NUMERICAL SIMULATION TECHNIQUE FOR AEROELASTIC RESPONSE USING INVERSE FOURIER TRANSFORM

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This paper reports a new simulation technique for aeroelastic systems which respond to external forces due to spatially distributed atmospheric turbulence. If the system equation includes the effects of unsteady aerodynamics which is analytically derived in the frequency domain, then the Inverse Discrete Fourier Transform (IDFT) can be utilized for simulating the response in the time domain. The response against the vertical gust is first calculated through a transfer function given in the frequency domain and then converted whole to in the time domain. The objective of the present study is to provide the system transfer function including the effects of unsteady aerodynamic characteristics and to simulate the response to external forces come from the random or spatially frozen gust. The technique may be utilized to establish the control law of active control device coping with discrete and/or random turbulence. The method can also be utilized to calculate mathematical time history data for evaluating the control performance against the realistic gust.

Keywords: Aeroelasticity, Random gust, Numerical simulation, Fourier transform, Flutter prediction, IDFT

1. INTRODUCTION

Wing flutter is one of the most critical problems to be solved before the final stage of aircraft design. After completing the design procedure, it must be demonstrated by flight tests that the airplane is completely free from fluttering. During the design procedure, flutter tests are also conducted with scaled models to confirm the flutter boundaries in the wind tunnel. In the following actual flight tests, it will be of importance to estimate the flutter boundary from the subcritical response data in the flight envelope[1)]. Even during the wind tunnel test, it could happen that a precious wing model would be lost by abrupt occurrence of fluttering. Therefore, in both cases, the reliable prediction of the critical speed before flutter onset is highly required. Although quite a few methods have been proposed, it is still difficult to predict flutter. Difficulties are also for evaluating various prediction methods because the data acquisition by experiments and/or by the analysis is not an easy task. The aeroelastic analysis includes the complicate calculation of unsteady aerodynamic forces for the response of the system. As long as to find the flutter point, the method is thought to be matured with the aid of the linear theory of unsteady lifting surfaces. There is no efficient method, however, to simulate the subcritical response since the unsteady aerodynamics is mostly provided in the frequency domain except for the costly CFD. The other reason of the lack of reliable method for prediction is that it is quite difficult to obtain subcritical response data together with actual flutter occurrence experimentally. Hence, the numerical simulation with random external loads and/or with random internal noise from instruments is eagerly desired.

The phenomena of flutter involve the unsteady forces which are induced by the wing motion itself. In order to analyse them, theoretical aerodynamic forces are calculated with the functions of the so-called reduced frequency which is non-dimensionalized by a flow speed and a representative length. In the practical process, these forces are computed with at most about twenty reduced frequencies for several wing-deflection modes. Values between frequencies are interpolated to reduce cumbersome calculations of the generalized forces which are obtained by solving the singular integral equation.

Nowadays, however, the performance of an electric computer progressed tremendously and has made it easy to compute the unsteady aerodynamic forces even with a small workstation. Therefore it has become feasible to give them with respect to literally thousands of frequencies. This leads us to an idea that the time response of an aeroelastic system can be simulated by using thousands of discrete digital data in the frequency domain through the Inverse Discrete Fourier Transform (IDFT). Furthermore, if we choose the number of data as, say 1024 or 2048 for example, we can utilize the technique of FFT[2)] directly even for the inverse transform without any approximation of interpolation between frequencies. This enables us to obtain the time series of response data with a constant flow speed below the

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flutter by converting the analysis in the frequency domain.

This paper reports the data handling in details of the conversion to simulate the aeroelastic response. It assumes the aeroelastic response is caused by the random turbulence or by spatially frozen gust.

The results can be applied to evaluate the reliability of various flutter prediction methods and to find the proper location of sensors which depends on the flutter characteristics. The simulation technique is also expected to contribute for examining the active control effects to attenuate the gust load.

