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A Research on the Improvement of Flying Qualities of Piloted Airplanes

—Reduced Stiffness Concept applied to Elevator Control System—

By

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Abstract. The present paper is intended to show that a deliberate reduction of the elevator control system stiffness effectively improves the flying qualities relevant to longitudinal maneuver of a piloted airplane (excluding a fully power controlled airplane with an at present commonly used feel mechanism) through the matching of the stability and control characteristics of the airplane with the physical and perceptive characteristics of the pilot. A properly selected stiffness of the control system will cause upward shift and flattening of the curves of stick travel per airplane response versus speed over wide ranges of speed and normal acceleration, and hence it will add a good measure of longitudinal maneuverability sensed by the pilot to the already established stick force per airplane response.

Analytical investigations are made of the effect of the stiffness on the stick travel per airplane response in steady longitudinal maneuvers, and on the frequency response characteristics of the airplane to stick movement during unaccelerated flight. Numerical calculations, including an analog computer analysis of the effect on the responses stated above in the initial or transient state of longitudinal control, are made based on the data of a prototype airplane for which the author was responsible. The developments of the concept and the flight-tests of the said airplane which took place about twenty-six years ago are briefly described. Both results are fairly coincident with each other, and are further backed-up by ample service experiences on this airplane and its successors. Thus the effectiveness of the concept is sufficiently demonstrated.

Also investigations are made of such characteristics as might be apprehended by some people to be adversely affected by the reduced stiffness. Confirmation is made that it is applicable to a subsonic airplane which is normally designed and not seriously affected by aerodynamic compressibility.

It may be remarked that, being easy to apply and extremely simple in its applied form, the concept may give neat solutions to some of the problems in the field of mechanical as well as aeronautical engineering, in which means to provide self-adaptability and/or to soften responses are required. A spring tab may be cited as the second successful application of the concept to an airplane within the author's knowledge.

INTRODUCTION AND SUMMARY

Control feel is born from the matching of the stability and control characteristics of an airplane with the physical and perceptive characteristics of a pilot. It is an essential element of the flying qualities, or a prerequisite for the safe, easy and precise flying, of a piloted airplane. But unfortunately the studies of the perceptivity and dynamic behavior of a human pilot have not yet been advanced to such a stage as to give sufficient data for the problems or items associated with them to be dealt with under through understanding in design practice or adequately provided in flying qualities requirements. Consequently, there seems to be still rather large room left for refining flying qualities through the effort at pilot-airplane matching. It is a longitudinal maneuver where pilot-airplane matching is most seriously required. Naturally, control force per airplane response and stick travel per airplane response should be potential measures of maneuverability of an airplane as sensed by a pilot. However, since the latter quantity varies theoretically inversely proportional to the square of the speed for each configuration of an airlane, it cannot give a really good measure of maneuverability through a wide range of speed, unless some means of compensation for the effect of dynamic pressure is provided.

In the present paper the author intends to show that it is possible to compensate for the said effect to a great extent by a deliberate reduction of the elevator control system stiffness, and hence to improve the pilot-airplane matching remarkably in longitudinal maneuver. It should act as an automatic, infinitely variable linkage, which increases the contribution of the deformation of the control system to the stick travel with increase in speed. This results in an upward shift and a flattening of the curves which give stick travel per airplane response versus speed over a wide range of speed from a high g maneuver to a precise corrective control.

Analytical investigations are made of the effects of the reduced stiffness of the elevator control system on the stick travel per g, stick travel per stick force, etc. over a wide range of speed in steady longitudinal maneuvers; and also of the said effects on the frequency response of the stick force, angle of attack and angle of pitch to the sinusoidal movement, with respect to time, of the control stick in unaccelerated flight. Numerical calculations, including an analog computer analysis of the effect on the responses stated above in the initial or transient state of longitudinal control, are made based on the actual dimensions, weights and aerodynamic data from wind-tunnel tests of the prototype airplane for which the author was responsible. The developments of the concept and the flight-tests of the said example are described. It is shown that the results of the analytical investigations fairly agree with those of the flight-tests, and that the author's object is efficiently attained by selecting the stiffness and adjusting it in accordance with test-flying. The nature of the problem and the above investigations suggest that, this concept is applicable to normally designed subsonic airplanes and most effective for those which are required good response characteristics over wide ranges of speed and normal acceleration.

The reduced stiffness of the elevator control system, on the other hand, might be apprehended by some people to have more or less adverse influences on some of the characteristics of an airplane. Investigations are made of such influences on elevator flutter, on the deterioration of the damping characteristics of a rotational vibration of the elevator system induced by a quick actuation of the stick, and on the possibility of coupling of the said vibration with the short-period oscillation of the airplane. Also a brief investigations are made of the said problems in the case of a faster and heavier hypothetical airplane. It is deduced from the results that these are not practically adversely affected by the application of the proposed principle, so long as the airplane is normally designed and flown at speed lower than that where the aerodynamic characteristics are affected by compressibility.

A general procedure in the mind of the author to find a proper stiffness is presented. It goes without saying that good aerodynamic characteristics are prerequisite for good flying qualities, and the reduced stiffness coffers an efficient means to refine them further. It should also be remarked that a "fully power controlled airplane" shall be excluded from a "piloted airplane" in this paper, unless a much better feel mechanism than commonly used at present is conceived in its powered control system.

Though the concept is apparently extremely simple in its applied form, it is based on the theory of dynamics of flight and dynamics of solid bodies, and backed-up by positive proof through flight-tests as well as through ample service eperiences—which is in line with the traditional Japanese philosophy of art and technology. It may be added that it could give neat solutions to some of the problems in the field of mechanical as well as aeronautical engineering, where means to provide self adaptability and/or to soften responses is required and elastic deformations of members to a certain degree are permissible. A spring tab may be cited as the second example of its successful application to an airplane within the author's knowledge. Herein the flexibility of the member connecting the balance tab and the main control surface is ingeniously utilized for automatically adjusting the tab ratio according to the variation of the control force or the speed.

It was about twenty-six years ago when the author first conceived this principle and verified it by the flight-test of the airplane mentioned above. He has long since conceived an advice to designers of airplanes (particularly of fighting, aerobatic and training categories) to incorporate into their designs, and to the government agency to stipulate as a recommendation clause in the airworthiness requirements, the policy of minimizing the variation of stick travel per airplane response with the change of speed. To begin with, he proposed to the government agency to withdraw from the airworthiness requirements the criterion calling for a control system stiffness not less than a specified value. After the lapse of some fifteen years, he had a chance of looking into the airworthiness regulations of the U.S.A. and of Great Britain, and found that the Americans had already withdrawn the criterion in question, but the British still retained it. The British attitude seemed to be as conservative as the Japanese government agency before WWII, remembering that a more general requirement was provided to reject such deformations of members which would impair the performance, structural strength, functions of the components, or the stability and control characteristics of an airplane. Today, about a quarter of a century since the author's conception and proposal, stick travel per response as a factor in the control feel problem is becoming a somewhat popular subject of discussion among the students of flight dynamics. But no simple and effective means to realize it has yet been proposed by anybody else, as far as the author knows.

CHAPTER 1

Flying Qualities Requirements and the Reduced Stiffness Concept

1-1. Nature and Requirements of Flying Qualities

The design of an airplane deals with problems which cover a wide range of subjects such as:

Performance
Structure and strength
Flying qualities
Equipment and installations (to match the desired performances, functions, safety, comfort, maintenance, etc.)
Fabrication (covering such items as material, production, cost, maintenance, etc.)

The former three constitute fundamental properties which are essential to flying, and the last two constitute supplementary properties which depend on the nature of utilization.

The "flying qualities" of a piloted airplane may be defined as the stability and control characteristics of the airplane collated with the physical and perceptive characteristics of the pilot. They are the most fundamental among the fundamental properties of an airplane, in that they characteristically distinguish it most definitely from the other, that they are basic to flying, and that the requisites for them are qualitatively identical for the whole family of airplanes.

It is obvious that an airplane to be flown by a human pilot has to be designed to match as far as possible the physical and perceptive characteristics of the pilot. For example, the forces, movements, physiological strength, and reaction required of him must be within the range of which he is capable; and the accelerations, change of attitude and of flight path of the airplane induced in response to the pilot's actuation of controls must be as nearly proportional as possible to the forces, displacements and speed of the movements exerted by him to the control stick. His dynamic behavior constitutes, as it were, a link in a closed-loop system, made-up of a man and an airplane. Designing ideal flying qualities into an airplane would require studies of dynamics of the pilot-plus-airplane system.

It is customary for the government agencies responsible for licensing civil airplanes, or for the procurement of military airplanes, to specify compliance with certain "flying qualities requirements" which constitute the core of the technical standards enacted to assure the safety and efficiency of aircraft and their equipment (generally called "airworthiness regulations"). These requirements are the extracts from extensive and continuing flight investigations, based on the opinions of research pilots and substantiated by careful instrumentation. They are subject to continuous study and modification in order to keep them up-to-date with the research and design informations. It is usual that only a few leading countries have their own "requirements" and the other countries follow the former. The requisites for flying qualities are basically much the same for the whole family of airplanes, with some variations from category to category only in details or in quantitative expressions. For example, for transport category from the standpoint of guaranteeing safety in normal or emergency conditions, and for military categories from the standpoint of giving capability of precise control in accordance with the missions, the corresponding items should be particularly strictly stipulated. It may be remarked that the military is a step ahead of the civil in introducing human physical and perceptive characteristics into the requisites for flying qualities, but that unfortunately there have been as yet insufficient observations made on the airplane-pilot matching to make average designers pay keen attention to this subject.

The recent progress of automatic pilot and powered control permits those airplanes to fly safely which have stability and control characteristics far from matching with those of human pilots. But in civil airplanes only those of high performance or those over medium-sized participate in this benefit. In the majority of civil airplanes, particularly small-sized ones, human pilots remain to be the master. And recently in advanced countries this category of airplanes is increasing rapidly in number.

The author learned from experiences that the most important and delicate characteristics among the flying qualities of an airplane are longitudinal stability and control, and, in particular, those in maneuvers for an airplane of fighting, aerobatic or training category. This is because, their missions usually demand of them more frequent and rapid change of speed, attitude and flight paths, and accordingly quicker, truer and smoother response to the actuation of controls in maneuvers than in airplanes of other categories.

In spite of the fact that the "control feel" in longitudinal maneuver has long been a subject of study for designers, test-pilots, and those interested in dynamics of flight, the requirements as such written in publications are rather limited, as far as the author knows. It seems to the author that little has been added recently to the requirements which have since long been familiar to the people concerned through regulations enacted by government agencies or through text-books and reports of established reputation.

Those for longitudinal maneuver are;

(a) Gradient of the curve of the elevator angle to trim against speed in level flight. [*3*]

(b) Gradient of the curve of the stick force to trim against speed in level flight. [*2*], [*3*], [*4*], [*5*], [*6*], [*9*]

(c) Elevator angle required per g of normal acceleration in pull-up over a certain range of speed. [*6*], [*7*], [*8*]

(d) Stick force required per g of normal acceleration in pull-up over a certain range of speed. [*5*], [*6*], [*7*], [*8*], [*9*]

The quantities (a) and (b) represent the "measures", sensed by the pilot, of static longitudinal stability in unaccelerated flight, and (c) and (d) the "criteria", sensed by the pilot, for static stability in longitudinal maneuver. If these quantities are positive, the corresponding stability is positive; if these quantities are large, he feels that the corresponding stability is large and the airplane is less maneuverable, and *vice versa.* (a) and (b) came into common knowledge pretty long ago (in 1920's); and the concept (c) and (d) are accepted to have been first introduced by S. B. Gate, a Britisher, in 1942. [6], [7], [8]

The author should like to remark as follows:

Among the four quantities stated above, (b) and (d) are proper criteria in view of their characters and expressions, and are referred to very frequently; while (a) and (c), being measured by the pilot through the control system which is more or less flexible, offer only indirect indices, and are referred to less frequently.

1-2. Recommendations for Better Matching of the Airplane and the Pilot

About twenty-six years ago just before the flight-test of a prototype airplane of fighting category was started, the author had conceived that the direct measure for a pilot to sense stability in longitudinal maneuver was to be represented by the stick travel and stick force required to produce a certain intensity level of maneuver (which is expressed by the value of normal acceleration, or rapidness of change of attitude or of change of flight path of an airplane). He had learned that, in order to attain good matching of the characteristics of an airplane and those of a pilot, stick travel and stick force should not vary very much with the airplane speed, but should vary nearly in proportion to the intensity level of maneuver or of the response of the airplane. He conceived that such characteristics could be realized by providing the control system with such properties that the stick travel and stick force represent the effectiveness, and not the deflection, of the elevator, preferably irrespective of the airplane speed. The beneficial effects resulting from such characteristics would be:

1) As stick force required per g of normal acceleration theoretically remains constant irrespective of speed, the two quantities, *i.e.* stick force and stick travel, would from now on provide measures of the intensity level of maneuver, and accordingly it would ease the pilot physically and psychologically.
2) As it would require a larger stick travel than before for the same intensity level of maneuver or the same degree of airplane response, it would enable the pilot to perform a smoother and more precise control, or save him from the liability to over- or under-control.
3) It would increase flexibility in the behavior of the airplane in response to a jerky actuation of control at high speeds, and relieve the pilot of the fear of inadvertently violent maneuver or unintentional stall.
4) As the limit normal acceleration would occur in correspondence with a stick travel which is not very much smaller than its maximum stroke even at

high speeds, it would do good to eliminate chances of unintentional over g maneuver.

5) But it goes without saying that good aerodynamic characteristics are prerequisite for good flying qualities, and they can hardly be compensated for by adjusting the control system stiffness.

Since the author reached a means which, simple to realize, almost answered the purpose, he has conceived recommendations to airplane designers, particularly of fighting, aerobatic and training categories, and such recommendations to be incorporated in the flying qualities requirements in the airworthiness regulations. They are summarized as follows:

1) The stick travel per g of normal acceleration and the stick travel per unit stick force must be adequately large, and preferably must not vary very much through the frequently used speed range.

2) The stick travel to produce the limit normal acceleration and, at the same time, a stall should desirably not leave a margin larger than reasonable against its maximum stroke. The reasonable margin means the allowance for exceeding the limit normal acceleration by a pre-determined small value in case of emergency, and the allowance for such physical construction of a human body that the arm force a human body can exert varies according to the position of the arm relative to the body.

These expressions may not be quantitative enough to be called criteria. But the author believes that they are a step advance compared with the present state of the art. Not many literatures, so far published, whether concerning a manually-controlled airplane or a powered-controlled one, have touched on the importance of the role of the travel of a cockpit control. And none of them seems to have gone into this subject far enough to make a convincing proposal thereon. The readers are asked to refer to the statements of a representative material [*33*], which are reproduced below:

"Although it is recognized that the amount of deflection of a control is certainly a factor in the control feel problem, there has been insufficient correlation of data to evaluate the importance of this factor at the present time."

"Another aspect of the transient feel problem which is often overlooked is the amount of stick deflection required in rapid maneuvering... it would appear either that this criterion should be extended to include stick deflection, or that additional criteria may be necessary."

1-3. Proposed Method of Realizing the Recommendations

Since flying qualities are born as a result of the combination of the pilot and the airplane, every element of the combination is worth deliberate examination. However, nobody might have examined the contribution of the elastic properties of the control system to the dynamic characteristics of the pilot-plus-airplane system, when the author conceived its utilization to attain proper matching of the pilot and the airplane and hence a remarkable improvement of flying qualities.

The control system transmits the force and displacement applied by the pilot to a stick or pedal at one end up to the control surfaces connected to the other end of it. If it is rigid, the deflection of the control surfaces is practically proportional to the displacement of the stick or pedal, and accordingly the control force and control effect, *i.e.*, the moment produced by the control surfaces about the center of gravity of the airplane, varies in proportion to the displacement of the control stick or pedal and the square of the airplane speed.

Now let us restrict the discussion to a longitudinal motion. Theory shows that for a given intensity level of maneuver (represented by the magnitude of normal acceleration or the rapidiness of change of angle of attack, and change of flight path), the force to be applied to the stick remains almost constant and the displacement to be applied to it changes almost inversely proportional to the square of the speed. On the other hand, both the largest elevator angle and the largest stick travel are usually required in landing, because the airplane has to be brought to the largest angle of attack at the lowest speed for the airplane, when an additional elevator power is required to compensate for the ground effect. Accordingly, the excess power of the elevator increases, as the speed goes upwards beyond the landing speed. Therefore, it can easily be seen that the recommendations proposed in the preceding section can not be realized without a special device. The greater the difference between the speed most frequently used and that for landing, and also the more frequent and the more rapid the change of speed, the more intense will be the demand for the realization of the proposal or the more beneficial its realization.

Everyone should agree that no other method can literally realize the proposal than a device which produces an elevator angle roughly inversely proportional to the square of the speed for a given stick travel. And everyone should think first of a manually-operated variable linkage or an automatic one which works upon the signal of dynamic pressure. However, the former would not be welcomed by pilots due to its imposing an extra burden on them, and the latter would be a heavy and complicated mechanism which adds chances of malfunction. The balance between gain and loss would not warrant the use of such mechanism for ordinary airplanes.

The author came to the idea of an automatic, infinitely variable linkage built in the control system, which would produce a larger ratio of elevator deflection to stick travel at low speeds and a smaller one at high speeds, and in which any desired ratio of the linkage would be obtained by providing the control system with a part having desired flexibility or by adjusting the elasticity of the whole system to a desired value. A control system could easily be designed to have any desired value of elasticity, without losing static and fatigue strength or causing structural complication and weight increase. This concept was first tried in the prototype of a fighter airplane mentioned in Section 1-2 (hereafter referred to as the A-1), and it proved to be as satisfactory as expected. Then it was successfully applied to several types of airplanes produced in quantity for which the author was responsible. Reports were made officially, and received due appreciation from those concerned, including an American and a Britisher who happened to lear learn it later. [*12*], [*13*], [*14*], [*15*], [*16*], [*17*], [*18*], [*19*]

The author intended to propose to the government agency to withdraw the criterion, calling for the deflection of the control system to be less than 12.5 percent of the maximum stroke of the stick under the limit loading, prescribed in the "Technical Standards for Airplane Design" [*1*], equivalent of the airworthiness regulations. He author believed that this criterion had been provided to reject those airplanes in which sufficient control surface deflections against the main wings or the stabilizers were not available due to the insufficient rigidity mainly in the supporting structures of the control surfaces and/or the control systems. The case of the author was quite contrary to this circumstance. Furthermore, at the time when the author faced this problem, there was a requirement provided, in priority to the criterion for the control system stiffness, to reject such deformations of members that would impair structural strength, functions of the components, or the stability and control characteristics of airplanes. This requirement alone would prevent the birth of such airplanes in which the effectiveness of control was deficient due to insufficient control system stiffness. It seemed then to the author that the radical change of the state of the art in the previous ten years or so (e.g. popularized all mental construction, doubled or tripled speed, etc.) had already put an end to the *raison d'être* of the requirement specifying the minimum permissible stiffness of the control system.

The proposal was made in the report on the reduced stiffness concept applied to the elevator control system at the regular technical meeting held by the government agency, in the autumn of 1939. But the agency was too conservative to agree to officially withdraw the criterion, but approval was given not to conform to it, if otherwise proved. This practically meant the withdrawal of the criterion.

Analytical investigations are made, in CHAPTER 2, of the effects of the reduced stiffness of the control system on the stick travel, stick travel per *g*, and stick travel per stick force in steady longitudinal maneuver, and also of the said effects on the frequency response characteristics of the elevator angle, stick force, angle of attack and angle of pitch of the airplane to the stick movement in unaccelerated flight. Numerical calculations, including an analog computor analysis of the effect on the responses stated above in the initial or transient state of longitudinal control, based on the dimensions, weights and wind-tunnel test data of the A-1 demonstrate a fine conformity with the results of the flight-test and service experiences, which are described in CHAPTER 3.

The nature of the problem, the results of the analytical investigations and of the flight-tests, and the characteristics of the airplane taken as the example of the numerical calculations suggest, that this concept is applicable to normally designed subsonic airplanes and most effective for those which are required good response characteristics over wide ranges of speed and normal acceleration.

The author has in mind a general procedure to find a proper stiffness as follows: First, calculate the stiffness which gives the maximum stick stroke obtainable minus a reasonable margin mentioned in item 2), Sec. 1-2 to just produce the limit normal acceleration and a stall at the same time in steady maneuver.

In this calculation, make a reasonable allowance for the inaccuracy of aerodynamic data, etc., and take the already existing flexibility of the control system into account in case a spring tab and the like is employed. If the stiffness calculated is too low to keep within the maximum stroke obtainable the travel which produces required elevator power in take-off, landing and other critical slow flying conditions, the latter requirement naturally has the priority. Then make the stiffness adjustable around the value thus determined by changing a number of parts of the control system. Remember that it is not always easy to accurately estimate the control system stiffness in design stage, and that it is rather more practical and not too late to wait this until it is obtained by a static test of the prototype (this latter feature should be one of the merits of the concept). Finally, select the best stiffness from overall consideration of the results of flight-tests, as is usually the case with the problems relevant to flying qualities.

Needless to say that this concept cannot be applied to an irreversible control system, where the hinge moment is not transmitted to the cockpit control. Also it may not be beneficial to a transonic and supersonic airplane, where the same proportionality relation between control effectiveness and hinge moment does not hold throughout the frequently used speed range.

1-4. Apprehensions that Might Arise

Apprehensions might arise of the possibility of the adverse effect of the reduced stiffness of the control system on some of the flight characteristics. Before putting this principle to flight-test, examinations were made of such problems as the effect on elevator flutter and on the dynamic response characteristics of the airplane to quick control stick actuation, e.g., in a pursuit action.

The maximum allowable speed of an airplane ought to be lower than the lowest of the critical speed of flutter with a reasonable margin throughout the whole range of constraint of the stick (from fixed to free). The author felt rather easy about the influence of the control system stiffness on the control surface flutter, because the critical speed for the fixed stick with any degree of the control system stiffness should come between those for the stick free and for the stick fixed with a higher stiffness, and because the pilot would be unable to hold the control stick steady in high frequency [*24*] and violent vibration characteristic of common flutter, if it should occur.

The characteristics of the rotational vibration of the elevator was then fairly known to the author, but he and his staff did not know how to estimate the behavior of the airplane, in case the longitudinal short-period oscillation of the airplane should couple with the rotational vibration of the elevator. The author decided to leave the matter to flight-test, because he knew that safe and simple flight-test would suffice to give the answer, which could not be spared even when the answer was obtained by an analytical method. Evidences were produced by the careful flight-test that it was only a groundless apprehension, and no claims as such were voiced by pilots through a long period of service operations which followed the flight-tests.

In CHAPTER 4, a review is given of the knowledge on control surface flutter established by the results of later researches. And also analytical investigations are made of the influence of the reduced stiffness on the characteristics of the rotational vibration of the elevator and of the lodgitudinal short-period oscillation of the airplane, and further the condition is discussed that a coupling does not occur between the two modes. Thus more general proofs are obtained than those deduced from the flight-tests of particular airplanes, that the reduced stiffness practically makes these problems no worse.

After fifteen years from then when the author inspected the civil airworthiness regulations of U.S.A. and Great Britain, he was surprised to notice that the British still retained a similar criterion, while the Americans had already withdrawn it. It seemed to the author that the reason why the British still sticked to this criterion was rather from the standpoint similar to that of the Japanese government agency before WWII than from radical consideration based on theory and practice of the advanced technology. An evidence for this view is found in the book review made in 1958 on "THE ZERO FIGHTER" [*18*], [*19*], in which J. W. Fozard, the reviewer, appreciated the author's reduced stiffness concept so well that he proceeded to say, "But perhaps the most astonishing technological ingenuity, by Western standards, was the use in the Zero, and later fighters, of deliberate flexibility in the elevator circuit in order to increase the stick movement per g at high indicated air speeds."

During WWII the Americans, having been forced to face the above-mentioned airplane, should have studied its characteristics more thoroughly than any other nation. The author reminds of the following facts: Special notices were given of the superior flying qualities of this airplane below 300 mph indicated air speed, particularly in logitudinal maneuver, in a U.S.A.A.F.'s report on the flight-tests of the captured Japanese Zero Fighter as early as in the fall of 1942. Also in his NACA Memorandum Report on the flying qualities measurements of this airplane, for the Bureau of Aeronautics, U.S. Navy Department [*17*], W. H. Phillips wrote a comment of recognition of increased stretch in the elevator control system at higher speed which caused a large stick travel to go from level flight at high speed to a stall, and an exceptionally large longitudinal stability in turning flight, as measured by the slope of the curve of stick travel (apparent elevator angle) against lift coefficient. But, of course, the author does not think that these facts alone motivated the Americans to abolish the criterion.

In a spring-tab finds the author a fine example of the reduced stiffness principle successfully applied to a control system, in which the tab is connected with the main control surface through a spring whose elaticity adjusts automatically the ratio of the deflection of the former to that of the latter, and hence the ratio of the stick travel to the main control surface deflection, according to the variation of the control force or of the dynamic pressure. But the author does not know whether the inventor was aware of the author's reduced stiffness concept or not.

CHAPTER 2

DEMONSTRATION BY ANALYSES OF THE EFFECT OF REDUCED STIFFNESS

2-1. *Introductory Remarks*

This chapter is devoted to the demonstration by analyses with numerical calculations of the contribution of the reduced stiffness concept to the matching of the airplane and the pilot. In the first half of the chapter, theoretical analyses and numerical calculations are made of the effects of the reduced stiffness of the elevator control system on the stick travels for various values of g, the stick travel per g and the stick travel per stick force through a wide range of speed in steady longitudinal maneuvers; and in the second half are made those of the effects of the reduced stiffness on the response of the airplane to cyclic (*e.g.* in a corrective control) or quick actuation (*e.g.* in a quick turn) of the control stick through the study of the said effects on the frequency response characteristics of the stick force, angle of attack and angle of pitch of the airplane to the movement of the control stick in unaccelerated flight, and, in addition, through numerical analysis with an analog computer of the effect on the responses stated above in the initial or transient state of longitudinal control.

As is often the case in this kind of a problem, we can expect higher order of accuracy in the differences or ratios of the values than in their absolute values. This comes from the fact that the correction of stability derivatives for power effect and of aerodynamic coefficients for different Reynolds numbers or the nature of flows are given up, because we have at present no simple, reliable method to accurately estimate them.* And the assumption of the linearity of the change of aerodynamic coefficients with respect to angle of attack, etc., and the simplification of equations of motion in the analyses from the order of magnitude consideration are other reasons. But fortunately, comparative investigations of the effects of the stiffness values on the stick travels, etc. and on the responses of angle of attack, etc. are sufficient for the present purpose in view of the nature of the problem, so that the author's policy in the analyses stated above may be justified. Further, it may allow us to infer that the general features of the effects of the stiffness concluded from the results of the above analyses are compatible with normally designed subsonic airplanes with few exceptions.

2-2. *Notations*

Notations generally or frequently used in the analyses of the present paper are listed below. Those less generally used are shown where they are used.

g	acceleration due to gravity, $9.80\,m\cdot\text{sec}^{-2}$
ρ	air density, $kg\cdot m^{-4}\cdot\text{sec}^2$
S	main win area

* N.B. For the purpose of a new design instead of general demonstration, the procedure usually followed is to allow a certain margin based on experiences to account for destabilizing effect of power, and then to analyze the final design in the wind tunnel by carefully testing a powered model on which the propeller characteristics, etc. are reproduced.

S_t	horizontal tail area (horizontal stabilizer and elevator)
S_e	elevator area
$\bar{c}$	mean aerodynamic chord length
$\bar{c}_t$	mean chord length of the horizontal tail
$\bar{c}_e$	mean chord length of the elevator
l_t	distance between the airplane center of gravity and the center of pressure of the horizontal tail
l_s	length of the control stick
W	gross weight of the airplane
$m=\frac{W}{g}$	mass of the airplane
I_{GY}	moment of inetia of the airplane about its lateral axis
$k_Y=\sqrt{\frac{I_{GY}}{m}}$	radius of gyration of the airplane about its lateral axis
V	true airspeed
V_1	true airspeed for trim at $n=1$
$q=\frac{1}{2}\rho V^2$	dynamic pressure
η_t	ratio of the dynamic pressure at the horizontal tail to that at infinity, unity for power-off flight
n	normal acceleration of the airplane in maneuver in terms of g
P	control stick force (taken as positive for pushing the stick forward)
$H.M.$	abbreviation for the moment acting on the elevator about its hinge point. (taken as positive when acting to lower the trailing edge of the elevator)
MAC	abbreviation for mean aerodynamic chord
$s=l_s\delta_s$	displacement of the control stick from the trim position (taken as positive when the stick moves forward)
δ_e	deflection of the elevator against the chord line of the horizontal tail (taken as positive for drooping trailing edge)
δ_{e0}	δ_e for trim at $C_L=0$
δ_{e1}	δ_e for trim at $n=1$
δ_s	deflection of the control stick from the trim position (taken as positive when the stick moves forward)
$G=\frac{\delta_e}{l_s\delta_s}$	elevator gearing ratio, $rad\cdot m^{-1}$
$K_1=\frac{1}{57.3G}$	elevator gearing ratio, $m\cdot deg^{-1}$
K_2	stiffness constant of the elevator control system, $m\cdot kg^{-1}$
$E=\frac{K_2P}{s}$	stiffness ratio of the elevator control system (See, Sec. 2-4)
α	angle of attack of the main wing

α_s angle of attack of the horizontal stabilizer against the local flow

α_t effective angle of attack of the horizontal tail against the local flow, including the influence of the elevator on the total lift of the horizontal tail

$\tau_e=\frac{\partial \alpha_t}{\partial \delta_e}$ elevator effectiveness factor (c.f. Fig. 5-33, ref. [6] or Fig. 15, ref. [23])

θ angle of pitch (angle of the longitudinal axis of the airplane against the space axis)

ϵ angle of downwash at the horizontal tail due to the main wing and body

$\omega=\frac{2\pi}{T}$ circular frequency, or 2π times the frequency of a cylic motion

T period of a cylic motion, or tensile force

C_L lift coefficient of the whole airplane

C_D drag coefficient of the whole airplane

$C_{L_\alpha}=\frac{\partial C_L}{\partial \alpha}$

$C_{D_\alpha}=\frac{\partial C_D}{\partial \alpha}$

$a_w=\left(\frac{\partial C_L}{\partial \alpha}\right)_{\text{Wing}}$ wing lift-curve slope

$a_t=\left(\frac{\partial C_L}{\partial \alpha}\right)_{\text{Tail}}$ tail lift-curve slope

C_m pitching moment coefficient of the whole airplane about the air-plane center of gravity

C_h hinge moment coefficient of the elevator

$C_{m_\alpha}=\frac{\partial C_m}{\partial \alpha}$ rate of change of pitching moment coefficient with respect to the change of angle of attack of the main wing

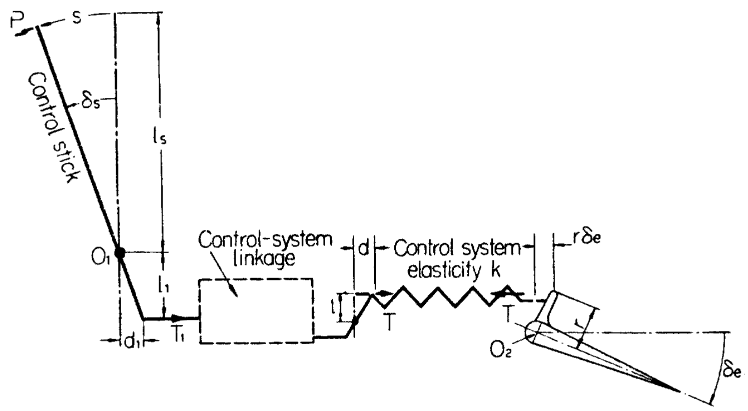
$C_{m_\delta}=\frac{\partial C_m}{\partial \delta_e}$ rate of change of pitching moment coefficient with respect to the change of elevator deflection

$C_{h_\delta}=\frac{\partial C_h}{\partial \delta_e}$ rate of change of elevator hinge moment coefficient with elevator deflection

$C_{h_{\alpha t}}=\frac{\partial C_h}{\partial \alpha_s}$ rate of change of elevator hinge moment coefficient with stabilizer angle of attack

2-3. Geometric and Elastic Parameters of Elevator Control System

The essential geometric and elastic characteristics of the elevator control system of a man-piloted airplane can generally be represented in such a dynamic model



O_1 : pivot point of the control stick.
O_2 : hinge axis of the elevator.
d_1 and d : displacement of the forward and aft end respectively of the control system linkage corresponding to the stick movement $s=l_s\delta_s$.
T_1 and T : force produced in the link at the forward and aft end respectively of the control system linkage corresponding to the stick force P.
T_1l_1 and Tl: moment acting on the control system linkage at its forward and aft end respectively.
l : length of the crank arm at the aft end of the control system linkage.

as shown below, in which the elasticity of the system is represented by a spring whose spring constant is k, and the part except for the spring is a rigid linkage.

