On the Asymptotic Solution of the Laminar Compressible Boundary Layer over a Circular Cylinder with Uniform Suction

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ABSTRACT: Asymptotic solutions for the flow of a laminar compressible boundary layer over a circular cylinder with uniform suction are presented in the present paper. These solutions may correspond to the flow far down-stream the leading edge of the circular cylinder with its axis laid in uniform stream direction, and with uniform suction.

The exact solutions for these flows are given in closed form for several cases, and their characteristics are discussed. Several numerical results will help to understand characteristics of these flows in contrast with the two dimensional cases. When the radius of the cylinder diminishes, the effects of suction become smaller as compared with the case of flat plate, being considered at the same distance from the body surface.

SYMBOLS AND NOTATION

x, r=coordinates parallel and perpendicular to the cylinder, respectively, where $r=0$ corresponds to the center of the cylinder
x, y=coordinates parallel and perpendicular to the flat plate, respectively
u, v=velocity components on x and r or y directions, respectively
p=absolute pressure
ρ=mass density of fluid
T=absolute temperature
μ=coefficient of viscosity
λ=coefficient of thermal conductivity
c_p=specific heat at constant pressure
R=gas constant
$P_r=(\mu c_p/\lambda)$=Prandtl number
γ=ratio of specific heat at constant pressure to specific heat at constant volume
M=Mach Number
ε=radius of cylinder
S=Sutherland constant
C=Chapman-Rubesin constant
V=suction speed at the wall, $\geqslant 0$
$R_{c\varepsilon}$=radius paramater defined by $\rho_\infty V\varepsilon/C\mu_\infty$
R_{er}=characteristic length defined by $\rho_\infty V r/C\mu_\infty$
$Y=R_{er}-R_{e\varepsilon}+1$
τ=shear stress

Subscripts

∞=conditions outside of the boundary layer
w=conditions at the wall of the cylinder

§ 1. Introduction

Recently, an increasing amount of attention has been paid to the flow of viscous compressible fluids over slender bodies of revolution, from the practical need of the aeronautical engineering for high speed flights. It is well known to accomplish the modification of the boundary layer equation of the body of revolution to that of the flat plate by using Mangler's transformation.[1] They have been set on the assumption that the boundary layer thickness is small compared with that of the body. But for extremely slender bodies the thickness of the boundary will become the same order compared with that of the body. The present paper treats one of such cases.

For incompressible fluids, the boundary layer over a circular cylinder near to the leading edge has already been attacked by R. A. Seban and R. Bond[2], and that at large distance from the leading edge where the thickness of the boundary layer may become the same order compared with the radius of the cylinder has recently been discussed by K. Stewartson[3], who pointed out that the solution tend to that of Oseen's approximation for the viscous fluid. Further Mark[4] has given an exact solution of the boundary boundary layer along a slender paraboloid as a similar solution. Wuest[5] and Lew[6] have also given the asymptotic solution over a circular cylinder with uniform suction in a closed form.

For compressible fluids, the problem is much complicated and there can be seen only few studies. Probstein and Elliot[7] have presented a method of calculation for bodies of revolution using Mangler's transformation as a zeroth approximation, and got several successful results in cases when the pressure gradient along the body is absent.

In the present paper, it will be shown that there is an interesting solution on a circular cylinder of infinite length. This solution may be referred as the asymptotic solution, and it is of interest because it is an exact solution of the cylindrical type boundary layer equations. We restrict our attention to the flow which all quantities are independent of x, the direction of axis of the circular cylinder. For cases of the flat plate, Young[8] and Lew and Fanucci[9] have shown refined methods of integration. Moreover Lew and Fanucci have shown the feature how the velocity profile of the finite flat plate tends to the above asymptotic solution with increasing x.

In the following articles asymptotic solutions of the high-speed viscous flow along the circular cylinder of infinite length where all quantities are independent of x will be presented.

§2. Equations of Motion and Solutions

We consider a circular cylinder of radius ε whose axis x is parallel to the direction of the undisturbed uniform flow. Let r denote the distance from this axis, and u, v, the components of the fluid velocities in the directions of x and r respectively. If μ is the coeffisient of viscosity, λ the coefficient of conductivity, c_p the specific heat at constant pressure, p the pressure, ρ the density, T the temperature and R the gas constant, the fundamental equations of the compres-

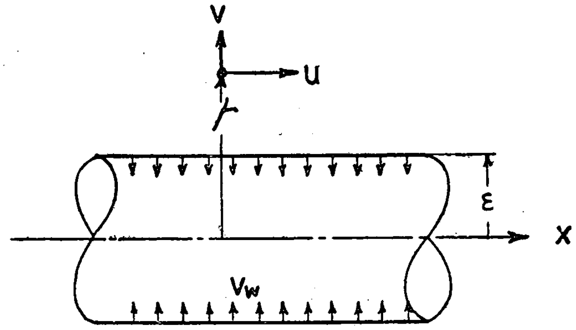

Fig. 1 Cylindrical coordinates

sible boundary-layer flow with axial symmetry are given as follows.