2. DISCRETE FOURIER TRANSFORM （DFT）

The discrete Fourier transform and its inverse[3)] are defined by the following pair of equations

$$G(m) = \frac{1}{N}\sum_{n=0}^{N-1} g(n)\,\mathrm{e}^{-i\frac{2\pi m n}{N}} , \tag{1}$$

$$g(n) = \sum_{m=0}^{N-1} G(m)\,\mathrm{e}^{i\frac{2\pi m n}{N}} , \tag{2}$$

where $g(n)$ denotes a series of digital signal with equally sampled in the time domain and $G(m)$ its DFT. Equation (2) is called as the inverse discrete Fourier transform which is abbreviated as IDFT. The variables m and n are integers and N is selected as multiple powers of 2 so as to utilize the efficient Fast Fourier Transform technique developed by Cooley and Tukey. Mathematically, DFT has the assumption that the time signal should be periodic. In practical cases, however, this restriction does not cause any problem because the duration of the actual signal is always finite and it can be treated as if it were one period.

3. IMPULSIVE RESPONSE AND TRANSFER FUNCTION

Generally, time history of the response can be obtained with a convolution integral of an impulsive response function and the external loads. An impulsive response function is equivalent to the inverse of the Laplace transform of the transfer function itself[4)]. Therefore it can be calculated with the inverse Fourier transform when the frequency response function of the system is given.

For the continuous signal, the impulsive response function of a system is directly related to the inverse Laplace transform of a transfer function $H(s)$.

$$g(t) = \mathfrak{L}^{-1}[H(s)] . \tag{3}$$

In the Laplace transformation domain, the impulse as an external force can be given by a unit function. Then the response becomes

$$G(s) = H(s)\cdot 1 . \tag{4}$$

In case of the steady state response by harmonic excitation, Eq.(4) yields, by putting $s = i\omega$,

$$G(i\omega) = H(i\omega)\cdot 1 . \tag{5}$$

This relationship describes the response with the uniformly distributed exciting force in the frequency domain. On the other hand, the definition of the Laplace transform is given by

$$G(s) = \int_0^{\infty} g(t)e^{-st}dt . \tag{6}$$

Putting $s = i\omega$ in Eq.(6) leads us to an expression for the steady state response by the harmonic excitation.

$$G(i\omega) = \int_0^{\infty} g(t)\mathrm{e}^{-i\omega t}dt . \tag{7}$$

Here, in order to clarify the relationship between the Laplace transform and the Fourier transform, we extend the impulsive response function to the negative region of time as

$$g(-t) = g(t), (t > 0) \quad (8)$$

Then, the corresponding part of the function in the frequency domain becomes

$$G(-i\omega) = \overline{G(i\omega)} = \int_0^\infty g(t)\mathrm{e}^{i\omega t}dt = \int_{-\infty}^0 g(-t)\mathrm{e}^{-i\omega t}dt = \int_{-\infty}^0 g(t)\mathrm{e}^{-i\omega t}dt\,. \quad (9)$$

Thus, we can obtain the Fourier transform as

$$G(i\omega) + \overline{G(i\omega)} = \int_{-\infty}^\infty g(t)\mathrm{e}^{-i\omega t}dt = \hat{g}(\omega) \quad (10)$$

If we assume the transfer function $G(s)$ for an aeroelastic system including the effects of unsteady aerodynamic forces, then the spectrum of frequency response function becomes

$$G(i\omega) = \hat{g}(\omega) \quad (11)$$

This enables us to write the impulsive response $g(t)$ as an inverse Fourier transform.

$$g(t) = \mathcal{F}^{-1}[G(i\omega)], (t > 0) \quad (12)$$

Equation (12) implies that the discrete data of the time history can be converted from the discrete frequency data and vice versa through the pair of relations, Eqs.(1) and (2).

It should be noted that the amplitudes of the response calculated form the transfer function in the frequency domain by using the digital data with finite values are different from those obtained with a unit impulse in the continuous time domain. We have to adjust the level of each input whenever comparing those results.

Once the impulsive response function of a system has been obtained, then the time history for arbitrary external forces can be generated by the convolution integral[5]. There is an alternative method, however, to obtain the response in the time domain, i.e. the application of inverse Fourier transform after the multiplication of the transfer function and the Fourier transform of the external forces. The present paper explains the latter technique since it is much efficient to simulate the response.

