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Aerodynamic Study of Stagnation Ablation

By

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Summary. Present paper gives detailed results of a theoretical approach to stagnation ablation associated with hypersonic reentry of blunt-nosed bodies of revolution. Fundamental equations are derived, under an assumption of sublimating ablation, for a non-reacting laminar boundary layer flow of binary gas mixture, in which transport properties are taken exactly into consideration by use of the atomic kinetic theory of gases.

It is shown that all physical properties concerning the ablating field can be uniquely determined under the given boundary conditions by matching of an aerodynamic solution with a static one obtained from chemical kinetics. Detailed examination reveals that error in estimation of Chapman-Rubesin number and Schmidt number across the boundary layer is most sensitive to the aerodynamic solutions, thus indicating that viscosity and diffusivity of gas mixture play a main role in controlling the flow characteristics of the ablating field.

Numerical calculation carried out for teflon ablator shows that the ablation rate increases with increase of stagnation temperature in free stream, while wall temperature does not rise so much, indicating that teflon is a fairly good ablator as a shield of aerodynamic heating for comparatively low enthalpy flow.

In order to confirm validity of the present approach, theoretical results are compared with experimental data for teflon. Agreement between theory and experiment is fairly good in the range of stagnation temperature from 800°C to 1200°C.

SYMBOLS

$(\bar{X}, \bar{Y})$	orthogonal coordinates system fixed in space
$(\bar{x}, \bar{y})$	orthogonal coordinates system fixed on body surface
$(\bar{U}, \bar{V})$	components of velocity vector in $(\bar{X}, \bar{Y})$ coordinates system
$(\bar{u}, \bar{v})$	components of velocity vector in $(\bar{x}, \bar{y})$ coordinates system
$\bar{v}_b(\bar{x})$	surface velocity due to ablation
$\bar{p}$	pressure
$\bar{\rho}$	density
$\bar{T}$	temperature
$\bar{\mu}_i$ $(i=1, 2)$	coefficient of viscosity for each pure species
$\bar{\mu}$	mean coefficient of viscosity
$\bar{\kappa}_i$ $(i=1, 2)$	coefficient of thermal conductivity for each pure species
$\bar{\kappa}$	mean coefficient of thermal conductivity
$\bar{D}$	binary diffusion coefficient
$\bar{R}$	universal gas constant

$\bar{R}_i$ $(i=1, 2)$	gas constant for each species
$\bar{R}_b$	radius of carvature of body
$\bar{r}_0$	cylindrical radius of body
$\bar{a}$	speed of sound
$\bar{L}$	latent heat for sublimation
$\bar{C}_{p_i}$ $(i=1, 2)$	specific heat at constant pressure for each pure species
$\bar{C}_p$	mean specific heat at constant pressure
$\bar{C}_b$	specific heat for solid material
$\dot{\bar{m}}$	ablation rate
$\dot{\bar{q}}$	heat transfer rate
$\bar{H}_{eff}$	effective heat of ablation
$\bar{\delta}$	boundary layer thickness
$\bar{\sigma}$	collision diameter
$\Omega^{(1,1)*}$, $\Omega^{(2,2)*}$	collision integral
(x, y)	non-dimensional coordinates system fixed on body surface
(s, η)	transformed coordinates system (see Eq. (4.6))
(u, v)	reduced components of velocity vector
p	reduced pressure
ρ	reduced density
T	reduced temperature
μ_i $(i=1, 2)$	reduced coefficient of viscosity for each pure species
μ	mean reduced coefficient of viscosity
κ	mean reduced coefficient of thermal conductivity
D	reduced binary diffusion coefficient
c_i	mole fraction of species i
r_0	reduced cylindrical radius of body
M_i $(i=1, 2)$	molecular weight of component gas
M_∞	free stream Mach number
ψ	stream function
H	total enthalpy function
K	concentration function
Re	Reynolds number
C	Chapman-Rubesin number
P	Prandtl number
S	Schmidt number
Le	Lewis number
l	non-dimensional latent heat for sublimation normalized by $\bar{C}_{p2}\bar{T}_{st}$
C_{p_i} $(i=1, 2)$	reduced specific heat at constant pressure for each pure species
C_p	mean reduced specific heat at constant pressure
γ	ratio of specific heats for air

Subscripts;

st	stagnation conditions in free stream
∞	conditions in free stream

s	conditions just aft of shock wave
e	conditions at outer edge of boundary layer
1	conditions of foreign gas
2	conditions of air
a	conditions with ablation
no	conditions without ablation
w	conditions at wall
b	conditions inside the body
$(\ )'$	differentiation with respect to argument

Superscripts:

$*$	conditions for non-dimensionalization

1. INTRODUCTION

With the advance of space exploration, aerodynamic heating problems at high speed reentry of space vehicles to the earth atmosphere have become increasingly important and a number of investigations [1],[2],[3] have been conducted to give informations, indicating that, in the range of altitude at which air is considered to be continuum, the aerodynamic heating, especially in stagnation region of blunt bodies, becomes so severe that some methods of heat absorption or cooling are required.

In case of a steep descent where reentry time is sufficiently short, it is well known from engineering viewpoint that ablation is one of the most practical methods for shielding space vehicles from severe aerodynamic heating in the sense that it has two predominant effects, which may be stated as follows:

(1) Since injected foreign gas species, which are generated through ablation of surface material, convect in the aerodynamic boundary layer over the surface of the body by the actions of existing pressure gradient and viscous force and diffuse further due to concentration gradient, thickness of the boundary layer grows so as to diminish the aerodynamic heating itself.

(2) A large amount of heat flow from boundary layer is absorbed efficiently in latent heat required for phase change of surface material such as liquefaction or sublimation, so that the heat flow conducted inside the body is automatically controlled to decrease considerably even for high enthalpy flow outside the boundary layer.

It is clear that the ablation applied to shield the aerodynamic heating to reentry bodies is a very complicated phenomenon associated with coupling of a non-equilibrium chemical process with aerodynamic one. Unfortunately, however, chemical kinetic theory seems to give poor information on this complicated process at the present time and, consequently, there does not seem in this field to exist many previous works on solid materials which are believed to be proper ablators.

On the other hand, the aerodynamic studies have already been developed approximately by many scientists. Theoretical works proposed, respectively, by Sutton [4] and Roberts [5] were concerned mainly with melting surface in which a liquid layer flows over the surface. The main conclusion in these works is that the convective shielding by liquid layer is limited by the temperature at which liquid boils. When vaporization occurs, however, the convective shield in the gas boundary layer is found to increase considerably.

By taking diffusion effect into consideration, Roberts [5],[6],[7] further presented a simple analysis concerning the sublimating ablation. The treatment of the boundary layer being based essentially upon an integral method in which distribution of physical properties is assumed to be linear across the boundary layer, this approach may show qualitatively important physical aspects associated with the mechanism of shielding by vaporization rather than the quantitative information.

Since, as pointed out by Swann [8], the aerodynamic mechanism of shielding by vaporization can be considered to be equivalent to that of the boundary layer flow with mass injection except for the effect of latent heat required for phase change of the material, comprehensive reviews [9],[10] and studies [11],[12],[13],[14] have been made of the convective heat transfer with mass addition. The main conclusion in these papers is in that the transpiration of foreign gas with light molecular weight may reduce the aerodynamic heating remarkably so far as the convective heat transfer is concerned. Moreover, the effects of transport properties on boundary layer behaviour involving chemical reactions such as combustion and dissociation were examined, respectively, in detail by Libby [15] for hydrogen injection and by Koh [16] for ammonia addition, indicating remarkable results that the effect of dissociation of the coolant with an associated heat absorption is to reduce the coolant flow rate required to maintain the surface at a specified temperature, while the prediction of analysis based on simple transport properties may be in serious error. Scala [17] further developed an analytical approach to sublimating ablation of graphite, in which effects of dissociation, recombination and combustion were included. Mass loss rate being relatively small even for high enthalpy flow, it will be found from Scala's results that graphite may become of a proper ablator in more advanced reentry technique.

Notwithstanding that the transport properties have serious effects on boundary layer behaviour as has been just mentioned above, it seems that there has been developed few theoretical work on boundary layer flow concerning ablation in which transport properties are exactly evaluated. This circumstance requires a more rigorous method of analytical approach to the aerodynamic relation, which is of a primary importance for clarifying the mechanism of ablation.

Present paper has a purpose to give an analytical approach to sublimating stagnation ablation associated with high speed reentry of blunt-nosed bodies of revolution, in which transport properties are taken exactly into consideration by use of the atomic kinetic theory of gases, and to show that the ablating field can be uniquely determined under the given boundary conditions by matching of a static relation from chemical kinetics with a dynamic relation from boundary layer theory. For the purpose of simplifying the analysis some assumptions are introduced such as non-reacting laminar boundary layer of binary gas mixture, etc. These assumptions, however, do not degenerate the essential feature of the problem.

Numerical calculations are carried out for teflon ablator and the results are compared with experimental data.

2. Mechanism for Determination of Ablating Field

As has been already mentioned briefly in the last section, ablation under consideration is a very complicated phenomenon associated with coupling of a non-equilibrium chemical process with an aerodynamic one and, therefore, it may be of a primary interest to clarify the mechanism for determining the ablating field.

Since ablation rate $\dot{m}$ (mass loss rate of solid material per unit surface area and unit time), which is one of the most characteristic quantities for predicting the ablation phenomenon, is, essentially, a kind of chemical reaction rates, it can be considered to be expressed in terms of two thermodynamic variables of state in a form

$$\dot{m}=F(\bar{T}_w, \bar{P}) , \tag{2.1}$$

where a functional form F may be determined from chemical kinetics. It must be noted that Eq. (2.1) includes substantial characteristics of solid material implicitly and may be considered to give a static relation necessary for all the fields where ablation occurs. In applying this relation to the problem under consideration, however, it will be easily found that Eq. (2.1) is insufficient to predict the ablating field in a closed form, since surface temperature is not known a priori. This circumstance requires a complemental relation between ablation rate and surface temperature, which will be derived aerodynamically through consideration of heat transfer balance at the ablating surface.

As is well known, the solutions of boundary layer equations, which are parabolic type from mathematical point of view, require either temperature or temperature gradient at the wall to be known a priori. This characteristic together with conservation of heat transfer at the ablating surface suggests clearly a possibility to derive an aerodynamic relation between $\dot{m}$ and $\bar{T}_w$, which may be expressed as

$$\dot{m}=G(\bar{T}_w, \bar{N}) , \tag{2.2}$$

where $\bar{N}$ means a number of parametric boundary conditions such as stagnation conditions in the free stream, body shape and substantial properties of the material, etc. Eq. (2.2) indicates a dynamic relation permissible aerodynamically for an ablating surface.

Since Eq (2.2) is independent of Eq. (2.1), these two equations can be considered to be simultaneous equations with respect to variables $\dot{m}$ and $\bar{T}_w$. The solutions thus obtained satisfy both static and dynamic relations at the same time and, consequently, ablating field can be determined uniquely under the given boundary conditions.

3. Fundamental Equations for Boundary Layer Flow

Being mainly concerned with a rate of phase change of the surface material, ablation phenomenon is essentially a non-steady problem involving change of body shape with time. However, if the rate of phase change is assumed to be either independent or dependent very slightly of time, then, the problem may be reasonably reduced to steady (or quasi-steady) ablation. From experimental point of view, the steady ablation seems to exist approximately near the stagnation region of blunt-nosed bodies of revolution after a certain transient time is passed. Therefore, the present approach is developed under several assumptions as follows;

(1) steady sublimating ablation,

(2) non-reacting laminar boundary layer of binary gas mixture,

(3) boundary layer thickness is very small compared with radius of body curvature,

(4) radiation is negligible.

