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Modern Aspects of Radiation Magnetogasdynamics

By

Shih-I PAI*

PREFACE

During the academic year 1966–1967, I am on sabbatical leave from the Institute for Fluid Dynamics and Applied Mathematics, University of Maryland and on a National Science Foundation Senior Postdoctoral Fellowship of U.S.A. to do research on high temperature flow of gasdynamics in Japan and Austria. With the kind invitations from Professor N. Takagi, director of the Institute of Space and Aeronautical Science (ISAS), University of Tokyo and my good friend Professor I. Tani, I have the opportunity to stay in ISAS, University of Tokyo as a visiting professor for five months from September, 1966 to January 1967. My main purpose to visit Japan is to exchange scientific ideas with Japanese scientists. Besides many scientific discussions with colleagues in ISAS, particularly the group under Professor R. Kawamura, H. Oguchi, K. Oshima and K. Karashima, I offer a series of eight lectures on Modern Aspects of Radiation Magnetogasdynamics which concern with the effects of ionization and thermal radiation on high temperature gas flow. This report contains main points of my lectures. There are many details of such analysis, particularly those of multifluid theory which have not been worked out yet. I hope that this report may give young scientists some inspirations to study this new field of gasdynamics.

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S. I. PAI

1. INTRODUCTION

The current trend of the flow problem in aerospace engineering is towards high temperature and low density of the gas. Under such conditions, the gas will be ionized and the electromagnetic forces are important. For a first approximation, the classical magnetogasdynamics of single fluid theory has been successfully used [*1*]**.

* Research Professor, Institute for Fluid Dynamics & Applied Mathematics, University of Maryland

Visiting Professor, Institute of Space and Aeronautical Science, University of Tokyo

** This number refers to the number of reference in section 9.

The basic assumptions for this set of equations are (i) a generalized Ohm's law is used instead of the exact differential equation of electrical current density, (ii) the fluid has a scalar electrical conductivity and (iii) the temperature of all species in the electrically conducting fluid are the same. Such a system of equations gives good results for electrically conducting liquid as well as ionized gas if the strength of magnetic field is not too large and the density of the gas is not too small. However in current practice, the strength of magnetic field gradually increases and the density of the gas decreases. As a result, the Hall current and ion slip will be important and we should not assume that the electrical conductivity of the gas is a scalar quantity. When the degree of ionization is large and the density of the gas is low, gasdynamic forces will affect the electric current density distribution and the temperature of electrons may be different appreciably from that of heavy particles. We have to improve our fundamental equations of magnetogasdynamics so that these new phenomena may be taken into account. One way to improve the classical magnetogasdynamic equations is to use a complicated generalized Ohm's law including the Hall current, ion slip and simple gasdynamic effects. Such improvements may still be insufficient for many other effects, such as different temperature between species, large diffusion velocity, etc. A better and more logic approach is the multifluid theory of magnetofluid dynamics.

At very high temperature and low density, the thermal radiation may become an important mode of heat transfer in comparison with heat conduction and heat convection. Hence in many advanced aerospace engineering problems, we should consider the thermal radiation effects which include (i) the radiation stresses (ii) the radiation energy density and (iii) the heat flux of radiation [2]. From macroscopic point of view, we may use the specific intensity of radiation to describe the thermal radiation field in the gas flow. In this report, we are going to discuss first the fundamental equations of multifluid theory of magnetogasdynamics including thermal radiation effects which may be called Radiation Magnetogasdynamics and then to analysis some flow problems based on these equations.

An ionized gas or a plasma may be considered as a mixture of N species which consists of ions, electrons and neutral particles. From a macroscopic point of view, a complete description of the flow field of a plasma should consist of the gasdynamic variables of all species, i.e., velocity vectors, pressure, density and temperature; the specific intensity of all species; and the electromagnetic fields. Such an analysis is known as multifluid theory of radiation magnetogasdynamics (RMGD). We are going to discuss the fundamental equations of RMGD in section II. For many engineering problems, we are mainly interested in the overall effects of the flow field. For instance, we would like to know the total pressure of the plasma as a whole on the surface of a body. We shall define the gross variables of the plasma as a whole from the partial variables of each species in the mixture and derive the fundamental equations of these gross variables. It is interesting to notice that the fundamental equations for these gross variables are identical to those of single fluid theory of RMGD. However, these fundamental equations for gross variables are, in general, not sufficient to describe all the flow phenomena in a plasma. They should be solved simultaneously with some equations of partial variables which represent diffusion phenomena, chemical reactions, ionization processes, different temperature of various species and other new phenomena. Since the complete set of equations of multifluid theory, whether they consist of all the equations of partial variables or the equations of gross variables together with those equations of some modified partial variables, is too complicated to be used for practical problems, we shall discuss various simplifications of these equations which have been successfully used in analyzing the flow problems of a plasma, including the classical magnetogasdynamic approximations. The basic principles and approximations used for the radiative transfer equation will be discussed in section 3 and those for the electromagnetic equations, particularly the equation of electrical current density will be discussed in section 4.

In the last four sections, we discuss some interesting flow problems with both thermal radiation effects and electromagnetic field effects. In sections 5 and 6, we discuss the wave motions in RMGD, while in sections 7 and 8, we discuss some problems associated with heat transfer in RMGD.

2. Multifluid Theory of Radiation Magnetogasdynamics [*1*], [*3*]

The variables in the multifluid theory of RMGD are:

The temperature of s th species T_s
The pressure of s th species p_s
The density of s th species $\rho_s = m_s n_s$
The velocity vector of s th species $\vec{q}_s$ with components u_s^i (2.1)
The specific intensity of thermal radiation of s th species $I_{\nu s}$
The electric field strength $\vec{E}$ with component E^i
and The magnetic field strength $\vec{H}$ with component H^i

where $s = 1, 2, \cdots, N$ and we assume that there are N-species in the plasma, $i =$ 1, 2, or 3 which represents one of the three spatial directions. The number density of s th species is n_s and the mass of a particle of s th species is m_s.

There are $7N+6$ variables in this theory which are functions of time t and spatial coordinates x^i. We should find $7N+6$ equations to govern these variables.

The variables T_s etc. are known as partial variables in multifluid theory. We may define the gross variables of the mixture as a whole from these partial variables as follows:

$$\text{Pressure of the mixture} = p = \sum_{s=1}^{N} p_s \tag{2.2}$$

$$\text{number density of the mixture} = n = \sum_{s=1}^{N} n_s \tag{2.3}$$

$$\text{density of the mixture} = \rho = mn = \sum_{s=1}^{N} m_s n_s \tag{2.4}$$

where m is the mean mass of the mixture which is a function of the composition of the mixture.

Temperature of the mixture $= T = (1/n) \sum_{s=1}^{N} n_s T_s$ (2.5)

i th velocity component of the mixture $= u^i = (1/\rho) \sum_{s=1}^{N} \rho_s u_s^i$ (2.6)

i th component of diffusion velocity of the mixture $= w_s^i = u_s^i - u^i$ (2.7)

From the definition of flow velocity u^i and diffusion velocity w_s^i, we have

$$\sum_{s=1}^{N} \rho_s w_s^i = 0 \tag{2.8}$$

Excess electric charge $= \rho_e = \sum_{s=1}^{N} \rho_{es} = \sum_{s=1}^{N} n_s e_s$ (2.9)

where e_s is the electric charge of a particle of s th species.

i th component of electrical current density $= J^i = \sum_{s=1}^{N} J_s^i = \sum_{s=1}^{N} \rho_{es} u_s^i = \sum_{s=1}^{N} \rho_{es} w_s^i$
$+ u^i \sum_{s=1}^{N} \rho_{es} = i^i + \rho_e u^i$ (2.10)

where $\vec{i}(i^i)$ is the electric conduction current density and $\rho_e \vec{q}$ is the electric convection current density.

Specific intensity of thermal radiation of the mixture $= I_\nu = \sum_{s=1}^{N} I_{\nu s}$ (2.11)

where ν is the frequency of the heat wave.

The fundamental equations which govern the partial variables and gross variables are the $6N$ gasdynamic equations, N radiative transfer equations and 6 electromagnetic field equations. They are as follows:

(i) *Equations of state.* The ideal gas law may be used and it is

$$p_s = R_A n_s T_s \tag{2.12}$$

where $R_A = 1.381 \times 10^{-16}$ cm-dyne/°C is the universal gas constant. The sum of N equations of the type of Eq. (2.12) gives the equation of state of the mixture as follows:

$$p = R_A n T = R \rho T \tag{2.13}$$

where R is the gas constant of the mixture which is a function of the composition of the mixture.

(ii) *Equation of continuity.* The conservation of mass of each species gives

$$\frac{\partial \rho_s}{\partial t} + \frac{\partial}{\partial x^i} (\rho_s u_s^i) = \sigma_s \tag{2.14}$$

where σ_s is the mass source per unit volume of s th species which is due to ionization process or other chemical reactions. The summation convention is used for the repeated tensorial indices i but not for indices distinguishing the species s. By the conservation of mass of the plasma, we have

$$\sum_{s=1}^{N} \sigma_s = 0 \tag{2.15}$$

The sum of N equations of the type of Eq. (2.14) gives the equation of continuity of the plasma as a whole:

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u^i}{\partial x^i}=0 \tag{2.16}$$

It is interesting to note that Eq. (2.14) is the diffusion equation used in ordinary gasdynamics but usually some simplified assumptions on the diffusion velocity are made so that we need not solve simultaneously the equation of diffusion velocity. Both Eqs. (2.13) and (2.16) are the same as those in single fluid theory.

If we multiply $\gamma_s=e_s/m_s$ to Eq. (2.14), we have

$$\frac{\partial \rho_{es}}{\partial t}+\frac{\partial}{\partial x^i}(\rho_{es}u_s^i)=\gamma_s\sigma_s \tag{2.17}$$

On principle, Eq. (2.17) is the same as Eq. (2.14). By conservation of total electric charge, we have

$$\sum_{s=1}^{N}\gamma_s\sigma_s=0 \tag{2.18}$$

From Eqs. (2.17) and (2.18), we have

$$\frac{\partial \rho_e}{\partial t}+\frac{\partial J^i}{\partial x^i}=0 \tag{2.19}$$

Eq. (2.19) is the well know equation of conservation of electric charge which is important in single fluid theory of RMGD. But in multifluid theory it is simply another form of the equation of continuity and may be used to replace one of the equation (2.14).

(iii) *Equation of motion.* The conservation of momentum of s th species gives the equation of motion of s th species as follows:

$$\frac{\partial \rho_s u_s^i}{\partial t}+\frac{\partial}{\partial x^j}(\rho_s u_s^i u_s^j-\tau_s^{ij})=X_s^i+\sigma_s Z_s^i \tag{2.20}$$

where $\sigma_s Z_s^i$ is the i th component of the momentum source per unit volume associated with the mass source σ_s. We have

$$\sum_{s=1}^{N}\sigma_s Z_s^i=0 \tag{2.21}$$

The term τ_s^{ij} is the ij th component of the stress tensor of s th species. In general the stress tensor is governed by a complicated partial differential equation. Within the approximation of the continuum theory, we may divide the stress tensor into two distinguish parts :one is due to thermal radiation which will be discussed in section 3 and the other is the viscous stress which may be expressed in terms of viscosity coefficient μ_s and the velocity gradient. The difference of the coefficient of viscosity of each species should be noted.

The body force X_s^i consists of electromagnetic force F_{es}^i, non-electric force such as gravitational force F_{gs}^i and the interaction forces between species F_{os}^i, which are given below:

$$F^i_{es}=\rho_{es}[E^i+(\vec{q}\times\vec{B})^i] \tag{2.22}$$

$$F^i_{gs}=\rho_s g^i \tag{2.23}$$

$$F^i_{os}=\sum_{s=1}^{N} K_{st}(u^i_t-u^i_s) \tag{2.24}$$

where $\vec{B}=\mu_e\vec{H}$ is the magnetic induction and μ_e is the magnetic permeability. K_{st} is known as the friction coefficient between species s and t and we have

$$\sum_{s=1}^{N} F^i_{os}=0 \tag{2.25}$$

The ith component of the gravitational acceleration is g^i. If we add all the N equations of the type of Eq. (2.20), we have the equation of motion of the mixture as follows:

$$\frac{\partial\rho u^i}{\partial t}+\frac{\partial\rho u^i u^j}{\partial x^j}=\rho\left(\frac{\partial u^i}{\partial t}+u^j\frac{\partial u^i}{\partial x^j}\right)=\rho\frac{Du^i}{Dt}=-\frac{\partial p_T}{\partial x^i}+\frac{\partial\tau^{ij}}{\partial x^j}+F^i_e+F^i_g \tag{2.26}$$

The form of Eq. (2.26) is exactly the same as that of the single fluid theory but it is interesting to find out the difference by examining the definition of various terms, particularly the stress tensor.

The non-electric body force such as gravitational force is simple and is

$$F^i_g=\sum_{s=1}^{N} E^i_{gs}=\rho^i_g \tag{2.27}$$

The electromagnetic force is

$$F^i_e=\sum_{s=1}^{N} F^i_{es}=\rho_e E^i+(\vec{J}\times\vec{B})^i \tag{2.28}$$

The stress tensor now consists of three parts: one is due to thermal radiation, the second is due to the viscous stress of each species and the third is due to the diffusion phenomena. The total pressure p_T is the sum of the radiation pressure and the gas pressure. Formally, we may write the stress tensor τ^{ij} as follows:

$$\tau^{ij}=\tau^{ij}_v+\tau^{ij}_R \tag{2.29}$$

where τ^{ij}_R is the radiative stress component which will be discussed in section 3. The term τ^{ij}_v is the viscous stress component of the mixture due to the molecular motion of the particles of the mixture which is

$$\tau^{ij}_v=\sum_{s=1}^{N}\tau^{ij}_{os}-\sum_{s=1}^{N}\rho_s w^i_s w^j_s \tag{2.30}$$

where τ^{ij}_{0s} is the ij th component of the viscous stress tensor of s th species. The interesting point is that the viscous stress τ^{ij}_v depends on the diffusion velocity. Only when the diffusion velocity is small and negligible, we may use the simple expression as Navier-Stokes relation for the viscous stress of the mixture as a whole. In general, we should solve the equations of diffusion velocity simultaneous with the equation of motion of the plasmas as a whole. The difference of Eqs. (2.20) and (2.26) gives the equation of diffusion velocity $w_r^i = u_r^i - u^i$ as follows:

$$\frac{\partial w_r^i}{\partial t} + u_r^j \frac{\partial u_r^i}{\partial x^j} - u^j \frac{\partial u^i}{\partial x^j} = \frac{1}{\rho_r} \frac{\partial \tau_r^{ij}}{\partial x^j} + \frac{1}{\rho} \frac{\partial p_T}{\partial x^i} - \frac{1}{\rho} \frac{\partial \tau^{ij}}{\partial x^j} + \frac{x_r^i}{\rho_r} - \frac{F_e^i + F_g^i}{\rho}$$
$$+ \frac{\sigma_r}{\rho_r}(Z_r^i - u_r^i) \qquad (2.31)$$

We shall show in section 4 that the ordinary generalized Ohm's law may be derived from Eq. (2.31) under a number of assumptions. It is always better if we could use Eq. (2.31) to replace the generalized Ohm's law as we shall see later.

