Institute of Space and Aeronautical Science, University of Tokyo
Report No. 436, February 1969

The Measurement of the Flexural Wave Propagation Velocity by Correlation Techniques

By

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Summary: The group delay time is defined for the dispersive wave propagation system. It is the time which is required for the envelope of the narrow band input signal to travel through the system. Three methods of detecting the group delay time by correlation techniques were developed and the experiments were performed in flexural wave propagation system.

1. INTRODUCTION

In considering signal flow problem, it would be desirable that the response of the system can be predicted for the arbitrary input signal. In the case of linear system, the superposition principle assures that by decomposing the input signal into the sum of elementary signals, the response can be calculated as the sum of the responses for each elementary signal. So if the responses of the system for the elementary signals are known, the response for the arbitrary signal can be predicted. Sinusoidal signals are usually adopted as elementary signals.

For sinusoidal signals the responses of the linear system are characterized by two quantities, those are amplitude ratio and the time delay, and in general frequency dependent. If they are constant in the frequency range of the input signal, the response for the arbitrary signal is the same form as the input signal except some multiple constant and time delay.

More generally, even the delay time of the sinusoidal signal depends upon the frequency, the group delay time can be defined for suitably narrow band arbitrary signal. The group delay time is the time required for the signal envelope to pass through the linear system and is connected with the energy or information transport problem. In this paper three ways of detecting the group delay time by correlation techniques are developed and experimental results are illustrated.

2. ENVELOPES AND PRE-ENVELOPES [1]

We denote the input signal of the linear system $G(s)$ as $f_1(t)$ and the output signal as $f_2(t)$. The pre-envelopes of $f_1(t)$ and $f_2(t)$ are defined as,

$$z_1(t)=f_1(t)+i\hat{f}_1(t) \qquad (1)$$

$$z_2(t)=f_2(t)+i\hat{f}_2(t) \tag{2}$$

Where $\hat{f}_1(t)$, $\hat{f}_2(t)$ are the Hilbert transforms of $f_1(t)$ and $f_2(t)$. The envelope of the signal is an absolute value of the pre-envelope. So $z_1(t)$ and $z_2(t)$ are written as follows.

$$z_1(t)=R_1(t)\cdot e^{i\theta_1(t)} \tag{3}$$

$$z_2(t)=R_2(t)\cdot e^{i\theta_2(t)} \tag{4}$$

$R_1(t)$ and $R_2(t)$ are the absolute values respectively. Generally, $f_1(t)$ can be written in the form

$$f_1(t)=\sum_{n=1}^{N} C_n \cos(\omega_n t-\varphi_n) \tag{5}$$

and $f_2(t)$ is

$$f_2(t)=\sum_{n=1}^{N} C_n\,|G(i\omega_n)|\cos\{\omega_n t-\varphi_n+\angle G(i\omega_n)\} \tag{6}$$

Where ω_n's are selected suitably in the signal frequency range to represent the wave form. $\angle G(i\omega)$ is the angle of the transfer function and when $G(s)$ represents the real physical system, it is supposed to be differentiable function of ω. Then in some narrow range of ω, the next approximation can be obtained.

$$\angle G(i\omega_n)-\angle G(i\omega_m)\fallingdotseq(\omega_n-\omega_m)\frac{d}{d\omega}\{\angle G(i\omega)\}\,|_{\omega=\omega_m} \tag{7}$$

Where ω_m is the center angular frequency of $f_1(t)$. The phase delay τ_m and the group delay τ_g at the angular frequency ω_m are defined as [2]

$$\tau_m=-\frac{\angle G(i\omega_m)}{\omega_m} \tag{8}$$

$$\tau_g=-\frac{d}{d\omega}\{\angle G(i\omega)\}\,|_{\omega=\omega_m} \tag{9}$$

Then for $f_1(t)$ and $f_2(t)$, we have

$$f_1(t)=\cos\omega_m t\sum_{n=1}^{N} C_n\cos\{(\omega_n-\omega_m)t-\varphi_n\}-\sin\omega_m t\sum_{n=1}^{N} C_n\sin\{(\omega_n-\omega_m)t-\varphi_n\} \tag{10}$$

$$\begin{aligned} f_2(t)=&\cos\{\omega_m(t-\tau_m)\}\sum_{n=1}^{N} C_n\,|G(i\omega_n)|\cos\{(\omega_n-\omega_m)(t-\tau_g)-\varphi_n\}\\ &-\sin\{\omega_m(t-\tau_m)\}\sum_{n=1}^{N} C_n\,|G(i\omega_n)|\sin\{(\omega_n-\omega_m)(t-\tau_g)-\varphi_n\} \end{aligned} \tag{11}$$

Since the Hilbert transform of $\cos x$ is $\sin x$, $\hat{f}_1(t)$ and $\hat{f}_2(t)$ are

$$\begin{aligned} \hat{f}_1(t)=&\sum_{n=1}^{N} C_n\sin(\omega_n t-\varphi_n)=\cos\omega_m t\sum_{n=1}^{N} C_n\sin\{(\omega_n-\omega_m)t-\varphi_n\}\\ &+\sin\omega_m t\sum_{n=1}^{N} C_n\cos\{(\omega_n-\omega_m)t-\varphi_n\} \end{aligned} \tag{12}$$

$$\hat{f}_2(t)=\sum_{n=1}^{N} C_n|G(i\omega_n)|\sin\{\omega_n t-\varphi_n+\angle G(i\omega_n)\}$$
$$=\cos\{\omega_m(t-\tau_m)\}\sum_{n=1}^{N} C_n|G(i\omega_n)|\sin\{(\omega_n-\omega_m)(t-\tau_g)-\varphi_n\}$$
$$+\sin\{\omega_m(t-\tau_m)\}\sum_{n=1}^{N} C_n|G(i\omega_n)|\cos\{(\omega_n-\omega_m)(t-\tau_g)-\varphi_n\} \tag{13}$$

Then the pre-envelopes of $f_1(t)$ and $f_2(t)$ are represented as follows.