P, s, δ_s and δ_e are defined to be positive in the directions shown by arrows.

By the definition, expressing δ_e and δ_{e1} in degrees,

$$s=K_1(\delta_e-\delta_{e1})+K_2P \tag{1}$$

From the geometry and the condition of equilibrium of forces and moments, we obtain

$$\frac{s}{l_s}=\frac{d_1}{l_1}=\frac{d}{l}\ , \tag{2}$$

$$\frac{l}{l_s}=Gr \tag{3}$$

$$\frac{s}{GK_2}=kdr \tag{4}$$

Eliminating s and d from (2) and (4), and making use of the relation (3), we obtain

$$\left.\begin{aligned} kr^2 &= \frac{1}{G^2K_2} \\ kl^2 &= \frac{l_s^2}{K_2} \\ krl &= \frac{l_s}{GK_2} \end{aligned}\right\} \qquad (5)$$

2-4. Stick Travel per g and Stick Travel per Stick Force in Steady Maneuvers

Elevator Angle, Stick Force, Stick Travel and Those per g

A general expression of the stick travel is reproduced from the equation (1), Section 2-3:

$$s = K_1(\delta_e - \delta_{e1}) + K_2 P \qquad (1)$$

By differentiating both sides of (1) with n, where n denotes the normal acceleration in terms of g we obtain the stick travel per g

$$\frac{\partial s}{\partial n} = K_1 \frac{\partial \delta_e}{\partial n} + K_2 \frac{\partial P}{\partial n} \qquad (2)$$

Equations (1) and (2) hold in steady maneuvers as well as in transient motion. In this section degree is used as the unit of angles.

For carrying out numerical calculations based on the data of an actual airplane, we shall take the expressions of δ_e, P, $\partial\delta_e/\partial n$ and $\partial P/\partial n$ from one of the representative text-books [6] and give them in the following.

For steady symmetrical pull-up (airplane in horizontal attitude),

$$\delta_e - \delta_{e0} = -\frac{1}{V^2}\left[\frac{2n(W/S)}{\rho C_{m\delta}} \cdot \left(\frac{\partial C_m}{\partial C_L}\right)_{\text{Elevator-Fixed}} + \frac{1.1 \times 57.3 g l_t}{\tau_e}(n-1)\right] \qquad (3)$$

$$\frac{\partial \delta_e}{\partial n} = -\frac{1}{V^2}\left[\frac{2(W/S)}{\rho C_{m\delta}} \cdot \left(\frac{\partial C_m}{\partial C_L}\right)_{\text{Elevator-Fixed}} + \frac{1.1 \times 57.3 g l_t}{\tau_e}\right] \qquad (4)$$

$$P = -G\eta_t S_e \bar{c}_e\left[\frac{W}{S} \cdot \frac{C_{h\delta}}{C_{m\delta}}\left(\frac{\partial C_m}{\partial C_L}\right)_{\text{Elevator-Free}} \cdot \left(\frac{V^2}{V_1^2} - n\right) + 57.3(n-1) g l_t \frac{\rho}{2} \cdot \left(C_{h\alpha t} - \frac{1.1 C_{h\delta}}{\tau_e}\right)\right] \qquad (5)$$

$$\frac{\partial P}{\partial n} = G\eta_t S_e \bar{c}_e\left[\frac{W}{S} \cdot \frac{C_{h\delta}}{C_{m\delta}}\left(\frac{\partial C_m}{\partial C_L}\right)_{\text{Elevator-Free}} - 57.3 g l_t \frac{\rho}{2}\left(C_{h\alpha t} - \frac{1.1 C_{h\delta}}{\tau_e}\right)\right] \qquad (6)$$

$$\text{angular velocity:} \quad q = \frac{g}{V}(n-1) \qquad (7)$$

For steady turn with no side-slip or steady coordinated turn (circling in a horizontal plane),

$$\delta_e-\delta_{e0}=-\frac{1}{V^2}\left[\frac{2n(W/S)}{\rho C_{m\delta}}\cdot\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Elevator-}\\ \text{Fixcd}}}+\frac{1.1\times 57.3gl_t}{\tau_e}\left(n-\frac{1}{n}\right)\right] \quad (8)$$

$$\frac{\partial\delta_e}{\partial n}=-\frac{1}{V^2}\left[\frac{2(W/S)}{\rho C_{m\delta}}\cdot\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Elevator-}\\ \text{Fixed}}}+\frac{1.1\times 57.3gl_t}{\tau_e}\left(1+\frac{1}{n^2}\right)\right] \quad (9)$$

$$P=-G\eta_t S_e\bar{c}_e\left[\frac{W}{S}\cdot\frac{C_{h\delta}}{C_{m\delta}}\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Elevator-}\\ \text{Free}}}\cdot\left(\frac{V^2}{V_1^2}-n\right)\right.$$
$$\left.+57.3\left(n-\frac{1}{n}\right)gl_t\frac{\rho}{2}\cdot\left(C_{h_{\alpha t}}-\frac{1.1C_{h\delta}}{\tau_e}\right)\right] \quad (10)$$

$$\frac{\partial P}{\partial n}=G\eta_t S_e\bar{c}_e\left[\frac{W}{S}\cdot\frac{C_{h\delta}}{C_{m\delta}}\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Elevator-}\\ \text{Free}}}\right.$$
$$\left.-57.3\left(1+\frac{1}{n^2}\right)gl_t\frac{\rho}{2}\left(C_{h_{\alpha t}}-\frac{1.1C_{h\delta}}{\tau_e}\right)\right] \quad (11)$$

$$\text{angular velocity:}\quad q=\frac{g}{V}\left(n-\frac{1}{n}\right). \quad (12)$$

Stick Travel per Stick Force ds/dP

The rate of change of stick travel with respect to stick force is nearly similar (exactly equal in symmetrical pull-up) to the ratio of stick travel per g to stick force per g in steady maneuver. This quantity is important, since it offers a measure of longitudinal stability in maneuvering flight, as sensed by the pilot. The pilot should feel comfortable, when it is sufficiently large and does not vary to a great extent with speed.

Expressing ds/dP in a form convenient for calculation,

$$\frac{ds}{dP}=\frac{\partial s}{\partial\delta_e}\cdot\frac{\partial\delta_e}{\partial(H.M.)}\cdot\frac{\partial(H.M.)}{\partial P}+\frac{\partial s}{\partial P} \quad (13)$$

where

$$H.M.=q\eta_t S_e\bar{c}_e C_h.$$

By differentiating (1) with δ_e and P respectively, we obtain

$$\frac{\partial s}{\partial\delta_e}=K_1,\quad \frac{\partial s}{\partial P}=K_2$$

By differentiating the expression of $H.M.$ with δ_e, we obtain

$$\frac{\partial(H.M.)}{\partial\delta_e}=q\eta_t S_e\bar{c}_e C_{h\delta}$$

By the definition

$$\frac{\partial(H.M.)}{\partial P}=-\frac{1}{G}$$

Substituting these relations in (13) and rewriting it, we obtain

$$\frac{ds}{dP}=-\frac{K_1}{G}\cdot\frac{1}{q\eta_t S_e\bar{c}_e C_{h\delta}}+K_2 \tag{14}$$

Numerical Calculations

The data of the airplane A-1 in which the reduced stiffness was first tested and put into practice are used for the numerical calculations. The geometric and weight data are picked up from the TABLE A.1, and the center of gravity is assumed to be at 26% MAC. The aerodynamic data are determined from the curves of the wind-tunnel test results given in Fig. A.3 to Fig. A.10.

$$\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Elevator-}\\\text{Fixed}}}=\left[\left(\frac{\partial C_m}{\partial \alpha}\right)\Big/\left(\frac{\partial C_L}{\partial \alpha}\right)\right]_{\substack{\text{Elevator-}\\\text{Fixed}}}=-0.140$$

$$C_{m\delta}=-0.0156/deg$$

$$C_{h\delta}=-0.010/deg$$

$$C_{h\alpha t}=-0.0021\sim-0.0035,\qquad \text{say } -0.0028/deg$$

$$\frac{C_{h\alpha t}}{C_{h\delta}}=0.21\sim0.35,\qquad \text{say } 0.28$$

The derivative $\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Elevator-}\\\text{Free}}}$ is calculated by the following formulas, using the wind-tunnel test data on the complete model with and without the horizontal tail, and the value of elevator effectiveness factor τ_e read from the curve given by Fig. 5-33 in ref. [*6*] or Fig. 15 in ref. [*23*].

$$\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Complete Airplane}\\\text{Elevator-Free}}}=\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Complete Airplane}\\\text{Elevator-Fixed}}}+\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Free-Elevator}\\\text{Effect}}} \tag{15}$$

where

$$\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Free-Elevator}\\\text{Effect}}}=-\frac{C_{h\alpha t}}{C_{h\delta}}\tau_e\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Tail Contribution}\\\text{Elevator-Fixed}}} \tag{16}$$

$$\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Tail Contr.}\\\text{Elevator-Fixed}}}=\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Complete Airplane}\\\text{Elevator-Fixed}}}-\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Complete Airplane}\\\text{less Tail}}} \tag{17}$$

The intermediate values in the process of calculations are

$$\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Tail Contr.}\\\delta e=0}}\doteqdot-0.193$$

$$\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Free-Elevator}\\\text{Effect}}}\doteqdot0.025$$

τ_e for $S_e/S_t=0.250$ is 0.46

The final $\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Elevator-}\\\text{Free}}}$ is -0.115.

The density of the air and the geometric and elastic parameters of the control system taken in the calculations are:

$$\rho=0.0927\ \text{kg}\cdot\text{m}^{-4}\cdot\text{sec}^2\ \text{(3000 m altitude)}$$
$$G=2.56\ \text{m}^{-1}$$
$$K_1=6.81\times10^{-3}\ \text{m}\cdot\text{deg}^{-1}$$
$$K_2=6.62\times E\times10^{-3}\ \text{m}\cdot\text{kg}^{-1}$$

where E denotes the ratio of the control stick deflection, caused by a force 60 kg applied to the stick grip when the elevator is fixed at 0°, to the maximum control stick travel for zero load [*1*]. Hence $E=0$ corresponds to a rigid control system, and $E=\infty$ to an elevator-free condition.

The results of the calculations are plotted with the speeds as abscissas on Fig. 2-4.1 to Fig. 2-4.16.

Discussions on the Results of Numerical Calculations

Examinations are made of the results of the numerical calculations with focuses laid on the variations, instead of the absolute values, of stick travel, etc. with the change of the control system stiffness. The values of E assumed in the calculations are 0.535, 0.262, and 0, where $E=0.535$ represents the stiffness ratio adopted in the A-1 (0.125 was required by the I.J.N.'s "Technical Standards for Airplane Design" [*1*]). The results are quite similar in symmetric pull-ups and coordinated turns; the values of $\delta_e-\delta_{e0}$ or $\delta_e-\delta_{e1}$, P, s, $\partial\delta_e/\partial n$, and $\partial s/\partial n$ are a little larger in turns than in pull-ups (as seen from the formulas), and the values of $\frac{\partial s}{\partial n}\Big/\left(\frac{\partial s}{\partial n}\right)_{180\,\text{kt}}$ are almost equal for both cases. Therefore the following features and conclusions picked up from the results of the analyses are common to both cases.

(1) The higher the speed, the markedly larger the effect of the reduced stiffness on $\delta_e-\delta_{e0}$ or $\delta_e-\delta_{e1}$, s and $\partial s/\partial n$. This is a very desirable characteristics for good control feel.

(2) At $n=6$ and $C_L=1.52$, the stick travel for $E=0.535$ comes pretty near to 235 mm. which is the maximum travel for zero load. It is larger than twice the travel for $E=0$ (rigid), and nearly equal to 1.4 times the travel for $E=0.262$. At $n=4$, $V=300$ kt, the stick travel for $E=0.535$ is nearly equal to four times that for $E=0$, and to 1.6 times that for $E=0.262$. At $n=6$, the ratio of the stick travel at 180 kt to that at 300 kt is approximately 2.0 for $E=0.535$, 2.4 for $E=0.262$, and over 3.5 for $E=0$. (Figs. 2-4. 3, 4, 10 and 11)

(3) At 180 kt, the stick travel per g for $E=0.535$ is more than twice that for $E=0$, and more than 1.3 times that for $E=0.262$. At 300 kt, the ratio is nearly equal to 4.0 to that for $E=0$, and 1.6 to that for $E=0.262$. (Figs. 2-4. 6 and 14)

(4) It is clearly shown that, the lower the stiffness, the less becomes the variation of $\frac{\partial s}{\partial n} \Big/ \left(\frac{\partial s}{\partial n}\right)_{180\,\mathrm{kt}}$ with the change of speed (Figs. 2-4. 7 and 15).

(5) The general feature of the curve ds/dP versus speed is much the same as that of the curve $\partial s/\partial n$ versus speed. (Figs. 2-4. 6, 14 and 16). These curves comply with the fact that pilots unanimously complained in high speed maneuvers that the airplane "felt too stiff", "lacked smoothness" or "was liable to respond more sharply than intended".

(6) As a whole the features described above of the results of the analyses conform very well to those of the flight-tests. The general features of the effect of the stiffness may not be much different from those described among normally designed subsonic airplanes.

2-5. Dynamic Response Characteristics to Stick Displacement Input

The frequency response of a dynamic system gives the variation with frequency of the steady-state amplitude and phase lag of the output when the input is a sinusoidal function, with respect to time, of a fixed amplitude. If we study the frequency response characteristics of elevator angle, stick force, airplane's angle of attack and angle of pitch to stick displacement input for various values of elevator control system stiffness, we may get the response of elevator angle, etc., and hence the variation of the same with the change of the stiffness to an arbitray stick movement. But our immediate object of this study is to find the effect of the stiffness on the responses mentioned above in a corrective control in pursuit action or in directing the airplane to a desired path, which is one of the most important maneuvers from the standpoint of airplane response to stick displacement. It is essentially a cyclic control of elevator with small, diminishing-amplitude, accompanied by proper aileron and rudder aid, and is, as it were, the opposite extreme of longitudinal maneuvers against high g pull-up or turn. We may consider that the responses in steady cyclic control, as given by the frequency response analysis, almost represent those in corrective control, because the rate of diminution in the latter is usually moderate. If we add analysis of the effect of the stiffness on the responses in the initial or transient state of longitudinal control, our present purpose of obtaining a general idea of the said effect on the responses covering such representative longitudinal maneuvers as steady pull-up, steady turn, abrupt pull-up, abrupt turn, corrective control, etc. may be attained. The range of the frequency studied in the frequency response analysis may be limited to—say, 0.1 to 1.5 cycles per second from a practical standpoint.

Derivation of the Equations of Motion

We shall assume that the airplane motion is disturbed by a small amount from steady horizontal flight; and that dynamic response of the airplane to the stick movement is resolved into the dynamic response of the airplane to the elevator deflection and the dynamic response of the elevator deflection to the stick movement.

For discussing the motion of the control system (in a broad sense), refer to the dynamic model in Section 2-3. The system can be divided into, (1) that of the front part of the control system including the control stick about the latter's pivot O_1, and (2) that of the elevator and the part of the control system adjacent to the elevator about its hinge O_2. In the analyses of this section radian is used as the unit of angles.

The equation of motion for the dynamic system (1) is

$$I_c\ddot{\delta}_s = Pl_s + Tl \tag{1}$$

The equation of motion for the dynamic system (2) is

$$I_e\ddot{\delta}_s = -Tr + H.M. \tag{2}$$

where

$$T = k(r\delta_e - d)$$

And where T is defined to be positive, when a tensile force is produced in the spring; I_e denotes the equivalent moment of inertia of the dynamic system (1), I_e the equivalent moment of inertia of the dynamic system (2); and *H.M.* the aerodynamic hinge moment on the elevator.

By the use of the relation (2), Section 2-3,

$$d = \frac{l}{l_s}s = l\delta_s$$

Hence

$$T = k(r\delta_e - l\delta_s) \tag{3}$$

The angle of attack of the horizontal tail against the local flow is expressed as follows:

$$\alpha_s = \alpha_w - \epsilon - i_w + i_s + \frac{\dot{\theta}l_t}{V} \tag{4}$$

where

α_w = angle of attack of the main wing

i_w, i_s = setting angle of the main wing and of the tail respectively against the datum line of the airplane

$\frac{\dot{\theta}l_t}{V}$ = change of angle of attack of the tail due to the angular velocity $\dot{\theta}$ of the airplane about the *C.G.*

Neglecting the terms in higher order time derivatives of δ_e and of α_s, the aerodynamic hinge moment is expressed as:

$$H.M. = q\eta_t S_e \bar{c}_e (C_{h\delta}\delta_e + C_{h\dot{\delta}}\dot{\delta}_e + C_{h\alpha t}\alpha_s) \tag{5}$$

where $q\eta_t S_e \bar{c}_e C_{h_{\dot{\delta}}} \dot{\delta}_e$ denotes the aerodynamic damping moment acting on the elevator when it rotates about its hinge with an angular velocity $\dot{\delta}_e$. Substituting (3) and (5) into (1) and (2),

$$I_c \ddot{\delta}_s = Pl_s + kl(r\delta_e - l\delta_s) \tag{6}$$

$$I_e \ddot{\delta}_e = -kr(r\delta_e - l\delta_s) + q\eta_t S_e \bar{c}_e (C_{h_\delta}\delta_e + C_{h_{\dot{\delta}}}\dot{\delta}_e + C_{h_{\alpha t}}\alpha_s) \tag{7}$$

The equations of longitudinal motion are taken from one of the representative text-books [6] together with the notations, thus

$$(C_D + d)u + \frac{1}{2}(C_{D_\alpha} - C_L)\alpha + \frac{1}{2}C_L\theta = 0 \tag{8}$$

$$C_L u + \left(\frac{1}{2}C_{L_\alpha} + d\right)\alpha - d\theta = 0 \tag{9}$$

$$C_{m_u}u + (C_{m_\alpha} + C_{m_{d\alpha}}d)\alpha + (C_{m_{d\theta}}d - hd^2)\theta + (C_{m_\delta} + C_{m_{d\delta}}d)\delta_e = 0 \tag{10}$$

where

d = differential operator $\dfrac{d}{d(t/\tau)}$ or $\tau\dfrac{d}{dt}$

u = speed ratio $\dfrac{\Delta V}{V}$

t = time $\qquad \tau = \dfrac{m}{\rho SV}$

$\mu = \dfrac{m}{\rho S\bar{c}} \qquad h = \dfrac{2I_{GY}}{m}\cdot\dfrac{1}{\mu\bar{c}^2}$

$C_{m_u} = \dfrac{\partial C_m}{\partial u} \qquad C_{m_{d\alpha}} = \dfrac{\partial C_m}{\partial\left(\dfrac{d\alpha}{d(t/\tau)}\right)}$

$C_{m_{d\theta}} = \dfrac{\partial C_m}{\partial\left(\dfrac{d\theta}{d(t/\tau)}\right)} \qquad C_{m_{d\delta}} = \dfrac{\partial C_m}{\partial\left(\dfrac{d\delta_e}{d(t/\tau)}\right)}$

(6) to (10) are the simultaneous equations of motion for the seven variables δ_s, δ_e, P, u, α, α_s and θ. An order-of-magnitude consideration may allow us, for approximately estimating the relative change of the responses due to the variation of control system stiffness at least in normally designed subsonic airplanes, to neglect the third term $q\eta_t S_e \bar{c}_e C_{h_{\alpha t}}\alpha_s$ in equation (7) and also the terms $C_{m_u}u$ and $C_{m_{d\delta}}d\delta_e$ in equation (10), as compared with the other terms. Therefore the simultaneous equations of motion for calculating the frequency response can be written as follows:

$$I_c \ddot{\delta}_s = Pl_s + kl(r\delta_e - l\delta_s) \tag{11-a}$$

$$I_e\ddot{\delta}_s = -kr(r\delta_e - l\delta_s) + q\eta_t S_e \bar{c}_e (C_{h_\delta}\delta_e + C_{h_{\dot{\delta}}}\dot{\delta}_e) \tag{11-b}$$

$$(C_D + d)u + \frac{1}{2}(C_{D_\alpha} - C_L)\alpha + \frac{1}{2}C_L\theta = 0 \tag{11-c}$$

$$C_L u + \left(\frac{1}{2}C_{L_\alpha} + d\right)\alpha - d\theta = 0 \tag{11-d}$$

$$(C_{m_\alpha} + C_{m_{d\alpha}}d)\alpha + (C_{m_{d\theta}}d - hd^2)\theta = -C_{m_\delta}\delta_e \tag{11-e}$$

These five simultaneous differential equations are grouped into (11-a) and (11-b) which represent the motion of the control system, and into (11-c), (11-d), and (11-e) which represent the longitudinal motion of the airplane.

The formulas for approximately evaluating the stability derivatives included in the above equations of motion are also taken from the same text-book [*6*] or from ref. [*23*], and written as follows:

$$C_{h_{\dot{\delta}}} = \frac{\partial C_h}{\partial \dot{\delta}_e} = \tau\frac{\partial C_h}{\partial(\tau\dot{\delta}_e)} = \tau\frac{\partial C_h}{\partial\left(\dfrac{d\delta_e}{d(t/\tau)}\right)} = \tau C_{h_{d\delta}} \doteqdot -\frac{\tau}{2\mu}(C + Da_t)\frac{\bar{c}_t}{\bar{c}} = -\frac{\bar{c}_t}{2V}(C + Da_t)$$

$$C_{m_{d\alpha}} \doteqdot \frac{\partial C_{mt}}{\partial\left(\dfrac{d\alpha}{d(t/\tau)}\right)} \doteqdot -a_t\frac{S_t l_t}{S\bar{c}}\eta_t \cdot \frac{l_t}{\bar{c}} \cdot \frac{1}{\mu} \cdot \frac{\partial\epsilon}{\partial\alpha}$$

$$C_{m_{d\theta}} \doteqdot 1.1\frac{\partial C_{mt}}{\partial\left(\dfrac{d\theta}{d(t/\tau)}\right)} \doteqdot -1.1a_t\frac{S_t l_t}{S\bar{c}}\eta_t \cdot \frac{l_t}{\bar{c}} \cdot \frac{1}{\mu}$$

where C_{m_t} denotes the horizontal tail contribution to the pitching moment coefficient of the airplane about its *C.G.*

Frequency Response of the Control System to Stick Displacement Input

The control stick is assumed to be displaced sinusoidally with respect to time, and if we put

$$\delta_s = \bar{\delta}_s e^{i\omega t} \tag{12-a}$$

then the frequency response of the stick force and of the elevator angle are expressed as follows:

$$P = \bar{P}e^{i\omega t} \tag{12b}$$

$$\delta_s = \bar{\delta}_e{}^{i\omega t} \tag{12c}$$

where $\bar{\delta}_s$, $\bar{P}$ and $\bar{\delta}_e$ denote the steady state amplitudes of δ_s, P and δ_e, respectively. Then from (11-a) and (11-b), and making use of the relation (5), Section 2-3, we obtain

$$\frac{\bar{\delta}_e}{\bar{\delta}_s} = -\frac{l_s}{GK_2} \cdot \frac{1}{M} \tag{13}$$

$$\frac{\bar{P}}{\bar{\delta}_s}=\frac{\left(-\frac{\omega^2 I_c}{l_s}+\frac{l_s}{K_2}\right)M+\frac{l_s}{G^2K_2^2}}{M} \tag{14}$$

where

$$M=(\omega^2 I_e+q\eta_t S_e\bar{c}_e C_{h_\delta}-kr^2)+i\omega q\eta_t S_e\bar{c}_e C_{h_{\dot{\delta}}} \tag{15}$$

For calculating the amplitude ratios and phase lags of the responses to the input, we may put

$$\frac{\bar{\delta}_e}{\bar{\delta}_s}=\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|e^{-i\phi(\delta_s,\delta_e)}, \tag{16}$$

$$\frac{\bar{P}}{\bar{\delta}_s}=\left|\frac{\bar{P}}{\bar{\delta}_s}\right|e^{-i\phi(\delta_s,P)}, \tag{17}$$

where $\phi_{(\delta_s,\delta_e)}$ and $\phi_{(\delta_s,P)}$ denote the phase lags of the responses δ_e and P to the input δ_s, respectively. $\phi_{(\delta_s,\delta_e)}$ is calculated by the following formula:

$$\phi_{(\delta_s,\delta_e)}=\tan^{-1}\frac{I_D}{R_D}-\tan^{-1}\frac{I_N}{R_N}$$

in which I_D and I_N denote the imaginary parts of the denominator and of the numerator, and R_D and R_N their real parts, respectively, of the equation (13). $\phi_{(\delta_s,P)}$ is calculated similarly with respect to (14).

Frequency Response of the Airplane to Elevator Angle Input

The frequency responses of the airplane to the elevator deflection input $\delta_e=\bar{\delta}_e e^{i\omega t}$ are expressed as follows:

$$u=\bar{u}e^{i\omega t} \tag{18-a}$$

$$\alpha=\bar{\alpha}e^{i\omega t} \tag{18-b}$$

$$\theta=\bar{\theta}e^{i\omega t} \tag{18-c}$$

Then the simultaneous equations of motion (11-c), (11-d) and (11-e) are reduced to:

$$(C_D+i\omega\tau)\bar{u}+\frac{1}{2}(C_{D_\alpha}-C_L)\bar{\alpha}+\frac{1}{2}C_L\bar{\theta}=0 \tag{19-a}$$

$$C_L\bar{u}-\left(\frac{1}{2}C_{L_\alpha}+i\omega\tau\right)\bar{\alpha}-i\omega\tau\bar{\theta}=0 \tag{19-b}$$

$$(C_{m_\alpha}+i\omega\tau C_{m_{\dot{\alpha}}})\alpha+[i\omega\tau C_{m_{\dot{\theta}}}+(\omega\tau)^2 h]\bar{\theta}=-C_{m_\delta}\bar{\delta}_e \tag{19-c}$$

If we put

$$\frac{\bar{\alpha}}{\bar{\delta}_e}=\frac{F_1}{N} \tag{20}$$

$$\frac{\bar{\theta}}{\bar{\delta}_e}=\frac{F_2}{N} \tag{21}$$

then from (19-a) to (19-c),

$$\left.\begin{aligned}
F_1&=C_{m_\delta}\left[(\omega\tau)^2-\frac{1}{2}C^2{}_L-i\omega\tau C_D\right]\\
F_2&=C_{m_\delta}\left[(\omega\tau)^2-\frac{1}{2}(C^2{}_L-C_LC_{D_\alpha}+C_DC_{L_\alpha})-i\omega\tau\left(C_D+\frac{1}{2}C_{L_\alpha}\right)\right]\\
N&=-h(\omega\tau)^4+(\omega\tau)^2\left[\frac{1}{2}h(C^2{}_L-C_LC_{D_\alpha}+C_DC_{L_\alpha})\right.\\
&\quad\left.-C_{m_\alpha}-C_DC_{m_{d\alpha}}-\left(C_D+\frac{1}{2}C_{L_\alpha}\right)C_{m_{d\theta}}\right]+\frac{1}{2}C^2{}_LC_{m_\alpha}\\
&\quad+i(\omega\tau)^3\left[h\left(C_D+\frac{1}{2}C_{L_\alpha}\right)-C_{m_{d\alpha}}-C_{m_{d\theta}}\right]\\
&\quad+i\omega\tau\left[C_DC_{m_\alpha}+\frac{1}{2}C^2{}_LC_{m_{d\alpha}}+\frac{1}{2}\left(C^2{}_L-C_LC_{D_\alpha}+C_DC_{L_\alpha}\right)C_{m_{d\theta}}\right]
\end{aligned}\right\} \tag{22}$$

For calculating the amplitude ratios and phase lags of the responses to the input, we may put

$$\frac{\bar{\alpha}}{\bar{\delta}_e}=\left|\frac{\bar{\alpha}}{\bar{\delta}_e}\right|e^{-i\phi(\delta_e,\alpha)} \tag{23}$$

$$\frac{\bar{\theta}}{\bar{\delta}_e}=\left|\frac{\bar{\theta}}{\bar{\delta}_e}\right|e^{-i\phi(\delta_e,\theta)} \tag{24}$$

where $\phi_{(\delta_e,\alpha)}$ and $\phi_{(\delta_e,\theta)}$ denote the phase lags of the responses α and θ to the input δ_e, respectively, and are calculated in the similar way as $\phi_{(\delta_s,\delta_e)}$ and $\phi_{(\delta_s,P)}$.

Frequency Response of the Airplane to Stick Displacement Input

For calculating the overall frequency responses of the airplane to the stick displacement input $\delta_s=\bar{\delta}_se^{i\omega t}$, we may put

$$\frac{\bar{\alpha}}{\bar{\delta}_s}=\left|\frac{\bar{\alpha}}{\bar{\delta}_s}\right|e^{-i\phi(\delta_s,\alpha)} \tag{25}$$

$$\frac{\bar{\theta}}{\bar{\delta}_s}=\left|\frac{\bar{\theta}}{\bar{\delta}_s}\right|e^{-i\phi(\delta_s,\theta)} \tag{26}$$

where

$$\left|\frac{\bar{\alpha}}{\bar{\delta}_s}\right|=\left|\frac{\bar{\alpha}}{\bar{\delta}_e}\right|\cdot\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|,\qquad \phi_{(\delta_s,\alpha)}=\phi_{(\delta_e,\alpha)}+\phi_{(\delta_s,\delta_e)} \tag{27}$$

$$\left|\frac{\bar{\theta}}{\bar{\delta}_s}\right| = \left|\frac{\bar{\theta}}{\bar{\delta}_e}\right| \cdot \left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|, \qquad \phi_{(\delta_s,\theta)} = \phi_{(\delta_e,\theta)} + \phi_{(\delta_s,\delta_e)} \tag{28}$$

The equations (25) and (26), or (27) and (28), together with the equations (16) and (17), are the final output from the stick displacement input.

Numerical Calculation of the Frequency Response

The data of the airplane A-1 are used in the numerical calculations. The calculations are made for the flying altitude of 3000 m, the flying speed of 50 m/sec (97.1 kt) and 150 m/sec (291.3 kt), and the control system stiffness ratio E of 0.535, 0.262, 0.125 and 0 (rigid control system). The influence of retention and variation of $C_{h_{at}}$ in equation (11-b) is examined. Those data used in these calculations and not given hitherto are listed as follows:

V	50 m/sec (97.1 kt)			150 m/sec (291.3 kt)		
E	0.535	0.262	0.125	0.535	0.262	0.125
l_s/K_2	189.2	386	810	189.2	386	810
l_s/GK_2	74.0	150.9	316	74.0	150.9	316
$1/G^2K_2$	43.1	88.0	184.3	43.1	88.0	184.3
$l_s/G^2K_2^2$	8160	34060	149400	8160	34060	149400
l_s	0.670			0.670		
I_e	0.020			0.020		
I_c	0.016			0.016		
I_c/l_s	0.0239			0.0239		
m	239.0			239.0		
τ	2.295			0.765		
μ	59.5			59.5		
k_Y^2	2.61			2.61		
h	0.0235			0.0235		
$l_t/\mu\bar{c}$	0.0418			0.0418		
a_t	3.44			3.44		
$\partial\varepsilon/\partial\alpha$	0.445			0.445		
α	~11°			~0.9°		
C_L	0.900			0.100		
C_D	0.072			0.018		
$C_{L\alpha}$	4.62			4.62		
$C_{D\alpha}$	0.53			0.045		
$C_{m\alpha}$	−0.647			−0.647		
C_{m_δ}	−0.90			−0.90		
C_{h_δ}	−0.573			−0.573		
$C_{md\alpha}$	−0.0283			−0.0283		
$C_{md\theta}$	−0.0700			−0.0700		
$C_{h_{\dot\delta}}$	−0.0108			−0.0036		

Units are in meter, kilogram, second and radian.