(iv) *Equation of energy.* The conservation of energy of s th species gives the energy equation for s th species:

$$\frac{\partial \bar{e}_s}{\partial t} + \frac{\partial}{\partial x^j}(\bar{e}_s u_s^j - u_s^i \tau_s^{ij} - Q_s^j) = \bar{\varepsilon}_s \qquad (2.32)$$

where $\bar{e}_s = \rho_s \bar{e}_{ms}$ is the total energy of s th species of the mixture per unit volume which consists of the internal energy of s th species, kinetic energy of s th species, potential energy of s th species and radiation energy of s th species [1]. The heat flux Q_s^i consists of the heat flux due to thermal radiation and that due to conduction. The energy source ε_s consists of the term due to electromagnetic field ε_{es}, that due to chemical reactions ε_{cs} and that due to elastic collision between species ε_{os}.

Now if we add all N equations of the type of Eq. (2.32), we have the energy equation for the mixture as a whole:

$$\frac{\partial \rho \bar{e}_m}{\partial t} + \frac{\partial \rho u^j \bar{e}_m}{\partial x^j} = -\frac{\partial p_T u^j}{\partial x^j} + \frac{\partial u^i \tau^{ij}}{\partial x^j} - \frac{\partial Q^j}{\partial x^j} + \varepsilon_T \qquad (2.33)$$

The form of Eq. (2.33) is the same as that of single fluid theory but the meanings of various terms are different.

The total energy $\bar{e}_m$ consists of the internal energy, kinetic energy, potential energy and radiation energy but the definition of the internal energy per unit mass of the mixture as a whole consists of the diffusion kinetic energy as well as the ordinary internal energy as follows:

$$U_m = \frac{1}{\rho} \sum_{s=1}^{N} (\rho_s U_{ms} + \frac{1}{2} \rho_s w_s^2) \qquad (2.34)$$

where U_{ms} is the internal enegy of s th species. The total internal energy is the sum of internal energy of all species and the diffusion energy of all species. If the diffusion velocities are not small, we have to solve this energy equation with the equation of diffusion velocities. Since the internal energy of each species depends on its partial temperature T_s, we have to solve the equation for T_s with Eq. (2.33) if the temperatures of all species are not equal.

The heat flux Q^j consists of the part due to heat conduction Q_c^j and the other part due to thermal radiation Q_R^j. Both Q_c^j and Q_R^j depend on w_s^j and T_s.

The total energy source ε_T consists of the part due to chemical reaction and the other part due to electromagnetic fields.

(v) *Maxwell's equations of electromagnetic fields.* The electromagnetic fields are governed by Maxwell's equations:

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{2.35}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2.36}$$

where

$$\vec{D} = \epsilon \vec{E} = \text{dielectric displacement} \tag{2.37}$$

and ϵ is the inductive capacity. ∇ is the gradient operator. The interaction of the electromagnetic field equations with the flow of a plasma is through the electrical current density $\vec{J}$ which represents the relative motion of various charged particles in the plasma.

(vi) *Radiative transfer equation* [2]. The conservation of radiative energy gives the radiative transfer equation:

$$\frac{1}{c}\frac{\partial I_{\nu s}}{\partial t} + n^i \frac{\partial I_{\nu s}}{\partial x^i} = \rho_s k_{\nu s}(J_{\nu s} - I_{\nu s}) \tag{2.38}$$

where n^i is the ith component of the direction cosine of the ray of radiation with respect to ith axis, $k_{\nu s}$ is the absorption coefficient of sth species, $J_{\nu s}$ is the source function of radiation of sth species.

The sum of N equations of the type of Eq. (2.38) gives the radiative transfer equation for the mixture:

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + n^i \frac{\partial I_\nu}{\partial x^i} = \rho k_\nu (J_\nu - I_\nu) \tag{2.39}$$

where c is the velocity of light. The absorption coefficient of the mixture as a whole is k_ν and the corresponding source function is J_ν. We are going to discuss the radiative transfer equation and the meaning of various terms in section 3.

3. Radiative Transfer • Basic Principles [2]

We may either consider the thermal radiation as a stream of photons or as electromagnetic waves. When we consider the thermal radiation as a stream of photons, we should use the relativistic Boltzmann equation to study the motion of these particles. When we consider the thermal radiation as electromagnetic waves, we may use the geometrical optics to study the behavior of thermal radiation from the macroscopic point of view. The results of these two approaches are the same (see Reference 2). In this report, we consider only the case of continuum theory and we shall use the treatment of geometrical optics. The thermal radiation may be expressed in terms of a specific intensity I_ν which is defined as follows:

$$I_\nu = \lim_{d\sigma_0, d\omega, dt, d\nu \to 0} \left(\frac{dE_\nu}{d\sigma_0 \cdot \cos\theta \cdot d\omega \cdot dt \cdot d\nu} \right) \tag{3.1}$$

where I_ν is a function of time, spatial coordinates, direction θ and the frequency of the wave ν. The amount of radiative energy flowing through the area $d\sigma_0$ in the frequency range ν and $\nu+d\nu$, in the direction L which makes an angle θ with the normal of $d\sigma_0$ within an solid angle $d\omega$ in the time interval dt is dE_ν. The total amount of energy radiated over the whole spectrum is

$$dE = \int_0^\infty \left(\frac{dE_\nu}{d\nu} \right) d\nu = I \cos\theta d\omega d\nu dt \tag{3.2}$$

where

$$I = \int_0^\infty I_\nu d\nu = \text{integrated intensity of thermal radiation.} \tag{3.3}$$

If the specific intensity I_ν is known, we may easily calculate the effects of thermal radiation on the flow field which are:

(i) The flux of heat energy by radiation is q_R^i, *i.e.*,

$$q_R^i = \int_{4\pi} I n^i d\omega \tag{3.4}$$

where n^i is the directional cosine of the radiation ray with respect to ith axis. We should add the divergence of this radiative heat flux in the energy equation i.e.,

$$Q_R = \nabla \cdot \vec{q}_R \tag{3.5}$$

In general, Q_R is a differentio-integral expression and the energy equation is a differentio-integral equation.

(ii) Energy density of radiation E_R. The energy density of radiation within the frequency range ν and $\nu+d\nu$ is

$$u_\nu = \frac{1}{c} \int_{4\pi} I_\nu d\omega \tag{3.6}$$

and the energy density of radiation for the whole spectrum is then

$$E_R = \int_0^\infty u_\nu d\nu = \frac{1}{c} \int_{4\pi} I d\omega \tag{3.7}$$

This radiative energy should be added to the total energy of the gas and it behaves similar to internal energy of the gas.

(iii) Stress tensor of radiation. The ijth component of stress tensor of radiation is

$$\tau_R^{ij} = -\frac{1}{c} \int_{4\pi} I n^i n^j d\omega \tag{3.8}$$

We may define a radiation pressure p_R as follows:

$$p_R = -\frac{1}{3}(\tau_R^{11} + \tau_R^{22} + \tau_R^{33}) = \frac{1}{3c}\int_{4\pi} I d\omega = \frac{1}{3} E_R \tag{3.9}$$

It should be noticed we consider the gas consisting of one species in the above formulas. If the gas consists of more than one species, we should have similar expressions for each species. For simplicity, in this section we consider the case for a single species only.

For macroscopic treatment, the specific intensity of radiation may be expressed in terms of two overall coefficients: one is the absorption coefficient of radiation k_ν and the other is emission coefficient of radiation j_ν. They are defined as follows:

(i) Absorption coefficient k_ν of radiation. The loss of specific intensity along the ray of radiation over a distance ds is

$$dI_\nu = -\rho k_\nu I_\nu ds \tag{3.10}$$

The integration of Eq. (3.10) gives

$$I_\nu(s) = I_\nu(s_0) \exp\left(-\int_{s_0}^{s} \rho k_\nu ds\right) = I_\nu(s_0) \exp(-\tau_\nu) \tag{3.11}$$

where τ_ν is known as optical thickness of radiation of the layer $(s - s_0)$ and

$$L_{R\nu} = \frac{1}{\rho k_\nu} = \text{mean free path of radiation} \tag{3.12}$$

The absorption coefficient k_ν is a function of the temperature and density of the medium as well as the frequency ν. In general k_ν consists of two parts: one is the true absorption and the other is that due to scattering. In macroscopic theory of radiation gasdynamics, we assume that k_ν is a known function of temperature, density and frequency ν. The determination of k_ν can be made by microscopic theory or experiment. The absorption coefficient in radiation gasdynamics has similar position as other transport coefficients such as that of viscosity, thermal conductivity in ordinary gasdynamics. It may be considered as a new transport coefficient.

(ii) Emission coefficient of radiation j_ν. The radiation energy emitted from a mass dm is

$$dE_e = j_\nu dm d\omega d\nu dt \tag{3.13}$$

If we have both the absorption coefficient and the emission coefficient of the medium, the conservation of radiative energy gives the radiative transfer equation which governs the specific intensity, i.e.,

$$dE_0 - dE_i = dE_e + dE_a - dE_t \tag{3.14}$$

The difference between the outgoing radiative energy dE_0 and the incoming radiative energy dE_i must be equal to the sum of the energy emitted and energy absorpted minus the net change of radiative energy in the volume with time. Eq. (3.14) in terms of k_ν and j_ν is

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t}+n^i\frac{\partial I_\nu}{\partial x^i}=\rho k_\nu(J_\nu-I_\nu) \tag{3.15}$$

where $J_\nu = j_\nu/k_\nu$ = source function of radiation (3.16)

One of the most difficult problems in radiation gasdynamics is to determine the source function of radiation. At present stage of investigation, we usually use the assumption of local thermodynamic equilibrium which may be considered as a first approximation of the actual case. After we know more about the results under local thermodynamic condition, we should study the source function in non-equilibrium condition.

Under complete thermodynamic equilibrium, the specific intensity of thermal radiation is given by Planck radiation function B_ν which is also known as black body radiation function and which is

$$B_\nu(\nu, T)=\frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1} \tag{3.17}$$

The properties of B_ν had been known before Planck found the correct expression (3.17). For instance, Kirchhoff knew that under thermodynamic equilibrium:

$$I_\nu=j_\nu/k_\nu=B_\nu(T) \tag{3.18}$$

The function $B_\nu(T)$ must be a function of temperature and independent of the material. Wien found the displacement law

$$\frac{U_\lambda}{T^5}=G\left(\frac{c}{\lambda T}\right) \tag{3.19}$$

where $U_\lambda d\lambda = U_\nu d\nu$ and λ is the wave length. Rayleigh-Jean found that at low frequency:

$$B_\nu(\nu, T)=(1/c^2)8\pi\nu^2 kT \tag{3.20}$$

Of course, Eq. (3.20) is not valid for high frequency which causes the untraviolet catastroph of Rayleigh-Jeans law.

If we assume that the gas is in local thermodynamic equilibrium, i.e., the emission is determined by the local temperature, equation (3.15) becomes

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t}+\frac{\partial I_\nu}{\partial s}=\rho k'_\nu(B_\nu-I_\nu) \tag{3.21}$$

where

$$k'_\nu=k_\nu[1-\exp(-h\nu/kT)] \tag{3.22}$$

where h is the Planck constant and k is the Boltzmann constant. The reduction of the absorption coefficient is due to induced emission.

In general we have to solve the radiative transfer equation (3.15) or (3.21) with other fundamental equations in RMGS with radiation terms in integral forms. These differentio-integral equations are very difficult to solve. In order to get some essential features of the effects of thermal radiation, some approximations have been used. They are discussed in some detail in reference 2. Here we shall list a few of the most common ones:

(i) Optical thick medium. When the mean free path of radiation is very small, the solution of Eq. (3.21) may be expressed as follows:

$$I_\nu = B_\nu - L_{R\nu}\left(n^i \frac{\partial I_\nu}{\partial x^i}\right) + O(L_{R\nu}^2) \tag{3.23}$$

where $L_{R\nu} = 1/\rho k'_\nu$. If we neglect higher order terms of $L_{R\nu}^2$, the radiation terms in gasdynamic equations can be easily evaluated, i.e.,

$$E_R = a_R T^4 = 3p_R \tag{3.24}$$

where a_R is the Stefan-Boltzmann constant and

$$\vec{q}_R = D_R \nabla E_R \tag{3.25}$$

and

$$D_R = c/(3\rho K_R) \tag{3.26}$$

and

$$K_R = \left(\int_0^\infty \frac{\partial B_\nu}{\partial T} d\nu\right) \Big/ \left(\int_0^\infty \frac{1}{k'_\nu} \frac{\partial B_\nu}{\partial T} d\nu\right)$$
$$= \text{Rosseland mean absorption coefficient} \tag{3.27}$$

For the optical thick medium, the differentio-integral equations are reduced to differential equations. The equations are essentially the same as ordinary Navier-Stokes equations of a compressible fluid with a few more terms.

(ii) One dimensional approximation. If the mean free path of radiation is not small, we have to use more terms in the series expansion of Eq. (3.23). It is questionable how many terms we should use and it is not known about the convergence of this series. Hence it is better to solve the integro-differential equations. In many practical problems such as boundary layer flows, the gradient of temperature in one direction is much larger than those in the other directions. Hence we may assume that the specific intensity I_ν is essential a function of this predominant coordinate, say y,i.e., $I_\nu(y, \theta)$. If we define the optical thickness in terms of y,i.e.,

$$\tau_\nu = \int_0^y \rho k'_\nu dy \tag{3.28}$$

we may carry out the integration in solid angle θ and ϕ. Hence the integral expressions of radiative terms will be simplified. We shall discuss the actual expression in section 7.

