An Aerodynamic Study of Ablation Near the Region of Stagnation Point of Axially Symmetric Bodies at Hypersonic Speeds

By

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Summary: A theoretical approach to aerodynamic mechanism of ablation is made for the region downstream of stagnation point of blunt-nosed axially symmetric bodies at hypersonic speeds on the basis of small perturbation approximation. Fundamental equations are derived, under an assumption of sublimating ablation, for a non-reacting laminar boundary layer flow of binary gas mixture, in which transport properties are taken exactly into consideration by use of atomic kinetic theory of gases.

It is shown that all physical properties concerning the ablative field can be uniquely determined by matching of an aerodynamic solution with a static one obtained from chemical kinetics.

An emphasis is laid on the fact that perturbation field has no longer similar solutions, while basic field has similar ones for given perimetric conditions.

Numerical calculation carried out for teflon ablator indicates that local ablation rate decreases gradually with increase of distance from stagnation point and the ratio of local ablation to stagnation ablation rate tends to increase with increase of stagnation temperature in free stream.

In order to confirm validity of the present approach, theoretical results are compared with experimental data for teflon. Agreement between theory and experiment is fairly good near the region of stagnation point at free stream stagnation temperature near 1200°C.

SYMBOLS

$(\bar{X}, \bar{Y})$	orthogonal coordinates system fixed in space
$(\bar{x}, \bar{y})$	orthogonal coordinates system fixed on body surface
$(\bar{U}, \bar{V})$	components of velocity vector in $(\bar{X}, \bar{Y})$ coordinates system
$(\bar{u}, v)$	components of velocity vector in $(\bar{x}, \bar{y})$ coodinates system
$\bar{v}_b(\bar{x})$	surface velocity due to ablation
$\bar{t}$	time
$\bar{p}$	pressure
$\bar{\rho}$	density
$\bar{T}$	temperature
$\bar{\mu}$	mean coefficient of viscosity
$\bar{\kappa}$	mean coefficient of thermal conductivity

[157]

$\bar{D}$	binary diffusion coefficient
$\bar{R}(\bar{x}, \bar{t})$	local radius of body curvature
$\bar{R}_0$	radius of body curvature at a reference time $\bar{t}=0$
$\bar{R}_b$	radius of body curvature used as reference length
$\bar{r}_0$	cylindrical radius of body
$\bar{a}$	speed of sound
$\bar{C}_{pi}$ $(i=1, 2)$	specific heat at constant pressure for each pure species
$\bar{C}_p$	mean specific heat at constant pressure
$\bar{C}_b$	specific heat for solid material
$\bar{L}$	latent heat for sublimation
$\dot{\bar{m}}$	ablation rate
$\bar{\delta}$	boundary layer thickness
$\bar{\sigma}$	collision diameter
$\Omega^{(1,1)*}, \Omega^{(2,2)*}$	collision integral
(x, y)	non-dimensional coordinates system fixed on body surface
(s, η)	transformed coordinates system (see Eq. (4.5))
(u, v)	reduced components of velocity vector
p	reduced pressure
ρ	reduced density
T	reduced temperature
μ_i $(i=1, 2)$	reduced coefficient of viscosity for each pure species
μ	mean reduced coefficient of viscosity
κ	mean reduced coefficient of thermal conductivity
D	reduced binary diffusion coefficient
r_0	reduced cylindrical radius of body
M_i $(i=1, 2)$	molecular weight of component gas
M_∞	free stream Mach number
ψ	stream function
H	total enthalpy function
K	concentration function
Re	Reynolds number
C	Chapman-Rubesin number
P	Prandtl number
S	Schmidt number
Le	Lewis number
l	non-dimensional latent heat for sublimation normalized by $\bar{C}_{p2}\bar{T}_{st}$
C_{pi} $(i=1, 2)$	reduced specific heat at constant pressure for each pure species
C_p	mean reduced specific heat at constant pressure
γ	ratio of specific heats for air
θ	local polar angle of body

Subscripts;

st	stagnation conditions in free stream

∞	conditions in free stream
s	conditions just aft of shock wave
e	conditions at outer edge of boundary layer
0	conditions for basic field
1	conditions for perturbation field
1	conditions of foreign gas
2	conditions of air
w	conditions at wall
b	conditions inside the body
$(\ )'$	differentiation with respect to argument

Superscripts;

$*$	conditions for non-dimensionalization

1. INTRODUCTION

At high speed reentry of space vehicles to atmospheric environment ablative heat shield has been found to be one of the practical methods for protection of the vehicles from severe aerodynamic heating. The device of alleviating aerodynamic heating by use of ablation is, of course, to sublimate or to melt surface material of the vehicle, so that a large amount of heat flow from boundary layer is absorbed efficiently in latent heat for phase change of the surface material so as to diminish heat conducting inside the vehicle considerably.

The aerodynamic study of ablation has already been developed in many papers [1], [2], [3], [4], [5], [6], [7], in which aerodynamic mechanism of shielding by vaporization is shown to be equivalent to that of the boundary layer flow with coolant mass addition except for the effect of the latent heat. However, these approaches are based upon simple evaluation of transport properties in boundary layer flow, which have been found to play an important role in controlling flow characteristics of the ablating field.

Nevertheless, simple analyses proposed by Roberts [2], [3], [4] seem to be remarkable in the sense that they may give a qualitative aspect in estimation of effect on ablation rate of various aerodynamic and substantial parameters, which come into ablation phenomenon, rather than quantitative information.

With emphasis being laid on an explicit prediction of aerodynamic mechanism of shielding by vaporization, Karashima and Kubota [8] proposed an analytical approach to stagnation point ablation associated with hypersonic flight of blunt-nosed bodies of revolution, in which transport coefficients in laminar boundary layer flow of binary gas mixture were exactly estimated by use of atomic kinetic theory of gases. This approach, although complicated in numerical computation, enables to evaluate all physical quantities concerning ablative field uniquely under the given perimetric conditions and may be useful one in the sense that it clarifies the essential aspect of aerodynamic mechanism of ablation not only qualitatively but also quantitatively.

On the other hand, in contrast with many existing works on stagnation point ablation, there seem to exist few previous approaches to downstream characteristics of the ablative field. Of course, this may be due to the fact that the aerodynamic heating downstream of stagnation point has already been revealed, from conventional boundary layer theory, to decrease considerably. Nevertheless, it seems further to be of much interest to investigate detailed characteristics of the ablative field downstream of the stagnation point in order to clarify the quantitative information.

It is a purpose of the present paper to propose an approximate, analytical approach to sublimating ablation downstream of stagnation point of blunt-nosed bodies of revolution. By introducing a concept of small perturbation, the development of the approach is made for extending the range of applicability of the stagnation point solutions proposed by Karashima and Kubota [8] to downstream region in such a way that local ablative field is considered to consist of a basic field corresponding to the stagnation point solutions upon which is superimposed a perturbation field due to body curvature.

In order to simplify the analysis, several assumptions are introduced which do not degenerate the essential feature of the problem. Numerical calculations are carried out for teflon ablator and the results are compared with experimental data.

2. Preliminary Discussion on Validity of the Assumption of Quasi-Steady Ablation

Since ablation rate $\dot{m}$ (mass loss rate of solid material per unit surface area and unit time) is a kind of chemical reaction rates, any physical phenomenon involving ablation is essentially non-steady in a sense that it may cause a net change of body shape with time. However, if the point of interest is restricted to the very vicinity of stagnation point of blunt-nosed bodies of revolution, it has been already found from experimental investigation [9] that the ablation rate becomes either independent or dependent very slightly of time after a certain transient time is passed, and this gives an experimental evidence to the assumption of steady ablation made in the theoretical approach proposed by Karashima and Kubota [8].

However, in case of developing a theoretical approach applicable further to the region downstream of stagnation point, the assumption of steady ablation seems, in exactness, to be incompatible with real physical feature of the problem in a sense that a net change of geometrical body contour may be inevitable with time, since local surface velocity due to ablation should be, in general, a function of both time and a local coordinate measured along body contour from stagnation point.

It must be noted here that an exact steady ablation may be specified as the state where local surface velocity $\bar{v}$ parallel to the axis of symmetry is constant everywhere so as to result in no net change of geometrical body contour with time, as is seen in Fig. 1. Consequently, local ablation rate may be obtained in a form

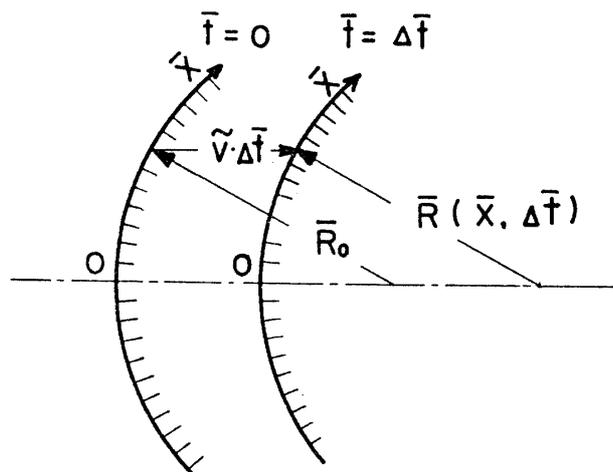$$(\bar{v})_{\text{steady}} = \bar{v}_0 = \text{const} ,$$

FIG. 1. Illustration of change of body curvature with time.

$$(\dot{m})_{\text{steady}} = \bar{\rho}_b \tilde{v} \cos \frac{\bar{x}}{\bar{R}_0} ,$$

$$= \dot{m}_0 \cos \frac{\bar{x}}{\bar{R}_0} , \tag{2.1}$$

where $\dot{m}_0$ denotes ablation rate just at the stagnation point.

As has been already mentioned previously, the local surface velocity $\tilde{v}$ should be, in general, a function of $\bar{x}$ and $\bar{t}$ and, therefore, the exact steady ablation cannot exist. However, in case that the rate of change of body shape with time can be considered to be very small, the assumption of quasi-steady ablation may be applicable for the reason that the geometrical body contour changes very slowly with time. Fortunately, an experimental study [10] has already revealed a remarkable fact that $\tilde{v}$ downstream of stagnation point tends to become either independent or dependent very slightly of time as well as that at the stagnation point after a certain transient time is passed. This fact together with small value of ablation rate seems to be capable of satisfying the requirement for quasi-steady assumption.

In order to examine this circumstance in more quantitative detail, consider the change of local radius of body curvature with time due to ablation. For the purpose of simplifying the argument, it is to be assumed that the body has initially a hemi-spherical contour with nose radius of $\bar{R}_0$ at a specified reference time $\bar{t} = 0$ (see Fig. 1) and the corresponding surface velocity is independent of time. Then, the surface velocity distribution may be expressed within the accuracy of the first-order perturbation, analogously to Eq. (2.1), in a form

$$\tilde{v}(\bar{x}) = \tilde{v}_0 - \tilde{v}_1 \left(\frac{\bar{x}}{\bar{R}_0} \right)^2 + O \left\{ \left(\frac{\bar{x}}{\bar{R}_0} \right)^4 \right\} , \tag{2.2}$$

$$\frac{\bar{x}}{\bar{R}_0} < 1 \quad \text{for stagnation region}$$

where $\tilde{v}_1$ is a constant with same order of magnitude as surface velocity just at the

stagnation point, $\bar{v}_0$. The first-order perturbation approach such as expressed by Eq. (2.2) may be reasonable, since it will be consistent with solutions of appropriate boundary layer equations predicting the ablative flow field near the region of stagnation point of blunt-nosed bodies of revolution. Thus, the change of local radius of body curvature with time may be obtained as

$$\frac{\bar{R}(\bar{x},\bar{t})}{\bar{R}_0}=\frac{1}{1-\tau}-\frac{3}{2}\frac{\tau(3-3\tau+\tau^2)}{(1-\tau)^2}\left(\frac{\bar{x}}{\bar{R}_0}\right)^2+\cdots,$$

$$\tau=\tau(\bar{t})=\frac{2\bar{v}_1\bar{t}}{\bar{R}_0}.$$

$$(2.3)$$

From Eq. (2.3), it will be easily known that, if $\tau(\Delta\bar{t})$ defined for a finite time interval of $\Delta\bar{t}$ is very small compared with unity and, hence, negligible, the second term in right hand side of Eq. (2.3) may also be negligible within the accuracy of the first-order perturbation approach, that is

$$\tau(\Delta\bar{t})=\frac{2\bar{v}_1\Delta\bar{t}}{\bar{R}_0}\ll 1, \qquad 1-\tau(\Delta\bar{t})\doteqdot 1,$$

$$\therefore \quad \tau(\Delta\bar{t})\left(\frac{\bar{x}}{\bar{R}_0}\right)^2\ll O\left\{\left(\frac{\bar{x}}{\bar{R}_0}\right)^2\right\}.$$

Hence,

$$\frac{\bar{R}}{\bar{R}_0}=1+O\left\{\frac{2\bar{v}_1\Delta\bar{t}}{\bar{R}_0}\left(\frac{\bar{x}}{\bar{R}_0}\right)^2\right\}.$$

$$(2.4)$$

Eq. (2.4) clearly indicates that the local radius of body curvature near the region of stagnation point is almost unchanged within a time interval of $\Delta\bar{t}$ which is finite but is not so large, if $|\bar{v}_1/\bar{R}_0|$ is very small. This, in turn, can be interpreted into a statement that the assumption of quasi-steady ablation may be applicable with a satisfactory accuracy. Thus, validity of the quasi-steady assumption depends mainly upon order of magnitude of $|\bar{v}_1/\bar{R}_0|$.

The order of magnitude of $|\bar{v}_1/\bar{R}_0|$ may be evaluated from the results given in reference [8], since $\bar{v}_1$ is of the same order of $\bar{v}_0$. For hemi-sphere models made of teflon with radius of nose curvature of order of 1 cm, $\bar{v}_0$ is found to be of order of 10^{-3} cm·sec^{-1} at free stream stagnation temperature near 10^3 degree in centigrade. Therefore,

$$\left|\frac{\tau(\Delta\bar{t})}{\Delta\bar{t}}\right|=\left|\frac{2\bar{v}_1}{\bar{R}_0}\right|\sim\left|\frac{\bar{v}_0}{\bar{R}_0}\right|\sim O(10^{-3}\ \mathrm{sec}^{-1}),$$

$$(2.5)$$

which clearly implies that the body shape changes very slowly with time, thus giving an evidence to the statement that the quasi-steady assumption may be available within the accuracy of the first-order perturbation approach.

