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An Analytical Approach to the Thermal Design of Spacecrafts

By

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Summary: The linearized thermal balance equations of two-nodes system, in which radiative heat dissipation to outer space from the each node and heat transfer between them are counted, are analytically solved for several special cases which closely relate to the real situations in spacecrafts. And general theorems on n-nodes system are presented with relations to the real spacecraft structure. Based on these discussions, a new method of thermal analysis of spacecrafts is proposed, which is essentially an experimental analysis of simplified thermal model. The construction method of this model and the data reduction process are shown with an illustrative example. Using this method, the thermal character of a spacecraft can be determined with minimum simulator time and without longsome numerical computation.

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1. INTRODUCTION

It has been passed over ten years since the first artificial satellite was orbited. During this period, much progress has been made in every field of space technology. The thermal analysis of the spacecrafts also has grown up from preliminary stage

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to today's sophisticated arts. Many reports on the analysis used and the results obtained with regard to the thermal design of the particular spacecrafts have appeared in academic publications [*1*~*3*], such as Explorer VII [*5*] Telstar [*6*], etc. of the early type of spacecrafts [*4*~*8*], and Pegasus, Mariner, Appolo [*9*~*13*], etc. of the more sophisticated spacecrafts. Now it seems that the thermal design method has been well-established, at least in the point of view of practical application. The work presented in this paper is a simplified method to determine the thermal character of a spacecraft based on the approximate analytical study as well as on simplified thermal model test by space simulator.

First step of the thermal analysis of the spacecraft is to determine the radiation energy input on the spacecraft from the outer space, which consists of the solar radiation, the planetary albedo and the infra-red radiation of the planet. The methods of calculations of these effects are well-established in [*5*~*18*]. Even on the most ambiguous factor of these, the earth albedo, too, some data by artificial satellite measurement were reported [*19*], and now it seems that one has reliable values on this effect.

Next step is to determine the optical character of the spacecraft surfaces which controls the radiation energy input to the surface and the radiative heat output from it to the outer space. A great deal of efforts has been put on this problem [*20*~*29*], some of which are summarized in [*24*] and [*28*]. Many kinds of space environmental testings on these materials as well as actual satellite-born testings were carried out with the emphasis on the in-situ measurements of optical characters. Thus almost any kind of surface optical characters has become available with reasonable reliability.

Third step is to determine the thermal parameters of each part of the spacecraft, such as heat capacity and heat transfer coefficient. Much difficulty in this step associates with the phenomena of thermal contact resistance of two surfaces and of the radiative heat exchange between two surfaces. The former problem has been treated more or less by phenomenological method and some data have been accumulated, though it is not conclusive yet [*1*] and [*30*]. The latter relates to an important application to the space radiator, such as thermal louver system, and has been enthusiastically worked out. However, due to its inherent mathematical difficulty, the final solution is far out of sight and one has to relay on the experimental measurement to each case [*31*].

Based on these knowledge, usually the so-called node analysis has been used to obtain the temperature distribution and its variation of each part of the spacecraft, although the more rigorous partial differential equations of heat transfer were solved for some simple cases, such as spherical hull or cylinder [*32*~*34*]. The node analysis is, in essence, a finite difference approximation to the original partial differential equations, then some ambiguities on the selection of the node points and on the assignment of the thermal characters to each node point are left open to question. Very few discussions on this point are found in the literature. Furthermore, many efforts to solve the thermal balance equations, which have applied various methods [*35*~*38*] including those using electrical analogy, have failed so far to give the complete analytical solution of these equations. Therefore this method applied to the real spacecraft often requires prohibitably large amount of numerical computation even for today's large electronic computors, though a short-cut method with a sacrifice of the accuracy has been proposed. On this point, in some extent, we have to trust on the experimental data.

The experiments on this problem are usually carried out in the one of the space environmental simulation facilities. Criteria of these simulation testing facilities were frequently discussed [*39*~*46*]. In general, a vacuum chamber with liquid nitrogen cooled shroud, now it is common, can sufficiently simulate the space environment with respect to the thermal balance character of the spacecraft tested. Probably, the biggest trouble in this simulation testing relates to the solar simulator, which yet is in a preliminary stage in this country, and is capable to produce simulated light source with quite insufficient uniformity and hopelessly large divergent angle, even besides its spectral mismatch to the real sun. Therefore, in order to simulate the heat input to each node point, electric heaters attached to the each node are much more accurate and useful than solar simulators. The amount of heat being supplied to the each node is calculated from the geometrical relation and the surface optical characters.

Nextly, the experimental data thus obtained are compared with the computed results by node analysis. The calculated results are not in analytical form, then there is no way to deduce the thermal characteristic parameters of each node point from the experimental data. On the other hand, the simulation test condition has to be the same as those computed, which means that the long simulator time, which sometimes runs over several days or even weeks, is necessary and that the model has to be built in the same way as the computed ones. Thus, for example, the effect of small modifications of the model characters is completely unknown from the experimental data.

With some relations to this point, the thermal similitude of the spacecraft testing has been discussed by many authors, who have treated this problem mainly with the relations to the possibility of the scale-down model experiment [*47*~*50*]. Many thermal similitude criteria in spacecraft testing have been established, but none of them is useful for the data reduction and comparison of the experiment to the model analysis.

Considering these difficulties of the thermal analysis of spacecrafts, a new process of thermal analysis is presented in this report. In chapter 2, the basic equations of node analysis is shown, and the solutions of it for the case of single-node system are derived in chapters 3 and 4. The solutions of two-nodes system are discussed in chapters 5 and 6. Based on these special cases, the general characters of the solutions of the n-nodes system are treated and some useful theorems are presented in chapter 7. In addition, the Fourier analysis applied to this problem is presented in chapter 8. In chapter 9, the practical process of thermal analysis is shown with an example.

2. Basic Equations

For the theoretical design of spacecrafts, which naturally have thermally complicated structures, the so-called node analysis has been traditionally used, in which the whole structures of the spacecraft are divided into several node points, as shown in Fig. 1, and the each node (i) is assumed to have a temperature T_i, a difinite heat capacity C_i and heat transfer rates between each others. Then the thermal balance equation of each node point is expressed as;

$$C_i \frac{dT_i}{dt} = Q_i - A_i \sigma \varepsilon_i T_i^4 - \sum_j K_{ij}(T_i - T_j) - \sum_j A_i \sigma F_{ij}(T_i^4 - T_j^4) \qquad (1)$$

where A, ε, σ, K and F are the thermal radiative area to the outer space, the thermal emissivity of the surface, the Stefan-Boltzman constant, the thermal conductance and the area factor of the radiative heat exchange, respectively. The heat input to this node, Q_i is composed of the solar radiation, $F'_{ai}\alpha_i I_s$, the earth albedo, $F''_{ai}\alpha_i I_a$, the infra-red radiation of the earth, $F'''_{ai}\varepsilon_i I_e$, and the internal heat dissipation within this node, P_i;

$$Q_i = F'_{ai}\alpha_i I_s + F''_{ai}\alpha_i I_a + F'''_{ai}\varepsilon_i I_e + P_i \qquad (2)$$

where F'_a, F''_a and F'''_a are the area receiving the respective radiations, and I_s, I_a and I_e are the radiation intensities of the sun, the earth albedo and the infra-red radiation of the earth, respectively.

In this report, we assume that all the material constants, such as the heat capacity

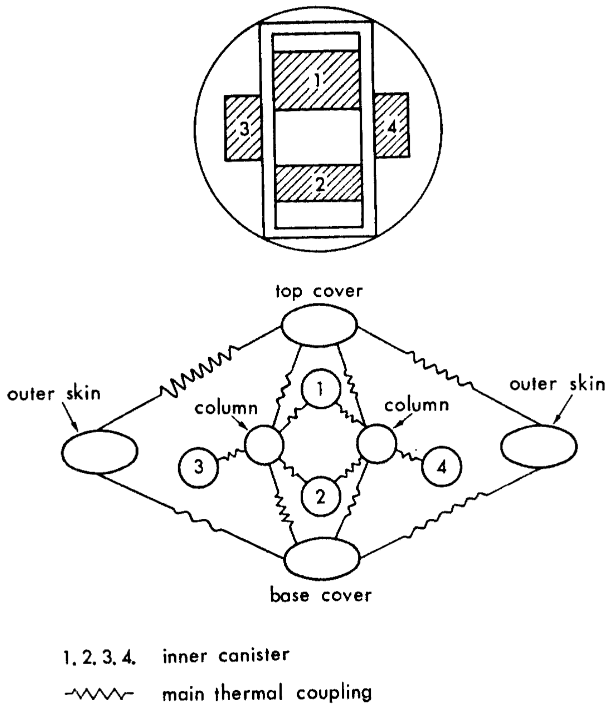


Fig. 1. Schematic diagram of the node system

or the thermal conductance, are constants regardless to the outer conditions, such as the temperature or the time, and that the heat input to the node point is definitely given.

The compatibility conditions of the problem are usually given as one of three ways; (1) find out the final balanced temperature distribution under a constant heat input after sufficiently long time, (2) follow the temperature variation of each node starting with a definite temperature distribution of system, (3) determine the quasi-equilibrium state under a cyclic heating during time t_1 and cooling during time t_2. Schematic presentations of these conditions are shown in Fig. 2. Now we can define the several definite temperatures; the final balanced temperature $T_{i\infty}$, the maximum temperature $T_{i\,\max}$, and the minimum temperature $T_{i\,\min}$, some of which are shown in the Figure. Furthermore, we introduce the mean temperature $T_{i\,\mathrm{mean}}$ which is

$$T_{i\,\mathrm{mean}}=\frac{1}{2}(T_{i\,\max}+T_{i\,\min})$$

and this temperature becomes asymptotically closer to a temperature when the cyclic period decreases $(t_1+t_2\to 0)$, which is called the limiting mean temperature. This temperature should be the same as the final balanced temperature under the time averaged heat input $(t_1Q_1+t_2Q_2)/(t_1+t_2)$.

Under a certain circumstance, the basic equations are linearized to

$$\frac{d\theta_i}{dt}=\frac{Q_i}{C_iT_i}-\frac{1}{C_iS_i}\theta_i-\sum_j\frac{1}{C_iR_{ij}}(\theta_i-\theta_j) \tag{3}$$

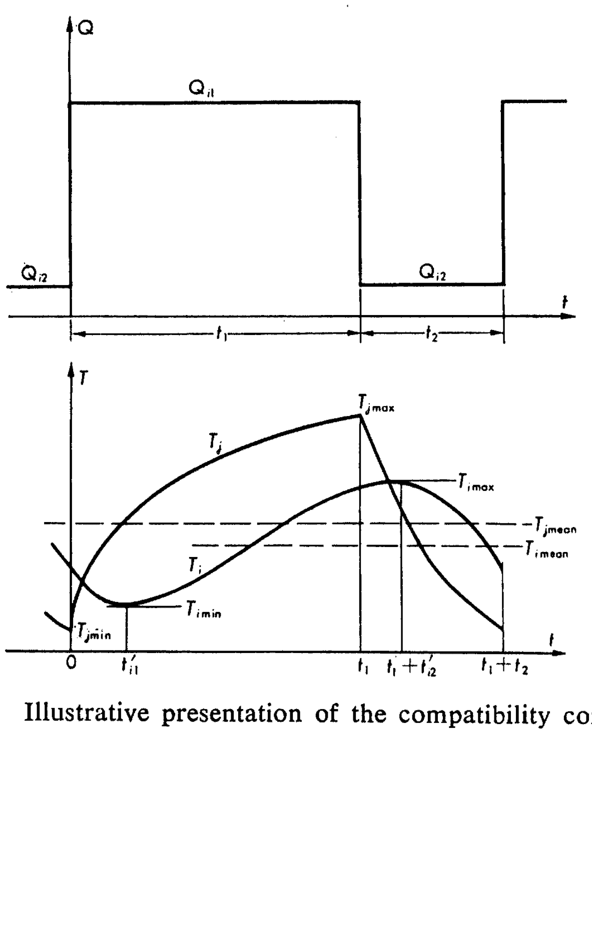


FIG. 2. Illustrative presentation of the compatibility condition

where

$$T_i = T_0\left(\frac{3}{4} + \theta_i\right)$$

and

$$S_i = \frac{1}{4A_i\sigma\varepsilon_i T_0^3}$$

$$R_{ij} = \frac{1}{K_{ij} + 4A_i\sigma F_{ij} T_0^3}$$

The temperature T_0 is to be defined in each problem. For example, if we define it as the temperature of node 1 in the case of the compatibility condition (1), then we have

$$\frac{T_i}{T_1} = \frac{3}{4} + \theta_i \tag{4}$$

or

$$\frac{T_i}{T_1} = \left(\frac{\theta_i}{\theta_1}\right)^{1/4} = (4\theta_i)^{1/4} \tag{5}$$

In general, the solution of Q_0 which corresponds to T_0 is found under the specified condition corresponding to the definition of T_0, which derives a general form of

$$\theta_0 = \sum_i F_i(S, R)\frac{Q_i}{T_0} \tag{6}$$

Since $\theta_0 = 1/4$, we have

$$T_0 = 4\sum_i F_i(S, R)\cdot Q_i \tag{7}$$

Because that S and R are the functions of T_0 and so F_i is, this relation indirectly defines the value of T_0 as the function of the thermal characters of the node system and the heat input. In experimental point of view, this relation is considered as the relation to obtain the value of F_i and then S and R from the measured values of T_0 and Q_i,

3. Single Node System—Exact Solution

As the simplest case, let us consider a single node system. This case corresponds to a small satellite or a meteor which has a uniform temperature in it. The thermal balance equation is

$$C\frac{dT}{dt} = Q - A\varepsilon\sigma T^4 \tag{8}$$

in which the suffix is dropped.

The final balanced temperature T_∞, which relates to the compatibility condition (1), is writen as

$$T_\infty=\left(\frac{Q}{A\varepsilon\sigma}\right)^{1/4} \tag{9}$$

Regarding to the compatibility condition (2), the temperature variation with the starting temperature $T_{(0)}$ at $t=0$ is expressed as; if $Q\neq 0$

$$\ln\frac{(T_\infty+T)(T_\infty-T_{(0)})}{(T_\infty-T)(T_\infty+T_{(0)})}+2\tan^{-1}\frac{T}{T_\infty}-2\tan^{-1}\frac{T_{(0)}}{T_\infty}=\frac{4\varepsilon\sigma AT_\infty}{C}t \tag{10}$$

where T_∞ takes the same value as in (9), and if $Q=0$

$$\left(\frac{T_{(0)}}{T}\right)^3=1+\frac{3\varepsilon\sigma AT_{(0)}^3}{C}t \tag{11}$$

It is interesting to find out the approximate solution with small t for these solutions. They are

$$\frac{T}{T_{(0)}}=1-\frac{\varepsilon\sigma A(T_{(0)}^3-T_\infty^3)}{C}t+\cdots\cdots \tag{12}$$

$$\frac{T}{T_{(0)}}=1-\frac{\varepsilon\sigma AT_{(0)}^3}{C}t+\cdots\cdots \tag{13}$$

for (10) and (11), respectively.