It must be noted that the additional assumptions from (2) to (4) are made only for the purpose of simplifying the analysis and do not degenerate the essential feature of the problem. If chemical reactions and associated radiation are involved in boundary layer of multi-components gas mixture, the problem can be treated in quite the same way by adding the terms representing those effects to the fundamental equations.

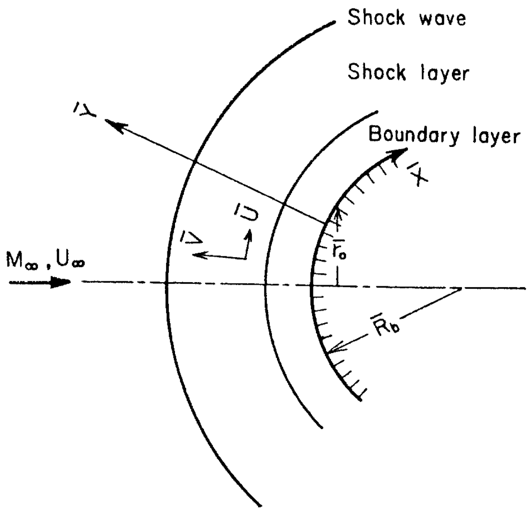

Fig. 1. Orthogonal coordinates system

Let the origin of orthogonal coordinates system $(\bar{X}, \bar{Y})$ fixed in space be taken at the stagnation point of blunt-nosed bodies of revolution at a specified reference time, $\bar{X}$-axis being taken along a meridian line of the surface as shown in Fig. 1, then, the boundary layer equations for binary gas mixture may be written as

$$\frac{\partial(\bar{\rho}\bar{r}_0)}{\partial t}+\frac{\partial(\bar{\rho}\bar{U}\bar{r}_0)}{\partial\bar{X}}+\frac{\partial(\bar{\rho}\bar{V}\bar{r}_0)}{\partial\bar{Y}}=0, \tag{3.1a}$$

$$\bar{\rho}\frac{\partial \bar{U}}{\partial t}+\bar{\rho}\bar{U}\frac{\partial \bar{U}}{\partial \bar{X}}+\bar{\rho}\bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}}=\bar{\rho}_e\bar{U}_e\frac{\partial \bar{U}_e}{\partial \bar{X}}+\frac{\partial}{\partial \bar{Y}}\left(\bar{\mu}\frac{\partial \bar{U}}{\partial \bar{Y}}\right), \tag{3.1b}$$

$$\frac{\partial \bar{p}}{\partial \bar{Y}}=0, \tag{3.1c}$$

$$\bar{\rho}\frac{\partial \bar{H}}{\partial t}+\bar{\rho}\bar{U}\frac{\partial \bar{H}}{\partial \bar{X}}+\bar{\rho}\bar{V}\frac{\partial \bar{H}}{\partial \bar{Y}}=\frac{\partial}{\partial \bar{Y}}\left[\frac{\bar{\mu}}{P}\frac{\partial \bar{H}}{\partial \bar{Y}}+\bar{\mu}\left(1-\frac{1}{P}\right)\frac{1}{2}\frac{\partial \bar{U}^2}{\partial \bar{Y}}\right]$$
$$-\frac{\partial}{\partial \bar{Y}}\left[\left(\frac{1}{Le}-1\right)\bar{\rho}\bar{D}(\bar{C}_{p1}-\bar{C}_{p2})\bar{T}\frac{\partial K}{\partial \bar{Y}}\right], \tag{3.1d}$$

$$\bar{\rho}\frac{\partial K}{\partial t}+\bar{\rho}\bar{U}\frac{\partial K}{\partial \bar{X}}+\bar{\rho}\bar{V}\frac{\partial K}{\partial \bar{Y}}=\frac{\partial}{\partial \bar{Y}}\left(\bar{\rho}\bar{D}\frac{\partial K}{\partial \bar{Y}}\right), \tag{3.1e}$$

where $\bar{H}$ is total enthalpy defined by the equation

$$\bar{H}=\bar{C}_p\bar{T}+\frac{1}{2}\bar{U}^2. \tag{3.1f}$$

From the assumption of steady ablation, since surface velocity $\bar{v}_b$ is independent of time, it is easily known that Eq. (3.1a) to (3.1f) can be interpreted into steady equations expressed in an orthogonal coordinates system ($\bar{x}$, $\bar{y}$) fixed on moving body surface by use of a transformation of variables such as

$$\left.\begin{aligned}\bar{x}&=\bar{X},\\ \bar{y}&=\bar{Y}-\bar{v}_b(\bar{x})t,\\ \bar{u}&=\bar{U},\\ \bar{v}&=\bar{V}-\bar{v}_b(\bar{x}).\end{aligned}\right\} \tag{3.2}$$

Thus, by introducing non-dimensional expressions

$$\left.\begin{aligned}&x=\frac{\bar{x}}{\bar{x}^*}(\bar{x}^*=\bar{R}_b),\quad y=\frac{\bar{y}}{\bar{y}^*},\quad r_0=\frac{\bar{r}_0}{\bar{R}_b},\quad u=\frac{\bar{u}}{\bar{u}^*},\quad v=\frac{\bar{v}}{\bar{u}^*},\\ &\rho=\frac{\bar{\rho}}{\bar{\rho}_s},\quad \mu=\frac{\bar{\mu}}{\bar{\mu}_s},\quad T=\frac{\bar{T}}{\bar{T}_{st}},\quad H=\frac{\bar{H}}{\bar{H}^*}=\frac{\bar{H}}{\bar{C}_{p2}\bar{T}_{st}},\\ &C_p=\frac{\bar{C}_p}{\bar{C}_{p2}},\quad \kappa=\frac{\bar{\kappa}}{\bar{\kappa}_2},\quad D=\frac{\bar{\rho}_s}{\bar{\mu}_s}\bar{D},\quad p=\frac{\bar{p}}{\bar{\rho}_s\bar{u}^{*2}},\end{aligned}\right\} \tag{3.3}$$

together with an additional condition

$$\bar{y}^*=\left(\frac{\bar{\mu}_s\bar{R}_b}{\bar{\rho}_s\bar{u}^*}\right)^{\frac{1}{2}}, \tag{3.4}$$

the boundary layer equations can be reduced to the forms

$$\frac{\partial(\rho u r_0)}{\partial x}+\frac{\bar{x}^*}{\bar{y}^*}\frac{\partial(\rho v r_0)}{\partial y}=0, \tag{3.5a}$$

$$\rho u\frac{\partial u}{\partial x}+\frac{\bar{x}^*}{\bar{y}^*}\rho v\frac{\partial u}{\partial y}=\rho_e u_e\frac{\partial u_e}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{3.5b}$$

$$\frac{\partial p}{\partial y}=0\,, \tag{3.5c}$$

$$\rho u\frac{\partial H}{\partial x}+\frac{\bar{x}^*}{\bar{y}^*}\rho v\frac{\partial H}{\partial y}=\frac{\partial}{\partial y}\left[\frac{\mu}{P}\frac{\partial H}{\partial y}+\frac{\bar{u}^{*2}}{\bar{H}^*}\mu\left(1-\frac{1}{P}\right)\frac{1}{2}\frac{\partial u^2}{\partial y}\right]$$
$$+\frac{\partial}{\partial y}\left[\left(\frac{1}{Le}-1\right)\alpha_2\rho DT\frac{\partial K}{\partial y}\right], \tag{3.5d}$$

$$\rho u\frac{\partial K}{\partial x}+\frac{\bar{x}^*}{\bar{y}^*}\rho v\frac{\partial K}{\partial y}=\frac{\partial}{\partial y}\left(\rho D\frac{\partial K}{\partial y}\right), \tag{3.5e}$$

where

$$H=C_pT+\frac{1}{2}u^2\frac{\bar{u}^{*2}}{\bar{H}^*}, \tag{3.5f}$$

and

$$\alpha_2=1-\frac{\bar{C}_{p1}}{\bar{C}_{p2}}\,. \tag{3.6}$$

4. Equations Near the Region of Stagnation Point

Continuity equation, Eq. (3.5a), may be accounted for by introducing a stream function defined as

$$\psi_x=-\frac{\bar{x}^*}{\bar{y}^*}\rho v r_0\,,\qquad \psi_y=\rho u r_0\,. \tag{4.1}$$

However, before presenting the detailed formulations, the inviscid flow conditions at outer edge of the boundary layer must be first specified near the stagnation region. In the present approach it is assumed that the inviscid flow conditions are given by the constant density solution for hypersonic flow past a sphere [18], although they may be evaluated exactly by use of a numerical method proposed by Van Dyke [19]. Therefore, the velocity along the body surface is expressed by the equation

$$\bar{u}_e=\bar{u}_\infty\sqrt{\frac{8\varepsilon}{3}}\frac{1}{1+\lambda_0}x+O(x^3)\,, \tag{4.2}$$

$$\varepsilon=\frac{\bar{\rho}_\infty}{\bar{\rho}_s}=\frac{(\gamma-1)M_\infty^2+2}{(\gamma+1)M_\infty^2}, \tag{4.3a}$$

where λ_0 is a ratio of shock-detachment distance to the radius of body curvature at stagnation point and is given by

$$\lambda_0=\frac{\varepsilon}{1+\sqrt{\dfrac{8\varepsilon}{3}-\varepsilon}}\,. \tag{4.3b}$$

Density and viscosity at the outer edge of the boundary layer may also be expressed in non-dimensional forms as

$$\left.\begin{aligned}\rho_e&=1+O(x^2)\,,\\ \mu_e&=1+O(x^2)\,.\end{aligned}\right\}\tag{4.4}$$

Moreover, since the body is blunt at its nose, its contour may be given by the equation

$$r_0=\frac{\bar{r}_0}{\bar{R}_b}=x+O(x^3)\,.\tag{4.5}$$

Introducing a well-known Lees-Dorodnitsyn transformation

$$s=\int_0^x \rho_e\mu_e r_0^2 dx\,,\qquad \eta=\frac{\rho_e r_0}{s^m}\int_0^y \frac{\rho}{\rho_e}dy\,,\tag{4.6}$$

then, gives a relation between x and s near the stagnation point such as

$$x=3^{\frac{1}{3}}s^{\frac{1}{3}}+O(s)\,.\tag{4.7}$$

By use of Eq. (4.7), Eq. (4.2) can be reexpressed as

$$\bar{u}_e=\bar{u}_\infty 3^{\frac{1}{3}}\sqrt{\frac{8\varepsilon}{3}}\,\frac{1}{1+\lambda_0}\,s^{\frac{1}{3}}+O(s)\,.\tag{4.8}$$

Therefore, it will be found convenient for subsequent manipulations to choose $\bar{u}^*$, which is used for normalization of the velocity components, as

$$\bar{u}^*=3^{\frac{1}{3}}\sqrt{\frac{8\varepsilon}{3}}\,\frac{\bar{u}_\infty}{1+\lambda_0}\,.\tag{4.9}$$

Thus, velocity, density and viscosity at the outer edge of the boundary layer can be written, respectively, as

$$\left.\begin{aligned}u_e&=s^{\frac{1}{3}}+O(s)\,,\\ \rho_e&=1+O(s^{\frac{2}{3}})\,,\\ \mu_e&=1+O(s^{\frac{2}{3}})\,.\end{aligned}\right\}\tag{4.10}$$

