EFFECT OF MODELING ON DYNAMIC STABILITY OF A FLEXIBLE CANTILEVERED PLATE SUBJECTED TO A PARALLEL FLOW

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The stability of a flexible cantilevered plate subjected to an axial flow is investigated. As the flexible flat plates, the papers in a high speed printing machine, the thin plastic and metal films, the fluttering flag are enumerated. The fluid is assumed to be treated as an ideal fluid in a subsonic domain, and the fluid pressure is calculated using the velocity potential theory. The coupled equation of motion of a flexible cantilevered plate is derived taking into consideration with the added mass, added damping and added stiffness respectively. The complex eigenvalue analysis is performed for the stability analysis. In order to consider the accuracy of dynamic stability analysis, three stability analysis methods are performed. Firstly, the analysis method based on boundary conditions in the half space surrounded by the leading edge and the trailing edge of a plate is performed. Hereafter, let's call it the coupled solution. Secondly, the analysis method based on the non circulatory aerodynamic theory is performed. Let's call it the non circulatory solution. Thirdly, the analysis method which fulfills the Kutta's condition is performed. Let's call it the circulatory solution. The following matters are clarified through the comparison of three solutions. When the mass ratio of a fluid system for a structure system is small, the flutter of the lower mode such as a second mode become predominant. And, when the mass ratio is large, the higher mode flutter appears. The critical flow velocity of the circulatory solution becomes lower than those of the coupled solution and the non circulatory solution when the mass ratio becomes low. On the other hand, the critical flow velocity of the circulatory solution becomes higher than those of the coupled solution and the non circulatory solution when the mass ratio becomes high.

Keyword: flutter, flow-induced vibration, self-excited vibration, panel flutter, parallel flow

1. INTRODUCTION

The energy-related machine and structure often show the performance through fluid and heat. The destructive vibration was activated to a machine and a structure by fluid flow, and it has been often experienced that it led to a great accident. Here, our attention is paid to the dynamic stability of a relatively flexible flat plate subjected to a parallel flow, namely flutter. For example, here, the vibrations of the paper in a high speed printing machine, the vibrations of the thin plastic and metal films, and in addition, the vibrations of turbo-machineries and the components of nuclear power plant are considered to be a problem.

There are already many studies in this field. For example, as a representative example of a flexible structure, Chang and Moretti[1), 2)] reported the experiment and the analysis about the instability of a paper. In addition, the study on the stability of the stationary flat plate subjected to a water power reported by Weaver and Unny[3)], and the study on stability of the cantilevered rectangular plate in the non-viscous passage flow reported by Guo and Paidoussis[4)] have been included. Moreover, many research papers such as Bidkar et al.[5)], Howell et al.[6)] and Eloy et al.[7)] have been reported. Also, the authors have reported the flutter and divergence of the stationary simply supported flexible plate subjected to a parallel flow and the simply supported plate moving through the stagnant fluid by Fujita and Imai[8)], Fujita[9)]. Successively to these, in this paper, we propose three kinds of analysis modelings on the dynamic stability of the flexible cantilevered plate subjected to a parallel flow from a simplified modeling to a more exact modeling.

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Three stability analysis methods are as follows. Firstly, the analysis method based on boundary conditions in the half space surrounded by the leading edge and the trailing edge of a flexible flat plate is performed as reported by Fujita and Imai[8]. Hereafter, let's call it the coupled solution. Secondly, Kornecki et al.[10] derived the unsteady velocity potential by applying the non circulatory steady aerodynamics theory, and proposed the fluid pressure acting on a flexible plate. Using this proposal, the stability analysis is performed. Let's call it the non circulatory solution. We have already reported the first method and the second method by Fujita and Matsumoto[11]. In order to study more, the third method is proposed and discussed.