Eq. of continuity: $$\frac{\partial(\rho u)}{\partial x}+\frac{1}{r}\frac{\partial(\rho v r)}{\partial r}=0 \tag{1}$$

Eq. of motion (x-component):

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial r}=-\frac{\partial p}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial u}{\partial r}\right) \tag{2}$$

Eq. of motion, (r-component): $$0=-\frac{\partial p}{\partial r} \tag{3}$$

Eq. of energy:

$$\rho u\frac{\partial(c_pT)}{\partial x}+\rho v\frac{\partial(c_pT)}{\partial r}-u\frac{\partial p}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r\frac{\partial T}{\partial r}\right)+\mu\left(\frac{\partial u}{\partial r}\right)^2 \tag{4}$$

Eq. of state: $$p=R\rho T \tag{5}$$

In addition, it is assumed that both specific heat at constant pressure and Prandtl number are constants.

$$c_p=\text{const.}$$

$$c_p\mu/\lambda=\text{const.} \tag{7}$$

As the fluid flows with axial symmetry, quantities are independent of x and determined as the only functions of r, and above equations become

$$\frac{1}{r}\frac{d}{dr}(\rho v r)=0 \tag{8}$$

$$\rho v\frac{du}{dr}=\frac{1}{r}\frac{d}{dr}\left(\mu r\frac{du}{dr}\right) \tag{9}$$

$$0=-\frac{dp}{dr} \tag{10}$$

$$c_p\rho v\frac{dT}{dr}=\frac{1}{r}\left(\frac{c_p\mu}{Pr}r\frac{dT}{dr}\right)+\mu\left(\frac{du}{dr}\right)^2 \tag{11}$$

$$p=R\rho T \tag{12}$$

In the present prper, quantities far from the cylinder are indicated by suffix ∞, and those on the surface of cylinder by suffix w. Integrating Eq. (8), we have

$$\rho v r=\rho_w v_w \varepsilon=\text{const.} \tag{13}$$

where v_w denotes the vertical velocity component on the surface of circular cylinder which is positive ($v_w<0$) for the injection and negative for the suction.
Eq. (9) is integrated by the aid of Eq. (13),

$$\rho_w v_w \varepsilon u+A=\mu r\frac{du}{dr}$$

where A represents the unknown constant.

To determine A, the boundary condition that as $r\to\infty$, then $u\to u_\infty$, $\mu\to\mu_\infty$, $du/dr\to 0$, and $r\cdot du/dr\to 0$ are used. The last condition will be plausible by the reason that, if $r\cdot du/dr$ has a finite value as $r\to\infty$, then as the limiting case, $r\cdot du/dr=C$, or $u=C\log r+D$, while according to the boundary condition, $u\to u_\infty$ when $r\to\infty$, thus C must be zero. Using above conditions, A is determined, and

$$\rho_w v_w \varepsilon(u-u_\infty)=\mu r\frac{du}{dr} \tag{14}$$

Then, following Lew-Fanucci's method, the independent variable in the Eq. (11) is transformed into u.

Using Eqs. (13) and (14), it becomes

$$(u_\infty-u)\frac{d^2(c_pT)}{du^2}-(1-Pr)\frac{d(c_pT)}{du}+(u_\infty-u)Pr=0 \tag{15}$$

with the condition that

$$\begin{aligned} u&=0, & T&=T_w, & &\text{at} & r&=\varepsilon \\ u&=u_\infty, & T&=T_\infty, & &\text{as} & r&\to\infty \end{aligned} \tag{16}$$

Eq. (15) shows a linear differential equation of second order, and thus it is integrable in a closed form. Then, solution of Eq. (15), which satisfys the condition (16) is

$$c_p(T_\infty-T)=\left(1-\frac{u}{u_\infty}\right)^{Pr}\left(c_pT_\infty-c_pT_w+\frac{u_\infty^2}{2}\right)+\frac{Pru_\infty^2}{2(2-Pr)}\left\{\left(1-\frac{u}{u_\infty}\right)^2-\frac{2}{Pr}\left(1-\frac{u}{u_\infty}\right)^{Pr}\right\} \tag{17}$$

The coefficient of viscosity is given by a function of temperature. The most accurate formula is Sutherland's one, which is given by,

$$\frac{\mu}{\mu_\infty}=\left(\frac{T}{T_\infty}\right)^{\frac{3}{2}}\frac{(T_\infty+S)}{(T+S)} \tag{18}$$

where S represents Sutherland's constant.

Thus μ is represented as a function of T, and by Eq. (17) as that of u. Therefore Eq. (14) can be integrated,

$$\rho_w v_w \varepsilon\int_\varepsilon^r\frac{dr}{r}=-\int_0^u\frac{\mu(u)}{(u_\infty-u)}du \tag{19}$$

The above relation is complicated to integrate in a closed form.

There is however an interesting case when above equations can be integrated analytically by an easy direct integration, if the viscosity is taken as a linear function of the temperature, which is so called Chapman and Rubesin's[6] approximation, that is

$$\frac{\mu}{\mu_\infty}=C\frac{T}{T_\infty} \tag{20}$$

where

$$C=\left(\frac{T_w}{T_\infty}\right)^{\frac{1}{2}}\cdot\frac{(T_\infty+S)}{(T_w+S)}$$

Eq. (17) can be, now, expressed as follows.

$$\frac{T}{T_\infty}=1-\left(1-\frac{u}{u_\infty}\right)^{Pr}\cdot\left(1-\frac{T_w}{T_\infty}+\frac{\gamma-1}{2}M_\infty^2\right)-\frac{(\gamma-1)Pr}{2(2-Pr)}M_\infty^2\left\{\left(1-\frac{u}{u_\infty}\right)^2-\frac{2}{Pr}\left(1-\frac{u}{u_\infty}\right)^{Pr}\right\} \tag{21}$$

Substituting Eqs. (2) and (21) into Eq. (19), and integrating, following solution can be obtained.

$$-\frac{T_\infty}{T_w}\frac{\rho_\infty v_w \varepsilon}{C\mu_\infty}\log\frac{r}{\varepsilon}=-\log\left(1-\frac{u}{u_\infty}\right)+\frac{(\gamma-1)Pr}{4(2-Pr)}M_\infty^2\left\{\left(1-\frac{u}{u_\infty}\right)^2-1\right\}$$
$$+\frac{1}{Pr}\left\{1-\frac{(\gamma-1)Pr}{2(2-Pr)}M_\infty^2\right\}\cdot\left\{\left(1-\frac{u}{u_\infty}\right)^{Pr}-1\right\} \tag{22}$$

Eqs. (17) and (22) are the results required.

