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Hydromagnetic Coupling Oscillations in Non-uniform, Collisionless Plasmas

By

Tsutomu TAMAO

Summary: The main object in the present paper is to study the thermal effects on resonant coupling hydromagnetic oscillations in inhomogenuous, low-β plasmas, such as the earth magnetosphere. Applying the drift kinetic approximation the fundamental equations for the coupling modes between the Alfvén and magnetosonic oscillations in a non-uniform, collisionless plasma are obtained. There are two thermal effects for such oscillations, the thermal dissipation due to wave-particle resonance interactions and the drift effect from plasma inhomogeneities. For the outer magnetospheric conditions, where $\beta \simeq m_e/m_i$, it is shown that electron thermal dissipation of hydromagnetic oscillations with $\omega/k_{\parallel} \simeq V_A$ is most effective within the resonance regions which appear in the cold plasma limit. This effect is proportional to ω/Ω_i for the case of continuous distribution of plasma parameters and $(\omega/\Omega_i)^2$ for the stratified distribution, respectively, where Ω_i is the ion gyrofrequency. Attenuation of hydromagnetic waves due to ion-wave interaction may be a possible process for ion heating in the plasma sheet of the geomagnetic tail and relevant equations are also derived. With a view to applying the results to the magnetospheric plasma, drift wave instabilities in long wave length limit are discussed for the most general case, including simultaneously the density, temperature and magnetic field gradients, as well as the ion Larmor radius effect.

LIST OF NOTATIONS

A: defined by (27) or (44),
B: defined by (27),
$\boldsymbol{B}=\boldsymbol{B}_0+\delta\boldsymbol{B}$,
$\boldsymbol{B}_0$: ambient magnetic field,
$\delta\boldsymbol{B}$: pertrubation magnetic field,
c: velocity of light,
C, D: defined by (27),
e: electric charge in e.m.u.
$\boldsymbol{E}$: perturbation electric field,
$\boldsymbol{E}_{\parallel}$: parallel component of electric field,
$\boldsymbol{E}_{\perp}$: perpendicular component of electric field,
$f_0=(\omega/V_A)^2-\eta k_{\parallel}^2$,
$f_m=(\omega/V_A)^2-\eta k_{\parallel}^2-(m/r)^2$,

$\bar{f}_m = (\omega / V_A)^2 - \eta k_\parallel^2 - (m/r)^2(1+\beta_\perp C)$,
$F = F_0 + \delta F$: distribution function,
F_{0j}: initial particle distribution function, $(j=e, i)$,
$\delta \boldsymbol{J}_\perp = \delta \boldsymbol{J}_D + \delta \boldsymbol{J}_M$: perpendicular component of perturbation current,
$\delta \boldsymbol{J}_D$: drift current, defined by (8),
$\delta \boldsymbol{J}_M$: diamagnetic current, defined by (9),
$\boldsymbol{k}$: wave number vector,
K: defined by (49),
$L_\perp$: typical scale length of plasma inhomogenuities,
$l_\perp$: typical scale length of perturbations,
m: azimuthal mode number $(m=0, \pm 1, \cdots)$,
m_j: mass of particles, $(j=e, i)$,
$\boldsymbol{n} = \boldsymbol{n}_0 + \delta \boldsymbol{n}$: unit vector in the direction of magnetic field,
$\boldsymbol{n}_0 = \boldsymbol{B}_0 / B_0$,
$\delta \boldsymbol{n}_0 \simeq \delta \boldsymbol{B} / B_0$,
$N_j = N_0 + \delta N_j$: particle number density $(j=e, i)$,
N_0: initial particle density,
$p_\parallel$: parallel gas pressure,
$p_\perp$: perpendicular gas pressure,
$p(r)$: defined by (30),
q_j: charge of particles $(j=e, i)$,
$q(r)$: defined by (31),
Q_1: defined by (37),
Q_2: defined by (40),
(r, φ, z): cylindrical coordinates,
$\boldsymbol{r}_j$: Lamor radius of particles $(j=e, i)$,
R_E: earth's radius,
T_j: temperature $(j=e, i)$,
$\boldsymbol{u}$: velocity of guiding center of a particle motion,
U: defined by (18),
U_i: defined by (43),
$\boldsymbol{v}$: particle velocity,
$\boldsymbol{v}_d$: perturbation drift velocity of a particle,
$\boldsymbol{v}_j$: thermal velocity of particles $(j=e, i)$,
$\boldsymbol{V}_d$: unperturbed drift velocity of a particle,
$\boldsymbol{V}_{Dj}$: drift velocity of particles $(j=e, i)$,
$V_A = B_0(4\pi N_0 m_i)^{-1/2}$: Alfvem speed,
$V_S = (T_e / m_i)^{1/2}$: ion sound speed,
V_D: defined by (22),
$V_{Bj} + V_{Nj} = (T_j / m_j \Omega_j)[(B_0'/B_0) - (N_0'/N_0)]$, $(j=e, i)$,
$V_{Tj} = (T_j / 2m_j \Omega_j)(T_j'/T_j)$, $(j=e, i)$,
$\boldsymbol{w}$: gyration velocity of a particle,
(x, y, z): cartesian coordinates,
$\alpha_\parallel = (T_i / T_e)_\parallel$,

$\alpha_\perp=(T_i/T_e)_\perp$,
$\beta_\perp=8\pi N_0 T_\perp/B_0^2$,
$\gamma=m_e/m_i$,
$\epsilon=\omega/\Omega_i$,
$\zeta_j=\omega/k_\parallel v_j$, $(j=e, i)$,
η_0: kinematic viscosity,
$\eta=1+(\beta_\perp/2)(1+\alpha_\perp)-(\beta_\parallel/2)(1+\alpha_\parallel)$,
$\mu=mw^2/2B$: magnetic moment of a particle,
μ^2: square of refractive index, see (33),
ν: inter-particles collision frequency,
$\xi=k_y U_i/kV_A$,
σ: electric conductivity,
τ_{ij}: collisional time,
$\tau_{tr}^{(j)}$: transit time of a particle along the field line,
φ: azimuthal angle in cylindrical coordinates,
ϕ: defined in (44),
$\Phi=\mathrm{div}\,\boldsymbol{E}_\perp$,
$\Psi=\mathrm{rot}_\parallel\boldsymbol{E}$,
ω: angular frequency of perturbations,
$\omega_{pe}=(4\pi N_0 e^2 c^2/m_e)^{1/2}$,
Ω_j: Larmor frequency of a particle, $(j=e, i)$,
$\mathbf{1}_z$: unit vector in z-direction,
$\mathbf{1}_r$: unit vector in r-direction,
$\parallel$: parallel direction to the magnetic field line,
$\perp$: perpendicular direction to the magnetic field line,
(e, i): for electrons or ions.

1. Introduction

With respect to geomagnetic pulsations there have been several investigations on eigen-oscillations and propagation of hydromagnetic waves in the non-uniform, cold magnetosphere [*1*]. It was shown from these studies under the macroscopic one-fluid approximation that axisymmetric perturbations are separated into the Alfvén (or transverse) and fast magnetosonic (or isotropic) modes [*2*]. The former corresponds to one-dimensional torsional oscillations of individual field lines, while the latter represents two-dimensional compressional oscillations in the meridional plane. In the uniform cold plasma limit, separation into two modes is also possible even for non-axisymmetric perturbations [*2*]. One is the transverse mode which carries $\mathrm{div}\,\boldsymbol{E}_\perp$ one-dimensionally along the ambient field line, the other is a carrier of $\mathrm{rot}_\parallel\boldsymbol{E}$ and propagates isotropically. $\boldsymbol{E}$ is the perturbation electric field, and subscripts $\parallel$ and $\perp$ denote the parallel and perpendicular directions to the magnetic field line.

In the present paper we consider only the case wherein the plasma inhomogeneity occurs in radial direction and the straight field line of the ambient magnetic field,

$\boldsymbol{B}_0$, coincides with z-direction in Cylindrical coordinates (r, φ, z).

Introducing the space inhomogeneity of the Alfvén speed, $V_A(r)=B_0(4\pi N_0 m_i)^{-1/2}$, where B_0 is the ambient magnetic field intensity, N_0 the plasma density and m_i the ion mass, respectively, we have the coupling mode of hydromagnetic waves between the local transverse and isotropic ones. Representing the disturbance in terms of normal modes of

$$R(r)\exp[i(k_{\parallel}z+m\varphi-\omega t)]$$

the resulting equation for the electric field component of coupling mode becomes [3]

$$\frac{d^2E_\varphi}{dr^2}+\frac{f_m}{f_0}\frac{d}{dr}\left(\frac{f_0}{f_m}\right)\frac{dE_\varphi}{dr}+f_mE_\varphi=0, \tag{1}$$

where

$$\begin{cases} f_0=(\omega/V_A)^2-k_{\parallel}^2 \\ f_m=(\omega/V_A)^2-k_{\parallel}^2-(m/r)^2. \end{cases} \tag{2}$$

In derivation of the above equation we neglect both effects of electron inertia and Hall current. The former is important only in short wave length such as $k_\perp c/\omega_{pe} \gtrsim 1$ and the latter for high frequency, $\omega \gtrsim \Omega_i$, where ω_{pe} is the electron plasma frequency, Ω_i, the ion Larmor frequency and c the velocity of light. From (1) we see that roots of $f_0=0$ or $f_m=0$ are both singular points and the intensity of perturbations becomes to be infinity due to coupling resonance between the local transverse and isotropic modes. For axisymmetric perturbations ($m=0$), no singularity occurs and the characteristic equation corresponding to (1) separates into the following two equations, $f_0E_r=0$ and $d^2E_\varphi/dr^2+f_0E_\varphi=0$, respectively.

What higher order correction must be introduced within the resonance region, in order to overcome the coupling resonance in the inhomogeneous cold plasma. There are following three possibilities.

1) In collision-dominated plasma, Ohmic and viscous attenuation due to inter-particle collisions will be effective. It is known that the relevant viscous coefficient for dissipation of the compressional mode of hydromannetic waves is not the reduced viscosity in a strong magnetic field, but the same order as with no field [4], η_0. This viscous dissipations is equivalent to the so-called gyrorelaxation damping [5]. The relative importance between Ohmic and viscous attenuation depends on $(4\pi\sigma\eta_0)^{1/2}$, where σ is the electrical conductivity in e.m.u. and η_0 the kinematic viscosity. When $(4\pi\sigma\eta_0)^{1/2}>1$, the viscous dissipation is more important [6]. If the temperature of a plasma is of the order of $T_e\simeq 0.1eV$, the above condition can be satisfied for $N_0<10^7\,\mathrm{cm}^{-3}$. Thus, for the inner magnetosphere where $T_e\simeq 0.1\sim 1\ eV$ and $N_0\simeq 10^3\sim 10^2\,\mathrm{cm}^{-3}$, the dominant collisional dissipation is viscous attenuation. The magnetospheric plasma density and geomagnetic field intensity decrease with increase of radial distances, while the temperature of thermal plasma will increase. Therefore, the collisional contribution will become less importance with radial distances. Referring to the observational results [7]

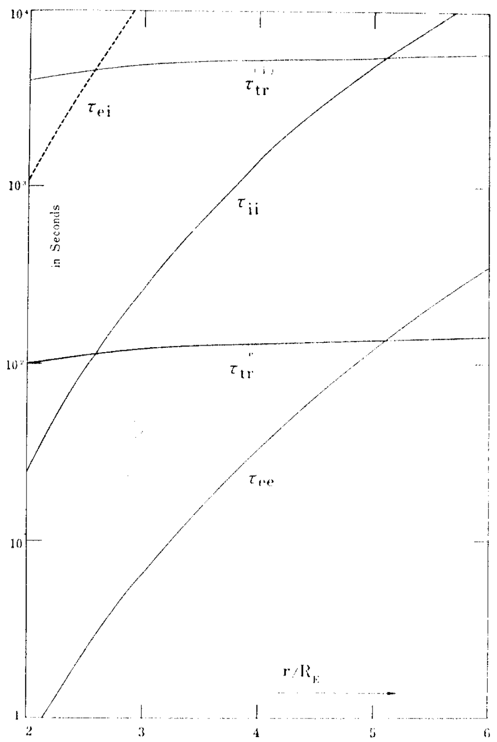

FIG. 1. Particle collision time, τ_{ij}, and transit time of particles $\tau_{tr}^{(j)}$, over the length of a field line in a model magnetosphere versus geocentric distances, r/R_E.

with IMP-satellite, collision time, τ_{ij}, between i and j species and transit time of particles, τ_{tr}, over the length of a field line with their thermal velocity are shown in Fig. 1. If we use a criterion of $\tau_{coll} < \tau_{tr}$ as a term of collision-dominated, the inner magnetosphere within about 5 earth radii may be the viscous dissipating region, and the outer magnetosphere beyond this distance can be regarded effectively as a collision-free plasma. It is also well believed that the thermal plasma density of the magnetosphere decreases suddenly by amounts of 10～100 at the density knee, [*8, 9*] which is now called plasmapause. Observed radial distances of the plasmapause is about 4～6 earth radii [*8, 10*] and its thickness is of the order of 1000 km. Whether there is a sharp transition of temperature increase corresponding to density decrease or not, furthermore if exist, which coincides with the density knee, are notclear at present. Results of several observations compiled by A'lpert [*11*] are given in Fig. 2 which show the temperature of 1～10 eV beyond the plasmapause. Fig. 3 shows radial variations of electron thermal speed, v_e and Alfvén speed V_A which calculated from the same distribu tions of the density and temperature used in Fig. 1.