(iii) Gray gas approximation. The absorption coefficient k'_ν is in general a function of frequency ν. As a result, the integral expression of radiation terms may not be integrated analytically. Gray gas approximation has been used by assuming that the absorption coefficient is independent of frequency. Actually we use some mean value of the absorption coefficient for the overall flow phenomena. Rosseland mean absorption coefficient (3.27) is the one for optically thick medium. Planck mean absorption coefficient K_P is the one used for finite mean free path of radiation which is defined as

$$K_p=\frac{\int_0^\infty k'_\nu B_\nu d\nu}{\int_0^\infty B_\nu d\nu} \tag{3.29}$$

4. GENERAL CONSIDERATIONS OF ELECTROMAGNETIC EQUATIONS [1], [3]

Among the electromagnetic equations, it is usually to make considerable simplifications in the equation of electrical current density in magnetogasdynamics. It is worthwhile to examine these simplifications and to find ways to improve the results. By definition, (Eq. (2.10)), the electrical current density is

$$J^i=\sum_{s=1}^{N}\rho_{es}w_s^i+\rho_e u^i=i^i+\rho_e u^i \tag{4.1}$$

The most difficult part is to find the proper equation for the conducting current i^i. Since i^i is essentially a function of the diffusion velocity w_s^i, we may easily calculate i^i if we know all the diffusion velocities w_s^i. The exact diffusion velocity equation from macroscopic point of view is as follows: (cf. Eq. (2.31))

$$\frac{\partial w_r^i}{\partial t}+u_r^j\frac{\partial u_r^i}{\partial x^j}-u^j\frac{\partial u^i}{\partial x^j}=\frac{1}{\rho_r}\frac{\partial \tau_r^{ij}}{\partial x^j}+\frac{1}{\rho}\frac{\partial p_r}{\partial x^i}-\frac{1}{\rho}\frac{\partial \tau^{ij}}{\partial x^j}$$
$$+\frac{x_r^i}{\rho_r}-\frac{F_e^i+F_q^i}{\rho}+\frac{\sigma_r}{\rho_r}(Z_r^i-u_r^i) \tag{4.2}$$

Since it is extremely difficult to solve Eq. (4.2) with other magnetogasdynamic equations, the following assumptions have been made in order to get some essential features of the flow of an electrically conducting fluid:

(i) The electrical conducting current or the diffusion velocity is explicitly independent of time t and spatial coordinates x^i.

(ii) The electromagnetic forces are the only dominant forces in determining the diffusion velocities and then the electrical conduction current.

(iii) There is no source term in the process.

Under these assumptions, Eq. (4.2) is reduced to the simple form [1]:

$$(\rho\rho_{er}-\rho_r\rho_e)\vec{E}_u+(\rho\rho_{er}\vec{w}_r-\rho_r\vec{i})\times\vec{B}=\rho\sum_{s=1}^{N}K_{sr}(\vec{w}_r-\vec{w}_s) \tag{4.3}$$

where

$$\vec{E}_u=\vec{E}+\vec{q}\times\vec{B} \tag{4.4}$$

Eq. (4.3) gives a set of linear algebraic equations for $\vec{w}_r$ if we assume that ρ_r, ρ_{er}, $\vec{E}_u$, $\vec{B}$ and K_{sr} are given. After we slove for w_r, we may easily find the conducotin current $\vec{i}$. However the general solution of Eq. (4.3) gives a result which is still more complicated than the simple generalized Ohm's law used in the classical magnetogasdynamics.

In order to find the relation of Eq. (4.3) with the well known simple generalized Ohm's law in classical magnetogasdynamics and their limitations, we consider the case of a slightly ionized monatomic gas such as argon, which consists of electrons (subscript e), ions (subscript i) and neutral atoms (subscript a). We assume that the gas is slightly ionized, the relations between the number densities of these three species are

$$n_e \cong n_i \ll n_a \tag{4.5}$$

We further assume that the ions are singly charged so that

$$\rho_e = -en + en_i \cong 0 \tag{4.6}$$

and the mass density of the plasma is

$$\rho = m_e n_e + m_i n_i + m_a n_a \cong m_a(n_e + n_a) \tag{4.7}$$

where the masses of these species have the relation: $m_e \ll m_i \cong m_a$.

The diffusion velocities of these species have the relation:

$$m_e n_e \vec{w}_e + m_i n_i \vec{w}_i + m_a n_a \vec{w}_a = 0 \tag{4.8}$$

Since the three terms of Eq. (4.8) should be of the same order of magnitude, we have

$$/\vec{w}_e/ \gg /\vec{w}_i/ \gg /\vec{w}_a/ \tag{4.9}$$

The general solution of Eq. (4.3) may be written in the following form:

$$\sigma\vec{E}_u = A_1\vec{i} + A_2(\vec{i}\times\vec{B}) + A_3(\vec{i}\times\vec{B})\times\vec{B} \tag{4.10}$$

The factors A_1, A_2 and A_3 depend on the collision frequencies between species, their number densities and the magnetic induction B. Usually the collision effects due to neutral particles are negligible, and then Eq. (4.10) may be written in the following form:

$$\vec{i} = \frac{1+\beta_i\beta_e+\beta_e^2}{(1+\beta_i\beta_e)^2+\beta_e^2}\cdot\sigma\vec{E}_u^{\,\prime\prime} + \frac{(1+\beta_i\beta_e)\sigma\vec{E}_u^{\perp}}{(1+\beta_i\beta_e)^2+\beta_e^2} + \frac{\sigma\beta_e}{(1+\beta_i\beta_e)^2+\beta_e^2}\left(\frac{\vec{B}}{B}\times\vec{E}_u\right) = \sigma_T\vec{E}_u \quad \text{(say)} \tag{4.11}$$

where $\vec{E}_u^{\,\prime\prime} = (\vec{E}_u\cdot\vec{B})(\vec{B}/B^2) =$ component of $\vec{E}_u$ parallel to $\vec{B}$

$\vec{E}_u^{\perp} = \vec{B}\times(\vec{E}_u\times\vec{B})/B^2 =$ component of $\vec{E}_u$ perpendicular to $\vec{B}$

$\sigma = n_e^2 e^2/K_{ei} =$ scalar electric conductivity (4.12)

$\beta_e = en_eB/K_{ei} = \omega_c/f =$ Hall factor

$\omega_c = en_eB/m_e =$ cyclotrone frequency of electrons

$f =$ collision frequency between ions and electrons $= K_{ei}/m_e$

$$\beta_i = \frac{en_e B}{\frac{n_e}{n_a} K_{ea} + \left(1 + \frac{n_e}{n_a}\right) K_{ai}} = \text{ion slip factor}$$

If the strength of the magnetic induction is not too large and the density of the plasma is not too low, we usually have the relation $1 \gg \beta_e \gg \beta_i$. If we neglect both β_e and β_i, we have the simple generalized Ohm's law of classical magnetogasdynamics from Eq. (4.11), i.e.,

$$\vec{i} = \sigma \vec{E}_u \tag{4.13}$$

and the electrical conductivity of the plasma is a scalar quantity σ. If the Hall factor β_e or both the Hall factor β_e and the ion slip factor β_i are not negligible, we should use Eq. (4.11), the electrical conductivity σ_T of the plasma is then a tensor quantity and the conduction current $\vec{i}$ is in general not parallel to the electric field $\vec{E}_u$.

Since the diffusion velocity and the electric conduction current depend also on other forces than the electromagnetic forces alone, Eq. (4.11) is only a first approximation. We should include other forces if more accurate results are required. Since the diffusion velocity of the electrons $\vec{w}_e$ is the major component of the conduction current, we may include the pressure gradient of the electrons in the equation of the diffusion velocity of electrons as a first improvement of the generalized Ohm's law. If we neglect β_i and include only β_e, the equation of electric conduction current becomes:

$$\vec{i} + \frac{1}{en_e}(\vec{i} \times \vec{B}) = \sigma \left(\vec{E}_u + \frac{1}{en_e} \nabla p_e\right) = \sigma \vec{E}_{uT} \tag{4.14}$$

The effect of the pressure gradient of electrons is to increase the effective electric field strength. If we consider the ion slip factor β_i, other additional terms due to ∇p_e will be added to Eq. (4.14). Since such a piecewise improvement of the electric current equation will not include all the gasdynamic effects, particularly the effects of different temperatures between species, the author prefers to the use of multifluid theory in which all the gasdynamic effects on the electric current density will be included.

I would like to point out that for different forms of the electric current density equation, the flow pattern of an electrical conducting fluid may be entirely different under the same geometric boundary and the same externally applied electromagnetic fields. For instance, we consider the flow of an electrically conducting fluid between two infinite parallel plates under a uniform magnetic field perpendicular to the plates. If the average flow direction is along the x-axis and the applied magnetic field is along the y-direction, the plates are in the planes of y=constant and the z-direction is perpendicular to both x- and y-axis. If the electric conductivity of the fluid is a scalar quantity, we may assume that only velocity component which is different from zero is the x-component $u(y)$ and the x-component of the magnetic field is not zero but the z-component of the magnetic field is zero. However, if the electric conductivity of the fluid is a tensor quantity, both the z-component of velocity and the z-component of magnetic field are not zero, even though the average mass flow of the fluid is in the x-direction only. We will find sinusoidal distribution of the z-component of velocity with zero net flow in the z-direction when the electric conductivity of the fluid is a tensor [*3*].

For more general form of the equation of electrical current density, we expect that many new phenemena will be found which will be missed in the classical theory of magnetogasdynamics where Eq. (4.13) is used.

In the last three sections, we discuss briefly the fundamental equations of radiation magnetogasdynamics. In the next four sections, we are going to consider some simple flow problems based on these fundamental equations or its simplified forms. We are especially interested in the effects of radiative transfer as well as the multi-fluid treatment on the flow field of a plasma or a high temperature gas.

5. Wave Motions in Radiation Magnetogasdynamics Single Fluid Theory

The study of wave propagation in a radiating and ionized gas has both academic interest and practical applications. The wave motion will bring out many characteristic features in radiation magnetogasdynamics which may differ considerably from those in ordinary gasdynamics. The practical applications of wave propagation are numerous. Some of the important applications are: (i) space communication systems, (ii) radio wave propagation in ionosphere, (iii) magnetofluid dynamic power generation, (iv) travelling wave tubes, (v) many geophysical problems such as geomagnetic storms, auroras and other ionospheric disturbances, (vi) many astrophysical problems such as solar flares, generation of cosmic rf radiation, etc. (vii) diagnostic and confinement schemes in nuclear fusion devices and (viii) other problems of plasma dynamics associated with wave phenomena such as plasma jets. Because of these wide range of interest, wave motion in a plasma has been extensively studied.

The properties of a wave in a radiating and electrically conducting fluid depend on the amplitude of the wave. The simplest type of wave is the wave of infinitesimal amplitude. Ordinary sound wave and radio waves belong to such a group. Both of these waves may be considered as a special case of magnetogasdynamic waves. In fact, the magnetogasdynamic wave is a resultant wave due to the interaction of a sound wave and an electromagnetic wave by the means of an externally applied magnetic field. Such an interaction will give us many new phenomena which are not in either ordinary gasdynamics nor in ordinary electrodynamics. Thermal radiation may interact with ordinary sound wave and electromagnetic wave too. One of the main features of these waves of infinitesimal amplitude is that superposition principle is applicable to these waves. Mathematically speaking, we may linearize the equations which govern these waves. Since these equations are linear, the sum of two solutions of them is also a solution of these equations. Thus we may study any typical solution of these wave equations which will give the general features of the wave propagation.

For waves of finite amplitude, the shape of the wave will distort as the wave propagates while the wave of infinitesimal amplitude will maintain its shape when it propagates. When the distortion is large, ordinary waves will develop into shock wave in which a large change of physical variables occurs in a very thin region. We are going to discuss the waves of infinitesimal amplitude in radiation magnetogasdynamics first and then the shock wave.

(i) *Waves of small amplitude in an optically thick and electrically conducting medium.* In the study of propagation of wave of small amplitude, we consider the equilibrium condition under small disturbances. Hence the details of the wave motion depend on the forces considered. The more detail of the flow field is investigated, more modes of wave will be found. First we consider a simple case but the effects of thermal radiation and electromagnetic fields will be brought out clearly. The case which we consider is that the governing equations are those given by the single fluid theory with simple generalized Ohm's law (4.13) and simple expressions of radiation terms by Eqs. (3.24) and (3.25).

We assume that originally the plasma is at rest with a pressure p_0, a temperature T_0 and a density ρ_0 and that it is subjected to an externally applied uniform magnetic field $\vec{H}_0=\vec{i}H_x+\vec{j}H_y+\vec{k}0$ where $\vec{i}$, $\vec{j}$, and k are respectively the x-, y- and z-wise unit vectors; H_x and H_y are constant. There is no electrical current, nor excess electric charge, nor applied electric field. The plasma is perturbed by a small disturbance so that in the resultant disturbed motion, we have

$$u=u(x,t);\ v=v(x,t);\ w=w(x,t);\ p=p_0+p'(x,t);\ T=T_0+T'(x,t)$$
$$\rho=\rho_0+\rho'(x,t);\ \vec{E}=\vec{E}(x,t);\ \vec{J}=\vec{J}(x,t);\ \rho_e=\rho_e(x,t);\ \vec{H}=\vec{H}_0+\vec{h}(x,t) \tag{5.1}$$

where u, v, and w are respectively the perturbed x-, y- and z-components, prime refers to the pertubed quantities, $\vec{E}$, $\vec{J}$, ρ_e and $\vec{h}$ are the electromagnetic pertubed quantities with their usual meanings. For simplicity, we assume that all the perturbed quantities are functions of one spatial coordinate x and time t only. Thus we consider the wave propagation in the x-direction. Substituting Eq. (5.1) into the fundamental equations and neglecting the higher order terms, we have 16 linear partial differential equations for the 16 magnetogasdynamic variables. These magnetogasdynamic variables may be divided into three independent groups:

(a) $h_x=0$ which is independent of all the other variables.

(b) Transverse waves: w, h_z, J_x, J_y, E_x, E_y and ρ_e.

(c) Longitudinal waves: u, v, p', T', ρ', h_y, J_z and E_z

We are looking for periodic solution in which all the perturbed quantities are proportional to

$$\exp[i(\omega t-\lambda x)]=\exp[-i\lambda_R(x-Vt)]\exp(\lambda_i x) \tag{5.2}$$

where ω is a given real angular frequency, $\lambda=\lambda_R+i\lambda_i$ is the complex wave number, $i=\sqrt{-1}$ and

$$V=\frac{\omega}{\lambda_R}=\text{speed of wave propagation} \tag{5.3}$$

Substituting the expressions in the form of Eq. (5.2) into the linearized equations of radiation magnetogasdynamics, we obtain the dispersion relation $\lambda(\omega)$ for both the transverse and the longitudinal waves.