4. PROCESSING OF DIGITAL SIGNALS

(1) Implementation

The Theodorsen function for a two dimensional airfoil, or the unsteady lifting surface theory[6] for a finite wing, provides the unsteady aerodynamic force due to the system motion as functions of the reduced frequency, i.e. given in the frequency domain. Therefore, they cannot be expressed directly with the Laplace operator s which is corresponding to the differentiation with respect to time. This means that the transfer function of an aerodynamic system is written in the matrix of complex numbers while its responses of Eq.(2) are real numbers. In the application of IDFT to obtain time history data, we have to take this consideration correctly into account. The procedure is described as follows.

(i) Assume $G(m)$ as an the aeroelastic response function in the frequency domain and calculate $(N/2+1)$ number of complex values for each frequency with an interval $\Delta\omega$ from $\omega = 0$ up to $\omega = (N/2)\Delta\omega$.

(ii) The corresponding duration time (a theoretical periodic interval in the time domain) and the sampling period of digital signal become, respectively,

$$T = 2\pi/\Delta\omega \quad \text{and} \quad \Delta t = T/N \quad (13)$$

(iii) In order to hold the causality valid, the following complex conjugates are allotted to $G(m)$ for $m = (N/2+1),..., (N-1)$.

$$G(m) = \mathrm{conj}[G(N-m)] \quad (14)$$

(iv) Particularly at a center of the data series, we enforce to put

$$\mathrm{Im}[G(N/2)] = 0 \quad (15)$$

(v) Since the present simulation is aimed for the aeroelastic subcritical analysis, there is no need to include the static displacement. Hence

$$G(0) = 0 \quad (16)$$

(vi) Appling IDFT to $G(0),...,G(N-1)$ thus formed yields a time history consisting of N number of digital data.

(2) Mathematical background

The discrete time signal can be written with the frequency response function $G(m)$ in the discrete frequency domain as

$$\begin{aligned} g(n) &= \sum_{m=0}^{N-1} G(m)\,\mathrm{e}^{+i\frac{2\pi nm}{N}} \\ &= G(0) + \sum_{m=1}^{\frac{N}{2}-1} G(m)\,\mathrm{e}^{+i\frac{2\pi nm}{N}} + G(\tfrac{N}{2})\mathrm{e}^{in\pi} + \sum_{m=\frac{N}{2}+1}^{N-1} G(m)\,\mathrm{e}^{+i\frac{2\pi nm}{N}} \\ &= G(0) + \sum_{m=1}^{\frac{N}{2}-1} G(m)\,\mathrm{e}^{+i\frac{2\pi nm}{N}} + G(\tfrac{N}{2})\mathrm{e}^{in\pi} + \sum_{m=1}^{\frac{N}{2}-1} G(N-m)\,\mathrm{e}^{-i\frac{2\pi nm}{N}}. \end{aligned} \quad (17)$$

The necessary and sufficient condition that the left hand side of Eq.(17) becomes a real number for any complex response data in the frequency domain can be given by

$$G(N-m) = \overline{G(m)}, \quad \text{for} \quad m = 1,..., \tfrac{N}{2}-1, \quad \text{and} \quad \mathrm{Im}[G(\tfrac{N}{2})] = 0. \quad (18)$$

5. GOVERNING EQUATION OF AEROELASTIC SYSTEM

Assuming $D(i\omega)$ as the impedance matrix of a mechanical system and $A(\omega)$ as the unsteady aerodynamic matrix, we can write the governing equation for an aeroelastic system with the generalized coordinates $\mathbf{q}$ as

$$[D(s) + A(\omega)]\,\mathbf{q} = \mathbf{f} \,, \quad (19)$$

where the $\mathbf{f}$ in the right hand side of the equation denotes the generalized external forces, which may also be random aerodynamic noise. If we re-define the transfer function of the system $H(i\omega)$ in the frequency domain as $H(\omega)$, then it can be obtained from Eq.(17) as

$$H(\omega) = [D(i\omega) + A(\omega)]^{-1} . \quad (20)$$

Then the impulsive response of the system including the effects of the unsteady aerodynamic force is written for each component as

$$h_{ij}(t) = \mathfrak{F}^{-1}[H_{ij}(\omega)], \quad (t > 0) . \quad (21)$$

6. TYPICAL SECTION AIRFOIL

As an example problem, we shall use a typical section of the two-dimensional airfoil shown in Fig.1. Each parameter