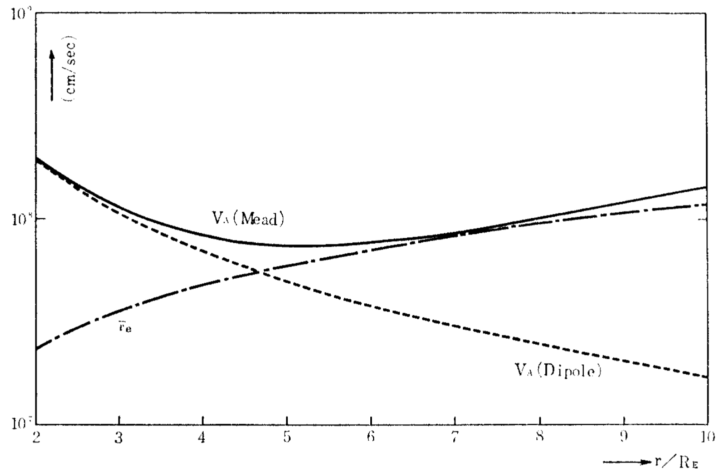

FIG. 2. Variations of the Alfvén speed, V_A and electron thermal velocity, $\bar{v}_e$ with radial distances, r/R_E.

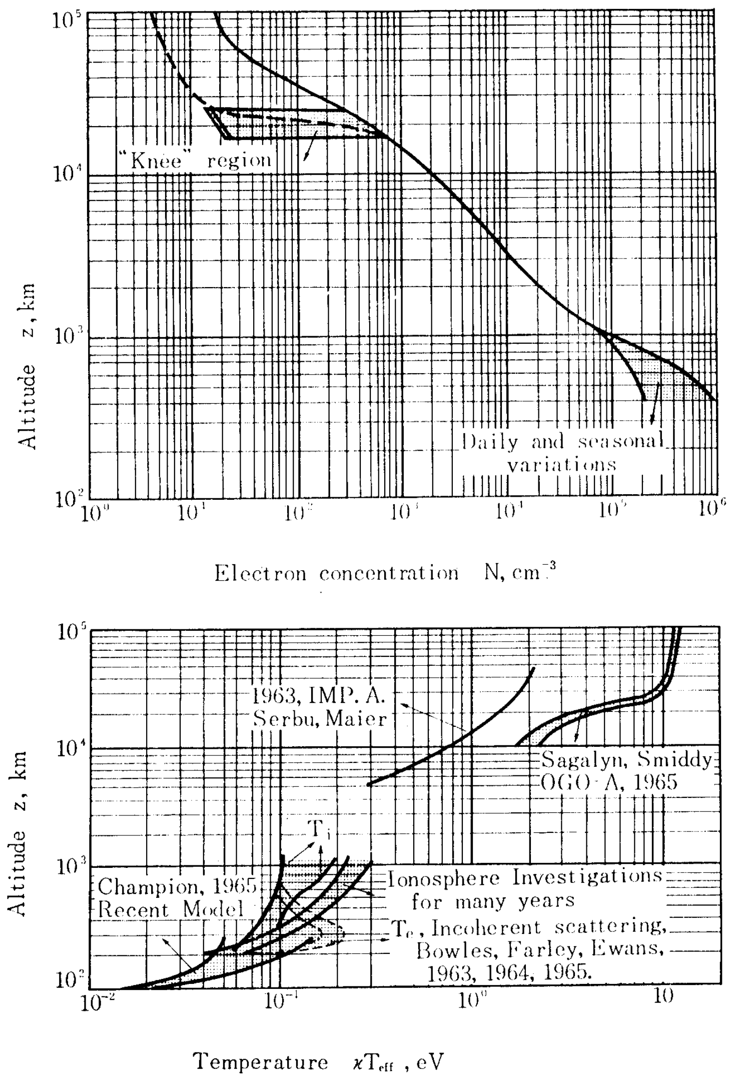

FIG. 3. Observational results of distributions of the thermal electron density and plasma temperature in the ionosphere and magnetosphere (after, Ja, L. Al'pert).

Summarizing the thermal plasma character in the magnetosphere mentioned above, we have

i) within plasmapause (inner magnetosphere); collision-dominated plasma, extremely cold, $\beta \ll m_e/m_i$.

ii) beyond plasmapause (outer magnetosphere); collision-free plasma, $v_e \gtrsim V_A$ which is equivalent to $\beta \gtrsim m_e/m_i$.

A trough of V_A near the plasmapause is useful as an energy trapping region of hydromagnetic oscillations. Although the main object of the present paper is to study the thermal effect on hydromagnetic oscillations in the inhomogeneous collision-free plasma, the effect of inter-particle collisions can be taken account by substitution of BGK collision term [*12*] into the perturbation kinetic equation, if necessary.

2) The important dissipation process of waves in a collisionless plasma is thermal dissipation resulting from wave particle resonance interactions. For hydromagnetic frequency range, $\omega \ll \Omega_i$, Cerenkov interaction with thermal particles which satisfies a resonance condition of $\omega - k_{\parallel} v_{th} = 0$ is most effective. Collisionless damping of hydromagnetic waves consists of two parts, Landau damping and transit-time damping [5, 13, 14]. The former comes from the parallel electric field perturbation and the latter the parallel magnetic force. Both damping process of hydromagnetic waves lead to acceleration of thermal particles along the ambient magnetic field line, and then particle precipitation from a trapped region. The associated perpendicular component of perturbation electric field also causes plasma diffusion across the ambient field line. These two effects, heating and diffusion due to hydromagnetic waves, may have appreciable contribution for the thermal particle distribution and energy transfer in the outer magnetosphere [15]. Under the quasi-linear approximation the rate of acceleration and diffusion coefficient are proportional to electric field energy of perturbations, which depends on the thermal dissipation of waves and a degree of the inhomogeneity of a plasma.

Cerenkov resonance interaction between thermal particles and hydromagnetic waves strongly depends on β. Thermal electrons may be important in the outer magnetosphere beyond the knee, since $\beta \gtrsim m_e/m_i$ in that region as was mentioned above. On the other hand, for the plasma sheet in the geomagnetic tail where particles with energy of 1 kev range are dominant constitution and $\beta \simeq 1$ [16, 17], collisionless dissipation of hydromagnetic waves due to interaction with thermal ions would be primarily important, unless there is any instability source such as a drift current or pressure anisotropy. If there is such a source, the ion- wave interaction does not lead to wave dissipation but to wave instability during the linear stage. At the non-linear stage, however, this interaction would be also useful for ion thermalization.

3) The third possibility of the higher order correction in the resonance region is the generation of secondary wave resulting from the finite pressure. With the one-fluid approximation, this wave corresponds to the slow mode of magnetosonic waves. However, consideration of this mode in the resonance region under the one-fluid picture cannot disappear the singularity but lead a shifting of the resonance point to the new point where f_0 or f_m is of the order of $\beta \sim (V_s/V_A)^2$ in the previously assumed cold plasma approximation. On the other hand, drift motions of particles in a collision-free inhomogeneous plasma modify the Alfvén and slow modes of pure hydromagnetic waves to the fast and slow modes of drift waves [18, 19], which have a strong anisotropy in their wave length between parallel and perpendicular directions to the ambient field in low-β plasma. Since we have $\omega \sim k_{\perp} V_D$ for drift waves where V_D is the drift speed of thermal particles due to the plasma inhomogeneity, coupling of drift waves with the primary hydromagnetic waves of $\omega \sim k_{\parallel} V_A$ can be expected within the resonance region if $k_{\perp}/k_{\parallel} \simeq V_A/V_D \gg 1$. Thus, the secondary generation of drift waves may have a important roll in the narrow region near the singular point, if the plasma parameters in this localized region

satisfy the condition of $v_i \ll \omega / k_{\parallel} \ll v_e$ under which Landau damping of waves can be neglected.

In order to study the thermal corrections described above, namely thermal dissipation and drift waves, the drift-kinetic approximation [18, 20] is a useful method for the low-frequency oscillations. Within an accuracy of the first order of $\epsilon = \omega / \Omega_i$ for the drift velocity of thermal particles, the perpendicular component of electric current and perturbation space charge density can be experessed in terms of perturbation electric field components from the drift-kinetic approximation. Substituting these source terms into Maxwell's equations we abtain fundamental equations for the perturbation electric field in a collisionless inhomogeneous plasma, which are coupled equations for $E_{\parallel}$, $\Phi = \mathrm{div}\,\boldsymbol{E}_{\perp}$ and $\Psi = \mathrm{rot}_{\parallel} \boldsymbol{E}_{\perp}$. Elimination of $E_{\parallel}$ leads, in general, to the fourth order differntial equation with respect to radial distance. In particular, for perturbations with $\omega \gg mV_D / r$ the effect of drift waves may be less important. Neglecting higher order terms of ϵ^2 or more, we have a second order differential equation of E_{φ} which is appropriate for the study of coupling resonance of hydromagnetic oscillations, and also reduced to (1) in the cold plasma limit. Based on this equation we can discuss qualitatively the effect of thermal dissipation of the non-axisymmetric hydromagnetic oscillations, especially in the coupling resonance region in the outer magnetosphere. In section 4 the coupled equations will be derived which valid for a non-uniform, hot plasma such as the geomagnetic tail. In this case no singular point such as $f_0 = 0$ or $f_m = 0$ appears, but due to a condition of $\beta \simeq 1$, appreciable attenuation from ion-wave interactions may be expected. A possibility of drift wave instabilities in the magnetosphere will be considered section 5 where the drift motion of particles is represented by the most general form including space derivatives of the plasma density, temperature and the magnetic field intensity, and also takes account of the so-called finite Larmor radius effect [21, 22]. Using the results developed in this paper, quantitative discussions concerning characters of hydromagnetic oscillations and the plasma heating in the magnetosphere will be possible if the magnetospheric plasma parameters, which are so definite that numerical calculations have a sufficient meaning, can be available.