The results of the calculations are plotted in Fig. 2-5.1 to Fig. 2-5.16. Fig. 2-5.1 to Fig. 2-5.12 show the amplitude ratios and the phase lags of $\frac{\bar{\delta}_e}{\bar{\delta}_s}$, $\frac{\bar{P}}{\bar{\delta}_s}$, $\frac{\bar{\alpha}}{\bar{\delta}_e}$, and $\frac{\bar{\theta}}{\bar{\delta}_e}$, respectively, versus ω over the range from 0 (steady) to 10 (very quick manipulation). Fig. 2-5.13 to Fig. 2-5.16 show the amplitude ratios of $\frac{\bar{\delta}_e}{\bar{\delta}_s}$, $\frac{\bar{P}}{\bar{\delta}_s}$, $\frac{\bar{\alpha}}{\bar{\delta}_s}$, and $\frac{\bar{\theta}}{\bar{\delta}_s}$, respectively, versus E over the range from 0 (rigid) to 0.535. The last four should give the final output or the objects for discussing the influence of the control system stiffness on the responses considered. The phase lags of the final output are not shown, because they vary little with the change of stiffness and are not important for the present purpose. The results of the examination of the influence of $C_{h_{at}}$ versus C_{h_δ} are not shown in curves, but are taken up in the following discussions.

Discussions on the Results of Numerical Calculations

Since our purpose is to investigate the effect of the control system stiffness on the responses, attention is focused on the relative change of the responses to input with the variation of the stiffness. Detailed discussions on the responses of the airplane to elevator deflection are outside our objective.

(1) The amplitude ratio $\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|$. For a given speed and stiffness it increases very slowly with the increase of ω, and for $E=0$ it remains at a fixed value, l_sG, irrespective of ω and speed. The influences of both the speed and the stiffness are distinct, and the latter (influence) increases markedly with the increase of speed. The variation of $\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|$ with the change of E and of V shows the effect of the stiffness. For the range of ω from 0 to 10, the ratio of $\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|$ for $E=0.125$, 0.262 and 0.535 to $\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|$ for $E=0$ ranges from 0.92 to 0.928, 0.844 to 0.86, and 0.725 to 0.75, respectively at 50 m/sec; and the same ratio is around 0.55, 0.37 and 0.225, respectively at 150 m/sec. (Figs. 2-5.1, 2 and 13).

Examination is made of the influence of $C_{h_{at}}$ on the effect of the stiffness on the responses. The behavior of the curves of $\frac{\bar{\alpha}}{\bar{\delta}_e}$ and $\frac{\bar{\theta}}{\bar{\delta}_e}$ versus ω reflects on the curve of $\frac{\bar{P}}{\bar{\delta}_s}$ versus ω through the floating of the elevator, and thence on the curve of $\frac{\bar{\delta}_e}{\bar{\delta}_s}$ versus ω. And since the natural modes, i.e., phugoid and short-period, of the airplane affect the $\frac{\bar{\alpha}}{\bar{\delta}_e}$ and $\frac{\bar{\theta}}{\bar{\delta}_e}$ curves over the lower half of the range of ω investigated at 50 m/sec, and almost over the whole range of ω investigated at 150 m/sec, the influence of $C_{h_{at}}$

versus C_{h_δ} on the effect of the stiffness, etc., appears markedly over the corresponding range of ω. As $-C_{h_{at}}$ increases relatively to $-C_{h_\delta}$, $\left|\frac{\bar{P}}{\bar{\delta}_e}\right|$ or $\left|\frac{\bar{P}}{\bar{\delta}_s}\right|$ decreases, and hence the effect of the stiffness on $\left|\frac{\bar{P}}{\bar{\delta}_s}\right|$ and on $\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|$ also decreases. For example, in the case where the term of $C_{h_{at}}$ is retained in equation (7), with the other characteristics unaltered from those of the A-1, the ratio of $\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|$ for $E=0.125$, 0.262, and 0.535 to that for $E=0$ increases, with the change of ω from zero to 10, by about 0.06 to almost zero, 0.06 to almost zero, and 0.045 to almost zero, respectively at 150 m/sec. In another case where $C_{h_{at}}$ is 1.57 times, C_{h_δ} 0.80 times, and $\frac{C_{h_{at}}}{C_{h_\delta}}$ almost twice that of the A-1, the same ratio increases, with the change of ω from zero to 10, by about 0.12 to 0.05, 0.13 to 0.05, and 0.12 to 0.035, respectively at 150 m/sec. And it is noteworthy, too, that the influence of $C_{h_{at}}$ on $\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|$ decreases markedly with decreasing speed and less so with increasing ω.

(2) The amplitude ratio $\left|\frac{\bar{P}}{\bar{\delta}_s}\right|$. The influences of the stiffness, speed and ω are similar to, but the sign of the last two being opposite to, those in the case of $\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|$. $\left|\frac{\bar{P}}{\bar{\delta}_s}\right|$ tends to increase nearly in proportion to the square of the speed, as E and ω approach to 0. For the range of ω from 0 to 10, the ratio of $\left|\frac{\bar{P}}{\bar{\delta}_s}\right|$ for $E=0.125$, 0.262 and 0.535 to $\left|\frac{\bar{P}}{\bar{\delta}_s}\right|$ for $E=0$ ranges from 0.91 to 0.92, 0.83 to 0.845, and 0.72 to 0.735, respectively at 50 m/sec; and the same ratio is around 0.55, 0.37 and 0.226, respectively at 150 m/sec. (Figs. 2-5.3, 4 and 14).

The influence of the floating of the elevator on $\frac{\bar{\delta}_e}{\bar{\delta}_s}$, $\frac{\bar{\alpha}}{\bar{\delta}_s}$, $\frac{\bar{\theta}}{\bar{\delta}_s}$, etc. shows itself through its influence on $\frac{\bar{P}}{\bar{\delta}_s}$. The influence is much the same as that on $\frac{\bar{\delta}_e}{\bar{\delta}_s}$. For example, the retaining of the $C_{h_{at}}$ term is the elevator motion of the A-1 makes the ratio of $\left|\frac{\bar{P}}{\bar{\delta}_s}\right|$ for $E=0.125$, 0.262 and 0.535 to that for $E=0$ increase, with the change of ω from zero to 10, by about 0.08 to 0.02, 0.08 to 0.01, and 0.05 to almost zero, respectively at 150 m/sec. In the case where $C_{h_{at}}$ is 1.57 times, C_{h_δ} 0.80 times, and $\frac{C_{h_{at}}}{C_{h_\delta}}$ almost twice that of the A-1, the same ratio increases, with the change of ω from zero to 10, by about 0.12 to 0.11, 0.13 to 0.09, and 0.12 to 0.06, respectively at 150 m/sec. It is noteworthy, too, that the influence of $C_{h\alpha t}$ on $\left|\frac{\bar{P}}{\bar{\delta}_s}\right|$, like that on $\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|$, decreases markedly with decreasing speed and less so with increasing ω.

(3) The amplitude ratio $\left|\frac{\bar{\alpha}}{\bar{\delta}_e}\right|$. Its variation with respect to the change of ω and speed, while seemingly peculiar, should be typical to normally designed subsonic airplanes. The first very steep peak and valley occur at ω which is very close to the natural frequency of the phugoid mode; then a gently sloping plateau continues up to ω which corresponds to the short-period mode; and after that the curve tends to descend gradually along a slope in proportion to ω^{-2}. The influence of speed is distinct only around the values of ω corresponding to the natural frequencies of the two modes and beyond that corresponding to the short-period mode. (Figs. 2-5.5 and 7).

(4) The amplitude ratio $\left|\frac{\bar{\theta}}{\bar{\delta}_e}\right|$. Its variation with respect to the change of ω and speed should also be typical. The important differences between the variation of $\left|\frac{\bar{\theta}}{\bar{\delta}_e}\right|$ and $\left|\frac{\bar{\alpha}}{\bar{\delta}_e}\right|$ with respect to the change of ω and speed are: In the former, (i) the curve starts at $\omega=0$ at the initial value which is higher than that proportional to the square of the speed; (ii) the first peak is much steeper, and more so as the speed increases; (iii) no valley exists at ω which is very close to the natural frequency of the phugoid mode; and (iv) it has an undulation along a descending slope, instead of a plateau, at ω corresponding to the natural frequency of the short-period mode. (Figs. 2-5.9 and 11).

(5) The phase lags $\phi_{(\delta e,\alpha)}$ and $\phi_{(\delta e,\theta)}$. Both curves start at $\omega=0$ at 180°; $\phi_{(\delta e,\alpha)}$ curve has a steep peak, while $\phi_{(\delta e,\theta)}$ curve first descends and then ascends steeply, both near the values of ω close to the natural frequencies of the phugoid mode; then both curves begin to ascend and undulate at ω corresponding to the natural frequencies of the short-period mode; and after that continue to ascend gradually. At ω where the amplitude ratio changes suddenly, the phase lag also changes suddenly. (Figs. 2-5.6, 8, 10 and 12).

(6) The amplitude ratios $\left|\frac{\bar{\alpha}}{\bar{\delta}_s}\right|$ and $\left|\frac{\bar{\theta}}{\bar{\delta}_s}\right|$. $\left|\frac{\bar{\alpha}}{\bar{\delta}_s}\right|$ represents the response of the airplane measured in the change of angle of attack, and $\left|\frac{\bar{\theta}}{\bar{\delta}_s}\right|$ that in the change of the direction of the airplane axis in space. Since the influences of the speed and stiffness on $\left|\frac{\bar{\alpha}}{\bar{\delta}_s}\right|$ and $\left|\frac{\bar{\theta}}{\bar{\delta}_s}\right|$ are represented by

those on $\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|$, as shown by equation (27) and (28), the repetition thereof is saved. It is clearly seen that both $\left|\frac{\bar{\alpha}}{\bar{\delta}_s}\right|$ and $\left|\frac{\bar{\theta}}{\bar{\delta}_s}\right|$ as well as their variations with respect to the change of speed are comfortably reduced and levelled off, as E increases. Since frequency response characteristics for the values of ω, say, much smaller than unity or larger than ten are essentially of little practical interest, the plotting of the curve of $\left|\frac{\bar{\theta}}{\bar{\delta}_s}\right|_{\substack{V=150\,\mathrm{m/sec.}\\ \omega=0}}$ which gose out of the scale of Fig. 2-5.16 is given up. (Fig. 2-5.13, 15 and 16).

(7) The steady-state amplitude ratios $\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|$, $\left|\frac{\bar{P}}{\bar{\delta}_s}\right|$, $\left|\frac{\bar{\alpha}}{\bar{\delta}_s}\right|$ and $\left|\frac{\bar{\theta}}{\bar{\delta}_s}\right|$ represent the corresponding responses, respectively, to a steady or slowly diminishing cyclic stick actuation, e.g. as in a continuing corrective control. For the range of ω investigated, the effect of the reduced stiffness on $\left|\frac{\bar{\alpha}}{\bar{\delta}_s}\right|$, $\left|\frac{\bar{\theta}}{\bar{\delta}_s}\right|$ and $\left|\frac{\bar{P}}{\bar{\delta}_s}\right|$, and its variation with the change of speed are almost similar to that on the reciprocal of $\frac{\partial s}{\partial n}$ and $\frac{ds}{dp}$, respectively and its variation with speed in steady maneuvers, as studied in the previous section. For example, the ratio of the reciprocal of $\frac{\partial s}{\partial n}$ for $E=0.262$ and 0.535 to that for $E=0$ is around 0.85～0.86, and 0.74, respectively at 51.5 m/sec, and the same ratio is around 0.38～0.39, and 0.23, respectively at 154.5 m/sec; the ratio of the reciprocal of $\frac{ds}{dp}$ for $E=0.262$, and 0.535 to that for $E=0$ is around 0.84 and 0.72, respectively at 51.5 m/sec, and the same ratio is around 0.37 and 0.22, respectively at 154.5 m/sec.

Thus confirmation is made that for the range of ω investigated the effect of the stiffness on the responses of the airplane to a steady or slowly diminishing cyclic stick acuation, e.g. as in a corrective control, is more or less nearly similar to the effect on the reciprocal of the stick travel per g and the stick travel per stick force in steady maneuvers, depending on the influence of $C_{h_{\alpha t}}$ versus C_{h_δ}, etc.

Response to Stick Displacement Input in the Initial Stage of Longitudinal Control —Numerical Analyses with an Analog Computor

To complete the picture of the effect of the control system stiffness on the dynamic response of the airplane to longitudinal control, numerical analyses of the response of elevator deflection, stick force, angle of attack and angle of pitch to stick displacement during some ten seconds after the start of the stick movement are made with an analog computor for four values of control system stiffness. The analyses are done based on the simultaneous equations of motion (6), (7), (11-c), (11-d) and (11-e),—same as in the frequency response analyses, except for the fact that the term $q\eta_t S_e \bar{c}_e C_{h_{\alpha t}} \alpha_s$ is retained in the hinge moment on the elevator.

The input is assumed to be one complete cycle of sinusoidal stick movement, which is expressed as

$$\delta_s(t) = \frac{1}{2}\bar{\delta}_s\left(1 - \cos 2\pi\frac{t}{T}\right) \qquad \text{for} \quad t = 0 \sim T$$

$$\delta_s(t) = 0 \qquad \text{for} \quad t > T$$

where

$\bar{\delta}_s$ = peak value of the control stick deflection
T = one cycle time of δ_s

This type of fore and aft movement of the stick, accompanied by proper application of the aileron and rudder control, may approximately represent that for turn. The peak value of the response for a given value of E, and angle of turn are almost defined by $\bar{\delta}_s$ and T, respectively. And if the other conditions are fixed, the intensity of the response as a function of $\frac{t}{T}$ is expected to be proportional to $\bar{\delta}_s$ from the linearity of the equations of motion used. Therefore analyses are executed for the combinations of T and E shown below, $\bar{\delta}_s$ being fixed at 2.36°. The velocity 150 m/sec is selected from among the two values assumed in the frequency response calculation, because the effect of the stiffness shows itself more clearly at higher speed.

E	0		0.125			0.262		0.535	
T (sec)	3.0	6.0	1.0	3.0	6.0	3.0	6.0	1.0	3.0

Samples of the analog computor recordings are reproduced on Fig. 2-5.21 (1), (2), (5) and (6). The peak values of the responses read from the records being denoted as $\bar{\delta}_e$, $\bar{P}$, $\bar{\alpha}$ and $\bar{\theta}$ respectively, we get the curves of $\frac{\bar{\delta}_e}{\bar{\delta}_s}$, $\frac{\bar{P}}{\bar{\delta}_s}$, $\frac{\bar{\alpha}}{\bar{\delta}_s}$ and $\frac{\bar{\theta}}{\bar{\delta}_s}$ plotted against E in Fig. 2-5.17 to Fig. 2-5.20, which compare with Fig. 2-5.13 to Fig. 2-5.16 for the frequency response analyses.

Discussions on the Results of Analog Computor Analyses

Discussions are made with focus laid on the effect of control system stiffness on the responses or on the variation of the latter with the change of the former.

(1) The curves of the responses versus time generally faithfully follow those of the input, as expected. Discontinuities or zigzagging of the curves around the spots where the curvature of the curves changes sharply may be attributable to the characteristics of the machine. Small, quickly damped fluctuation of the α-curve just before it diminishes to zero shows the exciting by the natural short-period mode of the airplane; and a gentle wave along a very slowly descending path of the θ-curve that starts just after the peak shows the exciting by the natural phugoid mode.

(2) If we call the plus or minus lapse of time from the peak of the input to those of the output "lag" or "lead", we observe lags of a few hundredths of a second for δ_e-curves, leads of a little less time for P-curves, and lags of fifteen to twenty hundredths of a second for α-curves, and a general tendency that the lag increases as E increases for δ_e- and α-curves. But we fail to observe orderly variation of the magnitude of lag or lead with respect to the change of T in δ_e-, α-, and P-curves. In θ-curves, the order of magnitude of the lag is two seconds for $T=6$ sec, one second for $T=3$ sec, and one third of a second for $T=1$ sec; but a tendency of increasing lag with increasing E is difficult to find. Generally the tendency of lags and leads observed above may be reasonable, although discussion on their magnitudes is unrealistic, considering the order of accuracy of the records and their reading.

(3) Direct comparison of the transient-state peak value ratios $\frac{\bar{\delta}_e}{\bar{\delta}_s}$, $\frac{\bar{P}}{\bar{\delta}_s}$, $\frac{\bar{\alpha}}{\bar{\delta}_s}$ and $\frac{\bar{\theta}}{\bar{\delta}_s}$ with the steady-state amplitude ratios of the frequency response $\left|\frac{\bar{\delta}_e}{\bar{\delta}_s}\right|$, $\left|\frac{\bar{P}}{\bar{\delta}_s}\right|$, $\left|\frac{\bar{\alpha}}{\bar{\delta}_s}\right|$ and $\left|\frac{\bar{\theta}}{\bar{\delta}_s}\right|$, respectively, may not be quite reasonable. But they are of comparable order for the same value of E and the same frequency of the input δ_s, except for the last ones. For the range of T investigated, $\frac{\bar{\theta}}{\bar{\delta}_s}$ from the transient response analysis is a few times larger than the corresponding $\left|\frac{\bar{\theta}}{\bar{\delta}_s}\right|$ from the frequency response analysis, and the multiple increases with increasing T; and the ratio of $\frac{\bar{\theta}}{\bar{\delta}_s}$ for $T=1, 3, 6$ sec. to that for $T=1$ sec. and the same value of E is roughly 1:2.4:4.5. The chief reason may be that, from the first order approximation, θ is proportional to the work done by the pitching moment produced by the elevator deflection during the time of its application. θ represents the angle of pitch in pure longitudinal maneuver or the angle of turn in turning.

(4) The effect of the stiffness shows itself most distinctly in the variation of the peak value ratio with the change of E, e.g. in the ratio of that for

$E=0.125$, 0.262 and 0.535 to that for $E=0$. In $\frac{\bar{\delta}_e}{\bar{\delta}_s}$ the ratio is 0.61, 0.41 and 0.26 for $T=3$ sec, and 0.59 (for $E=0.125$) and 0.42 (for $E=0.262$) for $T=6$sec; in $\frac{\bar{\alpha}}{\bar{\delta}_s}$ 0.58, 0.40 and 0.23 for $T=3$ sec, and 0.59 (for $E=0.125$) and 0.40 (for $E=0.262$) for $T=6$ sec; in $\frac{\bar{\theta}}{\bar{\delta}_s}$ 0.61, 0.42 and 0.26 for $T=3$ sec, and 0.61 ($E=0.125$) and 0.42 ($E=0.262$) for $T=6$ sec. P- curve for $E=0$ is not recorded, but the ratios of the same order are expectable from the peak value ratio $\frac{\bar{P}}{\bar{\delta}_s}$ for $E=0.125$, 0.262 and 0.535. From the comparison of the ratios studied above with the corresponding ratios in the frequency response, it is deduced that the effect of the stiffness on the responses is of the same order in the transient-state and in steady cyclic longitudinal control.

Summary of Conclusions of Sections 2-4 and 2-5 with Short Remarks

The effects of the reduced stiffness on the responses of the airplane to longitudinal control in such representative maneuvers as steady pull-up, steady turn, abrupt pull-up, abrupt turn, corrective control (which approximately corresponds to a slowly diminishing cyclic movement of the stick), etc. deduced from the foregoing analyses are summarized as follows:

(1) The response of the airplane to the stick movement is reduced with increasing speed and comfortably levelled off.

(2) The stick feel which is liable to become stiff at high speeds is made definitely softer.

(3) The marked decrease of $\left|\frac{\bar{\alpha}}{\bar{\delta}_s}\right|$ and $\left|\frac{\bar{\theta}}{\bar{\delta}_s}\right|$ or the increase of their reciprocals with increasing speed, assures smoother, truer, and more precise control even at high speeds, where such characteristics is otherwise very difficult to realize.

(4) The hinge moment coefficients, particularly $C_{h_{\alpha t}}$ versus C_{h_δ}, and other aerodynamic characteristics may alter appreciably, but not change drastically, the whole picture of the effect of the stiffness.

The above deductions except for the item (4) conform well to the demonstration by flight-tests and service experiences, as described in CHAPTER 3. The item (4) in its nature may not need proof by special flight-tests. The author should think that sufficient demonstration is made that the reduced stiffness is a remarkably efficient method to make the stick travel increase suitably with increasing speed and to make the curves of the stick travel per airplane response versus speed shift upward and flatten out, and thereby to attain excellent matching of the longitudinal control characteristics of the airplane and the physical and perceptive characteristics of the pilot.

The following remarks should be added to clarify the applicability of the concept: The nature of the problem, the results of the above investigations and of the flight-tests, and the characteristics of the samples taken in the numerical calculations suggest that this concept is applicable to normally designed subsonic airplanes with only a little reserve, though its effect may more or less vary according to the design. It is most beneficial to those airplanes which are required good response characteristics over wide ranges of speed and normal acceleration. A procedure the author has in mind to find a suitable stiffness for an individual model of airplane is presented in Sec. 1-3. The planned stiffness must be checked and adjusted in accordance with the flight-test, as is usually the case in the problems relevant to flying qualities.

CHAPTER 3

Conception and Verification of The Reduced Stiffness Principle by Flight-Tests

The test of maneuverability of the A-1 was begun about two weeks after the first flight. The senior test pilot made a report which touched the core of the problem just after his first maneuverability test. This was not a surprise to the author, because he remembered a problem of the same nature reservedly suggested by a few pilots in the model preceding the A-1. While in the preceding model it had been a suppressed complaint and hint from only one or two pilots, this time it was a definite objection to the A-1. By analysing the flight reports made by the senior and junior test pilots, the nature of the problem became clear. The features of the problem are summarised as follows:

(1) In pulling the stick in a normal way in a loop and in a turn, the response (increasing g, change of attitude and flight path) was too sharp, the stick was too heavy and the control feel was too stiff.

(2) To a quick pull of the stick, the airplane was liable to respond too violently, and the heaviness of the stick and the stiffness of the control feel grew sharply, as the flight speed increased.

(3) The stick travel required to perform a maneuver of the same intensity level (in g, and in the change of attitude and flight path) varied too much with the change of speed. It was considerably less in this airplane than in the preceding airplanes of the same category. The travel required for high g maneuvers was nearly a half or less than a half of the total stroke. Seventy per cent or more was desirable even at high speeds.

(4) The features described above were quite unfavorable for smooth and precise maneuver. This model would not make a good airplane of the specified category, if these features were not properly rectified.

(5) (After several modifications of the area of the horizontal stabilizer and elevator were test flown) Any combination of the areas of the stabilizer and elevator could not rectify these unfavorable features, so long as it

was mandatory to assure a necessary and sufficient effectiveness of the elevator in landing.

After investigating every factor which apparently contributed to the controllability of the airplane, which was in itself the result of the dynamic characteristics of the combined system of the airplane and the pilot, the author conceived the idea that the control system would offer a key to the solution of the problem. But so long as the control system was presumed to be rigid, it should be an automatic variable linkage which required a complicated mechanism to adapt itself to the change of speed. A careful study of the dynamics of control system lead the author to the conclusion that elastic properties properly controlled would, quite simply, produce an automatic, infinitely variable linkage which would serve as the solution to the problem.

Before starting the flight-test of a deliberate reduction of the control system stiffness, problems that might cause troubles were examined and dealt with as stated in Section *1-4*.

As a precaution, the reduction of the stiffness was progressively introduced in the flight-test, namely expressed in E, 0.262 (from the first flight up to the initial stage of the maneuverability test), uncertain (not measured), 0.430, and 0.535, respectively. In the third reduction all the unfavorable features mentioned were rectified with a reasonably small margin left between the maximum stroke of the stick at zero load and the actual travel for the limit g at or below medium speed. The finally reached stiffness ratio 0.535 was so low as compared with 0.125—the lowest ratio specified in the I.J.N.'s "Technical Standards for Airplane Design" [*1*] effective at that time. In the course of the tests, the customer's test pilots were invited to test-fly the airplane and discuss the results. The author was gratified to see that the first important object of the flight-tests was accomplished and the judgement of many a pilot was surprisingly coincident in this problem of such delicate nature.

The author did not ask the pilots to examine at every step of the reduction of the stiffness how they evaluated the reduction of the sensitivity of the airplane response to stick actuation in corrective control over the range of speed to be actually used, but asked them to report their findings in the change of the mentioned characteristics between the initial and final stiffness, partly based on memory. They reported that it was difficult to measure it instrumentally, but that they failed to find significant difference in it and felt it quite satisfactory both before and after the reduced stiffness was introduced.

But later the verification in flight was incidentally made by pilots who had chances of comparing the characteristics of the A-1 and those of a newer Japanese model and/or foreign airplanes of fighting category. They reported that the A-1's longitudinal response characteristics to stick actuation from a high g maneuver to a fine corrective control were definitely superior to those of any other ship they had ever flown, and that it was the only fighter among them that was controllable for a beginner pilot so smoothly, so precisely and so truly as he liked.

Among the characteristics of the airplane apprehensive of being adversely affected by the reduced stiffness, the author then saw absolute necessity of verifying by flight-test the deterioration, if any, of the longitudinal response characteristics to, or of the damping of the longitudinal disturbance induced by, a quick actuation of the control stick. This was easily and successfully done by both the company's and customer's test pilots. They reported that they found those characteristics quite satisfactory and found practically no difference between them in the initial and final stiffness.

The stiffness finally reached was adopted in all the production models of the A-1, which won later a unique reputation even among the opponent pilots in longitudinal maneuverability as well as in longitudinal response characteristics, although most of them hardly recognized what these desirable characteristics came from.

CHAPTER 4

Investigation of The Problems Apprehensive of being Adversely Affected by Reduced Stiffness

4-1. Introductory Notes

As stated in Section *1-4* and in CHAPTER 3, among the flight characteristics more or less feared by some people concerned of being adversely affected by the reduction of the control system stiffness, the following two are the most important. The one is the longitudinal response characteristics of the airplane to an abrupt actuation of the elevator, and the other is an elevator flutter. The author sees the necessity of proof by analytical or systematic experimental investigations of these problems in order to establish general validity of the reduced stiffness concept. Though not as serious as the two, the influence on the stick-fixed static longitudinal stability and the longitudinal short-period oscillation also may be the items on which due investigations are required.

In this chapter are made analytical investigations including numerical calculations of the problems mentioned above except for the control surface flutter, on which are presented reviews of information abstracted from recent representative theoretical and experimental researches.

4-2. Elevator Flutter

At the time when the reduced stiffness was first conceived and tested, general evidence was not obtainable that it did not adversely affect the flutter of control surfaces. At present, however, the quality and quantity of information acquired from later rigorous analytical and experimental researches on flutter offer us sufficiently general proof of this fact.

An elevator flutter usually is caused by coupling of the rotational vibration of the elevator with the vertical bending vibration of the fuselage or by that between the torsional vibrations of the elevator and the fuselage. Since the way of coupling of the individual modes of vibration is generally similar in the fuselage vertical bending-elevator rotational flutter and in the fuselage torsion-elevator torsional flutter, and since the effect of the control system stiffness is larger on the former than on the latter, explanation is here given of the former alone.

The natural frequency of the vertical bending vibration of the fuselage (f_χ) is determined by the distributions of mass and rigidity in the fuselage; and that of the rotational vibration of the elevator system (f_ζ) by the distributions of those in the elevator system. For an elevator system like the one treated in the present paper, f_ζ for the fixed-stick is given by

$$f_\zeta=\frac{\omega_n}{2\pi} \quad \text{or} \quad =\frac{1}{T_n}$$

where ω_n or T_n is obtained by putting $q=0$ in equation (4) or (5), Section *4-3*. Hence

$$f_\zeta=\frac{1}{2\pi\sqrt{G^2\cdot K_2\cdot I_e}} \quad \text{for the fixed-stick,}$$

and $$f_\zeta=0 \quad \text{for the free-elevator.}$$

The problem of the effect of the reduced stiffness on the elevator flutter reduces to that of the effect of f_ζ/f_χ on the critical speed of the said flutter. In the later researches by *T.* Matsudaira and others [*25*], [*26*], [*27*], [*28*], [*29*], [*30*], [*31*], investigations are made of the effect of f_ζ/f_χ on the critical speed of a coupled flutter v. A typical example of the subject is presented by the reproduction of Fig. 27 of the ref. [*28*]*, which shows the cases of the fuselage horizontal flexure-rudder rotational flutter and of the fuselage torsion-rudder rotational flutter. If we substitute "rudder" for "elevator" and strike off the figures on the curves and on the ordinate in the upper diagram of the said figure, it is applicable to the fuselage vertical bending-elevator rotational flutter. The characteristics shown by this diagram are at present universally accepted as applicable to common cases. It shows that, the lower the value of f_ζ/f_χ or the control system stiffness, the higher the critical speed of flutter, as the degree of mass balancing approaches to the ideal; and that this mode of flutter never occurs for the region of f_ζ/f_χ somewhat larger than that (usually around unity) corresponding to the lowest critical speed.

However, since the characteristics of an elevator flutter are governed by many factors such as the dimensions, the moment of inertia, the mass balancing, the mode of natural vibration of the elevator system, and by the mode of natural vibration of the fuselage and that of the horizontal stabilizer, it is possible, in some cases, that the relation between f_ζ/f_χ and v has somewhat different aspects from those shown by the referred diagram. For instance, there may be cases in which the critical speed v decreases monotonously as f_ζ/f_χ decreases from more than unity down to zero, or v changes along a concave upward curve having the minimum somewhere between unity and zero of f_ζ/f_χ.

* Fig. 27, p. 109 of the ref. [*28*] is reproduced and filed in the APPENDIXES.

Therefore it goes without saying, that analyses and model-tests of the characteristics of flutter for various control system stiffness and verification by flight-tests are essential to those categories of airplanes, for which a more than reasonable margin of the critical speed of flutter over the maximum allowable flying speed is not tolerable. Further, examination must be made on the possibility of increasing influence of the decreasing f_ζ or control system stiffness on the critical speed in such modes of flutter, *e.g.* the bending-torsional flutter of the horizontal tail, on which the degree of freedom of the elevator usually has little consequence.

In spite of the foregoing review, the author would like to remark that the stiffness of the control system may be chosen almost without regard to flutter in actual design of an airplane except for a small, slow one. The reasons are: (1) an airplane must be guaranteed against dangerous flutter, whether the control stick is held firm, loose or let free; and (2) the pilot may not be able to slow down the airplane speed by holding the control stick steady before coming to a catastrophe, in case a flutter, particularly an elevator flutter, begins.

4-3. Longitudinal Response Characteristics to Quick Actuation of the Control Stick

Note on the Nature of the Problem

As the subject of the present section is to investigate the effect of the reduction of elevator control system stiffness on the longitudinal response characteristics of an airplane to a quick actuation of the control stick, the examination of the effect as such on the damping characteristics of the rotational vibration of the elevator system induced by a quick actuation of the stick and on the interaction between the said vibration and the longitudinal short-period oscillation of the airplane will give the answer.

If proof is made that the reduced stiffness is not significantly injurious to the damping characteristics of the rotational vibration of the elevator system, and that it does not cause coupling of the said vibration with the longitudinal short-period oscillation of the airplane, it can be concluded that the reduced stiffness does not adversely affect the longitudinal response characteristics of the airplane to a quick actuation of the elevator. Hence the problem can be divided into three parts; *i.e.* the determination of the characteristics of the rotational vibration of the elevator system and that of the short-period oscillation of the airplane, as affected by the reduced stiffness, and the investigation of the possibility of coupling of the two modes.