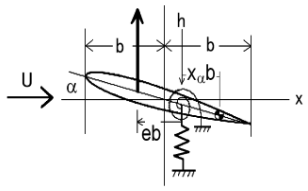


Figure 1: Typical Section Airfoil

is defined in the non-dimensionalized form. Different from the flutter analysis, the frequency Ω is normalized with respect to a pitching frequency ω_α instead of the so-called reduced frequency k. Defining the generalized coordinate vector as $\{h, \alpha\}^{\mathrm{T}}$, we obtain a part of the transfer function except for the aerodynamic force as

$$D(s) = s^2 \begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r_\alpha^2 \end{bmatrix} + \begin{bmatrix} R^2 & 0 \\ 0 & r_\alpha^2 \end{bmatrix}, \tag{22}$$

where R is a frequency ratio and r_α^2 is moment of inertia of the section which has been non-dimensionalized by the representative length and mass.

A textbook[7)] provides the two-dimensional incompressible unsteady aerodynamic matrix for Eq.(19) with Theodorsen function having the argument of the reduced frequency. We denote the mass ratio as μ and the non-dimensional speed $U^* = U/(b\omega_\alpha)$. Additionally, the non-dimensional dynamic pressure and time are introduced by $Q = 2U^{*2}/\mu$ and $t^* = \omega_\alpha t$, respectively. As the flow is fixed to a certain speed in the present problem, the Theodorsen function $C(k) = C(\Omega/U^*)$ is written as $C(\Omega)$. Then the aerodynamic matrix can be given by

$$\begin{aligned} \mu A(\Omega) = -\Omega^2 &\begin{bmatrix} 1 & -e+\frac{1}{2} \\ -e+\frac{1}{2} & e^2-e+\frac{3}{8} \end{bmatrix} + \\ i\Omega U^* &\begin{bmatrix} 2C(\Omega) & 1+2(1-e)C(\Omega) \\ -2eC(\Omega) & (1-e)-2e(1-e)C(\Omega) \end{bmatrix} + U^{*2}C(\Omega)\begin{bmatrix} 0 & 2 \\ 0 & -2e \end{bmatrix}. \end{aligned} \tag{23}$$

In the following numerical examples, the parameters of the system are set as the same as the case(n) on the P538 of the reference. Those are $\mu = 10,\ e = 0.2,\ x_\alpha = 0.1,\ r_\alpha^2 = 0.25,$ and $R = 0.3$. This combination of parameters results in the flutter critical dynamic pressure $Q_F = 0.80$ with the flutter frequency $\Omega_F = 0.62$.

7. FINITE STATE MODEL

In order to compare the results obtained by the present IDFT procedure, a time domain method using the finite state model is introduced. The aerodynamic effect is embedded in the system of differential equations approximately with the augmented state variables. For unsteady aerodynamic terms, the following form of the finite state[8)] is used,

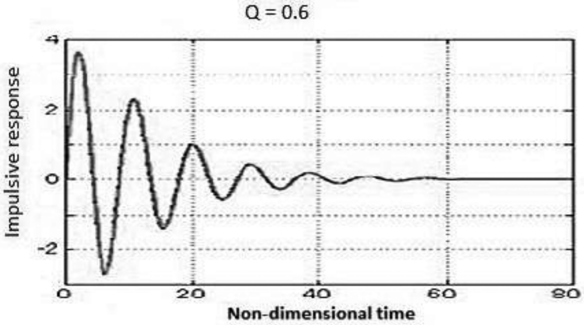
$$F_a(s, \mathbf{q}) = \left(A_2 s^2 + A_1 s + A_0 + \sum_{i=1}^{3} \frac{A_{L_i}}{s+\lambda_i} \right) \mathbf{q}, \tag{24}$$

where the symbol $\mathbf{q}$ denotes the generalized coordinate vector. The coefficients in Eq.(24), A_2, A_1, A_0, and A_{L_i} are determined with the aid of the least square method after the calculation by DPM[9)] for the frequencies from 0.01 to 2.0. The interval of the frequency is selected as 0.01 and the three arbitrary parameters λ_i's are selected as 0.1, 0.5, and 1.5.