If the stream function is assumed to have a form

$$\psi=s^{m+k}f_0(\eta)+q_1 s^{m+k+n}f_1(\eta)+\cdots\,,\tag{4.11}$$

where $q_1,\cdots$, etc. are constants, velocity component u in the boundary layer can be expressed as

$$u=s^k f_0'(\eta)+q_1 s^{k+n}f_1(\eta)+\cdots\,.\tag{4.12}$$

By comparing Eq. (4.12) with Eq. (4.10), it is easily found that the power indices k and n must be, respectively,

$$k=\frac{1}{3}, \qquad n=\frac{2}{3}, \tag{4.13}$$

and, hence,

$$u=s^{\frac{1}{3}}f_0'(\eta)+O(s). \tag{4.14}$$

In quite the same way, detailed examinations reveal that Taylor expansions of the total enthalpy function and concentration function consistent with a set of equations given by Eqs. (3.5b) to (3.5e) must have forms

$$\left.\begin{aligned} H&=g_0(\eta)+O(s^{\frac{2}{3}}),\\ K&=z_0(\eta)+O(s^{\frac{2}{3}}), \end{aligned}\right\} \tag{4.15}$$

where a power index, m, involved in Eq. (4.6), which is also consistent with the set of equations, has been found to be

$$m=\frac{1}{3}. \tag{4.16}$$

Since average specific heat for constant pressure of binary gas mixture is given by the equation

$$C_p=\frac{\bar{C}_p}{\bar{C}_{p2}}=\frac{1}{\bar{C}_{p2}}[\bar{C}_{p1}K+\bar{C}_{p2}(1-K)]=1-\alpha_2K, \tag{4.17}$$

the temperature may be expressed by use of Eqs. (3.5f) and (4.17) as

$$\frac{T}{T_e}=\frac{H-\dfrac{1}{2}u^2\dfrac{\bar{u}^{*2}}{\bar{H}^*}}{(1-\alpha_2K)\left(H_e-\dfrac{1}{2}u_e^2\dfrac{\bar{u}^{*2}}{\bar{H}^*}\right)}, \tag{4.18a}$$

which is further reduced, by use of Eq. (4.15), to

$$\frac{T}{T_e}=\frac{g_0}{H_e(1-\alpha_2z_0)}+O(s^{\frac{2}{3}}), \tag{4.18b}$$

where

$$H_e=1-\frac{\gamma-1}{\gamma+1}\varepsilon\approx1, \tag{4.19}$$

and is considered approximately to be unity for hypersonic flow.

An expression of density is a point of interest. It may be obtained from the equation of state for each component species by the following procedure;

$$p_1=\rho_1R_1T=\frac{K}{M_1}\rho RT, \qquad p_2=\rho_2R_2T=\frac{1-K}{M_2}\rho RT,$$

then $$p=p_1+p_2=\rho RT\left(\frac{K}{M_1}+\frac{1-K}{M_2}\right)=\rho_e RT_e\frac{1}{M_2}.$$

Hence, $$\frac{\rho_e}{\rho}=(1-\alpha_1 K)\frac{T}{T_e}$$
$$=\frac{(1-\alpha_1 z_0)g_0}{1-\alpha_2 z_0}+O(s^{\frac{2}{3}}), \tag{4.20}$$

where $$\alpha_1=1-\frac{M_2}{M_1}. \tag{4·21}$$

Estimation of the transport properties involved in Eqs. (3.5b) to (3.5e) is another point of interest. It can be done exactly by use of the atomic kinetic theory of gases and each transport property is obtained as a function of total enthalpy and concentration, which may be further expanded into a power series of s as well. The procedure for evaluation of the transport properties is shown in Appendices A and B in detail.

Substitution of Eqs. (4.1), (4.10), (4.14), (4.15), (4.18b) and (4.20) together with series expressions of transport properties developed in the appendices into Eqs. (3.5b) to (3.5e) and equating like power of s yields, as leading terms, the following simultaneous equations with respect to f_0, g_0 and z_0, which may predict the boundary layer flow near the region of stagnation of blunt-nosed bodies of revolution;

$$\left.\begin{aligned}
&(C_0 f_0'')'+\frac{2}{3}f_0 f_0''+\frac{1}{3}\left[\frac{1-\alpha_1 z_0}{1-\alpha_2 z_0}g_0-f_0'^2\right]=0,\\
&\left(\frac{C_0}{P_0}g_0'\right)'+\frac{2}{3}f_0 g_0'+(\phi_0 z_0')'=0,\\
&\left(\frac{C_0}{S_0}z_0'\right)'+\frac{2}{3}f_0 z_0'=0,\\
&\phi_0=\frac{\alpha_2 g_0}{1-\alpha_2 z_0}\,\frac{(S_0-P_0)C_0}{P_0 S_0},
\end{aligned}\right\} \tag{4.22}$$

where C_0, P_0 and S_0 indicate the leading terms of Taylor expansions of Chapman-Rubesin number, Prandtl number and Schmidt number with respect to s and are expressed as a function of g_0 and z_0, respectively (see Appendix B).

5. Boundary Conditions

It is clear that, in order to obtain the solutions to Eq. (4.22), seven boundary conditions are required. Unfortunately, however, there seem to exist only four explicit conditions such as a non-slip condition at the wall and other three conditions to make boundary layer flow compatible with outer inviscid flow. They are

$$\left.\begin{aligned}
&f_0'=0 \quad \text{at} \quad \eta=0,\\
&f_0'=1, \quad g_0=1, \quad z_0=0 \quad \text{at} \quad \eta=\infty.
\end{aligned}\right\} \tag{5.1}$$

The final condition in Eq. (5.1) indicates an additional restriction that convection and diffusion of the foreign gas species must be confined to inside of the boundary layer.

The other three boundary conditions required for Eq. (4.22) can be derived from discussions on relations associated with physical quantities at the ablating surface. It is evident that distribution of wall temperature consistent with the development of the boundary layer equations mentioned in the last section must have a form

$$T_w = T_{w0} + O(s^{\frac{2}{3}}) , \tag{5.2}$$

where T_{w0} denotes wall temperature just at the stagnation point and is unknown a priori. Substitution of Eqs. (4.15), (4.17) and (5.2) into Eq. (3.5f) and equating like power of s yields, as a leading term, a boundary condition indicating a relation between wall enthalpy and surface temperature at the stagnation point, that is

$$g_{0w} = (1 - \alpha_2 z_{0w}) T_{w0} . \tag{5.3}$$

In quite the same way, the physical condition that there is no net mass transfer of air into the wall gives a relation

$$-\left(\bar{\rho}\bar{D} \frac{\partial K}{\partial \bar{y}}\right)_w = (1 - K_w)(\bar{\rho}\bar{v})_w , \tag{5.4}$$

which may be further reduced, near the region of stagnation point, to

$$\left(\frac{C_0}{S_0} z_0'\right)_w = \frac{2}{3}(1 - z_{0w}) f_{0w} . \tag{5.5}$$

It remains to discuss the final condition. It can be obtained in such a way that heat transfer from the boundary layer must be equal to sum of the heat absorbed in latent heat for sublimation and the heat conducted inside the body, that is

$$\left(\bar{\kappa} \frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=+0} = \bar{L}(\bar{\rho}\bar{v})_w + \left(\bar{\kappa}_b \frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=-0} . \tag{5.6}$$

In order to reduce this equation, the heat conducting inside the body must be clarified. For this purpose it is assumed that the temperature gradient in $\bar{x}$-direction is negligible inside the body compared with that in $\bar{y}$-direction. With this assumption, the heat conduction equation can be written as

$$\begin{aligned} \bar{\kappa}_b \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} &= \bar{\rho}_b \bar{C}_b \frac{\partial \bar{T}}{\partial t} = \bar{\rho}_b \bar{C}_b \frac{\partial \bar{T}}{\partial \bar{y}} \frac{\partial \bar{y}}{\partial t} \\ &= -\bar{\rho}_b \bar{C}_b \bar{v}_b \frac{\partial \bar{T}}{\partial \bar{y}} . \end{aligned}$$

Consequently,

$$\bar{\kappa}_b \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} = \dot{\bar{m}} \bar{C}_b \frac{\partial \bar{T}}{\partial \bar{y}}, \tag{5.7}$$

where

$$\dot{\bar{m}} = -\bar{\rho}_b \bar{v}_b = (\bar{\rho}\bar{v})_w, \tag{5.8}$$

and the boundary conditions are given by

$$\left.\begin{array}{lll} \bar{T} = \bar{T}_w & \text{at} & \bar{y} = 0, \\ \bar{T} = \bar{T}_b & \text{at} & \bar{y} = -\infty. \end{array}\right\} \tag{5.9}$$

Therefore, the solution to Eq. (5.7) appropriate to Eq. (5.9) can be obtained as

$$\bar{T} = \bar{T}_b + (\bar{T}_w - \bar{T}_b) \exp\left(\frac{\bar{C}_b \dot{\bar{m}}\, \bar{y}}{\bar{\kappa}_b}\right). \tag{5.10}$$

Thus, by use of Eq. (5.10), Eq. (5.6) may be summarized near the region of stagnation point as (see Appendix C)

$$\frac{E_1(z_{0w})}{1-\alpha_1 z_{0w}} \left(\frac{T_s}{T_{w0}}\right)^{\frac{1}{2}} \left(\frac{g_0}{1-\alpha_2 z_0}\right)'_w = -\frac{2}{3} P_2 f_{0w} \left\{\frac{\bar{C}_b}{\bar{C}_{p2}} (T_{w0} - T_b) + l\right\},$$
$$l = \frac{\bar{L}}{\bar{C}_{p2} \bar{T}_{st}}, \tag{5.11}$$

where P_2 denotes Prandtl number for air and $E_1(z_{0w})$ is given by Eq. (A.2.9) in Appendix A.

Here an attention must be paid to the fact that the fundamental equations, Eq. (4.22), and the associated boundary conditions, Eqs. (5.1), (5.3), (5.5) and (5.11), does not involve any perimetric condition except for T_{w0} and T_b, since $T_s \doteqdot 1$ for any hypersonic stagnation flow. This clearly indicates that the solutions to Eq. (4.22) can be obtained uniquely irrespective of the perimetric conditions associated with the outer inviscid hypersonic flow.

6. Dynamic Relation for Ablation Rate

In the last two sections, the boundary layer equations and appropriate boundary conditions have been derived on the basis of the Taylor expansion of physical properties with respect to $\bar{x}$. In quite the same way, the aerodynamic relation for ablation rate can be obtained near the region of stagnation point by the following procedure.