Undamped Rotational Vibration of Elevator System

As the nature of the problem suggests that the control system stiffness has no significant effect on the damping characteristics of the rotational vibration of the control surface, the analyses are carried out first by neglecting damping terms and then by taking them into account.

The dynamic system shown in Section *2-3* is referred to. Then the equation of the rotational motion of an elevator system about its hinge caused by the displacement of the control stick δ_s is obtained from the equation (11-b), Section *2-5,* as

$$I_e\ddot{\delta}_e = q\eta_t S_e \bar{c}_e C_{h_\delta}\delta_e - kr(r\delta_e - l\delta_s) \qquad (1)$$

Rewriting (1) by making use of the relation (5), Section *2-3*

$$I_e\ddot{\delta}_e + \left(\frac{1}{G^2K_2} - q\eta_t S_e\bar{c}_e C_{h_\delta}\delta_e\right)\delta_e = \frac{l_s\delta_s}{GK_2} \qquad (2)$$

If δ_s is a step function with respect to time, the solution of the equation (2) is

$$\delta_e = \frac{l_s\delta_s}{GK_2}\cdot\frac{1}{\dfrac{1}{G^2K_2} - q\eta_t S_e\bar{c}_e C_{h_\delta}}(1-\cos\omega_n t)\,, \qquad (3)$$

where

$$\omega_n^2 = \frac{\dfrac{1}{G^2K_2} - q\eta_t S_e\bar{c}_e C_{h_\delta}}{I_e} \qquad (4)$$

The "undamped natural period" T_n is expressed by

$$T_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_e}{\dfrac{1}{G^2K_2} - q\eta_t S_e\bar{c}_e C_{h_\delta}}} \qquad (5)$$

Now numerical calculations are made of T_n from the equation (5) by using the data of the airplane *A-1* flying at an altitude of 3000 m. for various values of K_2. The results are:

K_2 \ $V(kt)$	60	97.1	180	291.3	400	500
$6.62\times0.535\times10^{-3}$	0.126	0.115	0.089	0.064	0.050	0.041
$6.62\times0.262\times10^{-3}$	0.091	0.087	0.074	0.058	0.0465	0.039
$6.62\times0.125\times10^{-3}$	0.064	0.062	0.057	0.049	0.041	0.036
0	0	0	0	0	0	0

It is pointed out that the effect of reduced stiffness on T_n is marked at low speeds and less so at high speeds. The effect of speed, *i.e.* that of aerodynamic hinge moment, is considerably large, and particularly so for low stiffness. The most important conclusion deduced from this investigation is that, within the usable range of control system stiffness, the period of the rotational vibration of an elevator system is generally less than one-tenth that of the longitudinal short-period oscillation of the same airplane. (c.f. Section *4-5*)

Brief Consideration on Faster and Heavier Airplanes

Since $q\eta_t S_e\bar{c}_e C_{h_\delta}$ is not allowed to vary widely for a man-piloted airplane, the denominator of the equation (5) should remain in a certain reasonable range of values. Hence the moment of inertia of the elevator is a dominant factor that makes T_n vary according to the speed, weight and size of an airplane. For a faster airplane whose density is more than twice that of the A-1, the period T_n would not considerably exceed 1.5 times that of the A-1. And in section *4-5* the period of the short-period oscillation for such an airplane is shown to be in the same order, and the time to damp to half amplitude to be not more than twice that of the A-1.

Damped Rotational Vibration of Elevator System

Aerodynamic resisting moment to the flapping of the elevator about its hinge and Coulomb friction in the control system resisting to its movement are the major damping factors in a damped rotational vibration of an elevator system. The friction is required to be within a specified value, and the resisting moment due to it is generally so small that it becomes comparable with that due to aerodynamic force only after the amplitude of the elevator flapping has diminished to a sufficiently small value. Hence it is reasonable and sufficient to make analytical investigation for the case where aerodynamic force is the only damping term.

The equation of motion for a damped rotational vibration of an elevator system for our present purpose is

$$I_e\ddot{\delta}_e - q\eta_t S_e\bar{c}_e C_{h\dot{\delta}}\dot{\delta}_e + \left(\frac{1}{G^2K_2} - q\eta_t S_e\bar{c}_e C_{h_\delta}\right)\delta_e = \frac{l_s\delta_s}{GK_2} \qquad (6)$$

Rewriting (6) by introducing a damping ratio ζ and using ω_n defined by (4),

$$\ddot{\delta}_e + 2\zeta\omega_n\dot{\delta}_e + \omega_n^2\delta_e = \frac{l_s\delta_s}{I_eGK_2} \qquad (7)$$

where

$$\zeta = -\frac{1}{2\omega_n}\cdot\frac{q\eta_t S_e\bar{c}_e C_{h\dot{\delta}}}{I_e} \qquad (8)$$

If the circular frequency of this mode of vibration is denoted by ω,

$$\omega = \omega_n\sqrt{1-\zeta^2} \qquad (9)$$

Numerical calculations are made of ω_n and ξ by using the data of the airplane A-1 flying at an altitude of 3000 m. for various values of K_2. The results are:

V (*kt*)	97.1	291.3	500
V (*m*/sec)	50	150	257.5
$q\eta_t S_e\bar{c}_e C_{h\dot{\delta}}$	−0.310	−0.930	−1.596
ω_n: $K_2=6.62\times0.535\times10^{-3}$	54.5	97.6	154.0
ω_n: $K_2=6.62\times0.262\times10^{-3}$	72.3	108.5	161.5
ω_n: $K_2=6.62\times0.125\times10^{-3}$	100.2	128.7	175.5

ζ	$K_2=6.62\times0.535\times10^{-3}$	0.142	0.238	0.259
	$K_2=6.62\times0.262\times10^{-3}$	0.107	0.214	0.247
	$K_2=6.62\times0.125\times10^{-3}$	0.077	0.181	0.227

ζ^2 is shown to be very small compared with unity. Therefore, ω is not significantly smaller than ω_n, and the period is only a very little longer than that for an undamped vibration.

Reading the values of $\frac{RT}{T_n}$ from Fig. 19-9 of the ref. [32]*, with the values of ζ obtained above, calculations are made of the values of RT, where RT denotes the time required for the amplitude to diminish to 0.05 of its initial value and never again exceed this value. The results of the calculations are shown as follows:

V (kt)		97.1	291.3	500
V (m/sec)		50	150	257.5
T_n	$K_2=6.62\times0.535\times10^{-3}$	0.115	0.064	0.041
	$K_2=6.62\times0.262\times10^{-3}$	0.087	0.058	0.039
	$K_2=6.62\times0.125\times10^{-3}$	0.063	0.049	0.036
$\frac{RT}{T_n}$	$K_2=6.62\times0.535\times10^{-3}$	3.1	1.8	1.7
	$K_2=6.62\times0.262\times10^{-3}$	4.2	2.1	1.8
	$K_2=6.62\times0.125\times10^{-3}$	6.2	2.6	2.0
RT	$K_2=6.62\times0.535\times10^{-3}$	0.36	0.12	0.071
	$K_2=6.62\times0.262\times10^{-3}$	0.36	0.12	0.071
	$K_2=6.62\times0.125\times10^{-3}$	0.38	0.125	0.072

The summary of the analyses is:

RT varies little with the change of the control system stiffness, so long as the latter remains within the usable range. The higher the speed, the smaller the values of T_n, T and RT; and the influence of speed on RT is much larger than on T_n and T. The number of cycles during RT decreases rapidly with the reduction of stiffness. The rotational vibration of an elevator system is well damped by aerodynamic damping alone, and is not adversely affected by the reduction of stiffness. In the case of this example, RT of the rotational vibration of the elevator is between ten to twenty per cent of the period of the longitudinal short-period oscillation. Because the data of the A-1 used in the numerical calculations are not exceptional, these conclusions are considered to be generally applicable to normally designed subsonic airplanes, which are free from such problems as compressibility effect due to high flying speed.

Conclusion

By investigating the rotational vibration of the elevator system and considering

* Fig. 19-9, p. 265 of the ref. [32] is reproduced and filed in the APPENDIXES.

the characteristics of the longitudinal short-period oscillation studied in Section *4-5*, it is concluded that the reduced stiffness concept is applicable to normally designed airplanes with little or no adverse effect on the longitudinal response characteristics of the airplane to a quick actuation of the control stick. This conclusion finely conforms to the results of the flight-tests (c.f. CHAPTER 3).

It can be deduced from a brief consideration in the previous paragraph, that a similar conclusion is valid for a much faster and heavier airplane, if it is free from such problems as compressibility effect, etc.

4-4. Stick-Fixed Static Longitudinal Stability

The contribution of the horizontal tail to the longitudinal stability varies with the floating characteristics of the elevator. The effect of free elevator on the static longitudinal stability is treated in several text-books on airplane stability and control, and is presented, for instance in ref. [*6*], as follows:

$$\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Stick-}\\\text{Fixed}}}-\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Stick-}\\\text{Free}}}=\frac{C_{h_{at}}}{C_{h_\delta}}\cdot\tau_e\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Tail Contr.}\\\delta e=0}} \qquad (1)$$

It seems to the author that no one has yet recognized a significant role the elasticity of the control system plays in this problem, and that "stick-fixed" and "stick-free" mean "elevator-fixed" and "elevator-free", respectively, in all the literatures hitherto published. However, if account is taken of the elasticity of the control system, it is easily understandable that the longitudinal stability may change with the increase of speed on account of the increase of the floating tendency of the elevator, even when the control stick is fixed.

This section is devoted to the investigation of the effect of the reduction of the elevator control system stiffness on the static longitudinal stability.

Assume that an equilibrium condition is held, during the airplane motion, among the moments about the elevator hinge due to the stick force and the aerodynamic force acting on the elevator. The change in the latter is caused by the change of the angle of attack of the horizontal tail and the change of the floating angle of the elevator. The equation for the condition of equilibrium can be expressed as follows, the kinetic and elastic energies in the control system being neglected:

$$\frac{\partial(H.M.)}{\partial P}\cdot\Delta P=\frac{\partial(H.M.)}{\partial\alpha_s}\cdot\Delta\alpha_s+\frac{\partial(H.M.)}{\partial\delta_e}\cdot\Delta\delta_e \qquad (2)$$

In the analyses of this section, degree is used as the unit of angles. Rewriting the equation (1), Section *2-4*,

$$P=\frac{s}{K_2}-\frac{K_1}{K_2}(\delta_e-\delta_{e1})$$

Assuming that the stick is held in the initial position, we obtain

$$\Delta P=\frac{\partial P}{\partial\delta_e}\cdot\Delta\delta_e=-\frac{K_1}{K_2}\cdot\Delta\delta_e$$

By the definition

$$\frac{\partial(H.M.)}{\partial P}=-\frac{1}{G}$$

Substituting the above relations into the left side of (2), and the expression $H.M.=q\eta_t S_e \bar{c}_e C_h$ into the right side,

$$\frac{K_1}{GK_2}\cdot\Delta\delta_e=q\eta_t S_e\bar{c}_e\left(\frac{\partial C_h}{\partial\alpha_s}\cdot\Delta\alpha_s+\frac{\partial C_h}{\partial\delta_e}\cdot\Delta\delta_e\right)$$

Rewriting the above equation, using the short-hand notations for $\frac{\partial C_h}{\partial\alpha_s}$ and $\frac{\partial C_h}{\partial\delta_e}$, we obtain

$$\frac{\Delta\delta_e}{\Delta\alpha_s}=-\frac{C_{h_{ct}}}{C_{h_\delta}}\left(\frac{1}{1-\frac{K_1}{GK_2}\cdot\frac{1}{q\eta_t S_e\bar{c}_e C_{h_\delta}}}\right)$$

Denoting the parenthesized term by $\frac{1}{K}$,

$$\frac{\Delta\delta_e}{\Delta\alpha_s}=-\frac{1}{K}\cdot\frac{C_{h\alpha t}}{C_{h\delta}} \tag{3}$$

where

$$K=1-\frac{K_1}{GK_2}\cdot\frac{1}{q\eta_t S_e\bar{c}_e C_{h\delta}} \tag{4}$$

The equation (3) represents the ratio of the change of floating angle of the elevator to that of the angle of attack of the horizontal tail during the motion of an airplane, when the control stick is fixed. The equation (4) shows that K is always positive and larger than unity, unless $C_{h\delta}>0$ or the elevator is aerodynamically over-balanced.

It is well known that the factor which acts to change the static longitudinal stability, when the elevator is freed, is the floating tendency of the elevator caused by the pitching motion of the airplane, and that the floating tendency of the elevator is expressed by the following equation:

$$\frac{\Delta\delta_e}{\Delta\alpha_s}=-\frac{C_{h\alpha t}}{C_{h\delta}} \tag{5}$$

Comparing (3) and (5), and remembering that the equation (1) represents the change of the static longitudinal stability due to the floating tendency of the elevator when it is freed, we can analogize that the change of the static longitudinal stability due to the flexibility of the control system, when the stick is fixed, is expressed by:

$$\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Elevator-}\\\text{Fixed}}}-\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Stick-}\\\text{Fixed}}}=\frac{C_{h\alpha t}}{C_{h\delta}}\cdot\frac{\tau_e}{K}\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Tail Contr.}\\\delta e=0}}$$

From now on we must distinguish "stick-fixed" from "control surface-fixed", when we take into account the flexibility of the control system.

It can be deduced, that the fact that the contribution of the horizontal tail to $\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Complete}\\ \text{Airplane}}}$ is reduced by $\frac{C_{h\alpha t}}{C_{h\delta}}\cdot\frac{\tau_e}{K}\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Tail.}\\ \text{Contr.}}}$ means that the tail contribution to other stability derivatives, such as C_{m_α}, $C_{m_{d\alpha}}$, and $C_{m_{d\theta}}$ for fixed-stick also are reduced by $\left(\frac{C_{h\alpha t}}{C_{h\delta}}\cdot\frac{\tau_e}{K}\right)\times 100$ per cent. This deduction will make a useful tool for the investigation of the longitudinal stability when the control system flexibility is taken into account. Fig. 4-4.1 shows $1-\frac{C_{h\alpha t}}{C_{h\delta}}\cdot\frac{\tau_e}{K}$ versus speed, taking the stiffness constant K_2 as a parameter. The influence of speed and of the stiffness constant on the decrement of the static longitudinal stability is very well displayed in this figure.

Influence of the Control System Stiffness on the Stick-Fixed Neutral Point

It is well known that the static longitudinal stability criterion $\frac{\partial C_m}{\partial C_L}$ decreases when the center of gravity of the airplane moves backward, that $\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Elevator-}\\ \text{Free}}}$ comes to zero for a certain $C.G.$ point, and that $\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Elevator}\\ \text{-Fixed}}}$ comes to zero for a certain $C.G.$ point further backward. The corresponding $C.G.$ points have been hitherto called "stick-free neutral point" and "stick-fixed neutral point" respectively. Hereafter they had better be re-named "elevator-free neutral point" and "elevator-fixed neutral point" respectively to avoid confusion. However, it should be remarked that the "elevator-fixed neutral point" does not exist in an actual airplane except for a hypothetical one.

If we denote the distances of the "neutral points", measured from the leading edge of the mean aerodynamic chord divided by the length of the latter, by N_0' and N_0 respectively, it is well known that the distance between them is approximately expressed by

$$N_0-N_0'=-\frac{C_{h\alpha t}}{C_{h\delta}}\cdot\tau_e\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Tail Contr.}\\ \delta e=0}} \qquad (7)$$

This is the same expression as that of $\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Free-Elevator}\\ \text{Effect}}}$ shown by (20), Section 2-4, or the right side of (1) in which the sign is reversed. It may be readily deduced that the distance between the hypothetical "elevator-fixed neutral point" and the "stick-fixed neutral point" for a finite stiffness can approximately be expressed by

$$N_0-N_0'=-\frac{C_{h\alpha t}}{C_{h\delta}}\cdot\frac{\tau_e}{K}\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Tail Contr.}\\ \delta e=0}} \qquad (8)$$

Numerical values of N_0-N_0' for the airplane A-1 are easily obtainable from Fig. 4-4.1 for a wide variey of K_2 for any value of $\left(\frac{\partial C_m}{\partial C_L}\right)_{\substack{\text{Tail Contr.}\\ \delta_e=0}}$.

4-5. *Dynamic Longitudinal Stability—Mode of Short-Period Oscillation*

The mode of long-period oscillation or a phugoid mode is understood to be unimportant because it has little correlation with pilots' evaluation of the flying qualities of an airplane. The short-period mode usually is heavily damped, and it is felt by the pilot only as a bump when a gust is encountered or in the longitudinal response of the airplane to an abrupt elevator control. It is also of very little consequence in itself, but there may be some who are apprehensive of the possibility of its coupling with the rotational vibration of the elevator system induced by an abrupt stick actuation. First numerical calculations are made of this mode in two cases, *i.e.* "hypothetical elevator-fixed" and "elevator-free"; and then from the results consideration is made on the effect of the reduced stiffness.

Elevator-Fixed Case

It is sufficient for the purpose stated above to approximately estimate the period and time to damp to half amplitude of the short-period oscillation. Excluding the phugoid mode out of the biquadratic characteristic equation by assuming that no change of speed occurs during the motion, the following quadratic equation is reached for the short-period mode, as for instance in ref. [6]:

$$\lambda^2-\left(\frac{1}{h}C_{m_{d\theta}}-\frac{1}{2}C_{L_\alpha}+\frac{1}{h}C_{m_{d\alpha}}\right)\lambda-\left(\frac{1}{h}C_{m_\alpha}+\frac{1}{h}C_{L_\alpha}C_{m_{d\theta}}\right)=0 \qquad (1)$$

where λ is in general a complex constant when the solutions of the original simultaneous equations of longitudinal motion in u, α, and θ are written as

$$\alpha=\bar{\alpha}e^{\lambda t/\tau}, \qquad \theta=\bar{\theta}e^{\lambda t/\tau}$$

and the other symbols are the same as those in Section *2-5*.

The roots of the equation (1) for a statically stable airplane in almost all cases are one complex pair. If we denote it by $\lambda=\xi+i\eta$, the period and damping of this mode of oscillation are:

$$\left.\begin{array}{lll}\text{Period} & T=\dfrac{2\pi}{\eta}\tau & \text{seconds}\\ \text{Time to damp to 1/2 amplitude} & T_{\frac{1}{2}}=\dfrac{0.693}{\xi}\tau & \text{seconds}\end{array}\right\} \qquad (2)$$

If use is made of the data of the airplane A-1 with *C.G.* at 26% MAC and flying at an altitude of 3000 m., which are given in Section *2-5*, the roots of the equation (1) are

$$\lambda=3.25\pm i\,4.88$$

Hence

$$\left.\begin{aligned} T &= \frac{148}{V} \quad \text{seconds} \\ T_{\frac{1}{2}} &= \frac{24.5}{V} \quad \text{seconds} \\ \text{where} \quad V &= \text{airplane speed in m/sec.} \end{aligned}\right\} \qquad (3)$$

Elevator-Free Case

The characteristic equation for the free elevator condition must be solved in order to investigate into this mode in detail. But it is well known that the solution of the equation (1), using stability derivatives modified for the free elevator condition, gives us approximate values of the period and time to damp to a half amplitude. These approximate values would sufficiently meet our present purpose. Those stability derivatives in (1) affected by freeing the elevator are C_{L_α}, C_{m_α}, C_{md_α}, and C_{md_θ}. The influence of freeing the elevator on C_{L_α} is to reduce it, but it is so small that C_{L_α} can be regarded to remain constant. The influence on C_{m_α} is almost equal to that on $\frac{\partial C_m}{\partial C_L}$, that is, to reduce it by $\frac{0.025}{0.140}\times 100\%$, as given in Section 2-4. The influence on C_{md_α} and C_{md_θ}, in which the tail contribution constitutes the major part, is to reduce them approximately by $\left(\frac{C_{h\alpha t}}{C_{h\delta}}\tau_e\right)\times 100\%$ from the fixed elevator condition, that is, to reduce them by $\frac{0.0028}{0.0100}\times 0.46\times 100\%$, also as given in Section 2-4. Thus for the free elevator condition,

$$C_{m_\alpha} = -0.531$$
$$C_{md_\alpha} = -0.0247$$
$$C_{md_\theta} = -0.0610$$

Solving the equation (1) with these data put into it, we obtain

$$\lambda = 2.98 \pm 4.44i$$

Hence

$$\left.\begin{aligned} T &= \frac{162.6}{V} \quad \text{seconds} \\ T_{\frac{1}{2}} &= \frac{26.7}{V} \quad \text{seconds} \end{aligned}\right\} \qquad (4)$$

The period and time to damp to 1/2 amplitude are calculated from (3) and (4), and shown in a tabular form as follows:

V (Kkt)	60	97.1	180	291.3	400	500
V (m/sec)		50		150		257.5

Elevator-fixed	T	4.79	2.96	1.60	0.99	0.72	0.57
	$T_{\frac{1}{2}}$	0.79	0.49	0.264	0.163	0.119	0.095
Elevator-free	T	5.26	3.25	1.75	1.08	0.79	0.63
	$T_{\frac{1}{2}}$	0.87	0.53	0.29	0.178	0.130	0.104

Reduced Stiffness of the Control System

The same stability derivatives as those affected by freeing the elevator are also affected by the flexibility of the control system. The rate of decrease of both C_m and $C_{m\alpha}$ due to the reduced stiffness is equal to the rate of decrease of $\frac{\partial C_m}{\partial C_L}$ due to the reduced stiffness, and the rate of decrease of $C_{m_{d\alpha}}$ and $C_{m_{d\theta}}$ is $\frac{C_{h_{\alpha t}}}{C_{h_\delta}} \cdot \frac{\tau_e}{K}$. It is sufficieut to indicate that the values of stability derivatives affected by the reduced stiffness lie between those for the fixed elevator and those for the free elevator. Therefore it is clear that the values of T, $T_{\frac{1}{2}}$, and other characteristics of the short-period mode of an airplane are between those for the two cases.

Coupling between the Short-Period Oscillation and the Elevator Rotational Vibration

An exact analyses of the mode of the short-period oscillation of the airplane and of the rotational vibration of the elevator system, as affected by their mutual interaction, if any, are possible by solving the simultaneous equations of motion in α, θ, and δ_e (*e.g.* (10-108) in ref. [*6*]) or a biquadratic equation in λ (*e.g.* (10-109) in ref. [*6*]). An approximate solution of the said biquadratic equation in λ, with the data of the same example put into it, gives us the periods and times to damp to half amplitude which are not very much different from those given in the preceding paragraph of this section and those in Section *4-3,* respectively, unless the *C.G.* of the elevator is far behind the hinge of the elevator. This is nothing but the confirmation that there is no possibility of the mutual interaction aggravating each other, or coupling of the two modes, so long as the airplane complies to the conditions therein set forth.

Brief Investigation into Faster and Heavier Airplanes

Numerical calculations of the period and time to damp to half amplitude of the short-period oscillation for the fixed elevator are made of a hypothetical example, whose data required for the calculations and different from those for the *A-1* are shown below:

W, μ	~ 2.3	times those for the A-1
$S_t l_t^2$, $C_{m\alpha}$	~ 0.70 to 0.75	times those for the A-1
$C_{m_{d\alpha}}$, $C_{m_{d\theta}}$	$\sim \frac{0.70 \sim 0.75}{2.3}$	times those for the A-1

h	$\sim\frac{1}{2.3}$	times those for the A-1
high speed	~ 1.8	times those for the A-1

The results show that the period is roughly equal to, and the time to damp to half amplitude is a little less than twice that of the A-1, respectively. It must be noted that the results are applicable to normally designed airplanes which are free from such phenomena as compressibility effect, etc.

Acknowledgement

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The last two diagrams of the Appendixes are reproduced, one each from the ref. [*28*] and the ref. [*32*] respectively, to which the author wishes to express his indebtedness.

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January 30, 1965

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CURVES

ILLUSTRATING THE RESULTS OF NUMERICAL CALCULATIONS

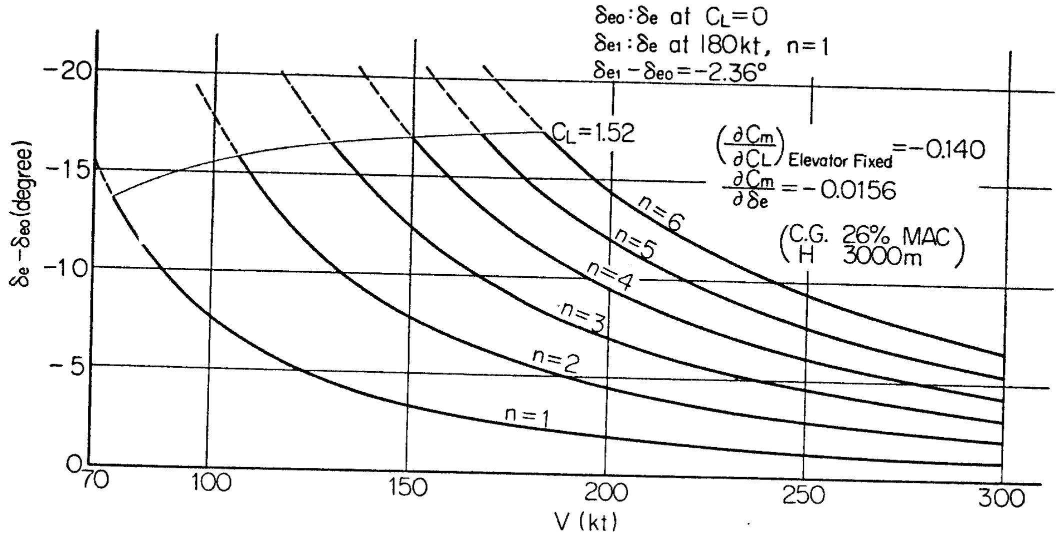

FIGURE 2-4.1. Elevator Angle required in Pul-Up.

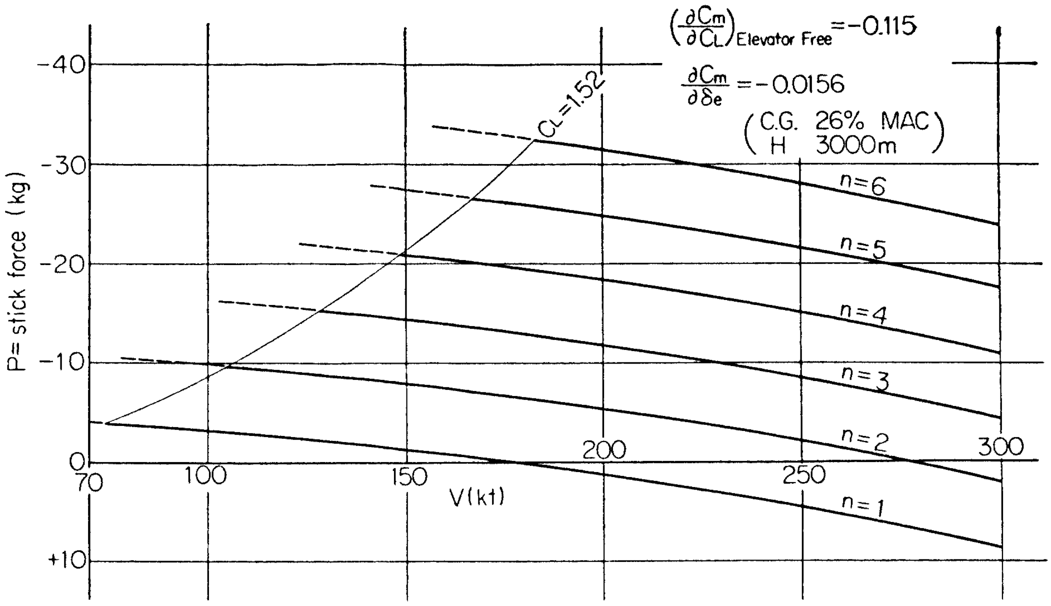

FIGURE 2-4.2. Stick Force required in Pull-Up.

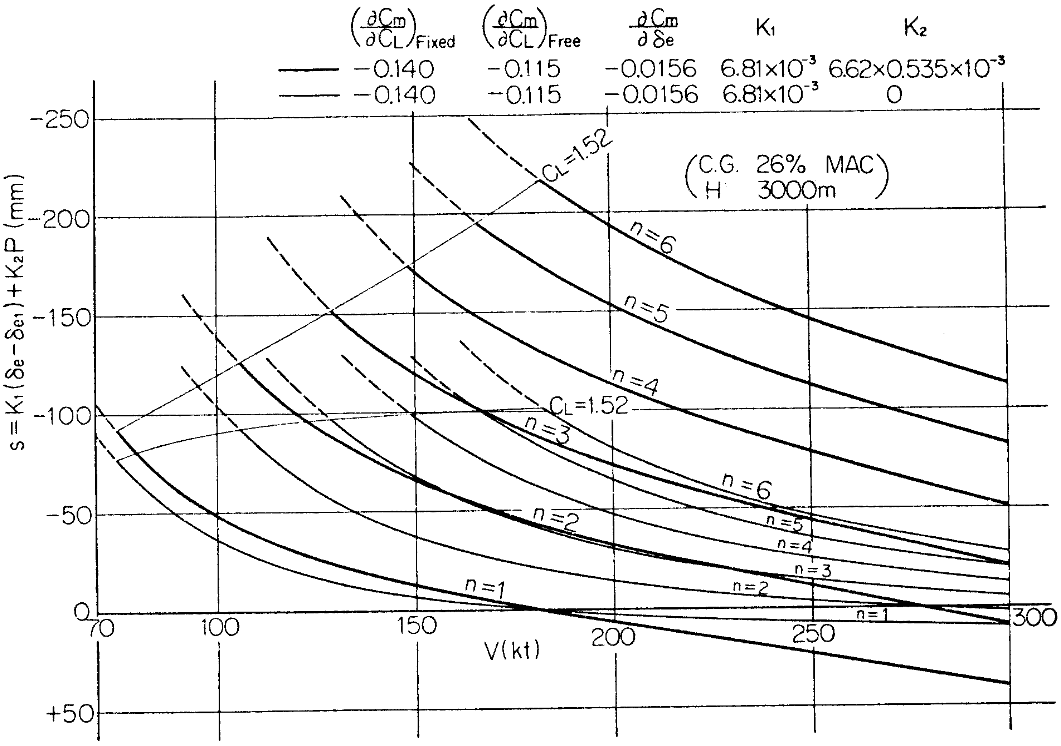

FIGURE 2-4.3. Stick Travel required in Pull-Up.

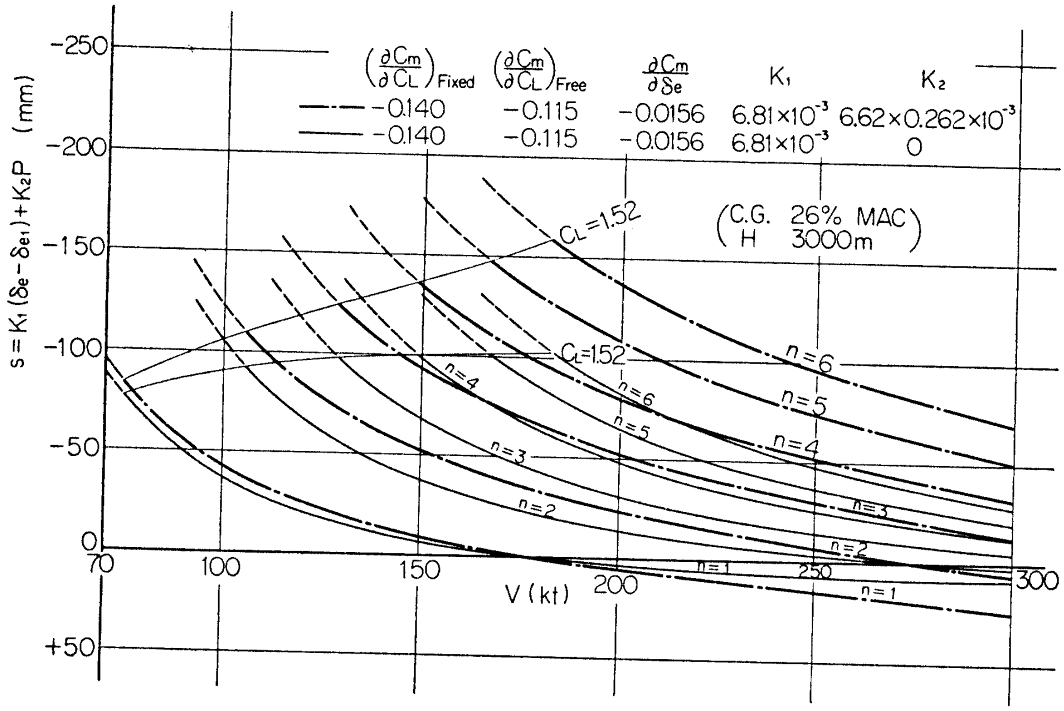

FIGURE 2-4.4. Stick Trave required in Pull-Up.