For the case of the compatibility condition (3), the solution is given as a combination of the former solution. That is, during the cool down period with Q_2

$$\ln\frac{(T_{\infty2}+T)(T_{\infty2}-T_{\max})}{(T_{\infty2}-T)(T_{\infty2}+T_{\max})}+2\tan^{-1}\frac{T}{T_{\infty2}}-2\tan^{-1}\frac{T_{\max}}{T_{\infty2}}=\frac{4\varepsilon\sigma AT_{\infty2}}{C}t \tag{14}$$

and during the heat up period with Q_1

$$\ln\frac{(T_{\infty1}+T)(T_{\infty1}-T_{\min})}{(T_{\infty1}-T)(T_{\infty1}+T_{\min})}+2\tan^{-1}\frac{T}{T_{\infty1}}-2\tan^{-1}\frac{T_{\min}}{T_{\infty1}}=\frac{4\varepsilon\sigma AT_{\infty1}}{C}t \tag{15}$$

where $T_{\infty1}$, $T_{\infty2}$ are the final balanced temperature with Q_1 or Q_2, respectively, If $Q_2=0$, the solution (14) should be repaced by

$$\left(\frac{T_{\max}}{T}\right)^3=1+\frac{3\varepsilon\sigma AT_{\max}^3}{C}t \tag{16}$$

The condition that the temperature of the node must be continuous at the beginning and the end of the each cycle gives the relations

$$\ln\frac{(T_{\infty1}+T_{\max})(T_{\infty1}-T_{\min})}{(T_{\infty1}-T_{\max})(T_{\infty1}+T_{\min})}+2\tan^{-1}\frac{T_{\max}}{T_{\infty1}}-2\tan^{-1}\frac{T_{\min}}{T_{\infty1}}=\frac{4\varepsilon\sigma AT_{\infty1}}{C}t_1 \tag{17}$$

$$\ln\frac{(T_{\infty 2}+T_{\min})(T_{\infty 2}-T_{\max})}{(T_{\infty 2}-T_{\min})(T_{\infty 2}+T_{\max})}+2\tan^{-1}\frac{T_{\min}}{T_{\infty 2}}-2\tan^{-1}\frac{T_{\max}}{T_{\infty 2}}=\frac{4\varepsilon\sigma AT_{\infty 2}}{C}t_2 \quad (18)$$

or if $Q_2=0$, instead of (18),

$$\left(\frac{T_{\max}}{T_{\min}}\right)^3=1+\frac{3\varepsilon\sigma AT^3_{\max}}{C}\cdot t_2 \quad (19)$$

These relations determine the value of $T_{\max}$ and $T_{\min}$.

From these results, the approximation for the case with small temperature amplitude ($T_{\max}$-$T_{\min}$) gives the relation

$$T_0=\lim_{t_1+t_2\to 0} T_{\text{mean}} \quad (20)$$

$$\frac{T_{\text{mean}}}{T_0}=1 \quad (21)$$

$$\frac{T_{\max}-T_{\min}}{T_0}=\frac{1}{4}\frac{t_2}{\tau}+\cdots\cdots \quad (22)$$

where

$$T_0=\left[\frac{Q}{\varepsilon\sigma A}\left(\frac{t_1}{t_1+t_2}\right)\right]^{1/4}$$

$$\tau=CS$$

$$S=\frac{1}{4\varepsilon\sigma AT_0^3}$$

On the other hand, if the period is sufficiently large, then the temperature at the end of the heating period becomes closer to the final balanced temperature with a constant heat input Q_1. The solution for this case is

$$\frac{T_{\text{mean}}}{T_0}=2\left(\frac{t_1}{t_1+t_2}\right)^{1/4}\left[1+\frac{1}{2}\left(\frac{\tau}{t_2}\right)^{1/3}\right]^{-1} \quad (23)$$

$$\frac{T_{\max}-T_{\min}}{T_0}=2\left[1-\left(\frac{\tau}{t_2}\right)^{1/3}\right] \quad (24)$$

4. Single Node Analysis—Linearized Solution

The linearized equation of the system is

$$\frac{d\theta}{dt}=\frac{Q}{CT_0}-\frac{1}{\tau}\theta \quad (25)$$

For the compatibility condition (1), the final balanced temperature under the heat input Q is

$$\theta_\infty = \frac{SQ}{T_0} \tag{26}$$

$$T_\infty = T_0\left(\frac{3}{4} + \frac{SQ}{T_0}\right) \tag{27}$$

If we take this temperature T_∞ as T_0, and $\theta_\infty = \theta_0 = 1/4$, then

$$T_0 = 4SQ \tag{28}$$

As mentioned before, this relation gives the means to experimentally determine the value of S.

The solution of the equation (25) is readily given by Laplace transformation. For the case of the compatibility condition (2), the temperature variation starting at a temperature $T_{(0)}$ and $\theta_{(0)}$ with Q is

$$\theta = \frac{SQ}{T_{(0)}} - \left\{\frac{SQ}{T_{(0)}} - \theta_{(0)}\right\} e^{-t/\tau} \tag{29}$$

If we here define the temperature $T_{(0)}$ as T_0 and so $\theta_{(0)} = 1/4$, then we have

$$\frac{T}{T_0} = \frac{3}{4} + \theta = \frac{3}{4} + \frac{SQ}{T_0} - \left\{\frac{SQ}{T_0} - \frac{1}{4}\right\} e^{-t/\tau} \tag{30}$$

or

$$\frac{T}{T_0} = \left(\frac{\theta}{\theta_0}\right)^{1/4} = \left[4\frac{SQ}{T_0} + \left\{1 - 4\frac{SQ}{T_0}\right\} e^{-t/\tau}\right]^{1/4} \tag{31}$$

The series expansion in t gives

$$\frac{T}{T_0} = 1 - \left(\frac{1}{4} - \frac{SQ}{T_0}\right)\frac{t}{\tau} + \left(\frac{1}{4} - \frac{SQ}{T_0}\right)\frac{t^2}{\tau^2} - \cdots\cdots \tag{32}$$

or

$$\frac{T}{T_0} = 1 - \left(\frac{1}{4} - \frac{SQ}{T_0}\right)\frac{t}{\tau} + \frac{5}{8}\left(\frac{1}{4} - \frac{SQ}{T_0}\right)\frac{t^2}{\tau^2} - \cdots\cdots \tag{33}$$

If $Q=0$, the solution is

$$\theta = \frac{1}{4} e^{-t/\tau} \tag{34}$$

$$\frac{T}{T_0} = \frac{3}{4} + \frac{1}{4} e^{-t/\tau} \tag{35}$$

or

$$\frac{T}{T_0} = e^{-t/4\tau} \tag{36}$$

Using the notation Q_0 which corresponds to the heat input giving the temperature $T_{(0)}$, one has

$$\frac{T}{T_0}=\left\{\frac{Q}{Q_0}+\left(1-\frac{Q}{Q_0}\right)e^{-t/\tau}\right\}^{1/4} \tag{37}$$

or

$$\frac{T}{T_0}=1-\frac{1}{4}\left(1-\frac{Q}{Q_0}\right)\frac{t}{\tau}+\cdots\cdots \tag{38}$$

Next let us solve the case of the compatibility condition (3), the solution θ_1 during the heating period with Q_1, and θ_2 during the cooling period with Q_2, are

$$\theta_1=\frac{SQ_1}{T_0}+A_1e^{-t/\tau} \tag{39}$$

$$\theta_2=\frac{SQ_2}{T_0}+A_2e^{-t/\tau} \tag{40}$$

respectively, where A_1 and A_2 are the integral constants to be determined from the initial conditions. The condition which the temperature is continuous at the beginning and the end of the each cycle is expressed as

$$\frac{SQ_1}{T_0}+A_1e^{-t_1/\tau}=\frac{SQ_2}{T_0}+A_2 \tag{41}$$

$$\frac{SQ_1}{T_0}+A_1=\frac{SQ_2}{T_0}+A_2e^{-t_2/\tau} \tag{42}$$

Then, we have

$$A_1=-\frac{(1-e^{-t_2/\tau})}{(1-e^{-(t_1+t_2)/\tau})}\frac{S(Q_1-Q_2)}{T_0} \tag{43}$$

$$A_2=-\frac{(1-e^{-t_1/\tau})}{(1-e^{-(t_1+t_2)/\tau})}\frac{S(Q_1-Q_2)}{T_0} \tag{44}$$

The mean temperature of the maximum and the minimum temperatures, and the corresponding value of θ are

$$T_{\text{mean}}=T_0\left(\frac{3}{4}+\theta_{\text{mean}}\right)$$

$$\theta_{\text{mean}}=\left[1-\frac{(1-e^{-t_2/\tau})(1+e^{-t_1/\tau})}{2(1-e^{-(t_1+t_2)/\tau})}\left(1-\frac{Q_2}{Q_1}\right)\right]\frac{SQ_1}{T_0} \tag{45}$$

$$\theta_{\text{max}}-\theta_{\text{min}}=\frac{(1-e^{-t_1/\tau})(1-e^{-t_2/\tau})}{(1-e^{-(t_1+t_2)/\tau})}\frac{S(Q_1-Q_2)}{T_0} \tag{46}$$

Let us define here T_0 as the limiting value of T_{mean} as $t_1+t_2\to 0$,

$$\theta_0 = \frac{t_1 Q_1 + t_2 Q_2}{t_1 + t_2} \frac{S}{T_0} = \frac{1}{4} \tag{47}$$

then

$$T_0 = 4S\left(\frac{t_1 Q_1 + t_2 Q_2}{t_1 + t_2}\right) \tag{48}$$

Thus we have

$$\frac{T_{\text{mean}}}{T_0} = \frac{3}{4} + \theta_{\text{mean}} = \frac{3}{4} + \left[1 - \frac{(1-e^{-t_2/\tau})(1+e^{-t_1/\tau})}{2(1-e^{-(t_1+t_2)/\tau})}\left(1 - \frac{Q_2}{Q_1}\right)\right] \frac{(t_1+t_2)}{4\left(t_1 + t_2 \dfrac{Q_2}{Q_1}\right)} \tag{49}$$

and

$$\frac{T_{\max} - T_{\min}}{T_0} = \theta_{\max} - \theta_{\min} = \frac{(1-e^{-t_1/\tau})(1-e^{-t_2/\tau})}{(1-e^{-(t_1+t_2)/\tau})} \frac{(t_1+t_2)}{4\left(t_1 + t_2 \dfrac{Q_2}{Q_1}\right)} \tag{50}$$

if one assumes $t_1 = t_2$

$$\frac{T_{\text{mean}}}{T_0} = 1 \tag{51}$$

$$\frac{T_{\max} - T_{\min}}{T_0} = \frac{1}{2} \frac{Q_1}{Q_1 + Q_2} \tanh \frac{t_1}{\tau} \tag{52}$$

if $Q_2 = 0$

$$\frac{T_{\text{mean}}}{T_0} = \frac{3}{4} + \left[1 - \frac{(1-e^{-t_2/\tau})(1+e^{-t_1/\tau})}{2(1-e^{-(t_1+t_2)/\tau})}\right] \frac{(t_1+t_2)}{4t_1} \tag{53}$$

$$\frac{T_{\max} - T_{\min}}{T_0} = \frac{(t_1+t_2)}{4t_1} \frac{(1-e^{-t_1/\tau})(1-e^{-t_2/\tau})}{(1-e^{-(t_1+t_2)/\tau})} \tag{54}$$

If $t_1 + t_2$ is small comparing to τ, we have the approximate expression as

$$\frac{T_{\text{mean}}}{T_0} = 1 - \frac{1}{48} \frac{(Q_1 - Q_2)(t_1 - t_2)}{(t_1 Q_1 + t_2 Q_2)} \frac{t_2^2}{\tau^2} + \cdots\cdots \tag{55}$$

$$\frac{T_{\max} - T_{\min}}{T_0} = \frac{t_2}{4\tau} \frac{1}{\left(1 - \dfrac{t_2 Q_2}{t_1 Q_1}\right)} \left(1 - \frac{1}{12} \frac{t_1 t_2}{\tau^2} + \cdots\cdots\right) \tag{56}$$

Some of these relations are shown in Figs. 3 and 4, together with the relations given in the previous chapter. As seen in Fig. 3, the relation (55) gives sufficiently accurate expression of T_{mean}/T_0. Rather it is practically usesul to take $T_{\text{mean}} = T_0$ for actual cases. And Fig. 4 shows that the first order expression is accurate enough for the case of $2t_2 < \tau$ and the third order correction is needed for $t_2 < 1.5\tau$.

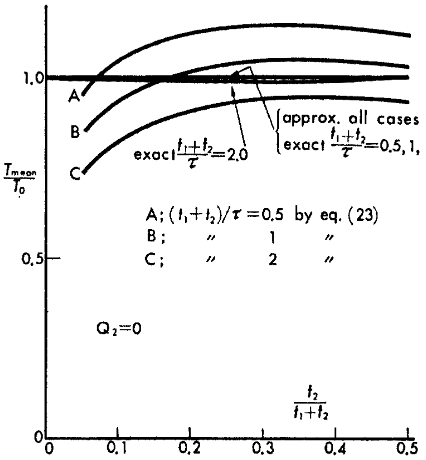


FIG. 3. The mean temperature of the single node system

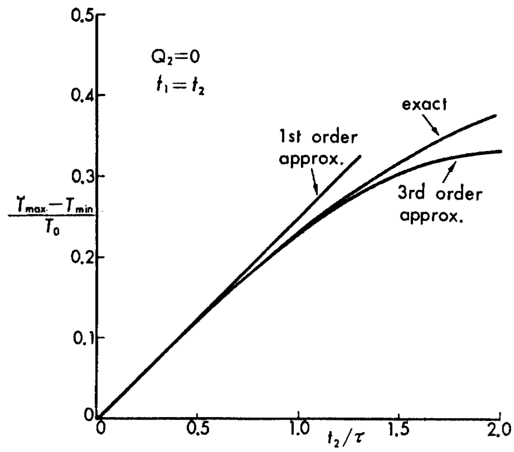


FIG. 4. The temperature variation of the single node system

5. Two-Nodes System—Cool Down Process

The equations of the two-nodes system are

$$\left.\begin{aligned} C_1\frac{dT_1}{dt}&=Q_1-A_1\varepsilon_1\sigma T_1^4-K_{12}(T_1-T_2)-A_1F_{12}\sigma(T_1^4-T_2^4)\\ C_2\frac{dT_2}{dt}&=Q_2-A_2\varepsilon_2\sigma T_2^4-K_{21}(T_2-T_1)-A_2F_{21}\sigma(T_2^4-T_1^4)\end{aligned}\right\} \tag{57}$$

After linearization, we have

$$\left.\begin{aligned} \frac{d\theta_1}{dt}&=\frac{Q_1}{C_1T_0}-\frac{1}{C_1S_1}\theta_1-\frac{1}{C_1R}(\theta_1-\theta_2)\\ \frac{d\theta_2}{dt}&=\frac{Q_2}{C_2T_0}-\frac{1}{C_2S_2}\theta_2-\frac{1}{C_2R}(\theta_2-\theta_1)\end{aligned}\right\} \tag{58}$$

where

$$S=\frac{1}{4\varepsilon\sigma AT_0^3}$$

$$R=\frac{1}{K+4AF\sigma T_0^3}$$

and the suffices 1 and 2 refer to the node 1 and 2, respectively. Naturally it is clear that $K_{12}=K_{21}$, $A_1F_{12}=A_2F_{21}$ and $R_{12}=R_{21}$.

