror radius of curvature is smaller than 1.5mm, and local tilt of mirror surface is smaller than 3μrad.

In the plan of constructing a 100m laser interferometric gravitational wave antenna (TENKO-100), not only the mirror fabrication but also the establishment of test will become important.

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