The dispersion relation for the *transverse wave* is

$$\left(i\omega-\nu_H\frac{\omega^2}{c^2}\right)\left[(i\omega+\nu_g\lambda^2)\left(i\omega+\nu_H\lambda^2-\nu_H\frac{\omega^2}{c^2}\right)+V_x^2\left(\lambda^2-\frac{\omega^2}{c^2}\right)\right]$$
$$-\frac{\omega^2}{c^2}V_y^2\left(i\omega+\nu_H\lambda^2-\nu_H\frac{\omega^2}{c^2}\right)=0 \tag{5.4}$$

where ν_g is the kinematic viscosity, $\nu_H=\frac{1}{\sigma\mu_e}$ is the magnetic diffusivity, $V_x=H_x(\mu_e/\rho_0)^{1/2}$ is the x-component of Alfven's wave velocity and $V_y=H_y(\mu_e/\rho_0)^{1/2}$ is the y-component of Alfven's wave velocity. From Eq. (5.4), we see that the transverse wave is independent of compressibility and thermal radiation. There are two basic transverse waves which are (i) viscous wave depending on ν_g and (ii) electromagnetic wave depending on ν_H. Eq. (5.4) gives two roots of λ^2 which represent the interaction of these two basic transverse waves due to the applied magnetic field. In classical magnetogasdynamics where the displacement current is neglected, all the terms with $(1/c^2)$ in Eq. (5.4) vanish. For an ideal plasma, i.e., $\nu_g=\nu_H=0$, Eq. (5.4) gives the Alfven's wave.

The dispersion relation of the *longitudinal wave* is

$$\left[K^*\left(\frac{1}{\rho_0}+i\frac{4\omega\nu_g}{3p_0}\right)\lambda^4-\left\{\frac{\omega^2K^*}{p_0}+\frac{4\nu_g\omega^2}{3T_0(\gamma-1)}(1+12(\gamma-1)R_p)\right.\right.$$
$$\left.\left.-i\omega(C_p+20RR_p+16RR_p^2)\right\}\lambda^2-\frac{i\omega^3}{T_0(\gamma-1)}(1+12(\gamma-1)R_p)\right]$$
$$\left[\left(\lambda^2\nu_H+i\omega-\nu_H\frac{\omega^2}{c^2}\right)(i\omega+\nu_g\lambda^2)+V_x^2\left(\lambda^2-\frac{\omega^2}{c^2}\right)\right]$$
$$-\left(\lambda^2-\frac{\omega^2}{c^2}\right)(i\omega+\nu_g\lambda^2)\left[\frac{\omega^2}{T_0(\gamma-1)}(1+12(\gamma-1)R_p)-\frac{i\omega K^*\lambda^2}{p_0}\right]V_y^2=0 \tag{5.5}$$

where $K^*=K+12RR_p\rho_0D_R$ is the effective coefficient of heat conductivity with thermal radiation and $R_p=a_RT_0^3/3R\rho_0$ is the ratio of radiation pressure to gas pressure.

The first square bracket of Eq. (5.5) gives the sound waves in radiation gasdynamics, the second square bracket is the transverse magnetogasdynamic waves and the last term gives the interaction between the sound wave and electromagnetic waves.

Some simple waves may be obtained from Eq. (5.5) as special cases:

(a) *Sound waves.* The first square bracket of Eq. (5.5) gives two roots of λ^2 which represent two modes of sound waves in a viscous, heat-conducting and radiating gas. These modes are the modifications of ordinary sound wave and the heat wave. The heat wave depends on the effective thermal conductivity K^*. If $K^*=0$, there is no heat wave. In an inviscid and non-heat-conducting gas, there is no heat wave if the thermal radiation is also negligible. If the thermal radiation is not negligible, it will introduce a heat wave even if the ordinary thermal conduction is negligible. Ordinary sound wave exists in an inviscid, non-heat-conducting and non-radiating heat flux medium. The sound wave speed will be increased by radiation pressure. If we take $\nu_g=0$, $K^*=0$, the first square bracket will give a sound speed, i.e., the speed of wave propagation V as follows:

$$V=\frac{\omega}{\lambda}=C_R=a_0\sqrt{\frac{1+20\left(\frac{\gamma-1}{\gamma}\right)R_p+16\left(\frac{\gamma-1}{\gamma}\right)R_p^2}{1+12(\gamma-1)R_p}} \tag{5.6}$$

where a_0 is ordinary sound speed with $R_p=0$. Hence thermal radiation increases the sound speed.

(b) Radiation magnetogasdynamic waves. If $V_y=0$, there is no interaction between sound wave and electromagnetic wave. If $V_y\neq 0$, there is interaction between sound wave and electromagnetic wave. In general, Eq. (5.5) gives four different roots of λ^2 which represent four different modes of the longitudinal wave of RMGD, which are due to the interaction of heat wave, sound wave, viscous wave and electromagnetic wave. For an ideal plasma $\nu_g=\nu_H=K^*=0$ with magnetogasdynamic approximation, Eq. (5.5) becomes

$$(C_R^2-V^2)(V_x^2-V^2)-V^2V_y^2=0 \tag{5.7}$$

where V is the speed of propagation of the RMGD waves in an ideal plasma. Eq. (5.7) is identical to the well known equation of fast and slow waves of magnetogasdynamics except that the radiation sound speed C_R replaces the ordinary sound speed a_0.

(ii) *Waves of small amplitude in an optically thin medium.* For optically thin medium, we have to use the integro-differential equations to derive the dispersion relation. Because of the dependence of the absorption coefficient with frequency, the speed of propagation depends on the frequency of the wave. The complete solution of this problem has not been obtained yet. However, some special cases have been studied which are given in reference [*2*].

(iii) *Shock waves in an optically thick medium.* One dimensional flow analysis may be used to study this problem. The fundamental equations are:

$$\rho u=\text{constant}=m \tag{5.8}$$

$$mu+p_t+(1/2)\mu_e H^2-(4/3)\mu(du/dx)=\text{constant}=mC_1 \tag{5.9}$$

$$mh_R+up_t-(4/3)\mu u(du/dx)-K^*(dT/dx)+EB=\text{constant}=mC_2 \tag{5.10}$$

$$uH-\nu_H(dH/dx)=\text{constant}=E \tag{5.11}$$

where $B=\mu_e H$ is the transverse magnetic induction, p_t is the sum of radiation pressure and gas pressure, and h_R is the effective enthalphy with radiation effect, i.e., the enthalphy plus radiation energy density.

We may obtain the Rankine-Hugoniot relation and the shock structure by solving the system of equations (5.8) to (5.11).

(a) *Rankine-Hugoniot relations.* From Eqs. (5.8) to (5.11), we have the following Rankine-Hugoniot relations for a normal shock in a radiating and electrically conducting medium under a transverse magnetic field.
The drop in velocity across the normal shock is given by the ratio:

$$\frac{u_2}{u_1}=\frac{1}{2}\left[\frac{\gamma_e-1}{\gamma_e+1}+\frac{2\gamma_e(P_e+h_1^2)}{\gamma_e+1}\right]+\frac{1}{2}\left\{\left[\frac{\gamma_e-1}{\gamma_e+1}+\frac{2\gamma_e(P_e+h_1^2)}{\gamma_e+1}\right]^2 +\gamma h_1^2\frac{(2-\gamma_e)}{(\gamma_e+1)}\right\}^{1/2} \tag{5.12}$$

where

$$\gamma_e=\frac{4(\gamma-1)R_{p2}+\gamma}{3(\gamma-1)R_{p2}+1}, \qquad P=(1/\gamma M_1^2)$$

$$P_e=(1+R_{p1})f(R_{p2})P, \qquad h_1=\frac{H_1}{\sqrt{2mu_1/\mu_e}}$$

The jump in temperature is given by the relation

$$\frac{T_2}{T_1}=\left\{\frac{1+R_{p1}}{R_{p1}}+\frac{6(M_e^2-1)[\gamma+20(\gamma-1)R_{p1}+16(\gamma-1)R_{p1}^2]}{7R_{p1}[1+12(\gamma-1)R_{p1}]}\right\}^{1/4} \tag{5.13}$$

where subscript 1 refers to the value in front of the shock and subscript 2 refers to the value behind the shock.

As the effective Mach number in front of the shock $M_{e1}=u_1/C_{R1}$ is very large, Eq. (5.13) becomes

$$T_2/T_1=1.033M_{e1}^{1/2} \tag{5.14}$$

where we take $\gamma=5/3$. It is interesting to notice that with radiation effects, the jump in temperature is proportional to the square root of the Mach number M_{e1} while without radiation effect, the jump of temperature is proportional to the square of the Mach number M_1 as M_1 is very large.

(b) *Shock wave structure.* There are three diffusion phenomena, i.e., (i) molecular diffusion (viscosity and heat conduction), (ii) radiation heat diffusion and (iii) charged particle diffusion (electrical conductivity). The complete problem has not been solved particularly for large radiation pressure number. (see reference [2]).

6. Wave Motions in Radiation Magnetogasdynamics Multifluid Theory [4], [5]

Since the multifluid theory considers more detail of flow field than the single fluid theory, we would expect that new phenomena will appear in multifluid theory which have been neglected in single fluid theory. The general treatment of multifluid theory is very complicated and has not been extensively studied yet, particularly when the thermal radiation effects are considered. In order to show the main features of multifluid theory, in this section, we consider a simple case in which the medium is assumed to be a fully ionized plasma which consists of electrons and one kind of singly charged ions ($N=2$) with the following assumptions:

(i) Both ions and electrons may be considered as inviscid and non-heat-conducting gas,

(ii) Thermal radiation effects are negligible,

(iii) Perfect gas law may be applied to all species, and

(iv) The interaction forces between ions and electrons are proportional to the difference between their mean velocities:

$$\vec{F}_{ie}=K_{ie}(\vec{q}_i-\vec{q}_e)=-\vec{F}_{ei} \tag{6.1}$$

where subscript e refers to the value of electrons and subscript i refers to the value of ions and $K_{ie}=K_{ei}$ is the friction coefficient of the plasma.

In a similar manner as in the single fluid theory, we assume that the disturbed flow field has the following quantities:

$$\left.\begin{aligned} &\vec{q}_s=\vec{q}_s(x,t);\ p_s=p_0+p'_s(x,t);\ T_s=T_0+T'_s(x,t);\\ &n_s=n_0+n'_s(x,t);\ \vec{E}=\vec{E}(x,t);\ \vec{H}=\vec{H}_0+h(x,t)\end{aligned}\right\} \tag{6.2}$$

where $s=i$ or e and subscript o refers to the partial variables in the undisturbed state.

Now there are 18 perturbed quantities. We may obtain 18 linearized equations for these pertubed quantities [3]. Similar to the case of single fluid theory, we find that $h_x=0$. However, all the other 17 variables are coupled if the external applied magnetic field $\vec{H}_0$ is of arbitrary orientation. We may consider again the periodic motion and find the dispersion relation in the same manner as in the single fluid theory. Of course, the dispersion relation is much more complicated than those in the single fluid theory.

(i) *Basic waves.* If there is no external magnetic field, i.e., $\vec{H}_0=0$, the 17 perturbed quantities may be separated into three independent groups which may be considered as three basic waves in the present case. These basic waves are:

(a) First basic transverse wave with variables v_i, v_e, E_y and h_z

(b) Second basic transverse wave with variables w_i, w_e, E_z and h_y

(c) Basic longitudinal waves with variables u_i, u_e, p'_i, p'_e, T'_i, T'_e, n'_i, n'_e, and E_x

We obtain the dispersion relations for these basic waves as follows:

(ii) *Basic transverse waves.* Since there is no external magnetic field, we cannot distinguish the two transverse waves and their dispersion relations are identical. The dispersion relation for the basic transverse wave is:

$$c^2\lambda^2=\omega^2-\omega_e^2\frac{1-iK_{12}^*}{L+iK_{12}^*} \tag{6.3}$$

where

$$\omega_e=e(n_0/\epsilon\ m_e)^{\frac{1}{2}}=\text{electron plasma frequency} \tag{6.4}$$

m_e is the mass of an electron and $K_{12}^*=K_{ie}/(m_e n_0\omega)$. For ideal plasma, $K_{ie}=0$, the velocity of propagation of these transverse waves is

$$V=\frac{\omega}{\lambda}=\frac{c}{\left(1-\frac{\omega_e^2}{\omega^2}\right)^{\frac{1}{2}}} \tag{6.5}$$

From Eq. (6.5), we see that the transverse wave is an undamped wave only where $\omega>\omega_e$ and a damped wave when $\omega<\omega_e$ which is the mechanism of black-out of radio communication during the reentry of a space vehicle. In vacuum, $n_0=0$ and $\omega_e=0$, Eq. (6.5) gives $V=c$, i.e., the electromagnetic wave is propagated at a speed of light.

(iii) *Basic longitudinal waves.* The dispersion relation for the basic longitudinal waves is

$$\lambda^2=\frac{\omega^2}{2a_i^2}\left\{\left[\left(1-2\frac{\omega_i^2}{\omega^2}\right)+2iK_{12}^*\frac{m_e}{m_i}\right]\pm\left\{\left[\left(1-2\frac{\omega_i^2}{\omega^2}\right)+2iK_{12}^*\frac{m_e}{m_i}\right]^2\right.\right.$$
$$\left.\left.-4\frac{a_i^2}{a_e^2}\left(1-\frac{\omega_e^2}{\omega^2}\right)+iK_{12}^*\right\}^{1/2}\right\} \tag{6.6}$$

where

$$a_s=(\gamma p_0/m_s n_0)^{\frac{1}{2}}=\text{sound speed of } s \text{ th species} \tag{6.7}$$

$$\omega_i=e(n_0/m_i\,\epsilon)^{\frac{1}{2}}=\text{ion plasma frequency} \tag{6.8}$$

There are two basic longitudinal waves from Eq. (6.6): One is the ion sound wave (Corresponding to $+$sign) which is always an undamped wave and as $\omega\to 0$, the velocity of propagation of this wave is $V_1=\sqrt{2}\,a_i=a_p$ which is the sound speed of the plasma as a whole. V_1 decreases as ω increases and as $\omega\to\infty$, $V_1=a_i$, the sound speed for ions alone. The other longitudinal wave is the electron sound wave (corresponding to $-$sign). This wave is a damped wave when $\omega<\omega_e$ and undamped wave when $\omega>\omega_e$. As $\omega\to\infty$, the speed of propagation of this second longitudinal wave is $V_2=a_e$, the sound speed of electrons alone. The single fluid theory gives only the results corresponding to $\omega\to 0$.