3. Basic Equations for Boundary Layer Flow

In the last section it has been confirmed that the assumption of quasi-steady

ablation may be reasonably applicable to the region near downstream of stagnation point of blunt-nosed bodies of revolution, if surface velocity due to ablation is small. This assumption seems to be very useful for the boundary layer flow involving ablation, since it enables to reduce the time-dependent boundary layer equations to the steady ones. Therefore, with this assumption the present approach is developed together with several additional assumptions which may be summarized as follows;

(1)   quasi-steady sublimating ablation,
(2)   boundary layer thickness is small compared with radius of body curvature,
(3)   non-reacting laminar boundary layer of binary gas mixture,
(4)   no radiation.

The assumptions [3] and [4] are made only for the purpose of simplifying the analysis and do not degenerate the essential mechanism of the ablation. The effect of chemical reactions and associated radiation involved in the boundary layer flow of multi-components gas mixture may be taken into consideration in quite the same way as will be presented in the subsequent development.

Let the origin of an orthogonal coordinates system $(\bar{X}, \bar{Y})$ fixed in space be taken at the stagnation point of blunt-noted axially symmetric bodies at a specified reference time, $\bar{X}$-axis being taken along a meridian line of the surface as shown in Fig. 2, then, the boundary layer equations may be written as

$$\frac{\partial(\bar{\rho}\bar{r}_0)}{\partial \bar{t}} + \frac{\partial(\bar{\rho}\bar{U}\bar{r}_0)}{\partial \bar{X}} + \frac{\partial(\bar{\rho}\bar{V}\bar{r}_0)}{\partial \bar{Y}} = 0 , \tag{3.1a}$$

$$\bar{\rho}\frac{\partial \bar{U}}{\partial \bar{t}} + \bar{\rho}\bar{U}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{\rho}\bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}} = -\frac{\partial \bar{P}}{\partial \bar{X}} + \frac{\partial}{\partial \bar{Y}}\left(\mu\frac{\partial \bar{U}}{\partial \bar{Y}}\right) , \tag{3.1b}$$

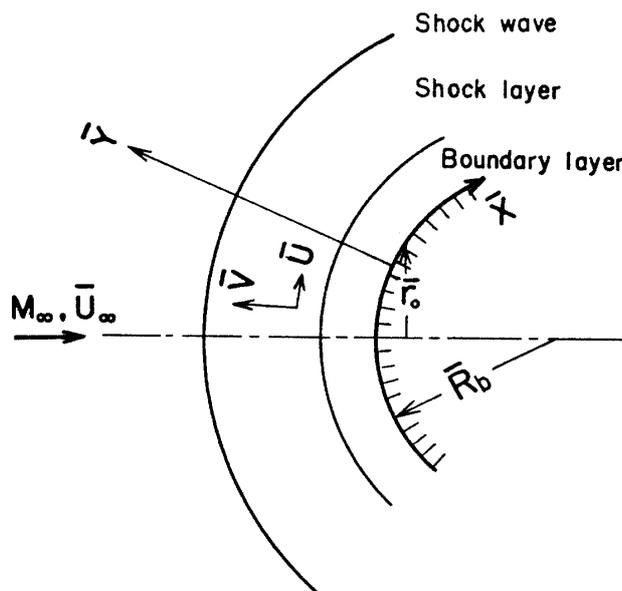$$\bar{\rho}\frac{\bar{U}^2}{R_b} = \frac{\partial \bar{P}}{\partial \bar{Y}} , \tag{3.1c}$$

FIG. 2.   Orthogonal coordinates system.

$$\bar{\rho}\frac{\partial \bar{H}}{\partial \bar{t}} + \bar{\rho}\bar{U}\frac{\partial \bar{H}}{\partial \bar{X}} + \bar{\rho}\bar{V}\frac{\partial \bar{H}}{\partial \bar{Y}} = \frac{\partial \bar{p}}{\partial \bar{t}} + \frac{\partial}{\partial \bar{Y}}\left[\frac{\bar{\mu}}{P}\frac{\partial \bar{H}}{\partial \bar{Y}} + \bar{\mu}\left(1 - \frac{1}{P}\right)\frac{1}{2}\frac{\partial \bar{U}^2}{\partial \bar{Y}}\right]$$

$$- \frac{\partial}{\partial \bar{Y}}\left[\left(\frac{1}{Le} - 1\right)\bar{\rho}\bar{D}(\bar{C}_{p1} - \bar{C}_{p2})\bar{T}\frac{\partial K}{\partial \bar{Y}}\right] , \quad (3.1d)$$

$$\bar{\rho}\frac{\partial K}{\partial \bar{t}} + \bar{\rho}\bar{U}\frac{\partial K}{\partial \bar{X}} + \bar{\rho}\bar{V}\frac{\partial K}{\partial \bar{Y}} = \frac{\partial}{\partial \bar{Y}}\left(\bar{\rho}\bar{D}\frac{\partial K}{\partial \bar{Y}}\right) , \quad (3.1e)$$

where $\bar{H}$ is total enthalpy defined by the equation

$$\bar{H} = \bar{C}_p\bar{T} + \frac{1}{2}\bar{U}^2 . \quad (3.2)$$

Since surface velocity $\bar{v}_b$ normal to the ablative surface is assumed to be independent of time, the above equations can be reduced to a set of steady equations expressed in an orthogonal coordinates system $(\bar{x}, \bar{y})$ fixed on moving body surface by use of a transformation of variables such as

$$\left.\begin{array}{l} \bar{t} = \bar{t} , \\ \bar{x} = \bar{X}, \\ \bar{y} = \bar{Y} - \bar{v}_b(\bar{x})\bar{t} , \\ \bar{u} = \bar{U} , \\ \bar{v} = \bar{V} - \bar{v}_b(\bar{x}) . \end{array}\right\} \quad (3.3)$$

Therefore, by use of the quasi-steady assumption, the time derivative can be transformed into

$$\frac{\partial}{\partial \bar{t}} = \frac{\partial}{\partial \bar{t}} + \frac{\partial}{\partial \bar{y}}\frac{\partial \bar{y}}{\partial \bar{t}} = -\bar{v}_b\frac{\partial}{\partial \bar{y}} .$$

Moreover, the following non-dimensional expressions are introduced for simplicity;

$$\left.\begin{array}{l} x = \dfrac{\bar{x}}{\bar{x}^*}(\bar{x}^* = \bar{R}_b) , \quad y = \dfrac{\bar{y}}{\bar{y}^*}\left(\bar{y}^* = \sqrt{\dfrac{\bar{\mu}_s\bar{R}_b}{\bar{\rho}_s\bar{u}^*}}\right) , \quad r_0 = \dfrac{\bar{r}_0}{\bar{R}_b} , \\[3mm] u = \dfrac{\bar{u}}{\bar{u}^*} , \quad v = \dfrac{\bar{v}}{\bar{u}^*} , \quad \rho = \dfrac{\bar{\rho}}{\bar{\rho}_s} , \quad T = \dfrac{\bar{T}}{\bar{T}_{st}} , \quad H = \dfrac{\bar{H}}{\bar{H}^*} = \dfrac{\bar{H}}{\bar{C}_{p2}\bar{T}_{st}} , \\[3mm] C_p = \dfrac{\bar{C}_p}{\bar{C}_{p2}} , \quad \mu = \dfrac{\bar{\mu}}{\bar{\mu}_s} , \quad \kappa = \dfrac{\bar{\kappa}}{\bar{\kappa}_s} , \quad D = \dfrac{\bar{\rho}_s}{\bar{\mu}_s}\bar{D} , \quad p = \dfrac{\bar{p}}{\bar{\rho}_s\bar{u}^{*2}} . \end{array}\right\} \quad (3.4)$$

Before presenting the transformed equations, an additional discussion must be made on $\bar{Y}$-momentum equation. By use of Eqs. (3.3) and (3.4), it can be reduced to

$$\frac{\partial p}{\partial y} = \frac{\bar{y}^*}{\bar{R}_b}\rho u^2 = \left(\frac{\bar{\mu}_s}{\bar{\rho}_s\bar{u}^*\bar{R}_b}\right)^{\frac{1}{2}}\rho u^2 . \quad (3.5)$$

Order estimation of the right-hand side of the above equation gives

$$\left(\frac{\bar{\mu}_s}{\bar{\rho}_s\bar{u}^*\bar{R}_b}\right)^{\frac{1}{2}} \sim O\left(\frac{1}{\sqrt{Re}}\right) \sim O\left(\frac{\bar{\delta}}{\bar{R}_b}\right) \; ,$$

$$\rho \sim O(1) \; , \qquad u \sim O\left(\frac{\bar{x}}{\bar{R}_b}\right) \; ,$$

where $\bar{\delta}$ and Re denote thickness of boundary layer and Reynolds number referred to free stream conditions and body radius of curvature, respectively. Consequently, Eq. (3.5) results, from the assumption [2], in

$$\frac{\partial p}{\partial y} \sim O\left\{\frac{\bar{\delta}}{\bar{R}_b}\left(\frac{\bar{x}}{\bar{R}_b}\right)^2\right\} \ll O\left\{\left(\frac{\bar{x}}{\bar{R}_b}\right)^2\right\} \; ,$$

which clearly indicates that y-momentum equation is negligible within the accuracy of the first-order perturbation approximation.

Thus, the boundary layer equations may be reduced to

$$\frac{\partial(\rho u r_0)}{\partial x} + \frac{\bar{x}^*}{\bar{y}^*}\frac{\partial(\rho v r_0)}{\partial y} = 0 \; , \tag{3.6a}$$

$$\rho u\frac{\partial u}{\partial x} + \frac{\bar{x}^*}{\bar{y}^*}\rho v\frac{\partial u}{\partial y} = \rho_e u_e\frac{\partial u_e}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \; , \tag{3.6b}$$

$$\frac{\partial p}{\partial y} = 0 \; , \tag{3.6c}$$

$$\rho u\frac{\partial H}{\partial x} + \frac{\bar{x}^*}{\bar{y}^*}\rho v\frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left[\frac{\mu}{P}\frac{\partial H}{\partial y} + \frac{\bar{u}^{*2}}{\bar{H}^*}\mu\left(1-\frac{1}{P}\right)\frac{1}{2}\frac{\partial u^2}{\partial y}\right]$$

$$+ \frac{\partial}{\partial y}\left[\left(\frac{1}{Le}-1\right)\alpha_2\rho D T\frac{\partial K}{\partial y}\right] \; , \tag{3.6d}$$

$$\rho u\frac{\partial K}{\partial x} + \frac{\bar{x}^*}{\bar{y}^*}\rho v\frac{\partial K}{\partial y} = \frac{\partial}{\partial y}\left(\rho D\frac{\partial K}{\partial y}\right) \; , \tag{3.6e}$$

where

$$\left.\begin{array}{l} H = C_p T + \dfrac{1}{2}u^2\dfrac{\bar{u}^{*2}}{\bar{H}^*} \; , \\[3mm] \alpha_2 = 1 - \dfrac{\bar{C}_{p1}}{\bar{C}_{p2}} \; . \end{array}\right\} \tag{3.7}$$

4. DEVELOPMENT OF THE FIRST-ORDER PERTURBATION THEORY

Since the assumption of quasi-steady ablation is applicable within the accuracy of the first-order perturbation scheme as has been already mentioned in section 2, the analytical approach to boundary layer flow downstream of stagnation point can be developed in such a way that the flow field is considered to consist of a basic field corresponding to the stagnation point upon which is superimposed a perturbation field due to body curvature.

However, before presenting detailed development, the inviscid flow conditions at outer edge of the boundary layer must be first specified near the stagnation point. In the present approach it is assumed that they are given by the constant density solutions of hypersonic flow past a sphere [11]. It must be noted that numerical solutions proposed by Van Dyke [12] may be available if a more rigorous specification of the inviscid flow conditions is required.

Therefore, velocity along surface is given by the equation

$$\bar{u}_e = \bar{u}_\infty \sqrt{\frac{8\varepsilon}{3}} \frac{\sin x}{1+\lambda_0} ,$$

which may be reduced, within the accuracy of the first-order perturbation, to

$$\bar{u}_e = \bar{u}_\infty \sqrt{\frac{8\varepsilon}{3}} \frac{1}{1+\lambda_0} \left(x - \frac{x^3}{6} \right) + O(x^5) , \qquad (4.1)$$

where

$$\left.\begin{array}{l}
\varepsilon = \dfrac{(\gamma-1)M_\infty^2 + 2}{(\gamma+1)M_\infty^2} , \\[4mm]
\lambda_0 = \dfrac{\varepsilon}{1 + \sqrt{\dfrac{8\varepsilon}{3} - \varepsilon}} .
\end{array}\right\} \qquad (4.2)$$

Density and viscosity at outer edge of the boundary layer and cylindrical radius of body contour may be expressed, respectively, as

$$\left.\begin{array}{l}
\rho_e = 1 . \\[1mm]
\mu_e = 1 + \mu_1 x^2 + O(x^4) , \\[2mm]
r_0 = \dfrac{\bar{r}_0}{R_b} = x - \dfrac{1}{6}x^3 + O(x^5) .
\end{array}\right\} \qquad (4.3)$$

where the body contour has been assumed to be hemi-sphereical and

$$\mu_1 = -\frac{1}{4} 3^{-\frac{2}{3}} W , \qquad (4.4)$$

where W will be expressed in Eq. (4.17)

Introducing Lees-Dorodnitsyn transformation

$$s = \int_0^x \rho_e \mu_e r_0^2 dx , \qquad \eta = \frac{\rho_e r_0}{s^m} \int_0^y \frac{\rho}{\rho_e} dy , \qquad (4.5)$$

then, gives a relation between x and s near the stagnation point such as

$$x = 3^{\frac{1}{3}} s^{\frac{1}{3}} - \frac{3}{5} \left(\mu_1 - \frac{1}{3} \right) s + O(s^{\frac{5}{3}}) . \qquad (4.6)$$

Therefore, by use of Eq. (4.6), Eq. (4.1) may be reexpressed as

$$\bar{u}_e = \bar{u}_\infty 3^{\frac{1}{3}} \sqrt{\frac{8\varepsilon}{3}} \frac{1}{1+\lambda_0} \left[s^{\frac{1}{3}} - \frac{3^{\frac{2}{3}}}{10} (2\mu_1 + 1)s \right] + O(s^{\frac{5}{3}}) .$$

It will be found convenient for subsequent manipulations to choose $\bar{u}^*$, which is used for normalization of velocity components, as

$$\bar{u}^* = 3^{\frac{1}{3}} \sqrt{\frac{8\varepsilon}{3}} \frac{\bar{u}_\infty}{1+\lambda_0} . \tag{4.7}$$

Thus, velocity, density and viscosity at outer edge of the boundary layer can be rewritten, respectively, as

$$\left.\begin{array}{l} u_e = s^{\frac{1}{3}} - \dfrac{3^{\frac{2}{3}}}{10}(2\mu_1 + 1)s + O(s^{\frac{5}{3}}) , \\[2mm] \rho_e = 1 , \\[2mm] \mu_e = 1 + 3^{\frac{2}{3}}\mu_1 s^{\frac{2}{3}} + O(s^{\frac{4}{3}}) \end{array}\right\} \tag{4.8}$$