$$z_1(t)=\left[\sum_{n=1}^{N} C_n\cos\{(\omega_n-\omega_m)t-\varphi_n\}+i\sum_{n=1}^{N} C_n\sin\{(\omega_n-\omega_m)t-\varphi_n\}\right]\exp(i\omega_m t) \tag{14}$$

$$z_2(t)=\left[\sum_{n=1}^{N} C_n|G(i\omega_n)|\cos\{\omega_n-\omega_m)(t-\tau_g)-\varphi_n\}\right.$$
$$\left.+i\sum_{n=1}^{N} C_n|G(i\omega_n)|\sin\{(\omega_n-\omega_m)(t-\tau_g)-\varphi_n\}\right]\exp\{i\omega_m(t-\tau_m)\} \tag{15}$$

When the signal frequency range is sufficiently narrow, $G(s)$ can be considered as $|G(s)|=k$ (constant). In such a case, the envelopes R_1, R_2 and the phasing functions θ_1, θ_2 are represented as

$$R_1(t)=\left[\left\{\sum_{n=1}^{N} C_n\cos\{(\omega_n-\omega_m)t-\varphi_n\}\right\}^2\right.$$
$$\left.+\left\{\sum_{n=1}^{N} C_n\sin\{(\omega_n-\omega_m)t-\varphi_n\}\right\}^2\right]^{\frac{1}{2}} \tag{16}$$

$$R_2(t)=k\left[\left\{\sum_{n=1}^{N} C_n\cos\{(\omega_n-\omega_m)(t-\tau_g)-\varphi_n\}\right\}^2\right.$$
$$\left.+\left\{\sum_{n=1}^{N} C_n\sin\{(\omega_n-\omega_m)(t-\tau_g)-\varphi_n\}\right\}^2\right]^{\frac{1}{2}} \tag{17}$$

$$\theta_1(t)=\tan^{-1}\left[\frac{\sum_{n=1}^{N} C_n\sin\{(\omega_n-\omega_m)t-\varphi_n\}}{\sum_{n=1}^{N} C_n\cos\{(\omega_n-\omega_m)t-\varphi_n\}}\right]+\omega_m t \tag{18}$$

$$\theta_2(t)=\tan^{-1}\left[\frac{\sum_{n=1}^{N} C_n\sin\{(\omega_n-\omega_m)(t-\tau_g)-\varphi_n\}}{\sum_{n=1}^{N} C_n\cos\{(\omega_n-\omega_m)(t-\tau_g)-\varphi_n\}}\right]+\omega_m(t-\tau_m) \tag{19}$$

Compairing (16) with (17), (18) with (19) next relations are obtained.

$$R_2(t)=kR_1(t-\tau_g) \tag{20}$$

$$\theta_2(t)=\theta_1(t-\tau_g)+\omega_m(\tau_g-\tau_m) \tag{21}$$

These results represent that the envelope travels at the velocity which is equal to the group velocity defined by the Eq. (9). In the following sections the measuring methods of τ_g by correlation techniques are described.

3. Measurement of τ_g by Ordinary Correlation Method [3]

By computing the cross-correlation functions of the input signal $f_1(t)$ and the output signal $f_2(t)$, the group delay τ_g of the linear system $G(s)$ can be obtained, assuming that the signal band width is so narrow that

(1) $\angle G(i\omega)$ can be considered as the linear function of ω and

(2) $|G(i\omega)|$ is constant in the frequency range considered.

Now from the Eq. (20) and Eq. (21), the cross-correlation function of the pre-envelope functions is expressed as follows.

$$\begin{aligned}\phi_{z_1z_2}(\tau)&=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}z_1^*(t)z_2(t+\tau)dt=\overline{R_1(t)e^{-i\theta_1(t)}R_2(t+\tau)e^{i\theta_2(t+\tau)}}\\&=k\overline{R_1(t)R_1(t+\tau-\tau_g)e^{i\theta_1(t+\tau-\tau_g)-i\theta_1(t)}}e^{i\omega m(\tau_g-\tau m)}\end{aligned} \quad (22)$$

Where the asterisk means the complex conjugate and the upper bar means to take average with respect to t. Since the autocorrelation function of $z_1(t)$ is

$$\phi_{z_1}(\tau)=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}z_1^*(t)z_1(t+\tau)dt=\overline{R_1(t)e^{-i\theta_1(t)}R_1(t+\tau)e^{i\theta_1(t+\tau)}} \quad (23)$$

(22) is written as

$$\phi_{z_1z_2}(\tau)=k\phi_{z_1}(\tau-\tau_g)e^{i\omega m(\tau_g-\tau m)} \quad (24)$$

Now we use next relation.

$$\phi_{z_1z_2}(\tau)=2\{\phi_{f_1f_2}(\tau)+i\hat{\phi}_{f_1f_2}(\tau)\} \quad (25)$$

(proof)

$$\begin{aligned}\phi_{z_1z_2}(\tau)&=\overline{\{f_1(t)-i\hat{f_1}(t)\}\{f_2(t+\tau)+i\hat{f_2}(t+\tau)\}}\\&=\phi_{f_1f_2}(\tau)+\phi_{\hat{f}_1\hat{f}_2}(\tau)-i\{\phi_{\hat{f}_1f_2}(\tau)-\phi_{f_1\hat{f}_2}(\tau)\}\end{aligned} \quad (26)$$

Since

$$\hat{f}(t)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(\xi)}{t-\xi}d\xi=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(t+\xi)}{-\xi}d\xi \quad (27)$$

assuming interchangeability of the order of integration and limit,

$$\begin{aligned}\phi_{\hat{f}_1\hat{f}_2}(\tau)&=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f_1(t+\xi)}{-\xi}d\xi\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f_2(t+\tau+\eta)}{-\eta}d\eta\\&=\frac{1}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\phi_{f_1f_2}(\tau+\eta-\xi)}{\xi\eta}d\xi d\eta=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\hat{\phi}_{f_1f_2}(\tau-\xi)}{-\xi}d\xi\\&=-\hat{\hat{\phi}}_{f_1f_2}(\tau)=\phi_{f_1f_2}(\tau)\end{aligned} \quad (28)$$

The last equality is a general property. In the same way

$$\begin{aligned}\phi_{\hat{f}_1f_2}(\tau)&=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt\cdot\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f_1(t+\xi)}{-\xi}d\xi\cdot f_2(t+\tau)\\&=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\phi_{f_1f_2}(\tau-\xi)}{-\xi}d\xi=-\hat{\phi}_{f_1f_2}(\tau)\end{aligned} \quad (29)$$