2. Drift Kinetic Approximation

Let us consider the drift motion of a charged particle under the influence of electromagnetic fields, $\boldsymbol{E}$ and $\boldsymbol{B} = \boldsymbol{B}_0 + \delta \boldsymbol{B}$. Separating the particle velocity into $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w}$, where $\boldsymbol{u}$ is a velocity of the guiding center and $\boldsymbol{w}$ a velocity of gyration, the equation of the guiding center becomes in first order of ω / Ω [21]

$$m d\boldsymbol{u} / dt = q(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) - \mu \nabla \boldsymbol{B}. \tag{3}^*$$

$\mu = m w_2 / 2B$ is a magnetic moment. Furthermore, we assume $\boldsymbol{u} = \boldsymbol{V}_d + \boldsymbol{v}_d$ and

* Electromagnetic fields must be evaluated at the instantaneous guiding center of drift motion.

$u_{\|}=v_{\|}$, where V_d and $\boldsymbol{v}_d$ are the drift velocity in the ambient field, B_0 and perturbation fields, respectively. Then, in first order of perturbations, the parallel and perpendicular components of (3) are reduced to

$$m(d\boldsymbol{u}/dt)_{\|}=q[E_{\|}+(\boldsymbol{V}_d\times\delta\boldsymbol{B})_{\|}]-\mu\nabla_{\|}\delta B$$

and

$$m(d\boldsymbol{u}_{\perp}/dt)=q[\boldsymbol{E}_{\perp}+\boldsymbol{v}_d\times\boldsymbol{B}_0+v_{\|}(\boldsymbol{n}_0\times\delta\boldsymbol{B})_{\|}]-\mu\nabla_{\perp}\delta B,$$

where $\boldsymbol{n}=\boldsymbol{n}_0+\delta\boldsymbol{n}$ is a unit vector in the direction of superposed magnetic field, $\boldsymbol{B}_0+\delta\boldsymbol{B}$. Since in the linear approximation we have $B=|\boldsymbol{B}_0+\delta\boldsymbol{B}|\cong B_0+\delta B_{\|}$ and $\delta\boldsymbol{n}\cong\delta\boldsymbol{B}_{\perp}/B_0$, the perturbation drift velocity becomes

$$\begin{aligned}\boldsymbol{v}_d=&B_0^{-2}\boldsymbol{E}\times\boldsymbol{B}_0+(m/qB_0^2)\partial\boldsymbol{E}/\partial t+(v_{\|}/B_0)\delta\boldsymbol{B}_{\perp}\\&+(mv_{\|}^2/qB_0)\mathrm{rot}_{\perp}(\delta\boldsymbol{B}_{\perp}/B_0)+(\mu/qB_0)\boldsymbol{n}_0\times\nabla_{\perp}\delta B_{\|}.\end{aligned}\qquad (4)^*$$

Neglecting terms of small order of $l_{\perp}/L_{\perp}\ll l$ where $l_{\perp}$ and $L_{\perp}$ are typical scale lengths of $\delta\boldsymbol{B}$ and B_0, we have

$$du_{\|}/dt=(q/m)[E_{\|}-(\mu/q)\nabla_{\|}\delta B_{\|}].\qquad (5)$$

In the drift approximation the particle velocity distribution function is expressed in terms of the guiding center position, $\boldsymbol{r}$, magnetic moment, μ, and the parallel component of particle velocity, $u_{\|}$. Then, as a drift kinetic equation we have

$$\frac{\partial F}{\partial t}+\mathrm{div}(\boldsymbol{u}F)+\frac{\partial}{\partial u_{\|}}\left(\frac{du_{\|}}{dt}F\right)=0.$$

Writing $F(v_{\|},\mu,\boldsymbol{r},t)=F_0+\delta F$ and substituting (4) and (5) into the above equation, the perturbation distribution function, δF, satisfys

$$i(\omega-k_{\|}v_{\|})\delta F=(\boldsymbol{v}_d\cdot\nabla_{\perp})F_0+F_0\,\mathrm{div}\,\boldsymbol{v}_d+(q/m)[E_{\|}-(\mu/q)\nabla_{\|}\delta B_{\|}]\partial F_0/\partial v_{\|},\quad (6)$$

here we assume a functional dependence of $\exp[i(k_{\|}z-\omega t)]$ and also that the stationary drift velocity, V_d, is much smaller than thermal speed of particles.

In order to cannot the guiding center motion of particles in the drift approximation with the macroscopic current density in plasma, in addition to the drift current, δJ_D, it is necessary to take into account the diamagnetic current, δJ_M, resulting from the particle gyration. Thus, the total current intensity perpendicular to the field line becomes

$$\delta\boldsymbol{J}_{\perp}=\delta\boldsymbol{J}_D+\delta\boldsymbol{J}_M,\qquad (7)$$

where

$$\delta\boldsymbol{J}_D=\sum_j q_j\int\boldsymbol{v}_{dj}F_{0j}d\mu dv_{\|}\qquad (8)$$

and

$$\delta\boldsymbol{J}_M=-\sum_j\mathrm{rot}_{\perp}\left[\delta\boldsymbol{n}\int\mu F_{0j}d\mu dv_{\|}+\boldsymbol{n}_0\int\mu\delta F_j d\mu dv_{\|}\right].\qquad (9)$$

* If the finite Larmor radius effect is taken into account, the higher order correction to the drift velocity is $q_j(r_j/2)^2\nabla^2{}_{\perp}\boldsymbol{E}\times\boldsymbol{n}_0$, (see, 21).

As an unperturbed distribution function we assume the local Maxwellian distrubution

$$F_{0j}=(N_0B_0/\pi^{1/2}T_{\perp j}v_j)\exp[-(v_{\parallel}/v_j)^2-(B_0/T_{\perp j})\mu], \tag{10}$$

$v_j=(2T_{\parallel}/m_j)^{1/2}$ is thermal speed in the parallel direction. After substitution of (4) and (10) into (8) and (9) we have

$$\begin{aligned}\delta J_D=&-(i/4\pi\omega)[(\omega/V_A)^2E_{\perp}+(\beta_{\parallel}/2)(1+\alpha_{\parallel})\,\mathrm{rot}_{\perp}\,\mathrm{rot}_{\perp}E\\&+(\beta_{\perp}/2)(1+\alpha_{\perp})\mathbf{1}_z\times\nabla_{\perp}\,\mathrm{rot}_{\parallel}E]\end{aligned} \tag{11}$$

and

$$\begin{aligned}\delta J_M=&\delta B_{\perp}\times\nabla_{\perp}[\beta_{\perp}(1+\alpha_{\perp})]-\beta_{\perp}(1+\alpha_{\perp})\,\mathrm{rot}_{\perp}\delta B\\&+\sum_j\mathbf{1}_z\times\nabla_{\perp}\int\mu\delta F_jd\mu dv_{\parallel}\end{aligned} \tag{12}$$

where

$$\beta_{\perp}=8\pi N_0T_{\perp e}/B_0^2,\qquad \beta_{\parallel}=8\pi N_0T_{\parallel e}/B_0^2$$

$\alpha_{\perp}=(T_i/T_e)_{\perp}$, $\alpha_{\parallel}=(T_i/T_e)_{\parallel}$ and $\mathbf{1}_z$ is a unit vector in z-direction. Expression for the last term in (12) and the perturbation space charge density will be derived in Appendix A.

From Maxwell equations we have

$$\mathrm{rot}_{\perp}\,\mathrm{rot}\,E=4\pi i\omega\delta J_{\perp}. \tag{13}$$

Poisson's equation is

$$\mathrm{div}\,E=4\pi c^2e(\delta N_i-\delta N_e), \tag{14}$$

where

$$\delta N_j=\int\delta F_jdv_{\parallel}d\mu.$$

As will be shown in Appendix A we have the quasi-neutral condition and then, (14) is replaced by

$$\delta N_i=\delta N_e.$$

After expressing δB by E from Maxwell equation, we can compile each terms in (A.1) and (A.2) into three terms of $E_{\parallel}$, $\mathrm{rot}_{\parallel}E$ and $\mathrm{div}E_{\perp}$, respectively. Performing some arrangements and neglecting terms of the small order, we have from (A. 1) and (A 2)

$$\begin{aligned}&(2k_{\parallel}/\omega)(\omega/k_{\parallel}V_S)^2\{[1+\zeta_eZ_e+\alpha_{\parallel}^{-1}(1+\zeta_iZ_i)]\omega+(m/r)(V_{Di}-V_{De})\}E_{\parallel}\\&\quad=(\beta_{\perp}/\gamma)(\omega/\Omega_i)(k_{\parallel}V_A/\omega)^2[\gamma\alpha_{\perp}\zeta_i^2(1+\zeta_iZ_i)-\zeta_e^2(1+\zeta_eZ_e)]\,\mathrm{rot}_{\parallel}E_{\perp}\\&\quad\quad-i(\omega/\Omega_i)^2[1+2\zeta_iZ_i+\gamma(1+\zeta_eZ_e)]\,\mathrm{div}\,E_{\perp},\end{aligned} \tag{15}$$

and

$$\begin{aligned}&4\pi i\omega\sum_j\int\mu\delta F_jd\mu dv_{\parallel}\\&\quad=-\beta_{\perp}(\Omega_i/k_{\parallel}V_S^2)[\omega(1+\zeta_eZ_e)-i(m/r)U]E_{\parallel}+\beta_{\perp}[(1/2)(1+\alpha_{\perp})\\&\quad\quad-\beta_{\perp}(V_A/V_S)^2(1+\zeta_eZ_e)]\,\mathrm{rot}_{\parallel}E_{\perp}+i\beta_{\perp}(\omega/2\Omega_i)[\alpha_{\perp}(1+2\zeta_iZ_i)\\&\quad\quad-\gamma(1+2\zeta_eZ_e)]\,\mathrm{div}\,E_{\perp}.\end{aligned} \tag{16}$$

Here $V_S=(2T_e/m_i)^{1/2}$ is the velocity of sound, $\xi_j=\omega/k_{\|}v_j$, $Z_j=Z(\xi_j)$ the plasma dispersion function and V_{Dj} the drift velocity of group of particles of species j,

$$V_{Dj}=(T_e/m_j\Omega_j)(1+\zeta_jZ_j)\Big\{(B_0'/B_0)-(N_0'/N_0) -\Big[\Big(\zeta_j^2-\frac{1}{2}\Big)+\frac{1}{2(1+\zeta_jZ_j)}\Big](T_j'/T_j)\Big\} \tag{17}$$

where prime means differentiation with respect to radial distance r. U in (16) is defined by

$$U=(T_e/m_i\Omega_i)\{[\alpha_\perp(1+\zeta_iZ_i)+1+\zeta_eZ_e]\beta_\perp^{-1}\nabla_\perp\beta_\perp +\nabla_\perp[\alpha_\perp\zeta_iZ_i+\zeta_eZ_e]. \tag{18}$$

In derivation of (15) and (16) we used that the azimuthal dependence of perturbations is $e^{im\varphi}$ and the differential relation, $dZ/d\zeta=-2(1+\zeta Z)$ [23].

Substitution of (16) into (12), then (11), (12) and (13) lead to

$$\begin{aligned}&[(\omega/V_A)^2-\eta k_\|^2]\boldsymbol{E}_\perp-i\eta k_\|\nabla_\perp E_\|+[1+(3/2)\beta_\perp(1+\alpha_\perp)]\mathbf{1}_z\times\nabla_\perp\,\mathrm{rot}_\|\boldsymbol{E}_\perp\\&+(\mathrm{rot}_\|\boldsymbol{E})\mathbf{1}_z\times\nabla_\perp[\beta_\perp(1+\alpha_\perp)]\\&-\mathbf{1}_z\times\nabla_\perp\{\beta_\perp(\Omega_i/k_\|V_s^2)[\omega(1+\zeta_eZ_e)-i(m/r)U]E_\|\\&+\beta_\perp^2(V_A/V_S)^2(1+\zeta_eZ_e)\,\mathrm{rot}_\|\boldsymbol{E}\\&-i\beta_\perp(\omega/2\Omega_i)[\alpha_\perp(1+2\zeta_iZ_i)-\gamma(1+2\zeta_eZ_e)\,\mathrm{div}\,\boldsymbol{E}_\perp\}=0,\end{aligned} \tag{19}$$

where

$$\eta=1+(\beta_\perp/2)(1+\alpha_\perp)-(\beta_\|/2)(1+\alpha_\|). \tag{20}$$

(15) and (19) are two fundamental equations for hydromagnetic perturbations in an inhomogeneous, finite-β Plasma.