The final balanced temperature under the constant heat inputs is

$$\left.\begin{aligned} \theta_{1\infty}&=\left\{\frac{(R+S_2)S_1Q_1+S_1S_2Q_2}{R+S_1+S_2}\right\}\frac{1}{T_0}\\ \theta_{2\infty}&=\left\{\frac{S_2S_1Q_1+(R+S_1)S_2Q_2}{R+S_1+S_2}\right\}\frac{1}{T_0}\end{aligned}\right\} \tag{59}$$

If we use this temperature $T_{1\infty}$ as T_0, we have

$$\left.\begin{aligned} T_{1\infty}&=T_0=4\frac{(R+S_2)S_1Q_1+S_1S_2Q_2}{R+S_1+S_2} \\ \frac{T_{2\infty}}{T_0}&=\frac{1}{4}\frac{(3R+4S_2)S_1Q_1+(R+4S_1)S_2Q_2}{(R+S_2)S_1Q_1+S_1S_2Q_2} \end{aligned}\right\} \quad (60)$$

Use of Laplace transformation is made on the equations (58) and the two constants are introduced, which are

$$\left.\begin{aligned} \alpha=&\frac{1}{2}\left(\frac{1}{C_1S_1}+\frac{1}{C_1R}+\frac{1}{C_2S_2}+\frac{1}{C_2R}\right)+\frac{1}{2}\left[\left(\frac{1}{C_1S_1}+\frac{1}{C_1R}+\frac{1}{C_2S_2}+\frac{1}{C_2R}\right)^2\right. \\ &\left.-4\left\{\left(\frac{1}{C_1S_1}+\frac{1}{C_1R}\right)\left(\frac{1}{C_2S_2}+\frac{1}{C_2R}\right)-\frac{1}{C_1R}\frac{1}{C_2R}\right\}\right]^{1/2} \\ \beta=&\frac{1}{2}\left(\frac{1}{C_1S_1}+\frac{1}{C_1R}+\frac{1}{C_2S_2}+\frac{1}{C_2R}\right)-\frac{1}{2}\left[\left(\frac{1}{C_1S_1}+\frac{1}{C_1R}+\frac{1}{C_2S_2}+\frac{1}{C_2R}\right)^2\right. \\ &\left.-4\left\{\left(\frac{1}{C_1S_1}+\frac{1}{C_1R}\right)\left(\frac{1}{C_2S_2}+\frac{1}{C_2R}\right)-\frac{1}{C_1R}\frac{1}{C_2R}\right\}\right]^{1/4} \end{aligned}\right\} \quad (61)$$

Some of these values are presented in Fig. 5. Then the solutions are

$$\begin{aligned} \theta_1=&\frac{1}{\alpha\beta}\left\{\left(\frac{1}{C_2S_2}+\frac{1}{C_2R}\right)\frac{Q_1}{C_1}+\frac{1}{C_1R}\frac{Q_2}{C_2}\right\}\frac{1}{T_0} \\ &+\frac{e^{-\alpha t}}{\alpha(\alpha-\beta)}\left\{\alpha^2\theta_1(0)-\alpha\theta_1(0)\left(\frac{1}{C_2S_2}+\frac{1}{C_2R}\right)-\alpha\theta_2(0)\frac{1}{C_1R}-\alpha\frac{Q_1}{C_1T_0}\right. \\ &\left.+\left(\frac{1}{C_2S_2}+\frac{1}{C_2R}\right)\frac{Q_1}{C_1T_0}+\frac{1}{C_1R}\frac{Q_2}{C_2T_0}\right\} \\ &+\frac{e^{-\beta t}}{\beta(\beta-\alpha)}\left\{\beta^2\theta_1(0)-\beta\theta_1(0)\left(\frac{1}{C_2S_2}+\frac{1}{C_2R}\right)-\beta\theta_2(0)\frac{1}{C_1R}-\beta\frac{Q_1}{C_1T_0}\right. \\ &\left.+\left(\frac{1}{C_2S_2}+\frac{1}{C_2R}\right)\frac{Q_1}{C_1T_0}+\frac{1}{C_1R}\frac{Q_2}{C_2T_0}\right\} \\ \theta_2=&\frac{1}{\alpha\beta}\left\{\left(\frac{1}{C_1S_1}+\frac{1}{C_1R}\right)\frac{Q_2}{C_2}+\frac{1}{C_2R}\frac{Q_1}{C_1}\right\}\frac{1}{T_0} \\ &+\frac{e^{-\alpha t}}{\alpha(\alpha-\beta)}\left\{\alpha^2\theta_2(0)-\alpha\theta_2(0)\left(\frac{1}{C_1S_1}+\frac{1}{C_1R}\right)-\alpha\theta_1(0)\frac{1}{C_2R}-\alpha\frac{Q_2}{C_2T_0}\right. \\ &\left.+\left(\frac{1}{C_1S_1}+\frac{1}{C_1R}\right)\frac{Q_2}{C_2T_0}+\frac{1}{C_2R}\frac{Q_1}{C_1T_0}\right\} \\ &+\frac{e^{-\beta t}}{\beta(\beta-\alpha)}\left\{\beta^2\theta_2(0)-\beta\theta_2(0)\left(\frac{1}{C_1S_1}+\frac{1}{C_1R}\right)-\alpha\theta_1(0)\frac{1}{C_2R}-\beta\frac{Q_2}{C_2T_0}\right. \\ &\left.+\left(\frac{1}{C_1S_1}+\frac{1}{C_1R}\right)\frac{Q_2}{C_2T_0}+\frac{1}{C_2R}\frac{Q_1}{C_1T_0}\right\} \end{aligned} \quad (62)$$

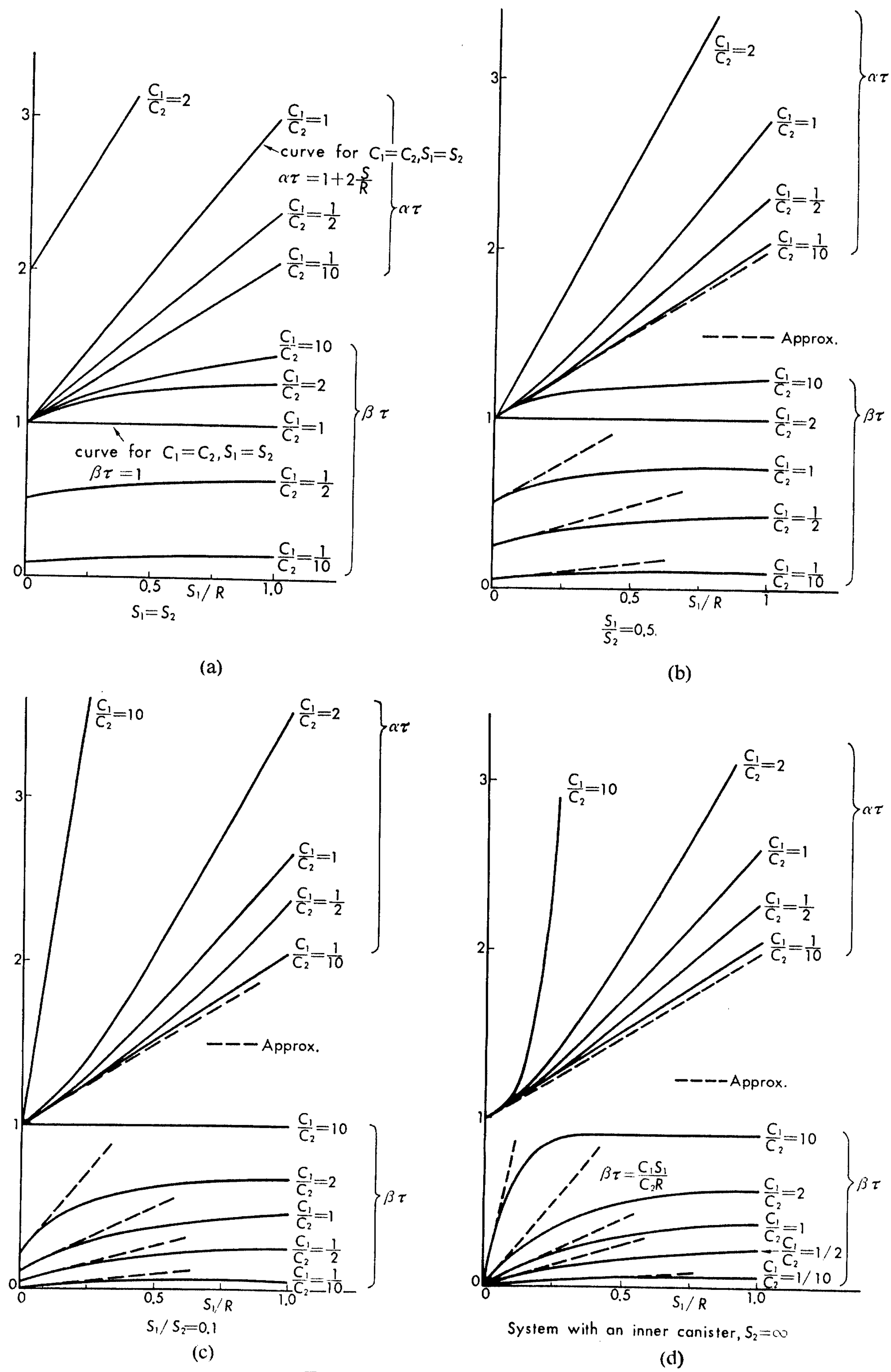


FIG. 5. The time constants

and

$$\left.\begin{aligned}\frac{T_1}{T_1(0)}&=\left[\frac{\theta_1}{\theta_1(0)}\right]^{1/4}\\ \frac{T_2}{T_2(0)}&=\left[\frac{\theta_2}{\theta_2(0)}\right]^{1/4}\end{aligned}\right\} \quad (63)$$

Expanding in the power series of t, we have

$$\left.\begin{aligned}\theta_1&=\theta_1(0)-\left[\left(\frac{1}{C_1S_1}+\frac{1}{C_1R}\right)\theta_1(0)-\frac{1}{C_1R}\theta_2(0)-\frac{Q_1}{C_1T_0}\right]\cdot t+\cdots\cdots\\ \theta_2&=\theta_2(0)-\left[\left(\frac{1}{C_2S_2}+\frac{1}{C_2R}\right)\theta_2(0)-\frac{1}{C_2R}\theta_1(0)-\frac{Q_2}{C_2T_0}\right]\cdot t+\cdots\cdots\end{aligned}\right\} \quad (64)$$

$$\left.\begin{aligned}\frac{T_1}{T_1(0)}&=1-\frac{1}{4}\left[\left(\frac{1}{C_1S_1}+\frac{1}{C_1R}\right)-\frac{1}{C_1R}\frac{\theta_1(0)}{\theta_2(0)}-\frac{Q_2}{C_2T_0}\frac{1}{\theta_2(0)}\right]\cdot t+\cdots\cdots\\ \frac{T_2}{T_2(0)}&=1-\frac{1}{4}\left[\left(\frac{1}{C_2S_2}+\frac{1}{C_2R}\right)-\frac{1}{C_2R}\frac{\theta_1(0)}{\theta_2(0)}-\frac{Q_2}{C_2T_0}\frac{1}{\theta_2(0)}\right]\cdot t+\cdots\cdots\end{aligned}\right\} \quad (65)$$

Let us consider cool down process without Q_1 and Q_2 starting with $\theta_1(0)=\theta_2(0)=1/4$, then

$$\left.\begin{aligned}\frac{T_1}{T_1(0)}&=1-\frac{1}{4}\frac{1}{C_1S_1}t+\cdots\cdots\\ \frac{T_2}{T_2(0)}&=1-\frac{1}{4}\frac{1}{C_2S_2}t+\cdots\cdots\end{aligned}\right\} \quad (66)$$

These relations show that the cool down of the each node begins linearly with the time provided with $S\neq 0$. If $S=0$, which means that the node is completely enclosed by the outer hull, the temperature decreases with the second or higher order of time.

6. Two-nodes System—Quasi-Equilibrium State

The basic equations (58) have the solution for the cyclic heating and cooling condition;

during the heat up period

$$\left.\begin{aligned}\theta_{11}&=\frac{(R+S_2)S_1Q_{11}+S_1S_2Q_{21}}{(R+S_1+S_2)T_0}+A_1e^{-\alpha t}+B_1e^{-\beta t}\\ \theta_{21}&=\frac{S_2S_1Q_{11}+(R+S_1)S_2Q_{21}}{(R+S_1+S_2)T_0}+A_2e^{-\alpha t}+B_2e^{-\beta t}\end{aligned}\right\} \quad (67)$$

during the cool down period

$$\left.\begin{aligned}\theta_{12}&=\frac{(R+S_2)S_1Q_{12}+S_1S_2Q_{22}}{(R+S_1+S_2)T_0}+A_1'e^{-\alpha t}+B_1'e^{-\beta t}\\ \theta_{22}&=\frac{S_2S_1Q_{11}+(R+S_1)S_2Q_{22}}{(R+S_1+S_2)T_0}+A_2'e^{-\alpha t}+B_2'e^{-\beta t}\end{aligned}\right\} \qquad (68)$$

where α and β are defined in (61) and the first suffix of θ and Q refers to the each node, and the second ones to the heating and the cooling period. Naturally, the values of Q_{12} and Q_{22} are likely to be zero. The values of A_1, A_2, A_1', A_2', B_1, B_2, B_1' and B_2' are determined by the initial conditions, which are

$$\left.\begin{aligned}\theta_{11}(t=0)&=\theta_{12}(t=t_2)\\ \theta_{11}(t=t_1)&=\theta_{12}(t=0)\\ \theta_{21}(t=0)&=\theta_{22}(t=t_2)\\ \theta_{21}(t=t_1)&=\theta_{22}(t=0)\end{aligned}\right\} \qquad (69)$$

and

$$\left.\begin{aligned}\left(\frac{d\theta_{11}}{dt}\right)_{t=0}&=\frac{Q_{11}}{C_1T_0}-\frac{1}{C_1S_1}\theta_{11}(t=0)-\frac{1}{C_1R}\{\theta_{11}(t=0)-\theta_{21}(t=0)\}\\ \left(\frac{d\theta_{21}}{dt}\right)_{t=0}&=\frac{Q_{21}}{C_2T_0}-\frac{1}{C_2S_2}\theta_{21}(t=0)-\frac{1}{C_2R}\{\theta_{21}(t=0)-\theta_{11}(t=0)\}\\ \left(\frac{d\theta_{12}}{dt}\right)_{t=0}&=\frac{Q_{12}}{C_1T_0}-\frac{1}{C_1S_1}\theta_{11}(t=0)-\frac{1}{C_1R}\{\theta_{12}(t=0)-\theta_{22}(t=0)\}\\ \left(\frac{d\theta_{22}}{dt}\right)_{t=0}&=\frac{Q_{22}}{C_2T_0}-\frac{1}{C_2S_2}\theta_{22}(t=0)-\frac{1}{C_2R}\{\theta_{22}(t=0)-\theta_{12}(t=0)\}\end{aligned}\right\} \qquad (70)$$

These are written in the explicit form as

$$\left.\begin{aligned}-A_1e^{-\alpha t_1}-B_1e^{-\beta t_1}+A_1'+B_1'&=\frac{(R+S_2)S_1(Q_{11}-Q_{12})+S_1S_2(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}\\ -A_2e^{-\alpha t_1}-B_2e^{-\beta t_1}+A_2'+B_2'&=\frac{S_1S_2(Q_{11}-Q_{12})+(R+S_1)S_2(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}\\ -A_1-B_1+A_1'e^{-\alpha t_2}+B_1'e^{-\beta t_2}&=\frac{(R+S_2)S_1(Q_{11}-Q_{12})+S_1S_2(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}\\ -A_2-B_2+A_2'e^{-\alpha t_2}+B_2'e^{-\beta t_2}&=\frac{S_2S_1(Q_{11}-Q_{12})+(R+S_1)S_2(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}\end{aligned}\right\} \qquad (71)$$

$$\begin{aligned}\left(\alpha\tau_1-1-\frac{S_1}{R}\right)A_1+\left(\beta\tau_1-1-\frac{S_1}{R}\right)B_1+\frac{S_1}{R}A_2+\frac{S_1}{R}B_2&=0\\ \left(\alpha\tau_2-1-\frac{S_2}{R}\right)A_2+\left(\beta\tau_2-1-\frac{S_2}{R}\right)B_2+\frac{S_2}{R}A_1+\frac{S_2}{R}B_1&=0\end{aligned}$$

$$\left.\begin{aligned}\left(\alpha\tau_1-1-\frac{S_1}{R}\right)A_1'+\left(\beta\tau_1-1-\frac{S_1}{R}\right)B_1'+\frac{S_1}{R}A_2'+\frac{S_1}{R}B_2'=0\\ \left(\alpha\tau_2-1-\frac{S_2}{R}\right)A_2'+\left(\beta\tau_2-1-\frac{S_2}{R}\right)B_2'+\frac{S_2}{R}A_1'+\frac{S_2}{R}B_1'=0\end{aligned}\right\} \tag{72}$$

These can be solved, but the results do not give any explanative presentation. Rather a few special cases show clear physical image of the problem. Let us study such cases.