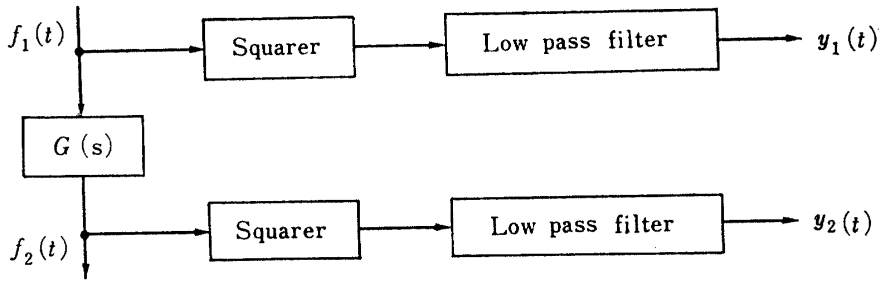


FIG. 1. Measuring system by squared signal correlation method.

$$\phi_{f_1f_2}(\tau)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\phi_{f_1f_2}(\tau+\eta)}{-\eta}d\eta=\hat{\phi}_{f_1f_2}(\tau) \tag{30}$$

From (26), (28), (29) and (30)

$$\phi_{z_1z_2}(\tau)=2\{\phi_{f_1f_2}(\tau)+i\hat{\phi}_{f_1f_2}(\tau)\} \qquad \text{Q.E.D.}$$

When $f_1=f_2$

$$\phi_{z_1}(\tau)=2\{\phi_{f_1}(\tau)+i\hat{\phi}_{f_1}(\tau)\} \tag{31}$$

Then from equations (24), (25) and (31) the next relation is obtained.

$$\phi_{f_1f_2}(\tau)+i\hat{\phi}_{f_1f_2}(\tau)=k\{\phi_{f_1}(\tau-\tau_g)+i\hat{\phi}_{f_1}(\tau-\tau_g)\}e^{i\omega_m(\tau_g-\tau_m)} \tag{32}$$

The envelopes $S_1(\tau)$, $S_2(\tau)$ and phasing angles $P_1(\tau)$, $P_2(\tau)$ of the autocorrelation function $\phi_{f_1}(\tau)$ and the cross-correlation function $\phi_{f_1f_2}(\tau)$ are,

$$\phi_{f_1}(\tau)+i\hat{\phi}_{f_1}(\tau)=S_1(\tau)e^{iP_1(\tau)} \tag{33}$$

$$\phi_{f_1f_2}(\tau)+i\hat{\phi}_{f_1f_2}(t)=S_2(\tau)e^{iP_2(\tau)} \tag{34}$$

(32) is written as

$$S_2(\tau)e^{iP_2(\tau)}=kS_1(\tau-\tau_g)e^{\{iP_1(\tau-\tau_g)+i\omega_m(\tau_g-\tau_m)\}} \tag{35}$$

which means

$$S_2(\tau)=kS_1(\tau-\tau_g) \tag{36}$$

$$P_2(\tau)=P_1(\tau-\tau_g)+\omega_m(\tau_g-\tau_m) \tag{37}$$

Equations (36), (37) are the same forms as (20), (21).

In other words, the relations of envelopes and phasing angles between the cross-correlation and autocorrelation function in τ domain are the same form as the relations of envelopes and phasing angles between the real input and the output signals in t domain. This is the basic relation for measuring the group delay τ_g by the envelope of the cross-correlation function.

4. MEASUREMENT OF τ_g BY SQUARED SIGNAL CORRELATION

The measuring system is shown in Fig. 1. $y_1(t)$, $y_2(t)$ are the outputs of the low pass filters. f_1(t) and $f_2(t)$ are written in analogous to (5) as

$$f(t)=\sum_{n=1}^{N} C_n \cos(\omega_n t-\varphi_n)=I_c \cos\omega_m t-I_s \sin\omega_m t \tag{38}$$

$$I_{\substack{c\\s}}=\sum_{n=1}^{N} C_n \begin{matrix}\cos\\ \sin\end{matrix}\{(\omega_n-\omega_m)t-\varphi_n\} \tag{39}$$

Then $f^2(t)$ is written as

$$\begin{aligned} f^2(t)&=I_c^2\cos^2\omega_m t+I_s^2\sin^2\omega_m t-2I_cI_s\cos\omega_m t\sin\omega_m t\\ &=\frac{1}{2}(I_c^2+I_s^2)+\frac{1}{2}(I_c^2-I_s^2)\cos 2\omega_m t-I_cI_s\sin 2\omega_m t \end{aligned} \tag{40}$$

On the other hand the envelope of $f(t)$ is expressed as follows.

$$\begin{aligned} R(t)&=|f(t)+i\hat{f}(t)|=\left|\sum_{n=1}^{N} C_n\cos(\omega_n t-\varphi_n)+i\sum_{n=1}^{N} C_n\sin(\omega_n t-\varphi_n)\right|\\ &=|(I_c\cos\omega_m t-I_s\sin\omega_m t)+i(I_s\cos\omega_m t+I_c\sin\omega_m t)|\\ &=|(I_c+iI_s)e^{i\omega_m t}|=(I_c^2+I_s^2)^{\frac{1}{2}} \end{aligned} \tag{41}$$

Then by combining (40) and (41),

$$f^2(t)=\frac{1}{2}R^2(t)+\frac{1}{2}(I_c^2-I_s^2)\cos 2\omega_m t-I_cI_s\sin 2\omega_m t \tag{42}$$

There is another property in regard to envelope function [1], that is, if the frequency spectrum of $f(t)$ is confined in the band $\omega_m-\frac{W}{2}<|\omega|<\omega_m+\frac{W}{2}$, then the square of the envelope is frequency limited to $|\omega|\leqq W$. So, if the cutoff frequency ω_c of the low pass filter is selected as $W<\omega_c\ll\omega_m$ the output of the low pass filter is,

$$y(t)=\frac{1}{2}R^2(t) \tag{43}$$

On the asumption (1), (2) in the article 3, there is the relation as

$$R_2(t)=kR_1(t-\tau_g) \tag{20}$$

Then the cross-correlation of $y_1(t)$ and $y_2(t)$ is calculated as follows.

$$\begin{aligned} \phi_{y_1y_2}(\tau)&=\overline{y_1(t)y_2(t+\tau)}=\frac{1}{4}\overline{R_1^2(t)R_2^2(t+\tau)}=\frac{k^2}{4}\overline{R_1^2(t)R_1^2(t+\tau-\tau_g)}\\ &=\frac{k^2}{4}\phi_{R_1^2}(\tau-\tau_g) \end{aligned} \tag{44}$$

Where $\phi_{R_1^2}(\tau)$ is the autocorrelation function of the square of the envelope of $f_1(t)$. (44) is the basic relation in measuring τ_g by squared signal correlation method.