3. Coupling Mode in the Inhomogeneous, Low-β Plasma

a. *Fundamental Equations for the Coupling Mode*

We consider the hydromagnetic perturbations which satisfy conditions that $\xi_i\gg1$ and $\zeta_e\lesssim1$ in the relatively low-β plasma of $1\gg\beta\gtrsim m_e/m_i$. These conditions may prevail in the outer magnetosphere beyond the density knee, and electron-wave interaction has the most dominant contribution to the thermal effect.

Using the asymptotic form for the plasma dispersion function, $Z(\zeta_i)=-(1/\zeta_i)(1+1/2\zeta_i^2+\cdots)$, and neglecting the higher order terms of ζ_i^{-2} and γ, we have from (15)

$$\begin{aligned}[\omega-(m/r)V_D]E_\|=&(\omega/2k_\|)(k_\|V_S/\omega)^2[-(\omega/\Omega_i)(\beta_\perp/\gamma)(k_\|V_A/\omega)^2\zeta_e^2\,\mathrm{rot}_\|\boldsymbol{E}\\&+i(\omega/\Omega_i)^2(1+\zeta_eZ_e)^{-1}\,\mathrm{div}\,\boldsymbol{E}_\perp],\end{aligned} \tag{21}$$

where V_D is a drift velocity of thermal electrons,

$$V_D=V_{De}/(1+\zeta_eZ_e). \tag{22}$$

After substitution of (21) into (19) we have

$$f_0 \boldsymbol{E}_\perp + i\epsilon A \nabla_\perp \Psi + \epsilon^2 B \nabla_\perp \Phi + (1+\beta_\perp C) \mathbf{1}_z \times \nabla_\perp \Psi + i\epsilon D \mathbf{1}_z \times \nabla_\perp \Phi = 0. \qquad (23)$$

Here,

$$\Phi = \mathrm{div}\, \boldsymbol{E}_\perp, \qquad \Psi = \mathrm{rot}_\parallel \boldsymbol{E}$$

and

$$A = \frac{\eta}{2} \gamma^{-1} \beta_\perp \left(\frac{k_\parallel V_S}{\omega}\right)^2 \left(\frac{V_A}{v_e}\right)^2 \frac{\omega}{\omega - (m/r)V_D}$$

$$B = \frac{\eta}{2} \frac{(k_\parallel V_S/\omega)^2}{1+\zeta_e Z_e} \cdot \frac{\omega}{\omega-(m/r)V_D} \qquad (24)$$

$$C = \frac{3}{2}(1+\alpha_\perp) - \gamma^{-1}\beta_\perp \left(\frac{V_A}{v_e}\right)^2 \left[1 - \frac{1}{2} \cdot \frac{(1+\zeta_e Z_e) - i(m/r)U}{\omega - (m/r)V_D}\right]$$

$$D = -\frac{1}{2}\beta_\perp \left[\alpha_\perp + \frac{\omega - i(m/r)U(1+\zeta_e Z_e)^{-1}}{\omega-(m/r)V_D}\right].$$

In reduction to (23) we neglected space derivatiyes of the stationary quantities comparing to that of perturbations.

In the cold plasma limit, $\beta_\perp = 0$ and $v_e = 0$, (21) and (23) reduce to

$$E_\parallel = -k_\parallel^{-1}(\omega^2/\Omega_e \Omega_i)\, \mathrm{div}\, \boldsymbol{E}_\perp$$

and

$$f_0 \boldsymbol{E}_\perp - \mathbf{1}_z \times \nabla_\perp \mathrm{rot}_\parallel \boldsymbol{E} - (\omega^2/\Omega_e\Omega_i)\nabla_\perp^2 \boldsymbol{E}_\perp = 0.$$

If we neglect the term of the order $\omega^2/\Omega_e\Omega_i$, the latter equation coincides to the coupling equation of (1) in a cold inhomogeneous plasma, after elimination of E_r.

In the following we consider the case where the drift effect of electrons is not important, namely the oscillations considered here have a property of $\omega \gg |(m/r)V_D|$. Because we have $v_e \gtrsim V_A$ in the outer magnetosphere, then for oscillations with $\omega \simeq k_\parallel V_A$ the primarily dominant collisionless process is electron thermal dissipation. On the other hand, if the drift effect may have an appreciable contribution to oscillations in the resonant region, the resulting oscillations must have the characteristics such as $\omega \ll k_\parallel V_A$ and $\omega \simeq (m/r)V_D$. Representing in terms of the electric field components, r- and φ-components of (23) under the above condition are given by

$$\begin{aligned} &\epsilon^2 B E_r'' + (m/r)(D+A)E_r' + [f_0 - (m/r)^2(1+\beta_\perp C)]E_r \\ &\quad + i\epsilon A E_\varphi'' + i(m/r)[\epsilon^2 B - (1+\beta_\perp C)]E_\varphi' + i\epsilon D(m/r)^2 E_\varphi = 0 \end{aligned} \qquad (25)$$

and

$$\begin{aligned} &i\epsilon D E_r'' + i(m/r)[\epsilon^2 B - (1+\beta_\perp C)]E_r' + i\epsilon A(m/r)^2 E_r \\ &\quad + (1+\beta_\perp C)E_\varphi'' - \epsilon(m/r)(D+A)E_\varphi' + [f_0 - \epsilon^2(m/r)^2 B]E_\varphi = 0, \end{aligned} \qquad (26)$$

where

$$\begin{aligned} &A = (\eta/2)\gamma^{-1}\beta_\perp(k_\parallel V_S/\omega)^2(V_A/v_e)^2, \\ &B = (\eta/2)(k_\parallel V_S/\omega)^2(1+\zeta_e Z_e)^{-1}, \\ &C = (3/2)(1+\alpha_\perp) - \gamma^{-1}\beta_\perp(V_A/v_e)[1-(1/2)(1+\zeta_e Z_e)], \\ &D = -(\beta_\perp/2)(1+\alpha_\perp). \end{aligned} \qquad (27)$$

Generally speaking, we have a fourth order ordinary differential equation from (25) and (26), which means that in addition to the coupling mode in the cold approximation the thermal correction creates a new mode of the higher order. However, as will be shown in the next section this new mode is highly dissipative for the outer magnetosphere.

b. *Many Layeered Approximation*

Before considering the general case in which plasma parameters for the stationary state vary continously, we would like to check a possibility of wave generation of higher order using many layer model where each layer has constant plasma parameters, but varies discontinously.

Let us represent perturbations in the form of plane wave, $e^{i(k_x x+k_y y+k_z z-\omega t)}$, in Cartesian coordinates wherein z-axis coincides with the direction of the ambient stationary magnetic field and plasma parameters vary discontinuously in x-direction. Then, perturbation electric field components satisfy the following two equations in each constant plasma layer

$$\{-\epsilon^2 Bk_x^2+ik_xk_y\epsilon(D+A)+[f_0-k_y^2(1+\beta_\perp C)]\}E_x$$
$$-\{i\epsilon Ak_x^2+k_xk_y[\epsilon^2 B-(1+\beta_\perp C)]-i\epsilon Dk_y^2\}E_y=0$$

and

$$\{i\epsilon Dk_x^2+k_xk_y[\epsilon^2 B-(1+\beta_\perp C)]+i\epsilon k_y^2 A\}E_x$$
$$-\{(1+\beta_\perp C)k_x^2+ik_xk_y\epsilon(D+A)-(f_0-\epsilon^2 Bk_y^2)\}E_y=0.$$

From these two equations we have the dispersion equation

$$\epsilon^2[B(1+\beta_\perp C)+AD]k_x^4$$
$$-\{[1+\beta_\perp C+\epsilon^2 B]f_0-2\epsilon^2 k_y^2[B(1+\beta_\perp C)+AD]\}k_x^2$$
$$-f_0[(1+\beta_\perp C)k_y^2-f_0]+\epsilon^2 k_y^2\{B[(1+\beta_\perp C)k_y^2-f_0]+ADk_y^2\}=0. \qquad (28)$$

When ω, k_y and k_z of the incident perturbations are given, we can determine the propagation character in x-direction in each layer by (28). For simplicity, let us consider a monotonic decreasing distribution of $V_A(x)$ and there is j-th layer which includes a point $x=x_0$ such as $f_0(x_0)=0$. Thus, we have $f_0<0$ for layers of $x>x_0$ and $f_0<0$ for $x<x_0$. Throughout the whole region considered we further assume that $\beta\simeq m_e/m_i\ll 1$, $\omega/k_z\simeq v_e$ and thus, $\zeta_e\simeq 1$. For this situation we have $|R_eB|\simeq|ImB|$ in (27) and (28), since the real and immaginary parts of the plasma dispersion function becomes tc be of the same order for its real argument of about unity[23]. In the following we discuss simply several limiting cases.

(i) in the layer of $f_0\gg\epsilon^2$, or two-dimentional perturbations ($k_y=0$):
From (28) there are two modes given by

$$k_x^2=\begin{cases}(f_0/\epsilon^2)(1+\beta_\perp C)[B(1+\beta_\perp C)-AD]^{-1}\\ f_0(1+\beta_\perp C)^{-1}-k_y^2,\end{cases}$$

$AD\simeq 0(\beta_\perp^2)$ and $B\simeq 0(\beta_\perp)$ by (27), then we have approximately

$$k_x^2=f_0/\epsilon^2 B \qquad \text{and} \qquad k_x^2\simeq\bar{f}_m(1+\beta_\perp C)^{-1},$$

where $$\bar{f}_m = f_0 - (1+\beta_\perp C)k_y^2.$$

Since $|R_eB| \simeq |ImB|$, the former mode, which corresponds to the slow mode of magnetosonic waves in the one-fluid picture, may be highly attenuative. The latter, on the other hand, is the fast magnetosonic mode and shows a weak absorption through $\beta_\perp ImC \simeq \gamma^{1-}\beta^2(V_A/v_e)Im(\zeta_e Z_e)$.

(ii) $f_0 \gg f_m$ and $|f_m| \lesssim \beta_\perp \ll 1$:

$$k_x^2 = \begin{cases} f_0/\epsilon^2 B \\ f_m(1+\beta_\perp C)^{-1} \simeq -k_y^2 \beta_\perp C(1+\beta_\perp C) \end{cases}$$

From $ImC \simeq \gamma^{-1}\beta_\perp(V_A/V_e)Im(\zeta_e Z_e) \simeq 1$ for $\beta_\perp \simeq m_e/m_i$, this region is a strong absorption region for the fast mode as well as the slow mode.

(iii) $|f_0|$, $|f_m| \gg \beta$, but $Ref_m < 0$:

$$k_x^2 = \begin{cases} f_0/\epsilon^2 B, \\ -|f_m|(1+\beta_\perp C)^{-1} \end{cases}$$

The fast mode becomes to be evanescent, but its thermal absorption is small owing to a factor of $\beta_\perp \gg 1$.

(iv) $|f_0| \lesssim \beta_\perp$ and $|f_m| \gg \beta_\perp$:

$$k_x^2 = -k_y^2 \pm [f_0(1+\beta_\perp C)^{-1}]^{1/2}.$$

Both modes are evanescence and degenerate to $k_x^2 = -k_y^2 < 0$ in the limit of $f_0 = 0$.

(v) $|f_0|$ and $|f_m| \gg \beta_\perp$, but their real part are negative:

$$k_x^2 = \begin{cases} -|f_0|/\epsilon^2 B, \\ -|f_m|(1+\beta_\perp C)^{-1}. \end{cases}$$

The fast mode is also evanescent. However, in consequence of a large value of $|f_m|$ its skin depth is relatively small.