If the boundary condition that $u=u_\infty$ as $r\to\infty$ is intoduced in the above equation, the right hand side tends to $+\log\infty$, so that v_w in the left hand side must have negative value to keep the equality of both sides as $r\to\infty$, therefore it is able to write

$$v_w\equiv -V, \qquad V\geqslant 0$$

Further if a parameter is introduced to be defined by

$$R_{e\varepsilon}=\frac{\rho_\infty V\varepsilon}{C\mu_\infty} \tag{23}$$

then, Eq. (22) become

$$\frac{\rho_\infty Vr}{C\mu_\infty}=\frac{R_{e\varepsilon}}{\left(1-\frac{u}{u_\infty}\right)^{\frac{T_w}{T_\infty}\cdot\frac{1}{R_{e\varepsilon}}}}\cdot\exp\left\{\frac{T_w}{T_\infty R_{e\varepsilon}}\cdot\left[\frac{(\gamma-1)Pr}{4(2-Pr)}M_\infty^2\left\{\left(1-\frac{u}{u_\infty}\right)^2-1\right\}\right.\right.$$
$$\left.\left.+\frac{1}{Pr}\left\{1-\frac{T_w}{T_\infty}-\frac{(\gamma-1)}{2(2-Pr)}M_\infty^2\right\}\cdot\left\{\left(1-\frac{u}{u_\infty}\right)^{Pr}-1\right\}\right]\right\} \tag{24}$$

Shear stress on the cylinder can be calculated easily by using Eq. (14), and has the same form as in the incompressible case,

$$\tau_w=\left(\mu\frac{du}{dr}\right)_w=\rho_w u_\infty V \tag{25}$$

The gradient of the temperature gives an indication of the quantity of heat transfer at the wall. It can be calculated by using Eqs. (21) and (24) as follows.

$$\left[\frac{d(T/T_\infty)}{d(\rho_\infty Vr/C\mu_\infty)}\right]_w=Pr\left(\frac{T_\infty}{T_w}\right)^2\cdot\left(1-\frac{T_w}{T_\infty}+\frac{\gamma-1}{2}M_\infty^2\right) \tag{26}$$

As shown above, the temperature gradient at the wall is directly proportional to the value of the Prandtl Number, which is the similar property with the case of flat plate.

When Prandle Number equals to unity, above relations reduce to,

$$\frac{T}{T_\infty}=\frac{T_w}{T_\infty}+\left(1-\frac{T_w}{T_\infty}-\frac{\gamma-1}{2}M_\infty^2\right)\cdot\frac{u}{u_\infty}-\frac{\gamma-1}{2}M_\infty^2\left(\frac{u}{u_\infty}\right)^2 \tag{27}$$

$$\frac{\rho_\infty Vr}{C\mu_\infty}=\frac{R_{e\varepsilon}}{\left(1-\frac{u}{u_\infty}\right)^{\frac{T_w}{T_\infty}\cdot\frac{1}{R_{e\varepsilon}}}}\cdot\exp\left[\frac{T_w}{T_\infty R_{e\varepsilon}}\left\{\frac{(\gamma-1)}{T_\infty R_{e\varepsilon}}M_\infty^2\left(\frac{u}{u_\infty}\right)^2+\left(\frac{T_w}{T_\infty}-1\right)\cdot\frac{u}{u_\infty}\right\}\right] \tag{28}$$

$$\tau_w=\rho_w u_\infty V \tag{29}$$

$$\left[\frac{d(T/T_\infty)}{d(\rho_\infty Vr/C\mu_\infty)}\right]_w=\left(\frac{T_\infty}{T_w}\right)^2\cdot\left(1-\frac{T_w}{T_\infty}+\frac{\gamma-1}{2}M_\infty^2\right) \tag{30}$$

When thermal isolation on the wall is imposed, $(dT/dr)_w=0$, and then $T_w/T_\infty=1+(\gamma-1)M_\infty^2/2$ from Eq. (26), and above relations are reduced to,

$$\frac{T}{T_\infty}=1+\frac{(\gamma-1)}{2}M_\infty^2\left(1-\frac{u^2}{u_\infty^2}\right) \tag{31}$$

$$\frac{\rho_\infty Vr}{C\mu_\infty}=\frac{R_{e\varepsilon}}{\left(1-\frac{u}{u_\infty}\right)^{\left(1+\frac{\gamma-1}{2}M_\infty^2\right)/R_{e\varepsilon}}}\cdot\exp\left[\frac{\left(1+\frac{\gamma-1}{2}M_\infty^2\right)}{R_{e\varepsilon}}\cdot\frac{(\gamma-1)M_\infty^2}{4}\left\{\left(1+\frac{u}{u_\infty}\right)^2-1\right\}\right] \tag{32}$$

$$\tau_w=\rho_\infty u_\infty V\Big/\left(1+\frac{\gamma-1}{2}M_\infty^2\right) \tag{33}$$

$$\left[\frac{d(T/T_\infty)}{d(\rho_\infty Vr/C\mu_\infty)}\right]_w=0 \tag{34}$$

Now, Lew and Fanucci[5] have given the asymtotic solution of the two-dimentional boundary layer equation of the flat plate with uniform suction. For purposes of comparison of above results with the solutions for two-dimentional case, it is usefull to write solutions of the flat plate case. These results are given in slightly modified form from their original paper as follows.