5. Measurement of τ_g by M-sequence Correlation [4],[5]

The product of the M-sequence signal and the band noise is the input signal to $G(s)$ in Fig. 1. The cross-correlation function of $y_2(t)$ and the M-sequence signal

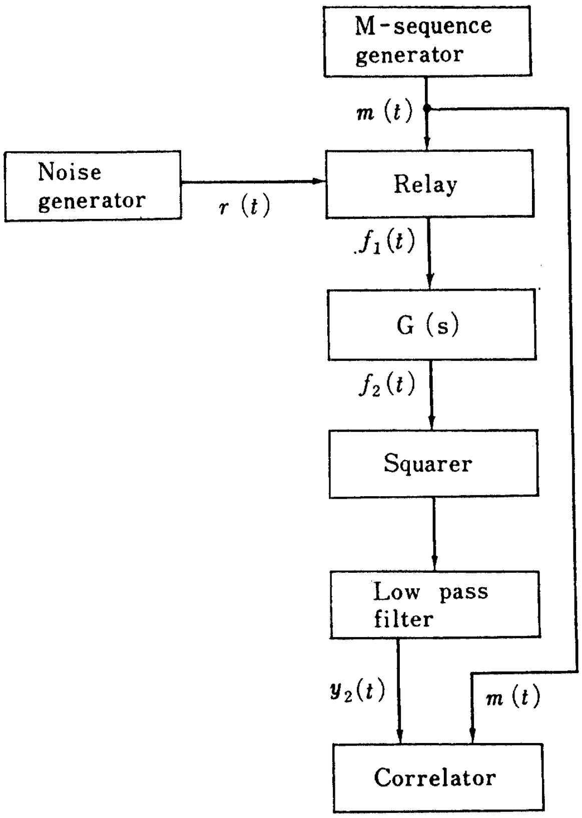


FIG. 2. Measuring system by M-sequence correlation method.

is calculated. That is identical with the M-sequence correlation method proposed by the authors before [*4*], [*5*]. The measuring system is shown in Fig. 2. The M-sequence [*6*] signal $m(t)$ is the two-valued pseudo-random artificial signal whose two values are assumed to be $+1$ and -1. The random signal $r(t)$ is intermitted by the relay according to the sign of the M-sequence signal. Then $f_1(t)$ in Fig. 2 is written as

$$f_1(t)=\frac{1}{2}\{m(t)+1\}r(t)=m'(t)r(t) \tag{45}$$

$$m'(t)=\frac{1}{2}\{m(t)+1\}$$

$r(t)$ is expressed as Eq. (5) or Eq. (38)

$$r(t)=\sum_{n=1}^{N} C'_n \cos(\omega'_n t-\varphi'_n)$$
$$=I'_c \cos\omega_m t - I'_s \sin\omega_m t \tag{46}$$

$$I'_{\begin{smallmatrix}c\\s\end{smallmatrix}}=\sum_{n=1}^{N} C'_n \begin{smallmatrix}\cos\\\sin\end{smallmatrix}\{(\omega'_n-\omega_m)t-\varphi'_n\} \tag{47}$$

Then
$$f_1(t)=m'(t)I'_c\cos\omega_m t-m'(t)I'_s\sin\omega_m t \tag{48}$$

On the other hand $f_1(t)$ is written as

$$f_1(t)=\sum_{n=1}^{N} C_n\cos(\omega_n t-\varphi_n)=I_c\cos\omega_m t-I_s\sin\omega_m t \tag{49}$$

$$I_{\begin{smallmatrix}c\\s\end{smallmatrix}}=C_n \begin{smallmatrix}\cos\\\sin\end{smallmatrix}\{(\omega_n-\omega_m)t-\varphi_n\} \tag{50}$$

From equating (48) and (49) next relations are obtained.

$$I_c=m'(t)I'_c, \qquad I_s=m'(t)I'_s \tag{51}$$

Then the envelope of $f_1(t)$ is written as

$$R_1(t)=(I_c^2+I_s^2)^{\frac{1}{2}}=m'(t)(I_c'^2+I_s'^2)^{\frac{1}{2}}=m'(t)R_r(t) \tag{52}$$

Where $R_r(t)$ is the envelope of the signal $r(t)$.

On the assumption (1), (2) in the article 3, there is a next relation between the envelopes of $f_1(t)$ and $f_2(t)$.

$$R_2(t)=kR_1(t-\tau_g) \tag{20}$$

Then considering (20), (43) and (52), $y_2(t)$ is expressed as follows.

$$y_2(t)=\frac{1}{2}R_2^2(t)=\frac{1}{2}k^2R_1^2(t-\tau_g)=\frac{k^2}{2}m'^2(t-\tau_g)R_r^2(t-\tau_g)$$
$$=\frac{k^2}{2}m'(t-\tau_g)R_r^2(t-\tau_g) \tag{53}$$

The last equality holds because the value of $m'(t)$ takes only $+1$ or 0. Now the cross-correlation function of $y_2(t)$ and $m(t)$ is calculated as

$$\phi_{my_2}(\tau)=\overline{m(t)y_2(t+\tau)}=\frac{k^2}{2}\overline{m(t)m'(t+\tau-\tau_g)R_r^2(t-\tau_g+\tau)}$$
$$=\frac{k^2}{2}\overline{m(t)\frac{1}{2}\{m(t+\tau-\tau_g)+1\}R_r^2(t+\tau-\tau_g)} \tag{54}$$

$r(t)$ is independent with $m(t)$. So $m(t)$ and $R_r(t)$ are mutually independent,

$$\phi_{my_2}(\tau)=\frac{k^2}{2}\overline{m(t)\frac{1}{2}\{m(t+\tau-\tau_g)+1\}}\;\overline{R_r^2(t+\tau-\tau_g)}=\frac{k^2}{4}\overline{R_r^2}\phi_m(\tau-\tau_g) \tag{55}$$

On the above derivation $r(t)$ is assumed to be stationary and the time average of $m(t)$ is considered to be approximately zero. (55) is the basic relation in measuring τ_g by M-sequence correlation method.