Thirdly, the velocity potential is derived by using the circulatory aerodynamics theory explained in the book by Bisplinghoff et al.[12]. Assuming the continuous vortex sheet from the trailing edge to the backward direction, the velocity potential based on the vortex sheet is obtained. And then, the circulatory solution is obtained by superposing this velocity potential based on the vortex sheet on the non circulatory velocity potential so that the Kutta's condition is satisfied at the trailing edge of a flat plate. Let's call it the circulatory solution. Comparing the three numerical results, the effect of analysis modeling on the dynamic stability of a cantilevered plate are discussed.

2. THEORY

Figure 1 shows the analysis model. x is the coordinate of a flowing direction, y is the coordinate parallel to a plate surface and perpendicular to a flowing direction, and z is the coordinate perpendicular to the plate. The plate is subjected to a parallel flow with the fluid density ρ_f , and the lateral deflection w to z direction of the plate is induced by an uniform flow velocity U. The width of a plate is assumed to be infinite. The leading edge of $x = 0$ is assumed to be fixed, and the trailing edge of $x = l$ is free. h_p is the thickness of a plate, and l is the length of a plate.

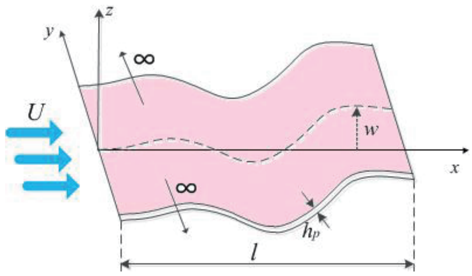


Figure 1: Analysis model in coupled solution and non-circulatory solution.

(1) Governing equation

As reported by Fujita and Imai[8], Fujita and Matsumoto[11], the equation of motion of a stationary flexible flat plate subjected to a parallel flow is obtained in the following expression.

$$\rho_p h_p \frac{\partial^2 w}{\partial t^2} + D\frac{\partial^4 w}{\partial x^4} + \left(p_U - p_L\right) = 0, \tag{1}$$

where, ρ_p is the mass per unit area of a plate. D is the bending rigidity of a plate, t is time. $\left(p_U - p_L\right)$ shows the difference between the pressure acting at upper surface and the one at lower surface. When a fluid is assumed to be inviscid, the unsteady Bernoulli's equation is given on the $z = 0$ plane of a plate as follows.

$$p_U - p_L = -2\rho_f \left(\frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial x} \right)_{z=0}, \tag{2}$$

where ϕ is a two dimensional velocity potential.

The following dimensionless parameters is introduced. The subscript p shows a plate, subscript f shows a fluid, and * shows a dimensionless system. Moreover, μ means the ratio of a fluid density for a plate density including the effect of l and h_p, let's call it the mass ratio.

$$x^* = x/l, \quad z^* = z/l, \quad w^*(x^*, t^*) = w(x,t)/l, \quad \mu = \rho_f l/\rho_p h_p$$
$$t^* = t\sqrt{D/\rho_p h_p}/l^2, \quad U^* = Ul\sqrt{\rho_p h_p/D}, \quad \phi^* = \phi\sqrt{\rho_p h_p/D}, \quad p^* = pl^3/D. \tag{3}$$

Using the dimensionless parameters system, Eqs. (1), (2) and the Laplace equation in which ϕ is satisfied can be obtained in the dimensionless form as follows. Besides, * is deleted for a simplicity of description only in this section.

$$\frac{\partial^2 w}{\partial t^2} + \frac{\partial^4 w}{\partial x^4} + (p_U - p_L) = 0. \tag{4}$$

$$(p_U - p_L) = -2\mu\left(\frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial x}\right)_{z=0}. \tag{5}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0. \tag{6}$$

The solution of $w(x, t)$ of equation (4) is assumed to be given by superposing the modes function in a vacuum.

$$w(x,t) = \sum_k w_k(x) q_k(t), \tag{7}$$

where $q_k(t)$ is the time function. $w_k(x)$ is the uncoupled k-th mode of a cantilevered plate.