1) general solution:

$$\frac{T}{T_\infty}=1-\left(1-\frac{u}{u_\infty}\right)^{Pr}\cdot\left(1-\frac{T}{T_\infty}+\frac{\gamma-1}{2}M_\infty^2\right)-\frac{(\gamma-1)Pr}{2(2-Pr)}M_\infty^2\cdot\left\{\left(1-\frac{u}{u_\infty}\right)^2-\frac{2}{Pr}\left(1-\frac{u}{u_\infty}\right)^{Pr}\right\} \tag{35}$$

$$\frac{\rho_\infty Vy}{C\mu_\infty}=\frac{T_w}{T_\infty}\left[-\log\left(1-\frac{u}{u_\infty}\right)+\frac{(\gamma-1)Pr}{4(2-Pr)}M_\infty^2\left\{\left(1-\frac{u}{u_\infty}\right)^2-1\right\}\right.$$
$$\left.+\frac{1}{Pr}\left\{1-\frac{T_w}{T_\infty}-\frac{(\gamma-1)Pr}{2(2-Pr)}M_\infty^2\right\}\cdot\left\{\left(1-\frac{u}{u_\infty}\right)^{Pr}-1\right\}\right] \tag{36}$$

$$\tau_w=\left(\mu\frac{du}{dr}\right)_w=\rho_w u_\infty V \tag{37}$$

$$\left[\frac{d(T/T_\infty)}{d(\rho_\infty Vy/C\mu_\infty)}\right]_w = Pr\left(\frac{T_\infty}{T_w}\right)^2\left(1-\frac{T_w}{T_\infty}+\frac{\gamma-1}{2}M_\infty^2\right) \tag{38}$$

2) when $Pr=1$:

$$\frac{T}{T_\infty}=\frac{T_w}{T_\infty}-\left(1-\frac{T_w}{T_\infty}+\frac{\gamma-1}{2}M_\infty^2\right)\cdot\frac{u}{u_\infty}-\frac{\gamma-1}{2}M_\infty^2\cdot\left(\frac{u}{u_\infty}\right)^2 \tag{39}$$

$$\frac{\rho_\infty Vy}{C\mu_\infty}=\frac{T_w}{T_\infty}\left[-\log\left(1-\frac{u}{u_\infty}\right)+\frac{\gamma-1}{4}M_\infty^2\left(\frac{u}{u_\infty}\right)^2+\left(\frac{T_w}{T_\infty}-1\right)\cdot\frac{u}{u_\infty}\right] \tag{40}$$

$$\tau_w=\rho_\infty u_\infty V \tag{41}$$

$$\left[\frac{d(T/T_\infty)}{d(\rho_\infty Vy/C\mu_\infty)}\right]_w=\left(\frac{T_\infty}{T_w}\right)^2\cdot\left(1-\frac{T_w}{T_\infty}+\frac{\gamma-1}{2}M_\infty^2\right) \tag{42}$$

3) when $Pr=1$ and $(dT/dr)_w=0$:

$$\frac{T}{T_\infty}=1+\frac{\gamma-1}{2}M_\infty^2\left\{1-\left(\frac{u}{u_\infty}\right)^2\right\} \tag{43}$$

$$\frac{\rho_\infty Vy}{C\mu_\infty}=\left(1+\frac{\gamma-1}{2}M_\infty^2\right)\cdot\left[-\log\left(1-\frac{u}{u_\infty}\right)+\frac{\gamma-1}{4}M_\infty^2\left\{\left(1+\frac{u}{u_\infty}\right)^2-1\right\}\right] \tag{44}$$

$$\tau_w=\rho_\infty u_\infty V\Big/\left(1+\frac{\gamma-1}{2}M_\infty^2\right) \tag{45}$$

$$\left[\frac{d(T/T_\infty)}{d(\rho_\infty Vy/C\mu_\infty)}\right]_w=0 \tag{46}$$

It is known from the comparison of Eqs. (21) and (24) with Eqs. (35) and (36), that the relation between T and u is same in both cases, and further it is easily shown that,

$$\lim_{\varepsilon\to\infty}\frac{\rho_\infty V(r-\varepsilon)}{C\mu_\infty}=\frac{\rho_\infty Vy}{C\mu_\infty}$$

which shows that in the limit $\varepsilon\to\infty$, the velocity profiles in two dimensional and axially symmetrical flow coincide in the same form.

Last, it should be noticed that, except the incompressible case where the boundary layer approximation is not contained, above calculations must be restricted to keep the displacement thickness finite. If the displacement thickness δ of a cylindrical flow is defined by

$$\pi(\delta^2-\varepsilon^2)=\int_\varepsilon^\infty 2\pi r\left(1-\frac{\rho u}{\rho_\infty u_\infty}\right)dr$$

then it may happen when δ become infinite in marked contrast with the two dimensional case where the displacement thickness has finite value.

In the present paper, the quantitative evaluation of the above integration is not presented beyond pointing out that the value of δ become infinite under the following condition

$$\frac{\rho_\infty V\varepsilon}{C\mu_\infty} \leq \frac{2T_w}{T_\infty} \tag{47}$$

which coincides with Wuest's result[5] for the incompressible flow.

In the next section, several numerical results and comparisons with the two-dimentional cases are presented.