6. EXPERIMENTS

To examine preceding considerations the experiments of flexural wave propagation measurements were performed. Flexural waves in solid body propagate with the velocity $v_p=(EI/A\rho)^{\frac{1}{4}}\omega^{\frac{1}{2}}$ which is the phase velocity and the group velocity is known as two times of the phase velocity, that is

$$v_g=2(EI/A\rho)^{\frac{1}{4}}\omega^{\frac{1}{2}} \tag{56}$$

where E is Young's modulous, I is moment of inertia of a section, A is a cross section and ρ is the density of the material [7].

The distance from the vibration excitor to the vibration pick-up is fixed as L, and it is assumed that the frequency response of the excitor and the pick-up are flat and there is no boundary nearby which reflects or disturbs the wave. Then the transfer function from the input of the excitor to the output of the pick-up can be written neglecting the multiple constant,

$$G(s)=e^{-\frac{L}{v_p}s} \tag{57}$$

From (8), (9) and (57),

$$\tau_m=\frac{\omega_m L(EI/A\rho)^{-\frac{1}{4}}\omega_m^{-\frac{1}{2}}}{\omega_m}=L(EI/A\rho)^{-\frac{1}{4}}\omega_m^{-\frac{1}{2}} \tag{58}$$

$$\tau_g = \frac{d}{d\omega}\{\omega L(EI/A\rho)^{-\frac{1}{4}}\omega^{-\frac{1}{2}}\}\Big|_{\omega=\omega_m} = \frac{1}{2}L(EI/A\rho)^{-\frac{1}{4}}\omega_m^{-\frac{1}{2}} = \frac{\tau_m}{2} \tag{59}$$

which confirms that the group velocity is two times of the phase velocity.

(1) *Ordinary cross-correlation method* [*3*]

The measuring system is shown in Fig. 3. The wave propagates in the steel strip whose cross section is $1^{\mathrm{mm}} \times 32^{\mathrm{mm}}$ and length is 9.5^{m}, which is suspended by the strings horizontally. The flexural wave is excited by an electromagnet and is detected by an accelerometer. The cross-correlation function of $f_1(t)$ and $f_2(t)$ is calculated by an electronic digital correlator [*8*].

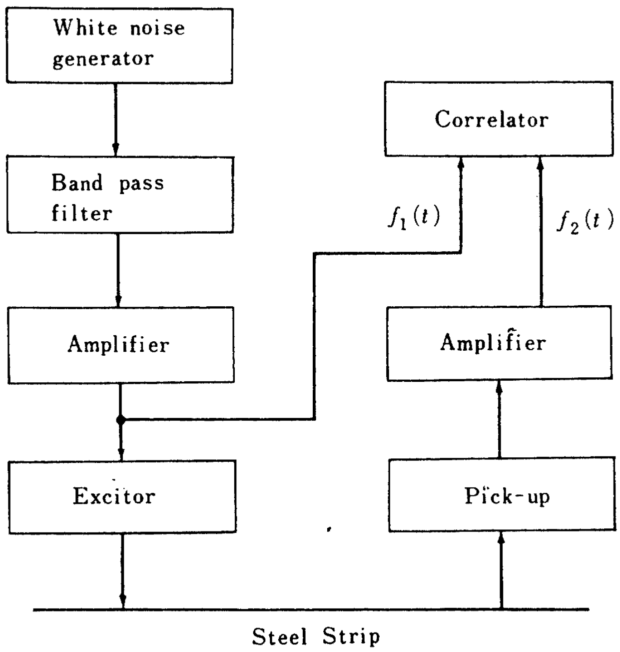


FIG. 3. Flexural wave propagation measurement by ordinary correlation method.

The measured cross-correlation functions are illustrated in Fig. 4, varying the distance from the excitor to the pick-up. In this example $f_1(t)$ is chosen as 1/3 octave band noise whose center frequency is 500 Hz. The sampling values of the correlation functions are represented by the length of the vertical bars. Since the sampling frequency is chosen as one half of the center frequency of $f_1(t)$, the detection of the correlation envelope is rather easy. As the distance increases the envelope pattern shifts to the right without serious distortion, which means from the discussion in article 3 that the envelope of the flexural wave propagates without serious distortion in this range of distance. That is, in considering the flexural wave propagation problem in a few meters, 500 Hz 1/3 octave band noise is narrow enough to assure the assumptions (1), (2) of the article 3.