The equation of motion of a plate is described by Eqs. (4) ~ (6) and Eq. (7). When the fluid force given by Eq. (5) is substituted into Eq. (4), the coupled equation of motion of a flexible cantilevered plate is derived in consideration of the added mass, added damping and added stiffness, respectively. As it is necessary to formulate the velocity potentials, let's explain the three kinds of solutions in the following 3 sections.

(2) Solution based on the boundary condition in the half-space

Firstly, the solution based on the boundary condition in the half-space is explained, that is the coupled solution. The analysis model shown in Fig. 1 is utilized. When the Laplace equation (6) is resolved by the variable separation method, and the boundary condition that the velocity potential must converge to zero at the infinity point of z direction is applied, the velocity potential ϕ can be obtained.

The boundary conditions at the ends of x = 0, 1 is considered. The unsteady fluid velocity in x direction is equal to the unsteady velocity of a plate in x direction. As the plate is fixed at x = 0, the unsteady fluid velocity becomes zero. On the other hand, the free edge can move freely. However, although there is the component of x direction generated by the movement of the free edge, it is assumed to be small as a second order. Therefore, as for the boundary condition of x direction at the free edge of x = 1, the unsteady fluid velocity in x direction can be assumed to be also equal to the unsteady velocity of a plate in x direction. Therefore, the following equation is given.

$$\left.\frac{\partial\phi}{\partial x}\right|_{z=0} = 0 \quad \text{for } x = 0,\ 1. \tag{8}$$

And, as for the boundary condition in z direction for $0 \le x \le L$, the flow velocity on the plate in z direction can be expressed in the following equation by Lighthill[13] when the steady flow velocity is U.

$$\left.\frac{\partial\phi}{\partial z}\right|_{z=0} = \frac{\partial w}{\partial t} + U\frac{\partial w}{\partial x}. \tag{9}$$

When Eq. (7) can be applied for $w\ (x,\ t\ )$, the velocity potential ϕ is determined by using the boundary conditions of Eqs. (8) and (9) as follows.

$$\phi(x,z,t) = \sum_{k}\sum_{i=1}^{\infty}\left[\begin{array}{l}\left(-\dfrac{2}{i\pi}\right)\exp(-i\pi z)\cos(i\pi x) \\ \times\left\{\displaystyle\int_0^1 w_k(x)\cos(i\pi x)dx\cdot\dot{q}_k(t) + \int_0^1\frac{dw_k(x)}{dx}\cos(i\pi x)dx\cdot Uq_k(t)\right\}\end{array}\right]. \tag{10}$$

The fluid pressure (p_U -p_L) can be determined by using Eq. (5). Here this solution is called as the coupled solution.

(3) Solution based on the non circulatory aerodynamic theory

Secondarily, the solution based on the non circulatory aerodynamic theory is explained. Similarly, the analysis model shown in Fig. 1 is utilized. As shown in Fig. 2, $H(\xi,t)$ is the intensity per unit length of local source or sink on the small length $d\xi$ of a flexible flat plate which carries out an unsteady motion. The velocity potential at the location (x, ξ) can be derived by using the local source and sink as follows.

$$d\phi = \frac{1}{4\pi}H(\xi,t)\ln\left\{(x-\xi)^2 + z^2\right\}d\xi. \tag{11}$$

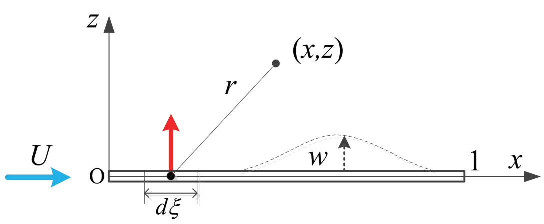


Figure 2: Local strength of source or sink at the microelement of a plate.