(1) *Symmetrical Hull*

Assuming $C_1=C_2$, $S_1=S_2$, which represents a symmetrical spacecraft having two nodes on the opposite point, such as a spinning spherical satellite hull irradiated by Sun with an angle between solar vector and the spinning axis, having the two nodes on the opposite end of the spin axis, then one has the following relations, which are noted in Fig. 5(a)

$$\left.\begin{aligned}\alpha\tau&=1+2\frac{S}{R}\\ \beta\tau&=1\end{aligned}\right\} \tag{73}$$

where we put $C_1=C_2=C$, $S_1=S_2=S$ and $\tau_1=\tau_2=\tau$, and

$$\left.\begin{aligned}A_1=-A_2&=-\frac{(1-e^{-\alpha t_2})}{2(1-e^{-\alpha(t_1+t_2)})}\,\frac{RS(Q_{11}-Q_{12}-Q_{21}+Q_{22})}{(R+2S)T_0}\\ B_1=B_2&=-\frac{(1-e^{-\beta t_2})}{2(1-e^{-\beta(t_1+t_2)})}\,\frac{S(Q_{11}-Q_{12}+Q_{21}-Q_{22})}{T_0}\\ A_1'=-A_2'&=\frac{(1-e^{-\alpha t_1})}{2(1-e^{-\alpha(t_1+t_2)})}\,\frac{RS(Q_{11}-Q_{12}-Q_{21}+Q_{22})}{(R+2S)T_0}\\ B_1'=B_2'&=\frac{(1-e^{-\beta t_1})}{2(1-e^{-\beta(t_1+t_2)})}\,\frac{S(Q_{11}-Q_{12}+Q_{21}-Q_{22})}{T_0}\end{aligned}\right\} \tag{74}$$

Now we define T_0 as the mean temperature of very short cyclic period under $Q_{11}=Q_{21}$ ($=Q_0$, say) and $Q_{12}=Q_{22}=0$, so

$$T_0=\frac{4t_1}{t_1+t_2}SQ_0 \tag{75}$$

The solutions are written as

$$\begin{aligned}\frac{T_{1\,\mathrm{mean}}}{T_0}=\frac{3}{4}+\frac{1}{4}\Bigg[&\frac{(R+S_2)S_1Q_{12}+S_1S_2Q_{22}}{(R+S_1+S_2)T_0}\\ &+\frac{(1-e^{-\alpha t_1})(1+e^{-\alpha t_2})}{(1-e^{-\alpha(t_1+t_2)})}\,\frac{RS(Q_{11}-Q_{12}-Q_{21}+Q_{22})}{(R+2S)T_0}\\ &+\frac{(1-e^{-\beta t_1})(1+e^{-\beta t_2})}{(1-e^{-\beta(t_1+t_2)})}\,\frac{S(Q_{11}-Q_{12}+Q_{21}-Q_{22})}{T_0}\Bigg]\end{aligned}$$

$$\frac{T_{2\,\mathrm{mean}}}{T_0}=\frac{3}{4}+\frac{1}{4}\left[\frac{S_2S_1Q_{12}+(R+S_1)S_2Q_{22}}{(R+S_1+S_2)T_0}\right.$$
$$+\frac{(1-e^{-\beta t_1})(1+e^{-\beta t_2})}{(1-e^{-\beta(t_1+t_2)})}\frac{S(Q_{11}-Q_{12}+Q_{21}-Q_{22})}{T_0}$$
$$\left.-\frac{(1-e^{-\alpha t_1})(1+e^{-\alpha t_2})}{(1-e^{-\alpha(t_1+t_2)})}\frac{RS(Q_{11}-Q_{12}-Q_{21}+Q_{22})}{(R+2S)T_0}\right] \tag{76}$$

$$\frac{T_{1\,\max}-T_{1\,\min}}{T_0}=\frac{(1-e^{-\beta t_1})(1-e^{-\beta t_2})}{2(1-e^{-\beta(t_1+t_2)})}\frac{S(Q_{11}-Q_{12}+Q_{21}-Q_{22})}{T_0}$$
$$+\frac{(1-e^{-\alpha t_1})(1-e^{-\alpha t_2})}{2(1-e^{-\alpha(t_1+t_2)})}\frac{RS(Q_{11}-Q_{12}-Q_{21}+Q_{22})}{(R+2S)T_0}$$
$$\frac{T_{2\,\max}-T_{2\,\min}}{T_0}=\frac{(1-e^{-\beta t_1})(1-e^{-\beta t_2})}{2(1-e^{-\beta(t_1+t_2)})}\frac{S(Q_{11}-Q_{12}+Q_{21}-Q_{22})}{T_0}$$
$$-\frac{(1-e^{-\alpha t_1})(1-e^{-\alpha t_2})}{2(1-e^{-\alpha(t_1+t_2)})}\frac{RS(Q_{11}-Q_{12}-Q_{21}+Q_{22})}{(R+2S)T_0} \tag{77}$$

For the sake of simplicity, we assume that $Q_{12}=Q_{22}=0$ in the following part of this section. For the case with $t_1=t_2$, we have

$$\left.\begin{aligned}\frac{T_{1\,\mathrm{mean}}}{T_0}&=1+\frac{R}{8(R+2S)}\left(\frac{Q_{11}-Q_{21}}{Q_0}\right)\\ \frac{T_{2\,\mathrm{mean}}}{T_0}&=1-\frac{R}{8(R+2S)}\left(\frac{Q_{11}-Q_{21}}{Q_0}\right)\end{aligned}\right\} \tag{78}$$

$$\left.\begin{aligned}\frac{T_{1\,\max}-T_{1\,\min}}{T_0}&=\frac{1}{2}\tanh\frac{\beta t_1}{2}+\frac{R}{4(R+2S)}\frac{(Q_{11}-Q_{21})}{Q_0}\tanh\frac{\alpha t_1}{2}\\ \frac{T_{2\,\max}-T_{2\,\min}}{T_0}&=\frac{1}{2}\tanh\frac{\beta t_1}{2}-\frac{R}{4(R+2S)}\frac{(Q_{11}-Q_{21})}{Q_0}\tanh\frac{\alpha t_1}{2}\end{aligned}\right\} \tag{79}$$

and for the case of $t_1+t_2\ll\tau$, the relations are expanded as

$$\left.\begin{aligned}\frac{T_{1\,\mathrm{mean}}}{T_0}&=1-\frac{1}{12}\frac{(t_1-t_2)t_2}{\tau^2}+\frac{R}{2(R+2S)}\left(\frac{Q_{11}-Q_{21}}{Q_0}\right)\\ &\quad-\frac{1}{24}\left(\frac{R+2S}{R}\right)\left(\frac{Q_{11}-Q_{21}}{Q_0}\right)\frac{(t_1-t_2)t_2}{\tau^2}+\cdots\cdots\\ \frac{T_{2\,\mathrm{mean}}}{T_0}&=1-\frac{1}{12}\frac{(t_1-t_2)t_2}{\tau^2}-\frac{R}{2(R+2S)}\left(\frac{Q_{11}-Q_{21}}{Q_0}\right)\\ &\quad+\frac{1}{24}\left(\frac{R+2S}{R}\right)\left(\frac{Q_{11}-Q_{21}}{Q_0}\right)\frac{(t_1-t_2)t_2}{\tau^2}+\cdots\cdots\end{aligned}\right\} \tag{80}$$

$$\left.\begin{aligned}
\frac{T_{1\,\max}-T_{1\,\min}}{T_0} &= \frac{1}{4}\frac{t_2}{\tau}\left(1-\frac{1}{12}\frac{t_1t_2}{\tau^2}\right)\\
&\quad+\left(\frac{Q_{11}-Q_{21}}{8Q_0}\right)\frac{t_2}{\tau}\left\{1-\frac{1}{12}\frac{(R+2S)^2}{R^2}\frac{t_1t_2}{\tau^2}\right\}+\cdots\cdots\\
&=\frac{1}{4}\frac{Q_{11}}{Q_0}\frac{t_2}{\tau}+\cdots\cdots\\
\frac{T_{2\,\max}-T_{2\,\min}}{T_0} &= \frac{1}{4}\frac{t_2}{\tau}\left(1-\frac{1}{12}\frac{t_1t_2}{\tau^2}\right)\\
&\quad-\frac{Q_{11}-Q_{21}}{8Q_0}\frac{t_2}{\tau}\left\{1-\frac{1}{12}\frac{(R+2S)^2}{R^2}\frac{t_1t_2}{\tau^2}\right\}+\cdots\cdots\\
&=\frac{1}{4}\frac{Q_{21}}{Q_0}\frac{t_2}{\tau}+\cdots\cdots
\end{aligned}\right\} \quad (81)$$

where use of the relation

$$Q_{11}+Q_{21}=2Q_0 \quad (82)$$

is made. Some of these relations are presented in Figs. 6 and 7.

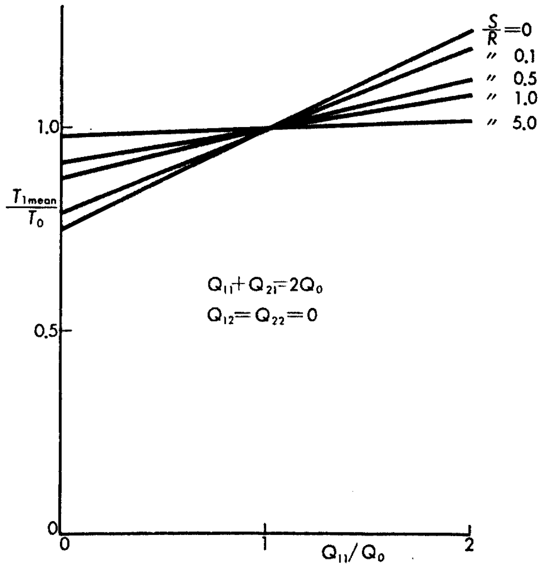


FIG. 6. The mean temperature of the symmetrical hull

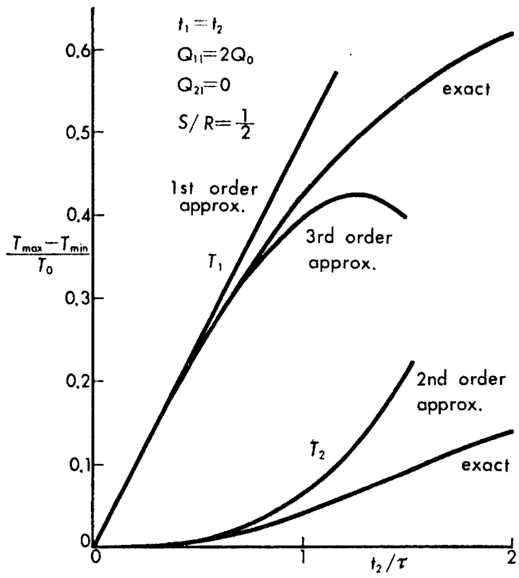


FIG. 7. The temperature variation of the symmetrical hull

As an example, let us consider a spherical satellite hull spinning around one axis, and take the two node points 1 and 2 on the opposite ends of the spin axis. We assume the case

$$T_0=280°\mathrm{K} \quad Q_0=500\,\mathrm{watt} \quad t_1=60\,\mathrm{min} \quad t_2=30\,\mathrm{min}$$
$$S=0.21°\mathrm{C/watt} \quad R=2.1°\mathrm{C/watt} \quad \tau=90\,\mathrm{min}$$

then the temperatures of the node 1 are plotted in Fig. 8 as the functions of $Q_{11}-Q_{21}/Q_0$, which directly relates to the solar incident angle to the spin axis.

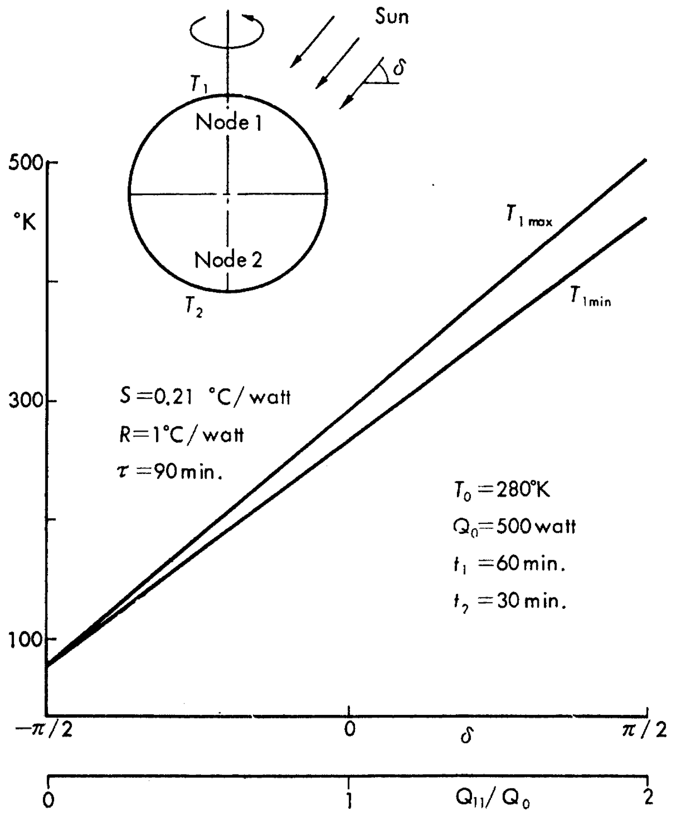


FIG. 8. An example of the symmetrical hull system

(2) *Small Appendage*

Here we assume that $S_1<R$, $S_2<R$, $C_1>C_2$ and $C_1S_1\cong C_2S_2$. This corresponds to a small appendage (node 2) attached to a large main structure (node 1) having a large heat capacity ($C_1>C_2$), both of which have made from the nearly same skin material and surface finish ($C_1S_1\cong C_2S_2$). Then we have

$$\left.\begin{aligned} \alpha\tau_1&=1+\frac{S_1}{R}\\ \beta\tau_1&=1+\frac{S_2}{R}\end{aligned}\right\}\tag{83}$$

These are shown in Fig. 5 (b) and (c) by dotted line, and

$$\left.\begin{aligned} A_1=A_2&=-\frac{(1-e^{-\alpha t_2})}{(1-e^{-\alpha(t_1+t_2)})}\left\{\frac{(R+S_2)S_1(Q_{11}-Q_{12})+S_1S_2(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}\right\}\\ A_1'=A_2'&=\frac{(1-e^{-\alpha t_1})}{(1-e^{-\alpha(t_1+t_2)})}\left\{\frac{(R+S_2)S_1(Q_{11}-Q_{12})+S_1S_2(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}\right\}\\ B_1=B_1'&=0\\ B_2&=\frac{(1-e^{-\beta t_2})}{(1-e^{-\beta(t_1+t_2)})}\frac{RS_1(Q_{11}-Q_{12})-RS_2(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}\\ B_2'&=-\frac{(1-e^{-\beta t_1})}{(1-e^{-\beta(t_1+t_2)})}\frac{RS_1(Q_{11}-Q_{12})-RS_2(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}\end{aligned}\right\}\tag{84}$$