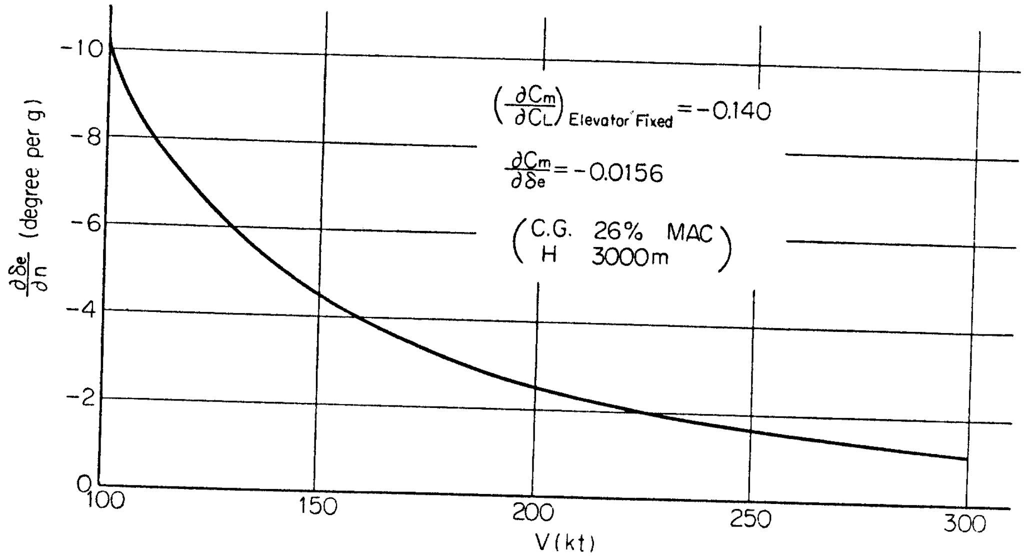

FIGURE 2-4.5. Elevator Angle per g in Pull-Up.

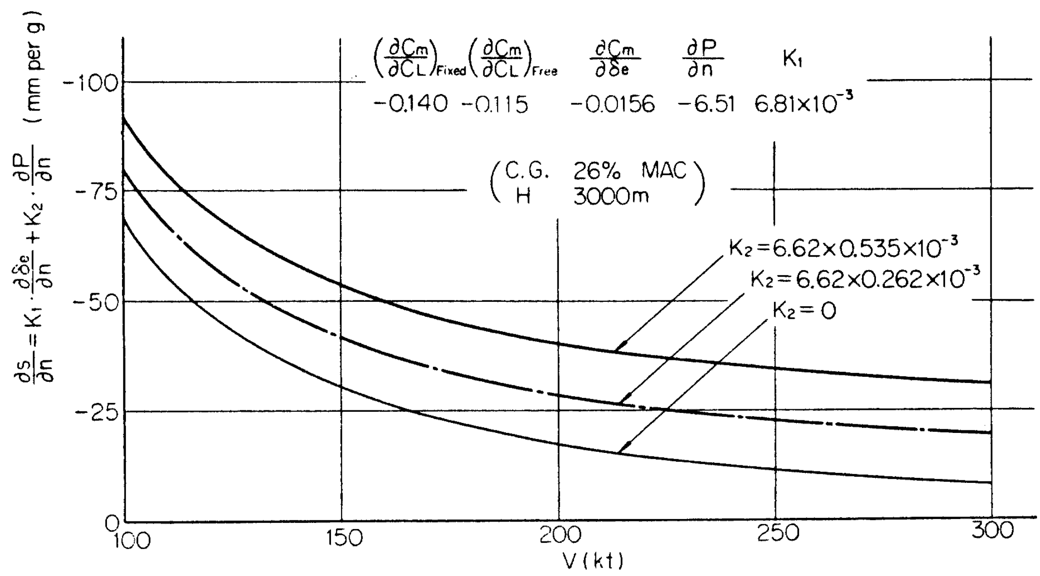

FIGURE 2-4.6. Stick Travel per g in Pull-Up.

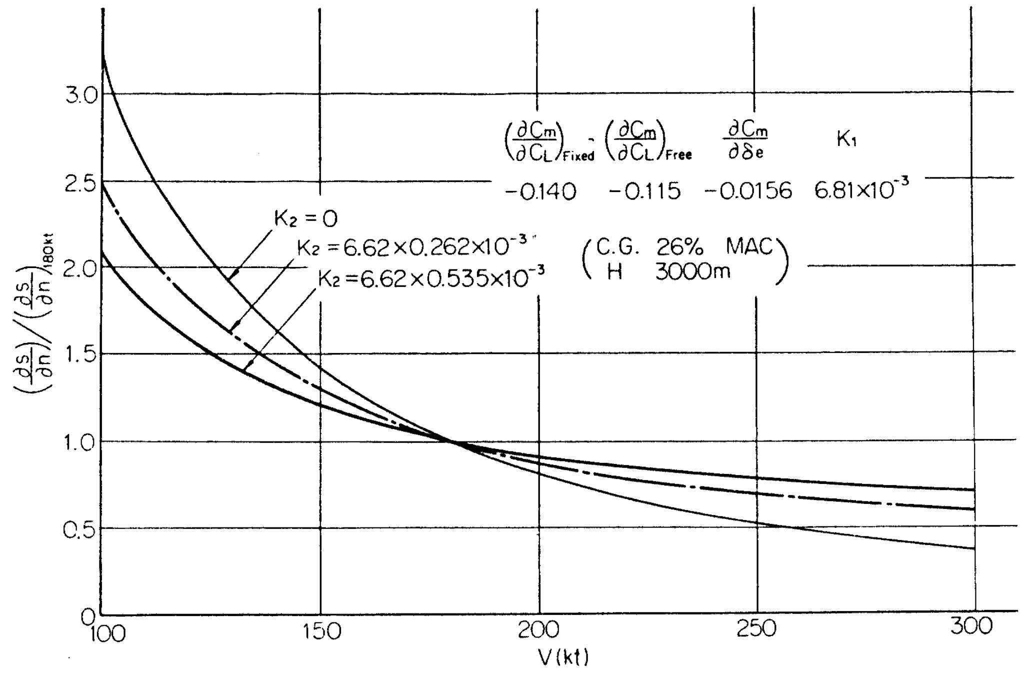

FIGURE 2-4.7. Ratio of Stick Travel per g in Pull-Up.

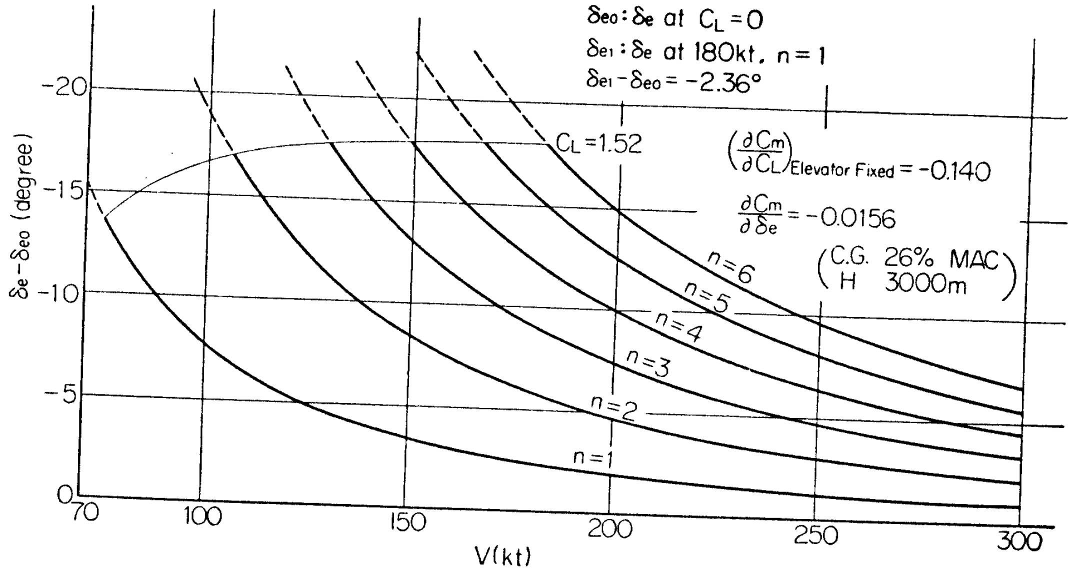

FIGURE 2-4.8. Elevator Angle required in Coordinated Turn.

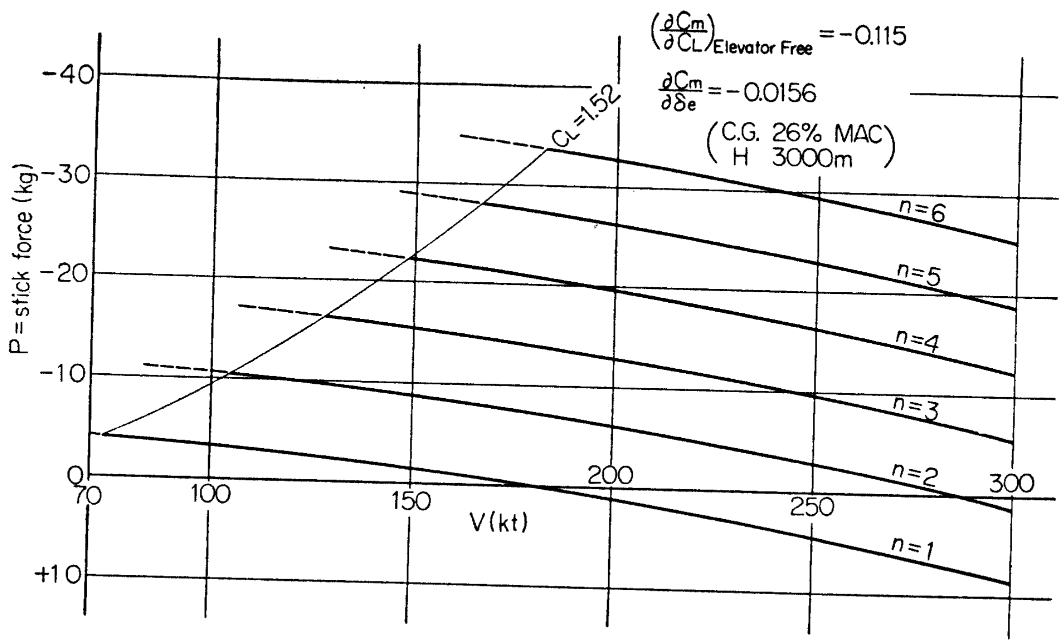

FIGURE 2-4.9. Stick Force required in Coordinated Turn.

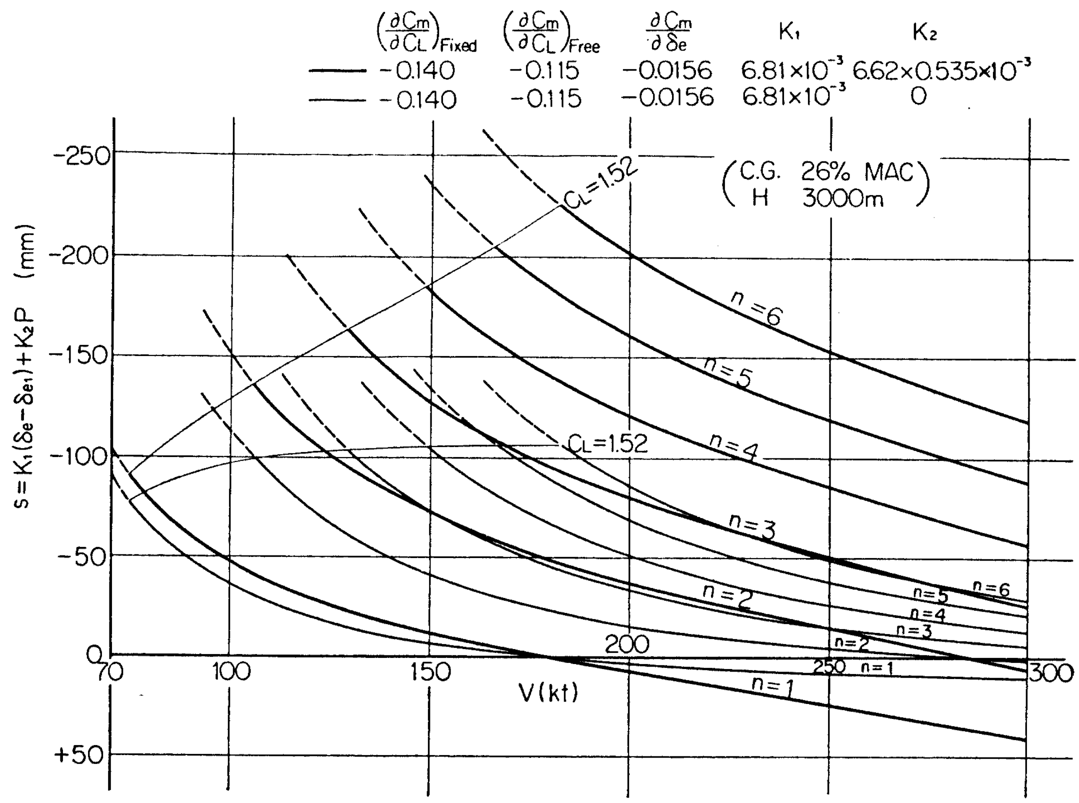

FIGURE 2-4.10. Stick Travel required in Coordinated Turn.

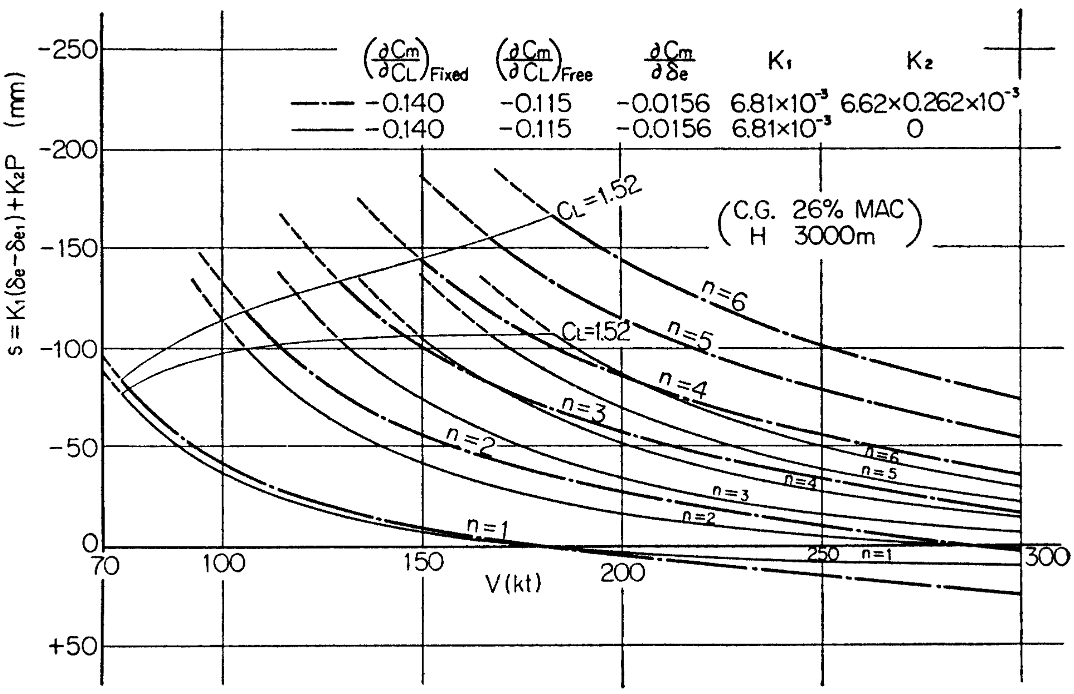

FIGURE 2-4.11. Stick Trauel reguired in Coordinated Turn.

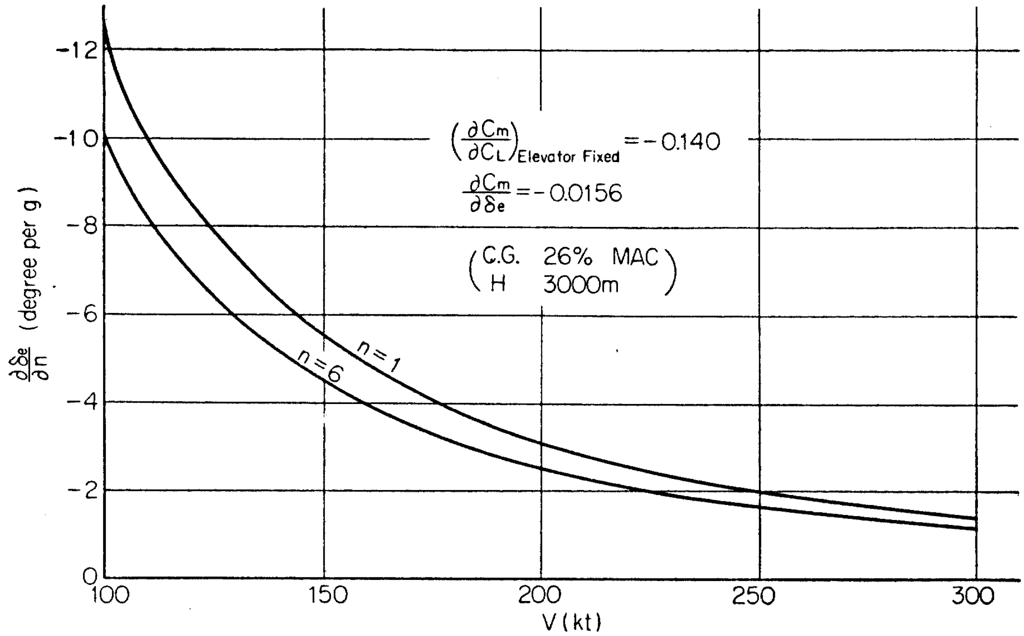

FIGURE 2-4.12. Elevator Angle per g in Coordinated Turn.

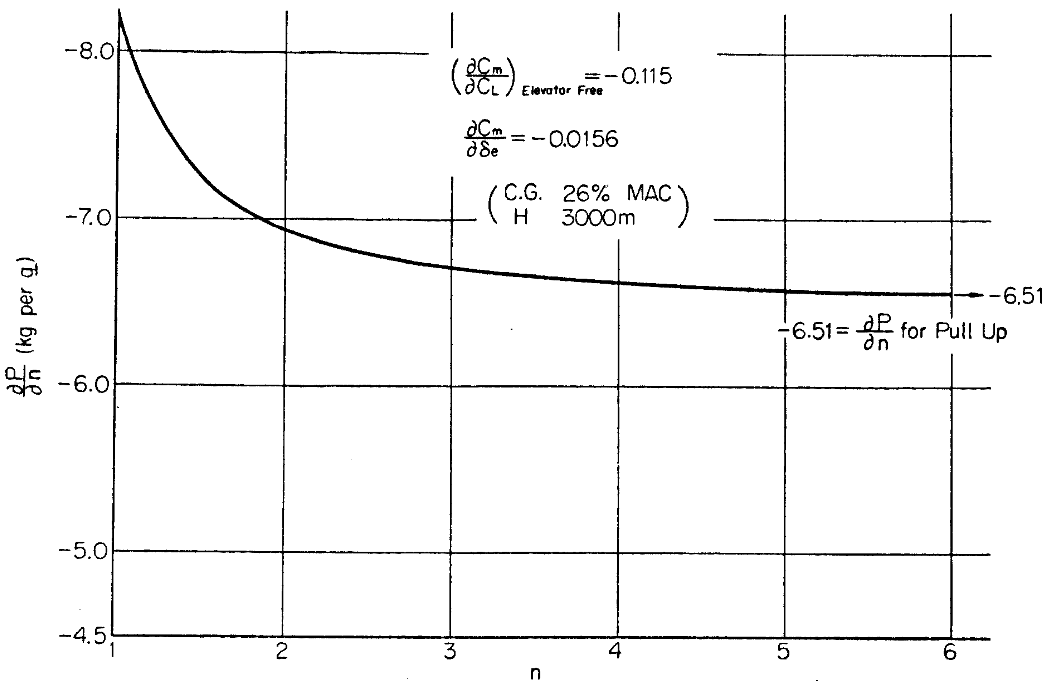

FIGURE 2-4.13. Stick Force per g in Coordinated Turn.

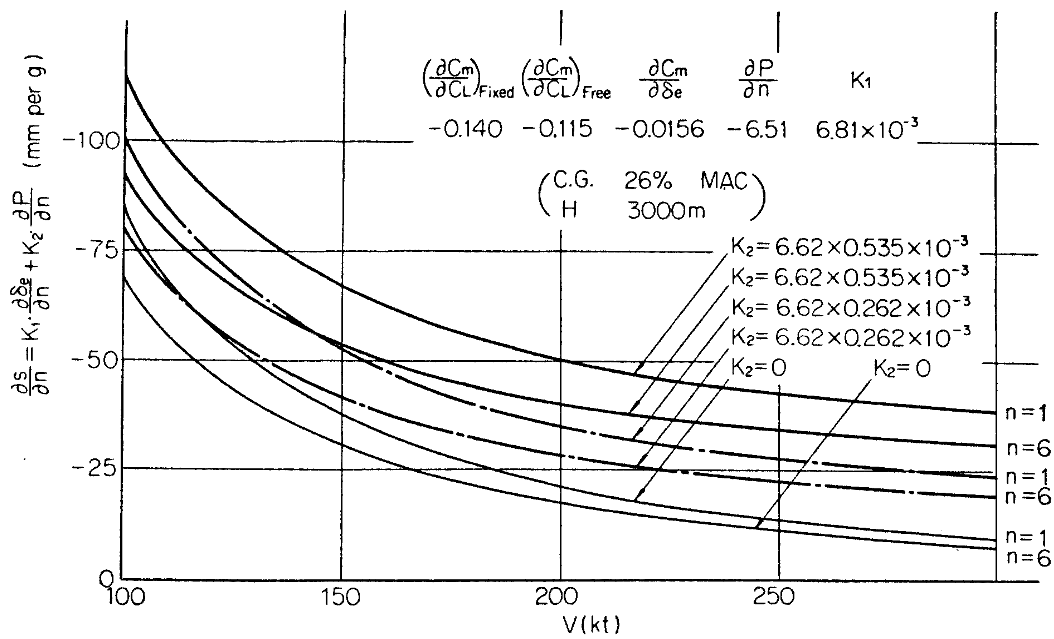

FIGURE 2-4.14. Stick Travel per g in Coordinated Turn.

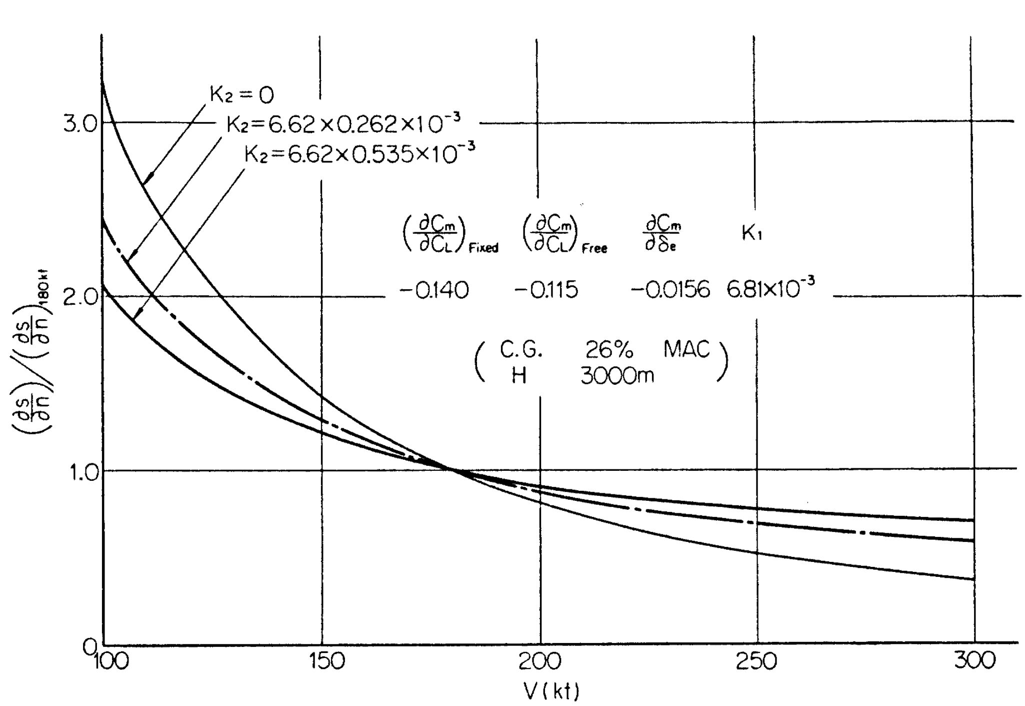

FIGURE 2-4.15. Ratio of Stick Travel per g in Coordinated Turn.

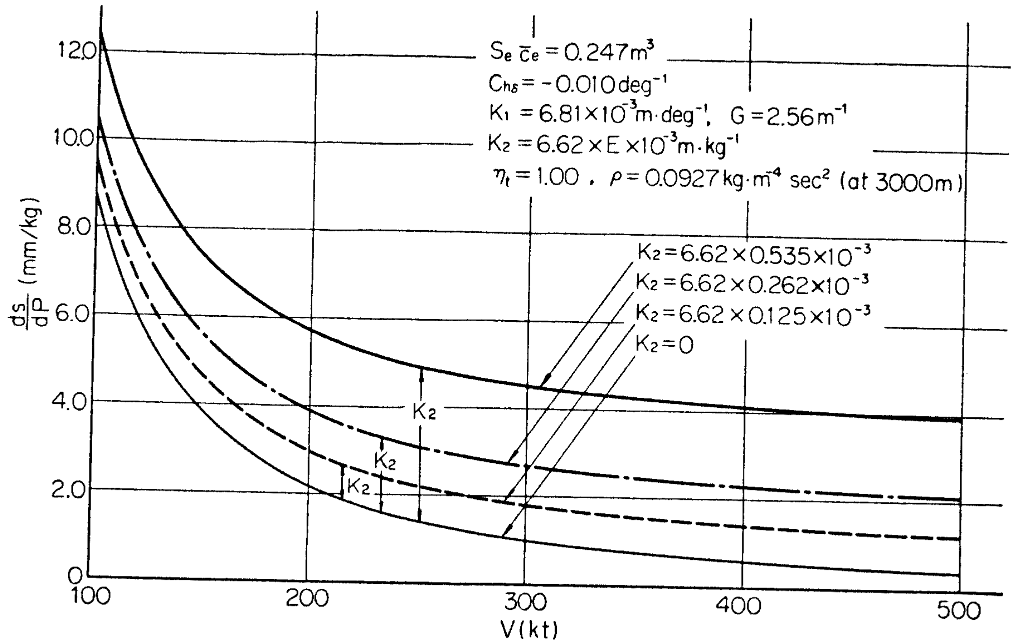

FIGURE 2-4.16. Stick Travel per Stick Force in Steady Maneuver.

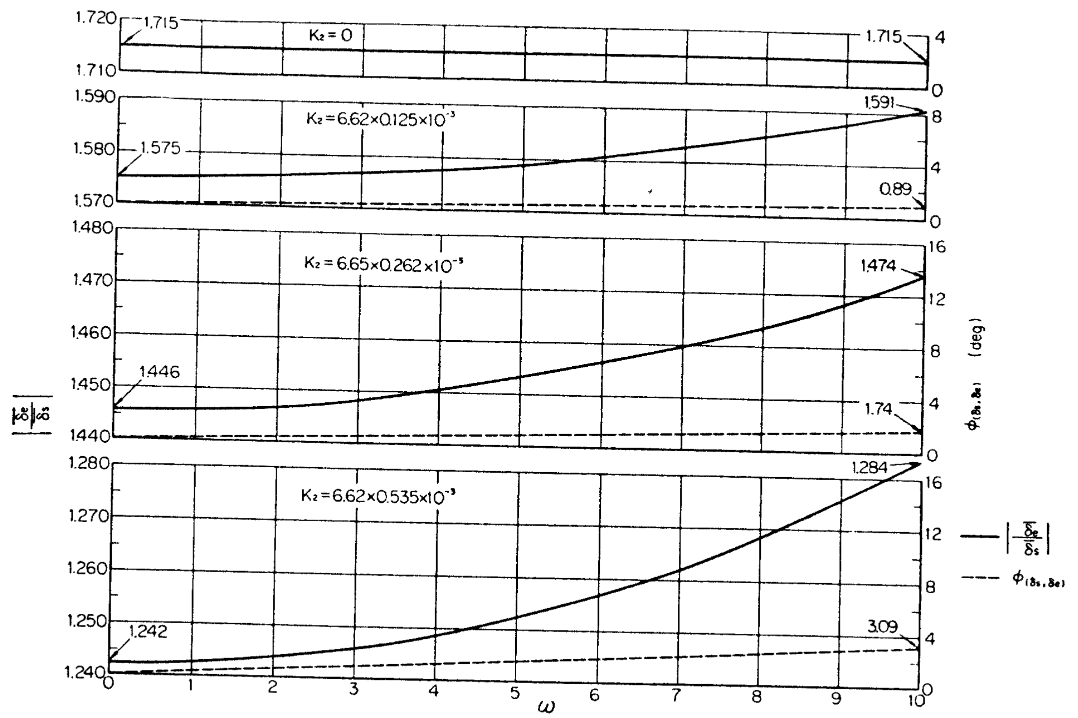

FIGURE 2-5.1. FREQUENCY RESPONSE.
Input : Stick Displacement $\bar{\delta}_s e^{i\omega t}$, Output : Elevator Angle $\bar{\delta}_e e^{i\omega t}$
$V = 50$ m/sec

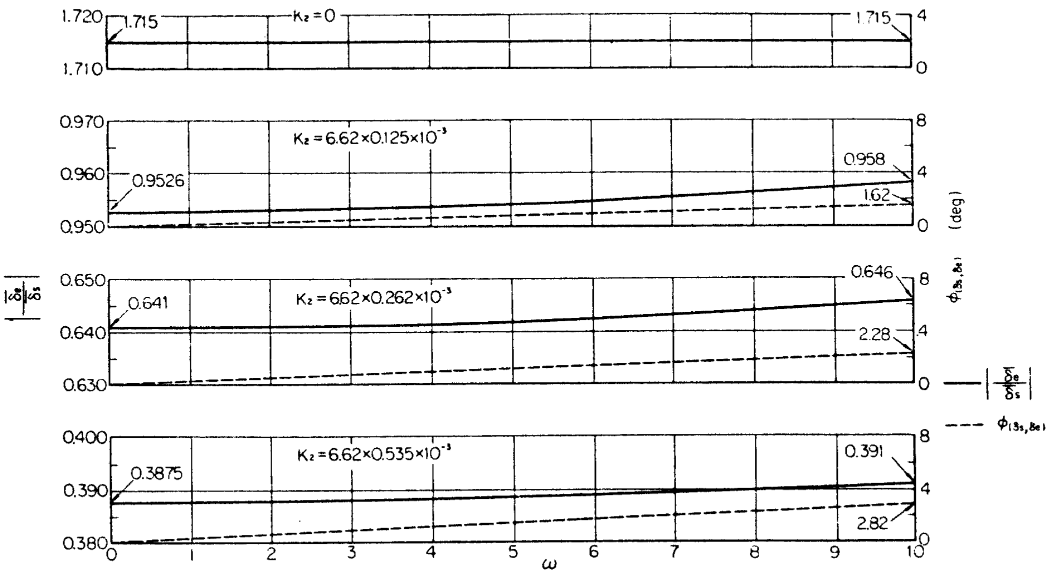

FIGURE 2-5.2. FREQUNCY RESPONSE.