(iv) *Waves under longitudinal external magnetic field* [*4*]. If $H_x\neq 0$ and $H_y=0$, the basic longitudinal waves are not affected by the magnetic field. The two basic transverse waves are interacted. The dispersion relation for these interacted transverse waves is

$$A_0\lambda^4+2A_2\lambda^2+A_4=0 \tag{6.9}$$

The coefficients A_0, A_2 and A_4 are functions of plasma frequencies, cyclotron frequencies of the species, frequency ω and friction coefficient K_{12}^*. For an ideal plasma, we have the following values for these coefficients:

$$\left.\begin{aligned}
A_0&=\left(1-\frac{\omega_{xi}^2}{\omega^2}\right)\left(1-\frac{\omega_{xe}^2}{\omega^2}\right)\\
A_2&=\frac{\omega_e^2}{c^2}\left[\left(1-\frac{\omega^2}{\omega_e^2}\right)-\frac{\omega_{xi}\omega_{xe}}{\omega^2}\left(1-\frac{\omega^2}{\omega_i^2}+\frac{\omega_{xi}\omega_{xe}}{\omega_e^2}\right)\right]\\
A_4&=\frac{\omega_e^4}{c^4}\left[\left(1-\frac{\omega^2}{\omega_e^2}+\frac{\omega_{xi}\omega_{xe}}{\omega^2}\right)^2-\frac{\omega_{xi}\omega_{xe}\omega^2}{\omega_i^2\omega_e^2}\right]
\end{aligned}\right\} \tag{6.10}$$

and

$$\omega_{xs}=\frac{eB_x}{m_s}=\text{cyclotron frequency of } s \text{ th species} \tag{6.11}$$

The two solutions of λ^2 of Eq. (6.9) are

$$\lambda_1^2=\frac{-A_2+\sqrt{A_2^2-A_0A_4}}{A_0}=\text{extraordinary waves (ions)} \tag{6.12}$$

$$\lambda_2^2=\frac{-A_2-\sqrt{A_2^2-A_0A_4}}{A_0}=\text{ordinary wave (electrons)} \tag{6.13}$$

The variations of λ_1^2 and λ_2^2 with frequency ω are shown in Fig. 6.1. One interesting result is that as $\omega\to 0$, the speed of wave propagation for these two interacted transverse waves is given by the same formula:

$$V=\frac{\omega}{\lambda}=\frac{V_x}{\sqrt{1+\frac{V_x^2}{c^2}}} \tag{6.14}$$

Hence with applied longitudinal magnetic field, we have undamped transverse waves at low frequency instead of damped waves for the case without applied magnetic

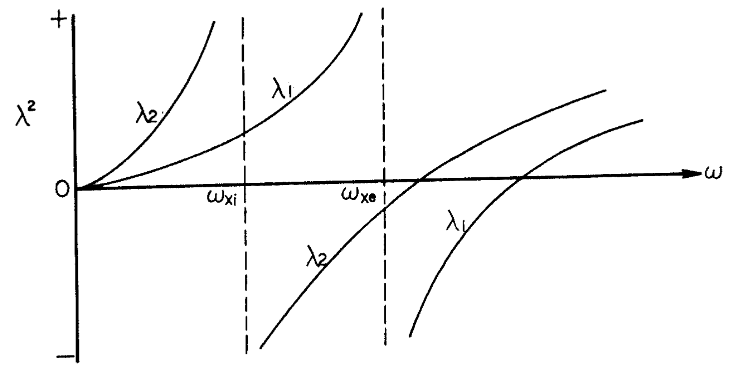

FIG. 6.1. Variation of λ^2 with frequency ω for interacted transverse waves given by Eqs. (6.12) and (6.13)

field. Hence the applied magnetic field improves the transverse wave propagation at low frequencies. The detailed discussions of these two interacted waves are given in reference [*4*].

(v) *Waves under transverse magnetic field* [*4*]. When $H_x=0$ and $H_y\neq 0$, the first transverse wave (v_i etc.) is not affected by the applied magnetic field and the second transverse wave is interacted with the two longitudinal waves and three new transverse-longitudinal waves result. The dispersion relations of these interacted waves is

$$\lambda^6+S_4\lambda^4+S_2\lambda^2+S_0=0 \tag{6.15}$$

where the coefficients S_0, S_2 and S_4 are functions of the sound speeds, plasma frequencies, and cyclotron frequencies of the two species and friction coefficients. For an ideal plasma, these coefficients have the following values:

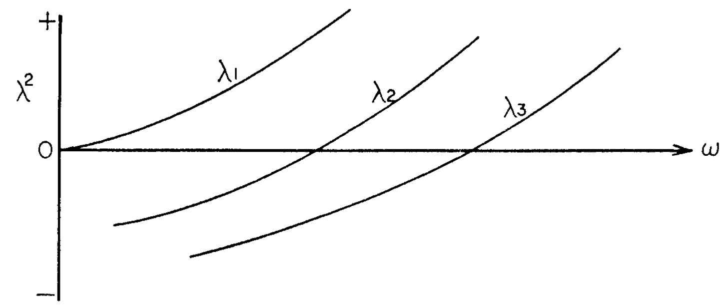

FIG. 6.2. Dispersion relations for the interacted transverse-longitudinal waves under transverse magnetic field in an ideal plasma

$$\left.\begin{aligned}
S_4 &= \frac{\omega^2}{c^2}\left(\frac{\omega_e^2}{\omega^2}-1\right)+\frac{\omega^2}{a_i^2}\left(2\frac{\omega_i^2}{\omega^2}-1+\frac{\omega_{yi}\omega_{ye}}{\omega^2}\right)\\
S_2 &= \frac{\omega^2}{c^2}\cdot\frac{\omega^2}{a_i^2}\left(2\frac{\omega_i^2}{\omega^2}-1\right)\left(\frac{\omega_e^2}{\omega^2}-1+\frac{\omega_{yi}\omega_{ye}}{\omega^2}\right)\\
&\quad+\frac{\omega^4}{a_i^2a_e^2}\left[\left(1-\frac{\omega_e^2}{\omega^2}\right)+\frac{\omega_{ye}^2}{\omega^2}\left(\frac{\omega_i^2}{\omega^2}+\frac{\omega_{yi}^2}{\omega^2}-1\right)\right]\\
S_0 &= -\frac{\omega^4}{c^4}\cdot\frac{\omega^2}{a_i^2a_e^2}\left[\left(\frac{\omega_e^2}{\omega^2}-1\right)^2+\frac{\omega_{ye}^2}{\omega^2}\left(2\frac{\omega_i^2}{\omega^2}+\frac{\omega_{yi}^2}{\omega^2}-1\right)\right]\\
\omega_{ys} &= \frac{eB_y}{m_s}
\end{aligned}\right\} \tag{6.16}$$

There are three roots of λ^2 obtained from Eq. (6.15) which represent three different modes of these interacted waves. These roots are sketched in Fig. 6.2. At low frequency $\omega\to 0$, only one mode is an undamped wave which has the speed of propagation as

$$V=\sqrt{a_p^2+V_y^2} \tag{6.17}$$

Hence the single fluid theory gives only the results at low frequency. At very high frequencies $\omega\to\infty$, the three basic modes are decoupled. In reference [*4*], detailed discussion of these modes are given.

(vi) *Waves under arbitrarily oriented magnetic field* [*4*]. If both H_x and H_y are different from zero, all the four basic waves interact and we have four new modes of the waves in a plasma whose dispersion relation [*4*] is

$$C_0\lambda^8+C_1\lambda^6+C_2\lambda^4+C_3\lambda^2+C_4=0 \tag{6.18}$$

The coefficients C_0 etc. are functions of plasma frequencies, cyclotron frequencies, sound speed of these two species and friction coefficient. In Fig. 6.3, we sketch the case for an ideal plasma. At low frequencies, $\omega\to 0$, there are three undamped waves and one damped waves. These three undamped waves are the fast wave, slow wave and transverse wave same as those given in single fluid theory. As $\omega>0$, the actual results differ greatly from the single fluid theory. The fourth mode is closely related with the sound waves of electrons which becomes undamped when $\omega>\omega_e$.

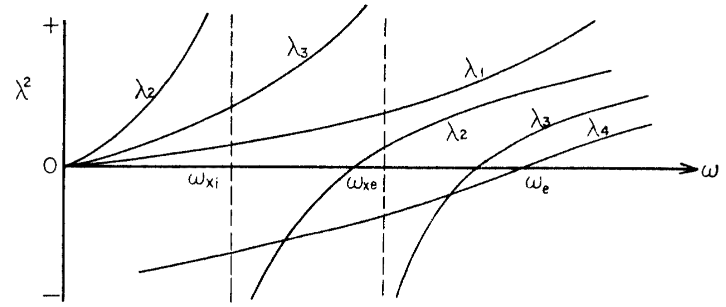

FIG. 6.3. Dispersion relations for the interacted waves under arbitrary oriented magnetic field in an ideal plasma

From the above results, we see that single fluid theory gives correct results for $\omega \to 0$ only. In general, the actual wave motion differs greatly from those given by the single fluid theory.

In the above study, we consider only the two fluid theory. If the plasma consists of more than two species, new modes of wave will results. For instance, if we consider a partially ionized plasma consisting of three species, electrons, one kind of ions and one kind of neutral particles, we will have a sound wave for the neutral particles which will interact with all the other waves discussed above. Some preliminary study has been given in reference [*5*].

7. HEAT TRANSFER IN RADIATION MAGNETOGASDYNAMICS SINGLE FLUID THEORY [*2*], [*6*]

One of the main topics of research for aerospace engineers is the heat transfer problem. Since at very high temperature both thermal radiation and ionization are important, we should include these effects in our actual analysis of heat transfer problems. It seems that the effect of thermal radiation has even less discussed than that due to electromagnetic field. We thus first discuss a case for thermal radiation only without the influence of electromagnetic field and then discuss a case with both thermal radiation and electromagnetic field effects.

(i) *Blasius problem in radiation gasdynamics* [*6*]. We consider a uniform flow of velocity U and temperature T_∞ over a semi-infinite flat plate. We assume that the temperature T_∞ is so high that the radiative heat transfer is of the same order of magnitude as the heat transfer by conduction and convection but the radiation stresses and radiation energy density are still negligible. We shall neglect those complicated boundary layer effects of high temperature due to chemical reaction, diffusion, etc. so that we consider only the essential effects of radiative heat transfer on the boundary layer over a flat plate. We assume that there is no external applied magnetic field and the induced electromagnetic field effects are negligible. Under these conditions, the fundamental equations of our problem are as follows:

$$p = R\rho T = \text{constant} \tag{7.1}$$

$$\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}=0 \tag{7.2}$$

$$\rho u \frac{\partial u}{\partial x}+\rho v \frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{7.3}$$

$$\rho u \frac{\partial h}{\partial x}+\rho v \frac{\partial h}{\partial y}=\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right)+\frac{\partial q_{Ry}}{\partial y} \tag{7.4}$$

where h is the enthalpy. We may solve for u, v, ρ and h from Eqs. (7.1) to (7.4) with proper boundary conditions if we can express the radiative heat flux q_{Ry} in terms of the state variables ρ and T.

(a) *Radiative heat flux*. In general, we should express q_{Ry} in terms of specific intensity of radiation I_ν and add the radiative transfer equation for I_ν in our fundamental equations as we have discussed before. For a first approximation, we may find an expression of q_{Ry} in terms of temperature T and the mean free path of radiation by making the following approximations:

(1) The thermal radiation is under local thermodynamic equilibrium.

(2) The gas is a gray gas so that the absorption coefficient k'_ν is independent of frequency ν. Actually we may use either the Rosseland mean absorption coefficient (Eq. (3.27)) for the optically thick medium or Planck mean absorption coefficient K_p for the general case or optically thin case. Planck mean absorption coefficient K_p is defined by the formula:

$$K_p=\frac{1}{B(T)}\int_0^\infty k'_\nu B_\nu d\nu \tag{7.5}$$

where

$$B(T)=\int_0^\infty B_\nu(\nu, T)d\nu$$

(3) One dimensional approximation, in which we assume that the temperature and the absorption coefficient depend only on one spatial coordinate y. This is a good approximation for boundary layer problems.

By the gray gas assumption, the integration of q_{Ry} with respect to ν may be carried out and by one dimensional approximation, the integration of q_{Ry} with respect to the angle θ may be carried. As a result, the radiative transfer term becomes [2]:

$$\frac{\partial q_{Ry}}{\partial y}=2\sigma\rho K_p\left[\int_0^t T^4\varepsilon_1(t-t')dt'+\int_t^\infty T^4\varepsilon_1(t'-t)dt'-2T^4+T_w^4\varepsilon_2(t)\right] \tag{7.6}$$

where

$$t=\int_0^y \rho K_p dy$$

$$\varepsilon_n(z)=\int_1^\infty \lambda^{-n}\exp(-\lambda z)d\lambda$$

$$\sigma=\frac{1}{4}ca_R=\text{Stefan-Boltzmann constant for radiative transfer}$$

(b) *Velocity profile.* In our study of the thermal radiation effects on the flow field, we find that the radiative heat flux has only a small effect on the velocity profile. Hence we use the well known assumptions of boundary layer flow of a compressible fluid such as the coefficient of viscosity is linearly proportional to the temperature and then the velocity profile is the Blasius profile in the following form:

$$\frac{u}{U}=f'(\xi), \qquad v=-\frac{\partial\phi}{\partial x}=\frac{1}{2}\sqrt{\frac{c_1\mu_\infty U}{\rho_\infty x}}(\xi f'-f) \tag{7.7}$$

where

$$\xi=\frac{Y}{x}\sqrt{\frac{\rho_\infty Ux}{\mu_\infty}}, \qquad Y=\int_0^y\frac{\rho}{\rho_\infty}dy$$

and $f(\xi)$ is the well known Blasius function and prime refers to the differentiation with respect to ξ. The velocity profile is similar [6].

(c) *Temperature profile.* After the velocity profile is obtained, we may calculate the temperature profile from the energy equation:

$$u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial Y}=\frac{c_1\mu_\infty}{\rho_\infty}\left(\frac{\partial u}{\partial Y}\right)^2+\frac{c_1\mu_\infty}{P_r\rho_\infty}\left(\frac{\partial^2 h}{\partial Y^2}\right)$$
$$+2\sigma\rho K_p\left[\int_0^t T^4\varepsilon_1(t-t')dt+\int_t^\infty T^4\varepsilon_1(t'-t)dt'+T_w^4\varepsilon_2(t)-2T^4\right] \tag{7.8}$$

where the boundary conditions are

$$x>0: \quad Y=0, \quad T=T_w; \quad Y\to\infty, \quad T\to T_\infty$$

Even though the velocity profile is similar, the temperature profile is in general not similar.

Eq. (7.8) has been numerically integrated for various case. Since we consider a large variation of temperature, the dependence of the mean absorption coefficient K_p with respect to state variables is important. The following formula is used:

$$K_p=4.5\times10^{-7}p^{1.31}\exp(5.18\times10^{-4}T-7.13\times10^{-7}T^2) \tag{7.9}$$

where K_p is in cm^{-1}, p is in atmospheres and T is in °K. The Prandtl number P_r is taken as 0.74 and the wall temperature is assumed to be 2,000°K. The numerical results are shown in Figs. 7.1 to 7.4.