The amount of shifts of correlation envelopes are illustrated as the function of

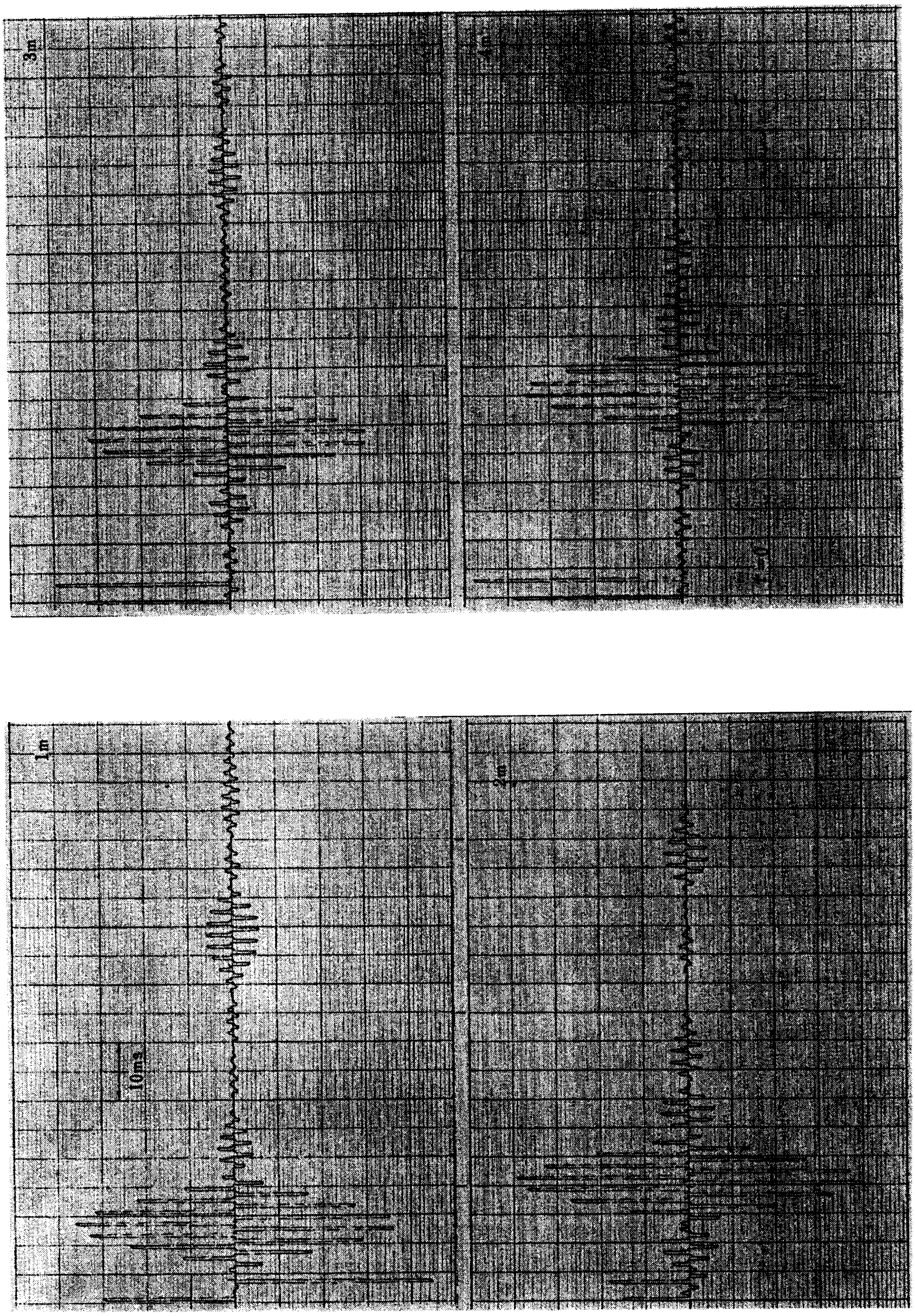


FIG. 4. Measured cross-correlation function by ordinary correlation method. $f_1(t)$ is 1/3 octave band noise whose center frequency is 500 Hz.

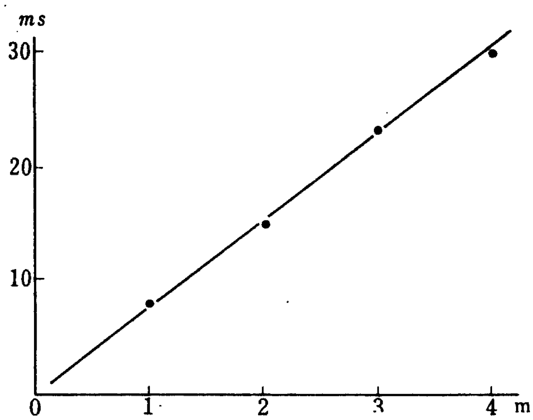


FIG. 5. Deley time vs, distance. Measured by ordinary correlation method.

distance in Fig 5. From the slope of this result, the propagation velocity of flexural wave envelope can be calculated as 1.33×10^2 m/s. On the other hand the theoretical value of the group velocity of the flexural wave propagation in this steel strip is calculated from the Eq. (56) as 1.36×10^2 m/s.

(2) *Squared signal correlation method*

The measuring system is shown in Fig. 6. As a squaring device, the nonlinear semiconductor resister called silister or thyrite is used [5]. The measured correlation functions are shown in Fig. 7. In this example the thickness of the steel strip is 0.7 mm, the signal $f_1(t)$ is 500 Hz 1/3 octave band noise and the cutoff frequency of the low pass filters is 150 Hz. As discussed in article 4, the cross-correlation pattern obtained in Fig. 7 is the same form as the autocorrelation function of the square of the envelope of $f_1(t)$ with some shift in τ axis. The amount of shift which corresponds to τ_g is plotted as the function of the distance in Fig. 8. From the slope, the propagation velocity is obtained as 1.14×10^2 m/s, on the other hand the theoretical value is 1.12×10^2 m/s. In this method the shift of the correlation function itself (not the envelope of the correlation function) represents τ_g, so it is easier to detect it.

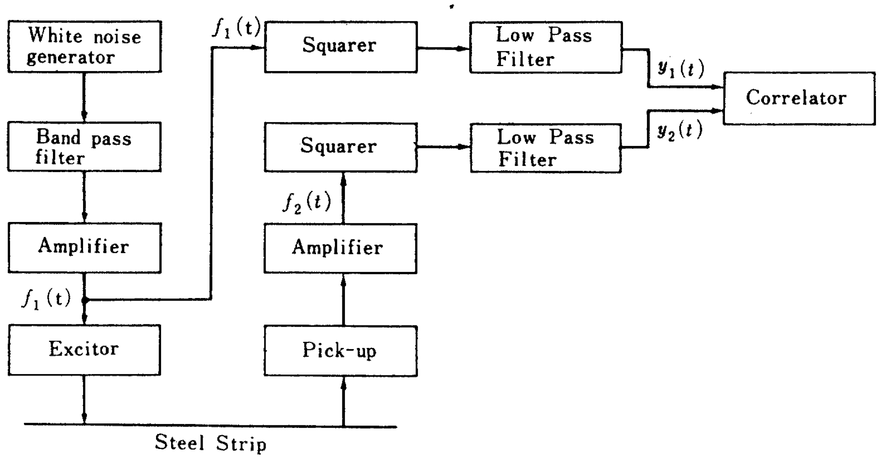


FIG. 6. Flexural wave propagation measurement by squared signal correlation method.