§ 3. Numerical Results and their Discussions with Two-Dimensional Case

For the purpose of comparisons with two dimensional cases, the nondimensional length in the boundary layer is conveniently given by the following form:

$$R_{er} \equiv \frac{\rho_\infty V r}{C\mu_\infty} \tag{48}$$

First, effects of Prandtl Number on the velocity profiles are shown in Fig. 2 when $\gamma=1.4$, $(dT/dr)_w=0$, and $R_{e\varepsilon}=1$. In the present paper coordinate R_{er} is shown by

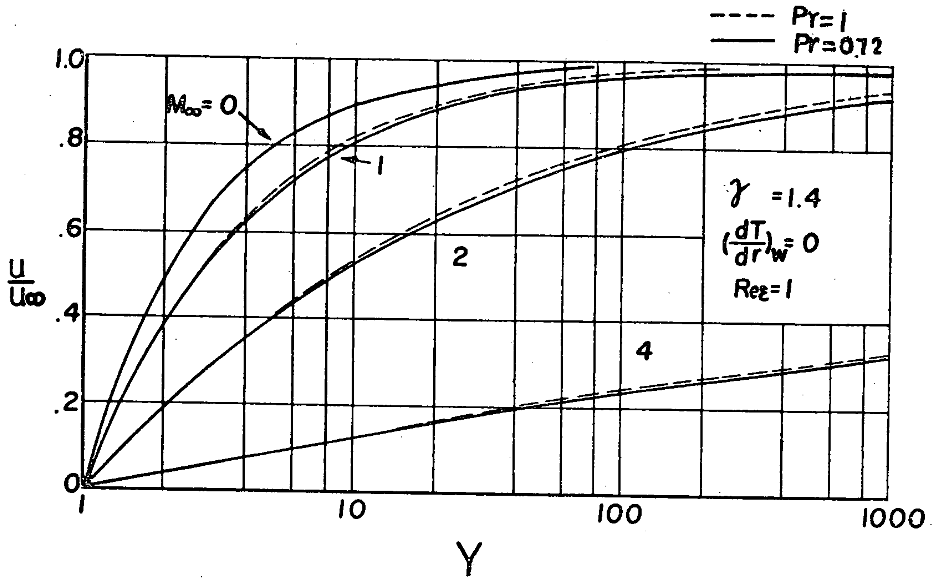

Fig. 2. Effects of Prandtl Number on the velocity profile.

modified logarithmic scale because of the need of very wide range value of R_{er} proper to show velocity and temperature profiles. Further, for the practical purpose to put the wall position at the same point both in the axially symmetric and in the two dimensional case, scales of length are chosen so as the wall to correspond to the position of the coordinate scale of unity. Thus the values of coordinate r in the figures are represented by

$$Y \equiv R_{er} - R_{e\varepsilon} + 1 = \frac{\rho_\infty V}{C\mu_\infty}(r-\varepsilon)+1 \tag{49}$$

For example, when $R_{e\varepsilon}=1$, Y equals to R_{er}.

Fig. 2 show that the effect of Prandtl Number on the velocity profile is very small when Mach Number of the main flow is, at least in the range of $M_\infty \leqq 4$.

Figs. 3-A, -B, -C, -and 3D show effects of the radins of the cylinder on the velocity profiles for various Mach Numbers under the condition that $\gamma=1.4$, $Pr=0.72$

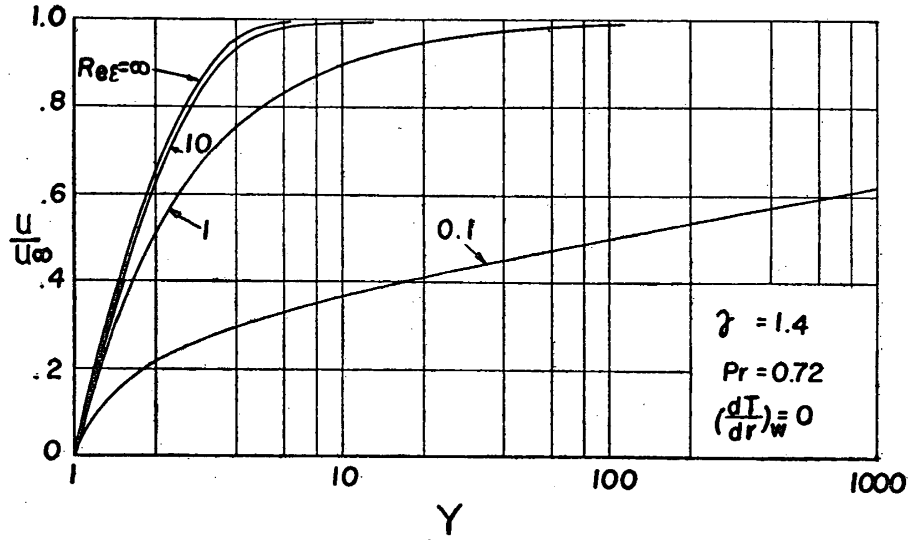

Fig. 3-A. Effects of $R_{\varepsilon\epsilon}$ on the velocity profile, $M_\infty=0$.