Since ablation rate is defined as the rate of mass loss of surface material per unit surface area and unit time, it is expressed as the mass flow rate of surface injection, that is

$$\dot{\bar{m}} = (\bar{\rho}\bar{v})_w = \bar{\rho}_s \bar{u}^* (\rho v)_w,$$

and this may be reduced by use of the stream function defined by Eqs. (4.1) and (4.11) to a series expansion form

$$\dot{\bar{m}} = -\bar{\rho}_s \bar{u}^* \frac{\bar{y}^*}{\bar{x}^*} \frac{1}{r_0} (\psi_x)_w$$
$$= -\bar{\rho}_s \bar{u}^* \left(\frac{\bar{\mu}_s}{\bar{\rho}_s \bar{u}^* \bar{R}_b} \right)^{\frac{1}{2}} \left[3^{\frac{1}{3}} \frac{2}{3} f_{0w} + O(s^{\frac{2}{3}}) \right]. \tag{6.1}$$

Therefore, the ablation rate at the stagnation point, $\dot{\bar{m}}_0$, can be obtained as

$$\dot{\bar{m}}_0 = -\frac{2}{3} 3^{\frac{1}{3}} \bar{\rho}_s \bar{u}^* \left(\frac{\bar{\mu}_s}{\bar{\rho}_s \bar{u}^* \bar{R}_b} \right)^{\frac{1}{2}} f_{0w} , \tag{6.2}$$

which may be further reexpressed by use of conditions in free stream as

$$\dot{\bar{m}}_0 = -\frac{2\sqrt{6}}{3} J(M_\infty) \frac{1}{\sqrt{Re}} \bar{\rho}_{st} \bar{a}_{st} f_{0w} , \tag{6.3}$$

where $\bar{\rho}_{st}$ and $\bar{a}_{st}$ denote density and speed of sound at stagnation in free stream, respectively, and Re is a Reynolds number defined by free stream conditions and radius of body curvature as

$$Re = \frac{\bar{\rho}_\infty \bar{u}_\infty \bar{R}_b}{\bar{\mu}_\infty} . \tag{6.4}$$

$J(M_\infty)$ denotes a function of free stream Mach number expressed by the equation

$$J(M_\infty) = \left(\sqrt{\frac{8\varepsilon}{3}} \frac{1}{1+\lambda_0} \right)^{\frac{1}{2}} \frac{M_\infty^{\frac{3}{2}} \{2\gamma M_\infty^2 - (\gamma-1)\}^{\frac{1}{4}}}{\{(\gamma-1)M_\infty^2 + 2\}^{\frac{3}{4}} \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\frac{1}{\gamma-1}}} , \tag{6.5}$$

where ε and λ_0 are given by Eqs. (4.3a) and (4.3b), respectively.

It must be noticed that, since f_{0w} in Eq. (6.3) can be obtained as a solution of the boundary layer equations, wall temperature, T_{w0}, is included implicitly in it. Therefore, Eq. (6.3) may be considered to correspond to a representative form of the dynamic relation between ablation rate and wall temperature such as expressed by Eq. (2.2).

7. Numerical Calculation for Teflon Ablator and Results

As has been already mentioned in section 5, the boundary value problem developed in the present approach is a closed one and the solutions to Eq. (4.22) can be, in principle, obtained under the given boundary conditions—four explicit conditions and three physical conditions. However, since Eq. (4.22) denotes very complicated non-linear simultaneous equations, it seems that only a numerical method of integration is available. Moreover, it must be noticed that a direct integration of Eq. (4.22) can not be made readily starting at $\eta=0$. This mathematical difficulty clearly arises from the fact that the present approach is of a two-points boundary value problem having its boundary conditions both at $\eta=0$ and $\eta=\infty$.

In order to avoid this difficulty some mathematical manipulations are introduced for starting numerical integrations outwards from the body surface, which may be done by the following procedure. First, three explicit conditions at $\eta=\infty$ are to be excluded from the boundary conditions. These are

$$f_0'=1\,,\quad g_0=1\,,\quad z_0=0\quad \text{at}\quad \eta=\infty\,. \tag{7.1}$$

On the other hand, for the purpose of making up for shortage of the boundary conditions just excluded, other three boundary values such as f_{0w}, f_{0w}'' and z_{0w} are then assumed at $\eta=0$. These assumed values, in turn, turn out three appropriate boundary conditions of g_{0w}, z_{0w}' and g_{0w}' for a fixed value of T_{w0} by use of Eqs. (5.3), and (5.5) and (5.11), respectively. Consequently, it will be easily found that these six boundary values together with the non-slip condition, $f_{0w}'=0$, give a set of boundary conditions sufficient for starting of the numerical integrations from $\eta=0$. The excluded conditions shown in Eq. (7.1) may be, therefore, used as conditions for convergency of the solutions in the present approach together with additional conditions

$$f_0''=f_0'''=g_0'=g_0''=z_0'=z_0''=0\,,\quad \text{at}\quad \eta=\infty\,. \tag{7.2}$$

It must be noted that these additional conditions may, at a glance, seem to result in over-determination of the solutions. However, this difficulty might be found from detailed examination of Eq. (4.22) to be avoided reasonably for the reason that the solutions to Eq. (4.22) have an exponential, asymptotic behaviour when η tends to infinity and, therefore, Eq. (7.2) must be satisfied automatically by the solutions required. Thus, Eq. (7.2) should be considered to be used as conditions for more rigorous justification of convergency of the solutions.

The actual calculation was carried out for teflon ablator by use of a HITAC 5020 high speed electronic computer. Integration of Eq. (4.22) was made step by step outwards starting from the body surface by assuming a proper set of boundary values (f_{0w}, $f_0''{}_w$, z_{0w}). If the solutions did not satisfy Eq. (7.1) for large value of η, then, these assumed values were slightly adjusted and the integration was carried out again. This trial and error method was continued to repeat again and again until the solutions converged sufficiently to satisfy both Eq. (7.1) and (7.2).

The reason for choosing teflon ablator as an example is clearly due to the fact that it may be a proper material satisfying all assumptions made in the present approach. For example, the volatile products from pyrolysis of polytetrafluoroethylene (teflon) cited from Madorsky [20] are shown in Table 1. As is seen in the table, the volatile products from teflon mainly consist of C_2F_4-gas and, consequently, the present assumption of non-reacting binary gas mixture may be confirmed approximately in the temperature range from 500°C to 800°C. Moreover,

TABLE 1. Volatile products from pyrolysis of polytetrafluoroethylene (in per cent weight (ref. 20))

Component	500°C	800°C
HF	0	0
CF_4	1.5	1.6
C_2F_4	94.8	92.5
C_3F_6	3.7	5.9
Vpyrolysis	0	0
Total	100.0	100.0

the static relation between $\dot{m}$ and $\bar{T}_w$ for teflon has already been proposed by Rashis and Hopko [*21*] such as shown in Fig. 2.

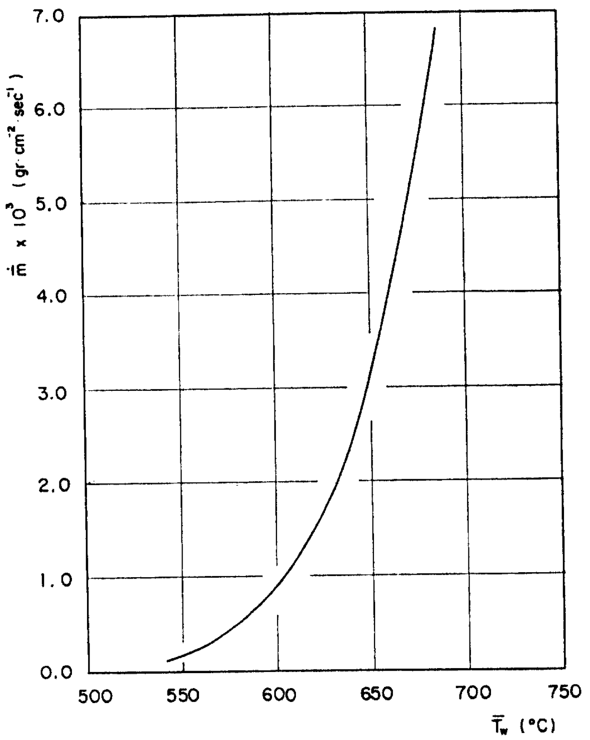

FIG. 2. Static relation between $\dot{m}$ and $\bar{T}_w$ for teflon (ref. 21)

The substantial properties for air, C_2F_4-gas and solid teflon, which are used in the present calculation, are

air

$$M_2 = 29$$
$$\bar{C}_{p2} = 0.28\ \mathrm{cal \cdot gr^{-1} \cdot deg^{-1}}$$
$$\bar{\sigma}_2 = 3.62\ \text{Å}$$
$$\Omega_2^{(2,2)*} = 0.79$$

C_2F_4-gas

$$M_1 = 100$$
$$\bar{C}_{p1} = 0.32\ \mathrm{cal \cdot gr^{-1} \cdot deg^{-1}}$$

$\sigma_1 = 5.00\ \text{Å}$
$\Omega_1^{(2,2)*} = 0.90$

teflon

$\bar{C}_b = 0.22\ \text{cal}\cdot\text{gr}^{-1}\cdot\text{deg}^{-1}$
$\bar{\rho}_b = 2.19\ \text{gr}\cdot\text{cm}^{-3}$
$\bar{\kappa}_b = 6.00\times 10^{-4}\ \text{cal}\cdot\text{cm}^{-1}\cdot\text{sec}^{-1}\cdot\text{deg}^{-1}$
$\bar{L} = 35\ \text{kcal}\cdot\text{mol}^{-1}$

Figs. 3a to 3d show several examples of the solutions of Eq. (4.22) for $T_{w0}=$ 0.15, 0.45, 0.65 and 0.90, respectively. It must be noticed here that T_{w0} in the figures denotes non-dimensional wall temperature defined by the ratio of wall temperature, $\bar{T}_{w0}$, at stagnation point, to stagnation temperature, $\bar{T}_{st}$, in free stream. In Fig. 4 are presented variations of f_{0w}, $f_0''{}_w$, z_{0w} with T_{w0} which are the most important results in the present approach. As is seen in the figure, $(-f_0)_w$, which corresponds to the surface injection of foreign gas species due to ablation, tends to increase as T_{w0} decreases. This clearly indicates a physical fact that the ablation rate must be increased in order to keep the ablating surface at low temperature, since a small value of T_{w0} corresponds to a large temperature difference between ablating surface and outer inviscid flow and, consequently, this may cause a large heat transfer to the wall.

Now that the solutions of boundary layer equations are obtained, the aerodynamic relation for ablation rate can be calculated under given perimetric conditions by use of Eq. (6.3). Thus, by matching of Eq. (6.3) with the static relation shown in Fig. 2, the ablation rate can be determined uniquely. Fig. 5 shows