Fig. 7.1 shows the temperature distribution at a low Mach number $M_\infty=1.25$. Since the temperature profile is not similar, we find that it is convenient to us the following Radiative Flux number R_f to denote the x-coordinate:

$$R_f=\frac{\sigma T_\infty^3 x^2}{K_\infty L_{R\infty}} \tag{7.10}$$

Without radiation effect, $R_f=0$ which is shown by the dotted curve in Fig. 7.1. The solution without radiation effect is a similar solution which holds for all x.

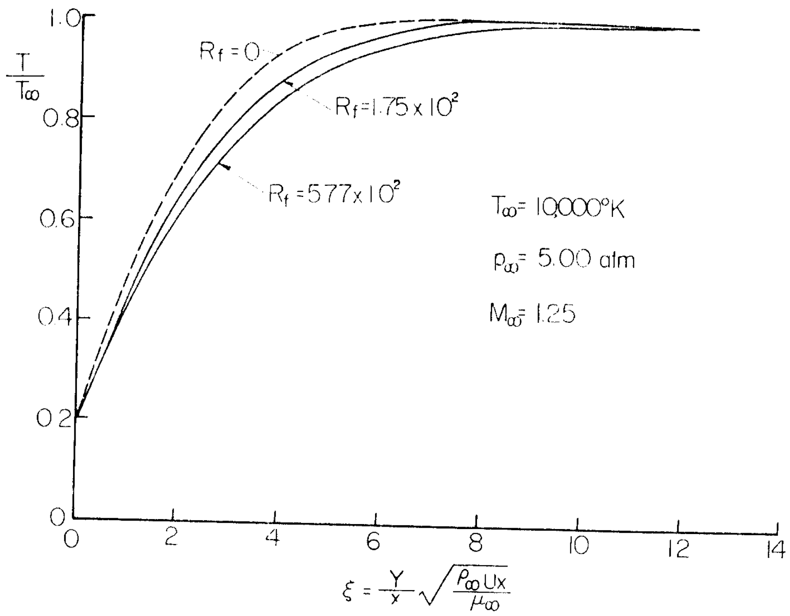

FIG. 7.1. Temperature Distribution of a Radiating Gas over a Flat Plate. Low Mach Number Case.

With radiation effect, the thermal boundary layer thickness increases with the radiative flux number R_f and the slope $dT/d\xi$ at the wall decreases with increase of R_f.

Fig. 7.2 shows the temperature distribution of a high Mach number case $M_\infty = 15.00$. It is well known that under high Mach number, viscous dissipation becomes important and the maximum temperature inside the boundary layer will be higher than those of the free stream temperature T and that of the wall T_w. The same situation exists for the boundary layer of a radiating gas. The effects of thermal radiation are (i) to decrease the maximum temperature, (ii) to increase the boundary layer thickness and (iii) to decrease the slope $dT/d\xi$ at the wall.

Fig. 7.3 shows the case of an optically thick medium in which the expression Eq. (3.25) is used for the radiative heat flux. In this case, we have similiar solution for the temperature profile. It is advisable in this case to use a radiative flux number R_f independent of x to show the effect of radiative heat transfer. We thus use the following definition for the radiative flux number:

$$R_f = \frac{\sigma T_\infty^3 L_{R\infty}}{K_\infty} \tag{7.11}$$

In comparing the results of optically thick approximation of Fig. 7.3 with the more accurate results of Fig. 7.1, we see that the optically thick approximation overestimate the effect of thermal radiation because for the values of same order of magnitude of radiative flux, the optically thick approximation gives larger boundary layer thickness and smaller slope $dT/d\xi$ at the wall than those of the accurate formula of integral expression.

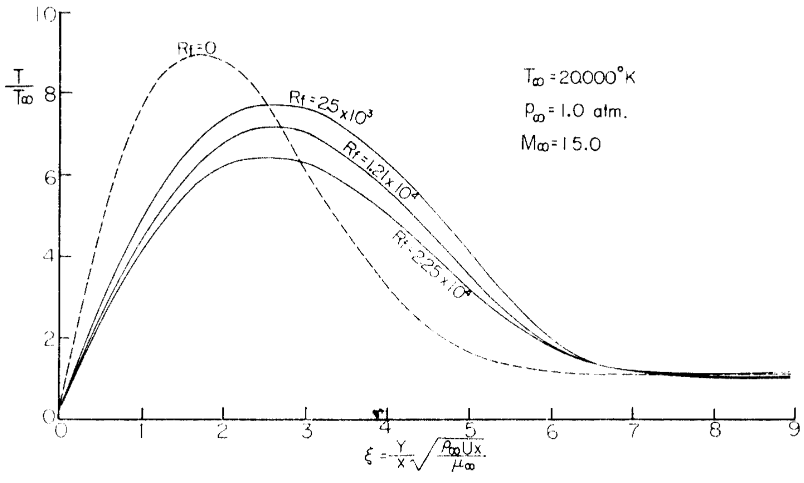

FIG. 7.2. Temperature Distribution of a Radiating Gas over a Flat Plate. High Mach Number Case.

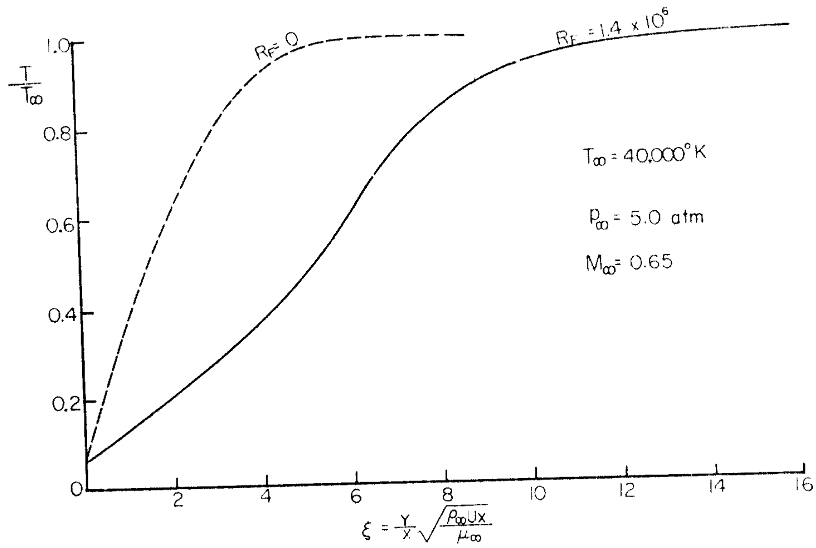

FIG. 7.3. Temperature Distribution of a Radiating Gas over a Flat Plate. (Optically Thick Approximation).

In many current literature of radiative heat transfer problems, an optically thin approximation has been used. For very small absorption, the integral terms in the accurate formula (7.6) may be neglected and the radiative heat transfer term for an optically thin medium may be approximated by the formula

$$\frac{\partial q_{Ry}}{\partial y} = 4\sigma\rho K_P\left(-T^4 + \frac{1}{2}T_w^4\right) \tag{7.12}$$

Since in our example, T_w is always much smaller than T except in the neighborhood of the wall, Eq. (7.12) shows that the radiative heat transfer acts as a heat source in the boundary layer. It increases the temperature inside the boundary layer but has little effect on the boundary layer thickness which depends mainly on the absorptivity of the gas. In Fig. 7.4, we show an example that the optically thin

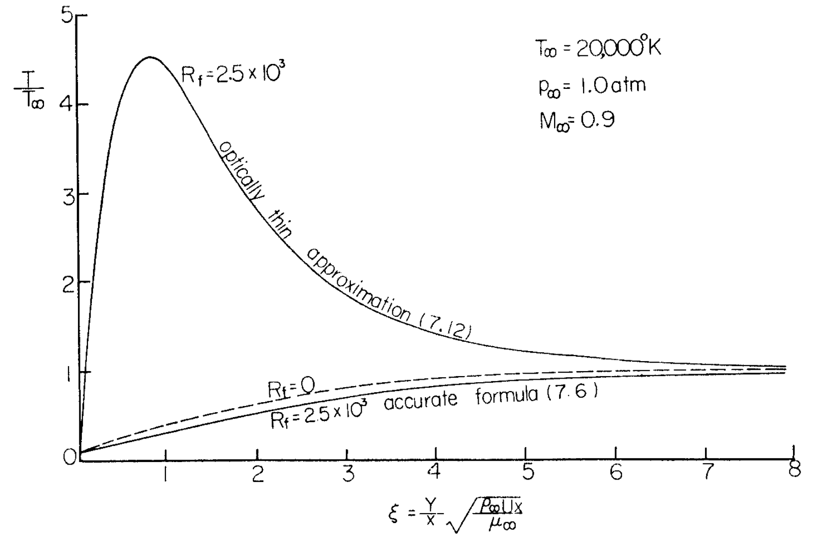

FIG. 7.4. Temperature Distribution of a Radiating Gas over a Flat Plate. (Optically Thin Approximation).

approximation gives entirely wrong results in comparison wtih the exact formula (7.6). Hence special care should be made for the application of optically thin approximation.

From the temperature distributions, we may calculate the conductive heat transfer to the wall by means of Nusselt number. In general, the Nusselt number increases with the pressure and the Mach number of the free stream.

When the radiation effect is very large, we may not have thermal boundary layer and the wall will have an upstream influence [6].

(ii) *Plane Couette Flow in Radiation Magnetogasdynamics.* Our second example is to consider an ionized gas flowing between two parallel plates: One of the plates is at rest and the other is of uniform motion with a velocity U. There is no pressure gradient. There is an externally applied transverse magnetic field $H_y = H_0 =$constant in the direction perpendicular to the plates and an externally applied electric field $E_z = E_0 =$constant in the direction perpendicular to both the plates and the direction of the flow which is in the x-direction. The ionized gas is assumed to be viscous, heat-conducting, thermal radiating and electrically conducting. The flow is assumed to be steady and laminar. For simplicity, we assume that all the transport coefficients are constant. Thus we may calculate the velocity and magnetic field first and then the temperature distributions from the energy equation with the known distributions of the velocity and the x-component of the induced magnetic field. The equations which govern the velocity u and the magnetic field H_x in non-dimensional form are

$$\mu \frac{du}{dy} + R_e R_H H_x = \text{constant} \tag{7.13}$$

$$\frac{dH_x}{dy} = R_\sigma(-u + R_E) \tag{7.14}$$

where $R_e=LU\rho_0/\mu_e$=Reynolds number of the flow with L as the length of the gap between the plates; $R_H=(\mu_e H_0^2)/(\rho_0 U^2)$=magnetic pressure number; $R_\sigma=UL/\nu_H$ =magnetic Reynolds number; $R_E=E_0/\mu_e H_0 U$=electric field number. The solutions of Eqs. (7.13) and (7.14) are given in Figs. 7.5 and 7.6 in terms of electric field number R_E and the Hartmann number $R_h=(R_\sigma R_H R_e)^{\frac{1}{2}}$. It is interesting to see the influence of the electric field number R_E which has not been emphasized in literature. Even though we call the flow "Magnetogasdynamics", the influence of the electric field is not negligible. The electric field behaves like a pressure gradient in the x-direction and it has significant influence in the induced magnetic field H_x.

After the velocity and the magnetic field distributions are obtained, we may calculate the temperature distribution from the energy equation:

$$\frac{1}{(\gamma-1)P_r}\frac{dT}{dy}+M^2\mu\frac{d}{dy}\left(\frac{1}{2}u^2\right)+R_e R_E R_H M^2 H_x + R_{p0}R_F R_e q_R(T)=\text{constant} \tag{7.15}$$

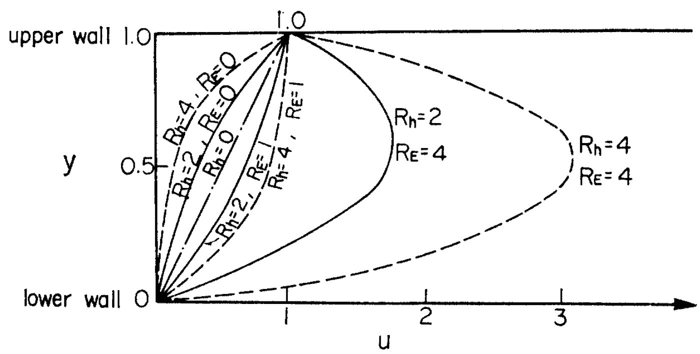

FIG. 7.5. Velocity Distribution of Plane Couette Flow in RMGD.

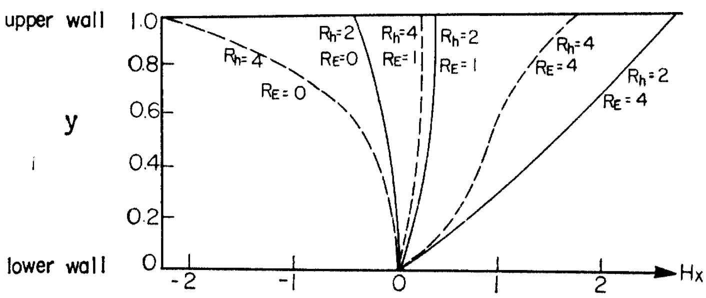

FIG. 7.6. X-wise Magnetic Field Distribution of Plane Couette Flow in RMGD.

where P_r is the Prandtl number of the gas, $M=U/a_0$ is the Mach number, $R_{p0}=p_{R0}/p_0$=radiation pressure number of the lower wall whose temperature is the reference temperature T_0, $R_F=cL/UL_R$ is the radiation flux number.

In order to slove Eq. (7.15), we have to know the expression of the radiative heat flux q_R. Two expressions have been used for q_R: One is the optically thick approximation which is

$$q_R = \frac{4}{\gamma} T^3 \frac{dT}{dy} \tag{7.16}$$

and the other is the modified optically thin approximation. Since we know from our previous example, we cannot neglect completely the absorption. If the mean free path of radiation is large in comparison of the typical length L, the following radiative heat flux expression may be used:

$$q_R(T) = \frac{3}{2\gamma}\left\{(T_1^4+1)\tau_a + \int_{\tau_a}^{\tau_{a2}} T^4(t)dt - \int_0^{\tau_a} T^4(t)dt\right. \tag{7.17}$$

where $\tau_a = \tau(L_R/L)$ and $\tau = \int_0^{y^*} pk_\nu' dy^*$, y^* is dimensional coordinate.