In this case let us define T_0 as a limiting mean temperature of the node 1 with Q_{11} but $Q_{12}=Q_{21}=Q_{22}=0$. This is

$$T_0=\frac{4t_1}{t_1+t_2}\frac{(R+S_2)S_1Q_{11}}{R+S_1+S_2} \tag{85}$$

The results are

$$\left.\begin{aligned}
\frac{T_{1\,\mathrm{mean}}}{T_0}&=\frac{3}{4}+\left\{\frac{(R+S_2)S_1Q_{12}+S_1S_2Q_{22}}{(R+S_1+S_2)T_0}\right\}\\
&\quad+\frac{1}{2}\frac{(1-e^{-\alpha t_1})(1+e^{-\alpha t_2})}{(1-e^{-\alpha(t_1+t_2)})}\left\{\frac{(R+S_2)S_1(Q_{11}-Q_{12})+S_1S_1(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}\right\}\\
\frac{T_{2\,\mathrm{mean}}}{T_0}&=\frac{3}{4}+\left\{\frac{S_2S_1Q_{12}+(R+S_1)S_2Q_{22}}{(R+S_1+S_2)T_0}\right\}\\
&\quad+\frac{1}{2}\frac{(1-e^{-\alpha t_1})(1+e^{-\alpha t_2})}{(1-e^{-\alpha(t_1+t_2)})}\left\{\frac{(R+S_2)S_1(Q_{11}-Q_{12})+S_1S_2(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}\right\}\\
&\quad-\frac{1}{2}\frac{(1-e^{-\beta t_1})(1+e^{-\beta t_2})}{(1-e^{-\beta(t_1+t_2)})}\left\{\frac{RS_1(Q_{11}-Q_{12})-RS_2(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}\right\}
\end{aligned}\right\} \tag{86}$$

$$\left.\begin{aligned}
\frac{T_{1\,\max}-T_{1\,\min}}{T_0}&=\frac{(1-e^{-\alpha t_1})(1-e^{-\alpha t_2})}{(1-e^{-\alpha(t_1+t_2)})}\,\frac{(R+S_2)S_1(Q_{11}-Q_{12})+S_1S_2(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}\\
\frac{T_{2\,\max}-T_{2\,\min}}{T_0}&=\frac{(1-e^{-\alpha t_1})(1-e^{-\alpha t_2})}{(1-e^{-\alpha(t_1+t_2)})}\,\frac{(R+S_2)S_1(Q_{11}-Q_{12})+S_1S_2(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}\\
&\quad-\frac{(1-e^{-\beta t_1})(1-e^{-\beta t_2})}{(1-e^{-\beta(t_1+t_2)})}\,\frac{RS_1(Q_{11}-Q_{12})-RS_2(Q_{21}-Q_{22})}{(R+S_1+S_2)T_0}
\end{aligned}\right\} \tag{87}$$

If $t_1=t_2$ and $Q_{12}=Q_{22}=0$, we have

$$\left.\begin{aligned}
\frac{T_{1\,\mathrm{mean}}}{T_0}&=1+\frac{1}{4}\frac{S_2}{R+S_2}\frac{Q_{21}}{Q_{11}}\\
\frac{T_{2\,\mathrm{mean}}}{T_0}&=1-\frac{1}{4}\frac{R}{R+S_2}+\frac{1}{4}\frac{(R+S_1)S_2}{(R+S_2)S_1}\frac{Q_{21}}{Q_{11}}
\end{aligned}\right\} \tag{88}$$

$$\left.\begin{aligned}
\frac{T_{1\,\max}-T_{1\,\min}}{T_0}&=\frac{1}{2}\left\{1+\frac{S_2}{R+S_2}\frac{Q_{21}}{Q_{11}}\right\}\tanh\frac{\alpha t_1}{2}\\
\frac{T_{2\,\max}-T_{2\,\min}}{T_0}&=\frac{1}{2}\left\{1+\frac{S_2}{R+S_2}\frac{Q_{21}}{Q_{11}}\right\}\tanh\frac{\alpha t_1}{2}\\
&\quad-\frac{1}{2}\frac{RS_1Q_{11}-RS_2Q_{21}}{(R+S_2)S_1Q_{11}}\tanh\frac{\beta t_1}{2}
\end{aligned}\right\} \tag{89}$$

If $t_1,t_2<\tau$, $Q_{12}=Q_{22}=0$, the expanded forms are

$$\left.\begin{aligned}
\frac{T_{1\,\mathrm{mean}}}{T_0} &= 1+\frac{S_2}{4(R+S_2)}\frac{Q_{21}}{Q_{11}}-\frac{1}{48}\frac{(R+S_1)^2}{R^2}\left(1+\frac{S_2}{R+S_2}\frac{Q_{21}}{Q_{11}}\right)\frac{(t_1-t_2)t_2}{\tau_1^2}\\
\frac{T_{2\,\mathrm{mean}}}{T_0} &= \frac{T_{1\,\mathrm{mean}}}{T_0}-\frac{R}{4(R+S_2)}+\frac{RS_2}{4(R+S_2)S_1}\frac{Q_{21}}{Q_{11}}\\
&\quad +\frac{1}{48}\frac{(R+S_2)^2}{R^2}\left(\frac{R}{R+S_2}-\frac{RS_2}{(R+S_2)S_1}\frac{Q_{21}}{Q_{11}}\right)\frac{(t_1-t_2)t_2}{\tau_1^2}
\end{aligned}\right\} \tag{90}$$

$$\left.\begin{aligned}
\frac{T_{1\,\max}-T_{1\,\min}}{T_0} &= \frac{1}{4}\frac{R+S_1}{R}\left(1+\frac{S_2}{R+S_2}\frac{Q_{21}}{Q_{11}}\right)\frac{t_2}{\tau_1}\left(1-\frac{1}{12}\frac{(R+S_1)^2}{R^2}\frac{t_1t_2}{\tau_1^2}\right)\\
&\quad +\cdots\cdots\\
\frac{T_{2\,\max}-T_{2\,\min}}{T_0} &= \frac{T_{1\,\max}-T_{1\,\min}}{T_0}-\frac{1}{4}\left\{1-\frac{S_2}{S_1}\frac{Q_{21}}{Q_{11}}\right\}\frac{t_2}{\tau_1}\\
&\quad \times\left\{1-\frac{1}{12}\frac{(R+S_2)^2}{R^2}\frac{t_1t_2}{\tau^2}\right\}+\cdots\cdots
\end{aligned}\right\} \tag{91}$$

Some of these are shown in Figs. 9 and 10.

The exact values are not distinguishable from the approx. values.

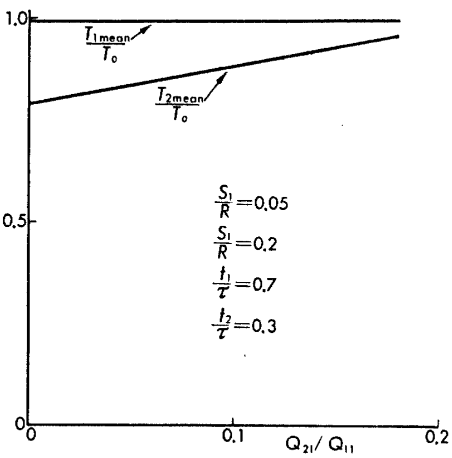


FIG. 9. The mean temperature of the small appendage system

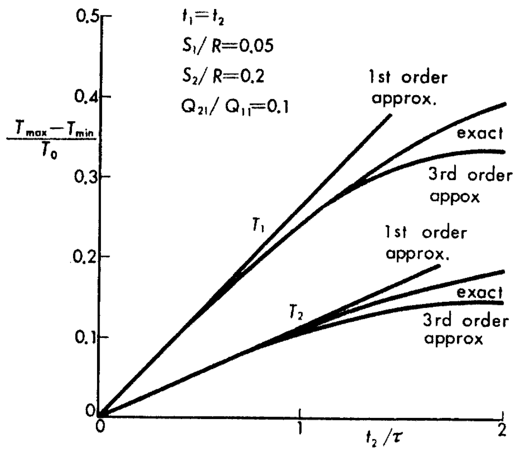


FIG. 10. The temperature variation of the small appendage system

As an example, a small artificial satellite which has a payload capsule on a spherical rocket chamber is studied. The schematic diagram of this satellite is shown in Fig. 11, as shown, this satellite is spinning around its axis and the heat input to the capsule and the spherical rocket relates to the solar incident angle to the spin axis. Then the ratio of the heat input to the spherical rocket (node 1) Q_{11} to the one of the capsule (node 2) Q_{21} relates to this angle, which is plotted in the figure. Let us consider a case with

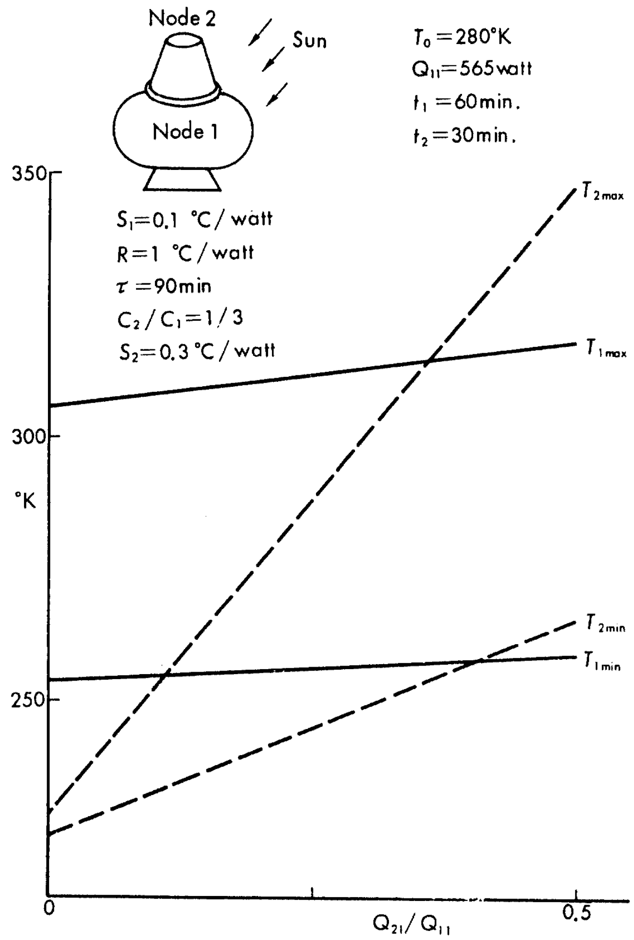


FIG. 11. An example of the small appendage system

$T_0=280°K$ $Q_{11}=565$ watt $t_1=60$ min $t_2=30$ min $S_1=0.1$ °C/watt
$C_2/C_1=1/3$ $S_2=0.3$ °C/watt $R=1$ °C/watt
then the mean temperature and the temperature variation of these cases are given as drawn in the diagram.

(3) *Inner Canister System*

Here an inner canister (node 2) enclosed by an outer hull (node 1) is analysed. In this system, the radiative heat transfer to the outer space from the node 2 is none ($S_2=\infty$) and the cyclic heat input is applied to only the outer hull and the heat input to the inner canister is usually considered to be constant.

In this case, we have

$$\left.\begin{aligned}\alpha\tau&=\frac{1}{2}\left[1+\left(1+\frac{C_1}{C_2}\right)\frac{S}{R}+\left\{1+2\left(1-\frac{C_1}{C_2}\right)\frac{S}{R}+\left(1+\frac{C_1}{C_2}\right)^2\left(\frac{S}{R}\right)^2\right\}^{1/2}\right]\\ \beta\tau&=\frac{1}{2}\left[1+\left(1+\frac{C_1}{C_2}\right)\frac{S}{R}-\left\{1+2\left(1-\frac{C_1}{C_2}\right)\frac{S}{R}+\left(1+\frac{C_1}{C_2}\right)^2\left(\frac{S}{R}\right)^2\right\}^{1/2}\right]\end{aligned}\right\}\tag{92}$$

where the unnecessary suffices are dropped, that is, we put $S_1=S$ and $\tau_1=\tau$. The maximum and the minimum temperatures of the outer hull may take place at the beginning and the end of the cooling period, respectively, but those of the inner canister may happen between these instants, which will be noted by $t=t_2'$ and $t=t_1'$, respectively.

For the sake of simplicity, let us solve the case with only Q_{11} but $Q_{12}=Q_{21}=Q_{22}=0$. Then the solutions have the forms

$$\left.\begin{aligned}
\theta_{11}&=\frac{SQ_{11}}{T_0}+A_1e^{-\alpha t}+B_1e^{-\beta t}\\
\theta_{21}&=\frac{SQ_{11}}{T_0}+A_2e^{-\alpha t}+B_2e^{-\beta t}\\
\theta_{12}&=A_1'e^{-\alpha t}+B_1'e^{-\beta t}\\
\theta_{22}&=A_2'e^{-\alpha t}+B_2'e^{-\beta t}
\end{aligned}\right\}\qquad(93)$$

The initial conditions will be given as

$$\left.\begin{aligned}
\theta_{11}(t=0)&=\theta_{12}(t=t_2)\\
\theta_{11}(t=t_1)&=\theta_{12}(t=0)\\
\theta_{21}(t=0)&=\theta_{22}(t=t_2)\\
\theta_{21}(t=t_1)&=\theta_{22}(t=0)
\end{aligned}\right\}\qquad(94)$$

$$\left.\begin{aligned}
\theta_{11}(t=t_1')&=\theta_{21}(t=t_1')\\
\theta_{12}(t=t_2')&=\theta_{22}(t=t_2')
\end{aligned}\right\}\qquad(95)$$

$$\left.\begin{aligned}
\left(\frac{d\theta_{11}}{dt}\right)_{(t=t'_1)}&=\frac{Q_{11}}{C_1T_0}-\frac{1}{\tau}\theta_{11}(t=t_1')\\
\left(\frac{d\theta_{21}}{dt}\right)_{(t=t'_1)}&=0\\
\left(\frac{d\theta_{12}}{dt}\right)_{(t=t'_2)}&=\frac{Q_{11}}{C_1T_0}-\frac{1}{\tau}\theta_{12}(t=t_2')\\
\left(\frac{d\theta_{22}}{dt}\right)_{(t=t'_2)}&=0
\end{aligned}\right\}\qquad(96)$$

It is needless to say about the physical meanings of these relations, which are slightly different from (69) and (70), but much more convenient to calculate the final results. The solutions of those are

$$\begin{aligned}
A_1&=\frac{(1-e^{-\alpha t_2})(\beta\tau-1)}{(1-e^{-\alpha(t_1+t_2)})(\alpha\tau-\beta\tau)}\,\frac{SQ_{11}}{T_0}\\
A_1'&=-\frac{(1-e^{-\alpha t_1})(\beta\tau-1)}{(1-e^{-\alpha(t_1+t_2)})(\alpha\tau-\beta\tau)}\,\frac{SQ_{11}}{T_0}
\end{aligned}$$

$$\left.\begin{aligned}
A_2 &= \frac{(1-e^{-\alpha t_2})\beta\tau}{(1-e^{-\alpha(t_1+t_2)})(\alpha\tau-\beta\tau)}\,\frac{SQ_{11}}{T_0}\\
A_2' &= -\frac{(1-e^{-\alpha t_1})\beta\tau}{(1-e^{-\alpha(t_1+t_2)})(\alpha\tau-\beta\tau)}\,\frac{SQ_{11}}{T_0}\\
B_1 &= -\frac{(1-e^{-\beta t_2})(\alpha\tau-1)}{(1-e^{-\beta(t_1+t_2)})(\alpha\tau-\beta\tau)}\,\frac{SQ_{11}}{T_0}\\
B_1' &= \frac{(1-e^{-\beta t_1})(\alpha\tau-1)}{(1-e^{-\beta(t_1+t_2)})(\alpha\tau-\beta\tau)}\,\frac{SQ_{11}}{T_0}\\
B_2 &= -\frac{(1-e^{-\beta t_2})\alpha\tau}{(1-e^{-\beta(t_1+t_2)})(\alpha\tau-\beta\tau)}\,\frac{SQ_{11}}{T_0}\\
B_2' &= \frac{(1-e^{-\beta t_1})\alpha\tau}{(1-e^{-\beta(t_1+t_2)})(\alpha\tau-\beta t)}\,\frac{SQ_{11}}{T_0}\\
e^{-t'_1} &= \left[\frac{(1-e^{-\alpha t_2})(1-e^{-\beta(t_1+t_2)})}{(1-e^{-\beta t_2})(1-e^{-\alpha(t_1+t_2)})}\right]^{-\frac{1}{\alpha-\beta}}\\
e^{-t'_2} &= \left[\frac{(1-e^{-\alpha t_1})(1-e^{-\beta(t_1+t_2)})}{(1-e^{-\beta t_1})(1-e^{-\alpha(t_1+t_2)})}\right]^{-\frac{1}{\alpha-\beta}}
\end{aligned}\right\}\qquad (97)$$