From the gross consideration given above we see that due to strong thermal absorption the generation of the slow magnetosonic mode is impossible under the assumed condition here. On the other hand, the fast mode, which have two resonance region ($f_m = 0$ and $f_0 = 0$) in its propagation from the oscillating region ($f_m > 0$) to the evanescent one ($f_m < 0$) in the cold plasma approximation, becomes to have the strong absorption within the region of $f_m = 0$. The thermal modification in the second resonance region ($f_0 = 0$) is small so far as we assume the low-β Plasma.

c. *Hydromagnetic Oscillations near the Trough of V_A*

As was mentioned in introduction a sudden decrease of the plasma density at the plasmapause gives a minimum of V_A. This trough of V_A is a trapping region of hydromagnetic waves and we may expect the characteristic oscillations in this region of which characters are determined by several plasma parmeters, such as the radial distribution of V_A, T_e and the length of the field lines. In Fig. 4a the schematic patterns of B_0, N_e and T_e in the magnetosphere are given. Fig. 4b shows a resulting distribution of V_A. In this section we consider hydromagnetic oscillations in the plasma with continously varying parameters. From discussion given in the previous section the modification for the coupling mode which is especially necessary in the resonant regions is thermal disipation of the oscillitions.

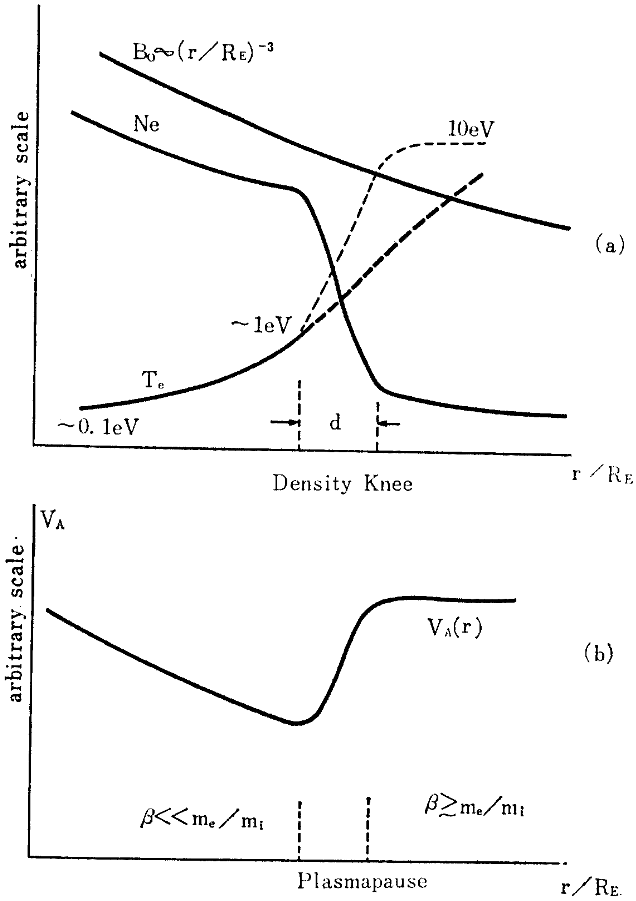

FIG. 4. (a) Diagram showing gross configurations of plasma parameters (including the density knee) in the magnetospheric core. (b) Variation of the Alfvén speed, V_A which is expected from (a).

Thus, a requiring examination is that what changes are brought into the coefficient of E''_φ, which is $f_0 f_m$ in the cold plasma approximation, if we consider electron-wave interactions.

We eliminate E_r and its derivatives from (25) and (26). In this reduction we neglect space derivatives of the stationary plasma parameteres except f'_0 and f'_m, since f_0 and f_m vanish at resonant points. After some lengthly calculation we have the following expressions for the coefficient of each derivative.
coefficient of $E_\varphi^{(\mathrm{iv})}$:

$$\epsilon^2 D[B(1+\beta_\perp C)+AD][D\bar{f}_m-\epsilon^2(m/r)^2 AB],$$

coefficient of $E_\varphi^{(\mathrm{iii})}$:

$$-\epsilon^2 D[C(1+\beta_\perp C)+AD][D\bar{f}_m-\epsilon^2(m/r)^2AB](\Delta'/\Delta),$$

coefficient of E''_φ:

$$(1+\beta_\perp C)D\{Df_0\bar{f}_m-\epsilon(m/r)[B(1+\beta_\perp C)+AD][f'_m+(\Delta'/\Delta)\bar{f}_m]\}+0(\epsilon^2),$$

coefficient of E'_φ:

$$-(m/r)^2(1+\beta_\perp C)^2D^2\bar{f}'_m+0(\epsilon),$$

coefficient of E_φ:

$$D^2f_0\bar{f}^2_m+0(\epsilon),$$

where

$$\Delta=[Df_m-\epsilon^2(m/r)^2AB]^2+[D(D+A)+B(1+\beta_\perp C-\epsilon^2B)]$$
$$\cdot\{(m/r)^2[(1+\beta_\perp C-\epsilon^2B)f_m+\epsilon^2(m/r)^2A(D+A)]-\epsilon(m/r)Df'_m\},$$

and

$$\bar{f}_m=f_0-(m/r)^2(1+\beta_\perp C).$$

In the limit of $|f_m|$, $|f_0|\ll 1$, we have $\bar{f}'_m+(\Delta'/\Delta)\bar{f}_m\simeq f'_m+0(\beta_\perp)$. Therefore neglecting higher order terms, the following requiring equation can be obtained

$$\frac{d^2E_\varphi}{dr^2}+p(r)\frac{dE_\varphi}{dr}+q(r)E_\varphi=0, \tag{29}$$

where

$$p(r)=-(m/r)^2f'_m[f_0f_m-\epsilon(m/r)(B/D)f'_m]^{-1} \tag{30}$$

and

$$q(r)=f_0f^2_m(1+\beta_\perp C)^{-1}[f_0f_m-\epsilon(m/r)(B/D)f'_m]^{-1}. \tag{31}$$

Since from (27) we have $B/D\simeq(1/2)(k_\parallel V_A/\omega)(1+\xi_eZ_e)^{-1}$ for $\alpha_\perp=1$ and $T_e=T_i$, we see that thermal attenuation in the resonant regions in the present case is much larger than that for the layered model.

Introducing a new variable

$$E_\varphi(r)=E(r)\exp\left[-\frac{1}{2}\int p(r)dr\right], \tag{32}$$

We rewrite (29) into the standard form

$$d^2E/dr^2+\mu^2(r)E=0. \tag{33}$$

$\mu^2=q-(1/2)'_p-(p^2/4)$ can be interpreted as the square of the refractive index for one-dimensional propagation along the radial direction. Particularly, we have

$$\mu^2\simeq-(1/2)[f_0+(1/2)(m/r)^2](\epsilon B/D)^{-2},\quad \text{for}\quad f_m=0$$

and

$$\mu^2\simeq-(1/2f'_m)[f'_0f_m+(1/2)(m/r)^2f'_m](\epsilon B/D)^{-2},\quad \text{for}\quad f_0=0.$$

For both expressions $Re\ \mu^2$ is negative and finite. While in the cold plasma limit μ^2 tends to plus infinity at $f_0=0$ and to minus infinity at $f_m=0$, respectively.

Fig. 5a is an extended pattern of the trough of V_A in the magnetosphere. When we consider the particular characteristic oscillations in the trapping region, of which frequency can be determined as an eigenvalue problem for given $k_{\parallel}$ and m, variations of f_0 and f_m are shown in Fig. 5b schematically. We have four vanishing points of f_0 or f_m, two is beyond the minimum of V_A and the remaining two within the density knee. For the axisymmetric oscillations ($m=0$) two points of $f_0=0$ are the turning points and a region of $f_0>0$ is the oscillating region. On the other hand, for three-dimensional oscillations we have the very complex configuration of μ^2 as is shown in Fig. 5c. There are four coupling resonance region between the oscillating and evanescent regions. It is noted that positions of $f_0=0$ or $f_m=0$ cannot be specified ápriόri but their determination must be

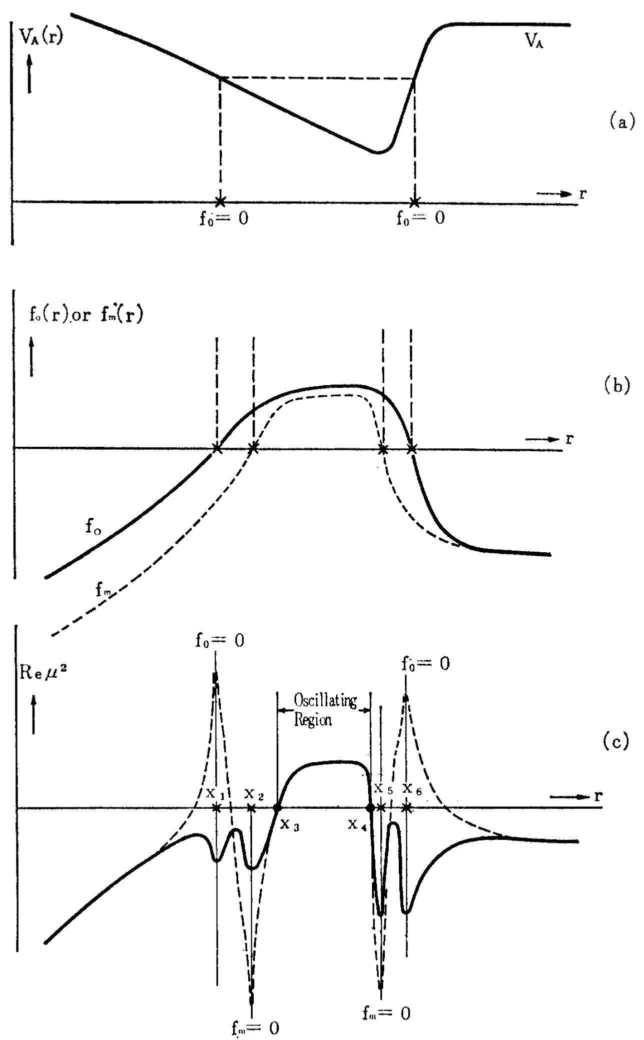

FIG. 5. (a) Extending distribution of V_A near the plasmapause. (b) Radial variations of f_0 and f_m. (c) Gross patterns of the square of equivalent refractive index, μ^2, near the oscillating region: the solid line for the case when thermal dissipation is considered and broken lines for the cold plasma limit.

done self-consistently in the eigen-value problem for characteristic oscillations. Roughly speaking, the characteristic frequency of a trapped oscillation will be given by the ratio of the mean value of V_A to the width of the oscillating region, while its damping is strongly dependent upon the rate of dissipation and the effective thickness of each coupling resonance region. The order of this thickness is given by the relation, $|f_0|$ or $|f_m| \lesssim |\epsilon(m/r)f'_m|$, form (30) or (31).

For the inner dissipating regions near x_1 and x_2 in Fig. 5c the dominant dissipation may come from inter-particle collisions as was mentioned in introduction, instead of collisionless dissipation. In that case we have a new argument of $Z(\zeta_j)$, $\xi_j=(\omega+i\nu)/k_{\|}v_j$, which results from substitution of the collision term, $-\nu(F-F_0)$, into the right-hand side of the perturbation kinetic equation, where ν is the effective collisional frequency. Then, $\epsilon(B/D)$ in (30) and (31) is about of $(\omega^2/\Omega_e\Omega_i)^{1/2}$ $(V_S/v_e)^2[1+i(\nu/\omega)]^2$.

Next we will give here formal soultions in the dissipating regions. we introduce a new non-dimensional variable x defind by $x=r/R_E$, R_E is the earth's radius.