Continuity equation, Eq. (3.6a), may be accounted for by introducing a stream function defined as

$$\phi_x = -\frac{\bar{x}^*}{\bar{y}^*}\rho v r_0 , \qquad \phi_y = \rho u r_0 . \tag{4.9}$$

If the stream function is assumed to have a form

$$\phi = s^{m+k} f_0(\eta) + q_1 s^{m+k+n} f_1(\eta) + \cdots , \tag{4.10}$$

where $q_1, \cdots$, etc. are constants, then, velocity component u in the boundary layer can be expressed as

$$u = s^k f_0'(\eta) + q_1 s^{k+n} f_1'(\eta) + \cdots . \tag{4.11}$$

Therefore, by comparing Eq. (4.11) with Eq. (4.8), it will be found that the power indices k and n must be, respectively,

$$k = \frac{1}{3} , \qquad n = \frac{2}{3} , \tag{4.12}$$

and, consequently,

$$u = s^{\frac{1}{3}} f_0'(\eta) + q_1 s f_1'(\eta) + \cdots . \tag{4.13}$$

In quite the same way, detailed examinations reveal that perturbation forms of total enthalpy function and concentration consistent with a set of equations given by Eqs. (3.6b) to (3.6e) must have forms

$$\left.\begin{array}{l} H = g_0(\eta) + s^{\frac{2}{3}} g_1(\eta) + O(s^{\frac{4}{3}}) , \\[2mm] K = z_0(\eta) + s^{\frac{2}{3}} z_1(\eta) + O(s^{\frac{4}{3}}) , \end{array}\right\} \tag{4.14}$$

where a power index m consistent with the set of equations has been found to be

$$m = \frac{1}{3} \, .$$
(4.15)

From Eq. (3.7), temperature may be expressed by

$$\frac{T}{T_e} = \frac{H - \frac{1}{2} u^2 \frac{\bar{u}^{*2}}{\bar{H}^*}}{(1 - \alpha_2 k) \left(H_e - \frac{1}{2} u_e^2 \frac{\bar{u}^{*2}}{\bar{H}^*} \right)} \, ,$$

which is further reduced, by use of Eqs. (4.13) and (4.14), to

$$\frac{T}{T_e} = \Gamma_0 + \Gamma_1 s^{\frac{2}{3}} + O(s^{\frac{4}{3}}) \, ,$$
(4.16)

where

$$\Gamma_0 = \frac{g_0}{1 - \alpha_2 z_0} \, ,$$

$$\Gamma_1 = \frac{1}{1 - \alpha_2 z_0} \left\{ g_1 - \frac{1}{2} W f_0''^2 + g_0 \left(\frac{1}{2} W + \frac{\alpha_2 z_1}{1 - \alpha_2 z_0} \right) \right\} \, , \qquad (4.17)$$

$$W = 3^{\frac{2}{3}} \frac{8\varepsilon}{3} \frac{M_\infty^2}{(1 + \lambda_0)^2} \left(1 + \frac{\gamma - 1}{2} M_\infty^2 \right)^{-1} (\gamma - 1) \, .$$

Here it must be noted that total enthalpy at outer edge of the boundary layer is given by

$$H_e = 1 - \frac{\gamma - 1}{\gamma + 1} \varepsilon \approx 1 \, ,$$

and is considered approximately to be unity for hypersonic flow.

An expression of density may be obtained from the equation of state in a form

$$\frac{\rho_e}{\rho} = (1 - \alpha_1 k) \frac{T}{T_e} \, ,$$

where

$$\alpha_1 = 1 - \frac{M_2}{M_1} \, ,$$
(4.18)

and is also reduced, within the accuracy of the first-order perturbation, to

$$\frac{\rho_e}{\rho} = \Lambda_0 + \Lambda_1 s^{\frac{2}{3}} + O(s^{\frac{4}{3}}) \, ,$$
(4.19)

where

$$\Lambda_0 = \Gamma_0 (1 - \alpha_1 z_0) \, ,$$
$$\Lambda_1 = \Gamma_1 (1 - \alpha_1 z_0) - \alpha_1 \Gamma_0 z_1 \, . \qquad (4.20)$$

Estimation of the transport coefficients involved in Eqs. (3.6b) to (3.6e) is another point of interest. The evaluation can be made by use of the atomic kinetic theory of gases and each transport coefficient is obtained as a function of total enthalpy and concentration, which may be further reduced to a first-order perturbation form as well. Detailed procedure for evaluation of transport coefficients is shown in Appendices A and B.

Substitution of Eqs. (4.13), (4.14), (4.16) and (4.19) together with transport coefficients developed in the appendices into Eqs. (3.6b) to (3.6e) and equating like power of s yields, as leading terms, the following simultaneous equations with respect to f_0, g_0 and z_0;

$$
\begin{aligned}
&(C_0 f_0'')' + \frac{2}{3} f_0 f_0'' + \frac{1}{3}\left[\frac{1-\alpha_1 z_0}{1-\alpha_2 z_0} g_0 - f_0'^2\right] = 0 , \\
&\left(\frac{C_0}{P_0} g_0'\right)' + \frac{2}{3} f_0 g_0' + (\phi_0 z_0')' = 0 , \\
&\left(\frac{C_0}{S_0} z_0'\right)' + \frac{2}{3} f_0 z_0' = 0 .
\end{aligned}
\qquad (4.21)
$$

where

$$
\phi_0 = \frac{\alpha_2 g_0}{1-\alpha_2 z_0} \frac{(S_0 - P_0)C_0}{P_0 S_0} , \qquad (4.22)
$$

and C_0, P_0 and S_0 denote the leading terms of the perturbation expression of Chapman-Rubesin number, Prandtl number and Schmidt number, respectively, and may be obtained as a function of g_0 and z_0. This set of equations, Eq. (4.21), predicts the basic field corresponding to the stagnation point flow and is quite the same one as has been already developed in reference [8].

Equations which predict the first-order perturbation field may be obtained in the following forms;

$$
\begin{aligned}
&(C_0 f_1'')' + \frac{1}{q_1}(C_1 f_0'')' + \frac{2}{3} f_0 f_1'' - \frac{4}{3} f_0' f_1' + \frac{4}{3} f_0'' f_1 \\
&\qquad + \frac{1}{3q_1}\left\{A_1 - \frac{2 \cdot 3^{\frac{2}{3}}}{5}(2\mu_1 + 1)A_0\right\} = 0 , \\
&\left(\frac{C_0}{P_0} g_1'\right)' + \left\{\frac{1}{P_0}\left(C_1 - \frac{C_0 P_1}{P_0}\right)g_0'\right\}' + W\left\{\left(C_0 - \frac{C_0}{P_0}\right)f_0' f_0''\right\}' \\
&\qquad + \phi_1' - \frac{2}{3} f_0' g_1 + \frac{2}{3} f_0 g_1' + \frac{4}{3} q_1 f_1 g_0' = 0 , \\
&\left(\frac{C_0}{S_0} z_1'\right)' + \left\{\frac{1}{S_0}\left(C_1 - \frac{C_0 S_1}{S_0}\right)z_0'\right\}' + \frac{2}{3} f_0 z_1' - \frac{2}{3} f_0' z_1 + \frac{4}{3} q_1 f_1 z_0' = 0 ,
\end{aligned}
\qquad (4.23)
$$

where

$$\phi_1 = \alpha_2 C_0 \left(\frac{1}{P_0} - \frac{1}{S_0}\right) \left\{ \Gamma_0 z_1' + \left(\Gamma_1 - \frac{1}{2} W\Gamma_0\right) z_0' \right\}$$

$$+ \alpha_2 \left\{ \frac{1}{P_0} \left(C_1 - \frac{C_0 P_1}{P_0}\right) - \frac{1}{S_0} \left(C_1 - \frac{C_0 S_1}{S_0}\right) \right\} \Gamma_0 z_1' , \qquad (4.24)$$

where C_1, P_1 and S_1 indicate perturbation coefficients of Chapman-Rubesin number, Prandtl number and Schmidt number, respectively, and are obtained as a function of f_0, g_0, g_1, z_0 and z_1. Eq. (4.23) denotes linear simultaneous equations with respect to f_1, g_1 and z_1 and the main purpose of the present study consists in finding out solutions to Eq. (4.23) under the given boundary conditions.

5. BOUNDARY CONDITIONS

It is clear that each of Eqs. (4.21) and (4.23) requires seven boundary conditions in order to settle a boundary value problem in a closed form. However, there seem to exist only four explicit conditions such as non-slip condition and other three conditions to make boundary layer flow compatible with outer inviscid flow. They are

$$\left.\begin{array}{ll} \bar{u}=0 & \text{at} \quad \bar{y}=0, \\[4pt] \bar{u}=\bar{u}_e, \quad \bar{H}=\bar{H}_e, \quad K=0 & \text{at} \quad \bar{y}=\bar{y}_e, \end{array}\right\} \qquad (5.1)$$

which can be readily reduced, by use of the perturbation scheme, to

$$\left.\begin{array}{l} f_0'=0 \quad \text{at} \quad \eta=0, \\[4pt] f_0'=1, \quad g_0^1=1, \quad z_0=0 \quad \text{at} \quad \eta=\infty \end{array}\right\} \text{ for basic field} \qquad (5.2a)$$

and

$$\left.\begin{array}{l} f_1'=0 \quad \text{at} \quad \eta=0 \\[4pt] f_1'=1, \quad g_1=0, \quad z_1=0 \quad \text{at} \quad \eta=\infty \end{array}\right\} \text{ for perturbation field,} \qquad (5.2b)$$

where q_1 in Eq. (4.11) has been chosen as

$$q_1 = -\frac{3^{\frac{2}{3}}}{10}(2\mu_1 + 1) . \qquad (5.3)$$

The other three boundary conditions required for each of Eqs. (4.21) and (4.23) may be derived from physical conditions at the ablative surface. The one is a relation between total enthalpy and temperature at the ablative surface. With the non-slip condition this relation can be directly obtained from Eq. (3.2) as

$$\bar{H}_w = (\bar{C}_p \bar{T})_w = \bar{C}_{p2}(1 - \alpha_2 K_w)\bar{T}_w . \qquad (5.4)$$

In quite the same way, a physical condition that there is no net mass transfer of air into the wall may provide a relation

$$-\left(\bar{\rho}\bar{D}\,\frac{\partial K}{\partial \bar{y}}\right)_w = (1 - K_w)(\bar{\rho}\bar{v})_w \,. \tag{5.5}$$

The final condition may be given by the heat transfer balance at the ablative surface such as

$$\left(\bar{\kappa}\,\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=+0} = \bar{L}(\bar{\rho}\bar{v})_w + \bar{\kappa}_b\sqrt{\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)^2_{\bar{y}=-0} + \left(\frac{\partial \bar{T}}{\partial \bar{x}}\right)^2_{\bar{y}=-0}} \,. \tag{5.6}$$

In order to reduce Eq. (5.6), the heat flow conducting inside the body must be first clarified. For this purpose it is assumed that diffusion of heat in $\bar{x}$-direction is negligible inside the body compared with that in $\bar{y}$-direction. In the range of free stream stagnation temperature beyond $1000°C$, this assumption is found to be reasonable near the region of stagnation point for hemi-sphere models made of teflon with nose radius of curvature of order of 1 cm. The brief argument on validity of this assumption is made in Appendix C in detail. Thus, the heat conduction equation may be expressed by

$$\bar{\kappa}_b\,\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} = \dot{\bar{m}}\bar{C}_b\,\frac{\partial \bar{T}}{\partial \bar{y}} \,, \tag{5.7}$$

where
$$\dot{\bar{m}} = -\bar{\rho}_b\bar{v}_b = (\bar{\rho}v)_w \,. \tag{5.8}$$

Under the boundary conditions

$$\left.\begin{array}{l} \bar{T} = \bar{T}_w(\bar{x}) \quad \text{at} \quad \bar{y}=0 \,, \\[2mm] \bar{T} = \bar{T}_b = \text{const} \quad \text{at} \quad \bar{y}=-\infty \,, \end{array}\right\} \tag{5.9}$$

the solution to Eq. (5.7) can be obtained as

$$\bar{T} = \bar{T}_b + \{\bar{T}_w(\bar{x}) - \bar{T}_b\}\,\exp\left[\frac{\bar{C}_b\dot{\bar{m}}(\bar{x})}{\bar{\kappa}_b}\bar{y}\right] \,. \tag{5.10}$$

On the other hand, the distribution of wall temperature consistent with the present development must have a form

$$T_w = T_{w_0} + T_{w_1}x^2 + O(x^4) \,, \tag{5.11}$$

where T_{w_0} and T_{w_1} are constants and are unknown a priori. Thus, by use of Eqs. (5.10) and (5.11), three physical conditions, Eqs. (5.4), (5.5) and (5.6), can be reduced to the following two sets of boundary conditions;
for basic flow

$$\left.\begin{array}{l} g_{0w} = (1 - \alpha_2 z_{0w})T_{w_0} \,, \\[3mm] \left(\dfrac{C_0}{S_0}z_0'\right)_w = \dfrac{2}{3}(1 - z_{0w})f_{0w} \,, \\[4mm] \dfrac{F_0(z_{0w})}{1 - \alpha_1 z_{0w}}\left(\dfrac{1}{T_{w_0}}\right)^{\frac{1}{2}}\left(\dfrac{g_0}{1 - \alpha_1 z_0}\right)'_w = -\dfrac{2}{3}P_2 f_{0w}\left\{\dfrac{\bar{C}_b}{\bar{C}_{p_2}}(T_{w_0} - T_b) + l\right\} \,, \end{array}\right\} \tag{5.12a}$$

for perturbation flow

$$g_{1w} = 3^{\frac{2}{3}}(1 - \alpha_2 z_{0w})T_{w_1} - \frac{\alpha_2 z_{1w}}{1 - \alpha_2 z_{0w}} g_{0w} \,,$$

$$\left(\frac{C_0}{S_0}\right)_w \left(z_1' + \frac{z_0' z_1}{1 - z_0}\right)_w = \frac{4}{3} q_1 f_{1w}(1 - z_{0w}) - \left\{\frac{1}{S_0}\left(C_0 - \frac{C_0 S_1}{S_0}\right)\right\}_w z_{0w}' \,,$$

$$\left(\Gamma_{1w}' - \frac{3^{\frac{2}{3}} T_{w_1} \Gamma_{0w}'}{T_{w_0}}\right) \frac{F_{0w} g_{0w}^{\frac{1}{2}}}{\sqrt{1 - \alpha_2 z_{0w}}} = -\Gamma_{0w}' F_{1w}$$

$$- P_2 T_{w0}\left[(1 - \alpha_1 z_{0w})\left[\frac{1}{3}\left\{2 \cdot 3^{\frac{2}{3}} \frac{\bar{C}_b}{\bar{C}_{p_2}} T_{w_1} + \frac{\bar{C}_{p_2}}{\bar{C}_b(T_{w_0} - T_b)}\left(\frac{3N\kappa_b T_{w_1}}{P_2 f_{0w}}\right)^2\right\}\right.$$

$$\left. + \left\{l + \frac{\bar{C}_b}{\bar{C}_{p_2}}(T_{w_0} - T_b)\right\}\left(\frac{1}{2} Wf_{0w} + \frac{4}{3} q_1 f_{1w}\right)\right]$$

$$\left. - \frac{2}{3} \alpha_1 f_{0w} z_{1w}\left\{l + \frac{\bar{C}_b}{\bar{C}_{p_2}}(T_w - T_b)\right\}\right) \,,$$

(5.12b)

where

$$l = \frac{\bar{L}}{\bar{C}_{p_2}\bar{T}_{st}} \,, \qquad N = \left[\frac{\{2\gamma M_\infty^2 - (\gamma - 1)\}^{\frac{1}{2}}\{(\gamma - 1)M_\infty^2 + 2\}^{\frac{3}{2}}}{3^{\frac{1}{2}}\sqrt{\frac{8\varepsilon}{3}\frac{1}{1 + \lambda_0}}(\gamma + 1)^2 M_\infty^3}\right]^{\frac{1}{2}} \frac{1}{\sqrt{R_e}} \,, \quad (5.13)$$

and $F_0(z_{0w})$, $F_1(g_{0w}, g_{1w}, z_{0w}, z_{1w})$ and P_2 are given in Appendices A and B.