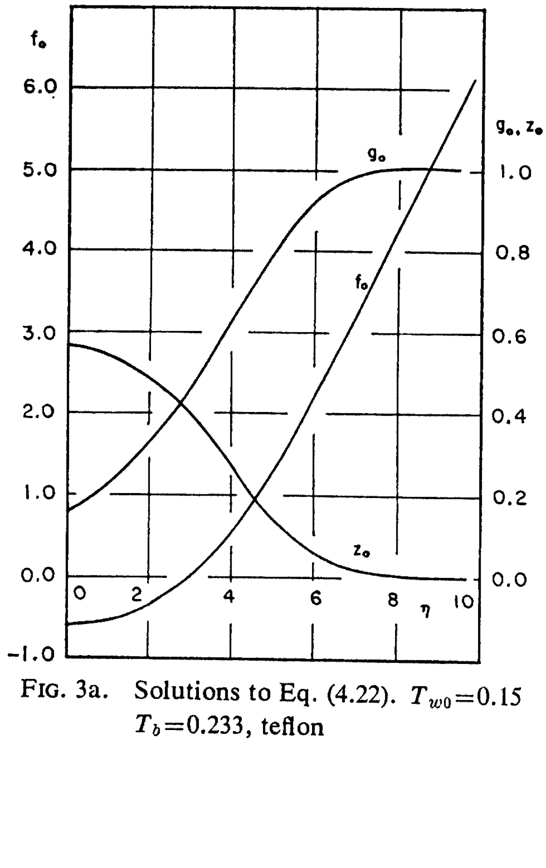

FIG. 3a. Solutions to Eq. (4.22). $T_{w0}=0.15$ $T_b=0.233$, teflon

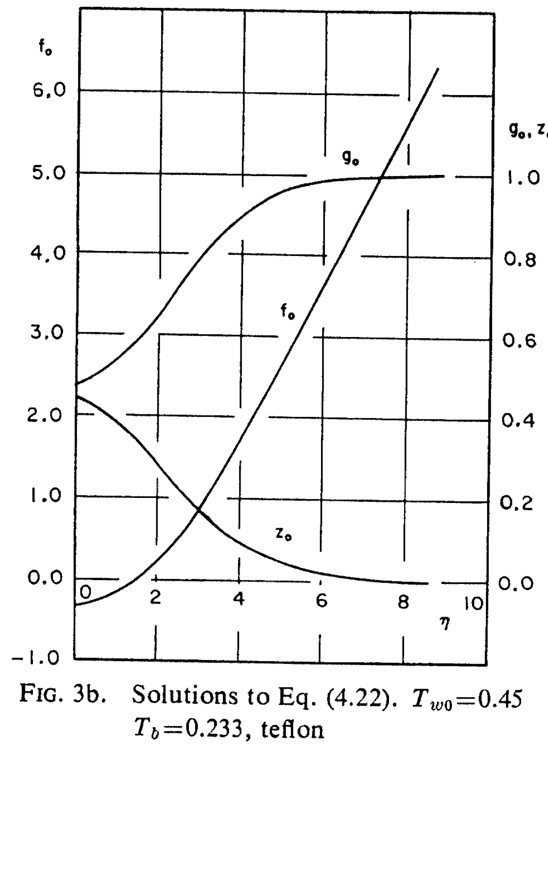

FIG. 3b. Solutions to Eq. (4.22). $T_{w0}=0.45$ $T_b=0.233$, teflon

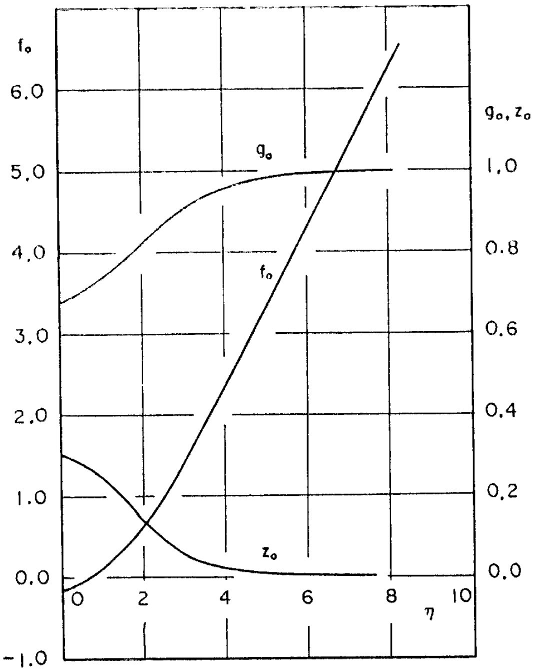

FIG. 3c. Solutions to Eq. (4.22). $T_{w0}=0.65$ $T_b=0.233$, teflon

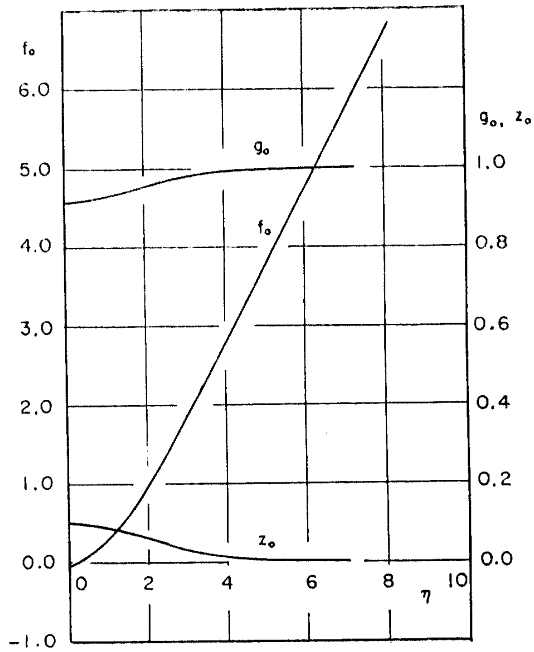

FIG. 3d. Solutions to Eq. (4.22). $T_{w0}=0.90$ $T_b=0.233$, teflon

variation of ablation rate with stagnation temperature in free stream, in which the radius of body curvature at stagnation point is included as a parameter. The figure shows that the ablation rate increases gradually with increase of stagnation temperature, while it decreases as the radius of body curvature grows. However, this trend

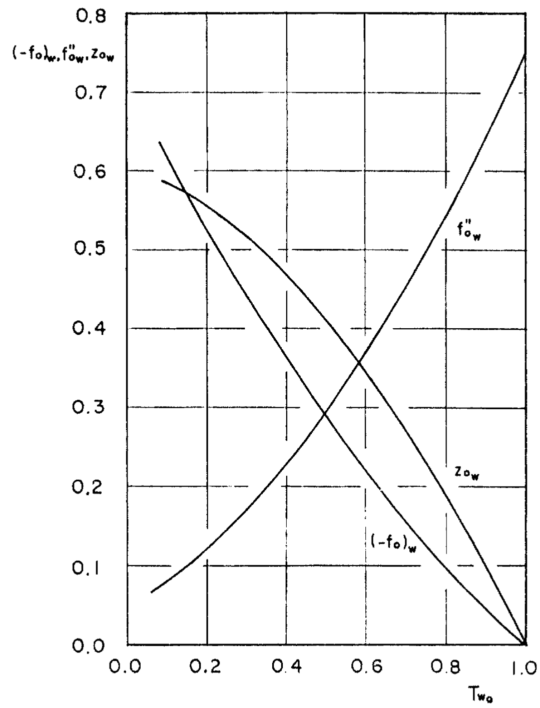

FIG. 4. Variation of f_{0w}, f''_{0w} and z_{0w} with T_{w0} for teflon

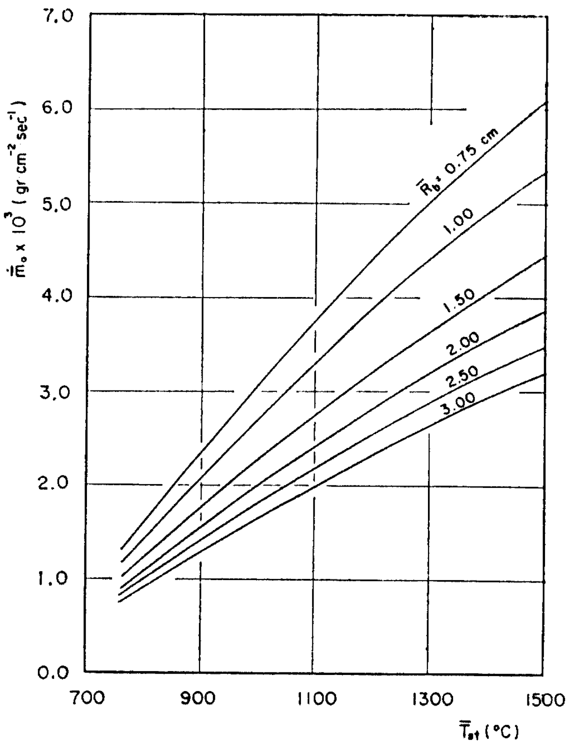

FIG. 5. Variation of ablation rate with stagnation temperature in free stream. Teflon, $M_\infty=5.74$, $\bar{p}_{st}=1$ atm, $T_b=0.233$

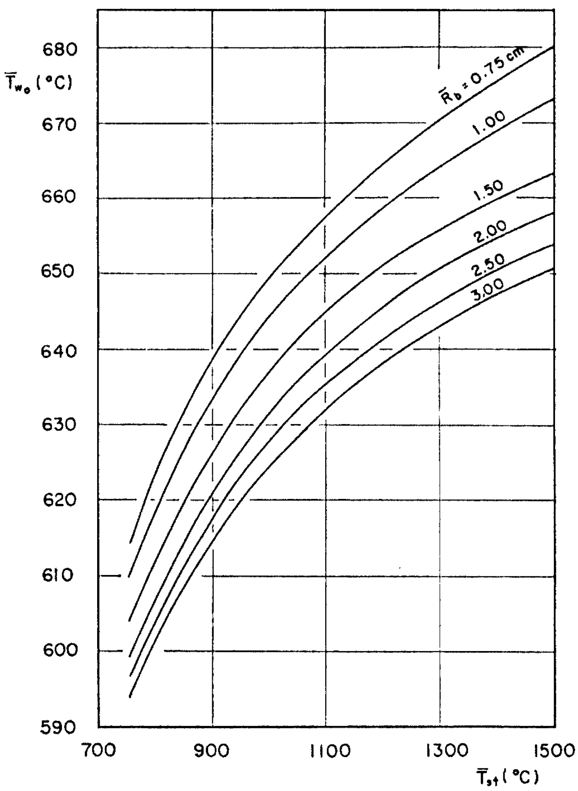

FIG. 6. Variation of wall temperature with stagnation temperature in free stream. Teflon, $M_\infty=5.74$, $\bar{p}_{st}=1$ atm, $T_b=0.233$

may be easily recognized from the result of the conventional laminar boundary layer theory that the surface heat transfer rate is inversely proportional to square root of Reynolds number referred to the radius of body curvature.

In Fig. 6 is presented variation of wall temperature with stagnation temperature for teflon ablator, in which the radius of body curvature is involved as a parameter. The figure clearly indicates a remarkable result that the wall temperature does not rise so much as the stagnation temperature increases, thus giving an quantitative evidence on effect of ablation shielding that a large amount of heat flow from boundary layer is absorbed efficiently in the latent heat required for phase change of the surface material, as has been already mentioned in section 1. Fig. 7 indicates alternative results for ablation rate in order to clarify the effect of body curvature.

Fig. 8 shows the effect of free stream Mach number on ablation rate. As is seen in the figure, it decreases with increasing Mach number. This trend, however, is clearly due to the fact that the heat transfer rate is proportional to the density in boundary layer which decreases with increase of Mach number, if the stagnation

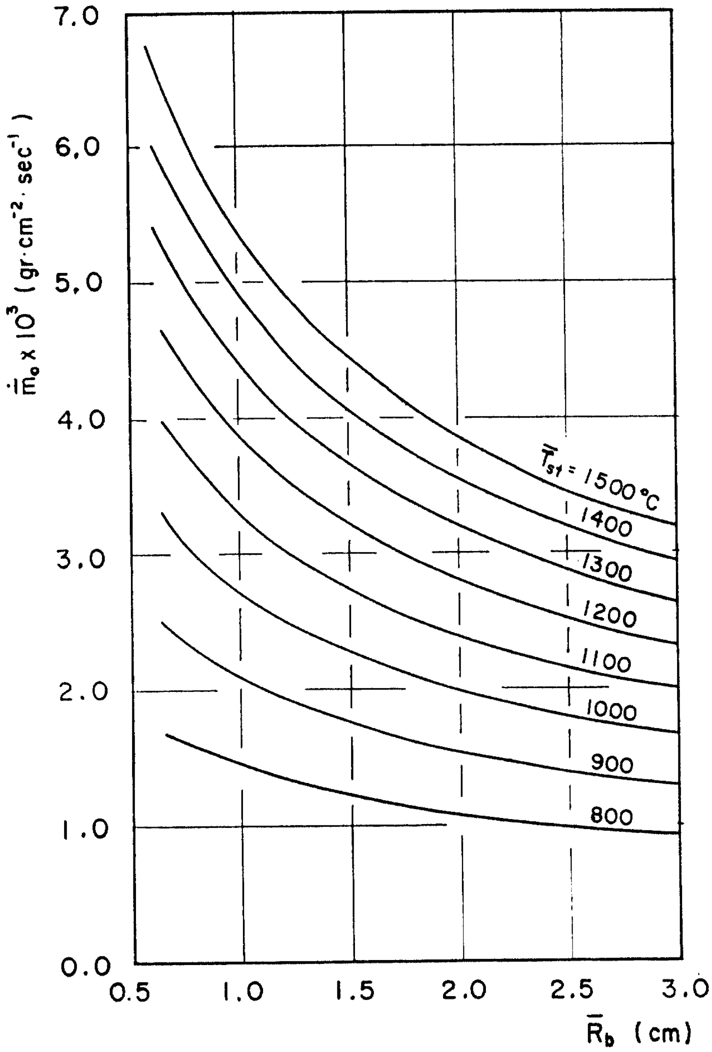

FIG. 7. Variation of ablation rate with radius of body curvature. Teflon, $M_\infty=5.74$, $\bar{p}_{st}=1$ atm, $T_b=0.233$

conditions in free stream are fixed.