(3) *M-sequence correlation method* [4],[5]

The measuring system is shown in Fig. 2. *M*-sequence signal generator is made by transister logic circuit and can produce the *M*-sequence signal whose maximum order is tenth and minimum time unit is approximately 3 ms. $G(s)$ is the flexural

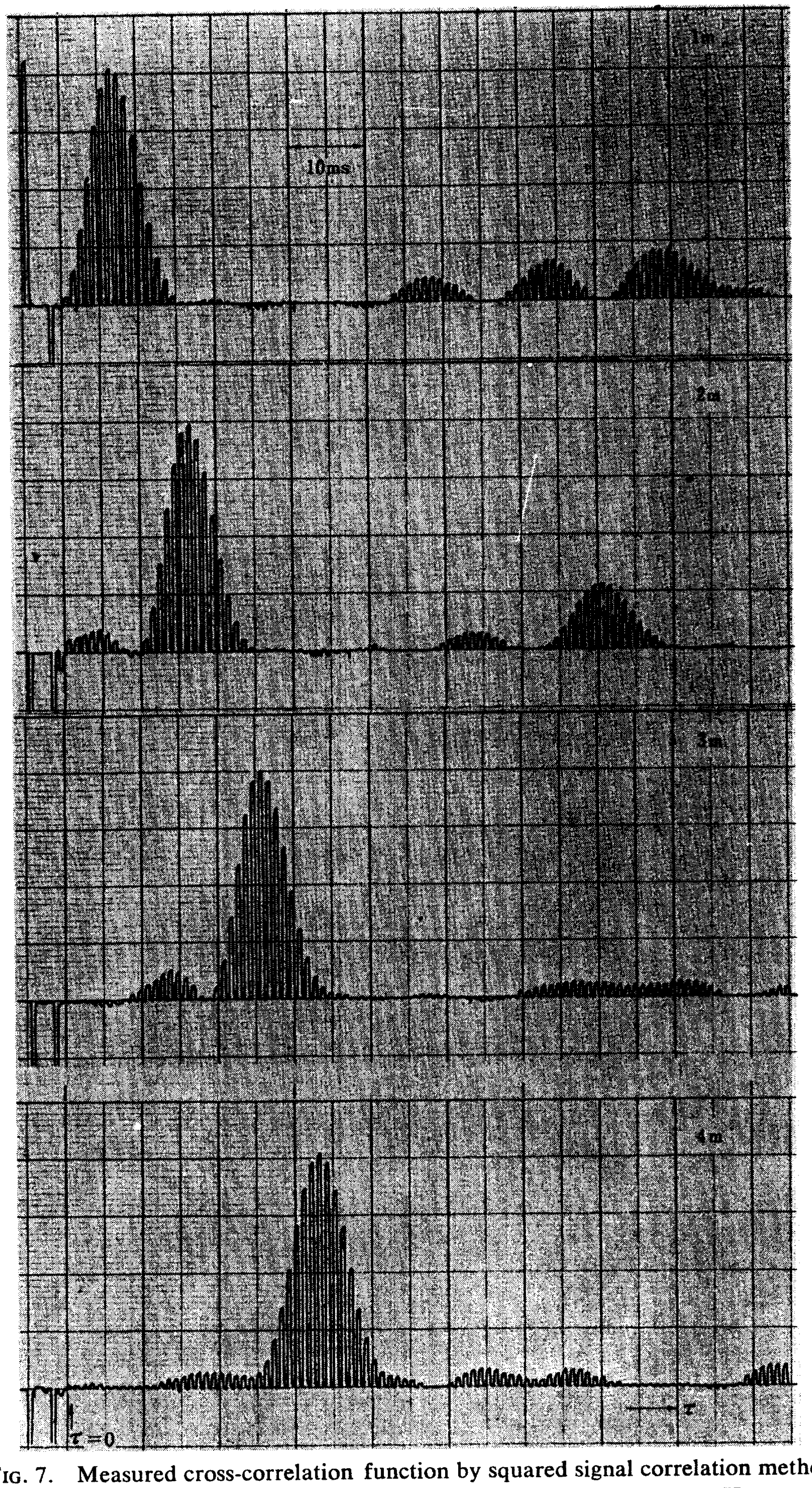
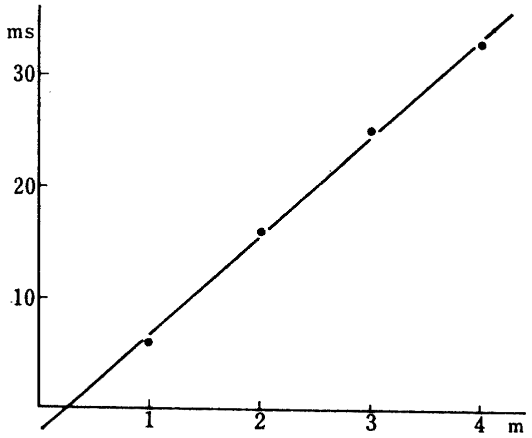


FIG. 7. Measured cross-correlation function by squared signal correlation method. $f_1(t)$ is 1/3 octave band noise whose center frequency is 500 Hz.



FGI. 8. Delay time vs. distance. Measured by squared signal correlation method.

wave propagation system as before.

Measured correlation functions are illustrated in Fig. 9. In this example the thickness of the steel strip is 1 mm, the signal $r(t)$ is 800 Hz 1/3 octave band noise and the time unit of the M-sequence signal $m(t)$ is 5 ms. The shift of the correlation function represents τ_g, as mentioned in the Eq. (55). From the slope of τ_g vs. distance, the propagation velocity of the envelope of the flexural wave is measured. By varying the center frequency of the 1/3 octave band noise, wave velocities are plotted as the function of the center frequencies in Fig. 10. The relation between the wave velocity v and the frequency f can be obtained as $v=6.0\sqrt{f}$. In this case the theoretical relation of (56) is $v=6.1\sqrt{f}$.

7. Comparison of Three Methods and General Properties

In the preceding articles three methods of measuring the group delay time τ_g were mentioned. The measurements are based on the next three equations.

(A) Ordinary correlation method

$$S_2(\tau)=kS_1(\tau-\tau_g) \tag{36}$$

(B) Squared signal correlation method

$$\phi_{y_1y_2}(\tau)=\frac{k^2}{4}\phi_{R^2}(\tau-\tau_g) \tag{44}$$

(C) M-sequence signal correlation method

$$\phi_{my_2}(\tau)=\frac{k^2}{4}\overline{R_r^2}\phi_m(\tau-\tau_g) \tag{55}$$

It is necessary to discuss the points of these methods.

(1) Method (A) requires the envelopes of the correlation functions, but the envelopes of the arbitrary functions are in general not easy to detect. On the other hand the equations of (B), (C) are the relations of the correlation functions themselves.

(2) The shapes of the correlation functions of (A) and (B) depend on the statistical properties of the input signal $f_1(t)$, but the correlation shape of (C) is identical with the shape of autocorrelation function of the M-sequence signal, which is triangular in form and it is very easy to detect the time delay.