6. ABLATION RATE

Since ablation rate, which is one of the most characteristic quantities for predicting the ablation phenomenon, is a kind of chemical reaction rates, it can be considered, in principle, to be expressed in terms of two thermodynamic variables of state in a form

$$\dot{m} = F(\bar{T}_w, \bar{P}) \,,$$ (6.1)

where a functional form F may be determined from chemical kinetics. However, in applying this relation to the problem under consideration, Eq. (6.1) is not sufficient to predict the ablative field in a closed form, since surface temperature $\bar{T}_w$, is not known a priori. As has been already pointed out in reference [8], the ablative field can be uniquely determined under the given perimetric conditions by matching of the static relation, Eq. (6.1), with the dynamic one derived aerodynamically through heat transfer balance at the ablative surface. Thus, the expression of aerodynamic relation for ablation rate becomes a point of interest.

Since ablation rate is defined as the mass loss rate of surface material per unit surface area and unit time, it is expressed as the mass flow rate of surface injection of coolant gas species due to ablation, that is

$$\dot{m} = (\overline{\rho v})_w = \overline{\rho}_s \overline{u}^*(\rho v)_w \ ,$$

and this may be reduced by use of Eqs. (4.9) and (4.10) to a perturbation form

$$\dot{m} = -\overline{\rho}_s \overline{u}^* \frac{\overline{y}^*}{\overline{x}^*} \frac{1}{r_0}(\phi_x)_w$$

$$= \dot{m}_0 + \dot{m}_1 x^2 + O(x^4)$$

$$= \dot{m}_0 + 3^{\frac{2}{3}} \dot{m}_1 s^{\frac{2}{3}} + O(s^{\frac{4}{3}}) \ , \tag{6.2}$$

where

$$\dot{m}_0 = -\frac{2\sqrt{6}}{3} J(M_\infty) \frac{1}{\sqrt{Re}} \overline{\rho}_{st} \overline{a}_{st} f_{0w} \ , \tag{6.3}$$

$$\dot{m}_1 = -\frac{2\sqrt{6}}{3} J(M_\infty) \frac{1}{\sqrt{Re}} \overline{\rho}_{st} \overline{a}_{st} \left\{ \frac{1}{10}(8\mu_1 - 1)f_{0w} + \frac{2}{3^{\frac{2}{3}}} q_1 f_{1w} \right\} \ . \tag{6.4}$$

$\overline{\rho}_{st}$ and $\overline{a}_{st}$ indicate density and speed of sound at stagnation in free stream, repectively, and Re is a Reynolds number defined by free stream conditions and radius of body curvature at stagnation point, that is

$$Re = \frac{\overline{\rho}_\infty \overline{u}_\infty \overline{R}_b}{\overline{\mu}_\infty} \ . \tag{6.5}$$

$J(M_\infty)$ denotes a function of free stream Mach number expressed by the equation

$$J(M_\infty) = \left(\sqrt{\frac{8\varepsilon}{3}} \frac{1}{1+\lambda_0} \right)^{\frac{1}{2}} \frac{M_\infty^{\frac{3}{2}} \{2\gamma M_\infty^2 - (\gamma-1)\}^{\frac{1}{4}}}{\{(\gamma-1)M_\infty^2 + 2\}^{\frac{3}{4}} \left(1 + \frac{\gamma-1}{2} M_\infty^2 \right)^{\frac{1}{\gamma-1}}} \ , \tag{6.6}$$

where ε and λ_0 are given by Eq. (4.2).

Since f_{0w} and f_{1w} involved in Eqs. (6.3) and (6.4) can be obtained as solutions of boundary layer equations, wall temperature T_w defined by Eq. (5.11) is included implicitly in them. Therefore, Eq. (6.2) may be considered as a dynamic relation between ablation rate and wall temperature expressed in a perturbation form of the first-order. Thus, the ablative field can be determined uniquely by solving simultaneous equations, Eqs. (6.1) and (6.2), with respect to $\dot{m}$ and $\overline{T}_w$.

7. SOLUTIONS FOR BASIC FIELD

The boundary value problem appropriate to the basic field—stagnation point—consists of a set of fundamental equations given by Eq. (4.21) and boundary conditions expressed by Eqs. (5.2a) and (5.12a). Here an emphasis must be laid on the fact that both fundamental equations and boundary conditions do not involve any perimetric condition except for T_{w_0} and T_b, since $T_s \fallingdotseq 1$ for any hypersonic stagnation flow. This clearly indicates that the solutions to Eq. (4.21) have a simi-

lar form irrespective of the perimetric conditions associated with the outer inviscid hypersonic flow.

The solutions to Eq. (4.21) have already been obtained numerically by Karashima and Kubota [8] for teflon ablator with the assumption that the boundary layer flow consists mainly of a binary gas mixture of air and C_2F_4-gas. This assumption has been confirmed quantitatively by Madorsky [13]. The static relation, Eq. (6.1), for teflon ablator has already been proposed by Rashis and Hopko [14], and the respective substantial properties for air, C_2F_4-gas and solid teflon used in the calculation are

air :

$$M_2 = 29$$

$$\bar{C}_{p2} = 0.28 \text{ cal} \cdot \text{gr}^{-1} \cdot \text{deg}^{-1}$$

$$\bar{\sigma}_2 = 3.62 \text{Å}$$

$$\Omega_2^{(2,2)*} = 0.79 ,$$

C_2F_4-gas :

$$M_1 = 100$$

$$\bar{C}_{p1} = 0.32 \text{ cal} \cdot \text{gr}^{-1} \cdot \text{deg}^{-1}$$

$$\bar{\sigma}_1 = 5.00 \text{Å}$$

$$\Omega_1^{(2,2)*} = 0.90 ,$$

solid teflon :

$$\bar{C}_b = 0.22 \text{ cal} \cdot \text{gr}^{-1} \cdot \text{deg}^{-1}$$

$$\bar{\rho}_b = 2.19 \text{ gr} \cdot \text{cm}^{-3}$$

$$\bar{\kappa}_b = 6.00 \times 10^{-4} \text{ cal} \cdot \text{cm}^{-1} \cdot \text{sec}^{-1} \cdot \text{deg}^{-1}$$

$$\bar{L} = 35 \text{ kcal} \cdot \text{mol}^{-1} .$$

In this section are presented several numerical results associated with the basic field, which are cited directly from reference [8], since they will become significant in the subsequent development concerning the first-order perturbation field. Fig. 3 shows an example of the solutions to Eq. (4.21) for $T_{w_0} = 0.633$. In Fig. 4 are presented variations of f_{0w}, f''_{0w}, and z_{0w} with T_{w_0} which are the most important results for the basic field. Fig. 5 shows variation of ablation rate at stagnation point with stagnation temperature in free stream and Fig.6 presents wall temperature at stagnation point. The last two figures are consistent with one another, since they are obtained as solutions of the simultaneous equations consisting of the static relation shown in Fig. 7 and the aerodynamic relation given by Eq. (6.3).

8. SOLUTIONS FOR THE FIRST-ORDER PERTURBATION FIELD

The boundary value problem appropriate to the first-order perturbation field consists of a set of fundamental equations expressed by Eq. (4.23) and boundary conditions given by Eqs. (5.2b) and (5.12b), in which all quantities subscribed by 0 have already been known from the solutions for the basic field. It must be noted that, in contrast with the basic field, the perturbation field does no longer be similar
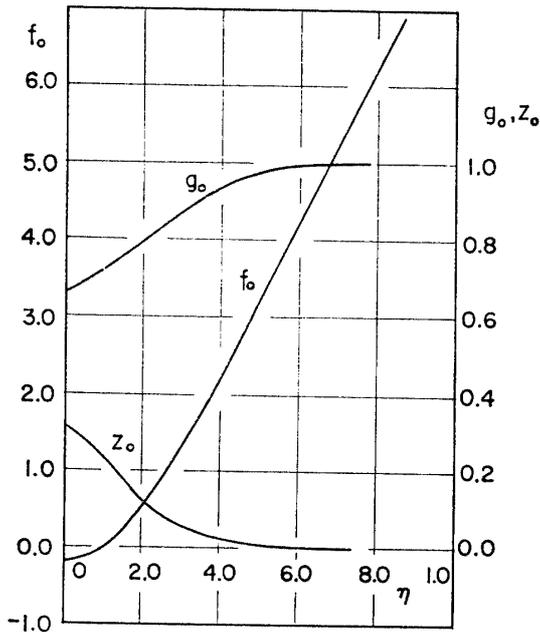
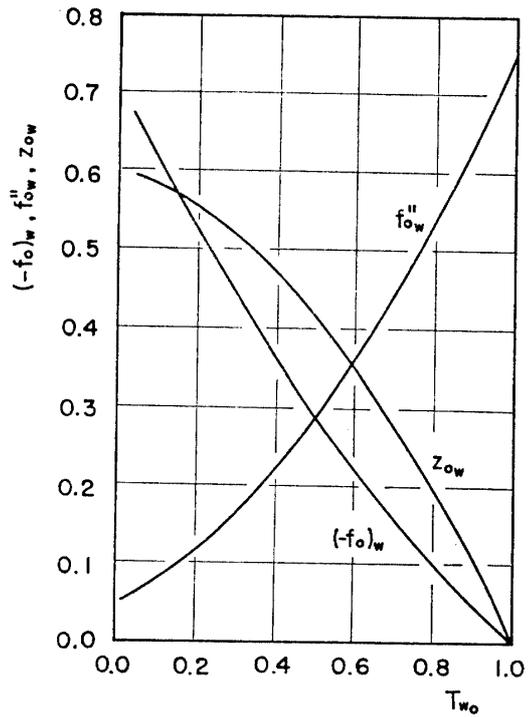FIG. 3.   Solutions to Eq. (4.21). $T_{w_0}=0.633$, $T_b=0.233$, teflon.

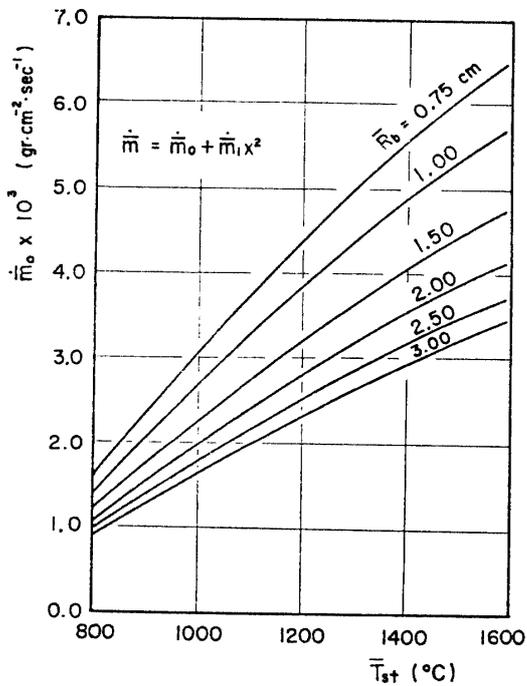FIG. 4.   Variation of f_{0w}, f''_{0w} and z_{0w} with T_{w_0} for teflon.

FIG. 5.   Variation of ablation rate at stagnation point with stagnation temperature in free stream. Teflon, $M_\infty=5.74$, $\bar{p}_{st}=1$ atm, $T_b=0.233$.
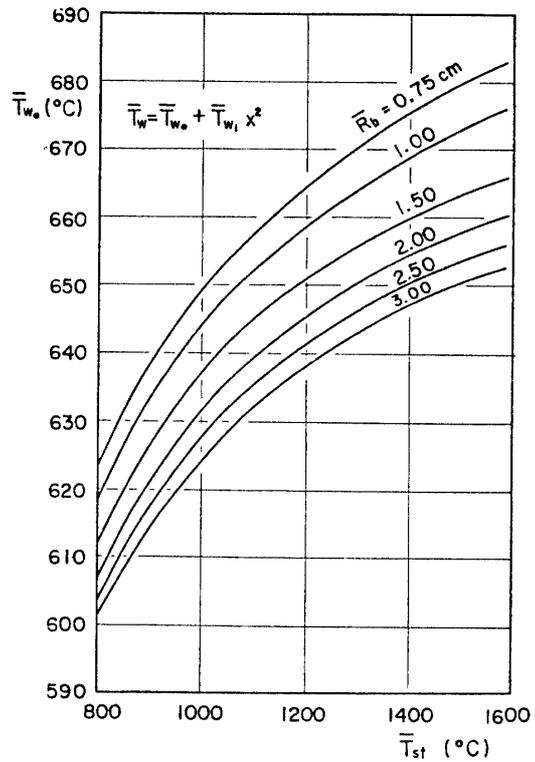
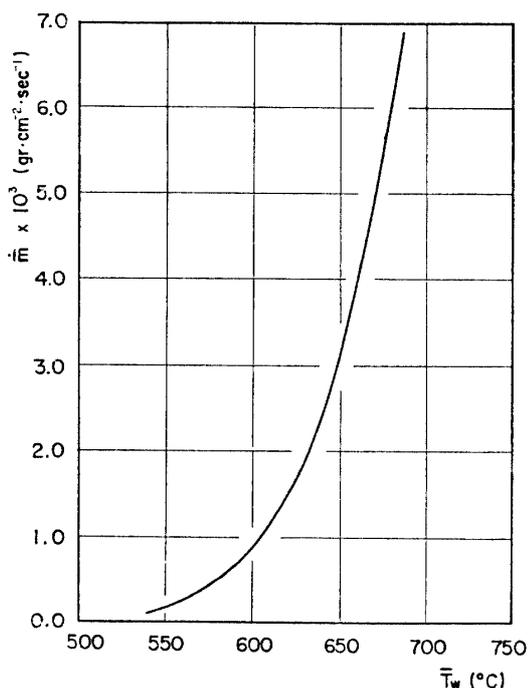FIG. 6.   Variation of wall temperature at stagnation point with stagnation temperature in free stream. Teflon, $M_\infty=5.74$, $\bar{p}_{st}=1$ atm, $T=0.233$.