In general, the flow around the object subjected to an uniform flow can be expressed by a combination of source and sink. The unsteady velocity potential can be obtained by putting the source line along the chord of airfoil. That is, it is given by integrating Eq. (11) along the chord of airfoil in the following.

$$\phi(x,z,t) = \frac{1}{4\pi}\int_0^1 H(\xi,t)\ln\left\{(x-\xi)^2 + z^2\right\}d\xi. \tag{12}$$

It is confirmed easily that this velocity potential satisfies the Laplace equation (6). When $H(\xi, t)$ is determined so that Eq. (12) satisfies the boundary condition Eq. (9) on the range $0 < x < 1$ of a plate, the following equation is obtained.

$$\phi(x,z,t) = \frac{1}{2\pi}\int_0^1 \left\{ \frac{\partial w(\xi,t)}{\partial t} + U\frac{\partial w(\xi,t)}{\partial \xi} \right\} \ln\left\{ (x-\xi)^2 + z^2 \right\} d\xi. \tag{13}$$

The fluid pressure (p_U -p_L) is derived by putting this velocity potential into Eq. (5). Here this solution is called as the non circulatory solution.

(4) Solution based on the circulatory aerodynamic theory

As the third solution, the solution based on circulatory aerodynamic theory is explained. In general, the unsteady fluid velocity becomes infinite at the trailing edge of a plate in the flow field with the velocity potential which is obtained by assuming an ideal fluid. However, as it is not a practical flow to become infinite, the solution is obtained using the velocity potential which satisfies the Kutta's condition, that is, "the value of flow velocity is finite at the trailing edge of a plate". In order to get the velocity potential by using the Joukowski transformation, the analysis model shown in Fig. 3 is utilized instead of Fig. 1. Similarly, x is the coordinate of a flowing direction, y is the coordinate parallel to a plate surface and perpendicular to a flowing direction, and z is the coordinate perpendicular to a plate. The leading edge of $x = - b$ is assumed to be fixed, and the trailing edge of $x = b$ is free, and $2b$ is the length of a plate. Besides, in the analysis model of Fig. 3, the representative length l must be replaced by $2b$ for the dimensionless system of Eq. (3).

When the flow field around a flat plate is replaced with the flow field around a circle, the analysis handling becomes convenient. By using the Joukowski transform, the flow field around a plate in the orthogonality coordinate system of Fig. 4 (a) is rewritten to the flow field around a circle in the polar coordinates system of Fig. 4 (b), and then the velocity potential is formulated by the polar coordinate system.

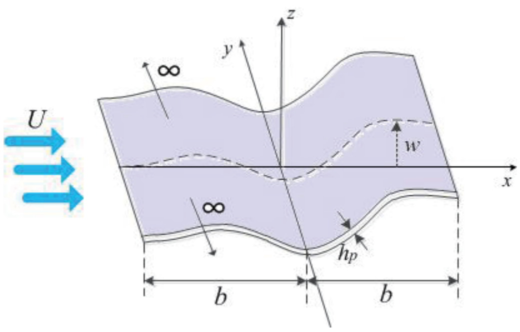
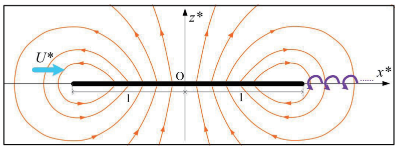
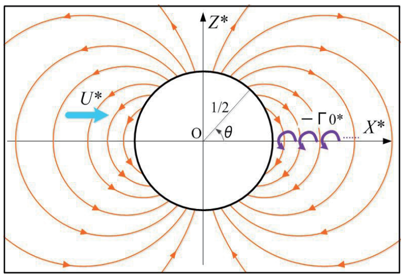


Figure 3: Analysis model in circulatory solution. U is an uniform flow velocity, w is the deflection of a plate, $2b$ is the length and h_p is the thickness.