The temperature T_0 in this case is defined as the limiting mean temperature of the node 1, which is

$$T_0=4\left(\frac{t_1}{t_1+t_2}\right)SQ_{11} \qquad (98)$$

The results are

$$\left.\begin{aligned}
\frac{T_{1\,\mathrm{mean}}}{T_0} &= \frac{3}{4}+\frac{1}{2}\{\theta_{12}(t=0)+\theta_{12}(t=t_2)\}\\
&= \frac{3}{4}+\frac{1}{2}\left\{\frac{(1-e^{-\alpha t_1})(1+e^{-\alpha t_2})(1-\beta\tau)}{(1-e^{-\alpha(t_1+t_2)})(\alpha\tau-\beta\tau)}\right.\\
&\qquad \left.+\frac{(1-e^{-\beta t_1})(1+e^{-\beta t_2})(\alpha\tau-1)}{(1-e^{-\beta(t_1+t_2)})(\alpha\tau-\beta\tau)}\right\}\frac{SQ_{11}}{T_0}\\
\frac{T_{2\,\mathrm{mean}}}{T_0} &= \frac{3}{4}+\frac{1}{2}\{\theta_{21}(t=t_1')+\theta_{22}(t=t_2')\}=\frac{T_{1\,\mathrm{mean}}}{T_0}
\end{aligned}\right\}\qquad (99)$$

$$\begin{aligned}
\frac{T_{1\,\max}-T_{1\,\min}}{T_0} &= \{\theta_{12}(t=0)-\theta_{12}(t=t_2)\}\\
&= \left\{\frac{(1-e^{-\alpha t_1})(1-e^{-\alpha t_2})(1-\beta\tau)}{(1-e^{-\alpha(t_1+t_2)})(\alpha\tau-\beta\tau)}\right.\\
&\qquad \left.+\frac{(1-e^{-\beta t_1})(1-e^{-\beta t_2})(\alpha\tau-1)}{(1-e^{-\beta(t_1+t_2)})(\alpha\tau-\beta\tau)}\right\}\frac{SQ_{11}}{T_0}
\end{aligned}$$

$$\left.\begin{aligned}\frac{T_{2\,\max}-T_{2\,\min}}{T_0}&=\{\theta_{22}(t=t_2')-\theta_{21}(t=t_1')\}\\&=\left\{\frac{-(1-e^{-\alpha t_1})\beta\tau}{(1-e^{-\alpha(t_1+t_2)})(\alpha\tau-\beta\tau)}e^{-\alpha t'_2}+\frac{(1-e^{-\beta t_1})\alpha\tau}{(1-e^{-\beta(t_1+t_2)})(\alpha\tau-\beta\tau)}e^{-\beta t'_2}\right.\\&\quad-1-\frac{(1-e^{-\alpha t_2})\beta\tau}{(1-e^{-\alpha(t_1+t_2)})(\alpha\tau-\beta\tau)}e^{-\alpha t'_1}\\&\quad\left.+\frac{(1-e^{-\beta t_2})\alpha\tau}{(1-e^{-\beta(t_1+t_2)})(\alpha\tau-\beta\tau)}e^{-\beta t'_1}\right\}\frac{SQ_{11}}{T_0}\end{aligned}\right\}\tag{100}$$

For the case of $t_1=t_2$, we have

$$\left.\begin{aligned}&\frac{T_{1\,\mathrm{mean}}}{T_0}=\frac{T_{2\,\mathrm{mean}}}{T_0}=1\\&\frac{T_{1\,\max}-T_{1\,\min}}{T_0}=\frac{1}{2}\frac{(1-\beta\tau)}{(\alpha\tau-\beta\tau)}\tanh\frac{\alpha t_1}{2}+\frac{1}{2}\frac{(\alpha\tau-1)}{(\alpha\tau-\beta\tau)}\tanh\frac{\beta t_1}{2}\end{aligned}\right\}\tag{101}$$

$$\frac{T_{2\,\max}-T_{2\,\min}}{T_0}=(1+e^{-\alpha t_1})^{\frac{\beta}{\alpha-\beta}}(1+e^{-\beta t_1})^{\frac{-\alpha}{\alpha-\beta}}-\frac{1}{2}\tag{102}$$

Now, for the case with $S\ll R$, one has an approximate from of

$$\left.\begin{aligned}\alpha\tau&=1+\frac{S}{R}\\\beta\tau&=\frac{C_1S}{C_2R}\end{aligned}\right\}\tag{103}$$

These values are plotted in Fig. 5 (d). For the case of t_1, $t_2\ll\tau$ and $S\ll R$, we have

$$\left.\begin{aligned}\frac{T_{1\,\mathrm{mean}}}{T_0}&=1-\frac{1}{48}\left(1+\frac{S}{R}\right)\frac{(t_1-t_2)t_1t_2}{(t_1+t_2)\tau^2}+\cdots\cdots.\\\frac{T_{2\,\mathrm{mean}}}{T_0}&=\frac{T_{1\,\mathrm{mean}}}{T_0}\end{aligned}\right\}\tag{104}$$

$$\left.\begin{aligned}\frac{T_{1\,\max}-T_{1\,\min}}{T_0}&=\frac{t_2}{4\tau}\left\{1-\frac{1}{12}\left(1+\frac{2S}{R}\right)\frac{t_1t_2}{\tau^2}\right.\\&\quad\left.+\frac{1}{12}\left(1+\frac{S}{R}\right)\frac{(t_1-t_2)t_2}{\tau^2}+\cdots\cdots\right\}\\\frac{T_{2\,\max}-T_{2\,\min}}{T_0}&=\frac{t_1t_2}{16C_1SC_2R}+\cdots\cdots\end{aligned}\right\}\tag{105}$$

The case with a constant heat input to the inner canister can be treated using the law of superposition, the results are

$$\left.\begin{aligned}\frac{T_{1\,\mathrm{mean}}}{T_0}&=\left|\frac{T_{1\,\mathrm{mean}}}{T_0}\right|_{Q_2=0}+\frac{(t_1+t_2)Q_2}{4t_1Q_{11}}\\ \frac{T_{2\,\mathrm{mean}}}{T_0}&=\left|\frac{T_{2\,\mathrm{mean}}}{T_0}\right|_{Q_2=0}+\frac{(t_1+t_2)(R+S)Q_2}{4t_1SQ_{11}}\end{aligned}\right\} \quad (106)$$

$$\left.\begin{aligned}\frac{T_{1\,\max}-T_{1\,\min}}{T_0}&=\left|\frac{T_{1\,\max}-T_{1\,\min}}{T_0}\right|_{Q_2=0}\\ \frac{T_{2\,\max}-T_{2\,\min}}{T_0}&=\left|\frac{T_{2\,\max}-T_{2\,\min}}{T_0}\right|_{Q_2=0}\end{aligned}\right\} \quad (107)$$

where we used the same definition for T_0 as in (98).

Some examples of these relations are shown in Figs. 12 and 13.

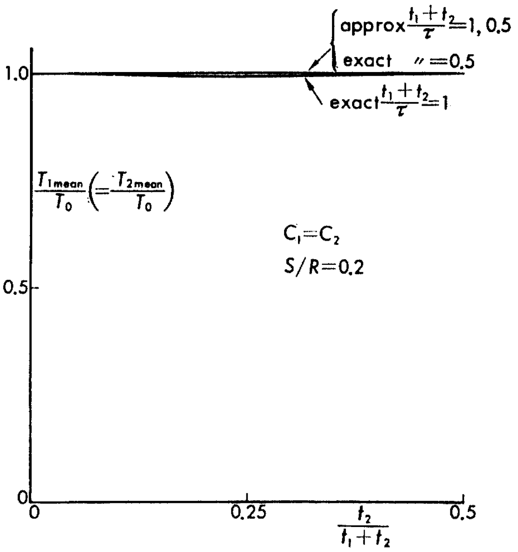


FIG. 12. The mean temperature of the inner canister system

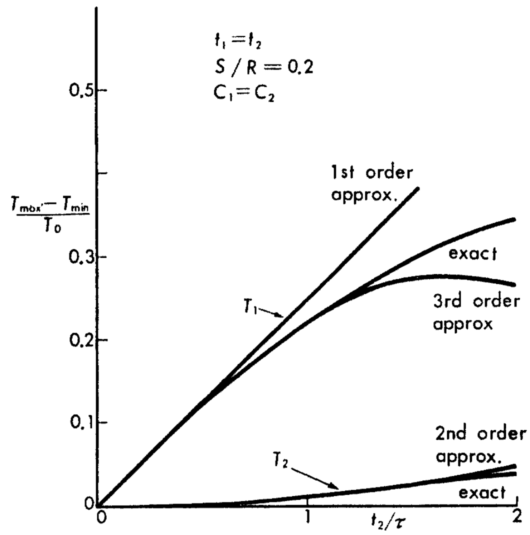


FIG. 13. The temperature variation of the inner canister system

Let us consider an example with

$T_0=280°\mathrm{K}$ $\quad Q_{11}=1\,\mathrm{kW}$ $\quad t_1=60\,\mathrm{min}$ $\quad t_2=30\,\mathrm{min}$ $\quad \tau=90\,\mathrm{min}$
$R=1\ °\mathrm{C/watt}$ $\quad C_2/C_1=10,$

then the temperature variations of the nodes 1 and 2 are plotted in Fig. 14, as the functions of S/R.

7. N-NODES SYSTEM

The general equations of the n-nodes system are expressed as, after linearization, in a system of n first order ordinary differential equations

$$C_i\frac{d\theta_i}{dt}=\frac{Q_i}{T_0}-\frac{1}{S_i}\theta_i-\sum_{j=1}^{n}\frac{1}{R_{ij}}(\theta_i-\theta_j) \quad (108)$$

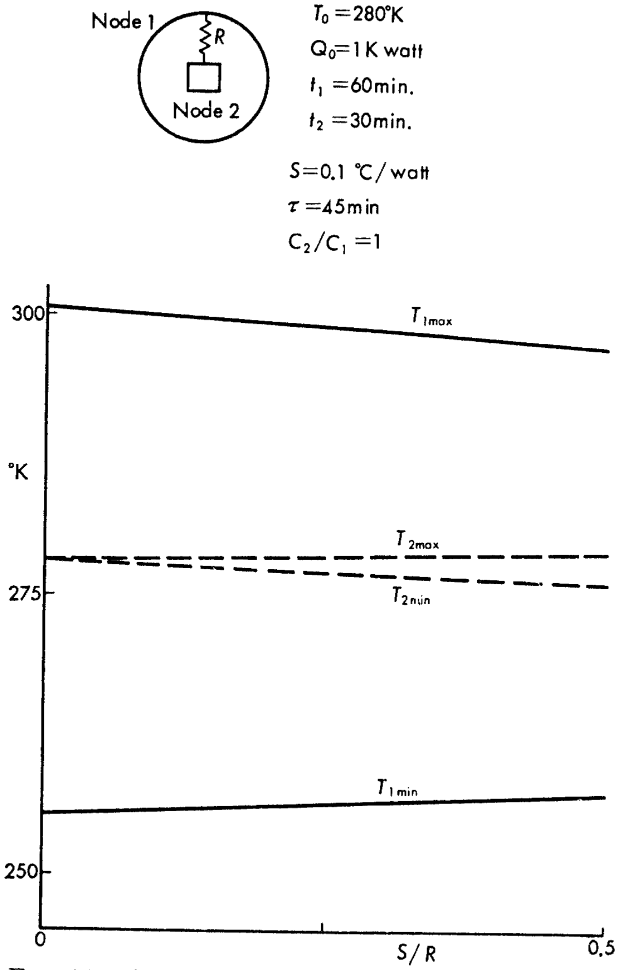


FIG. 14. An example of the inner canister system

This is one of the examples of the linear network analysis. Then it may be convenient to compare them with electric network analysis. The each quantity has the analogy to;

θ_i;	electric voltage of the node i
C_i;	electric capacity of the node i
Q_i/T_0;	electric current flowing into the node i
S_i;	electric resistance from the node i to the ground (leak resistance of the capacitor)
R_{ij};	electric resistance between the nodes i and j

Then the equations (108) say the conservation of electric current in the node i, which is one of the forms of Kirchhoff's law.

The solution of this system is written as

$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & & & \vdots \\ \vdots & & & \vdots \\ Z_{n1} & \cdots & \cdots & Z_{nn} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{pmatrix} \frac{1}{T_0} \tag{109}$$

where Z_{ij} are usually called as the impedance, and are functions of C, S, R and the heating condition (that is, t_1 and t_2 in this case). Some forms of them are already shown in the two-nodes system in the previous section. Furthermore the general theorems of the linear network analysis are also applied in this case. Some of them are

(1) *Law of Superposition*

If

$$\begin{pmatrix}\theta_{11}\\ \theta_{12}\\ \vdots\\ \theta_{1n}\end{pmatrix}=\begin{pmatrix}Z_{11}\cdots\cdots & \\ \vdots & \vdots\\ & \cdots\cdots Z_{nn}\end{pmatrix}\begin{pmatrix}Q_1\\ 0\\ \vdots\\ 0\end{pmatrix}\frac{1}{T_0},\quad \begin{pmatrix}\theta_{21}\\ \theta_{22}\\ \vdots\\ \theta_{2n}\end{pmatrix}=\begin{pmatrix}Z_{11}\cdots\cdots & \\ \vdots & \vdots\\ & \cdots\cdots Z_{nn}\end{pmatrix}\begin{pmatrix}0\\ Q_2\\ \vdots\\ 0\end{pmatrix}\frac{1}{T_0}$$

$$\cdots\cdots\cdots\cdots \quad \begin{pmatrix}\theta_{n1}\\ \theta_{n2}\\ \vdots\\ \theta_{nn}\end{pmatrix}=\begin{pmatrix}Z_{11}\cdots\cdots & \\ \vdots & \vdots\\ & \cdots\cdots Z_{nn}\end{pmatrix}\begin{pmatrix}0\\ 0\\ \vdots\\ 0\\ Q_{nn}\end{pmatrix}\frac{1}{T_0}$$

Then

$$\begin{pmatrix}\theta_1\\ \theta_2\\ \vdots\\ \theta_n\end{pmatrix}=\begin{pmatrix}\theta_{11}+\theta_{21}+\cdots+\theta_{n1}\\ \cdots\cdots\cdots\cdots\\ \cdots\cdots\cdots\cdots\\ \cdots\cdots\cdots\cdots\\ \theta_{1n}+\theta_{2n}+\cdots+\theta_{nn}\end{pmatrix}=\begin{pmatrix}Z_{11}\cdots\cdots & \\ \vdots & \vdots\\ & \cdots\cdots Z_{nn}\end{pmatrix}\begin{pmatrix}Q_1\\ Q_2\\ \vdots\\ Q_{nn}\end{pmatrix}\frac{1}{T_0} \tag{110}$$

This theorem is already used in order to find out the effect of the heat dissipation in the inner canister (106) (107). And, in the same way, this is useful to find the effect of variations of the heat input.