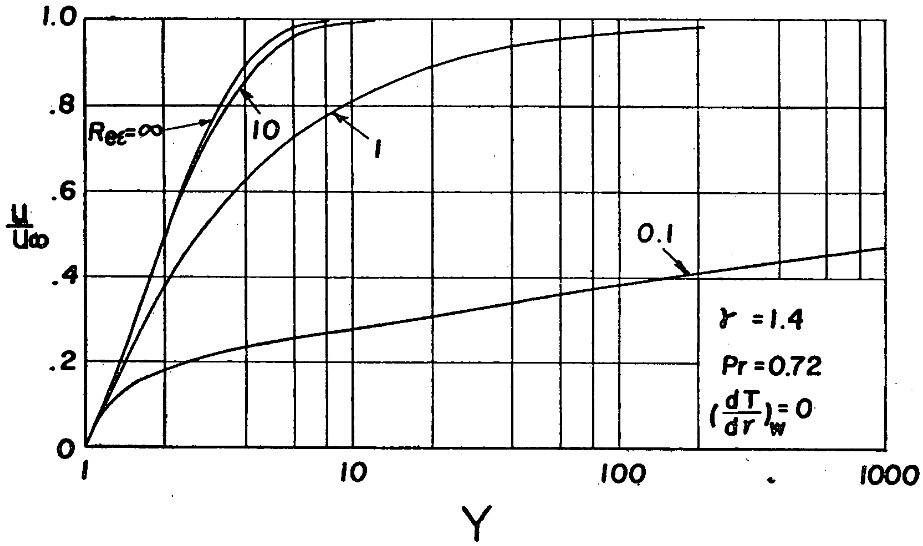

Fig. 3-B. Effects of $R_{\epsilon\varepsilon}$ on the velocity profile, $M_\infty=1$.

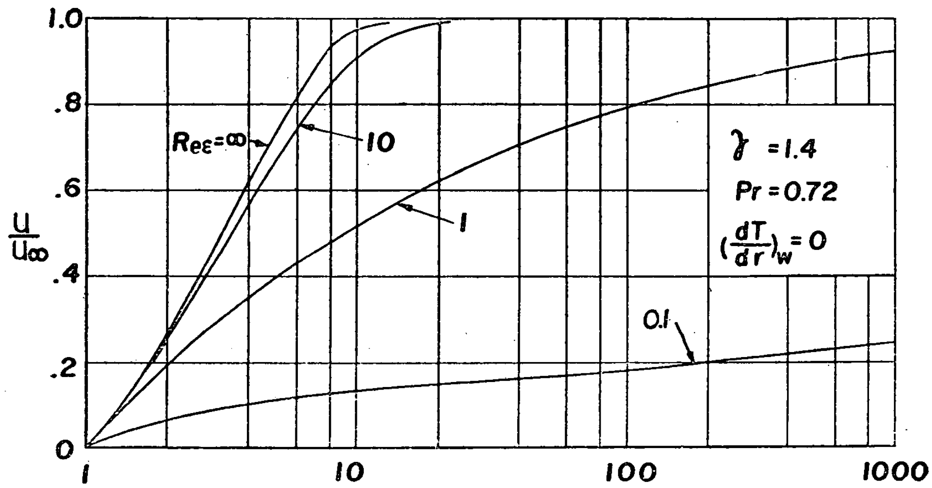

Fig. 3-C. Effects of $R_{\epsilon\varepsilon}$ on the velocity profile, $M_\infty=2$.

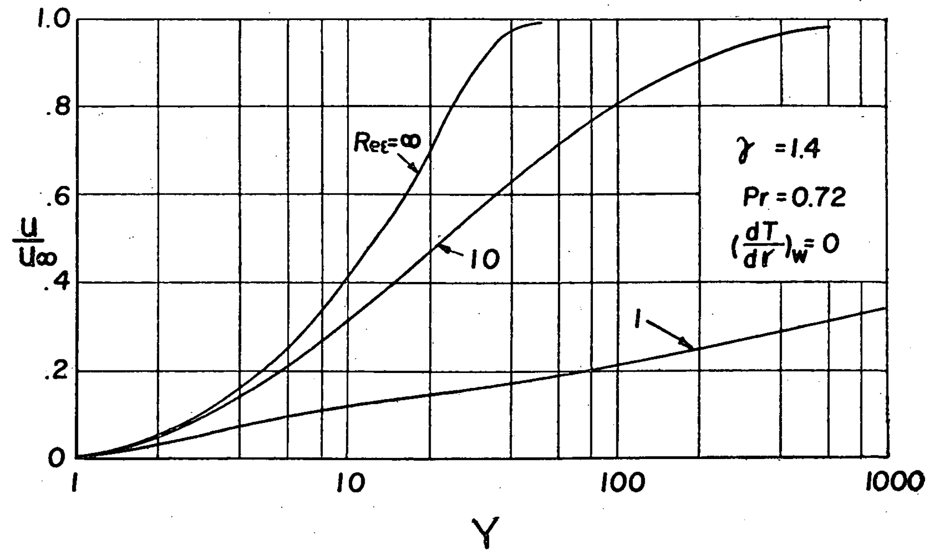

Fig. 3-D. Effects of $R_{e\varepsilon}$ on the velocity profile, $M_\infty=4$.