Concerning the flow field around a circle, the continuous distribution of source is assumed to be on upper half circumference and that of sink is assumed to be on lower half circumference. Deriving the velocity potential from these source and sink, the velocity potential which is equivalent with the non circulatory velocity potential shown in Eq. (13) can be formulated in the polar coordinate system as follows.



(a) Flow around a plate in orthogonal coordinate system



(b) Flow around a circle in polar coordinate system

Figure 4: Joukowski transformation. * shows a dimensionless system as shown in Eq. (3). *U* is an uniform velocity, Γ_0 is the circulation per unit length.

$$\phi_U(\theta,t) = -\frac{1}{\pi}\int_{\theta}^{\pi}\int_{0}^{\pi}\frac{w_a \sin^2\phi d\phi}{(\cos\phi - \cos\theta)}d\theta, \tag{14}$$

where ϕ, θ is the argument in the circumference, w_a is equal with the righthand side of Eq. (9). ϕ_U is the velocity potential on upper half circumference, and has a different relation on the plus and minus with the velocity potential in the lower half circumference each other. The fluid pressure which is caused by the non circulatory velocity potential can be obtained substituting Eq. (14) into Eq. (5) in the following.

$$(p_U - p_L)_{\mathrm{NC}} = \frac{2\mu}{\pi}\left[\int_{\theta}^{\pi}\left\{\int_{0}^{\pi}\frac{(\partial w_a/\partial t)\sin^2\phi d\phi}{(\cos\phi - \cos\theta)}\right\}d\theta + \frac{U}{\sin\theta}\int_{0}^{\pi}\frac{w_a \sin^2\phi d\phi}{(\cos\phi - \cos\theta)}\right], \tag{15}$$

where the subscript NC means the non circulatory. As shown in Fig. 4, the velocity potential can be derived by assuming the existence of the continuous vortex sheet from the trailing edge of a plate to the flow behind from it. Similarly, the following equation is given by formulating it in the polar coordinate system.

$$\phi_U^{all\Gamma_o}(\theta,t) = -\frac{1}{\pi}\int_{1}^{\infty}\gamma_w(\xi,t)\times\tan^{-1}\left[\sqrt{\frac{(\xi-1)(1+\cos\theta)}{(\xi+1)(1-\cos\theta)}}\right]d\xi, \tag{16}$$

where ξ is the variable which denotes the location from the trailing edge of a plate to the flow behind from it ($1 \le \xi < \infty$), $\gamma_w(\xi,t)$ is the intensity of circulation per unit length at the location ξ. And, the subscript $all\Gamma_0$ denotes the total of the circulation Γ_0 per unit length. Substituting Eq. (16) into Eq. (5), the following fluid pressure is obtained.

$$(p_U - p_L)_{all\Gamma_o} = \frac{\mu U}{\pi\sin\theta}\int_{1}^{\infty}\left\{\frac{\xi}{\sqrt{\xi^* - 1}}(1-\cos\theta) + \sqrt{\frac{\xi+1}{\xi-1}}\cos\theta\right\}\gamma_w(\xi,t)d\xi. \tag{17}$$

The velocity potential of Eq. (14) and that of Eq. (16) are superposed so that the Kutta's condition is satisfied. Concerning the relationship between the unsteady flow velocity on a plate and that on a circumference, the condition that the flow velocity at the trailing edge must be finite introduces the following equation.

$$Q = \frac{1}{\pi}\int_0^{\pi} \frac{w_a \sin^2\phi d\phi}{(\cos\phi - 1)} = -\frac{1}{2\pi}\int_1^{\infty}\sqrt{\frac{\xi+1}{\xi-1}}\gamma_w(\xi,t)d\xi. \tag{18}$$

Using Eq. (18), the fluid pressure of Eq. (17) can be rewritten. As a result, the total fluid pressure, that is the sum of Eq. (15) and Eq. (17) which is rewritten, is given as follows.