Thickness of the boundary layer and the associated heat transfer rate to the ablating surface are other points of interest. The former can be evaluated by use of Eqs. (3.4) and (4.6) as

$$\bar{\delta}=\beta\frac{\bar{R}_b}{\sqrt{Re}}\delta\,, \tag{7.3}$$

where

$$\left.\begin{aligned}\delta&=3^{-\frac{1}{3}}\int_0^{\eta_e}\frac{(1-\alpha_1 z_0)g_0}{1-\alpha_2 z_0}d\eta\,,\\ \beta&=\left(\frac{3}{8\varepsilon}\right)^{\frac{1}{4}}\frac{(1+\lambda_0)^{\frac{1}{2}}\{2\gamma M_\infty^2-(\gamma-1)\}^{\frac{1}{4}}\{(\gamma-1)M_\infty^2+2\}^{\frac{3}{4}}}{3^{\frac{1}{3}}(\gamma+1)M_\infty^{\frac{3}{2}}}\,.\end{aligned}\right\} \tag{7.4}$$

In Fig. 9 is presented variation of boundary layer thickness for $T_{w0}=0.65$ with stagnation temperature, in which $\bar{\delta}_a$ and $\bar{\delta}_{no}$ denote boundary layer thickness with and without ablation, respectively. The thickness with ablation is found to grow by 33 per cents than that without ablation, as is seen in the figure. Fig. 10 shows an example of relative decrease of aerodynamic heating itself due to boundary layer

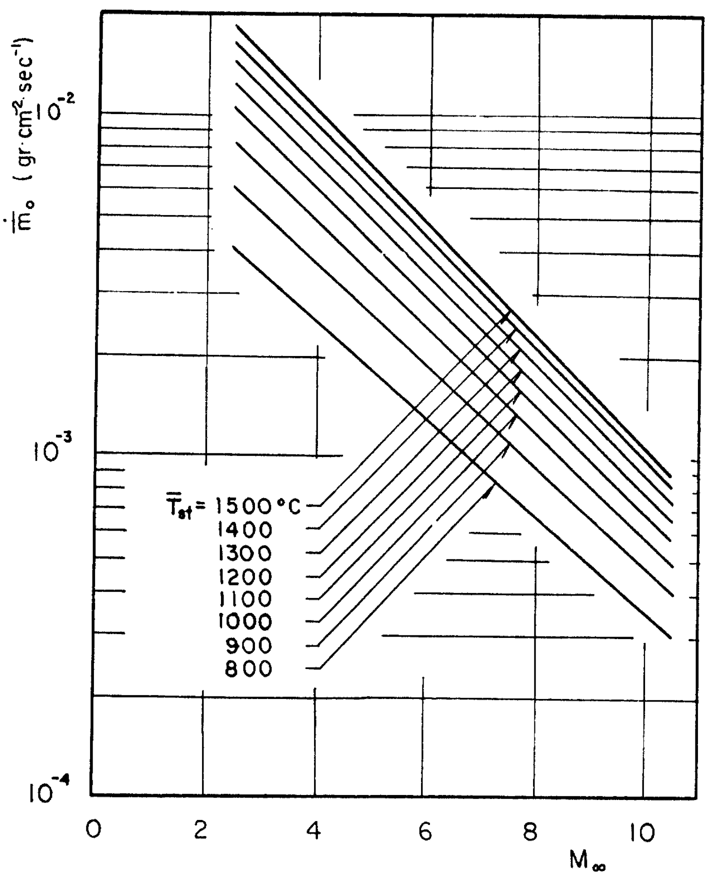

FIG. 8. Effect of free stream Mach number on ablation rate Teflon, $\bar{R}_b=1$ cm, $\bar{p}_{st}=1$ atm, $T_b=0.233$

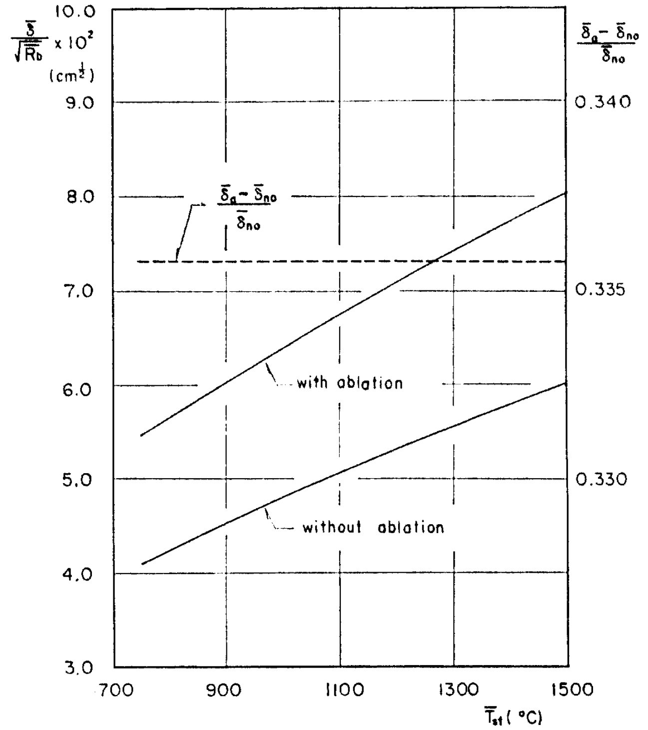

FIG. 9. Variation of boundary layer thickness with stagnation temperature in free stream. Teflon, $M_\infty=5.74$, $\bar{p}_{st}=1$ atm, $T_{w0}=0.65$, $T_b=0.233$

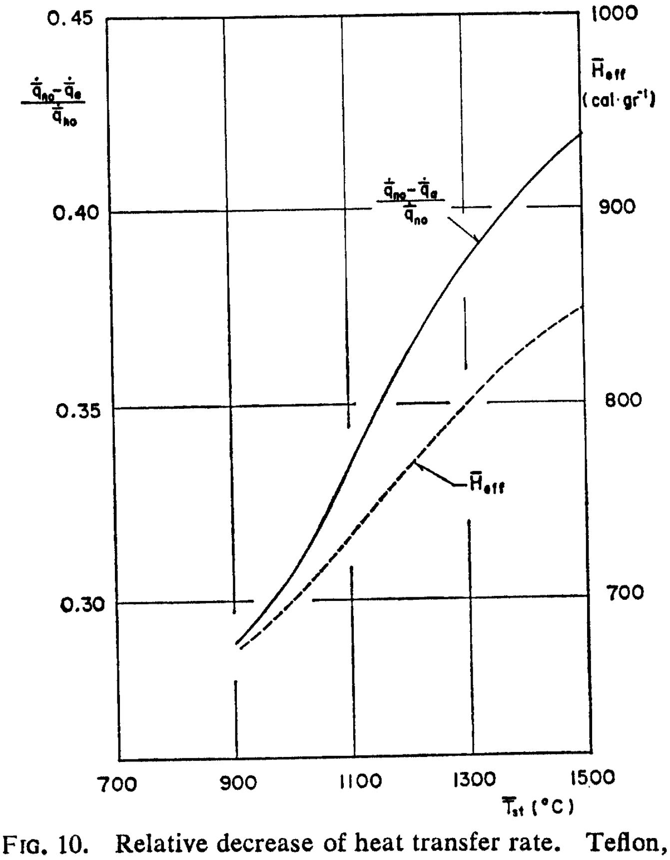

FIG. 10. Relative decrease of heat transfer rate. Teflon, $M_\infty=5.74$, $\bar{p}_{st}=1$ atm, $\bar{R}_b=1$ cm, $T_b=0.233$

growth. In the figure, $\dot{\bar{q}}_a$ and $\dot{\bar{q}}_{no}$ indicate heat transfer rates with and without ablation, respectively, which may be defined by the equations

$$\dot{\bar{q}}_a=\left(\bar{\kappa}\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=+0}=[\bar{C}_b(\bar{T}_w-\bar{T}_b)+\bar{L}]\dot{\bar{m}}\ , \tag{7.5}$$

$$\dot{\bar{q}}_{no}=\bar{C}_{p2}\frac{0.765}{P_w^{0.6}}(\bar{T}_e-\bar{T}_w)(\bar{\rho}_e\bar{\mu}_e\tilde{u}_0)^{\frac{1}{2}}\left(\frac{\bar{\rho}_w\bar{\mu}_w}{\bar{\rho}_e\bar{\mu}_e}\right)^{0.1}, \tag{7.6}$$

where

$$\tilde{u}_0=\bar{u}_\infty\sqrt{\frac{8\varepsilon}{3}}\frac{1}{(1+\lambda_0)\bar{R}_b}\ . \tag{7.7}$$

It must, however, be noted that, in Fig. 10, the heat transfer rate without ablation, $\dot{\bar{q}}_{no}$, is estimated under the same wall temperature that will be obtained when ablation occurs. Since the heat transfer rate with ablation is essentially different in physical feature from that without ablation and resulting wall temperature may be different from one another, such a comparison as shown in Fig. 10 might, exactly speaking, be non-sense, although it may be of a tentative measure for relative reduction of the heat transfer itself due to growth of the boundary layer thickness. Any way, those results presented in Figs. 9 and 10 might be considered to be a quantitative confirmation on shielding effect of boundary layer growth mentioned in section 1.

In Fig. 10 is also presented the effective heat of ablation defined by the equation

$$\bar{H}_{\mathrm{eff}}=\frac{\dot{\bar{q}}_{no}}{\dot{\bar{m}}_0}\ . \tag{7.8}$$

Although the effective heat of ablation may have been used in many existing papers for the purpose of representing the effectiveness of the ablators, it seems that such a definition as expressed by Eq. (7.8) might lead to no rigorous quantitative information about the effectiveness of the ablators, since $\dot{\bar{q}}_{no}$ (which is defined as the heat transfer rate without ablation) and $\dot{\bar{m}}_0$ can not be defined at the same time. Furthermore, if it were to be done, another curiosity arises that $\dot{\bar{q}}_{no}$ can not be single-valued in the sense that it depends upon the choice of either material or wall temperature or both. Therefore, the effective heat of ablation should be considered as representative measure of the effectiveness of the ablators.

A comparison of the theoretical results with experimental data is another point of interest in order to confirm the validity of the present approach. In Fig. 11 are plotted the unpublished experimental data for teflon obtained by Kawamura, Karashima and Sato [22] together with theoretical results for comparison. As is seen in the figure, the present approach agrees well with the experimental data over wide ranges of stagnation temperature in free stream and the radius of body curvature, thus giving an experimental evidence on validity of the theory. A slight deviation of the experimental data at comparatively high stagnation temperatures may be due to the error in measurement of the stagnation temperature.