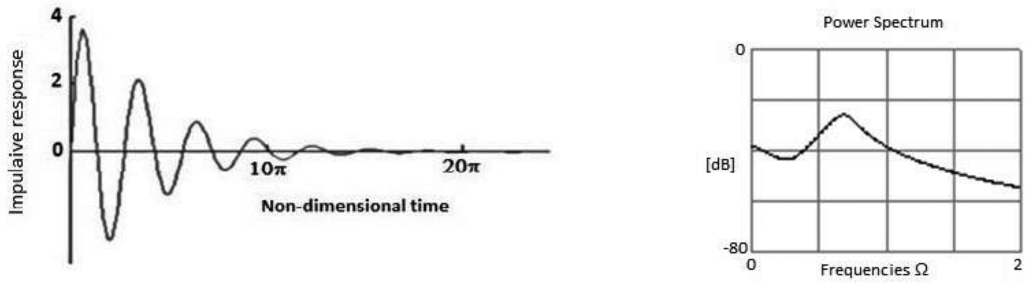
8. NUMERICAL EXAMPLES

(1) Impulsive response

For a certain dynamic pressure below the flutter critical speed, the impulsive response is calculated by applying IDFT to the transfer function which is given by Eq.(20). The discrete values of the function are computed for $N/2 = 1024$ frequencies with a frequency increment $\Delta\Omega = 0.01$. This yields 2048 data in the time domain with the sampling rate $\Delta t^* = 0.3068$. In the practical calculation, this distribution may be appropriately cut for higher frequencies above $\Omega = 5$ since the two natural frequencies of the system in this case are well below as $\Omega = 0.3$ and 1. Figure 2 illustrates the response of $h(t^*) = h_{\alpha h} + h_{\alpha\alpha}$ due to the α impulse.



(a) Impulsive response by the finite state model



(b) Impulsive response by the inverse Fourier transform

Figure 2: Comparisons of two Methods

It should be noted that a unit impulse, which is Dirac's delta function mathematically in the continuous domain, corresponds to a single finite value of $1/\Delta t$ at the starting point of the discrete data series and that the constant amplitude in the frequency domain must be $1/(N\Delta t)$ to make the powers of both signals equal to each other. It can be seen from the figures that the results obtained by IDFT agree well with those by the finite state model in the time domain.

(2) Discrete gust response

As mentioned before, once the impulsive response becomes known, it enables us to calculate a system response against general shapes of the external input by using the convolution integral. If we put the input and the impulsive response of the system as $f(t)$ and $h(t)$, respectively, then the response $x(t)$ is given by so-called Duhamel's integral.

$$x(t) = \int_0^t h(t-\tau)\, f(\tau) d\tau \tag{25}$$

In the frequency domain, this relationship becomes

$$\hat{x}(\omega) = \mathfrak{F}[\, h * f(t)\,] = \hat{h}(\omega)\hat{f}(\omega)\,. \tag{26}$$

Therefore, the response in the time domain for the discrete data can be calculated as

$$x(n) = \mathrm{IDFT}[\ \hat{x}(m)\,] = \mathrm{IDFT}[\ \hat{h}(m)\hat{f}(m)\,]\,. \tag{27}$$

An example result of this procedure is shown in Fig. 3.