Input: Stick Displacement $\bar{\delta}_s e^{i\omega t}$, Output: Elevator Angle $\bar{\delta}_e e^{i\omega t}$

$V=150$ m/sec

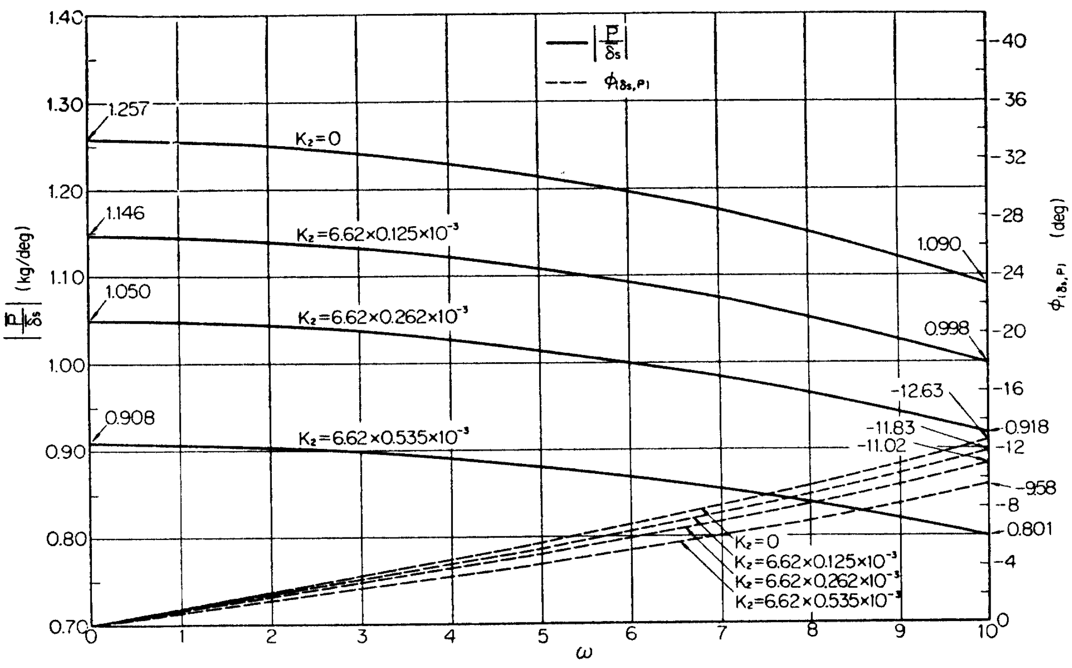

FIGURE 2-5.3. FREQUENCY RESPONSE.

Input: Stick Displacement $\bar{\delta}_s e^{i\omega t}$, Output: Stick Force $\bar{P} e^{i\omega t}$

$V=50$ m/sec.

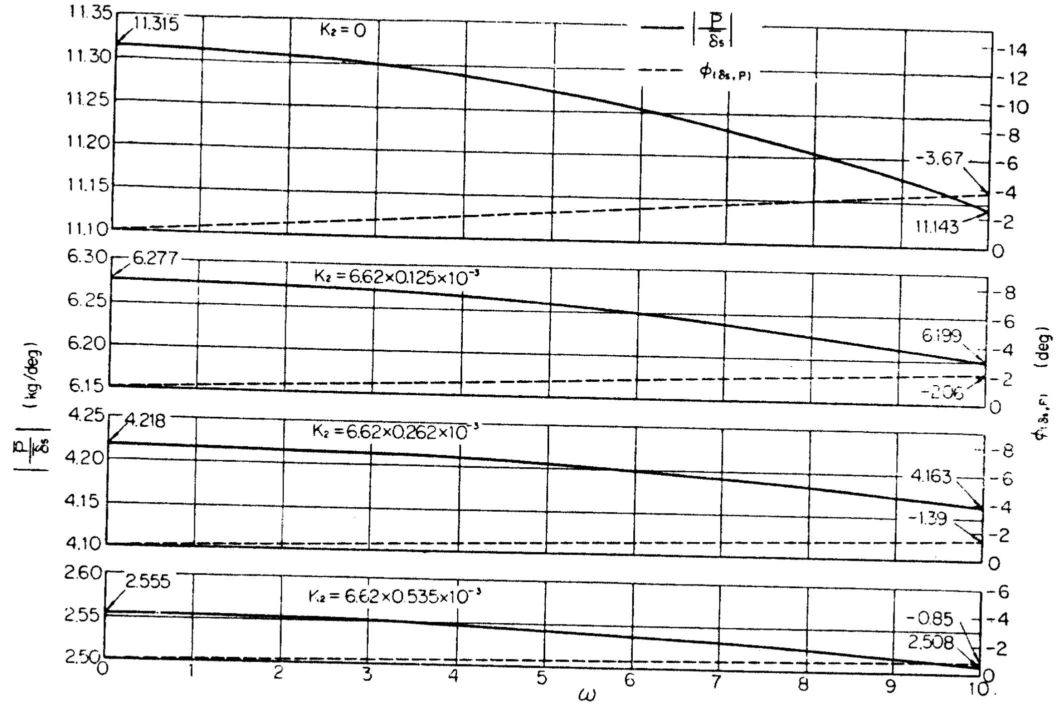

FIGURE 2-5.4. FREQUENCY RESPONSE.

Input: Stick Displacement $\delta_s e^{i\omega t}$, Output: Stick Force $\bar{P}e^{i\omega t}$

$V = 150$ m/sec.

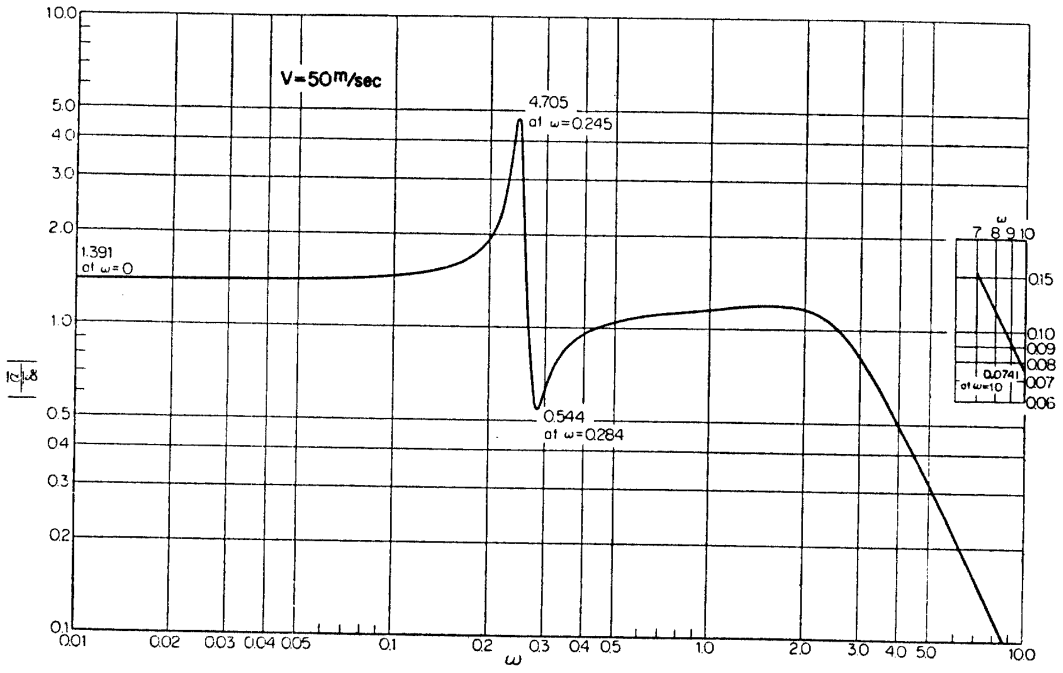

FIGURE 2-5.5. FREQUENCY RESPONSE.

Input: Elevator Angle $\bar{\delta}_e e^{i\omega t}$, Output: Angle of Attack $\bar{\alpha}e^{i\omega t}$

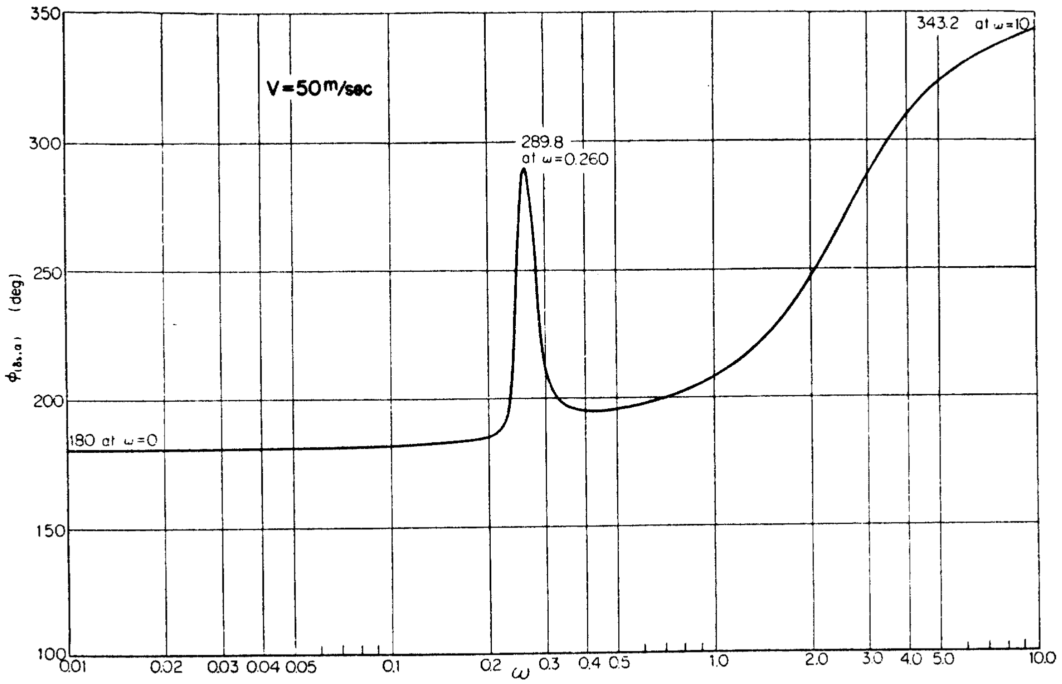

FIGURE 2-5.6. FREQUENCY RESPONSE.
Input: Elevator Angle $\bar{\delta}_e e^{i\omega t}$, Output: Angle of Attack $\bar{\alpha} e^{i\omega t}$

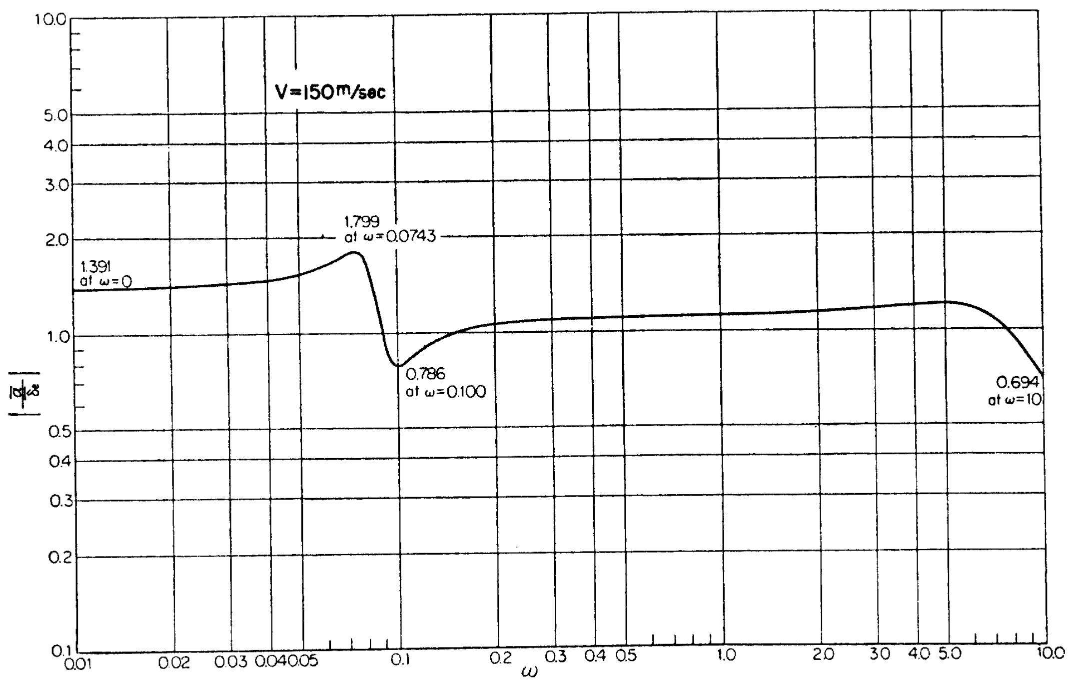

FIGURE 2-5.7. FREQUENCY RESPONSE.
Input: Elevator Angle $\bar{\delta}_e e^{i\omega t}$, Output: Angle of Attack $\bar{\alpha} e^{i\omega t}$

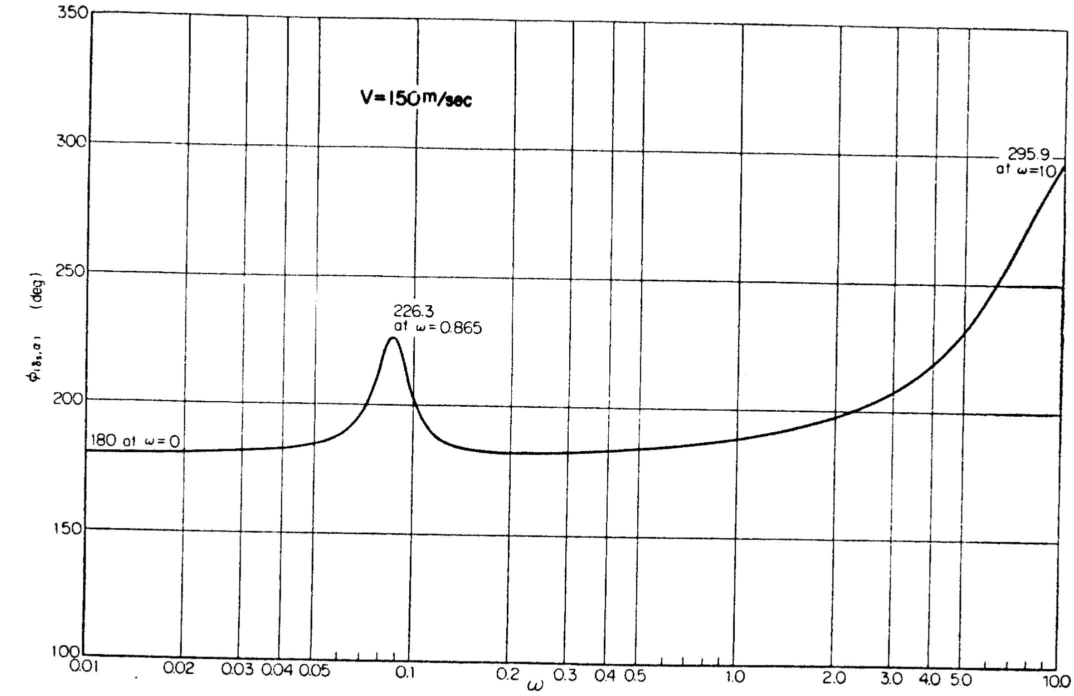

FIGURE 2-5.8. FREQUENCY RESPONSE.

Input: Elevator Angle $\bar{\delta}_e e^{i\omega t}$, Output: Angle of Attack $\bar{\alpha} e^{i\omega t}$

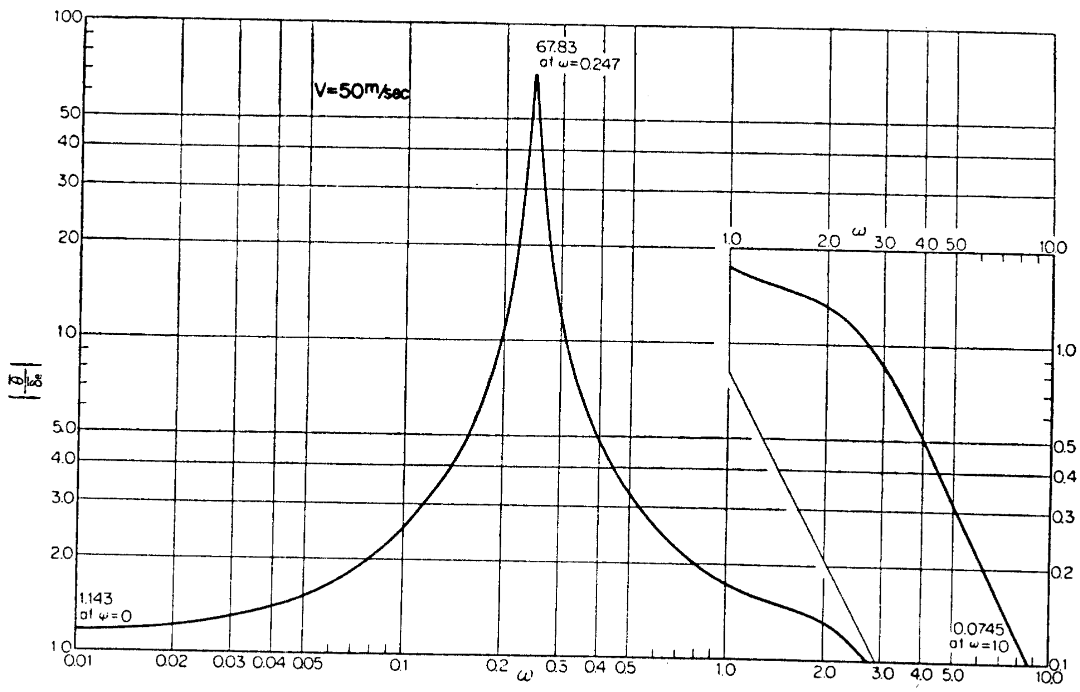

FIGURE 2-5.9. FREQUENCY RESPONSE.

Input: Elevator Angle $\bar{\delta}_e e^{i\omega t}$, Output: Angle of Pitch $\bar{\theta} e^{i\omega t}$

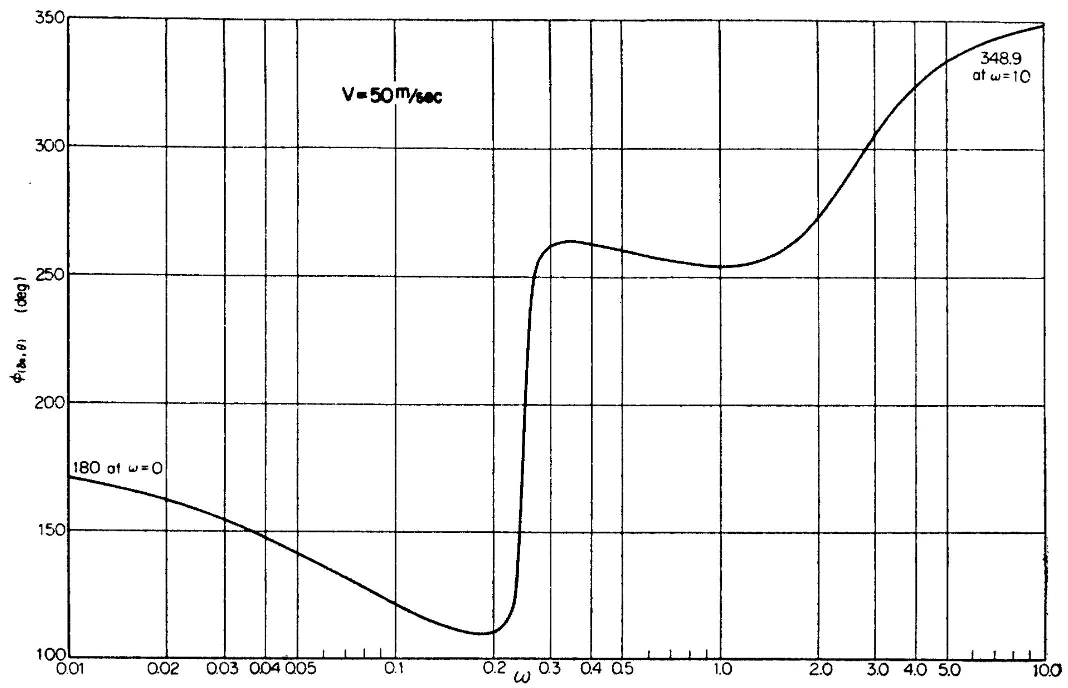

FIGURE 2-5.10. FREQUENCY RESPONSE.

Input: Elevator Angle $\bar{\delta}_e e^{i\omega t}$, Output: Angle of Pitch $\bar{\theta} e^{i\omega t}$

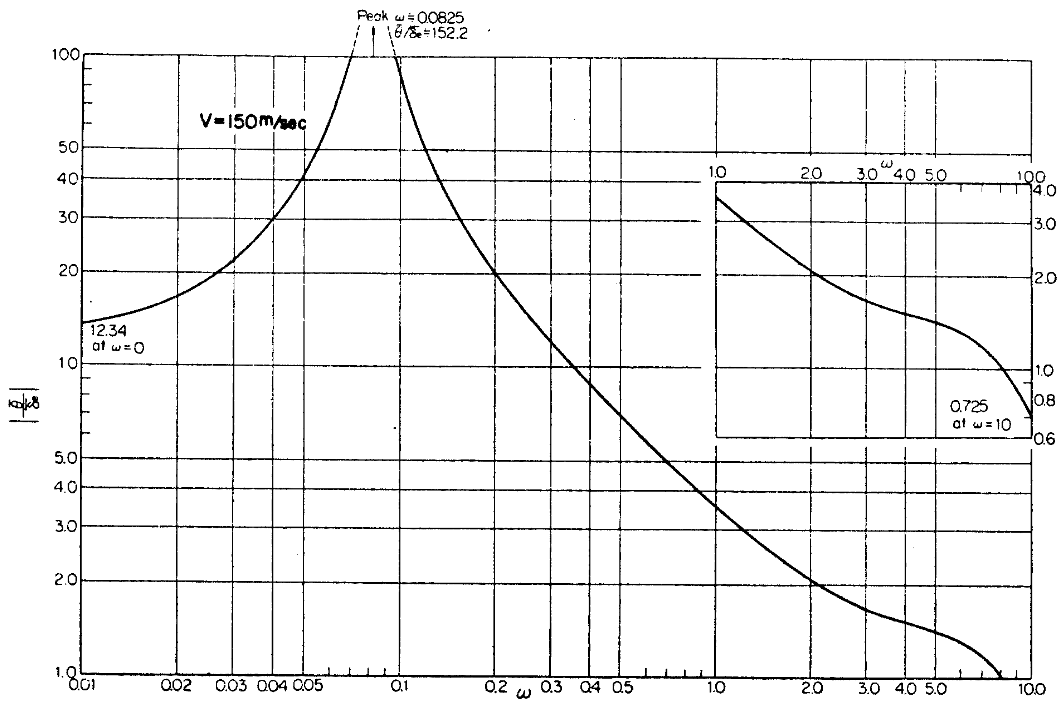

FIGURE 2-5.11. FREQUENCY RESPONSE.

Input: Elevator Angle $\bar{\delta}_e e^{i\omega t}$, Output: Angle of Pitch $\bar{\theta} e^{i\omega t}$

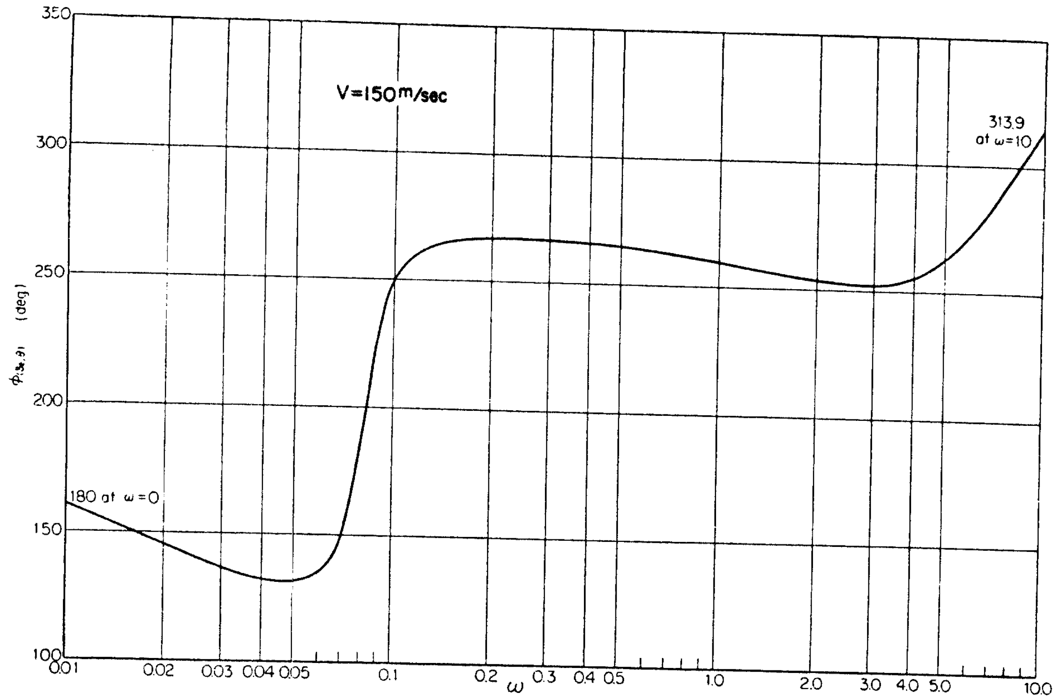

FIGURE 2-5.12. FREQUENCY RESPONSE.
Input: Elevator Angle $\bar{\delta}_e e^{i\omega t}$, Output: Angle of Pitch $\bar{\theta} e^{i\omega t}$

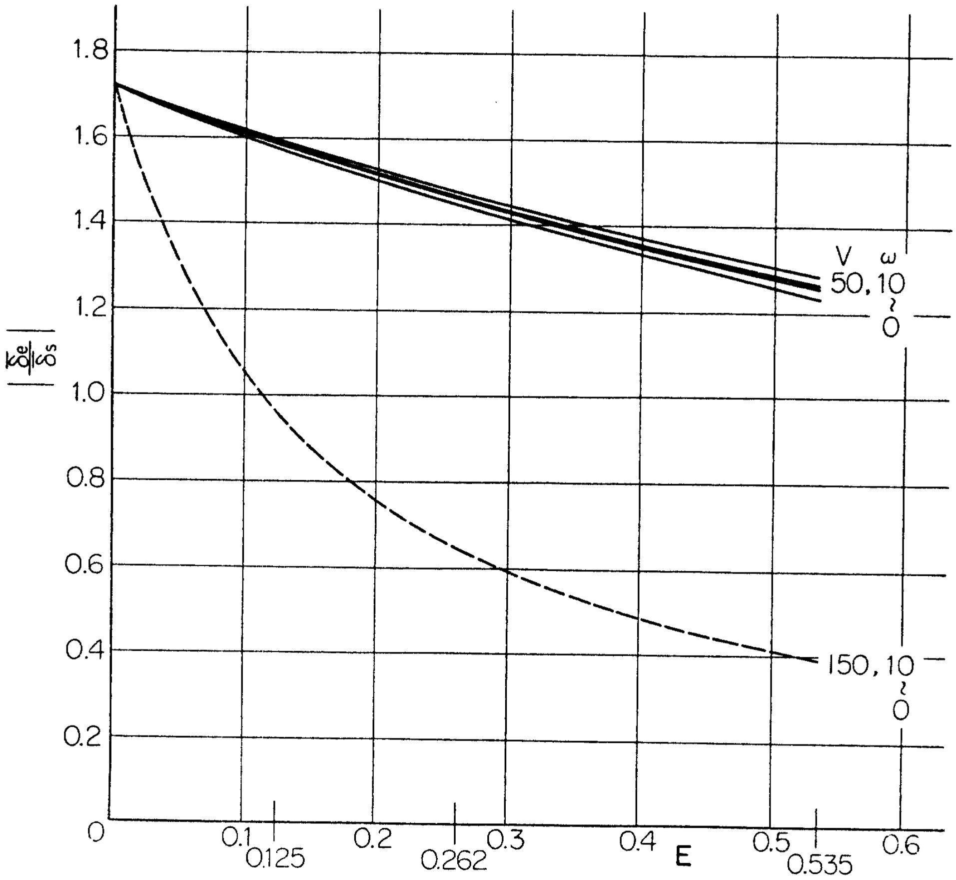

FIGURE 2-5.13. FREQUENCY RESPONSE.
Input: Stick Displacement $\bar{\delta}_s e^{i\omega t}$, Output: Elevator Angle $\bar{\delta}_e e^{i\omega t}$

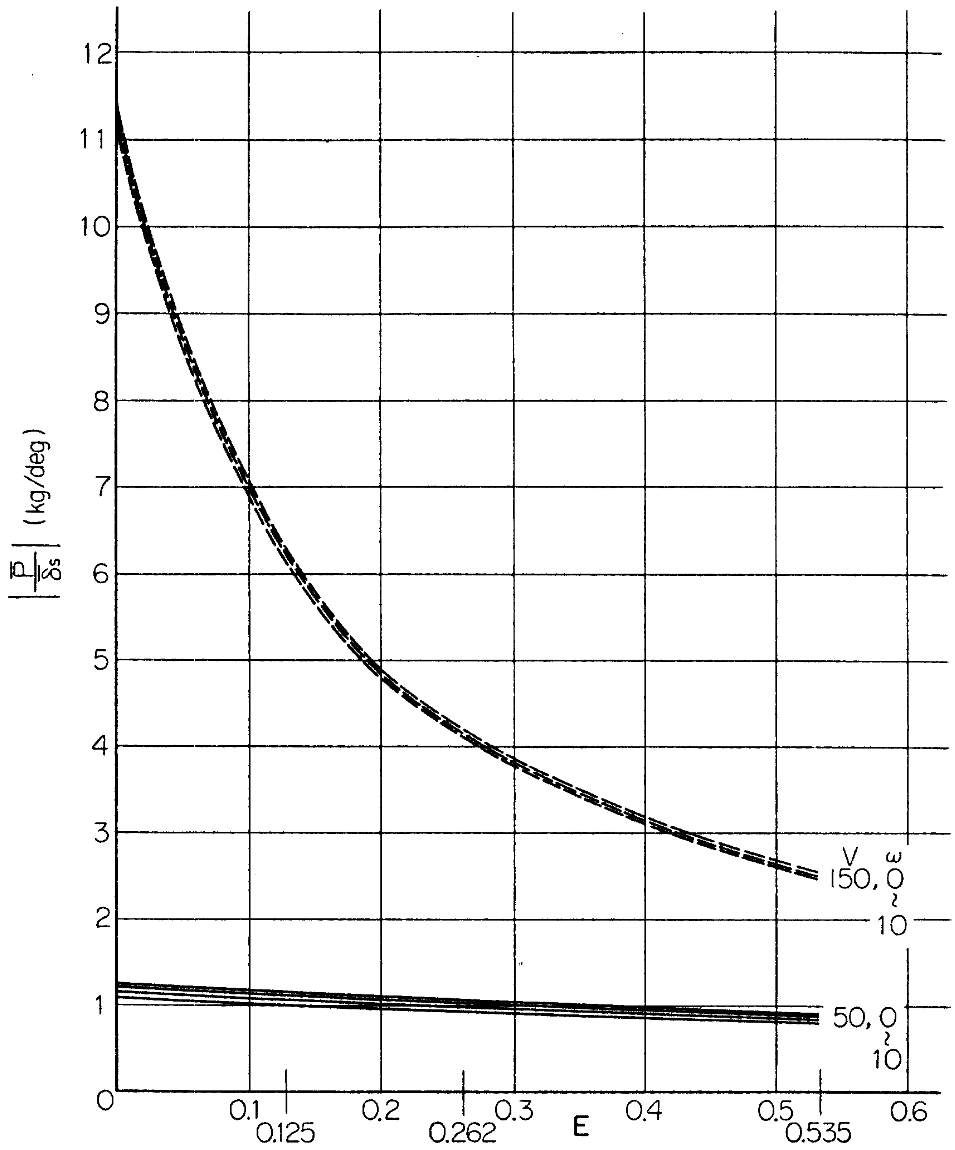

FIGURE 2-5.14. FREQUCY RESPONSE.