FIG. 7.   Static relation between $\dot{m}$ and T_w for teflon (ref. 14).

with respect to the given perimetric conditions, since free stream Mach number and Reynolds number are involved, as parameters, in both the fundamental equations and boundary conditions. Therefore, the solutions to Eq. (4.23) must be obtained case by case for given perimetric conditions.

Although Eq. (4.23) may denote linear simultaneous equations with respect to f_1, g_1 and z_1, it seems that only a numerical method of integration is available. Furthermore, it is clear that the direct integration of Eq. (4.23) cannot be made readily starting at $\eta = 0$, since a mathematical difficulty arises from the fact of a two-point boundary value problem having its boundary conditions both at $\eta = 0$ and $\eta = \infty$. This characteristic is quite the same as is seen in the boundary value problem appropriate to the basic field.

In order to avoid this difficulty the same mathematical manipulations as were made in reference [8] for basic field are introduced for starting of the numerical integrations outwards from the body surface, which may be done by the following procedure. First, three explicit conditions at $\eta = \infty$ are to be excluded from the boundary conditions. These are

$$f_1' = 1, \quad g_1 = 0, \quad z_1 = 0 \quad \text{at} \quad \eta = \infty . \tag{8.1}$$

On the other hand, for the purpose of making up for shortage of the boundary conditions just excluded, other three alternate boundary values such as f_{1w}, f_{1w}'' and z_{1w} are then assumed at $\eta = 0$. These assumed values, in turn, turn out three appropriate boundary values of g_{1w}, z_{1w}' and g_{1w}' for the fixed perimetric conditions by use of Eqs. (5.12a) and (5.12b). Therefore, these six boundary values together with the non-slip condition, $f_{1w}' = 0$, give a set of boundary values sufficient for

starting of the numerical integration of Eq. (4.23) from $\eta = 0$.

It must be noticed here that the coefficient of perturbed wall temperature T_{w_1} (see Eq. (5.11)) involved in the condition of heat flow balance, Eq. (5.12b), is still unknown a priori. Although this fact may, at a glance, seem to result in shortage of a boundary value necessary for evaluating g'_{1w}, it will be easily found that T_{w_1} can be, in principle, determined uniquely by matching of a dynamic relation of the ablation rate given by Eq. (6.2) with the static one, Eq. (6.1). However, in the first-order perturbation field, it will be found more convenient to introduce this matching process in estimation of the boundary values given by Eq. (5.12b), since T_{w_1} cannot be independent of f_{1w}.

The facts that the basic field has already been determined for given perimetric conditions and the wall temperature distribution has been assumed in such a form as is given by Eq. (5.11) enable to expand Eq. (6.1) in a Taylor's series in the form

$$
\begin{aligned}
\dot{m} &= F(\overline{T}_w) \\
&= F(\overline{T}_{w_0}) + F'(\overline{T}_{w_0})\overline{T}_{w_1}x^2 + O(x^4) \\
&= F(\overline{T}_{w_0}) + 3^{\frac{2}{3}} F'(\overline{T}_{w_0})\overline{T}_{w_1}s^{\frac{2}{3}} + O(s^{\frac{4}{3}}) ,
\end{aligned} \tag{8.2}
$$

where

$$
F'(\overline{T}_{w_0}) = \left[\frac{dF}{d\overline{T}_w} \right]_{\overline{T}_w = \overline{T}_{w_0}} . \tag{8.3}
$$

Therefore, by comparing Eq. (8.2) with Eq. (6.2), T_{w_1} may be expressed as

$$
T_{w_1} = -\frac{2\sqrt{6}}{3} J(M_\infty) \frac{1}{\sqrt{Re}} \frac{\bar{\rho}_{st}\bar{a}_{st}}{T_{st}F'(\overline{T}_{w_0})} \left[\frac{1}{10}(8\mu_1 - 1)f_{0w} + \frac{2}{3^{\frac{2}{3}}}q_1 f_{1w} \right] \tag{8.4}
$$

Now that a complemental relation T_{w_1} and f_{1w} is obtained, the three boundary values $(g_{1w}, z'_{1w}, g'_{1w})$ can be uniquely determined from a set of assumed boundary values $(f_{1w}, f''_{1w}, z_{1w})$.

The excluded conditions shown in Eq. (8.1) may be used as conditions for convergency of the solutions to Eq. (4.23) together with additional conditions

$$
f''_1 = f'''_1 = g'_1 = g''_1 = z'_1 = z''_1 = 0 \quad \text{at} \quad \eta = \infty . \tag{8.5}
$$

It must be noted that these additional conditions may seem to result in over-determination of the solutions to Eq. (4.23). However, this difficulty might be found to be avoided reasonably for the reason that the solutions have an exponential, asymptotic behaviour when η tends to infinity and, therefore, Eq. (8.5) must be satisfied automatically by the solutions required. This characteristic is quite the same as is seen in solutions appropriate to the basic field (see ref. [8]). Thus, the additional conditions shown in Eq. (8.5) should be, therefore, considered as conditions for more rigorous justification of convergency of the solutions.

The numerical calculation was carried out for teflon ablator by use of a HITAC 5020 high speed electronic computer. Integration of Eq. (4.23) was made step by

step outwards starting from the body surface by assuming a proper set of boundary values (f_{1w}, f''_{1w}, z_{1w}). If the solutions thus obtained did not satisfy Eq. (8.1) for large value of η, then, these assumed values were slightly adjusted and the integration was carried out again. This trial and error calculation was continued to repeat again and again until the solutions converged sufficiently to satisfy both Eqs. (8.1) and (8.5). The static relation between $\dot{m}$ and T_w which is necessary for evaluating Eq. (8.3) has already been shown in Fig. 7 for teflon ablator.

Figs. 8a to 8d show several examples of the solutions to Eq. (4.23) for respective perimetric conditions. By comparing Fig. 8c with Fig. 3, it will be easily found that the solutions for perturbation field converge more quickly than those for the basic field. The same is true for solutions obtained under the other perimetric conditions. This characteristic clearly indicates a physical meaning that the solution to Eq. (4.23) corresponds to a correction term relative to the respective solutions for the basic field. In Figs. 9 to 11 are presented the variations of respective results of f_{1w}, f''_{1w} and z_{1w} with stagnation temperature in free stream, in which radius of body curvature at stagnation point is included as a parameter. As is seen in Fig. 9, the absolute value of f_{1w} increase with increase of radius of body curvature, indicating a physical fact that the local mass flow rate of coolant gas injection due to ablation must be decreased as radius of body curvature grows. This characteristic is quite reasonable from the result of conventional boundary layer theory that the local heat transfer rate decreases with increase of the radius of body curvature. In quite the same sense, the result of perturbation of concentration, z_{1w}, shown in Fig. 11 is found to be consistent with the behaviour of f_{1w}. Thus, by use of the results shown in Fig. 9, perturbation coefficient of local ablation rate and wall temperature can be calculated from Eqs. (6.4) and (8.4), respectively, and the respective results are shown in Figs. 12 and 13.

In Fig. 14 is presented the variation of perturbation coefficient of local ablation rate defined by $\dot{m}_1/\dot{m}_0$ with stagnation temperature in free stream, in which the effect of radius of body curvature is indicated as a parameter. The corresponding result of perturbation coefficient of the local wall temperature defined by T_{w_1}/T_{w_0} is shown in Fig. 15. Since the perturbation coefficient of local ablation rate has negative sign, it will be easily found that the local ablation rate decreases monotonously onto downstream. The fact that the absolute value of $\dot{m}_1/\dot{m}_0$ increases as radius of body curvature grows is quite reasonable and is consistent with the dependency of surface heat transfer rate upon Reynolds number referred to the radius of body curvature. Moreover, it is interesting to see in the figure that $|\dot{m}_1/\dot{m}_0|$ decreases with increase of stagnation temperature in free stream. This, in turn, can be interpreted into a statement that the ratio of local ablation rate to that at the stagnation point tends to increase with increase of flow enthalpy outside the boundary layer. Although this fact may, at a glance, seem to be curious because of $|\dot{m}_1/\dot{m}_0|$ being non-dimensional, it is clearly due to the non-similar characteristics of the perturbation field downstream of stagnation point, as has been already mentioned in previous paragraph. In quite the same physical reasoning, the result
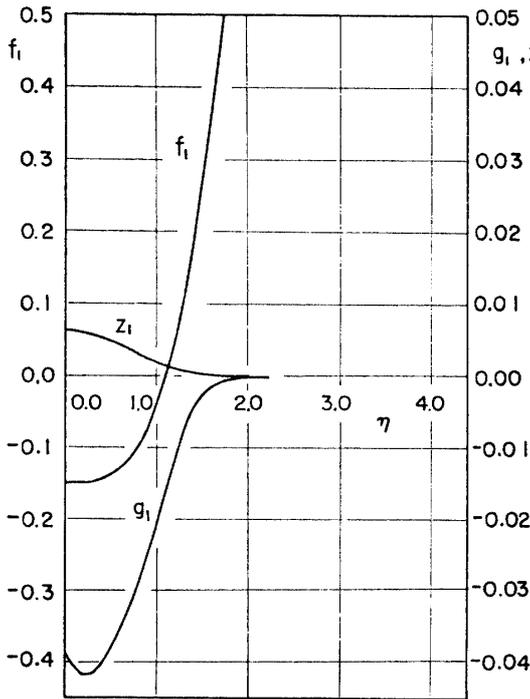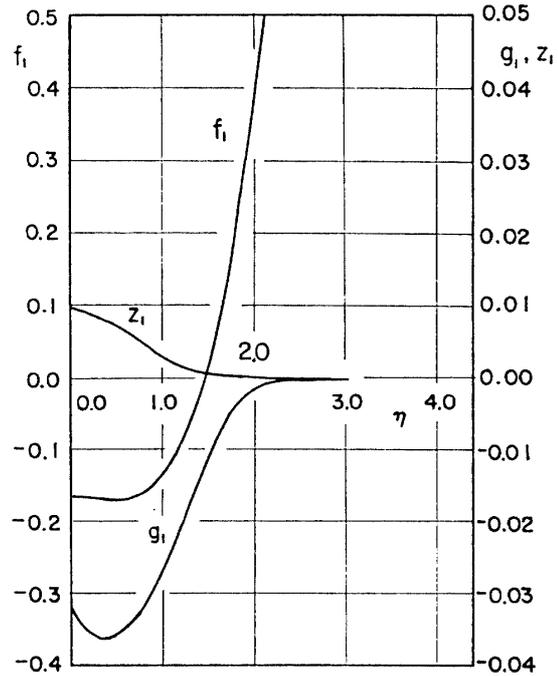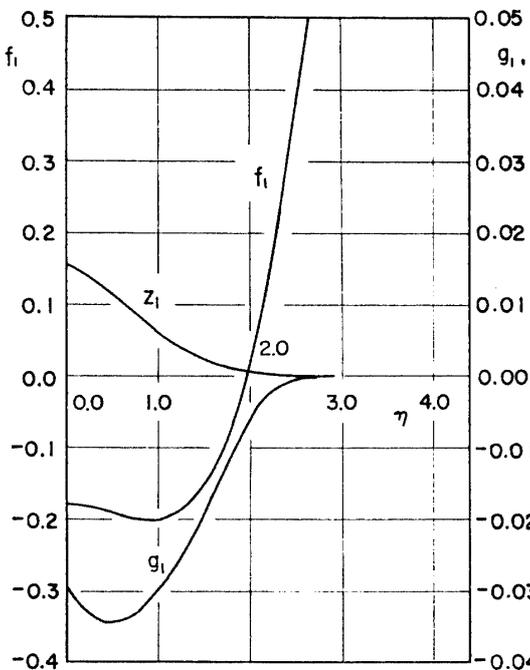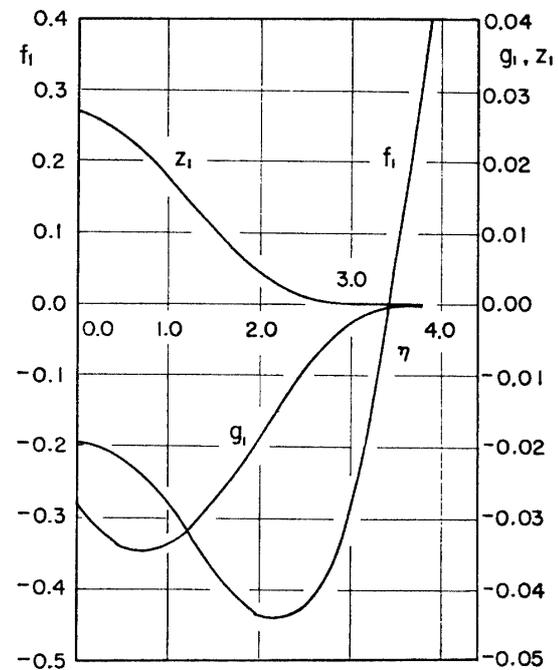
FIG. 8a. Solutions to Eq. (4.23). $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $\bar{T}_{st} = 800°C$, $\bar{R}_b = 1.0$ cm, $\bar{T}_{w_0} = 618°C$, $T_b = 0.233$, teflon.

FIG. 8b. Solutions to Eq. (4.23). $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $\bar{T}_{st} = 1000°C$, $\bar{R}_b = 1.0$ cm, $\bar{T}_{w_0} = 644°C$, $T_b = 0.233$, teflon.

FIG. 8c. Solutions to Eq. (4.23). $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $\bar{T}_{st} = 1200°C$, $\bar{R}_b = 1.0$ cm, $\bar{T}_{w_0} = 659°C$, $T_b = 0.233$, teflon.