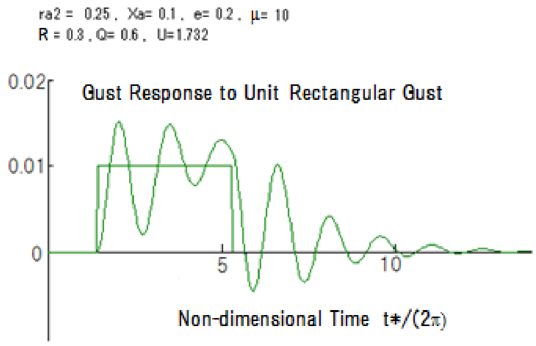


Figure 3: Transient response of the aeroelastic system

(3) Random noise

The random noise can be generated by the present method as follows. First, assume the transfer function of Eq.(20) as a unit matrix. Then, the components of random external forces in Eq.(19) are calculated in the frequency domain as

$$F(m) = \Phi(m)\mathrm{e}^{i\phi_m} , \tag{28}$$

where the phase ϕ_m can be provided with the uniformly distributed random number between 0 and 2π. In case of the white Gaussian noise, the spectrum $\Phi(m)$ must be the Gaussian distribution. The IDFT conversion after these calculations gives a series of the random signal. The similar noise can be also generated in the time domain by superimposing the cosine function for the entire interval[10]. Results by both methods are compared in Fig.4 for a series of random signal having the unit average amplitude and the standard deviation of 0.3.