Input : Stick Displacement $\bar{\delta}_s e^{i\omega t}$, Output : Stick Force $\bar{P}e^{i\omega t}$

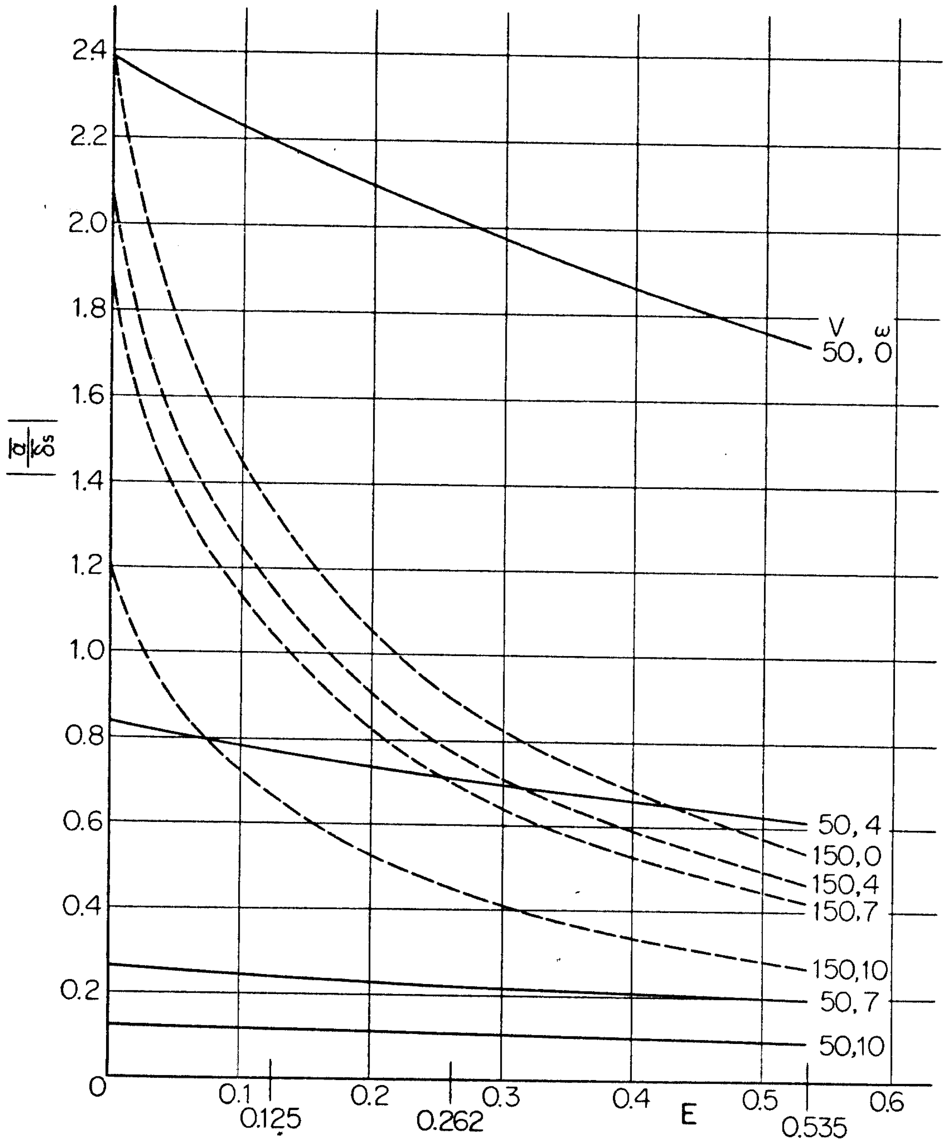

FIGURE 2-5.15. FREQUENCY RESPONSE.
Input: Stick Displacement $\bar{\delta}_s e^{i\omega t}$, Output: Angle of Attack $\bar{\alpha} e^{i\omega t}$

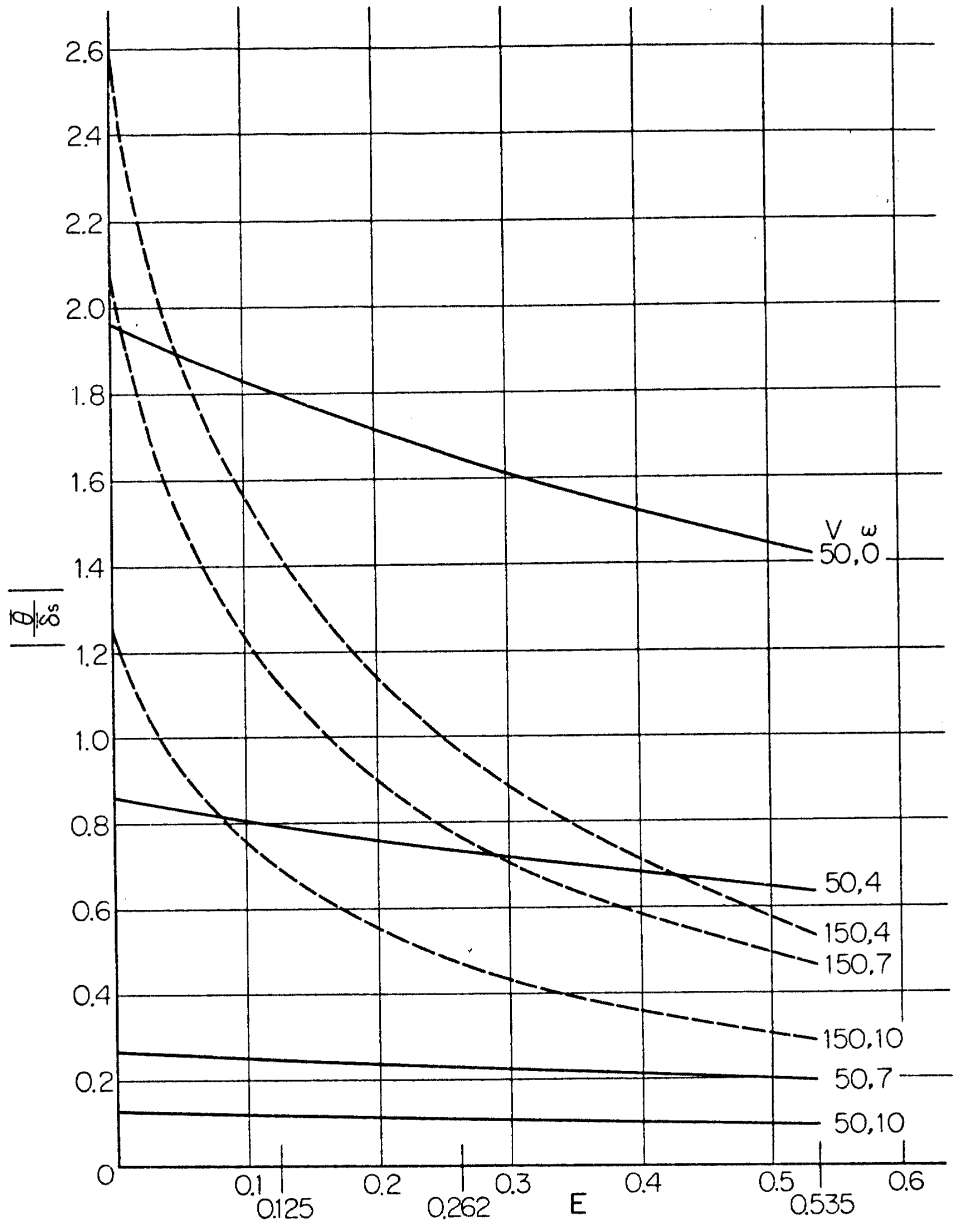

FIGURE 2-5.16. FREQUENCY RESPONSE.
Input: Stick Displacement $\bar{\delta}_s e^{i\omega t}$, Output: Angle of Pitch $\bar{\theta} e^{i\omega t}$

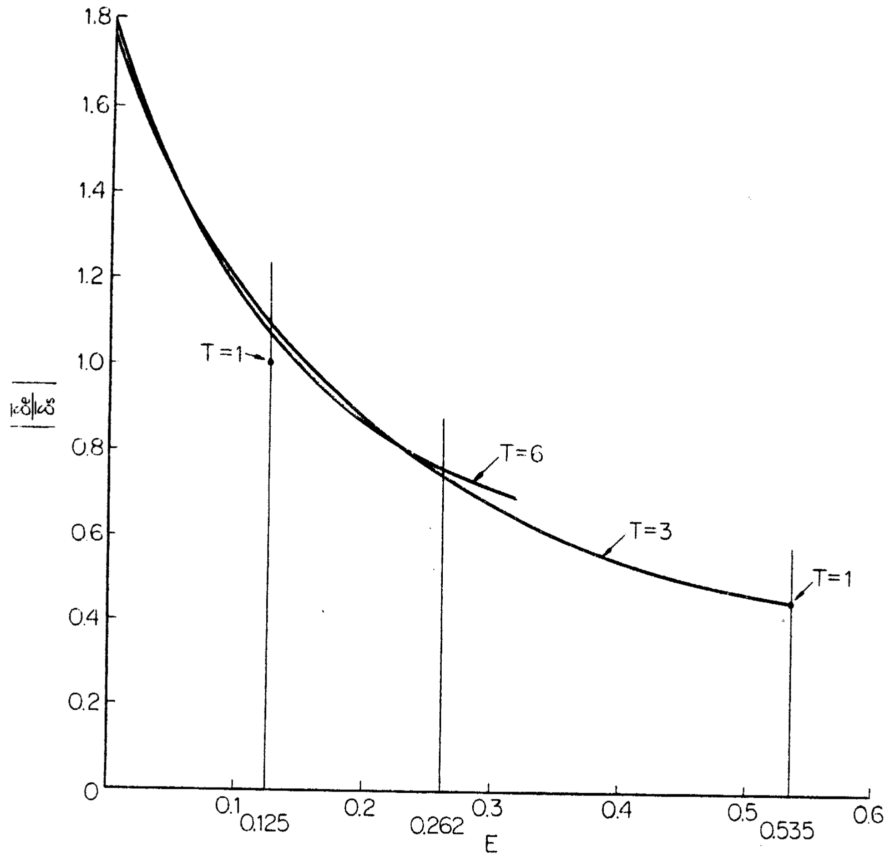

FIGURE 2-5.17. RESPONSE IN INITIAL STAGE OF LONGITUDINAL CONTROL, $V=150$ m/sec.

Input : Stick Displacement $\delta_s(t)=\frac{\bar{\delta}_s}{2}\left(1-\cos 2\pi\frac{t}{T}\right)$

Output : Elevator Angle $\bar{\delta}_e$=Peak Value

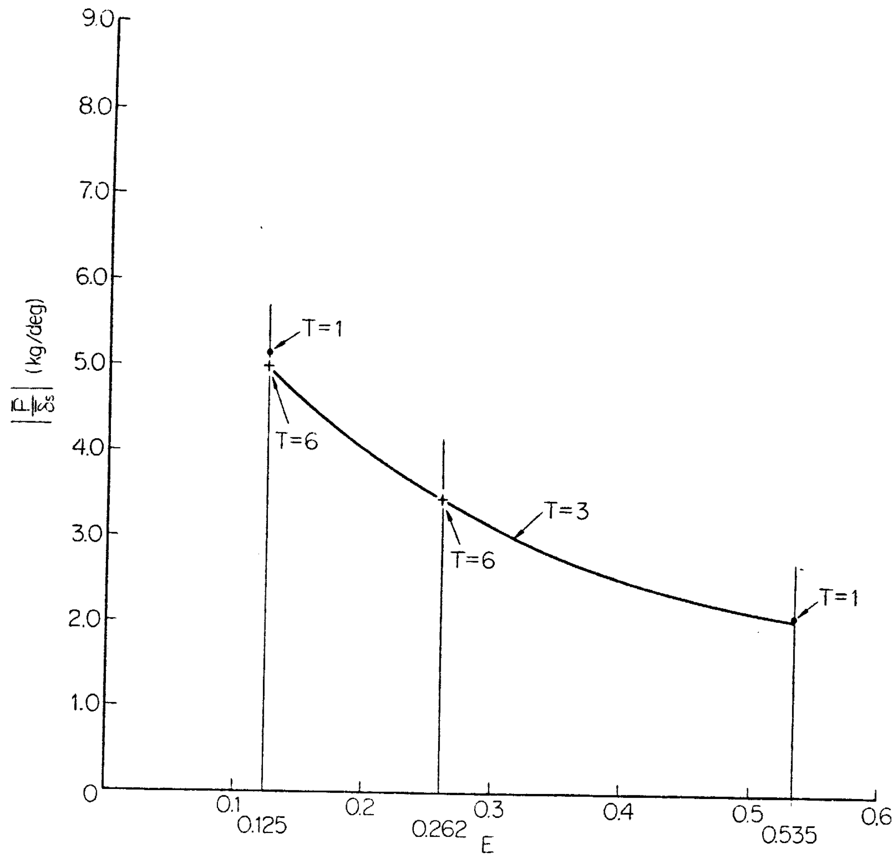

FIGURE 2-5.18. RESPONSE IN INITIAL STAGE OF LONGITUDINAL CONTROL, $V=150$ m/sec.

Input : Stick Displacement $\delta_s(t)=\frac{\bar{\delta}_s}{2}\left(1-\cos\frac{2\pi t}{T}\right)$

Output : Stick Forse $\bar{P}$=Peak Value

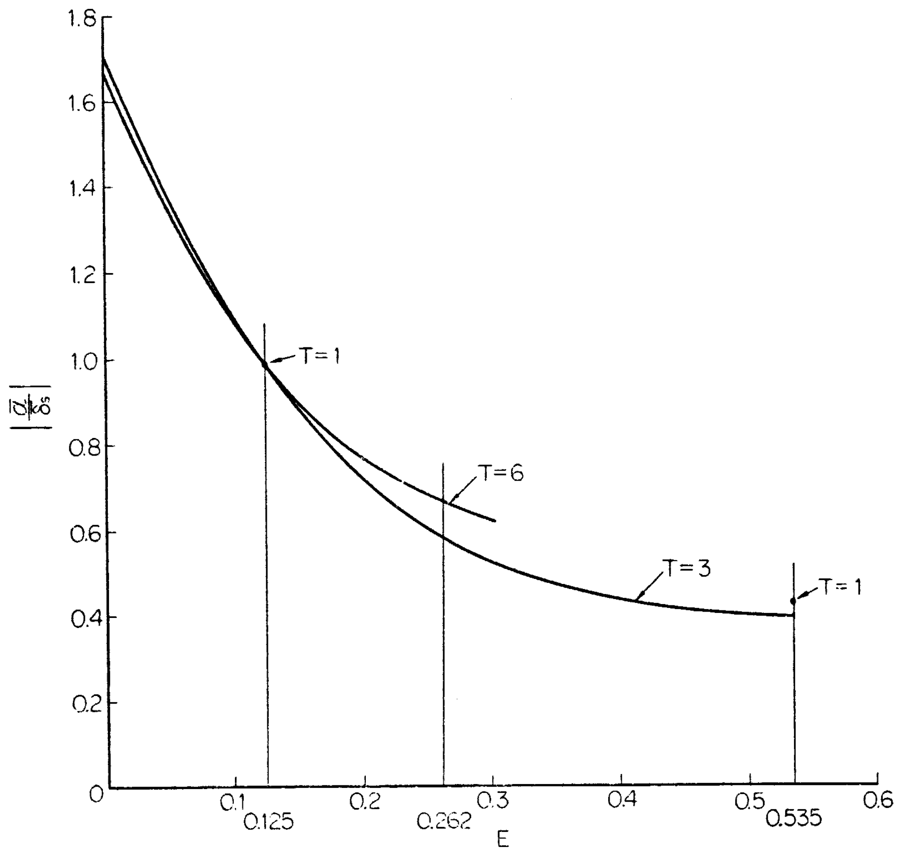

FIGURE 2-5.19. RESPONSE IN INITIAL STAGE OF LONGITUDINAL CONTROL, $V=150$ m/sec.

Input : Stick Displacement $\delta_s(t)=\frac{\bar{\delta}_s}{2}\left(1-\cos\frac{2\pi t}{T}\right)$

Output: Angle of Attack $\bar{\alpha}$=Peak Value

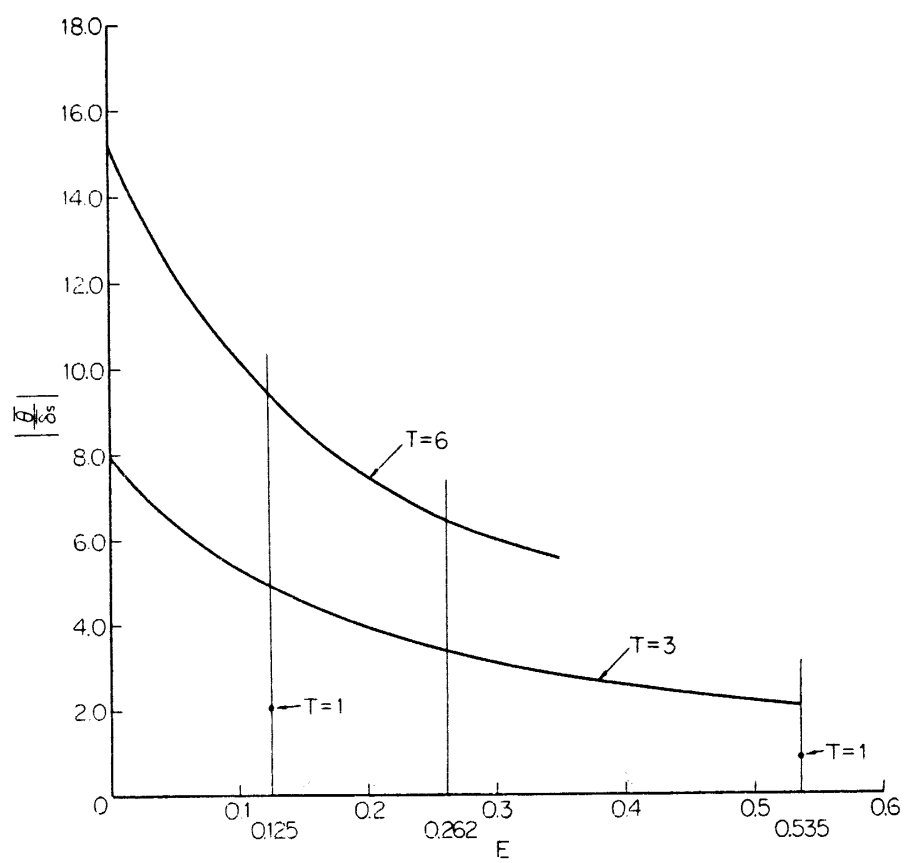

FIGURE 2-5.20. RESPONSE IN INITIAL STAGE OF LONGITUDINAL CONTROL, $V=150$ m/sec.

Input : Stick Displacement $\delta_s(t)=\frac{\bar{\delta}_s}{2}\left(1-\cos\frac{2\pi t}{T}\right)$

Output: Angle of Pitch $\bar{\theta}$=Peak Value

SAMPLES OF ANALOG COMPUTOR RECORDS OF THE RESPONES TO STICK DISPLACEMENT INPUT IN THE INITIAL STAGE OF LONGITUDINAL CONTROL

Input : $\delta_s = \frac{\bar{\delta}_s}{2}\left(1-\cos\frac{2\pi t}{T}\right)$

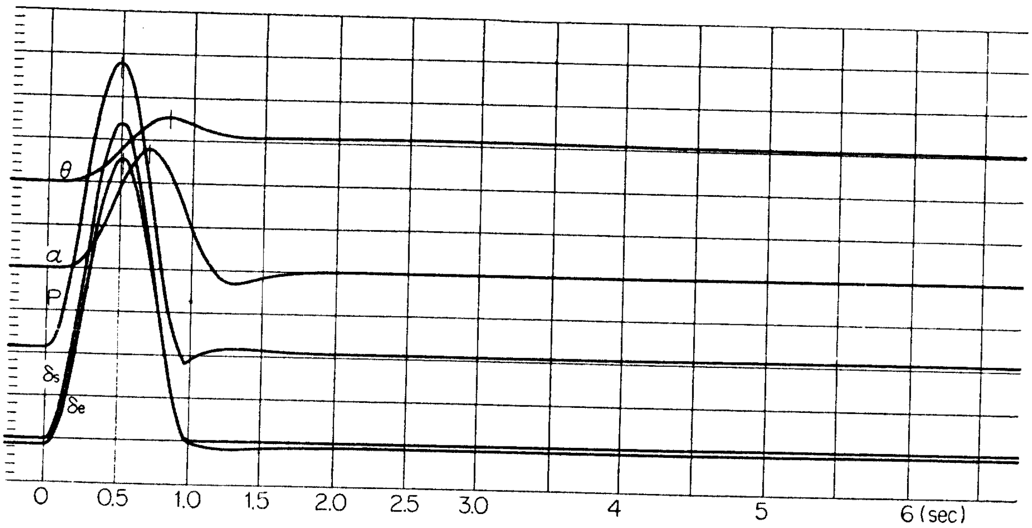

FIGURE 2-5.21 (1). $V=150$ m/sec., $T=1$ sec., $E=0.125$

θ : 6.36 deg/cm, α : 1.664 deg/cm, P : 3.70 kg/cm, δ_e : 0.754 deg/cm, δ_s : 0.646 deg/cm

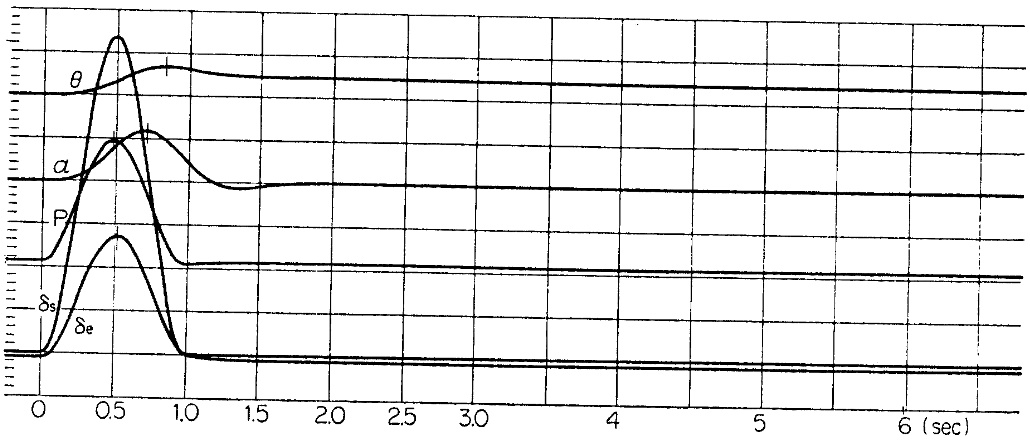

FIGURE 2-5.21 (2). $V=150$ m/sec., $T=1$ sec., $E=0.535$

θ : 6.36 deg/cm. α : 1.664 deg/cm, P : 3.70 kg/cm, δ_e : 0.754 deg/cm, δ_s : 0.646 deg/cm

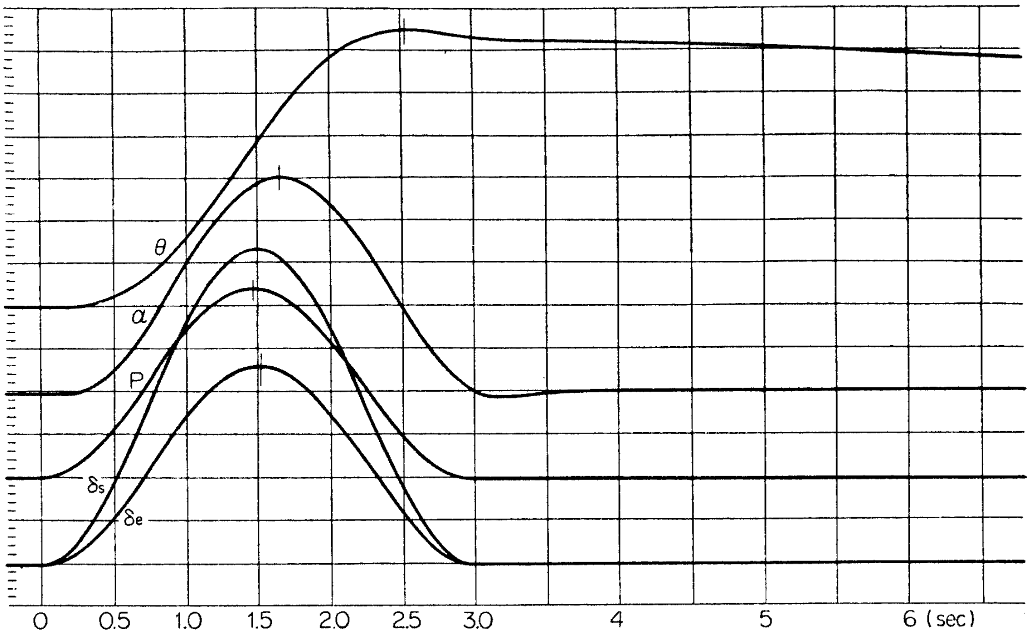

FIGURE 2-5.21 (5). $V=150$ m/sec., $T=3$ sec., $E=0.262$

θ : 2.44 deg/cm, α : 0.624 deg/cm, P : 3.70 kg/cm, δ_e : 0.754 deg/cm, δ_s : 0.646 deg/cm

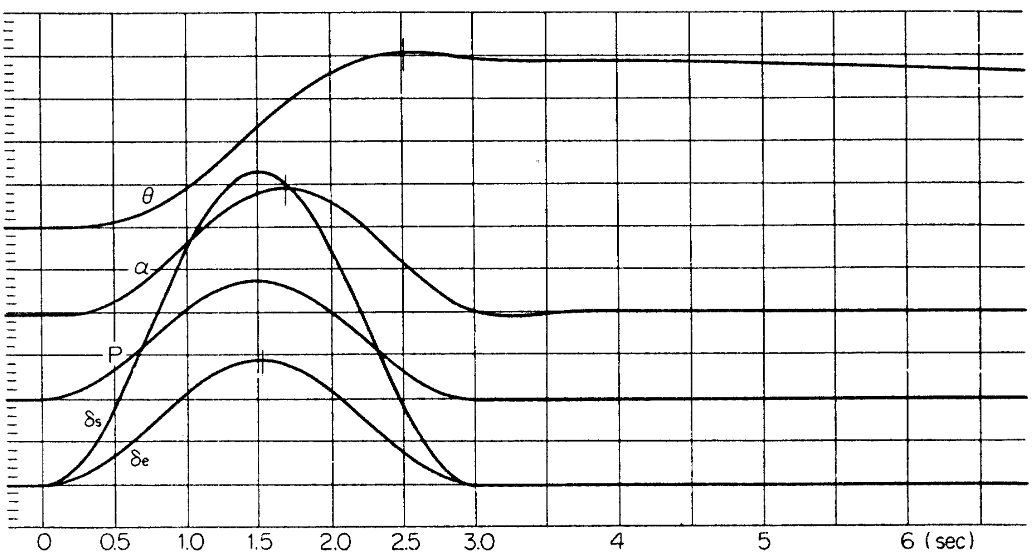

FIGURE 2-5.21 (6). $V=150$ m/sec., $T=3$ sec., $E=0.535$

θ : 2.44 deg/cm, α : 0.624 deg/cm, P : 3.70 kg/cm, δ_e : 7.54 deg/cm, δ_s : 0.646 deg/cm

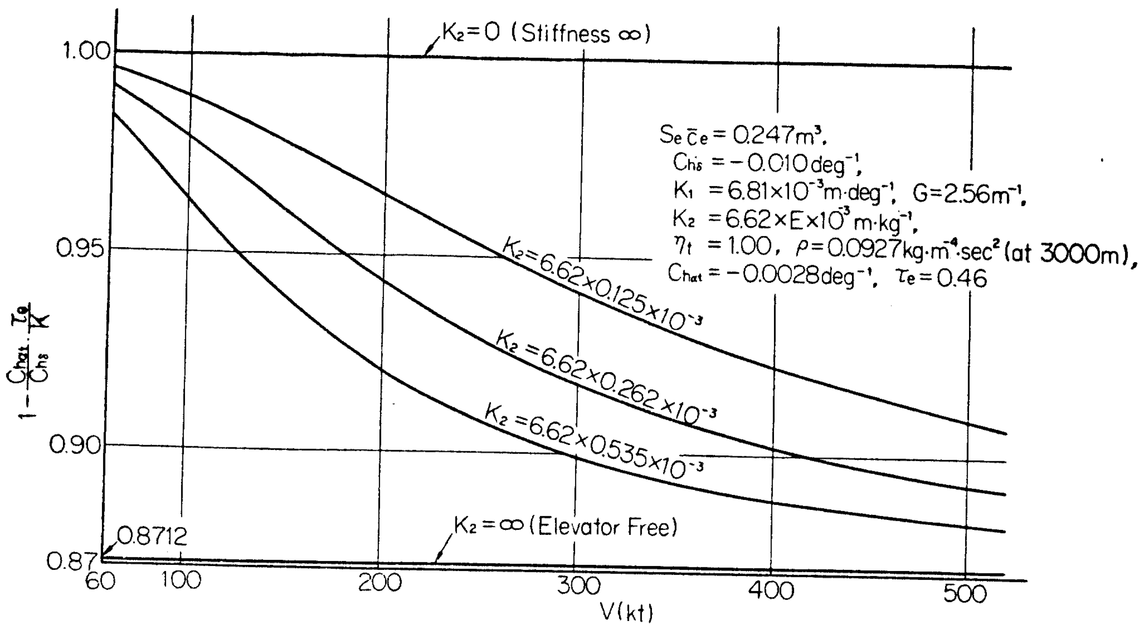

FIGURE 4-4.1. Influence of the Elevator Control System Stiffnes on $\left(\frac{\partial C_m}{\partial C_L}\right)_{\text{Tail Contr.}}$

APPENDIXES

TABLE A.1. GENERAL CHARACTERISTICS OF THE AIRPLANE A-1

Main wing span	12.00 m
Main wing area	22.44 m^2
Mean aerodynamic chord	1.93 m
Horizontal tail area (excluding the fusegage)	3.98 m^2
Elevator area	0.99 m^2
Mean chord of the horizontal tail	1.00 m
Mean chord of the elevator	0.25 m
Distance between the airplane center of gravity and the center of pressure of the horizontal tail	4.80 m
Horizontal tail volume $(S_t l_t / S\bar{c})$	0.443
S_e/S_t for the estimation of the elevator effectiveness factor τ_e	0.250
Angle between the chord-line of the horizontal tail and the airplane datum line	−1.0°
Gross weight	2343 kg.
Weight empty	1652 kg.
Type of powerplant	14-cylinder, Radial, Air-cooled
Max. rated horse power/rated altitude	875 hp/3600 m
Max. rated horse power/S.L.	780 hp/S.L.
Max. speed	over 275 kt/3600 m
Take off run at zero wind	180 m
Length of the control stick	0.670
Max. travel of the control stick	Backward 20°/Forward 14°
Max. deflection of the elevator	Up 34.3°/Down 24.0°

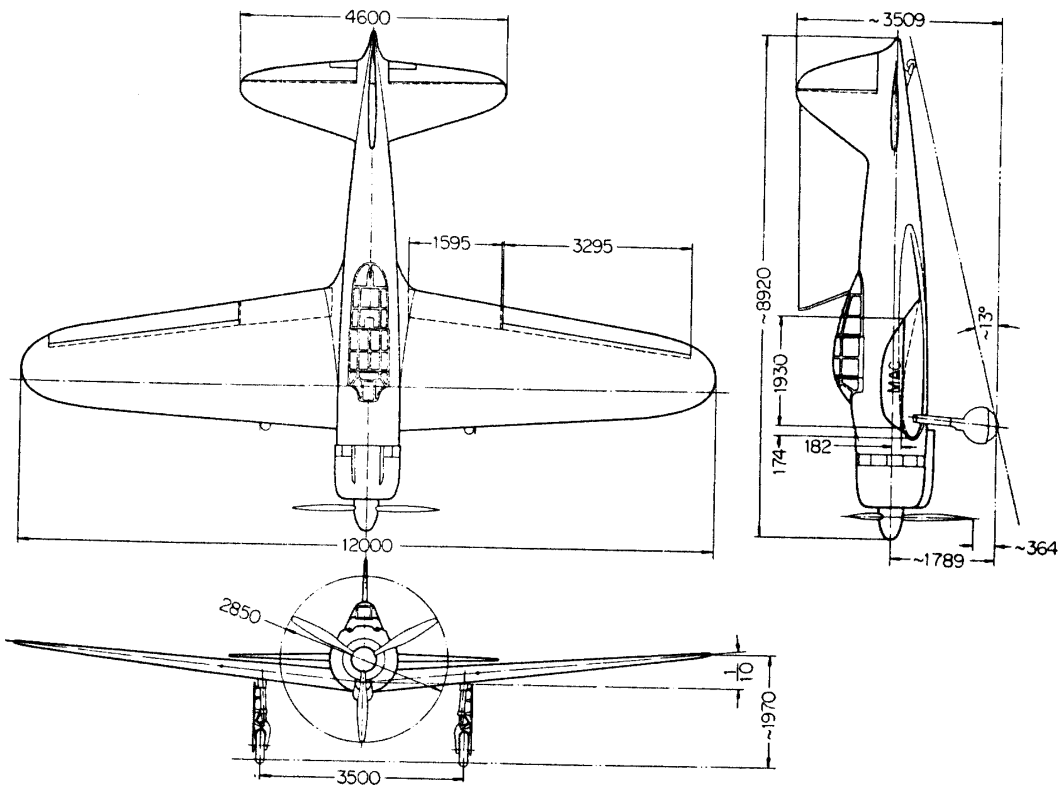

FIGURE A.1. A-1 General Arrangement.