(2) *Reciprocity Theorem*

If

$$\begin{pmatrix}\theta_1\\ \theta_2\\ \vdots\\ \theta_n\end{pmatrix}=\begin{pmatrix}Z_{11}\cdots\cdots & \\ \vdots & \vdots\\ & \cdots\cdots Z_{nn}\end{pmatrix}\begin{pmatrix}Q_1\\ Q_2\\ \vdots\\ Q_n\end{pmatrix}\frac{1}{T_0}$$

and

$$\begin{pmatrix}\theta_1'\\ \theta_2'\\ \vdots\\ \theta_n'\end{pmatrix}=\begin{pmatrix}Z_{11}\cdots\cdots & \\ \vdots & \vdots\\ & \cdots\cdots Z_{nn}\end{pmatrix}\begin{pmatrix}Q_1'\\ Q_2'\\ \vdots\\ Q_n'\end{pmatrix}\frac{1}{T_0}$$

then

$$\theta_1 Q_1' + \theta_2 Q_2' + \cdots + \theta_n Q_n' = \theta_1' Q_1 + \theta_2' Q_2 + \cdots + \theta_n' Q_n \tag{111}$$

As a special case of this, if

$$(Q_i)_{i \neq j} = 0 \qquad (Q_i')_{i \neq k} = 0$$

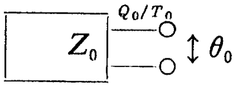
then

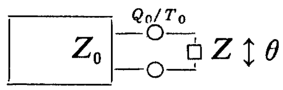
$$\theta_j Q_j' = \theta_k' Q_k \tag{112}$$

This theorem confirms the results of the symmetrical hull analysis in general form, and is useful to calculate the effect of heat input to a small appendage, one of the special cases of which was treated in 6-(2).

(3) *Thévenin's Theorem*

If





$$\theta_0 = Z_0 \frac{Q_0}{T_0}$$

then

$$\theta = (Z + Z_0) \frac{Q_0}{T_0} \tag{113}$$

This theorem is indispensable to predict the effect of small modification of the model. Especially, last-minute design change has to be evaluated using this theorem. And, based on this theorem, the influence of several canisters is analysed using the result of single canister system presented in 6-(3).

Generally say, these relations are useful to understand the general character of the system.

For the final balanced temperature with constant heat input, we have the relation

$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{pmatrix} = \begin{pmatrix} Z_{11\,0} & \cdots & \\ \vdots & & \vdots \\ & \cdots & Z_{nn0} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{pmatrix} \frac{1}{T_0} \tag{114}$$

where Z_{ij0} is the function of S and R (and t_1, t_2 but not C). This temperature is equal to the limiting mean temperature with averaged heat input $(t_1 Q_{i1} + t_2 Q_{i2})/(t_1 + t_2)$. Thus the temperature T' with KQ_1, KQ_2, $\cdots$ KQ_i, $\cdots$ is expressed as

$$\frac{T_i'}{T_i} = \left[\frac{\theta_i'}{\theta_i} \right]^{1/4} = [K]^{1/4} \tag{115}$$

and the temperature T_i is the function of only S, R and t_1, t_2, but not C.

Based on the above mentioned general theorems of the linear network system and followed the extrapolatory thoughts from the results of the two-nodes system, we can reduce the following general expressions for the system with $R \gg S$ and $t_1, t_2 \ll C_i S_i$

$$T_0 = \sum_i F_i\left(\frac{S}{R}\right) S_i Q_i \tag{116}$$

$$\frac{T_{i\,\mathrm{mean}}}{T_0} = G_{i0}\left(\frac{S}{R}\right) + G_{i2}\left(\frac{S}{R}\right)\frac{(t_1-t_2)t_2}{\tau^2} + \cdots\cdots \tag{117}$$

$$\left.\begin{aligned} \frac{T_{i\,\max}-T_{i\,\min}}{T_0} &= H_{i1}\left(\frac{S}{R}\right)\frac{t_2}{C_i S_i} + H_{i2S}\left(\frac{S}{R}\right)\frac{t_1 t_2}{(C_i S_i)^2} \\ &\quad + H_{i2R}\left(\frac{S}{R}\right)\frac{t_1 t_2}{C_i S_i C_i R_{ij}} + \cdots\cdots \end{aligned}\right\} \tag{118}$$

That is, the zeroth order term with regard to t_1 and t_2 appears in $T_{i\,\mathrm{mean}}$ which has a second order correction term. The temperature variation of the outer part of the spacecraft, on which S_i is relatively small and periodically irradiated by Sun, has the first order term $t_2/C_i S_i$. On the other hand, the temperature variation of the inner canister enclosed by the outer hull on which $(S_2=\infty)$, or of the outer hull on which no solar irradiation takes place, begins with the second order term such as $t_1 t_2/C_i R_{ij} C_j S_j$.

8. Solution by Fourier Analysis

The solution of the linearized system may be obtained by Fourier analysis. Here we treat a thermostat for space use, which consists of triple containers. Each container, called the outer, middle and inner canister, respectively, is separated by thermal insulator, as shown in Fig. 15, and three node points 1, 2 and 3 are assigned to the respective canister. The linearized equation of this system is

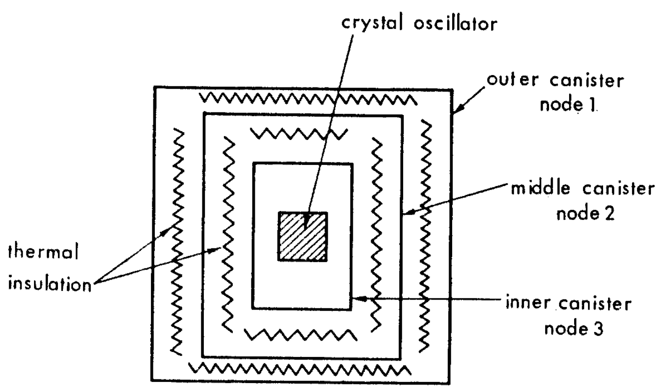


FIG. 15. Schematic diagram of a thermos

$$\left.\begin{aligned}
C_1\frac{d\theta_1}{dt}&=\frac{Q_1}{T_0}-\frac{\theta_1}{S}-\frac{1}{R_2}(\theta_1-\theta_2)\\
C_2\frac{d\theta_2}{dt}&=\frac{Q_2}{T_0}-\frac{1}{R_2}(\theta_2-\theta_1)-\frac{1}{R_3}(\theta_2-\theta_3)\\
C_3\frac{d\theta_3}{dt}&=\frac{Q_3}{T_0}-\frac{1}{R_3}(\theta_3-\theta_2)
\end{aligned}\right\}\tag{119}$$

Assuming the Fourier expansion of Q_1, Q_2 and Q_3, as

$$Q_1=\sum_{n=0}^{\infty}\bar{Q}_{1n}e^{i\omega t}$$

$$Q_2=\sum_{n=0}^{\infty}\bar{Q}_{2n}e^{i\omega t}$$

$$Q_3=\sum_{n=0}^{\infty}\bar{Q}_{3n}e^{i\omega t}$$

and let us find the solution in the forms of

$$\theta_1=\sum_{n=0}^{\infty}\bar{\theta}_{1n}e^{i\omega t}$$

$$\theta_2=\sum_{n=0}^{\infty}\bar{\theta}_{2n}e^{i\omega t}$$

$$\theta_3=\sum_{n=0}^{\infty}\bar{\theta}_{3n}e^{i\omega t}$$

The results are

$$\left.\begin{aligned}
\bar{\theta}_{1n}=&\frac{1}{\Delta}\left\{\left(jn\omega C_2+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(jn\omega C_3+\frac{1}{R_3}\right)-\frac{1}{R_3^2}\right\}\frac{\bar{Q}_{1n}}{T_0}\\
&+\frac{1}{\Delta}\left(jn\omega C_3+\frac{1}{R_3}\right)\frac{1}{R_2}\frac{\bar{Q}_{2n}}{T_0}+\frac{1}{\Delta}\frac{1}{R_2R_3}\frac{\bar{Q}_{3n}}{T_0}\\
\bar{\theta}_{2n}=&\frac{1}{\Delta}\left(jn\omega C_3+\frac{1}{R_3}\right)\frac{1}{R_2}\frac{\bar{Q}_{1n}}{T_0}+\frac{1}{\Delta}\left(jn\omega C_1+\frac{1}{S}+\frac{1}{R_2}\right)\left(jn\omega C_3+\frac{1}{R_3}\right)\frac{\bar{Q}_{2n}}{T_0}\\
&+\frac{1}{\Delta}\left(jn\omega C_1+\frac{1}{S}+\frac{1}{R_2}\right)\frac{1}{R_3}\frac{\bar{Q}_{3n}}{T_0}\\
\bar{\theta}_{3n}=&\frac{1}{\Delta}\frac{1}{R_2R_3}\frac{\bar{Q}_{1n}}{T_0}+\frac{1}{\Delta}\left(jn\omega C_1+\frac{1}{S}+\frac{1}{R_2}\right)\frac{1}{R_3}\frac{\bar{Q}_{2n}}{T_0}\\
&+\frac{1}{\Delta}\left\{\left(jn\omega C_1+\frac{1}{S}+\frac{1}{R_2}\right)\left(jn\omega C_2+\frac{1}{R_2}+\frac{1}{R_3}\right)-\frac{1}{R_2^2}\right\}\frac{\bar{Q}_{3n}}{T_0}
\end{aligned}\right\}\tag{120}$$

where

$$\Delta = -jn^3\omega^3 C_1C_2C_3 + jn\omega C_1\frac{1}{R_2R_3} + jn\omega C_2\frac{1}{R_3}\left(\frac{1}{S}+\frac{1}{R_2}\right)$$
$$+jn\omega C_3\left(\frac{1}{R_2S}+\frac{1}{R_3S}+\frac{1}{R_2R_3}\right) - n^2\omega^2 C_1C_3\left(\frac{1}{R_2}+\frac{1}{R_3}\right)$$
$$-n^2\omega^2 C_1C_2\frac{1}{R_3} - n^2\omega^2 C_2C_3\left(\frac{1}{S}+\frac{1}{R_2}\right) + \frac{1}{SR_2R_3}$$

As before, we define T_0 as the mean temperature of the node 1, then

$$\bar{\theta}_{10} = (\bar{Q}_{1n}+\bar{Q}_{2n}+\bar{Q}_{3n})\frac{S}{T_0}$$
$$=\frac{1}{4} \tag{121}$$

and

$$T_0 = (\bar{Q}_{1n}+\bar{Q}_{2n}+\bar{Q}_{3n})S \tag{122}$$

If we consider the case with the heat input Q_{11} during the time t_1 and $Q_{12}=0$ during t_2, as treated in 6-(3), then we have

$$Q_1 = \frac{t_1Q_{11}}{t_1+t_2} + \frac{Q_{11}}{\pi}\left[(1-\cos\omega t_1)\sin\omega t + \left(1-\frac{1}{2}\cos 2\omega t_1\right)\sin 2\omega t + \cdots\cdots\right]$$
$$+\frac{Q_{11}}{\pi}\left[\sin\omega t_1\cos\omega t + \frac{1}{2}\sin 2\omega t_1\cos 2\omega t + \cdots\cdots\right] \tag{123}$$

Thus the solution of this system is obtained.

This gives several useful relations as follows

$$\left.\begin{aligned}
\frac{\theta_{2n}}{\theta_{1n}} &= \frac{(jn\omega C_3R_3+1)\bar{Q}_{1n} + \left(jn\omega C_1R_2+\frac{R_2}{S_1}+1\right)(jn\omega C_3R_3+1)\bar{Q}_{2n} + \left(jn\omega C_1R_2+\frac{R_2}{S}+1\right)\bar{Q}_{3n}}{\left\{\left(jn\omega C_2R_2+\frac{R_2}{R_3}+1\right)(jn\omega C_3R_3+1)-\frac{R_2}{R_3}\right\}\bar{Q}_{1n} + (jn\omega C_3R_3+1)\bar{Q}_{2n}+\bar{Q}_{3n}} \\
\frac{\theta_{3n}}{\theta_{2n}} &= \frac{\bar{Q}_{1n} + \left(jn\omega C_1R_2+\frac{R_2}{S}+1\right)\bar{Q}_{2n} + \left(jn\omega C_1R_2+\frac{R_2}{S}+1\right)\bar{Q}_{3n}}{(jn\omega C_3R_3+1)\bar{Q}_{1n} + \left(jn\omega C_1R_2+\frac{R_2}{S}+1\right)(jn\omega C_3R_3+1)\bar{Q}_{2n} + \left\{\left(jn\omega C_1R_2+\frac{R_2}{S}+1\right)\left(jn\omega C_2R_3+\frac{R_3}{R_2}+1\right)-\frac{R_3}{R_2}\right\}\bar{Q}_{3n}}
\end{aligned}\right\} \tag{124}$$

For the case with $Q_2=Q_3=0$ and $n\omega C_3R_3\gg 1$, $n\omega C_2R_2\gg 1$, we have

$$\left.\begin{aligned}\left|\frac{\theta_{2n}}{\theta_{1n}}\right| &\approx \frac{1}{\sqrt{1+n^2\omega^2C_2^2R_2^2}} \approx \frac{t_1+t_2}{2n\pi C_2R_2}\\ \left|\frac{\theta_{3n}}{\theta_{2n}}\right| &= \frac{1}{\sqrt{1+n^2\omega^2C_3^2R_3^2}} \approx \frac{t_1+t_2}{2n\pi C_3R_3}\end{aligned}\right\} \quad (125)$$

If the higher mode terms are omitted, we have the approximate forms of

$$\left.\begin{aligned}\frac{T_{2\,\max}-T_{2\,\min}}{T_{1\,\max}-T_{1\,\min}} &= \frac{t_1+t_2}{2\pi C_2R_2}\\ \frac{T_{3\,\max}-T_{3\,\min}}{T_{2\,\max}-T_{2\,\min}} &= \frac{t_1+t_2}{2\pi C_3R_3}\end{aligned}\right\} \quad (126)$$

These are especially interesting to give the temperature compression ratio by the thermostat system.

Now let us find the effects of the heat input to the nodes 2 and 3, these are expressed as

$$\left.\begin{aligned}\frac{(\theta_2)_{Q_2,Q_3\neq 0}}{(\theta_2)_{Q_2=Q_3=0}} &= \Big[(jn\omega C_3R_3+1)\bar{Q}_{1n} + \Big(jn\omega C_1R_2+\frac{R_2}{S}+1\Big)(jn\omega C_3R_3+1)\bar{Q}_{2n}\\ &\quad + \Big(jn\omega C_1R_2+\frac{R_2}{S}+1\Big)\bar{Q}_{3n}\Big] \div [(jn\omega C_3R_3+1)\bar{Q}_{1n}]\\ \frac{(\theta_3)_{Q_2,Q_3\neq 0}}{(\theta_3)_{Q_2=Q_3=0}} &= \Big[\bar{Q}_{1n} + \Big(jn\omega C_1R_2+\frac{R_2}{S}+1\Big)\bar{Q}_{2n}\\ &\quad + \Big\{\Big(jn\omega C_1R_2+\frac{R_2}{S}+1\Big)\Big(jn\omega C_2R_3+\frac{R_3}{R_2}+1\Big)-\frac{R_3}{R_2}\Big\}\bar{Q}_{3n}\Big] \div \bar{Q}_{1n}\end{aligned}\right\} \quad (127)$$

These relations are used for the analysis of the effect of surface contamination which causes some heat input to the container surfaces. For such practical cases, the following assumptions may hold;

$$\begin{aligned}&Q_2 \ll Q_1, \quad Q_3 \ll Q_1\\ &n\omega C_2R_2 \gg 1 \quad n\omega C_3R_3 \gg 1\\ &R_2 \approx R_3 > S\end{aligned}$$

Because the phase relations of Q_2 and Q_3 with respect to Q_1 are not known, the only thing we can predict is the maximum possible temperature variations due to Q_2 and Q_3, which are

$$\begin{aligned}\frac{(T_{2\,\max}-T_{2\,\min})_{Q_2,Q_3\neq 0}}{(T_{2\,\max}-T_{2\,\min})_{Q_2=Q_3=0}} &= \frac{(\theta_2)_{Q_2,Q_3=0}}{(\theta_2)_{Q_2=Q_3=0}}\\ &\approx 1+j\omega C_1R_2\frac{Q_2}{Q_1}+\frac{C_2R_2}{C_3R_3}\frac{Q_3}{Q_1}\end{aligned}$$

$$\left.\begin{aligned} &\approx \left(\frac{2\pi C_1 R_2}{t_1+t_2}\right)\frac{Q_2}{Q_1} \\ \frac{(T_{3\,\max}-T_{3\,\min})_{Q_2,\,Q_3\neq 0}}{(T_{3\,\max}-T_{3\,\min})_{Q_2=Q_3=0}} &= \frac{(\theta_3)_{Q_2,\,Q_3=0}}{(\theta_3)_{Q_2=Q_3=0}} \\ &\approx 1+j\omega C_1 R_2\frac{Q_2}{Q_1}-\omega^2 C_1 R_2 C_2 R_3\frac{Q_3}{Q_1} \\ &\approx \left(\frac{2\pi}{t_1+t_2}\right)^2 C_1 R_2 C_2 R_3\frac{Q_3}{Q_1} \end{aligned}\right\} \quad (128)$$

Using these relations, we can guess the values of Q_2/Q_1 and Q_3/Q_1 from the measured temperature variations.