The integration of the energy equation with q_R given by Eq. (7.16) is

$$A[T^4 - 1 - y(T_1^4 - 1)] + \frac{\gamma}{(\gamma-1)P_r}[T - 1 - y(T_1 - 1)]$$

$$= \gamma M^2 (R_E/R_\sigma) R_h^2 \left[y \int_0^1 H_x dy - \int_0^y H_x dy\right] + \frac{1}{2}\gamma M^2 (y - u^2) \tag{7.18}$$

where $A = R_{p0} R_F R_e$ and with the expression of q_R by Eq. (7.17) is

$$\frac{3}{2} A \left\{\frac{1}{2}(T_1^4 + 1)(y^2 - y) + \int_0^y I_R dy - y \int_0^1 I_R dy\right\}$$

$$+ \frac{\gamma}{(\gamma-1)P_r}[T - 1 - y(T_1 - 1)]$$

$$= \gamma M^2 (R_E/R_\sigma) R_h^2 \left[y \int_0^1 H_x dy - \int_0^y H_x dy\right] + \frac{1}{2}\gamma M^2 (y - u^2) \tag{7.19}$$

where

$$I_R = \int_y^1 T^4(y)dy - \int_0^y T^4(y)dy$$

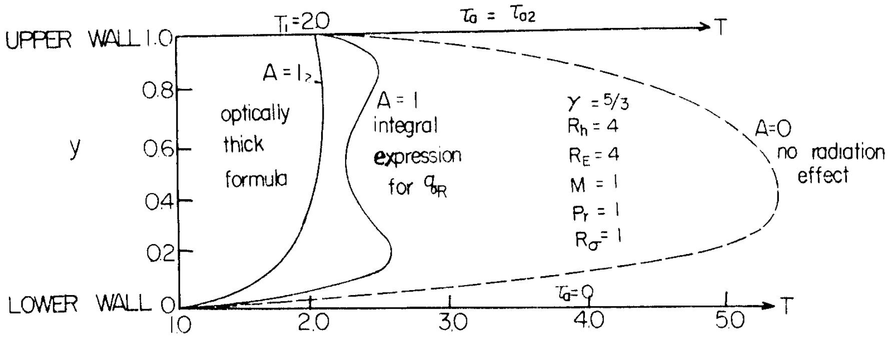

FIG. 7.7. Temperature Distributions fo Plane Couette Flow in RMGD.

Fig. 7.7 shows a typical case of temperature distribution. Without radiation effect, $A=0$, the temperature in the flow field is enormously high for the case of large R_h and R_E. With the radiation effects considered, the maximum temperature in the flow field drops a great deal. For the optical thick approximation, the formula overestimate the effect of radiation. It is better to use the integral expression. Due to the effect of thermal radiation, there is a tendency that the temperature in the central portion of the flow field becomes more or less uniform.

If we want to get more accurate numerical results, we should consider that the transport coefficients are functions of temperature and pressure. The resulting equations can be integrated numerically by the help of high speed computing machine.

8. HEAT TRANSFER IN RADIATION MAGNETOGASDYNAMICS MULTIFLUID THEORY [7], [8]

The complete analysis of heat transfer problem in radiation magnetogasdynamics by multifluid theory has not been worked out yet. In this section we are limited ourselves to the influence of ionization on the heat transfer problem by the help of multifluid theory. The continued interest in high energy gas flow phenomena has led to an increasing number of investigations directed at improving our knowledge and understanding of the influence of ionization on the fluid flows. These investigations have, in general, been concentrated in two areas: namely, investigations of the electrical characteristics of flows and investigations of surface heat transfer in ionized flows. The studies of plasma electrical characteristics are, in general, directed toward the improvement of diagnostic techniques such as the Langmuir-type probe, which is used to measure ion and electron number densities. The heat transfer investigations are generally directed toward the solution of problems of planetary entry or toward the solution of problems of design of very high temperature experimental facilities. In this section, we are concerned primarily with the study of heat transfer mechanisms. Although these areas of investigation can in some cases be explored separately, it is probable that the electric effects, whether induced or applied, will be present and hence, will contribute to the heat transfer process.

In this section, we seek to assess the effects of (a) the difference of temperature of electrons and that of heavy particles, (b) the non-equilibrium in degree of ionization due to electrical field and (c) the contribution of heat transfer by thermal conduction, diffusion and viscous dissipation in the flow field of an ionized gas. Multifluid theory will be used in the analysis.

We consider a partially ionized monatomic argon gas flowing between two parallel plates, a distance L apart, with the lower plate stationary and the upper plate moving at an appreciate velocity in the positive x-direction. Fig. 8.1 shows a sketch of this problem. The lower plate is considered to be the cooler surface and is completely catalytic in recombination of the ion-electron flux reaching it by diffusion. The upper moving plate is at quite a high temperature and is assumed to be capable of ionizing the incident atom flux with a specified efficiency. Although the surface ionization mechanism was not specified, its hypothesis was necessary, since only heterogeneous chemical reactions were assumed. Otherwise expressed, the diffusional flow normal to the plates consists of a flux of atoms, originating at the cool plate by the deionization, which flows to the hot plate and is ionized, thereby initiat-

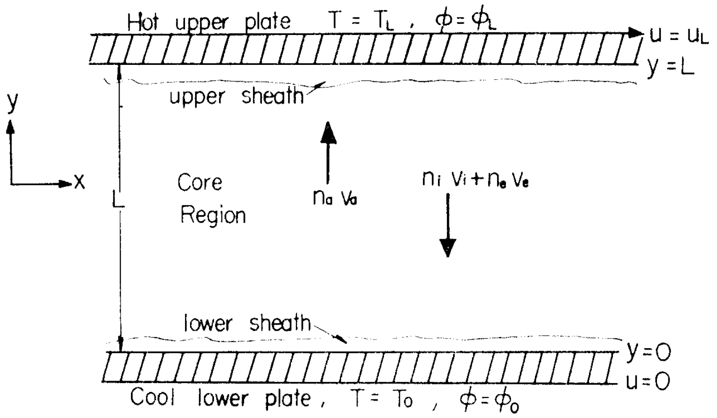

FIG. 8.1. Plane Couette Flow of a Partially Ionized Gas. Flow Model.

ing a flux of ions and electrons from the hot to cooled plates. For the multifluid theory investigations, we make the further assumption that the cool plate has a small negative potential, whereas the hot plate has a slightly positive potential, but their magnitude is such that there is zero net current flow.

Two further assumptions were made which are generally considered consistent with the assumed physical model. First, it was considered that the ions and atoms are mutually in thermal equilibrium. This assumption is based on the fact that particles of approximately equal mass exchange a large percentage of the initial difference in the kinetic energy in a collision, and, hence, very few collisions would be necessary for thermal equilibrium. The converse of this reasoning, when applied to electron-heavy particle collisions, is the basis of the concept of electron thermal nonequilibrium. The second assumption was that the x-component of all species velocities is considered equal.

The solution of our problem depends on the accuracy of the determination of the transport coefficients μ_s, k_s, K_{st} and K^T_{st}. Considerable effort has been made in determination of these coefficients. The details are given by Powers [8]. The variations of these transport coefficients for ionized Argon with the state variables are as follows:

(i) Coefficients of viscosity:

$$10^6\mu_a = 0.816T_a - 3.903\times10^{-4}T_a^2 + 1.073\times10^{-9}T_a^3 \text{ g/cm-sec. for } T_a < 1500°\text{K} \tag{8.1a}$$

$$10^4\mu_a = 0.0249T_a^{0.774} \text{ g/cm-sec. for } T_a > 1500°\text{K} \tag{8.1b}$$

$$\mu_e = 0.434\times10^{-16}T_e^{5/2}/\ln(1.24\times10^4T_e^{3/2}/n_e^{1/2}) \text{ g/cm-sec.} \tag{8.1c}$$

$$\mu_i = 1.171\times10^{-14}T_i^{5/2}/\ln(1.24\times10^4T_i^{3/2}/n_i^{1/2}) \text{ g/cm-sec.} \tag{8.1d}$$

(ii) Coefficients of thermal conductivity:

$$k_a = 0.1867\mu_a \text{ cal/cm-sec.°K} \tag{8.2a}$$

$$k_e = 1.334\times10^4\mu_e \text{ cal/cm-sec.°K} \tag{8.2b}$$

$$k_i = 0.1861\mu_i \text{ cal/cm-sec.°K} \tag{8.2c}$$

where subscript "a" refers to neutral atoms of argon, subscript "i" refers to singly charged ions and subscript "e" refers to electrons. Even though Eqs. (8.1) and (8.2) may be used without reservation for high-temperature. Low-density plasma, it is apparent that Eqs. (8.1c) and (8.1d) are singular when $T_s=1.87\times 10^{-3}n_s^{1/3}$. Physically, this corresponds to the condition that exists when the kinetic energy of the collision is qual to the Coulombic potential energy at a distance of a Debye radius, and implies that such low-energy collisions should not be properly included. To circumvent this difficulty, the low-temperature values of ion or electron transport coefficients were assumed to be linear in temperature with constant of proportionality chosen as a function of number density n_s in such a manner as to insure continuity in the first derivatives with respect to temperature T_s at the point of transition from low- to high- temperature expression, i.e.,

$$\mu_e\sim k_e\sim\mu_i\sim k_i\sim K\left\{\exp\frac{3}{2}\left[1-\ln\left(1.24\times 10^4/n_s^{\frac{1}{2}}\right)\right]\right\}^{3/2} T_s \tag{8.3}$$

When the transport properties of either the atoms, ions, or electrons in the presence of other species were required, these properties were derived in manner that mixture law based on mean free path considerations will be used [8], i.e., the viscosity and thermal conductivity in the presence of other species was the pure species value multiplied by the ratio of the pure species mean free path to the mean free path taking into account collisions with other species.

(iii) Resistance coefficients.

$$\log_{10}\left[\frac{K_{ia}\times 10^{32}}{n_a n_i}\right]=0.168\log_{10}T_a-0.0356$$
$$+(0.168\log_{10}T_a-0.0356)^2-0.154\log_{10}T_a+2.54^{1/2} \tag{8.4a}$$

where T_a is in degrees Kelvin.

$$K_{ie}=\frac{8\pi^{\frac{1}{2}}e^4 n_i n_e\ln(3R_A T_e r_D/e^2)}{\mu_{ie}q^3}\left[\frac{\pi^{\frac{1}{2}}}{2}\operatorname{erf}\frac{q}{\alpha^*}-\frac{q}{\alpha^*}e^{-q^2/\alpha^{*2}}\right] \tag{8.4b}$$

where

$$\mu_{ie}=m_i m_e/(m_i+m_e)\,,\qquad r_D=(R_A T_e/4\pi n_e e)^{\frac{1}{2}}$$
$$\alpha^*=[2R_A(m_i T_e+m_e T_i)/m_e m_i]^{\frac{1}{2}}\,,\qquad q=|v_i-v_e|$$

$$K_{ie}^T=\frac{4\pi e^4 n_i n_e\ln(3R_A T_e r_D/e^2)}{\mu_{ia}q}\operatorname{erf}\left(\frac{q}{\alpha^*}\right) \tag{8.4c}$$

We neglect K_{ea} and K_{ea}^T, which are found to be very small.

The important parameters in our problem are as follows:

(i) Reynolds number of sth species based on velocity v_s:

$$R_{ev_s}=m_s n_s v_s L/\mu_{sL}$$

(ii) Mach numbers:

$$M_{v_s}=\frac{v_{sL}}{(2R_A T_{sL}/m_s)^{\frac{1}{2}}}, \qquad M_u=\frac{u_L}{(2R_A T_{sL}/m_s)^{\frac{1}{2}}}$$

(iii) Prandtl number:

$$P_{rs}=\frac{3R_A\mu_{sL}}{2m_s k_{sL}}$$

(iv) Electrical, potential parameters:

$$P_i=\frac{4\pi e^2 n_{eL}L^2}{m_i v_{iL}^2}, \qquad P_e=\frac{4\pi e n_{eL}L^2}{m_e v_{eL}^2}$$

(v) Interaction force parameters:

$$R_{\text{Ist}}=\frac{K_{st}L^2}{\mu_{sL}}, \qquad R_{\text{IIst}}=\frac{K_{ST}^T L^2}{k_{sL}T_{sL}}$$

where subscript L refers to the value on the upper plate, i.e., $y=L$.

In order to define our problem, we have to know the range of these non-dimensional parameters. What we have considered are the following physical conditions [7]:

(i) Temperature range is from 5,000°K to 10,000°K.

(ii) u_L is from 1.2×10^3 to 7×10^3 meters/sec.

(iii) Pressure is of the order of 10^{-3} atmosphere.

(iv) $L=1$ cm.

As we have mentioned, the following assumptions are made:

(a) The lower plate is a cool plate and is completely catalytic in recombination of the ion-electron flux reaching it by diffusion.

(b) The upper plate is at quite a high temperature and is assumed to be capable of ionizing the incident atom flux with a specified efficiency.

(c) The temperature of heavy particles are equal, $T_a=T_i$.

(d) The x-components of velocity of all species are equal, $u_s=u$.

(e) There is no source term.

Under these conditions, the basic differential equations for the unknowns u, n_s, v_s, T_e, T_a and E_y are as follows:

$$\frac{dn_s v_s}{dy}=0 \tag{8.5}$$

$$(\rho_a v_a+\rho_i v_i+\rho_e v_e)\frac{du}{dy}=\frac{d}{dy}\left[(\mu_a+\mu_i+\mu_e)\frac{du}{dy}\right] \tag{8.6}$$

$$\rho_s v_s\frac{dv_s}{dy}-\frac{4}{3}\frac{d}{dy}\left(\mu_s\frac{dv_s}{dy}\right)+R_A\frac{dn_sT_s}{dy}=\sum_{t=1}^{3}K_{st}(v_s-v_t) \tag{8.7}$$

$$\frac{3}{2}R_A(n_a v_a+n_i v_i)\frac{dT_a}{dy}-\frac{d}{dy}\left[(k_a+k_i)\frac{dT_a}{dy}\right]-(\mu_a+\mu_i)\left(\frac{du}{dy}\right)^2$$
$$-\frac{4}{3}\left[\mu_a\left(\frac{dv_a}{dy}\right)^2+\mu_i\left(\frac{dv_i}{dy}\right)^2\right]+R_AT_a\left(n_a\frac{dv_a}{dy}+n_i\frac{dv_i}{dy}\right)$$

$$=K_{ai}(v_i-v_a)^2+\frac{m_e}{m_a}\frac{T_e}{T_a}K_{ie}(v_i-v_e)^2+\frac{m_e}{m_a}\left(\frac{T_e-T_a}{T_e}\right)K_{ei}^T\,. \tag{8.8}$$

$$\frac{3}{2}R_A n_e v_e\frac{dT_e}{dy}-\frac{d}{dy}\left(k_e\frac{dT_e}{dy}\right)-\mu_e\left[\left(\frac{du}{dy}\right)^2+\frac{4}{3}\left(\frac{dv_e}{dy}\right)^2\right]$$

$$+n_e R_A T_e\frac{dv_e}{dy}=K_{ei}(v_i-v_e)^2+\frac{m_e}{m_a}\left(\frac{T_e-T_a}{T_e}\right)K_{ei}^T \tag{8.9}$$

$$\frac{dE_y}{dy}=4\pi e(n_i-n_e) \tag{8.10}$$

The above equations are suitable for the physical conditions of our problem in which $M_{vs}^2\ll 1$, $M_{vs}/R_{evs}\ll 1$ and $M_{vs}^2 P_s\gg 1$ and the inertial and viscous terms are negligible in the y-wise momentum equations and ambipolar diffusion exists over a major portion of the core region. There is a very thin sheath layer near the upper and lower plates, which are of a thickness of a few Debye lengths. Hence it is sufficient to study the flow in the core region given by Eqs. (8.5) to (8.10) with the sheath layer as a boundary condition to determine the electron temperature.