$$(p_U - p_L)_C = \frac{2\mu}{\pi}\left[\int_\theta^{\pi}\left\{\int_0^{\pi}\frac{(\partial w_a/\partial t)\sin^2\phi d\phi}{(\cos\phi - \cos\theta)}\right\}d\theta + \frac{U}{\sin\theta}\int_0^{\pi}\frac{w_a \sin^2\phi d\phi}{(\cos\phi - \cos\theta)}\right]$$
$$-2\mu UQ\left\{\cot\theta + \left[\frac{1-\cos\theta}{\sin\theta}\right]C(k)\right\}, \tag{19}$$

where the subscript C means a circulation. Using the Hankel functions of the second kind $H_0^{(2)}$, $H_1^{(2)}$, $C(k)$ is shown in the following.

$$C(k) = F(k) + iG(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + iH_0^{(2)}(k)}, \tag{20}$$

where $C(k)$ is the Theodorsen function, it has a variable of the reduced frequency k. k is given by a dimensionless natural circular frequency ω and a dimensionless flow velocity U as follows.

$$k = \frac{\omega}{U}. \tag{21}$$

Here this solution is called as the circulatory solution.

(5) Fluid-structure coupled equation of motion

Substituting the fluid pressure into the equation of motion of a plate shown by Eq. (4), the fluid-structure coupled equation of motion can be derived. Truncating the mode number k as n, and then multiplying $w_j(x)$ (j = 1, 2 --- n) on the both sides of the fluid-structure coupled equation of motion, it is integrated from 0 to 1 as for x using the orthogonal characteristics. When the order of expressions is arranged further, the equation of motion can be obtained.

Changing the fluid velocity U, the complex eigenvalue analysis is performed and the root locus is obtained. When the real part $\Re(\lambda)$ of the eigenvalue λ is positive and the imaginary part $\Im(\lambda)$ is not zero, the system becomes flutter. And, when $\Re(\lambda)$ is positive and $\Im(\lambda)$ is zero, the system is divergence. Besides, $\Re(\lambda)$ means the growth rate of vibration, and $\Im(\lambda)$ means the coupled frequency.

3. ANALYSIS RESULTS AND CONSIDERATIONS

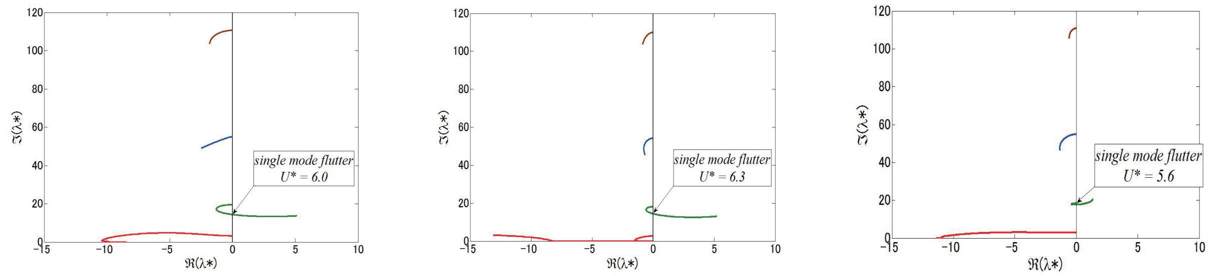
(1) Numerical analysis results on the stability of a cantilevered plate

Firstly, the numerical analysis results on the stability of a cantilevered flexible plate by performing the complex eigenvalue analysis. The solution depends on the dimensionless fluid velocity U^* and the dimensionless parameter μ. This μ means the ratio of the density of a fluid for the density of a structure as shown in Eq. (3), and it is called as a mass ratio here. When μ is small, the system of a structure governs the stability, and when it is large, the system of a fluid governs the stability. Here, changing μ, the comparison among the coupled solution, the non circulatory solution, and the circulatory solution is performed.