9. Experimental Analysis

Keeping in mind the general expressions of the node's temperature (117) (118), let us find the process to analyse the over-all thermal character of a spacecraft using minimum simulator time. Here we assume that the values of Q_i are known and the provisions to supply the respective heat energy to the every node point using solar simulator or suitable electric heater are ready.

First of all, a thermal model of the spacecraft will be constructed, which must have

(1) the same values of S and R

(2) the smaller values of τ.

Therefore it is most convenient in the practical point of view;

(1) to use the same main structural frame

(2) to replace the outer hull by plates with the same transverse heat conductivity and with the low heat capacity,

(3) to simulate the inner canister by a dammy box with the same size but small heat capacity, and

(4) to assemble them so as to have the same values of R as the real spacecraft.

Then this model is put in a suitable simulator and tested under the simulated environmental conditions. We assume here that the data obtained are sufficiently reliable. Firstly, the cyclic heating and cooling with relatively short time period (say 15 minute) are applied and it continues until the quasi-equilibrium state is obtained. From this data, the zeroth order term of the mean temperature and the first order term of the temperature variation are obtained. In the most cases of the real spacecraft design, these data obtained by the carefully constructed thermal model are sufficient to predict the thermal character of the spacecraft. However, if necessary, another cyclic heating experiment with a longer period is repeated and the data obtained will give the higher order correction terms. These measurements are repeated to the each heat input conditions with different proportion to each other.

As an example of this method, an experimental result of a thermal model is

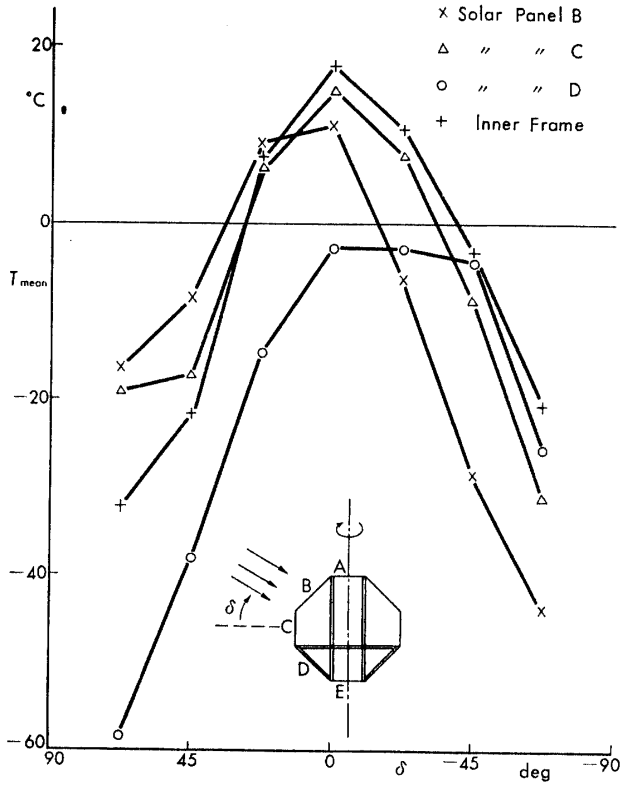


FIG. 16. The mean temperature of a thermal model

presented. This thermal model, schematically shown in Fig. 16, is constructed using the same structural frame but with copper surface panels painted black and with dammy loads. Thus this model has the same R and S as the real spacecraft but with one quarter of C of the real one on the nodes of the outer panel and more smaller fractions of C on the nodes of the inner canister. The simulator test was carried out using the simulated solar light with 0.16 watt/cm^2 strength for cyclic heating and cooling of 15 min. each. Fig. 16 shows the values of the zeroth order mean temperature (T_{i1}) of the model with respect to the sunlit angle, and Fig. 17 gives the data of the first order temperature variations.

From these data, for example, we can derive the mean temperature of the panel C under the sunlit angle 34° and the strength of 0.14 watt/cm^2 with 90 min. orbital period and 30 min. ecliptic time as follows

(1) read out the temperature 0°C from Fig. 16

(2)
$$(0+273)\times\left[\frac{0.14\times\dfrac{60}{90}}{0.16\times\dfrac{15}{30}}\right]^{1/4}-273=11.$$

The temperature variation of this panel is

(1) read out the values of 63°C in Fig. 17

(2)
$$63\times\frac{30}{15}\times\frac{1}{4}=22$$

The mean temperature of the frame is calculated by the same process as the panel

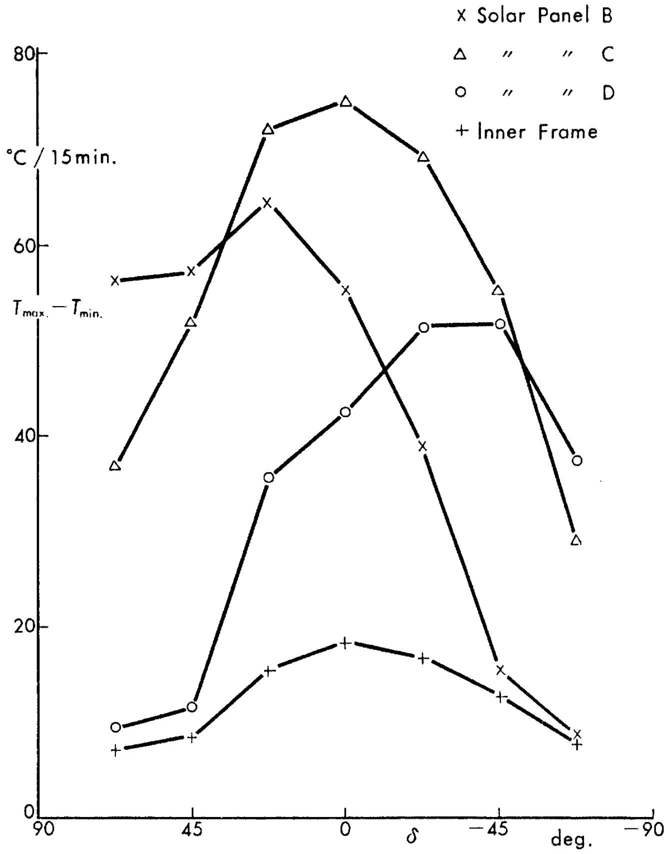


FIG. 17. The temperature variation of a thermal model

temperature, which happen to be the same as the one of the panel C. The temperature variation is

(1) read out the value of 10° in Fig. 17

(2) $$10\times\frac{30\times 60}{15\times 15}\times\frac{1}{4}=20$$

10. CONCLUSION

Here, the procedure of the thermal analysis of a spacecraft is summarized. Firstly, the suitable nodes are assigned on the points of the spacecraft where the large values of S (or R) and CS (or CR) exist. On the other words, two points which are connected with small R or have small τ are not necessary to be distinguished. Secondly, the thermal model which has the same R and S but small τ is constructed and tested in space simulator, finding out the quasi-equilibrium state under a cyclic heating and cooling condition with a short period. This test will give the lowest order term of the mean temperature and the temperature variation. Thirdly, further simulation testing with longer cyclic period is repeated, if necessary, and the higher order correction terms are found. Forthly, the temperatures at various orbital conditions are reduced from these data. And at later date, the additional predictions of the temperature change due to the design modification of the satellite, if any, are derived using the general theorems of the linear network.

This method has been used for several satellite designs, and proved to be useful by many simulation testings, though any actual flight does not take place yet.

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References

[1] G. B. Heller ed.: Thermophysics and temperature control of spacecrafts and entry vehicles: Academic Press. N.Y. (1966)
[2] Proc. 15th International Astronautical Congress (1964)
[3] Proc. 16th International Astronautical Congress (1965)
[4] G. P. Wood & A. F. Carter: Predicted characteristics of an inflatable aluminized plastic spherical earth satellite with regard to temperature, visibility, reflection of radar waves and protection from ultraviolet radiation. NASA TN D-115 (1959)
[5] G. Heller & B. P. Jones: Thermal properties of Explorer VII. NASA TN D-608 (1963)
[6] P. Hrycak: Thermal design of Telstar. Asrto. Aero. Eng., Dec. (1963) p. 44–49
[7] L. C. Clark & K. A. Laband: Orbital station temperature control. Astronautics Sept. (1962) p. 40–43
[8] C. V. Woerner & G. M. Keating: Temperature control of the Explorer IX satellite. NASA TN D-1369 (1962)
[9] L. C. Clark: Temperature balance of manned space stations. NASA TN D-1504 (1963) p. 21–31
[10] L. H. Hammerdinger: Thermal design of the orbiting astronautical observatory. J. Spacecraft vol. 1 (1964) p. 477–483
[11] L. Testaguzza, G. L. Wolfe & J. DiGiorgio: Flight verification of satellite thermodynamic analytical techniques and application to manned spacecraft. J. Spacecraft vol. 2 (1965) p. 537–542
[12] F. A. Carr: Flight report—Interplanetary monitoring platform IMP-I—Explorer XVIII. NASA TN D-3352 (1966)
[13] T. C. Bannister & R. J. Eby: Pegasus thermal design. NASA TN D-3625 (1966)
[14] R. Linton: Thermal design evaluation of Pegasus. NASA TN D-3642 (1966)
[15] P. Hrycak: Effect of secondary radiation on an orbiting satellite. ARS Journal (1962) p. 1924–1925
[16] F. G. Cunningham: Earth reflected solar radiation input to spherical satellites. NASA TN D-1099 (1961)
[17] F. G. Cunningham: Calculation of the eclipse factor for elliptical satellite orbits. NASA TN D-1347 (1962)
[18] F. G. Cunningham: Earth-reflected solar radiation input upon spherical satellite in general elliptical orbits. NASA TN D-1472 (1963)
[19] J. P. Millard & C. B. Neel: Measurements of albedo and earth radiation from OSO-1. AIAA Journal vol. 3 (1965) p. 1317–1329
[20] G. D. Gordon: Measurement of ratio of absorptivity of sunlight to thermal emissivity. Rev. Sci. Inst. vol. 31 (1960) p. 1204–1208
[21] W. B. Fussell, J. J. Triolo & J. H. Henninger: A dynamic thermal vacuum technique for measuring the solar absorptance and thermal emittance of spacecraft coatings. NASA TN D-1716 (1963)
[22] M. G. Hoke: A thermal vacuum technique for measuring the solar absorptance of satellite coatings as a function of angle of incidence. AIAA Journal Vol. 3 (1965)

p. 947–951
[23] R. R. Hibbard: Method for estimating ratio of absorptance to emittance. NASA TN D-2751 (1965)
[24] R. A. Perkins: Status of coated refractory metals. J. Spacecraft vol. 2, p. 520–523
[25] R. A. Bruech, N. J. Douglas & D. Vance: Effects of electron bomberdment on the optical properties of spacecraft temperature control coatings. AIAA Journal vol. 3, p. 2318–2327
[26] K. Schocken: Optical Properties of satellite materials—the theory of optical and infrared properties of metals. NASA TN D-1523 (1963)
[27] H. B. Curtis: Measurement of emittance and absorptance of selected material between 280 and 600°K. J. Spacecraft vol. 3 (1966) p. 383–387
[28] J. H. Diedrich & H. B. Curtis: Experimental investigation of total emittance and solar absorptance of several coatings between 300 and 575°K. NASA TN D-3381 (1966)
[29] H. Herzig: Uniform vacuum ultraviolet reflecting coatings on large surfaces. NASA TN D-3357 (1966)
[30] C. W. Stroud: Experimental verification of an analytical determination of overall thermal conductivity of honeycomb-core panel. NASA TN D-2866 (1965)
[31] J. R. Branstetter: Formulas for radiant heat transfer between nongrey parallel plates of polished refractory metals. NASA TN D-2902 (1965)
[32] A. Charnes & S. Raynor: Solar heating of a rotating cylindrical space vehicle. ARS Journal (1960) p. 479–484
[33] L. D. Nicols: Surface-temperature distribution on thin-walled bodies subjected to solar radiation in interplanetary space. NASA TN D-584 (1961)
[34] W. E. Olmstead & S. Raynor: Solar heating of a rotating spherical space vehicle. Int. J. Heat Mass Trans. vol. 5 (1962) p. 1165–1177
[35] L. A. Glenn & S. DeSoto: An analogue method for the solution of unsteady radiant heat-transfer problems with combined conduction and convection. J. Spacecraft vol. 3 (1966) p. 224–229
[36] F. J. Cepollina: Use of analogue computation in predicting dynamic temperature excursions of orbiting spacecraft. NASA TN D-3712 (1967)
[37] D. Berman: Satellite thermal simulation by a hybrid computor. J. Spacecraft vol. 4 (1967) p. 243–248
[38] E. W. Adams & H. Spreuer: Theoretical solution of the nonlinear problem of transient cooling of an opaque sphere in space. NASA TN D-4046 (1967)
[39] R. T. Hollingworth: A survey of large space chambers. NASA TN D-1673 (1963)
[40] M. P. Hnilicka & K. A. Geiger: Simulating interplanetary space. Astro. Aero. Eng. July (1963) p. 31–35
[41] B. H. Goethert & H. M. Cook: High ground for aerospace simulation. Astro. Aero. Nov. (1964) p. 96–103
[42] J. E. A. John & W. F. Hardgrove: Design of an ultrahigh-vacuum space simulator. J. Spacecraft vol. 3 (1966) p. 427–428
[43] J. P. Dawson: Prediction of cryopumping speeds in space simulation chambers. J. Spacecraft vol. 3 (1966) p. 218–223
[44] J. T. Gedargren, R. E. Harrington & M. R. Riek: Recent advances in the adaptation of carbon arcs to solar simulation. J. Spacecraft vol. 3 (1966) p. 230–234
[45] J. W. Anderson, E. A. LaBlanc & H. Cohan: Experimental and analytical assessment of space thermal and vacuum environment simulation requirement. J. Spacecraft vol. 3 (1966) p. 976–982
[46] J. C. New & A. R. Timmins: Effectiveness of environmental simulation testing for satellite. J. Spacecraft vol. 3 (1966) p. 1711–1715
[47] S. Katzoff: Simulation in thermal models of spacecraft. NASA TN D-1631 (1963)
[48] B. P. Jones: Thermal similitude studies. J. Spacecraft vol. 1 (1964) p. 364–369
[49] B. T. Chao & G. L. Wedekind: Similarity criteria for thermal modeling of spacecraft. J. Spacecraft vol. 2 (1965) p. 146–152
[50] J. M. F. Vickers: Thermal scale modeling. Aero. Astro. (1965) p. 34–39