Eqs. (8.5) to (8.10) should be solved for proper boundary conditions. Since the mean free path of the gas is much smaller than the characteristic length L in our problem, no slip of all velocities on the wall and no temperature jump for the heavy particles on the wall may be used as the proper boundary conditions. The boundary condition on the electron temperature was evaluated by applying a very elementary sheath layer concept. Within the sheath layer, charge neutrality breaks down and the sheath potential varies to its wall value in such a manner that there is zero net current flowing to the walls. Under the physical model considered, the lower wall was assumed to be at a small negative potential, and, hence, only electrons possessing kinetic energy, which is greater than the electrostatic potential energy of the wall, will be able to reach the wall. Similarly, the upper wall is assumed to be at a small positive potential. In this case, only electrons of sufficient kinetic energy can overcome the attractive potential and enter the core region where they come under the influence of diffusive forces. Using these concepts, one can compute the energy of electrons at the edge of a sheath layer, and then, accounting for the energy change across the sheath because of the wall potential ϕ, it is possible to obtain an electron energy balance between the core and sheath layers. From the boundary condition, the electron temperature is given in terms of the wall potentials.

The last boundary condition is to assume a fixed number flux $n_s v_s$ for a given case.

The solution of the core system of equations consists mathematically of solving five ordinary differential equations for the five functions: $u(y)$, $v_a(y)$, $v_i=v_e(y)$, $T_a(y)$ and $T_e(y)$. The equations are coupled, and the boundary conditions are split; hence, they must be integrated simultaneously, and one must deduce the correct values of a given number of initial conditions at one wall to satisfy the boundary conditions at the other wall.

In our computations, the given boundary conditions and a set of trial values were used at one wall to start the numerical integration of the equation system. If the trial values did not satisfy the boundary conditions at the other wall, the difference between the desired and obtained values was retained. Then small perturbations in each of the trial values were made successively, and the complete system of differential equations was integrated for each perturbation to obtain increments. Thus, it was possible to define approximate partial derivatives of the correction in the dependent variable with respect to a change in the trial values. After a few iterations, a satisfactory solution was generally obtained.

After the functional dependence of the flow variables has been determined by the preceding numerical procedures, the description of the flow was essentially complete, and the quantities of interest, namely the heat transfer rate and skin friction coefficient could be expressed in terms of these variables. The expressions for the heat transfer are a bit more complex because we have to consider the conduction by the atoms and electrons and the diffusion of deionization energy. The conduction of ions was negligible.

We compare our results of multifluid theory with the following two well known single fluid theories:

(a) Equilibrium single fluid theory in which all properties of the flowing partially ionized gas can be determined by relations describing a stationary gas in thermochemical equilibrium.

(b) Frozen flow single fluid theory in which all chemical reactions are frozen in the flow. This is the same assumption used in the multifluid theory and hence, the frozen single fluid theory might be expected to serve as a better criterion for comparison with the multifluid results.

Numerical computations have been carried out for the physical conditions described above by IBM 7094. The following are some of our main results [7], [8]:

(i) *Dependence of upper wall temperature.* Fig. 8.2 shows the heat transfer rate q with upper wall temperature T_L at a given lower wall temperature T_0= 2500°K. It is seen that at lower temperature with moderate thermal non-equilibrium, the multifluid theory results are similar to the single fluid theory results and that at temperature above 8000°K, there is a divergence of the theories. First the diffusion of deionization energy is greater at higher temperature for the frozen single fluid theory because the diffusive mass flux is concentration controlled rather than chemically rated controlled. Second the contribution due to electron conduction is important.

(ii) *Thermal Non-equilibrium effects.* As it became apparent that increased degrees of thermal non-equilibrium had a large effect on the electron conductive heat-transfer contributions, it appears desirable to investigate the phenomenon in detail. The thermal non-equilibrium depends on the electrical potential. Fig. 8.3 shows the variation of heat transfer rate with wall potential difference. It shows that the heat transfer rate is decreased with more positive potential differences. This results is consistent with the postulated physical situation and, by examination of the figure, it is seen to result directly from the decrease in electron conduction contribution.

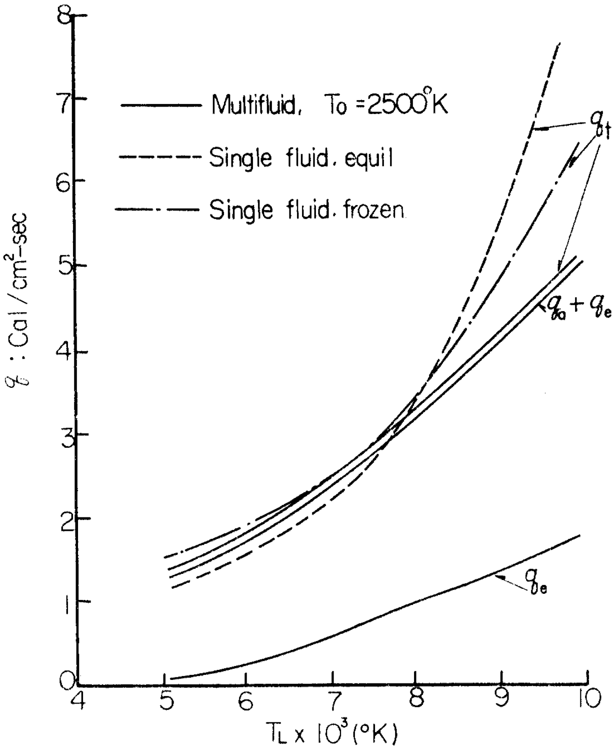

FIG. 8.2. Heat transfer rate *vs* upper wall temperature: Comparison of theories. q_t is the total heat transfer rate.

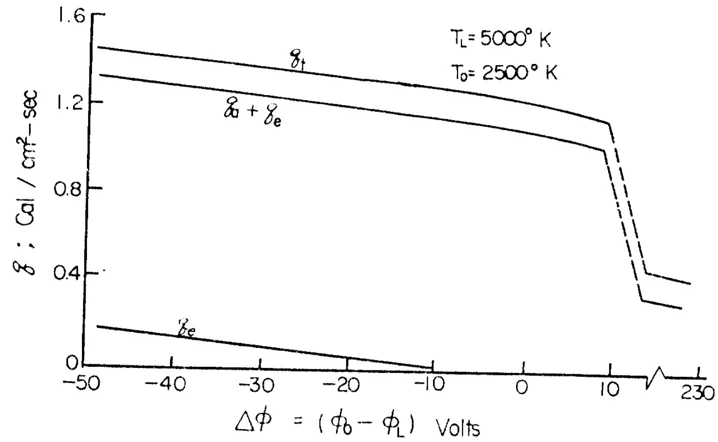

FIG. 8.3. Heat transfer rate *vs* wall potential difference.

(iii) *Mach number effects and skin friction.* Fig. 8.4 shows the variation of the heat transfer rate with the velocity of the upper plate. The contribution of the viscous coupling depends on the Mach number of the species. Since the electron sound speed is much larger than that of the atoms, the electron conductive heat transfer contribution is thus insensitive to the velocity u_L.

Fig. 8.5 shows the variation of the skin friction coefficient with the velocity of the upper wall. There appears to be reasonable similarity between the single- and multi-fluid frozen flow calculations.

(iv) *Dependence of solution on ionization mechanism.* In connection with the

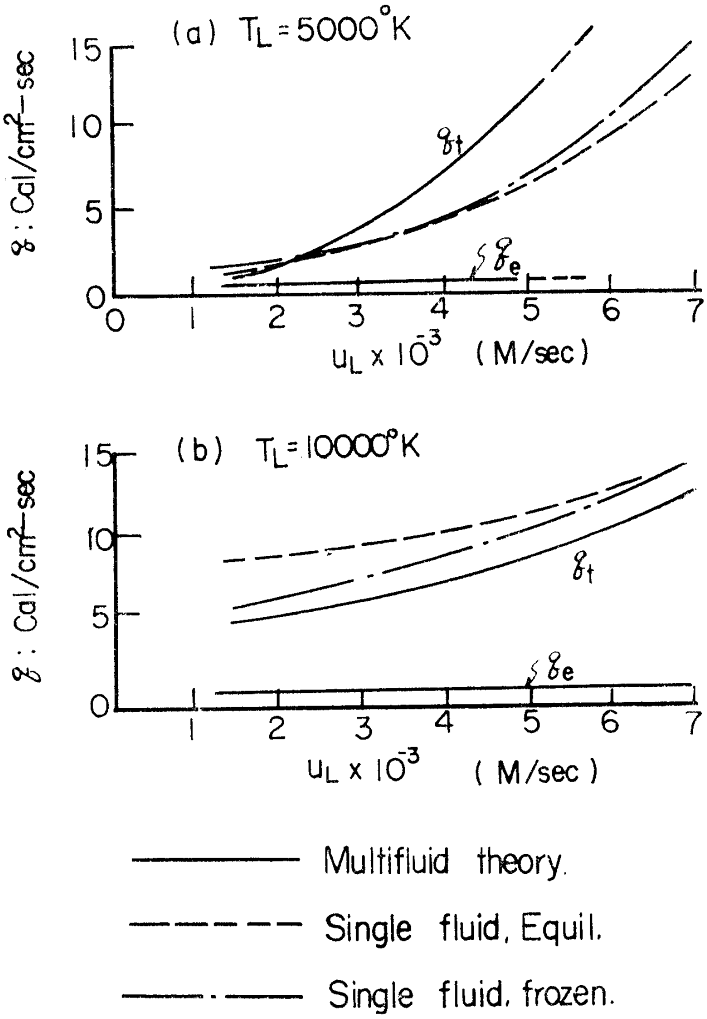

FIG. 8.4. Heat transfer rate *vs* velocity of the upper plate.

boundary conditions, the concept of an ionizing upper wall was introduced. This concept was based on the assumption that the upper wall chemical reactions could be described by a series of equations that define the species mass flux in terms of a non-equilibrium number density and a phenomenalogical efficiency which is related to the probability of incident atoms being converted to ions at the wall. The variation of ionization mechanism is equivalent to the variation of the mass flux. Fig. 8.6 shows the variation of heat transfer rate with the ion number flux. At the lower pressure level, both of the conductive contribution q_a and q_e were fairly independent of the number flux. The diffusive contribution resulting from the release of ionization energy by electron-ion recombination necessarily increases linearly with the number density flux, causing the total heat transfer q_t to increase similarly. At the higher pressure level, a somewhat unexpected results occurs. For both upper wall temperatures we calculate T_L=5000°K and 10000°K, the computations indicate a negative conductive contribution from the electrons and, hence a reduction in the total energy transfer.

Summaring the results in which we have sought to evaluate some of the effects of

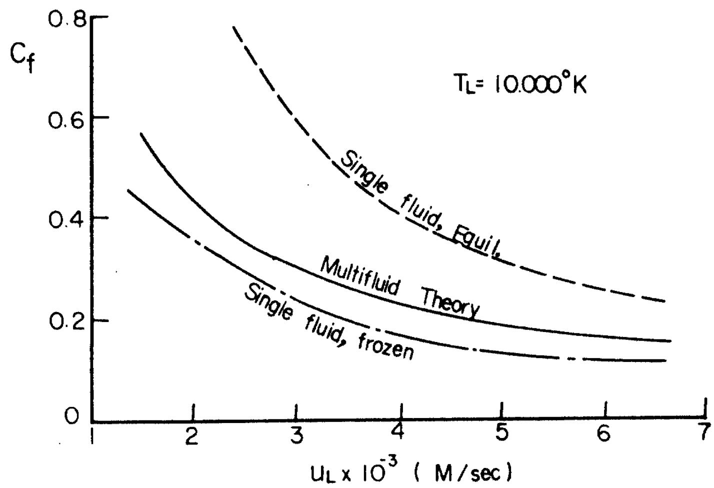

FIG. 8.5. Skin friction coefficient *vs* upper wall velocity.

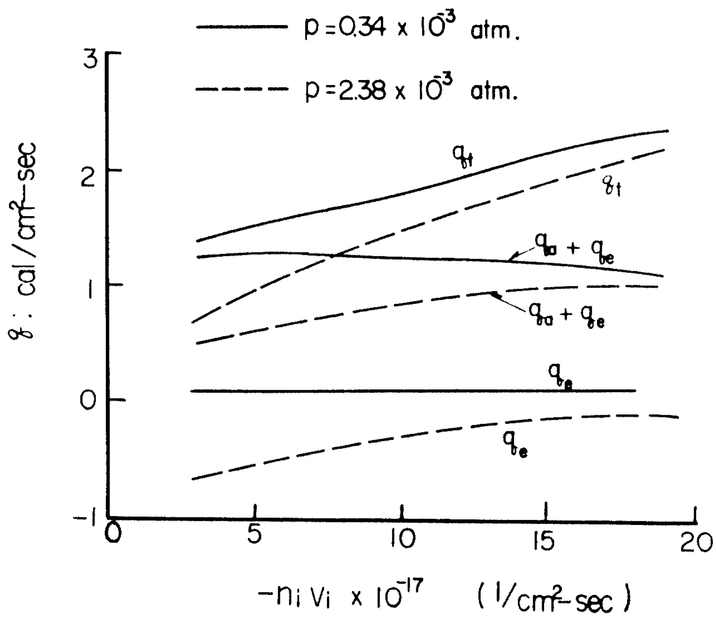

FIG. 8.6. Heat transfer rate *vs* ion number flux at two pressure levels.

non-equilibrium ionization on surface heat transfer in a fluid flow situation, we have come to the following general conclusions:

(1) The multifluid theory equation system, when judiciously used, is consistent with single fluid procedures and is particularly effective in evaluating the relative importance of different energy transfer mechanisms.

(2) Thermal non-equilibrium, which is characterized by electron temperature greater than heavy particle temperatures, results in a reduced total heat transfer, and the possibility of controlling the wall heat transfer by inducing thermal non-equilibrium through variations in the wall electrical potential is suggested.

(3) The coupling of the viscous flow variables is shown by the multifluid theory to have a significant effect on the heavy-particle temperature profiles and has a

negligible effect on the electron temperature profiles.

(4) Finally, the pressure level of the investigation strongly influences the degree of thermal non-equilibrium which may be realized.

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