(i) region near $f_m=0$ ($x_j=x_2$ or x_5):

We approximate $f_m(x)$ by the Taylor expansion

$$f_m(x_j+\delta x)=\begin{cases}(x-x_2)f'_m(x_2), & f'_m>0\\(x-x_5)f'_m(x_5), & f'_m<0.\end{cases}$$

Then, (33) can be reduced to

$$\frac{d^2E}{dx^2}+\frac{g_j}{(x-x_j-is_j)^2}E=0, \tag{34}$$

where

$$g_j=-(1/2f_0^2)(m/x_j)^2[f_0+(1/2)(m/x_j)^2]$$

and

$$s_j=Im[(\epsilon/f_0)(m/x)(B/D)]_{x=x_j}.$$

Since f_m is defined by $f_m=f_0-(m/x)^2$, we have approximately $f_0\simeq(m/x_j)^2$, near the point of $f_m=0$, and thus $g_j\simeq-3/4$.

The formal solution of (34) is

$$E=E_1(x-x_j-is_j)^{r_1}+E_2(x-x_j-is_j)^{r_2}, \tag{35}$$

E_1 and E_2 are constants and

$$r_{1,2}=\frac{1}{2}\pm\left(\frac{1}{4}-g_j\right)^{1/2}\approx\frac{1}{2}\pm1.$$

Thus, we have a resonance-type distribution of E, $(x-x_j-is_j)^{-1/2}$, which shows a strong peak at $x=x_j$, although the rate of attenuation of waves in this region is strongly larger than that for other regions.

(ii) Region near $f_0=0$ ($x_j=x_1$ or x_6):

Similarly we have

$$f_0 \simeq \begin{cases} (x-x_1)f_0'(x_1), & f_0'>0 \\ (x-x_6)f_0'(x_6), & f_0'<0, \end{cases}$$

and $\mu^2 = g_j(x-x_j-is_j)^{-2}$ where constants, g_j and s_j are given by

$$g_j \simeq -(1/2)(m/x_j)^2 f_m'[f_0' f_m + (1/2)(m/x)^2 f_m'](f_m f_0')^{-2}$$

and

$$s_j \simeq Im[\epsilon(m/x)(B/D)(f_m'/f_0' f_m)]_{x=x_j}.$$

By connecting the above solutions in the resonant regions to WKB-solutions with two turning points, we can quantitatively discuss the characters of three-dimensional hydromagnetic oscillations in the outer magnetosphere, such as their eigen-periods, Q-values and the radial distribution of perturbation fields. We don't attempt here to carry out such a numerical calculation, however, it may be expected that the large thermal dissipation and strong peak of electric field within the narrow resononce region must be closely related to the observed latitudinal distribution of geomagnetic pulsations in high latitudes and the associated particle precipitation.

4. Collisionless Absorption of Hydromagnetic Waves in the Geomagnetic Tail

Recent satellite observations [*16, 17*] confirmed that there is a hot plasma region in the geomagnetic tail which is called the plasma sheet, centered in the equatorial plane with thickness of several earth radii. Mean energy of electrons and ions in the plasma sheet is about from several hundreds of eV to 1 keV and their density is about of 1 cm^{-3}. Observed energy spectra of electrons and ions show a shifted peak centered at about 1 keV [*17*], although their peaked positions do not coincide between electrons and ions. What means this shifted peak, for example, a drift motion of thermal particles or a loss-cone type distribution, is not clear at present time. But it is no doubt that the strong acceleration and thermalization process occurs in this region. Outside the plasma sheet there are low-β plasma regions, tail magnetosphere, which are in contact with the solar wind plasma at the magnetopause. For simplisity we assume a simple model that the ambient magnetic field is parallel or anti-parallel to the sun-earth direction (z-axis) and x-axis is a vertical from the equatorial plane. In Fig. 6 we show gross patterns of plasma parameters in the geomagnetic tail, and resulting distributions of V_A and V_S schematically.

In contrast with the low-β plasma in the magnetospheric core discussed in section 3, the thermal effect is very dominant in the tail, especially within the plasma sheet where $\beta_\perp \simeq 1$.

As long as $l_\perp \gg r_i$, equations (15) and (19) will be valid in the hot plasma such as the tail plasma sheet. In this case $V_A \simeq V_S \simeq v_i$, consequently we have $|\xi_i| \simeq 1$ and $|\zeta_e| \ll 1$ if $\omega/k_\parallel = V_A$. If we consider penetration of hydromagnetic waves into the tail magnetosphere which generated at the magnetopause or the rear region of

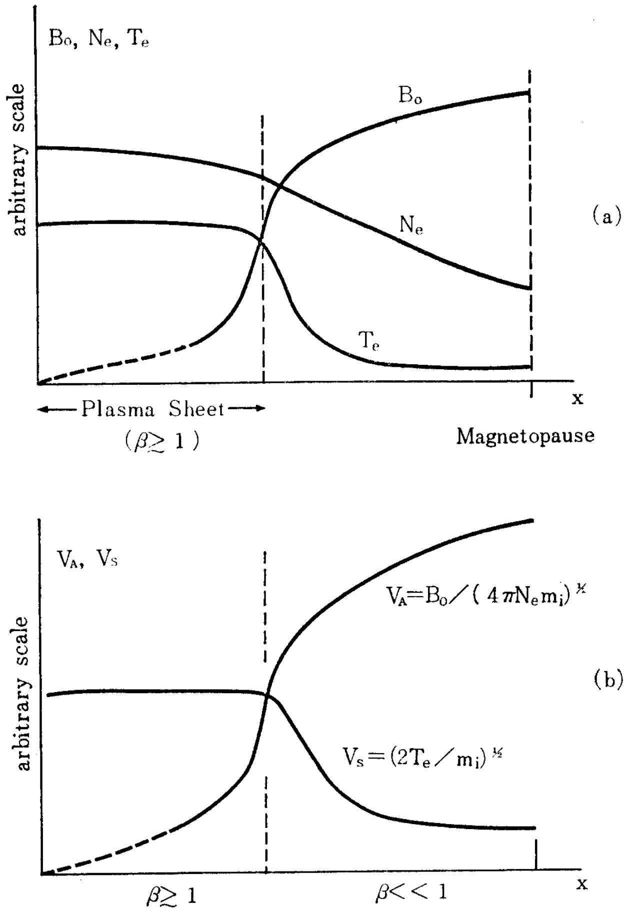

FIG. 6. (a) Schematic diagram showing space distribution of magnetic field, plasma density and temperature in the geomagnetic tail. (b) Variations of the Alfvén velocity, V_A, and ion sound velocity, V_S, being supposed from (a).

the magnetosheath [*24*], their propagation character would change with distance from the magnetopause, especially from the tail magnetosphere into the plasma sheet. However, for the wave with $\omega/k_\parallel$ such that $f_0>0$ at the magnetopause, we have no singular point of $f_0=0$ within the geomagnetic tail, since V_A decreases monotonously with approaching to the equatorial plane as is seen from Fig. 6. Therefore, except for the drift effect we will neglect here space derivatives of the equlibrium quantities.

From (15) we have

$$E_\parallel=(\omega/\Omega_i)(\omega/2k_\parallel)(k_\parallel V_S/\omega)^2\{[1+\zeta_e Z_e+\alpha_\parallel^{-1}(1+\zeta_i Z_i)]\omega+\omega^*\}^{-1}$$
$$\cdot\{(T_{\perp e}/T_{\parallel i})[\alpha_\perp(1+\zeta_i Z_i)-(1+\zeta_e Z_e)]\Psi-i(\omega/\Omega_i)(1+2\zeta_i Z_i)\Phi\},$$

where $\omega^*=(m/r)(V_{Di}-V_{De})$ is the drift frequency. Substituting the above expression of $E_\parallel$ into (19) and neglecting terms of the higher order of ω/Ω_i, we obtain

$$Q_1\mathbf{1}_z\times\nabla_\perp\Psi+f_0\mathbf{E}_\perp+0(\omega/\Omega_i)=0, \qquad (36)$$

$$Q_1 = 1 + \beta_\perp\{(3/2)(1+\alpha_\perp) - \beta_\perp(V_A/V_S)^2(1+\zeta_e Z_e) - \beta_\perp\left(\frac{V_A}{v_i}\right)^2 \cdot \frac{[(1+\zeta_e Z_e) - i(m/r)U][\alpha_\perp(1+\zeta_i Z_i) - (1+\zeta_e Z_e)]}{[1+\zeta_e Z_e + \alpha_\parallel^{-1}(1+\zeta_i Z_i)]\omega + \omega^*}\Big\}. \quad (37)$$

Divergence of (36) results to

$$r^{-1}d[r(d\Psi/dr)]/dr + [(f_0/Q_1) - (m/r)^2]\Psi = 0. \quad (38)$$

This is the basic equation for the study of absorption and instabilities of hydromagnetic waves within the inhomogenous, hot tail plasma. In a homogenous case this equation can be reduced to the dispersion relation for the so-called mirror instability due to the pressure anisotropy, $p_\perp > p_\parallel$, when $v_i \ll \omega/k_\parallel \ll v_e$. On the other hand, penetration of waves with $\omega/k_\parallel \simeq V_A$ into the geomagnetic tail may have a considerable contribution for the ion heating within the plasma sheet.

For the particlar mode of $\mathrm{rot}_\parallel E = 0$ we must retain the higher order term of $(\omega/\Omega_i)^2$ and thus approximately.

$$r^{-1}d[r(d\Phi/dr)]/dr + [(f_0/Q_2) - (m/r)^2]\Phi = 0, \quad (39)$$

where

$$Q_2 = -\frac{\eta}{2}\left(\frac{k_\parallel V_S}{\Omega_i}\right)^2 \frac{(1+2\zeta_i Z_i)\omega}{[1+\zeta_e Z_e + \alpha_\parallel(1+\zeta_i Z_i)]\omega + \omega^*}. \quad (40)$$

If we can take the quasi-localized approximation in both cases, the formal solutions of (38) and (39) are Hankel functions with order m, and furthermore if $|f_0 r/Q_j| \gg 1$, we obtain the attenuation depth

$$\delta = ImQ_j/f_0, \quad (j=1 \text{ or } 2)$$

from the asymptotic expansion of Hankel function.

5. Drift Instabilities

In this section we consider drift instabilities of the particular modes of $\delta B_\parallel = 0$, using the generalized equations for the coupling mode derived in section 2. In order to compare the results in this section to that of recent studies [25] in a limiting case, we introduce here the so-called finite Larmor radius effect.

Omitting small order terms of $E_\parallel$ and $E_\perp$ in $(B.3)$ we obtain

$$\begin{aligned}2(\omega/k_\parallel V_S)^2\{[1+\zeta_e Z_e + \alpha_\parallel^{-1}(1+\zeta_i Z_i)] + (m/r\omega)(V_{Di} - V_{De})\}E_\parallel \\ = -(i/k_\parallel)(\omega/\Omega_i)^2(1+2\zeta_i Z_i)\,\mathrm{div}\,\boldsymbol{E}_\perp \\ + (1/4k_\parallel^3)(\omega/\Omega_i)^3(\nabla_\perp^2 \boldsymbol{E}_\perp \times \mathbf{1}_z)\cdot[N_0^{-1}\nabla_\perp(N_0 Z_i/\zeta_i) \\ - 3(Z_i/\zeta_i)B_0^{-1}\nabla_\perp B_0]. \quad (41)\end{aligned}$$

For simplicity the isotropic temperature distribution, $T_\parallel = T_\perp$, was assumed. In the limit of $|\zeta_i| \gg 1$,

$$N_0^{-1}\nabla_\perp(N_0 Z_i/\zeta_i) = -\zeta_i^{-2}(N_0^{-1}\nabla_\perp N_0 + T_i^{-1}\nabla_\perp T_i) + 0(\zeta_i^{-3}),$$

and thus (41) can be reduced to

$$(\omega/k_\parallel)^2(m_i/T_e)[1+\zeta_e Z_e+\alpha_\parallel^{-1}(1+\zeta_i Z_i)+(m/r\omega)(V_{Di}-V_{De})]E_\parallel$$
$$=(i/k_\parallel)(\omega/\Omega_i)^2[\text{div}\,\boldsymbol{E}_\perp+i(U_i/\omega)\mathbf{1}_r\cdot(\nabla_\perp^2\boldsymbol{E}_\perp\times\mathbf{1}_z)], \tag{42}$$

where

$$U_i=(2T_i/m_i\Omega_i)[3(B_0'/B_0)-(N_0'/N_0)-(T_i'/T_i)], \tag{43}$$

$\mathbf{1}_r$ is a unit vector in the radial direction.