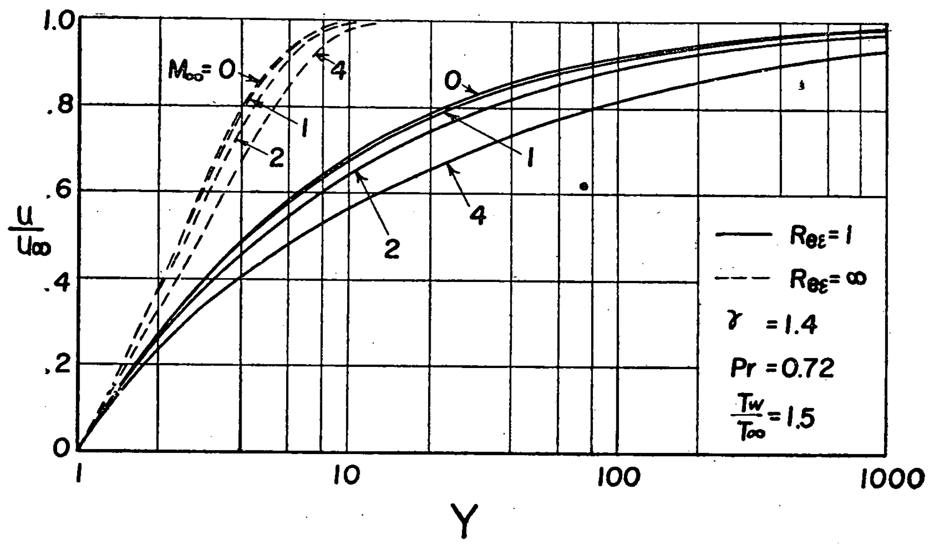

Fig. 4. Velocity profiles when $T_w/T_\infty=1.5$.

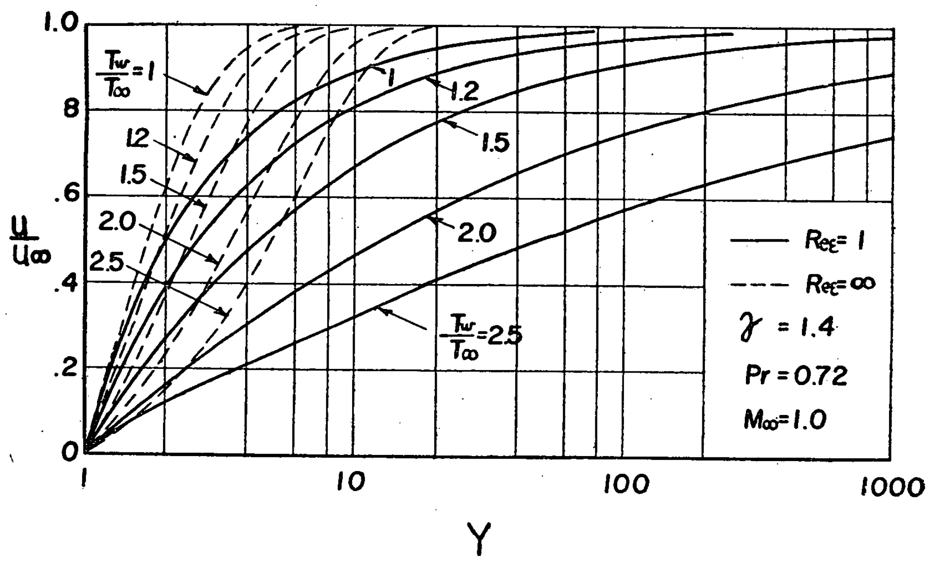

Fig. 5. Effects of the value of T_w/T_∞ on the velocity profile when $M_\infty=1.0$.

and $(dT/dr)_w=0$. In these figures, $R_{e\varepsilon}=\infty$ correspond to the two dimensional cases. General tendencies are as follows. When $R_{e\varepsilon}$ decrease from infinity to zero, the velocity profiles become far away to recover to the uniform velocity than two dimensional case. On the contrary when $R_{e\varepsilon}$ approaches to infinity, the velocity profile approaches to the two dimensional cases rapidly. Discussions of these results will be performed in the following section. One more remarkable tendency can be noted

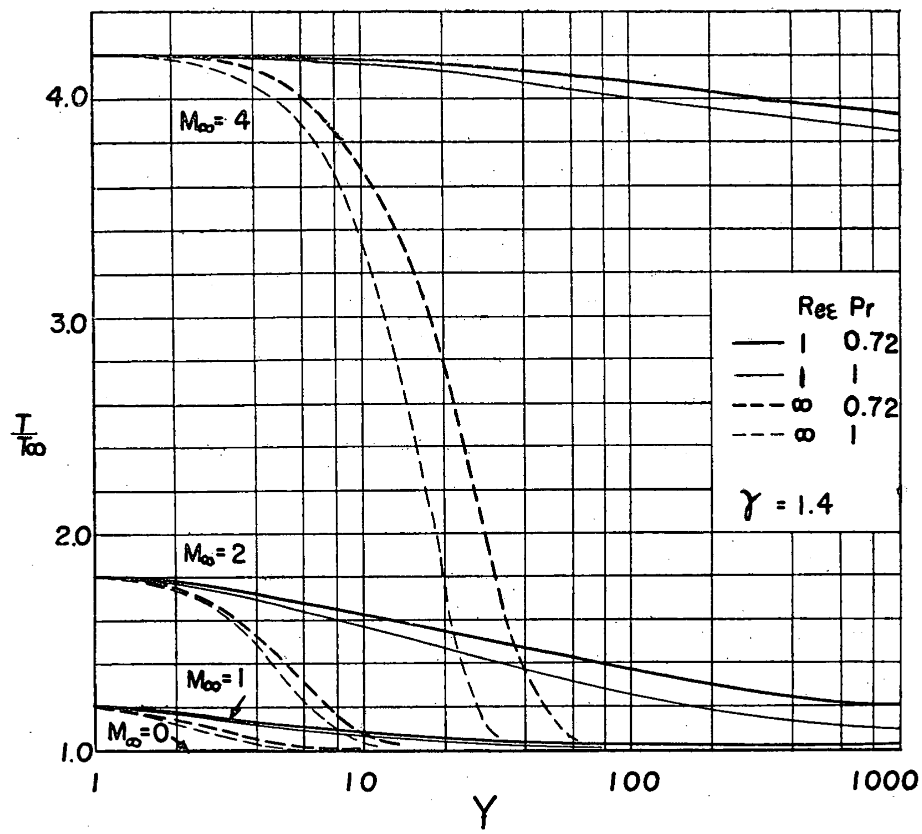

Fig. 6. Temperature profiles when $(dT/dr)_w=0$.