FIG. 8d. Solutions to Eq. (4.23). $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $\bar{T}_{st} = 1500°C$, $\bar{R}_b = 1.0$ cm, $\bar{T}_{w_0} = 674°C$, $T_b = 0.233$, teflon.
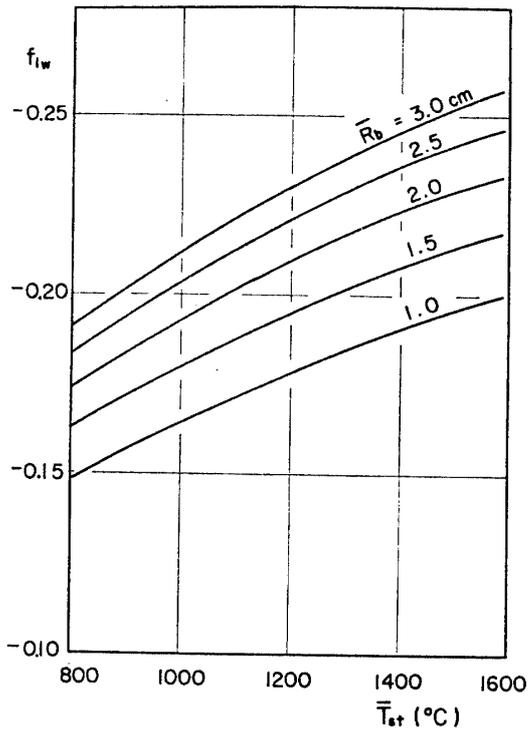
FIG. 9. Variation of f_{1w} with stagnation temperature in free stream. $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $T_b = 0.233$, teflon.
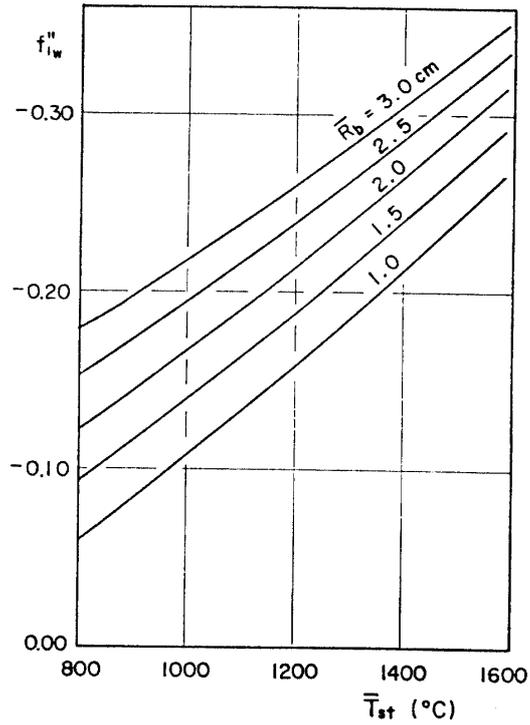
FIG. 10. Variation of f''_{1w} with stagnation temperature in free stream. $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $T_b = 0.233$, teflon.
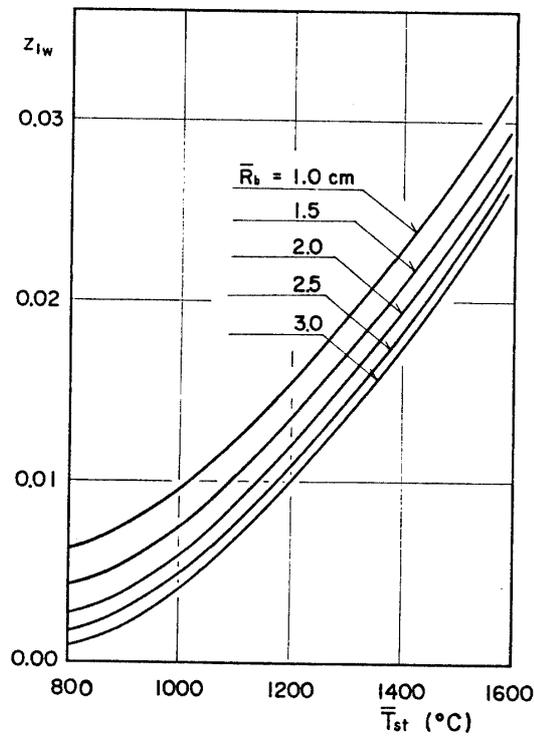

FIG. 11. Variation of z_{1w} with stagnation temperature in free stream. $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $T_b = 0.233$, teflon.
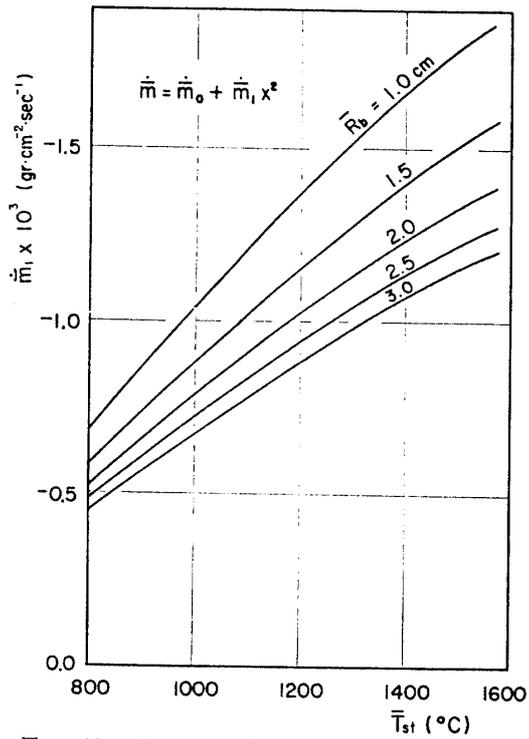
FIG. 12. Perturbation coefficient of local ablation rate. Teflon, $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $T_b = 0.233$.
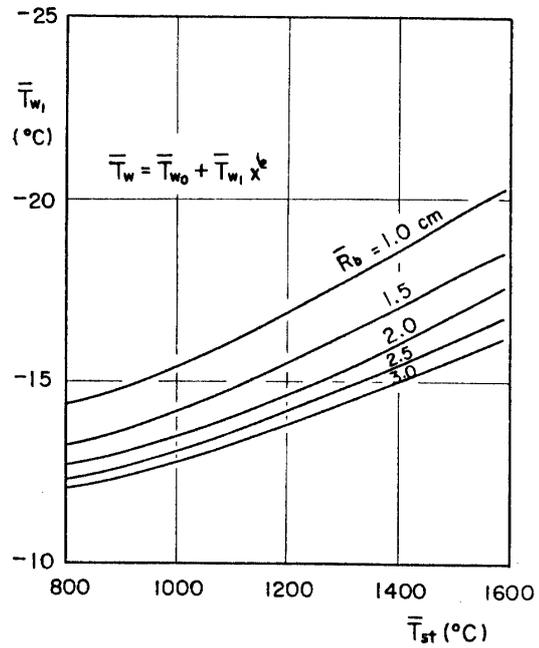
FIG. 13. Perturbation coefficient of local wall temperature. Teflon, $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $T_b = 0.233$.
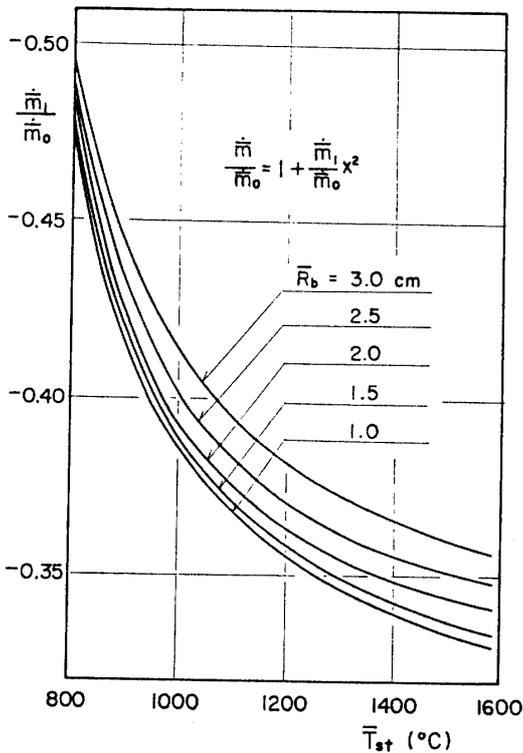
FIG. 14. Perturbation coefficient of ratio of local to stagnation ablation rate. Teflon, $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $T_b = 0.233$.
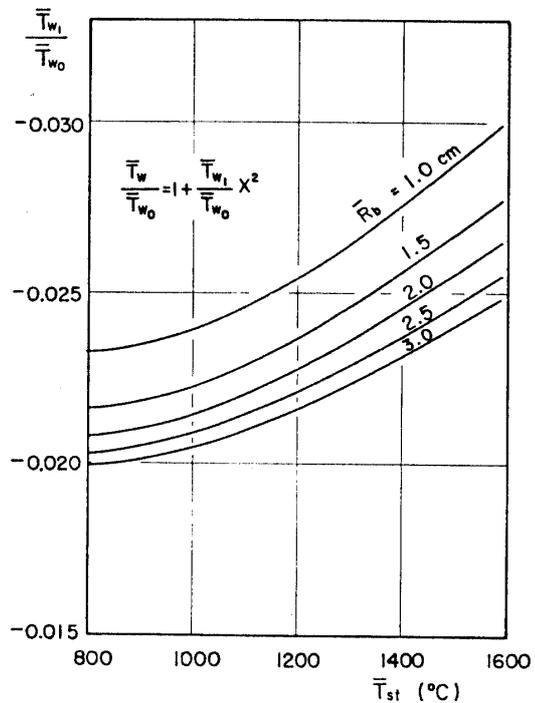
FIG. 15. Perturbation coefficient of ratio of local to stagnation wall temperature. Teflon, $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $T_b = 0.233$.

shown in Fig. 15 may be reasonably consistent with the characteristics of the abla-tive field downstream of stagnation point. Figs. 16 and 17 show alternate results concerning local ablation rate and the associated surface temperature, respectively, for the purpose of clarifying the effect of body curvature.

A comparison of the theoretical results with experimental data is of a point of interest in order to confirm the validity of the present development. In Figs. 18a to 18d are plotted the experimental data [10] on the ratio of local ablation to that at stagnation point obtained under various perimetric conditions together with respec-tive theoretical results for comparison. In each figure the ratio of local polar angle θ, to the maximum one, θ_{sh}, defined at the shoulder of the model is used as repre-sentation of the location downstream of stagnation point. For $\bar{T}_{st} > 1000°C$, the present theory is found to agree fairly well with experiment in the range of θ/θ_{sh} below 0.6, beyond which the agreement fails. However, this clarly arises from the inaccuracy of the first-order perturbation approach for large values of θ/θ_{sh} and, consequently, the theory will be reasonably improved to obtain a wider range of applicability if higher-order perturbation is taken into consideration.

As is seen in Fig. 18a, the present approach seems to have a little poor agreement with experiment for comparatively low stagnation temperature in free stream. Because of relatively small ablation rate at stagnation point in such temperature range, the disagreement may be due to the invalidity of the present assumption made in evaluation of the first-order temperature distribution inside the body.
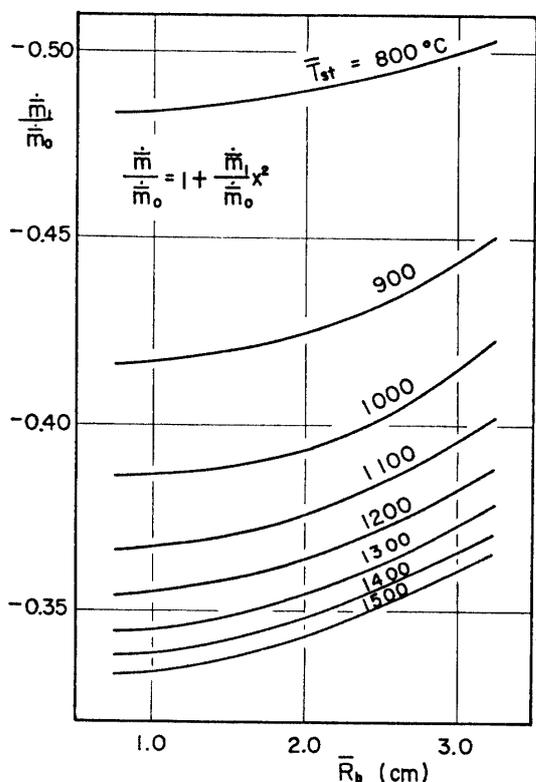

FIG. 16.   Alternate result of Fig. 14. Teflon,
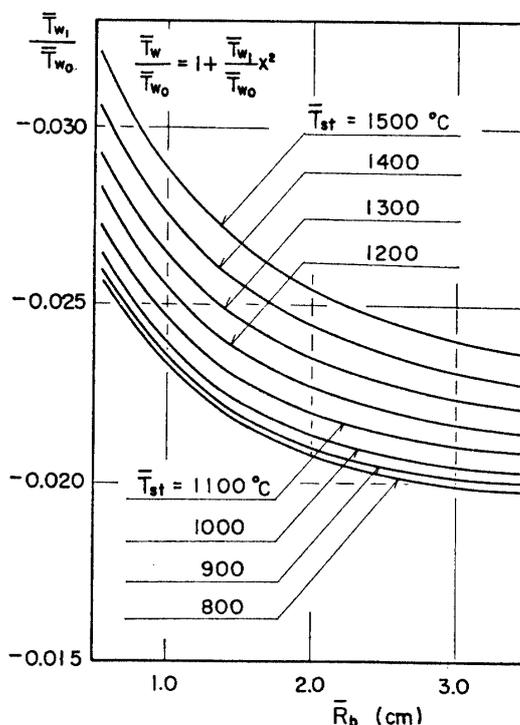 $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $T_b = 0.233$.

FIG. 17.   Alternate result of Fig. 15. Teflon,
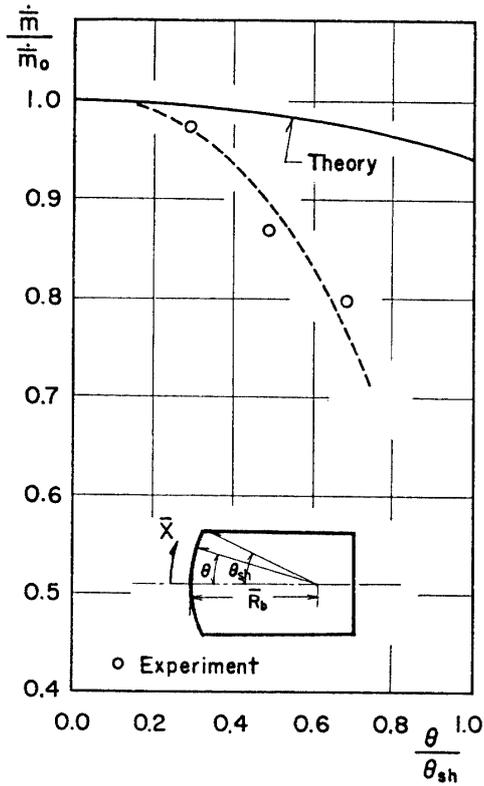 $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $T_b = 0.233$.