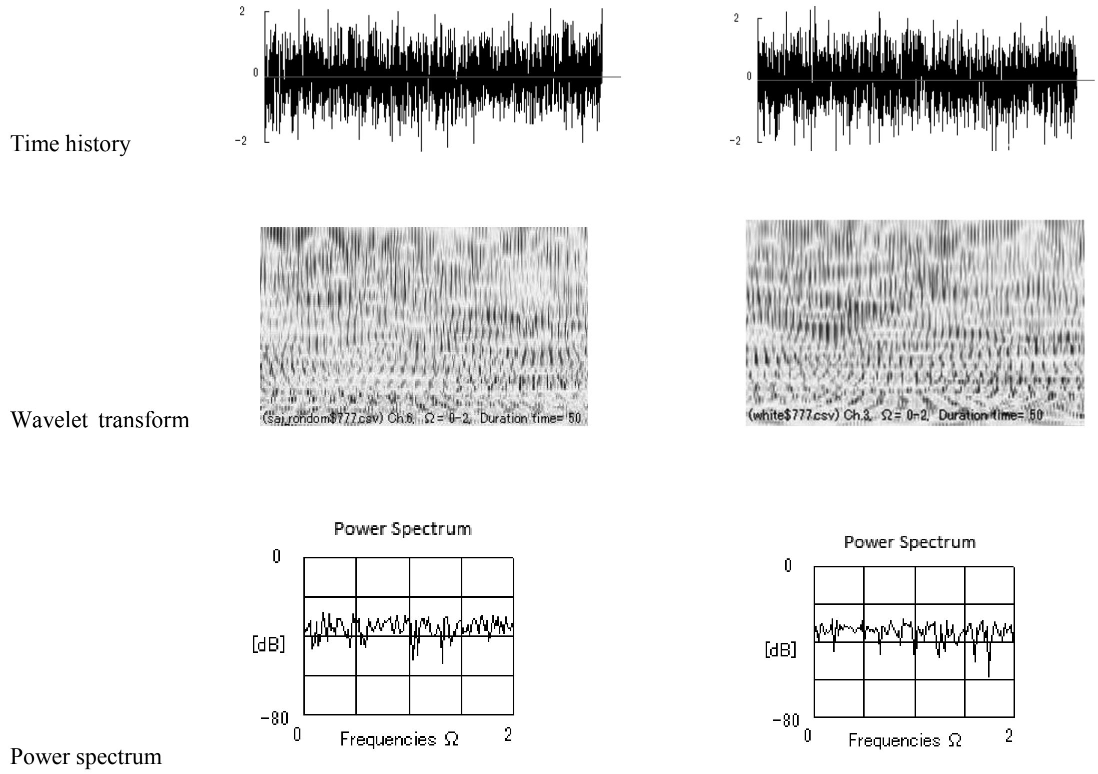


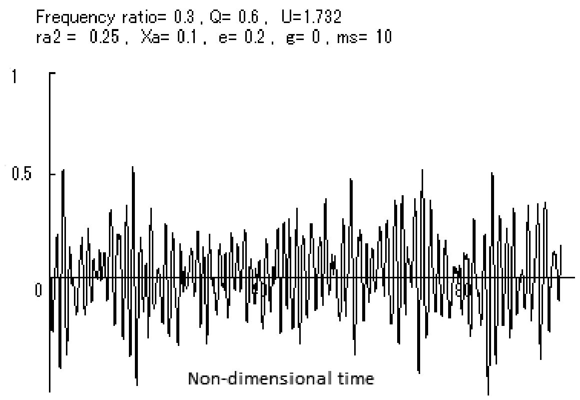
Figure 4: (a) Time Domain Method (b) Present IDFT Method

The result looks being agreed well to each other because the both procedures are theoretically equivalent. It should be noted that the present method is much more efficient in view of the computation time due to the FFT algorithm.

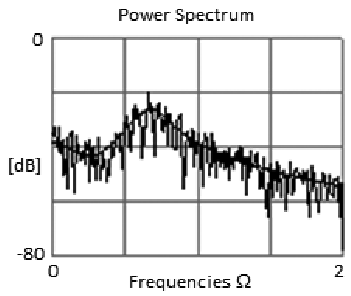
(4) Simulation of random response of a typical section airfoil

For the same dynamic pressure as that in the example of 8.1, an aeroelastic response due to the random noise has been calculated. The result is depicted in Fig. 5(a) with its power spectrum 5(b). The smooth curve in the figure indicates the power spectrum of the impulsive response, i.e. the response without noise. It can be seen that the random noise is properly included in the response.

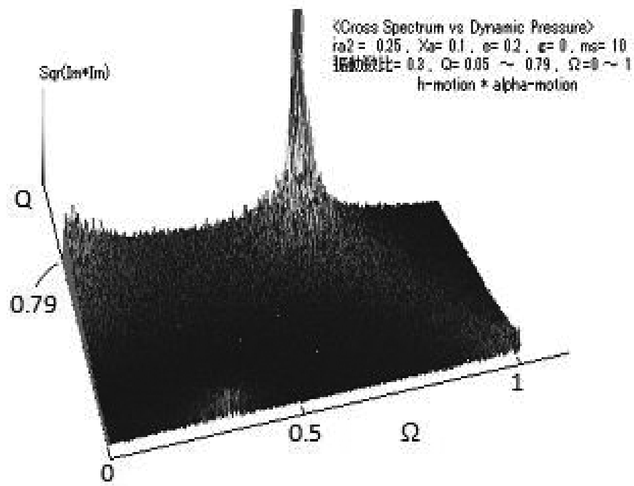
Furthermore, the dependency of the cross spectrum between the h and α of the response on the dynamic pressure is illustrated in Fig.5(c). The figure reveals the coupling of two modes which is going into flutter at the dynamic pressure of Q=0.8.



(a) Time History of Random Response (Q=0.6)



(b) Power Spectrum



(c) Dependency of Cross Spectra on Q

Figure 5: Simulation results

9. APPLICATION OF IDFT SIMULATION METHOD FOR RANDOM GUSTS

An example application of the present technique has been carried out on the flutter prediction using the wavelet transform[11]. The definition of the wavelet is given by

$$f_\psi^W(b,a) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt \quad , \tag{29}$$

$$\psi(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{t^2}{2\sigma^2}\right) \exp(i\omega_0 t) \quad . \tag{30}$$

Here we use Gabor's mother wavelet as described in Eq.(30).

In Fig.6, the simulated signals of the α motion are displayed for eight different dynamic pressures.