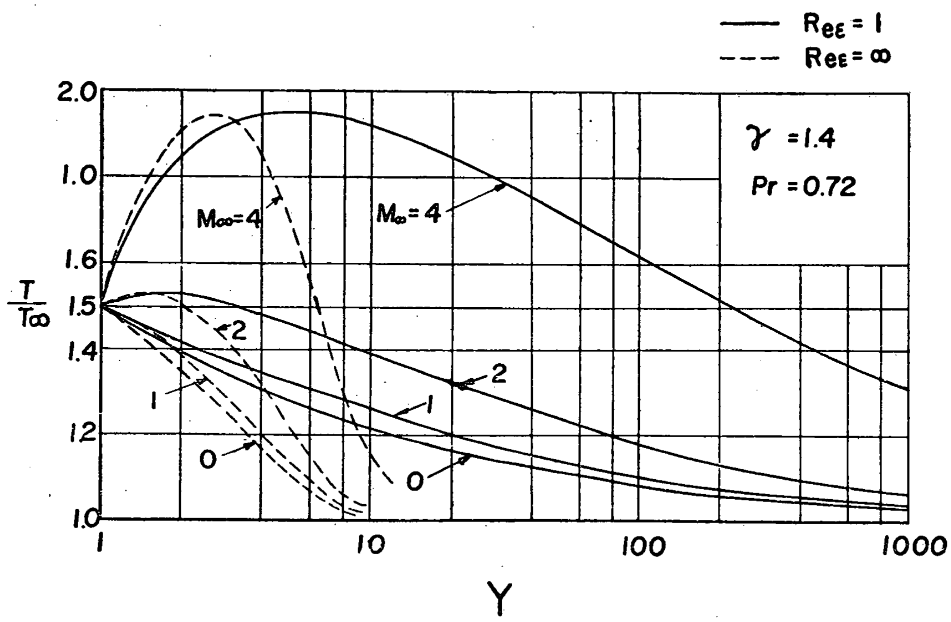

Fig. 7. Temperature profiles when $T_w/T_\infty=1.5$.

that, the curve corresponding to $R_{e\varepsilon}=10$ and that of $R_{e\varepsilon}=\infty$ are close each other near the wall and then apart with increasing Y. Figs. 4 and 5 show effects of Mach Number on the velocity profile when $T_w/T_\infty=1.5$, and effects of T_w/T_∞ when $M_\infty=1$, respectively. Their tendencies are similar to that of the two dimensional case except the order of Y.

Figs. 6 and 7 show temperature profiles when $(dT/dr)_w=0$, and when $T_w/T_\infty=1.5$, respectively.

§4. Discussions

As shown by Eq. (17), the relation between velocity and temperature is the same as the two dimensional case. The principal difference between the flow along circular cylinder and that of flat plate can be seen in the velocity profile. The equation for the case of a flat plate corresponding to Eq. (14) is,

$$\rho_w v_w(u-u_\infty)=\mu\frac{du}{dy} \tag{50}$$

When ε has large value compared with r in Eq. (14), then ε/r approaches to one for wide range of r because $r>\varepsilon$ always, therefore Eq. (14) become close to Eq. (50) of the two dimensional case. On the contrary, when ε has small valve compared with r, then ε/r also become small, and thus du/dr become smaller than du/dy for the same value of u, and in the limiting case when ε approaches to zero, du/dr tends to zero except when $r=\varepsilon=0$, that is, u does not increase for all values of r at last.

Such properties can be interpreted as follows. In the case of flat plate suction distributes laterally as well as longitudinally over infimite length, but in the cylindrical case, suctions distaibute on the cylinder surface only in the lateral direction, and effects of suction on a point in the flow will be small compared with the case of flat plate.

While the asymptotic solution shows the limiting case where the boundary layer is prevented to increase its thickness by means of suction, and then, for the flow with weak suction the asymprtic state will be realized far behind from the leading edge than the case of strong suction. In such cases, the boundary layer thickness of the asympotic state will become large as compared with the case having large suction effects.

For points near the cylinder surface velocity profiles will be close to that of the flate plate when the radius of the cylinder is not small, because main effects of suction come from nearest portion of the surface for each cases. It will be clear when curves of $R_{e\varepsilon}=10$ are compared with that of ∞ in Figs 3–A, –B, –C and 3–D.

§5. Conclutions

Asympotic solutions for the flow of a laminar compressible boundary layer over a circular cylinder with uniform suction are analized and discussed. These solutions will correspond to the flow far downstream from the leading edge of the cylinder with uniform suction. The exact solutions of such flows are given in closed form for several cases, and numerical examples and discussions are presented. From these, it will be concluded that, (1): effects of suction are small compared mith the case of flat plate, and therefore, (2): asymtotic state will be reached later than the case of flat plate under the same suction quantities, further, (3): the thickness of the asympotic boundary layer is larger compared with that case, (4): relations between velocities and temperatures are the same as the case of flat plate and (5): effects of the Prandtl Number on the velocity profile are comparatively small at least when Mach Numbers of the main flow are small.

It must be noted that the previous solutions are imposed by the following limitations, i.e. flow properties treated should be contained in the allowable range of boundary layer approximations, and superaerodynamic region should be excluded.

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