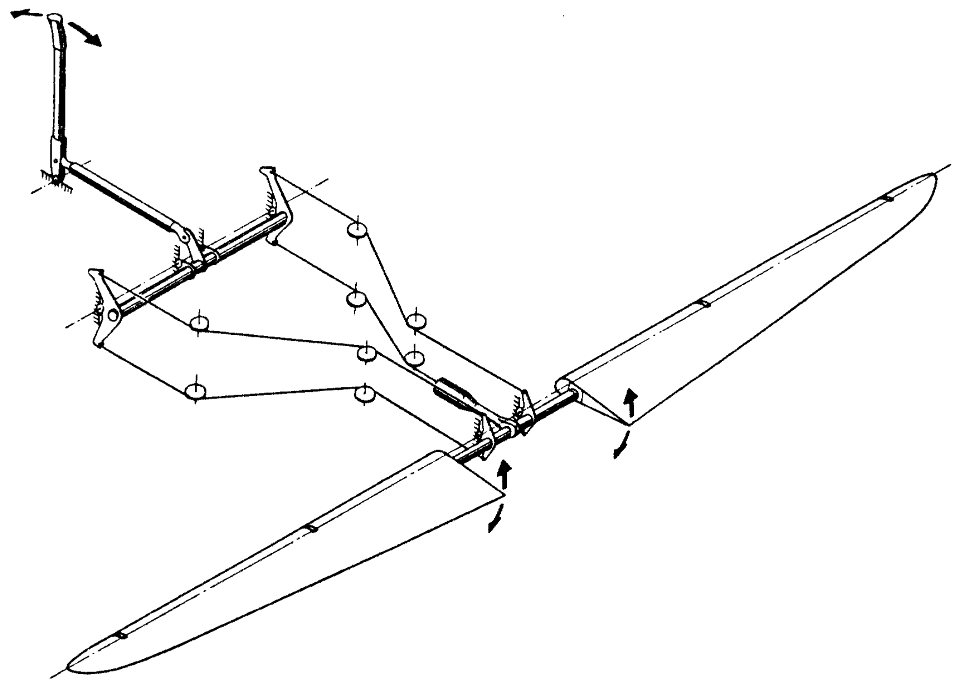

FIGURE A.2. A-1 Elevator Control System.

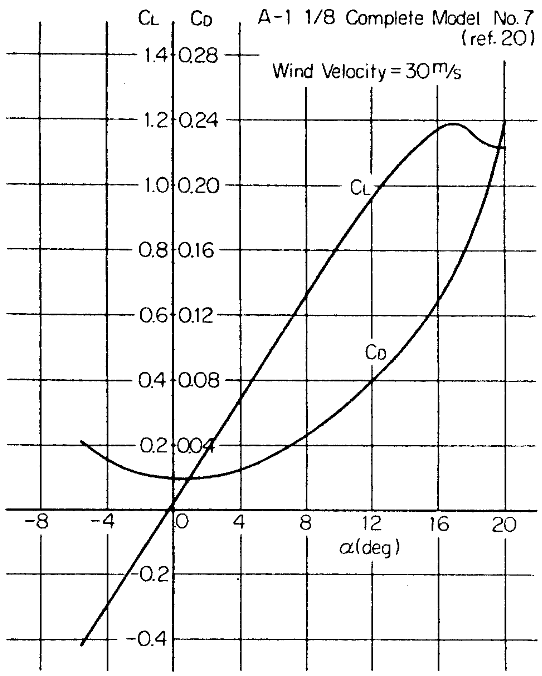

FIGURE A.3. C_L, C_D vs. α—Wind Tunnel Test.

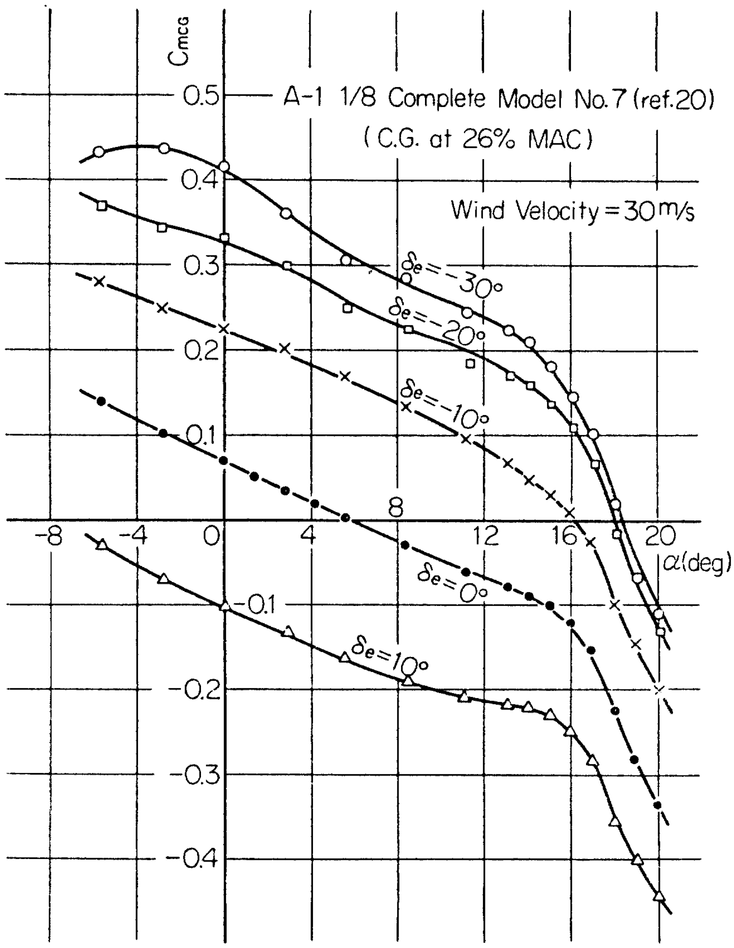

FIGURE A.4. C_{mCG} vs. α—Wind Tunnel Test.

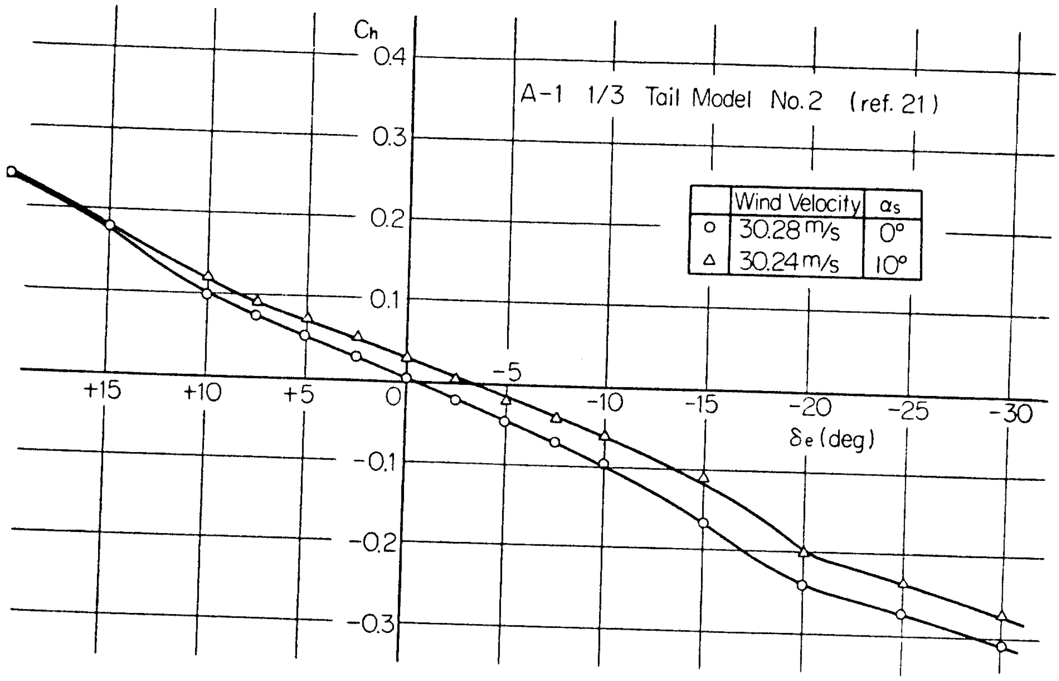

FIGURE A.5. $C_{h\delta}$ vs. δ_e—Wind Tunnel Test.

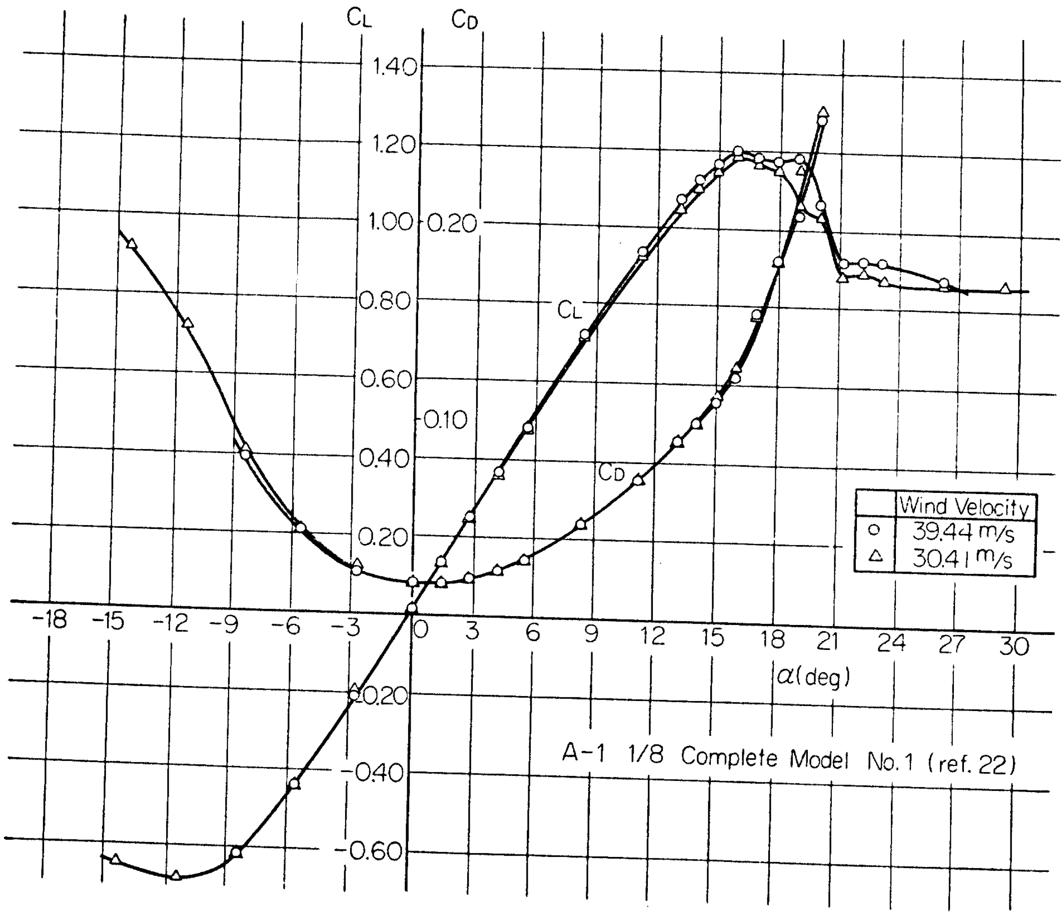

FIGURE A.6. C_L, C_D vs. α—Wind Tunnel Test.

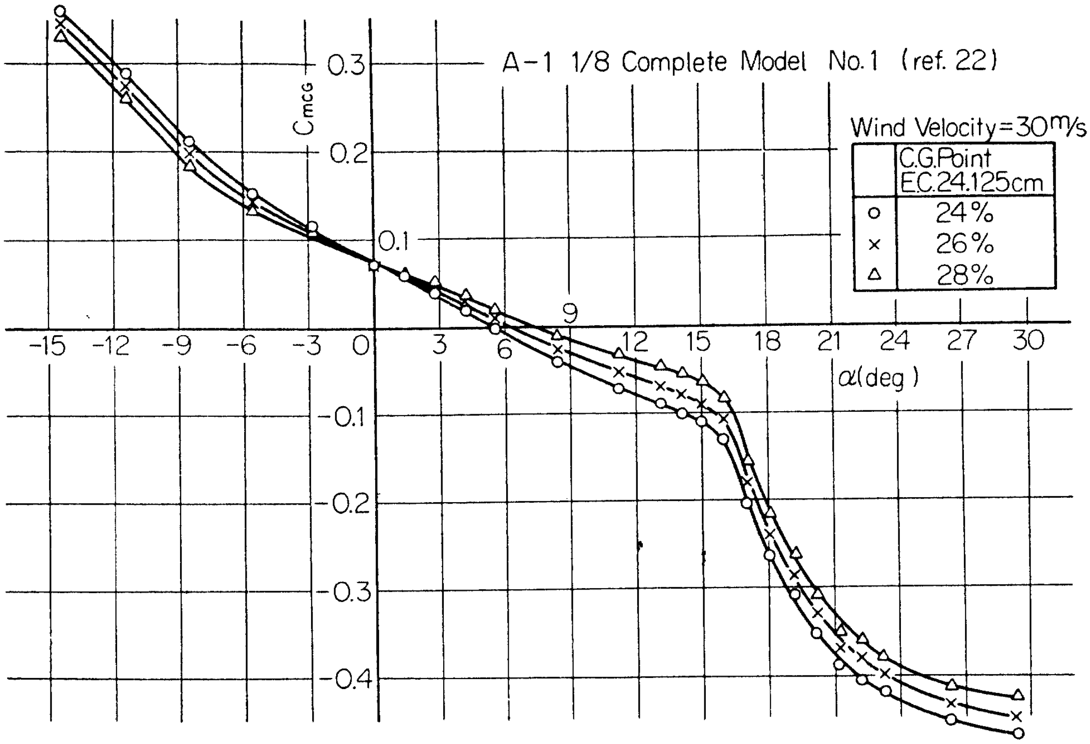

FIGURE A.7. C_{mCG} vs. α—Wind Tunnel Test.

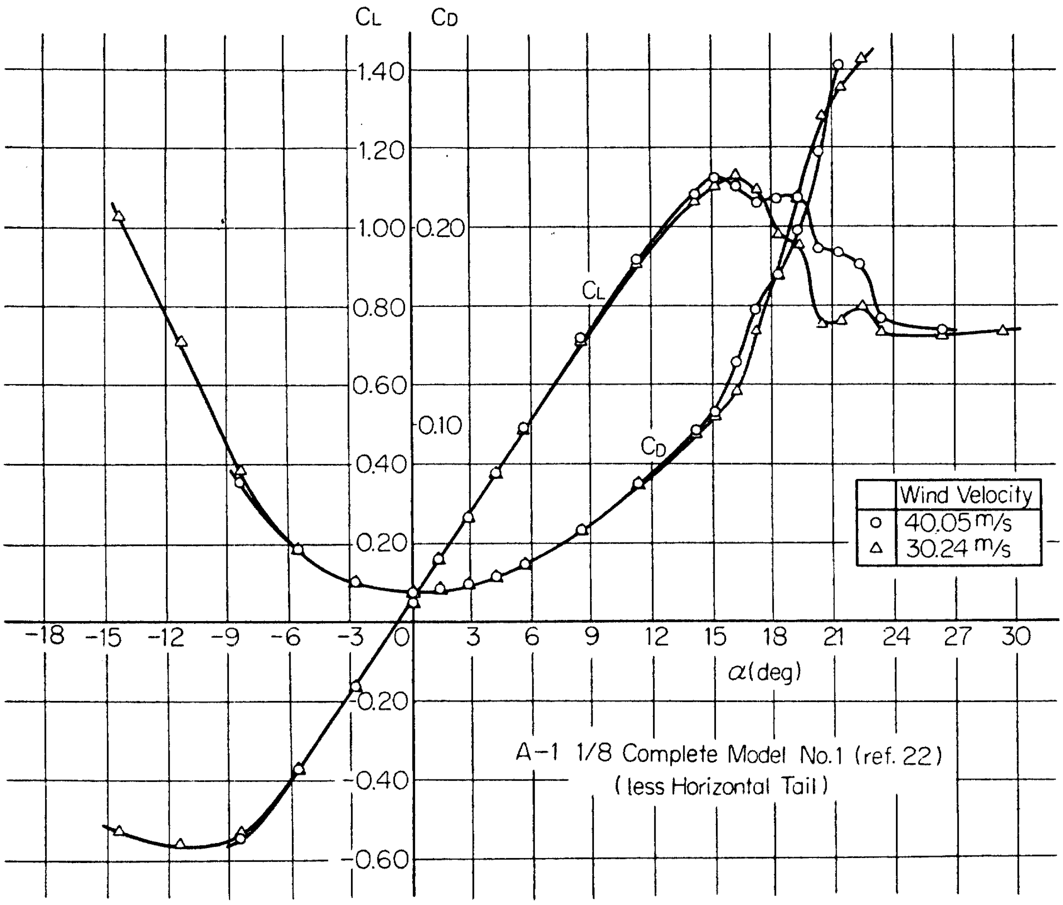

FIGURE A.8. C_L, C_D vs. α—Wind Tunnel Test.

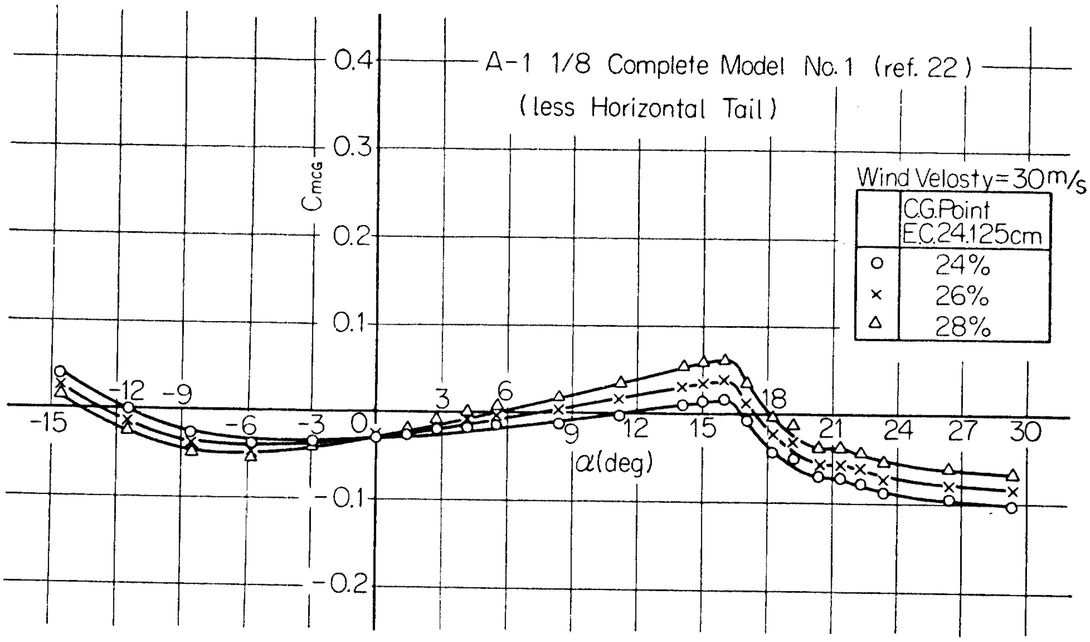

Figure A.9. C_{mCG} *vs.* α—Wind Tunnel Test.

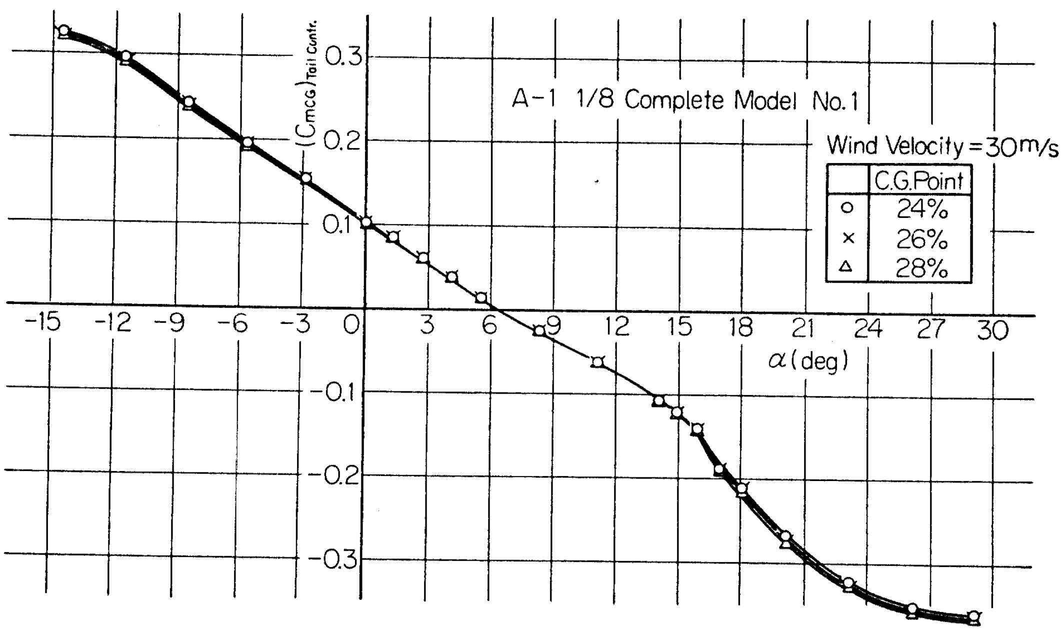

Figure A.10. (C_{mCG})Tail Contr. *vs.* α—Calculated from Fig. A.7 and Fig. A.9.

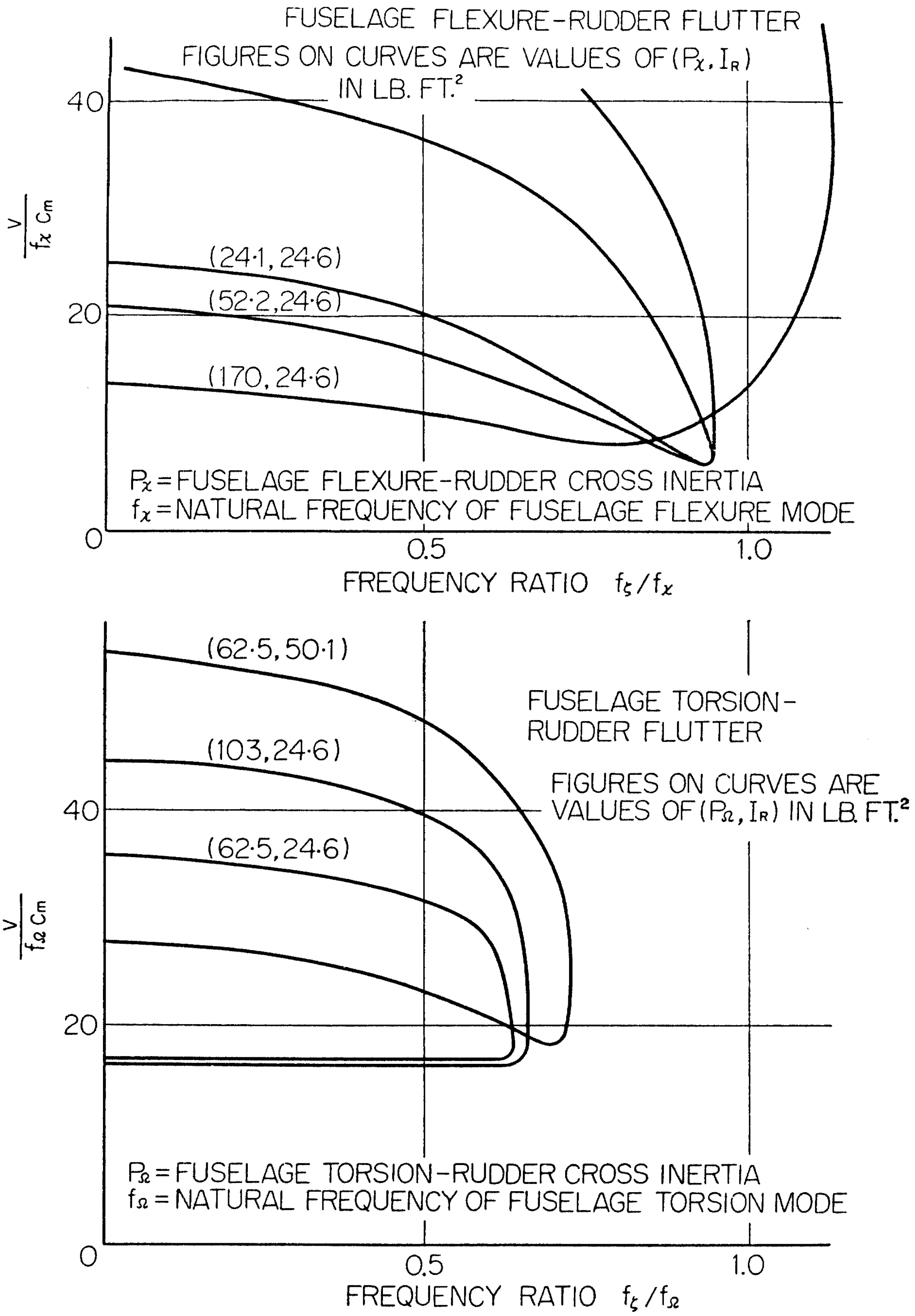

FIGURE 27. Effect of Rudder Constraint on Fuselage Rudder Flutter. (Reproduced from p. 109, ref. [*28*])

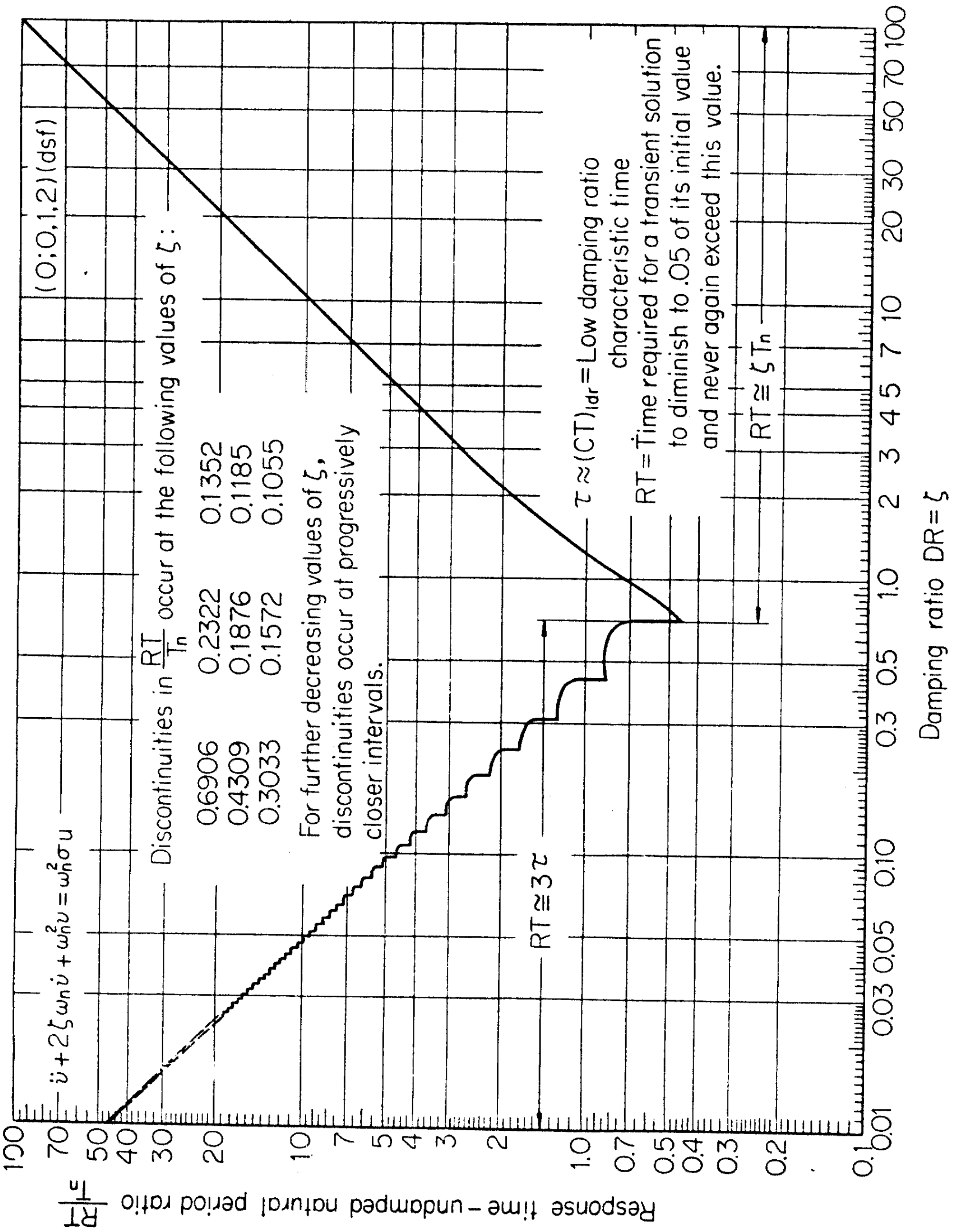

FIGURE 19.9. Relationship of Response Time-Undamped Natural Period Ratio to Damping Ratio for the Decreasing Step Function Response Associated with the (0, 0, 1, 2) Linear Second-Order Differential Equation with Constant Coefficients (Reproduced from p. 265, ref. [32]).