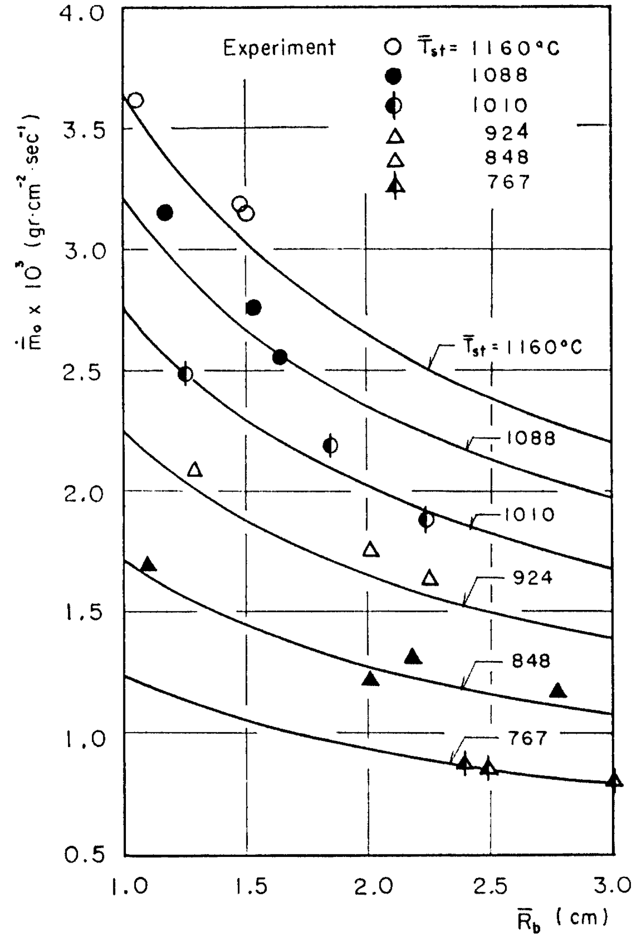

FIG. 11. Comparison of theory with experiment. Teflon, $M_\infty=5.74$, $\bar{p}_{st}=1$ atm

For the purpose of clarifying the effect of transport properties on boundary layer behaviour concerning the ablating field, numerical calculation was further carried out, as an example, for $T_{w0}=0.764$ by keeping either Chapman-Rubesin number C_0, Prandtl number P_0 or Schmidt number S_0 constant across the boundary layer at their wall values obtained from exact calculation, respectively, and the results for f_{0w} are shown in Table 2a to 2c together with its exact value for comparison. It is clear from the tables that a simple evaluation of Chapman-Rubesin number and Schmidt number results in a serious error in solutions of the boundary layer equations, while Prandtl number has less influence. In order to examine this circumstance in more detail, variations of C_0, P_0 and S_0 with concentration z_0 are plotted in Fig. 12. As is seen in the figure, a small change of concentration leads to a relatively large change of C_0 and S_0 rather than P_0. Since average viscosity and thermal conductivity of binary gas mixture have the same trend of dependence on concentration, the effect of change in concentration on Prandtl number seems to be cancelled to keep its value almost constant across the boundary layer. The reason for slight dependence of solutions of the boundary layer equations on a simple evaluation of Prandtl number may be, therefore, laid on this point.

TABLE 2a. Effect of simple evaluation of Chapman-Rubesin number on a solution of boundary layer equations

	$(-f_0)_w$	Error
Exact	0.120	
$C_0=C_{0_w}$	0.134	+11.7%

TABLE 2b. Effect of simple evaluation of Prandtl number on a solution of boundary layer equations

	$(-f_0)_w$	Error
Exact	0.120	
$P_0=P_{0_w}$	0.123	+2.50%

TABLE 2c. Effect of simple evaluation of Schmidt number on a solution of boundary layer equations

	$(-f_0)_w$	Error
Exact	0.120	
$S_0=S_{0_w}$	0.110	−8.34%

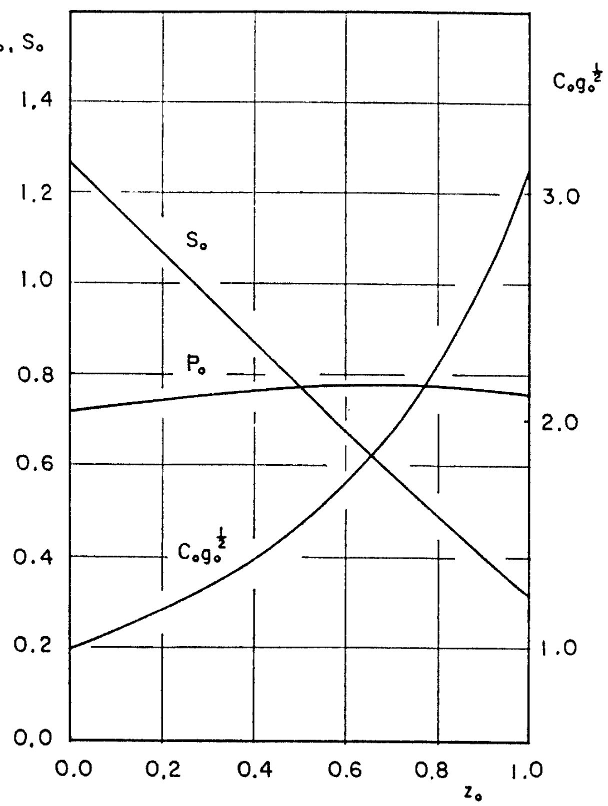

FIG. 12. Variation of C_0, P_0 and S_0 with z_0 for C_2F_4-air mixture

8. Conclusion

An analytical approach has been presented to stagnation ablation associated with hypersonic reentry of blunt-nosed bodies of revolution, indicating that all physical properties concerning the ablating field can be uniquely determined under given boundary conditions by matching of an aerodynamic relation with a static one obtained from chemical kinetics.

It was shown that a simple evaluation of transport coefficients except for Prandtl number results in serious errors in solutions of boundary layer equations, thus indicating that viscosity and diffusivity of binary gas mixture play a main role in controlling the aerodynamic characteristics of the ablating field.

Numerical calculation carried out for teflon ablator showed that boundary layer thickness with ablation grows larger than that without ablation and the associated heat transfer rate decreases considerably. This trend together with a remarkable result that the wall temperature does not rise so much as the stagnation temperature in free stream increases clearly gives a quantitative evidence on the effects of shielding the aerodynamic heating by vaporization.

From the practical point of view, the efficient ablator seems to be so defined as the material that can keep its surface at comparatively low temperature by means of ablation with small mass loss rate even for high enthalpy flow outside the boundary layer. With this definition, it can be easily found from qualitative examination of the aerodynamic shielding mechanism that there exist several substantial factors coming into play in controlling the shielding characteristics, which may be summarized as follows;

(1) light molecular weight of generated gas,
(2) large latent heat for sublimation,
(3) large specific heat and small thermal conductivity of solid material.

Acknowledgement

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April 18, 1967

Appendix A Transport Coefficients

A-1. *Coefficient of Viscosity*

For a pure gas species i the kinetic theory gives

$$\bar{\mu}_i = 266.93 \times 10^{-7} \frac{(M_i \bar{T})^{\frac{1}{2}}}{\bar{\sigma}_i^2 \Omega_i^{(2,2)*}} \qquad (\mathrm{gr \cdot cm^{-1} \cdot sec^{-1}}), \tag{A.1.1}$$

where $\bar{\sigma}_i$ = collision diameter (Å),

$\Omega_i^{(2,2)*}$ = non-dimensional collision integral.

By use of Eq. (A.1.1), Wilke [23] gives an expression for mean coefficient of viscosity of a gas mixture containing ν-components as

$$\bar{\mu} = \sum_{i=1}^{\nu} \bar{\mu}_i \left(1 + \sum_{\substack{k=1 \\ k \neq i}}^{\nu} G_{ik} \frac{c_k}{c_i}\right)^{-1} \qquad (\mathrm{gr \cdot cm^{-1} \cdot sec^{-1}}), \tag{A.1.2}$$

where c_i denotes mole fraction of species i defined by the equation

$$c_i = \frac{K_i}{M_i} \left(\sum_i \frac{K_i}{M_i}\right)^{-1}, \tag{A.1.3}$$

and

$$G_{ik} = \frac{\left[1 + \left(\frac{\bar{\mu}_i}{\bar{\mu}_k}\right)^{\frac{1}{2}} \left(\frac{M_k}{M_i}\right)^{\frac{1}{4}}\right]^2}{2^{\frac{3}{2}} \left(1 + \frac{M_i}{M_k}\right)^{\frac{1}{2}}}. \tag{A.1.4}$$

Therefore, for a binary gas mixture Eq. (A.1.2) can be reduced, together with an additional assumption that each species occupying the same space has the same temperature, to

$$\frac{\mu}{\mu_e} = \left[\frac{\tilde{\mu} K}{\tilde{\mu} G + (1 - \tilde{\mu} G) K} + \frac{1 - K}{1 + (\tilde{m} G - 1) K}\right] \left(\frac{T}{T_e}\right)^{\frac{1}{2}}, \tag{A.1.5}$$

where

$$\left.\begin{aligned} &G = G_{21}, \qquad \tilde{\mu} = \frac{\mu_1}{\mu_2}, \qquad \tilde{m} = \frac{M_2}{M_1}, \\ &K = K_1, \qquad K_2 = 1 - K. \end{aligned}\right\} \tag{A.1.6}$$

This additional assumption enables to evaluate the ratio of viscosities for each pure species as

$$\tilde{\mu} = \frac{\mu_1}{\mu_2} = \left(\frac{M_1}{M_2}\right)^{\frac{1}{2}} \left(\frac{\bar{\sigma}_2}{\bar{\sigma}_1}\right)^2 \frac{\Omega_2^{(2,2)*}}{\Omega_1^{(2,2)*}} = \mathrm{const}. \tag{A.1.7}$$

Taylor expansion of Eq. (A.1.5) by use of Eqs. (4.15) and (4.18b) leads to

$$\frac{\mu}{\mu_e} = E_0(z_0) g_0^{\frac{1}{2}} + O(s^{\frac{2}{3}}) , \tag{A.1.8}$$

where

$$E_0(z_0) = \frac{1}{\sqrt{1-\alpha_2 z_0}} \left[\frac{\tilde{\mu} z_0}{\tilde{\mu} G + (1-\tilde{\mu} G) z_0} + \frac{1-z_0}{1+(\tilde{m} G - 1) z_0} \right] . \tag{A.1.9}$$

A-2. *Coefficient of Thermal Conductivity*

The kinetic theory concerning thermal conductivity for polyatomic gases is not so complete as for monatomic ones. However, if certain assumptions are made in order to apply Chapman-Enskog theory available strictly for monatomic gases at low temperature, the theory may be reasonably extended to gas mixtures of polyatomic molecules. It can be shown that there exist many cases in which velocity distribution function for particles having no internal energy can represent the velocity distribution function for the ones having internal energy. Dorrance [*24*] suggests from detailed examinations that almost all simple polyatomic molecules may belong to such cases.