Introducing new variables, ϕ and A,

$$\boldsymbol{E}=-\nabla\phi-\mathbf{1}_z\frac{\partial A}{\partial t}, \tag{44}$$

(42) becomes

$$K[\phi-(\omega/k_\parallel)A]=(T_e/T_i)r_i^2[\omega+(m/r)U_i]\nabla_\perp^2\phi, \tag{45}$$

where

$$K=[1+\zeta_e Z_e+\alpha_\parallel^{-1}(1+\zeta_i Z_i)]\omega+(m/r)(V_{Di}-V_{De}).$$

In the same approximation, on the other hand, (19) in section II can be written as

$$f_0\boldsymbol{E}_\perp-i\eta k_\parallel\nabla_\perp E_\parallel+2\pi i(\omega/\Omega_i)(N_0T_i/B_0^2)(\nabla_\perp^2\boldsymbol{E}\times\mathbf{1}_z)$$
$$-\mathbf{1}_z\times\nabla\{\beta_\perp(\Omega_i/k_\parallel V_S)^2[\omega(1+\xi_e Z_e)-i(m/r)U]E_\parallel$$
$$+\alpha_\perp\beta_\perp(\omega/2\Omega_i)\,\text{div}\,\boldsymbol{E}_\perp\}=0,$$

where the third term is FLR contribution.
Taking divergence of the above equation yields

$$[\omega+(m/r)U_i]\nabla_\perp^2\phi-2\omega(\nabla_\perp V_A/V_A)\cdot\nabla\phi=\eta k_\parallel V_A^2\nabla_\perp^2 A. \tag{46}$$

(45) and (46) are coupling equations for drift waves. If we replace $\nabla_\perp^2$ and m/r by $-k_\perp^2$ and k_y respectively, and further assume $\nabla_\perp V_A=0$, (45) and (46) become

$$[T_i/T_e)k_\perp^2 r_i^2(\omega+k_yU_i)+K]\phi=(\omega/k_\parallel)KA$$

and

$$(\omega+k_yU_i)\phi=\eta k_\parallel V_A^2 A.$$

From these equations we have the following dispersion equation of drift waves

$$[\omega(\omega+k_yU_i)-k_\parallel^2V_A^2]K=(T_i/T_e)k_\parallel^2V_A^2r_i^2k_\perp^2(\omega+k_yU_i). \tag{47}$$

For the limiting case, $T_e'=T_i'=B_0'=0$ and $|\zeta_e|\ll 1$, (47) is equivalent to Eq. (2.13) in Mikhailovskii's review article [25], in the long wave length limit, $k_\perp^2 r_i^2\ll 1$.

For general case, elliminating A from (45) and (46) we have

$$\eta\omega(k_\parallel V_A/\omega)^2(T_e/T_i)r_i^2[1+(m/r\omega)U_i]\nabla_\perp^4\phi$$
$$+[1+(m/r\omega)U_i-\eta(k_\parallel V_A/\omega)^2]K\nabla_\perp^2\phi-2K(\nabla_\perp V_A/V_A)\cdot\nabla_\perp\phi=0. \tag{48}$$

Hereinafter we will restrict our discussion on the localized oscillations in Cartesian coordinates, where inhomogeneities is in x-direction, and discuss the localized drift instabilities. If $|\zeta_i| \gg 1$ and $|\zeta_e| \ll 1$, K can be expressed approximately by

$$K \simeq (1+i\pi^{1/2}\zeta_e)(\omega-k_yV_e)-(k_\parallel V_S/\omega)^2(\omega+k_yV_i). \tag{49}$$

V_e and V_i are defined by

$$V_e = V_{Ne}+V_{Be}+i\pi^{1/2}\zeta_e V_{Te}$$

and

$$V_i = V_{Ni}+V_{Bi}+V_{Ti},$$

where

$$V_{Bj}+V_{Nj}=(T_j/m_j\Omega_j)(B_0^{-1}\nabla_\perp B_0-N_0^{-1}\nabla_\perp N_0),$$
$$V_{Tj}=(T_j/2m_j\Omega_j)T_j^{-1}\nabla_\perp T_j.$$

Under these approximations mentioned above, the following dispersion equation can be obtained from (48)

$$\begin{aligned}&\{[1+2i(V'_A/k_\perp V_A)(k_x/k_\perp)]\omega^2+k_yU_i\omega-\eta(k_\parallel V_A)^2\}\\ &\quad\cdot\{[1+i\pi^{1/2}(\omega/k_\parallel v_e)][1-i\pi^{1/2}k_yk_\parallel^{-1}V_{Te}v_e^{-1})\omega-k_y(V_{Ne}+V_{Be})]\omega^2\\ &\quad-(k_\parallel V_S)^2[\omega+k_y(V_{Bi}+V_{Ni}+V_{Ti})]\}\\ &\quad-\eta(T_e/T_i)k_\perp^2r_i^2(k_\parallel V_A)^2\omega^2(\omega+k_yU_i)=0.\end{aligned} \tag{50}$$

Furthermore if $T_e=T_i$, then V_S is of the order of ion thermal velocity. Now, from a condition of $|\zeta_i|\gg 1$ we can neglect the term of V_S^2 in (50). However, in order to satisfy the above condition the following anisotropy between $k_\parallel$ and $k_\perp$ must be required

$$|k_\parallel/k_\perp| \ll (\omega/\Omega_i)(k_\perp r_i)^{-1},$$

since

$$(k_\parallel V_S)^2=(T_e/T_i)(\Omega_i k_\parallel/k_\perp)^2k_\perp^2r_i^2.$$

Thus, (50) reduces to

$$\begin{aligned}&D(\omega, k_\parallel, k_\perp)\\ &\equiv\{[1+2i(V'_A/k_\perp V_A)(k_x/k_\perp)]\omega^2+k_yU_i\omega-(k_\parallel V_A)^2\}\\ &\quad\cdot\ \{(\pi k_yV_{Te}/k_\parallel^2v_e^2)\omega^2+[1-i(\pi^{1/2}k_y/k_\parallel v_e)(V_{Be}+V_{Ne}+V_{Te})]\omega\\ &\quad-k_y(V_{Be}+V_{Ne})\}-(k_\parallel V_Ak_\perp r_i)^2(\omega+k_yU_i)=0.\end{aligned} \tag{51}$$

For long wave length perturbations, $k_\perp^2r_i^2\ll 1$, we expand ω and D in terms of the small quantity, $k_\perp^2r_i^2$,

$$\omega=\omega_0+(k_\perp r_i)^2\omega_1+\cdots$$

and

$$\begin{aligned}D&=D_0+(k_\perp r_i)^2D_1+\cdots\\ &\simeq D(\omega_0)+\omega_1(k_\perp r_i)^2(dD/d\omega)_{\omega=\omega_0}\cdots.\end{aligned}$$

Within first order approximation we have

$$\omega_1=-[D_1/(dD/d\omega)]_{\omega=\omega_0}.$$

Zeroth Order Dispersion

(51) yields the following two dispersion equations in zeroth order approximation

$$[1+2i(V'_A/k_\perp V_A)(k_x/k_\perp)]\omega^2+k_yU_i\omega-(k_\parallel V_A)^2=0 \tag{52}$$

and

$$[1+i\pi^{1/2}(\omega/k_\parallel v_e)][(1-i\pi^{1/2}k_yk_\parallel^{-1}V_{Te}v_e^{-1})\omega-k_y(V_{Be}+V_{Ne})=0 \tag{53}$$

where we omitted suffix zero of ω, for simplicity.

From (52) we have

$$\begin{aligned}2[1+2i(V'_A/k_\perp V_A)(k_x/k_\perp)]\omega=&-k_yU_i\\ &\pm\{k_y^2U_i^2+4(k_\parallel V_A)^2[1+2i(V'_A/k_\perp V_A)(k_x/k_\perp)]\}^{1/2}.\end{aligned}$$

Neglecting higher order terms in expansion of the squre root we obtain approximately

$$\begin{aligned}&[1+4(V'_A/k_\perp V_A)^2(k_x/k_\perp)^2](\omega/k_\parallel V_A)\\ &\simeq-(\xi/2)\pm(4+\xi^2)^{1/2}[1/2+(V'_A/k_\perp V_A)^2(k_x/k_\perp)^2(4+\xi^2)^{-1}]\\ &\quad-i(V'_A/k_\perp V_A)(k_x/k_\perp)\{\xi\mp(4+\xi^2)^{1/2}[1+(1/2)(4+\xi^2)^{-1}]\},\end{aligned} \tag{54}$$

where ξ is defined by

$$\xi=k_yU_i/k_\parallel V_A.$$

Thus, when $V'_A>0$, the mode corresponding to the upper sign is unstable. On the other hand, if $V'_A<0$, the situation becomes reverse.

(53) is a dispersion equation for the slow mode of drift waves and its roots are $\omega=k_y(V_{Be}+V_{Ne})[1-i\pi^{1/2}(k_yV_{Te}/k_\parallel v_e)]^{-1}$ and $\omega=-i\pi^{-1/2}k_\parallel v_e$.
The first root is written to

$$\omega=k_y(V_{Be}+V_{Ne})[1+i\pi^{1/2}(k_yV_{Te}/k_\parallel v_e)][1+\pi(k_yV_{Te}/k_\parallel v_e)^2]^{-1}. \tag{55}$$

If $V_{Te}=0$, there are stable oscillations. While the mode associated with the second root is always damped. Since V_{Ne} and V_{Te} are proportional to $-N'_e$ and T'_e, respectively, the instability condition, $V_{Ne}V_{Te}>0$, can be satisfied if $N'_eT'_e<0$. This requirement will be realized in the outer magnetosphere, as was mentioned in introduction.

Since from consideration of the mechanical balance in the equilibrium state we have $|V_{Be}|\simeq\beta_\perp|V_{Ne}+V_{Te}|$, the magnetic drift effect does not be important for low-β plasmas. Applying the results obtained above to the plasma distribution in the magnetosphere we can make a preliminary study on the drift instabilities in the magnetospheric plasma, especially in a region near the plasmapause. However, for quantitative study it is necessary to take into account the effect of the ionospheric Pedersen current which constitutes a part of the closed current circuit associated with drift instabilities and has a tendency to reduce space charges in the magnetosphere. The parallel electric field of drift waves may play a decisive role on acceleration and bombardment of trapped particles. From this expectation it is highly desirable to make a quantitative study of drift instabilities in the inner boundary layer of the cusped region* of the magnetosphere, with connection to auroral particle precipitation during geomagnetic storms.

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APPENDIX A.