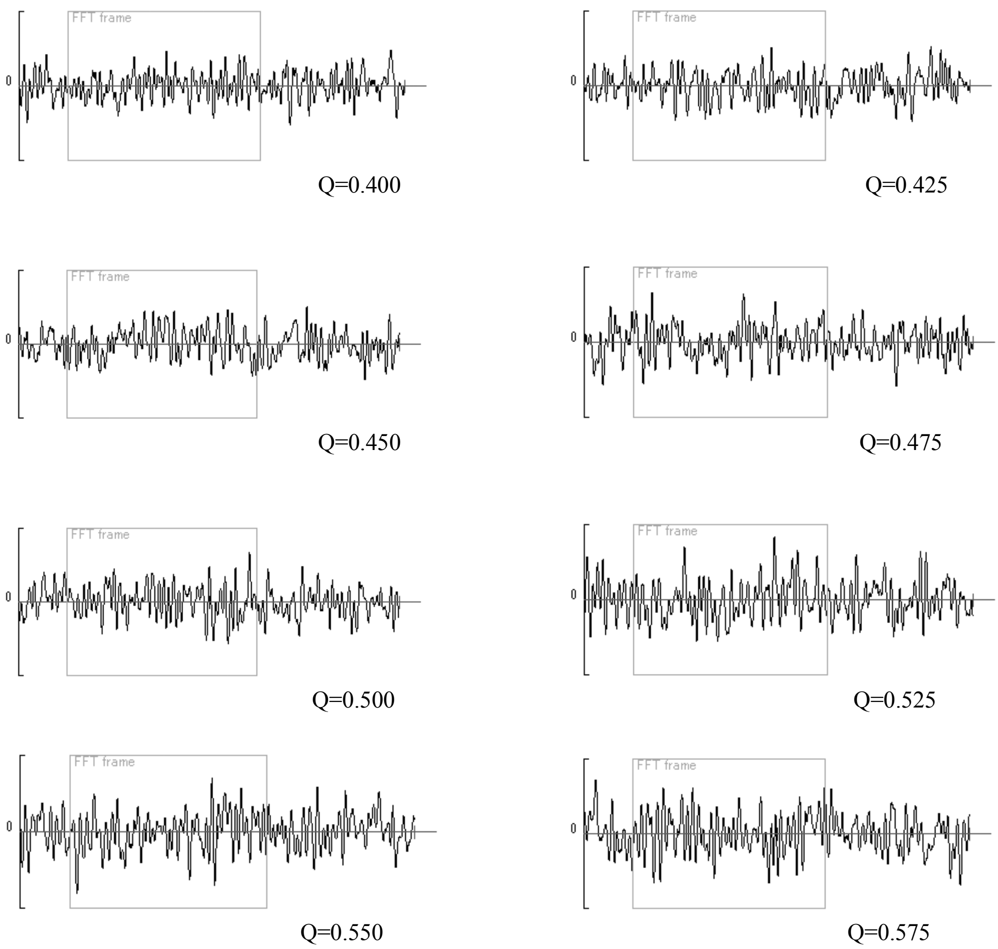


Figure 6: Simulated Response Signals for each Dynaic Pressure

These may be regarded as the output of 'virtual experiments' for the purpose of examining flutter prediction methods. The data in the frameworks above have been applied to the wavelet prediction method. The result is shown in Fig.7.

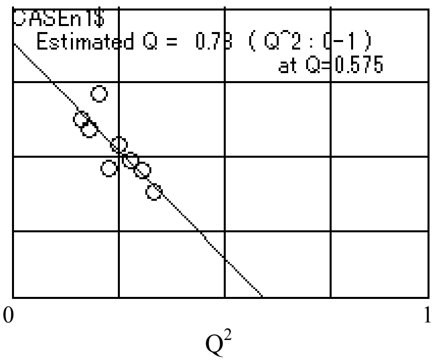


Figure 7: Wavelet Flutter Prediction from the Simulated Signals

It can be seen from the figure that the present simulation technique provides realistic output of the virtual experiment.

10. CONCLUDING REMARKS

The method using the inverse discrete Fourier transform to simulate the aeroelastic response to the random and/or discrete gust has been proposed and demonstrated. It is expected to be utilized for evaluating various methods to predict flutter and the performance of the active control technique to attenuate the response against the atmospheric turbulence.

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