Besides, as the Theodorsen function is included in the circulatory solution due to the Kutta's condition, the reduced frequency k^* is a variable in the numerical calculation. Therefore, the iteration calculation is performed by using the relationship $k_{cr}^* = \omega_{cr}^* / U_{cr}^*$ in order to get the converged solution. Besides, * shows a dimensionless system.

a) Root locus

Figure 5 shows the root loci of the coupled solution, the non circulatory solution, and the circulatory solution, respectively. Besides, the fluid velocity U^* is changed from 0 to 10.0. As the eigenvalues are investigated from 1st mode to 4th mode. The red color shows 1st mode, green color shows 2nd mode, blue color shows 3rd mode, and brown color shows 4th mode, respectively.



(a) Coupled solution (b) Non circulatory solution (c) Circulatory solution

Figure 5: Root locus at $\mu = 1$. μ is the mass ratio of a fluid density for a plate density.

In all of the coupled solution, the non circulatory solution and the circulatory solution, the real part of the first mode which means the growth rate of vibration is negative. So, it is found to be always stable.

Next, concerning with the 2nd mode in all of the coupled solution, the non circulatory solution and the circulatory solution, the growth rate and the frequency decrease gently increasing the flow velocity. However, the growth rate begins to increase when the flow velocity surpasses a certain value, and the coupled solution, non circulatory solution and circulatory solution become a single mode flutter at $U^* = 6.0$, $U^* = 6.3$, and $U^* = 5.6$, respectively. Further, increasing the flow velocity, they move to the more unstable side.

The 3rd mode and 4th mode of the coupled solution, the non circulatory solution and the circulatory solution are stable, respectively.

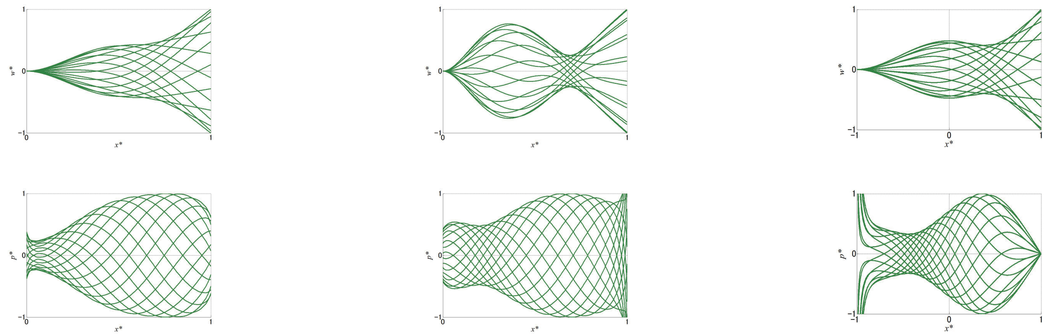
The mode number at which a flutter generates and the characteristics of a root locus show considerably a good agreement among three solutions qualitatively, although the critical flow velocities show a little differences quantitatively.

b) Vibration modes and fluid pressure distribution

Figure 6 shows the vibration mode and the fluid pressure distribution just after the outbreak of a flutter is investigated. These figures show the 2nd mode of the coupled solution, the non circulatory solution and the circulatory solution for $\mu = 1$, respectively. The upper row shows the vibration mode, and the lower row shows the fluid pressure distribution.

When the flow velocity increases, the fluid pressure of the non circulatory solution at the trailing edge is found to become very large. On the other hand, that of the circulatory solution converges to zero. This difference is thought to be due to the Kutta's condition which is applied to the circulatory solution, not to the non circulatory solution. That is, a fluid flowing along the upper surface of a plate and that along the lower surface become the same velocity, and then flow backward smoothly due to the Kutta's condition.

Moreover, at the leading edge, the fluid pressure of the circulatory solution is found to be very large comparing with the non circulatory solution. In the circulatory solution, it is thought that the flow field in which the leading edge becomes a stagnant point is formed strongly.