Space Charge Density and Diamagnetic Current Intensity

Integrating (6) with respect to $v_{\parallel}$ and μ and substituting the resulting perturbation densities for ions and electrons into (14), we have

$$
\begin{aligned}
(\omega/\omega_{pe})^2 \operatorname{div} \boldsymbol{E} = & -2ik_{\parallel}[\zeta_e^2(1+\zeta_e Z_e)+\zeta_i^2(1+\zeta_i Z_i)]E_{\parallel} \\
& -\beta_{\perp}k_{\parallel}^2(c^2\Omega_e/\omega_{pe}^2)[\gamma\alpha_{\perp}\zeta_i^2(1+\zeta_i Z_i)-\zeta_e^2(1+\zeta_e Z_e)]\delta B_{\parallel} \\
& +i(\omega/\Omega_e)(\zeta_i Z_i-\zeta_e Z_e)B_0 \operatorname{div}(B_0^{-1}\boldsymbol{E}\times \mathbf{1}_z) \\
& +(\omega^2/\Omega_e)(\zeta_i Z_i+\gamma\zeta_e Z_e)B_0 \operatorname{div}(\boldsymbol{E}_{\perp}/B_0\Omega_i) \\
& +i(\beta_{\perp}/2)(c^2\omega/\omega_{pe}^2)(\alpha_{\perp}\zeta_i Z_i+\zeta_e Z_e)B_0 \operatorname{div}(B_0^{-1}\mathbf{1}_z\times \nabla_{\perp}\delta B_{\parallel}) \\
& +i(\omega/k_{\parallel})^2(\omega/\Omega_e\Omega_i)B_0^2[1+\zeta_i Z_i+\gamma(1+\zeta_e Z_e)] \operatorname{div}[B_0^{-1}\operatorname{rot}_{\perp}(\delta\boldsymbol{B}_{\perp}/B_0)] \\
& +i(\omega^2/k_{\parallel}\Omega_e)(\zeta_i Z_i-\zeta_e Z_e)B_0 \operatorname{div}(\delta\boldsymbol{B}_{\perp}/B_0) \\
& +(\omega/\Omega_e)N_0^{-1}\nabla_{\perp}[N_0(\zeta_i Z_i-\zeta_e Z_e)]\cdot(\boldsymbol{E}_{\perp}\times\mathbf{1}_z) \\
& +(\omega^2/\Omega_e\Omega_i)N_0^{-1}\nabla_{\perp}[N_0(\zeta_i Z_i+\zeta_e Z_e)]\cdot\boldsymbol{E}_{\perp} \\
& +i(\omega^2/k_{\parallel}\Omega_e)N_0^{-1}\nabla_{\perp}[N_0(\zeta_i Z_i-\zeta_e Z_e)]\cdot\delta\boldsymbol{B}_{\perp} \\
& +i(\omega/k_{\parallel})(\omega^2/\Omega_e\Omega_i)(B_0/N_0)\nabla_{\perp}[N_0(1+\zeta_i Z_i)+\gamma N_0(1+\zeta_e Z_e)]\cdot\operatorname{rot}_{\perp}(\delta\boldsymbol{B}_{\perp}/B_0) \\
& +i(\omega c^2/2\omega_{pe}^2B_0)N_0^{-1}\nabla_{\perp}[\beta_{\perp}B_0(\alpha_{\perp}\zeta_i Z_i+\zeta_e Z_e)]\cdot(\mathbf{1}_z\times\nabla_{\perp}\delta B_{\parallel})\,. \qquad \text{(A.1)}
\end{aligned}
$$

where $\omega_{pe}^2=4\pi N_0e^2c^2/m_e$, $\Omega_i=eB_0/m_i$, $\Omega_e=eB_0/m_e$, $\gamma=m_e/m_i$, $\zeta_j=\omega/k_{\parallel}v_j$, $\zeta_jZ_j=\zeta_jZ(\zeta_j)$ and $Z(\zeta_j)$ is the so-called Plasma Dispersion Function defined by [23]

$$
Z(\zeta_j)=\pi^{-1/2}\int_{-\infty}^{\infty}(u-\zeta_j)^{-1}e^{-u^2}du\,.
$$

Comparing with the right hand side, the left hand side of (A.1) can be neglected in the order of $(\omega/\omega_{pe})^2$. Thus, we see that quasi- neutrality of electric charge is valid for low-frequancy perturbations considered here.

In the same way, the contribution of δF to the diamagnetic current can be obtained as

$$
\begin{aligned}
& \sum_j (4\pi\omega/\beta_{\perp})\int \mu\delta F_j d\mu dv_{\parallel} \\
& \quad = -i(k_{\parallel}\Omega_e/\omega)[\gamma\alpha_{\perp}\zeta_i^2(1+\zeta_i Z_i)-\zeta_e^2(1+\zeta_e Z_e)]E_{\parallel}
\end{aligned}
$$

---

* There is a relatively high-β plasma region between the earth's magnetosperic core and plasma sheet in the tail, beyond 8 earth radii on the night side. Main constituents in this region are trapped electrons and protons with the relatively low energy, several hundreds eV, their energy density is comparable with the magnetic field energy, see 17.

$$
\begin{aligned}
&-(k_\parallel^2\beta/\gamma\omega)V_A^2[\gamma\alpha_\perp^2\zeta_i^2(1+\zeta_i Z_i)+\zeta_e^2(1+\zeta_e Z_e)]\delta B_\parallel \\
&+i(B_0/2)(\alpha_\perp\zeta_i Z_i+\zeta_e Z_e)\ \mathrm{div}\ (B_0^{-1}\boldsymbol{E}\times\mathbf{1}_z) \\
&+(\omega/\Omega_i)(B_0^2/2)(\alpha_\perp\zeta_i Z_i-\gamma\zeta_e Z_e)\ \mathrm{div}\ (B_0^{-2}\boldsymbol{E}_\perp \\
&+i(\beta_\perp V_A^2/2\Omega_i)\alpha_\perp^2\zeta_i Z_i-\zeta_e Z_e)B_0\ \mathrm{div}\ [B_0^{-1}\mathbf{1}_z\times\nabla_\perp\delta B_\parallel] \\
&+i(\omega^2/2k_\parallel^2\Omega_i)B_0^2[\alpha_\perp(1+\zeta_i Z_i)-\gamma(1+\zeta_e Z_e)]\cdot\mathrm{div}\ [B_0^{-1}\ \mathrm{rot}_\perp(B_0^{-1}\delta\boldsymbol{B}_\perp)] \\
&+i(\omega/2k_\parallel)B_0[\alpha_\perp(1+\zeta_i Z_i)+(1+\zeta_e Z_e)]\ \mathrm{div}\ (B_0^{-1}\delta\boldsymbol{B}_\perp) \\
&+i(1/2\beta_\perp B_0)\nabla_\perp[\beta_\perp B_0(\alpha_\perp\zeta_i Z_i+\zeta_e Z_e)]\cdot(\boldsymbol{E}\times\mathbf{1}_z) \\
&+(\omega/\Omega_i)(1/2\beta_\perp B_0)\nabla_\perp[\beta_\perp B_0(\alpha_\perp\zeta_i Z_i-\gamma\zeta_e Z_e)]\cdot\boldsymbol{E}_\perp \\
&+i(\omega/2k_\parallel)(1/\beta_\perp B_0)\nabla_\perp\{\beta_\perp B_0[\alpha_\perp(1+\zeta_i Z_i)+1+\zeta_e Z_e]\}\cdot\delta\boldsymbol{B}_\perp \\
&+i(\omega^2/2k_\parallel^2\Omega_i)\beta_\perp^{-1}\nabla_\perp\{\beta_\perp B_0[\alpha_\perp(1+\zeta_i Z_i)-\gamma(1+\zeta_e Z_e)]\}\cdot\mathrm{rot}_\perp(\delta\boldsymbol{B}_\perp/B_0) \\
&+i(1/2\beta_\perp\Omega_i)\nabla_\perp[\beta_\perp^2 V_A^2(\alpha_\perp^2\zeta_i Z_i-\zeta_e Z_e)]\cdot(\mathbf{1}_z\times\nabla\delta B_\parallel)\ . \qquad \text{(A.2)}
\end{aligned}
$$

Appendix B.
Finite Larmor Radius Corrections

If we take account a finite Larmor radius effect on the drift motion, the correction to the perturbed disribution function, δF, is

$$
\begin{aligned}
(\delta F)_{FLR}=&-(i/\omega-k_\parallel v_\parallel)(\mu/2qB_0\Omega)[(\nabla_\perp^2\boldsymbol{E}\times\mathbf{1}_z)\cdot\nabla_\perp F_0 \\
&+F_0\ \mathrm{div}\ (\nabla_\perp^2\boldsymbol{E}\times\mathbf{1}_z)]\ . \qquad \text{(B.1)}
\end{aligned}
$$

Then, the corresponding space charge density becomes to

$$
\begin{aligned}
4\pi c^2 e(\delta N_i-\delta N_e)_{FLR}=&(i/2eB_0)(\omega/\Omega_e\Omega_i)(\omega_{pe}/\omega)^2 \\
&\cdot\{(T_{\perp i}\zeta_i Z_i-\gamma T_{\perp e}\zeta_e Z_e)\ \mathrm{div}\ (\nabla_\perp^2\boldsymbol{E}\times\mathbf{1}_z)+(\nabla_\perp^2\boldsymbol{E}\times\mathbf{1}_z) \\
&\cdot[N_0^{-1}\nabla_\perp[N_0(T_{\perp i}\zeta_i Z_i-\gamma T_{\perp e}\zeta_e Z_e)] \\
&\quad-3(B_0^{-1}\nabla_\perp B_0)(T_{\perp i}\zeta_i Z_i-\gamma T_{\perp e}\zeta_e Z_e)]\}\ . \qquad \text{(B.2)}
\end{aligned}
$$

Adding these FLR correction terms into the right hand side of (A.1) and neglecting small terms of the order of γ, we have

$$
\begin{aligned}
&2(\omega/k_\parallel V_S)^2[1+\zeta_e Z_e+\alpha_\parallel^{-1}(1+\zeta_i Z_i)]E_\parallel \\
&\quad+ik_\parallel^{-2}(\omega/\Omega_i)\{(\zeta_i Z_i-\zeta_e Z_e)B_0^{-1}\nabla B_0-N_0^{-1}\nabla[N_0(\zeta_i Z_i-\zeta_e Z_e)]\}\cdot(\mathbf{1}_z\times\nabla_\perp E_\parallel) \\
&\quad+ik_\parallel^{-1}(\omega/\Omega_i)^2\{(1+2\zeta_i Z_i)\ \mathrm{div}\ \boldsymbol{E}_\perp+N_0^{-1}\nabla_\perp[N_0(1+2\zeta_i Z_i)]\cdot\boldsymbol{E}_\perp\} \\
&\quad-k_\parallel^{-2}(\omega/\Omega_i)^2\{(1+\zeta_i Z_i)\ \mathrm{div}\ \nabla_\perp E_\parallel+N_0^{-1}\nabla_\perp[N_0(1+\zeta_i Z_i)]\cdot\nabla_\perp E_\parallel\} \\
&\quad-(\beta_\perp/k_\parallel\gamma)(k_\parallel V_A/\omega)^2(\omega/\Omega_i)[\gamma\alpha_\perp\zeta_i^2(1+\zeta_i Z_i)-\zeta_e^2(1+\zeta_e Z_e)]\ \mathrm{rot}_\parallel\boldsymbol{E}_\perp \\
&\quad-(1/4k_\parallel^3)(\omega/\Omega_i)^3\{(T_\perp/T_\parallel)_i(Z_i/\zeta_i)\ \mathrm{div}\ (\nabla_\perp^2\boldsymbol{E}\times\mathbf{1}_z) \\
&\quad+(\nabla_\perp^2\boldsymbol{E}\times\mathbf{1}_z)\cdot[N_0^{-1}\nabla_\perp(N_0T_\perp Z_i/\zeta_i T_\parallel)-3(T_\perp/T_\parallel)_i(Z_i/\zeta_i)B_0^{-1}\nabla_\perp B_0]\}=0. \qquad \text{(B.3)}
\end{aligned}
$$

A contribution from the FLR-effect to the drift current is

$$
(\boldsymbol{J}_D)_{FLR}=q\int(\boldsymbol{v}_d)_{FLR}F_0 d\mu dv_\parallel=(N_0T_{\perp i}/2\Omega_i B_0^2)\nabla_\perp^2\boldsymbol{E}\times\mathbf{1}_z\ , \qquad \text{(B.4)}
$$

and thus we have

$$
2\pi i(\omega/\Omega_i)(N_0T_{\perp i}/B_0^2)(\nabla_\perp^2\boldsymbol{E}\times\mathbf{1}_z)\ ,
$$

as an additional term to the left hand side of (19). The FLR effect to the diamagnetic current is small order compared to that of the drift current.

Department of Space Science
Institute of Space and Aeronautical Science
University of Tokyo, Tokyo
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