FIG. 18a. Comparison of theory with experiment. Teflon, $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $\bar{T}_{st} = 970°C$, $\bar{R}_b = 2.83$ cm, $T_b = 0.233$.

FIG. 18b. Comparison of theory with experiment. Teflon, $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $\bar{T}_{st} = 1045°C$, $\bar{R}_b = 1.40$ cm, $T_b = 0.233$.

FIG 18c. Comparison of theory with experiment. Teflon, $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $\bar{T}_{st} = 1120°C$, $\bar{R}_b = 1.63$ cm, $T_b = 0.233$.
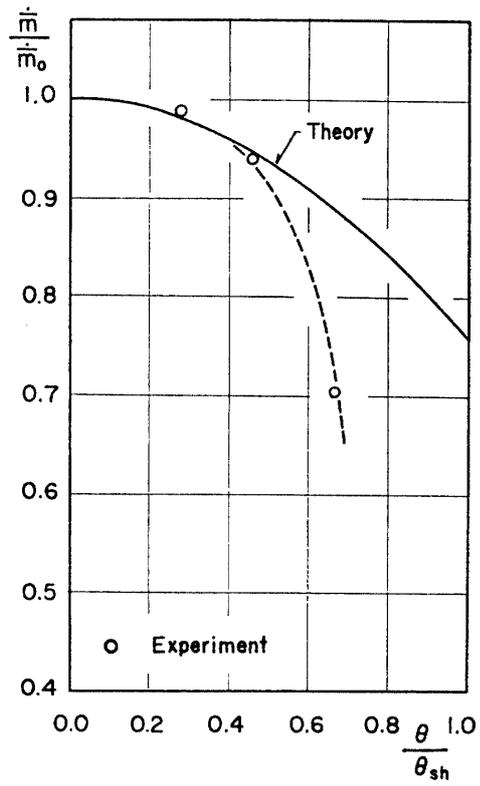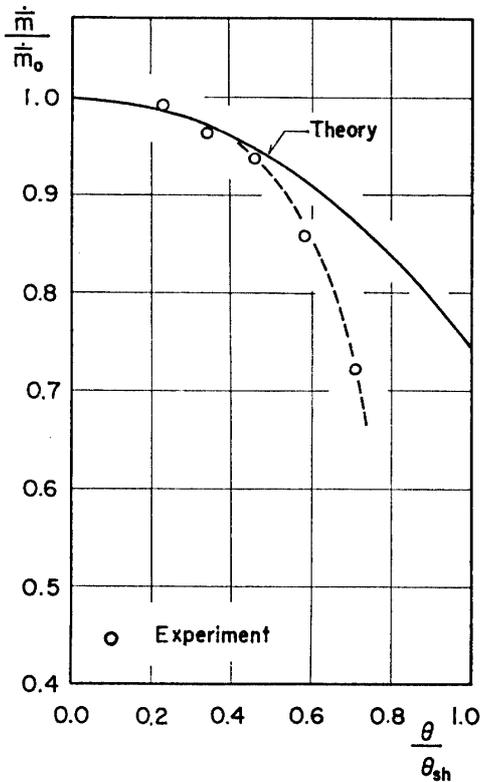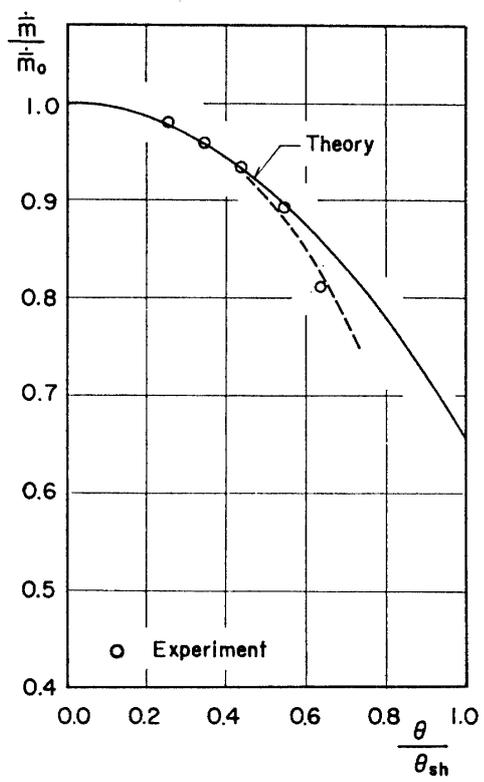

FIG. 18d. Comparison of theory with experiment. Teflon, $M_\infty = 5.74$, $\bar{p}_{st} = 1$ atm, $\bar{T}_{st} = 1200°C$, $\bar{R}_b = 1.21$ cm, $T_b = 0.233$.

9. Conclusion

An analytical approach has been presented to aerodynamic study of ablation downstream of stagnation point of blunt-nosed bodies of revolution at hypersonic speeds. The development of the theory was made based upon the first-order perturbation scheme, indicating that all physical quantities concerning the ablative field can be uniquely determined under the given perimetric conditions by matching of an aerodynamic relation with a static one obtained from chemical kinetics.

It was shown that the first-order perturbation field is no longer similar with respect to given perimetric conditions, while the basic field corresponding to the stagnation point solutions remains similar.

Numerical calculation carried out for teflon ablator revealed that the perturbation coefficient of local ablation rate, $\dot{m}_1$, is negative, so that local ablation rate tends to decrease onto the downstream from stagnation point. This trend is reasonably consistent with the distribution of local heat transfer rate obtained from conventional boundary layer theory.

Moreover, it was shown that the local ablation rate and the associated surface temperature have the same trend of dependency on radius of body curvature as those at the stagnation point. These characteristics together with a remarkable result that the local wall temperature does not rise so much as the stagnation temperature in free stream increases clearly indicate a quantitative evidence on the effect of shielding the aerodynamic heating by vaporization.

Comparison of the theoretical results with experimental data made for teflon ablator revealed that the present approach agrees fairly well with experiment near the region of stagnation point, thus giving an experimental evidence to the validity of the present development. Disagreement between theory and experiment in the region far away from the stagnation point may be due to the inaccuracy of the first-order perturbation approximation developed in the present approach.

ACKNOWLEDGEMENT

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April 15, 1968

APPENDIX A   TRANSPORT COEFFICIENTS

A-1. *Coefficient of Viscosity*

For a binary gas mixture the mean coefficient of viscosity across boundary layer

has been found, with an assumption that each species occupying the same space has the same temperature, to have the form (see ref. [8], Appendix A-1).

$$\frac{\mu}{\mu_e} = \left[\frac{\tilde{\mu}K}{\tilde{\mu}G + (1 - \tilde{\mu}G)K} + \frac{1 - K}{1 + (\tilde{m}G - 1)K} \right] \left(\frac{T}{T_e} \right)^{\frac{1}{2}}, \qquad \text{(A.1.1)}$$

where

$$\tilde{\mu} = \frac{\mu_1}{\mu_2}, \qquad \tilde{m} = \frac{M_2}{M_1},$$

$$G = \frac{\left[1 + \left(\frac{1}{\tilde{\mu}} \right)^{\frac{1}{2}} \left(\frac{1}{\tilde{m}} \right)^{\frac{1}{4}} \right]^2}{2^{\frac{3}{2}} (1 + \tilde{m})^{\frac{1}{2}}} . \qquad \text{(A.1.2)}$$

Therefore, Taylor expansion of Eq. (A.1.1) by use of Eqs. (4.14) and (4.16) leads to

$$\frac{\mu}{\mu_e} = E_0(z_0) g_0^{\frac{1}{2}} + E_1(f_0, g_0, g_1, z_0, z_1) s^{\frac{2}{3}} + O(s^{\frac{4}{3}}), \qquad \text{(A.1.3)}$$

where

$$E_0(z_0) = \frac{1}{\sqrt{1 - \alpha_2 z_0}} \left[\frac{\tilde{\mu} z_0}{\tilde{\mu}G + (1 - \tilde{\mu}G)z_0} + \frac{1 - z_0}{1 + (\tilde{m}G - 1)z_0} \right],$$

$$E_1(f_0, g_0, g_1, z_0, z_1) = \Gamma_0^{\frac{1}{2}} \left[\frac{\tilde{\mu}^2 G z_1}{\{\tilde{\mu}G + (1 - \tilde{\mu}G)z_0\}^2} - \frac{\tilde{m}G z_1}{\{1 + (\tilde{m}G - 1)z_0\}^2} \right] \qquad \text{(A.1.4)}$$
$$+ \frac{\Gamma_1}{2\Gamma_0^{\frac{1}{2}}} \left[\frac{\tilde{\mu} z_0}{\tilde{\mu}G + (1 - \tilde{\mu}G)z_0} + \frac{1 - z_0}{1 + (\tilde{m}G - 1)z_0} \right].$$

A-2. Coefficient of Thermal Conductivity

The mean thermal conductivity for a binary gas mixture can be evaluated across the boundary layer as (see ref. [8], Appendix A-2).

$$\frac{\kappa}{\kappa_e} = \left[\frac{\tilde{\kappa}K}{\tilde{\mu}G' + (1 - \tilde{\mu}G')K} + \frac{1 - K}{1 + (\tilde{m}G' - 1)K} \right] \left(\frac{T}{T_e} \right)^{\frac{1}{2}}, \qquad \text{(A.2.1)}$$

where

$$\tilde{\kappa} = \tilde{m}\tilde{\mu} \frac{0.115\bar{R} + 0.354 M_1 \bar{C}_{p1}}{0.115\bar{R} + 0.354 M_2 \bar{C}_{p2}},$$
$$G' = 1.065 G . \qquad \text{(A.2.2)}$$

Thus, Taylor expansion of Eq. (A.2.1) yields

$$\frac{\kappa}{\kappa_e} = \frac{F_0(z_0)}{\sqrt{1 - \alpha_2 z_0}} g_0^{\frac{1}{2}} + F_1(f_0, g_0, g_1, z_0, z_1) s^{\frac{2}{3}} + O(s^{\frac{4}{3}}), \qquad \text{(A.2.3)}$$

where

$$
\left.
\begin{aligned}
F_0(z_0) &= \left[\frac{\tilde{\kappa} z_0}{\tilde{\mu} G' + (1 - \tilde{\mu} G') z_0} + \frac{1 - z_0}{1 + (\tilde{m} G' - 1) z_0} \right] , \\
F_1(f_0, g_0, g_1, z_0, z_1) &= \Gamma_0^{\frac{1}{2}} \left[\frac{\tilde{\kappa} \tilde{\mu} G' z_1}{\{\tilde{\mu} G' + (1 - \tilde{\mu} G') z_0\}^2} - \frac{\tilde{m} G' z_1}{\{1 + (\tilde{m} G' - 1) z_0\}^2} \right] \\
&\quad + \frac{\Gamma_1}{2\Gamma_0^{\frac{1}{2}}} \left[\frac{\tilde{\kappa} z_0}{\tilde{\mu} G' + (1 - \tilde{\mu} G') z_0} + \frac{1 - z_0}{1 + (\tilde{m} G' - 1) z_0} \right] .
\end{aligned}
\right\} \quad \text{(A.2.4)}
$$

APPENDIX B   NON-DIMENSIONAL CHARACTERISTIC NUMBERS

B-1. *Chapman-Rubesin Number*

Chapman-Rubesin number is defined by the equation

$$
C = \frac{\rho \mu}{\rho_e \mu_e} . \tag{B.1.1}
$$

Substitution of Eqs. (4.19) and (A.1.3) into Eq. (B.1.1) leads to

$$
C = C_0 + C_1 s^{\frac{2}{3}} + O(s^{\frac{4}{3}}) , \tag{B.1.2}
$$

where

$$
\left.
\begin{aligned}
C_0 &= \frac{\sqrt{1 - \alpha_2 z_0}}{1 - \alpha_1 z_0} \left[\frac{\tilde{\mu} z_0}{\tilde{\mu} G + (1 - \tilde{\mu} G) z_0} + \frac{1 - z_0}{1 + (\tilde{m} G - 1) z_0} \right] \frac{1}{\sqrt{g_0}} , \\
C_1 &= \frac{1}{\Lambda_0^2} (E_1 \Lambda_0 - E_0 \Lambda_1 g_0^{\frac{1}{2}}) .
\end{aligned}
\right\} \quad \text{(B.1.3)}
$$

B-2. *Prandtl Number*

Prandtl number is defined by the equation

$$
P = \frac{\bar{C}_p \bar{\mu}}{\bar{\kappa}} , \tag{B.2.1}
$$

and is expressed as

$$
P = P_2 C_p \frac{\dfrac{\mu}{\mu_e}}{\dfrac{\kappa}{\kappa_e}} , \tag{B.2.2}
$$

where P_2 denotes Prandtl number for pure air, since it is defined by conditions only at outer edge of the boundary layer where concentration of foreign gas species vanishes. Substitution of Eqs. (A.1.3) and (A.2.3) into Eq. (B.2.2) gives

$$
P = P_0 + P_1 s^{\frac{2}{3}} + O(s^{\frac{4}{3}}) , \tag{B.2.3}
$$

where

$$P_0 = \frac{P_2(1-\alpha_2 z_0)\left[\dfrac{\bar{\mu}z_0}{\bar{\mu}G+(1-\bar{\mu}G)z_0} + \dfrac{1-z_0}{1+(\bar{m}G-1)z_0}\right]}{\dfrac{\bar{\kappa}z_0}{\bar{\mu}G'+(1-\bar{\mu}G')z_0} + \dfrac{1-z_0}{1+(\bar{m}G'-1)z_0}} \,,$$

$$P_1 = P_2 \frac{\sqrt{1-\alpha_2 z_0}}{F_0 g_0^{\frac{1}{2}}}\left[(1-\alpha_2 z_0)\left(E_1 - \frac{\sqrt{1-\alpha_2 z_0}}{F_0}E_0 F_1\right) - \alpha_2 E_0 g_0^{\frac{1}{2}} z_1\right] \,,$$

(B.2.4)

and where mean specific heat for a binary gas mixture, C_p, has been found to have form

$$C_p = 1 - \alpha_2 K \,. \tag{B.2.5}$$

B-3. *Schmidt Number*

Schmidt number has been found to have form (see ref. [8], Appendix B-3)

$$S = S_2(1-\alpha_1 K)\frac{\mu}{\mu_e}\left(\frac{T_e}{T}\right)^{\frac{1}{2}} \,, \tag{B.3.1}$$

where S_2 denotes Schmidt number for pure air. Taylor expansion of Eq. (B.3.1) gives

$$S = S_0 + S_1 s^{\frac{2}{3}} + O(s^{\frac{4}{3}}) \,, \tag{B.3.2}$$

where

$$S_0 = S_2(1-\alpha_1 z_0)\left[\frac{\bar{\mu}z_0}{\bar{\mu}G+(1-\bar{\mu}G)z_0} + \frac{1-z_0}{1+(\bar{m}G-1)z_0}\right] \,,$$

$$S_1 = S_2 \frac{1}{\Gamma_0^{\frac{1}{2}}}\left[(1-\alpha_1 z_0)E_1 - \alpha_1 E_0 g_0^{\frac{1}{2}} z_1 - \frac{(1-\alpha_1 z_0)E_0 g_0^{\frac{1}{2}}\Gamma_1}{2\Gamma_0}\right] \,.$$

(B.3.3)

B-4. *Lewis Number*

Lewis number is defined by the equation

$$Le = \frac{P}{S} \,. \tag{B.4.1}$$

Taylor expansion of Eq. (B.4.1) leads to

$$Le = \frac{P_0}{S_0} + \frac{P_1 S_0 - P_0 S_1}{S_0^2} s^{\frac{2}{3}} + O(s^{\frac{4}{3}}) \,, \tag{B.4.2}$$

where P_0, P_1, S_0 and S_1 are given by Eqs. (B.2.4) and (B.3.3), respectively.