(a) Coupled solution, $U^* = 6.0$ (b) Non circulatory solution, $U^* = 6.3$ (c) Circulatory solution, $U^* = 5.6$

Figure 6: Vibration mode and fluid pressure distribution just after flutter at $\mu = 1$.

(2) Relationship between mass ratio and critical velocity

Figure 7 shows the relationship of the critical velocity versus the mass ratio. The horizontal axis of Fig. 7 means the mass ratio, and the vertical axis of Fig. 7 shows the dimensionless critical flow velocity. Here, changing the dimensionless velocity from 0 to 20.0, the dimensionless critical velocity is determined.

The dotted line is the coupled solution, the one-dotted line is the non circulatory solution, and the solid line is the circulatory solution, respectively. It is found that the critical velocities of the coupled solution, the non circulatory solution, and the circulatory solution change as the mass ratio μ change. When the mass ratio becomes smaller, it is found that the critical flow velocity of the circulatory solution becomes lowest. When the mass ratio becomes about 1, three solutions approach to the same value. When the mass ratio increases from 1 to 10, the circulatory solution is found to be most stable, other two solutions show the stability of same degree. Furthermore, the step-like phenomena appear in three critical curves. When the mass ratio is small, the flutter of the lower mode such as a second mode become predominant. And, when the mass ratio is large, the higher mode appears. The step-like phenomena express the change of the flutter mode from the lower mode to the higher mode. As for the value of the mass ratio μ where the mode changes, it is found to be different among the coupled solution, the non circulatory solution, and the circulatory solution.

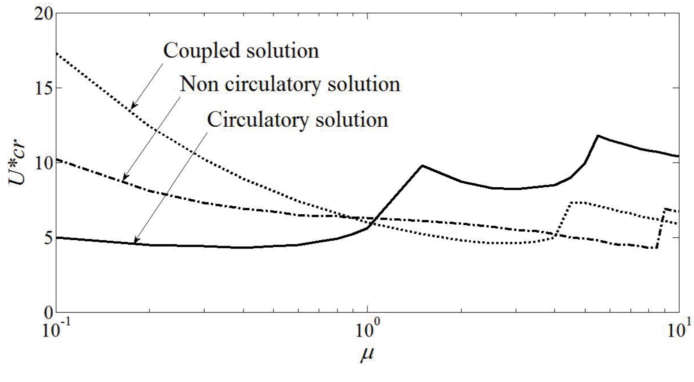


Figure7: Critical velocity versus mass ratio. μ is mass ratio.

In the range of $\mu \leq 1$ where the 2nd mode flutter generates, the critical velocity of the circulatory solution is smaller than those of the coupled solution and the non circulatory solution. And, in the range of $\mu > 1$ where the 3rd mode flutter generates, the critical velocity of the circulatory solution is larger than those of the coupled solution and the non circulatory solution. A cause of such a difference is thought to include the Kutta's condition. As the flow at the trailing edge becomes smooth by considering the Kutta's condition in the circulatory solution, the stability at this region is thought to become good. When the mass ratio becomes large, the effect of a fluid increases relatively and the fluctuation at the neighborhood of the trailing edge is apt to be active. In such situation that the fluctuation of the neighborhood of the trailing edge is large, the effect considering the Kutta's condition is activated more, and as a result, it is thought that the critical velocity increase. Therefore, the critical flow velocity is thought to become larger in the case that the Kutta's condition is considered when the mass ratio increases.

4. CONCLUSIONS

Being common to three solutions, the flutter of the lower mode such as 2nd mode occurs when the mass ratio is small. And, the flutter of the higher mode such as 3rd mode and 4th mode occurs when the mass ratio is large. The critical flow velocity of the circulatory solution becomes lower than those of the coupled solution and the non circulatory solution when the mass ratio becomes low. On the other hand, the critical flow velocity of the circulatory solution becomes higher than those of the coupled solution and the non circulatory solution when the mass ratio becomes high. The reason why such a difference generates is that the Kutta's condition is considered.

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