For a pure monatomic gas of species i, Chapman-Enskog theory yields for thermal conductivity

$$\bar{\kappa}_i = \frac{15}{4} \frac{\bar{R}}{M_i} \bar{\mu}_i \quad (\mathrm{cal \cdot cm^{-1} \cdot sec^{-1} \cdot deg^{-1}}) . \tag{A.2.1}$$

Here an assumption is introduced that the thermal conductivity denoted by Eq. (A.2.1) can be considered as that of the idealized polyatomic gas having no internal energy mode and the thermal conductivity of actual polyatomic gas $\bar{\kappa}'_i$ may be given by use of a correction factor of Eucken as

$$\bar{\kappa}'_i = \bar{\kappa}_i Eu_i , \tag{A.2.2}$$

where

$$Eu_i = \text{Eucken factor} = 0.115 + 0.354 \frac{\bar{C}_{pi}}{\bar{R}_i} . \tag{A.2.3}$$

Therefore, the thermal conductivity for a gas mixture containing ν-components may be expressed as

$$\bar{\kappa} = \sum_{i=1}^{\nu} \bar{\kappa}'_i \left(1 + 1.065 \sum_{\substack{k=1 \\ k \neq i}}^{\nu} G'_{ik} \frac{c_k}{c_i} \right)^{-1} \quad (\mathrm{cal \cdot cm^{-1} \cdot sec^{-1} \cdot deg^{-1}}) , \tag{A.2.4}$$

where

$$G'_{ik}=\frac{\left[1+\left(\frac{\kappa_i}{\kappa_k}\right)^{\frac{1}{2}}\left(\frac{M_i}{M_k}\right)^{\frac{1}{4}}\right]^2}{2^{\frac{3}{2}}\left(1+\frac{M_i}{M_k}\right)^{\frac{1}{2}}}. \tag{A.2.5}$$

By use of Eq. (A.2.4), the mean thermal conductivity can be obtained for a binary gas mixture as

$$\frac{\kappa}{\kappa_e}=\left[\frac{\tilde{\kappa}K}{\tilde{\mu}G'+(1-\tilde{\mu}G')K}+\frac{1-K}{1+(\tilde{m}G'-1)K}\right]\left(\frac{T}{T_e}\right)^{\frac{1}{2}}, \tag{A.2.6}$$

where

$$\left.\begin{aligned}&\tilde{\kappa}=\frac{\bar{\kappa}'_1}{\bar{\kappa}'_2}=\frac{\bar{\kappa}_1}{\bar{\kappa}_2}\frac{Eu_1}{Eu_2}=\frac{M_2}{M_1}\frac{\mu_1}{\mu_2}\frac{0.115\bar{R}+0.354M_1\bar{C}_{p1}}{0.115\bar{R}+0.354M_2\bar{C}_{p2}},\\&G'=1.065G'_{21}.\end{aligned}\right\} \tag{A.2.7}$$

Thus, Taylor expansion of Eq. (A.2.6) yields

$$\frac{\kappa}{\kappa_e}=\frac{E_1(z_0)}{\sqrt{1-\alpha_2 z_0}}g_0^{\frac{1}{3}}+O(s^{\frac{2}{3}}), \tag{A.2.8}$$

where

$$E_1(z_0)=\left[\frac{\tilde{\kappa}z_0}{\tilde{\mu}G'+(1-\tilde{\mu}G')z_0}+\frac{1-z_0}{1+(\tilde{m}G'-1)z_0}\right]. \tag{A.2.9}$$

Appendix B Non-dimensional Characteristic Numbers

B-1. *Chapman-Rubesin Number*

Chapman-Rubesin number is defined by the equation

$$C=\frac{\rho\mu}{\rho_e\mu_e}. \tag{B.1.1}$$

Substitution of Eqs. (4.20) and (A.1.8) into Eq. (B.1.1) leads to

$$C=C_0+O(s^{\frac{2}{3}}), \tag{B.1.2}$$

where

$$C_0=\frac{\sqrt{1-\alpha_2 z_0}}{1-\alpha_1 z_0}\left[\frac{\tilde{\mu}z_0}{\tilde{\mu}G+(1-\tilde{\mu}G)z_0}+\frac{1-z_0}{1+(\tilde{m}G-1)z_0}\right]g_0^{-\frac{1}{3}}. \tag{B.1.3}$$

B-2. *Prandtl Number*

Prandtl number is defined by the equation

$$P=\frac{\bar{C}_p\bar{\mu}}{\bar{\kappa}}, \tag{B.2.1}$$

and is rewritten as

$$P=\left(\frac{\bar{C}_p\bar{\mu}}{\bar{\kappa}}\right)_e\frac{C_p\dfrac{\mu}{\mu_e}}{\dfrac{\kappa}{\kappa_e}}=P_2\frac{C_p\dfrac{\mu}{\mu_e}}{\dfrac{\kappa}{\kappa_e}},$$

where P_2 denotes Prandtl number for air, since it is defined by conditions only at outer edge of the boundary layer where concentration of foreign gas species vanishes. Substitution of Eqs. (4.17), (A.1.8) and (A.2.8) into the above equation gives

$$P=P_0+O(s^{\frac{2}{3}}), \tag{B.2.2}$$

where

$$P_0=\frac{P_2(1-\alpha_2 z_0)\left[\dfrac{\tilde{\mu}z_0}{\tilde{\mu}G+(1-\tilde{\mu}G)z_0}+\dfrac{1-z_0}{1+(\tilde{m}G-1)z_0}\right]}{\dfrac{\tilde{\kappa}z_0}{\tilde{\mu}G'+(1-\tilde{\mu}G')z_0}+\dfrac{1-z_0}{1+(\tilde{m}G'-1)z_0}}. \tag{B.2.3}$$

B-3. *Schmidt Number*

Before presenting the expression of Schmidt number, binary diffusion coefficient must be first defined. It may be given by the equation (see ref. 24)

$$\bar{D}=262.8\times10^{-5}\frac{\left(\dfrac{M_1+M_2}{2M_1M_2}\right)^{\frac{1}{2}}\bar{T}^{\frac{3}{2}}}{\bar{p}\bar{\sigma}_{12}^2\Omega^{(1,1)*}}\qquad(\mathrm{cm^2\cdot sec^{-1}}), \tag{B.3.1}$$

where $\bar{\sigma}_{12}=\frac{1}{2}(\bar{\sigma}_1+\bar{\sigma}_2)$ (Å),

$\Omega^{(1,1)*}=$ non-dimensional collision integral.

By use of pressure given by the equation

$$\bar{p}=\bar{\rho}\bar{R}\bar{T}\left(\frac{K}{M_1}+\frac{1-K}{M_2}\right),$$

Eq. (B.3.1) can be rewritten as

$$\bar{D}=\tilde{D}\frac{M_2\bar{T}^{\frac{1}{2}}}{\bar{\rho}(1-\alpha_1 K)}, \tag{B.3.2}$$

where

$$\tilde{D}=262.8\times10^{-5}\frac{\left(\dfrac{M_1+M_2}{2M_1M_2}\right)^{\frac{1}{2}}}{\bar{R}\bar{\sigma}_{12}^2\Omega^{(1,1)*}}.$$

Schmidt number is defined as

$$S=\frac{\bar{\mu}}{\bar{\rho}\bar{D}}. \tag{B.3.3}$$

Rewriting visocity coefficient as

$$\bar{\mu}=\frac{\mu}{\mu_e}\bar{\mu}_e=\frac{\mu}{\mu_e}\tilde{\mu}_e\bar{T}_e^{\frac{1}{2}}, \tag{B.3.4}$$

$$\tilde{\mu}_e=266.93\times10^{-7}\frac{M_2^{\frac{1}{2}}}{\bar{\sigma}_2^2\Omega_2^{(2,2)*}}, \tag{B.3.5}$$

and substituting Eqs. (B.3.2) and (B.3.4) into Eq. (B.3.3) then leads to

$$S=\frac{\tilde{\mu}_e}{M_2\tilde{D}}(1-\alpha_1K)\frac{\mu}{\mu_e}\left(\frac{T_e}{T}\right)^{\frac{1}{2}}.$$

It is clear from the above equation that the term $\tilde{\mu}_e/M_2\tilde{D}$ denotes Schmidt number defined for pure air, S_2, since S must tend to S_2 as the outer edge of the boundary layer is approached. Therefore,

$$S=S_2(1-\alpha_1K)\frac{u}{\mu_e}\left(\frac{T_e}{T}\right)^{\frac{1}{2}}. \tag{B.3.6}$$

Taylor expansion of Eq. (B.3.6) gives

$$S=S_0+O(s^{\frac{2}{3}}), \tag{B.3.7}$$

where

$$S_0=S_2(1-\alpha_1z_0)\left[\frac{\tilde{\mu}z_0}{\tilde{\mu}G+(1-\tilde{\mu}G)z_0}+\frac{1-z_0}{1+(\tilde{m}G-1)z_0}\right]. \tag{B.3.8}$$

B-4. *Lewis Number*

Lewis number is defined by the equation

$$Le=\frac{P}{S}. \tag{B.4.1}$$

Taylor expansion of Eq. (B.4.1) leads to

$$Le=\frac{P_0}{S_0}+O(s^{\frac{2}{3}}), \tag{B.4.2}$$

where P_0 and S_0 are given by Eqs. (B.2.3) and (B.3.8), respectively.

APPENDIX C

Each term in Eq. (5.6) can be rewritten, respectively, after some manipulations as

$$\left(\bar{\kappa}\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=+0} = \frac{3^{\frac{1}{3}}\bar{\kappa}_e \bar{T}_{st}\rho_w T_e}{\bar{y}^*}\left(\frac{\kappa}{\kappa_e}\right)_{w^+}\left[\frac{\partial}{\partial \eta}\left(\frac{T}{T_e}\right)\right]_{w^+}, \tag{C.1}$$

$$\bar{L}(\bar{\rho}\bar{v})_w = -\frac{\bar{\kappa}_e \bar{T}_{st}\rho_w T_e}{\bar{y}^*}P_2\frac{l}{\rho_w T_e r_0}(\psi_x)_w, \tag{C.2}$$

$$\left(\kappa_b\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=-0} = -\frac{\bar{\kappa}_e \bar{T}_{st}\rho_w T_e}{\bar{y}^*}(T_w - T_b)P_2\frac{\bar{C}_b}{\bar{C}_{p2}}\frac{1}{\rho_w T_e r_0}(\psi_x)_w. \tag{C.3}$$

Substitution of these equations into Eq. (5.6) gives

$$3^{\frac{1}{3}}\left(\frac{\kappa}{\kappa_e}\right)_{w^+}\left[\frac{\partial}{\partial \eta}\left(\frac{T}{T_e}\right)\right]_{w^+} = -\frac{P_2}{\rho_w T_e r_0}\left[\frac{\bar{C}_b}{\bar{C}_{p2}}(T_w - T_b) + l\right](\psi_x)_w, \tag{C.4}$$

where $\frac{1}{r_0}(\psi_x)_w$ may be expressed as

$$\frac{1}{r_0}(\psi_x)_w = \frac{1}{r_0}\left(\frac{\partial \psi}{\partial s} + \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial s}\right)_w\frac{\partial s}{\partial x}$$

$$= \rho_e \mu_e r_0\left[\frac{2}{3}s^{-\frac{1}{3}}f_{0w} + \frac{4}{3}q_1 s^{\frac{1}{3}}f_{1w} + \cdots\right]$$

$$= 3^{\frac{1}{3}}\frac{2}{3}\rho_e f_{0w} + O(s^{\frac{2}{3}}). \tag{C.5}$$

Therefore, substitution of Eqs. (A.2.8), (4.18b), (5.2) and (C.5) into Eq. (C.4) and equating like power of s, yields after some manipulations,

$$\frac{E_1(z_{0w})}{1-\alpha_1 z_{0w}}\left(\frac{T_s}{T_{w0}}\right)^{\frac{1}{2}}\left(\frac{g_0}{1-\alpha_2 z_0}\right)'_w = -\frac{2}{3}P_2 f_{0w}\left[\frac{\bar{C}_b}{\bar{C}_{p2}}(T_{w0} - T_b) + l\right]. \tag{C.6}$$

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