APPENDIX C   APPROXIMATE SOLUTION OF HEAT CONDUCTION EQUATION

In order to estimate the heat flow conducting inside the body, diffusion of heat in

$\bar{x}$-direction must be taken into consideration near the region of stagnation point of blunt-nosed bodies of revolution. Therefore, the heat conduction equation may be written as

$$\bar{\rho}_b \bar{C}_b \frac{\partial \bar{T}}{\partial \bar{t}} = \bar{\kappa}_b \left(\frac{\partial^2 \bar{T}}{\partial \bar{Y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{X}^2} \right) , \qquad (C.1)$$

which can be reexpressed by use of transformation of variables, Eq. (3.3) as

$$-\bar{\rho}_b \bar{v}_b \bar{C}_b \frac{\partial \bar{T}}{\partial \bar{y}} = \dot{\bar{m}} \bar{C}_b \frac{\partial \bar{T}}{\partial \bar{y}} = \bar{\kappa}_b \left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} \right) . \qquad (C.2)$$

The appropriate boundary conditions may be given by

$$\left. \begin{array}{lll} \bar{T} = \bar{T}_w(\bar{x}) & \text{at} & \bar{y}=0 , \\ \bar{T} = \bar{T}_b & \text{at} & \bar{y}=-\infty . \end{array} \right\} \qquad (C.3)$$

However, it seems to be very difficult to get an exact analytical solution to Eq. (C.2) consistent with the given boundary conditions, Eq. (C.3), so that an approximate solution is to be obtained by use of a conventional method in the present approach. The procedure of obtaining the solution to Eq. (C.2) is as follows.

First, the diffusion term in $\bar{x}$-direction is assumed to be small compared with the others as a zeroth-order approximation. Therefore, the heat conduction equation may be reduced to

$$\bar{\kappa}_b \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \bar{C}_b \dot{\bar{m}} \frac{\partial \bar{T}}{\partial \bar{y}} = 0 , \qquad (C.4)$$

The solution to Eq. (C.4) consistent with the given boundary condition, Eq. (C.3), may be given by

$$\bar{T} = \bar{T}_b + (\bar{T}_w - \bar{T}_b) \exp \left(\frac{\bar{C}_b \dot{\bar{m}}}{\bar{\kappa}_b} \bar{y} \right) , \qquad (C.5)$$

where both $\bar{T}_w$ and $\dot{\bar{m}}$ are functions of $\bar{x}$ and have been assumed, within the accuracy of the first-order perturbation, to have respective forms

$$\left. \begin{array}{l} \bar{T}_w = \bar{T}_w(\bar{x}) = \bar{T}_{w_0} + \bar{T}_{w_1} \left(\dfrac{\bar{x}}{\bar{R}_b} \right)^2 + O\left\{ \left(\dfrac{\bar{x}}{\bar{R}_b} \right)^4 \right\} , \\[4mm] \dot{\bar{m}} = \dot{\bar{m}}(\bar{x}) = \dot{\bar{m}}_0 + \dot{\bar{m}}_1 \left(\dfrac{\bar{x}}{\bar{R}_b} \right)^2 + O\left\{ \left(\dfrac{\bar{x}}{\bar{R}_b} \right)^4 \right\} , \end{array} \right\} \qquad (C.6)$$

where $\bar{T}_{w_0}$, $\bar{T}_{w_1}$, $\dot{\bar{m}}_0$ and $\dot{\bar{m}}_1$ are constants.

Consequently, substitution of Eq. (C.5) into the diffusion term in $\bar{x}$-direction of Eq. (C.2) then yields an ordinary differential equation of the second-order with respect to $\bar{y}$, which may be written in a form

$$\bar{\kappa}_b \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \bar{C}_b \left(\dot{\bar{m}}_0 + \dot{\bar{m}}_1 \frac{\bar{x}^2}{\bar{R}_b^2} \right) \frac{\partial \bar{T}}{\partial \bar{y}} = -\bar{\kappa}_b \left[2 \frac{\bar{T}_{w_1}}{\bar{R}_b^2} + \left\{ \frac{2\bar{C}_b \dot{\bar{m}}_1 (\bar{T}_{w_0} - \bar{T}_b)}{\bar{\kappa}_b \bar{R}_b^2} \right. \right.$$

$$\left. \left. + \frac{10\bar{C}_b \bar{T}_{w_1} \dot{\bar{m}}_1}{\bar{\kappa}_b \bar{R}_b^4} \bar{x}^2 \right\} \bar{y} + \frac{4\bar{C}_b^2 \dot{\bar{m}}_1^2 (\bar{T}_{w_0} - \bar{T}_b)}{\bar{\kappa}_b^2 \bar{R}_b^4} \bar{x}^2 \bar{y}^2 \right] \exp \left(\frac{\bar{C}_b \dot{\bar{m}}}{\bar{\kappa}_b} \bar{y} \right) \qquad \text{(C.7)}$$

where the terms of order less than $\left(\dfrac{\bar{x}}{\bar{R}_b} \right)^3$ have been neglected. Eq. (C.7) may be considered as the first-order approximation form of the heat conduction equation. Thus, the solution to Eq. (C.7) can be obtained under the boundary conditions given by Eq. (C.3) as

$$\bar{T} = \bar{T}_b + (\bar{T}_w - \bar{T}_b) \exp \left(\frac{\bar{C}_b \dot{\bar{m}}}{\bar{\kappa}_b} \bar{y} \right) + (a\bar{y}^3 + b\bar{y}^2 + c\bar{y}) \exp \left(\frac{\bar{C}_b \dot{\bar{m}}}{\bar{\kappa}_b} \bar{y} \right) , \qquad \text{(C.8)}$$

where

$$\left. \begin{aligned} a &= -\frac{4}{3} \frac{\bar{C}_b \dot{\bar{m}}_1^2 (\bar{T}_{w_0} - \bar{T}_b) \bar{x}^2}{\bar{\kappa}_b \left(\dot{\bar{m}}_0 + \dot{\bar{m}}_1 \dfrac{\bar{x}^2}{\bar{R}_b^2} \right) \bar{R}_b^4} , \\[2mm] b &= \frac{4\dot{\bar{m}}_1^2 (\bar{T}_{w_0} - \bar{T}_b) \bar{x}^2}{(\dot{\bar{m}}_0 \bar{R}_b^2 + \dot{\bar{m}}_1 \bar{x}^2)^2} - \frac{\bar{R}_b^2 \dot{\bar{m}}_1 (\bar{T}_{w_0} - \bar{T}_b) + 5\dot{\bar{m}}_1 \bar{T}_{w_1} \bar{x}^2}{(\dot{\bar{m}}_0 \bar{R}_b^2 + \dot{\bar{m}}_1 \bar{x}^2) \bar{R}_b^2} , \\[2mm] c &= -\frac{2\bar{\kappa}_b \bar{T}_{w_1}}{\bar{C}_b (\dot{\bar{m}}_0 \bar{R}_b^2 + \dot{\bar{m}}_1 \bar{x}^2)} + \frac{2\bar{\kappa}_b \{ \dot{\bar{m}}_1 (\bar{T}_{w_0} - \bar{T}_b) \bar{R}_b^2 + 5\dot{\bar{m}}_1 \bar{T}_{w_1} \bar{x}^2 \}}{\bar{C}_b (\dot{\bar{m}}_0 \bar{R}_b^2 + \dot{\bar{m}}_1 \bar{x}^2)^2} \\[2mm] &\quad - \frac{8\bar{\kappa}_b \dot{\bar{m}}_1^2 (\bar{T}_{w_0} - \bar{T}_b) \bar{R}_b^2 \bar{x}^2}{\bar{C}_b (\dot{\bar{m}}_0 \bar{R}_b^2 + \dot{\bar{m}}_1 \bar{x}^2)^3} . \end{aligned} \right\} \qquad \text{(C.9)}$$

Now that the first-order approximate solution of temperature distribution near the region of stagnation point is clarified, each term in square root of Eq. (5.6) may be evaluated within the accuracy of the first-order perturbation as

$$\left. \begin{aligned} \left(\frac{\partial \bar{T}}{\partial \bar{x}} \right)_{\bar{y}=-0} &= 2\bar{T}_{w_1} \frac{\bar{x}}{\bar{R}_b^2} , \\[2mm] \left(\frac{\partial \bar{T}}{\partial \bar{y}} \right)_{\bar{y}=-0} &= \left[\frac{\bar{C}_b \dot{\bar{m}}_0 (\bar{T}_{w_0} - \bar{T}_b)}{\bar{\kappa}_b} - \frac{2\bar{\kappa}_b \{ \dot{\bar{m}}_0 \bar{T}_{w_1} - \dot{\bar{m}}_1 (\bar{T}_{w_0} - \bar{T}_b) \}}{\bar{C}_b \dot{\bar{m}}_0 \bar{R}_b^2} \right] \\[2mm] &\quad + \left[\frac{\bar{C}_b \{ \dot{\bar{m}}_1 (\bar{T}_{w_0} - \bar{T}_b) + \dot{\bar{m}}_0 \bar{T}_{w_1} \}}{\bar{\kappa}_b} \right. \\[2mm] &\quad \left. + \frac{12\bar{\kappa}_b \dot{\bar{m}}_1 \{ \dot{\bar{m}}_0 \bar{T}_{w_1} - \dot{\bar{m}}_1 (\bar{T}_{w_0} - \bar{T}_b) \}}{\bar{C}_b \dot{\bar{m}}_0^3 \bar{R}_b^2} \right] \frac{\bar{x}^2}{\bar{R}_b^2} . \end{aligned} \right\} \qquad \text{(C.10)}$$

On the other hand, if the temperature distribution is assumed to be given by Eq. (5.10), the corresponding temperature gradient in each direction will be given by the equations

$$
\left. \left(\frac{\partial \overline{T}}{\partial \overline{x}} \right) \right|_{\overline{y}=-0} = 2 \overline{T}_{w_1} \frac{\overline{x}}{\overline{R}_b^2} \, ,
$$

$$
\left. \left(\frac{\partial \overline{T}}{\partial \overline{y}} \right) \right|_{\overline{y}=-0} = \frac{\overline{C}_b \dot{m}_0 (\overline{T}_{w_0} - \overline{T}_b)}{\overline{\kappa}_b} + \frac{\overline{C}_b \{ \dot{m}_0 \overline{T}_{w_1} + \dot{m}_1 (\overline{T}_{w_0} - \overline{T}_b) \}}{\overline{\kappa}_b} \, \frac{\overline{x}^2}{\overline{R}_b^2} \, .
$$

$$\text{(C.11)}$$

Therefore, it will be easily found by comparing Eq. (C.11) with Eq. (C.10) that the temperature gradient in $\overline{x}$-direction is quite the same, while that in $\overline{y}$-direction is slightly different from one another. However, this discrepancy clearly arises from the fact that the diffusion term in $\overline{x}$-direction involved in the heat conduction equation, Eq. (C.2), contributes to result in the second term in each square bracket in Eq. (C.10).

Here an attention must be paid to the fact that $\dot{m}_1$ must be of the same order of magnitude as $\dot{m}_0$ but has opposite sign to $\dot{m}_0$. Furthermore, $\overline{T}_{w_1}$ must also be of the same order of magnitude as $(\overline{T}_{w_0} - \overline{T}_b)$ but has opposite sign to $(\overline{T}_{w_0} - \overline{T}_b)$. These facts clearly indicate possibility that the second term in each square bracket in Eq. (C.10) may become very small compared with the first term. If this is true, Eq. (C.10) will be reasonably reduced to Eq. (C.11) and, consequently, the assumption of negligible diffusion of heat in $\overline{x}$-direction near the region of stagnation point may be applicable within the accuracy of the first-order perturbation approximation. Thus, the validity of the assumption just mentioned above depends mainly upon the order of magnitude of the second term in each square bracket in Eq. (C.10).

In order to examine this circumstance in quantitative detail, consider the ratio of the second term to the first one in the first square bracket in Eq. (C.10). That is

$$
\left[\frac{2 \overline{\kappa}_b \{ \dot{m}_0 \overline{T}_{w_1} - \dot{m}_1 (\overline{T}_{w_0} - \overline{T}_b) \}}{\overline{C}_b \dot{m}_0 \overline{R}_b^2} \right] \Big/ \left[\frac{\overline{C}_b \dot{m}_0 (\overline{T}_{w_0} - \overline{T}_b)}{\overline{\kappa}_b} \right] \sim O\left\{ \left(\frac{\overline{\kappa}_b}{\overline{C}_b \dot{m}_0 \overline{R}_b} \right)^2 \right\}
$$

From the experiment [9] made for hemi-sphere models of teflon with nose radius of curvature of order of 1 cm, it has been found that $\dot{m}_0$ is of the order of $10^{-3} \mathrm{gr} \cdot \mathrm{cm}^{-2} \cdot \mathrm{sec}^{-1}$ in the range of stagnation temperature in free stream above $1000^\circ \mathrm{C}$. Therefore

$$
O\left\{ \left(\frac{\overline{\kappa}_b}{\overline{C}_b \dot{m}_0 \overline{R}_b} \right)^2 \right\} = O(10^{-2}) \, ,
$$

$$\text{(C.12)}$$

which is very small compared with unity. The same is true for terms in the second square bracket in Eq. (C.10), indicating that the second term in the bracket is negligibly small compared with the first. Thus, it has been confirmed that Eq. (5.7) may be reasonably applicable near the region of stagnation point and, consequently, Eq. (5.10) may be used to estimate Eq. (5.6) within the accuracy of the first-order perturbation approximation.

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