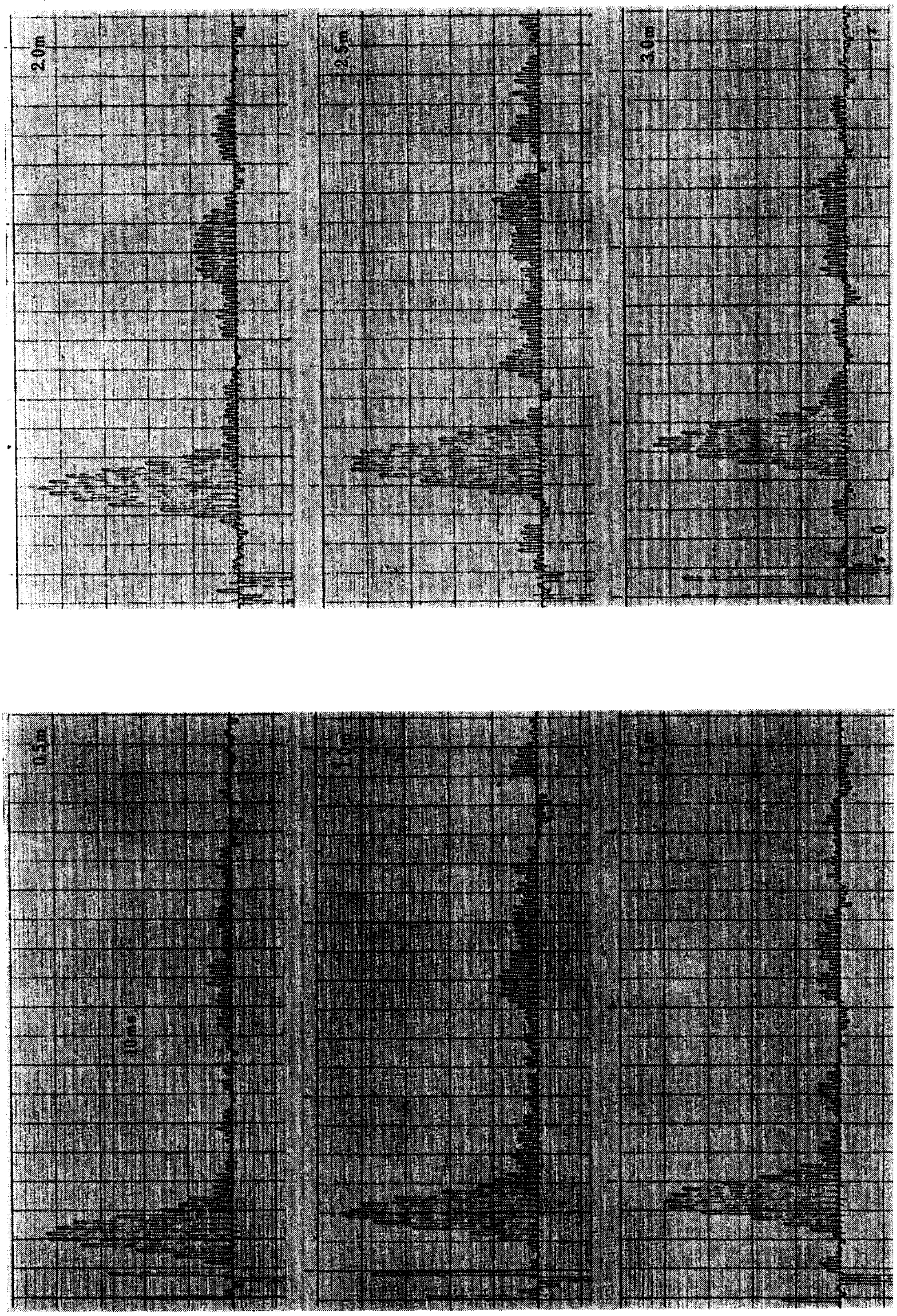


FIG. 9. Measured cross-correlation function by M-sequence correlation method. $r(t)$ is 1/3 octave band noise whose center frequency is 800 Hz.

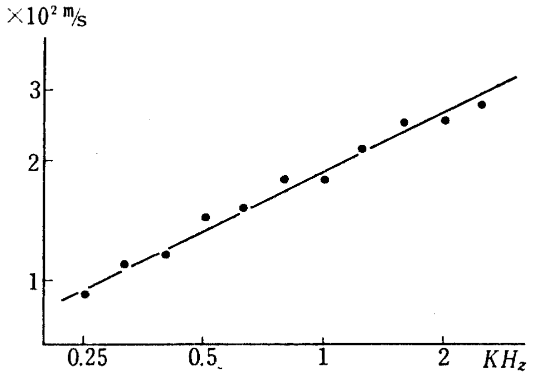


FIG. 10. Wave velocity vs. frequency. Measured by M-sequence signal correlation method.

(3) In measuring the group velocity as a function of frequency, the test signal $f_1(t)$ should vary as wide as possible in frequency. Then the method (A) requires wide range operation for frequency characteristics of the correlator. On the other hand in (B) and (C), input signals to the correlator are the output of the low pass filters, and the correlator handles with low frequency signals whose upper cutoff frequencies are some definite values. This is favorable for the digital type correlator. Especially by (C), one of the correlator inputs is the two-valued signal, and the procedure of the calculation can be simplified.

(4) As the measuring signal, method (C) requires a special source which generates the intermitted random signal.

(5) By (B) and (C), the squaring devise is needed, and an accurate high speed squarer is not easy to construct.

The following properties are common to these methods because of the correlation techniques.

(6) The influence of the external noise which is incoherent with the input signal $f_1(t)$ can be eliminated.

(7) If there are wave reflecting boundaries, wave rays can be detected separately unless the pass differences are too small. In this case because of the randomness of the signal $f_1(t)$, wave rays do not interfere each other in computing cross-correlation functions. The example of the multiple wave paths is illustrated in Fig. 11. These are the measurements of the flexural wave in steel strip whose width is 0.7 mm with the signal 500 Hz, 1/3 octave band noise. The first peak represents the direct wave, the second peak is the reflection at the left end and the third peak is the reflection at the right end. From these peaks not only the delay times but the relative intensities of the rays can be measured.

8. CONCLUSION

In this paper the group delay time is defined for general physical systems and three measuring methods of the group delay time by correlation techniques were developed. The M-sequence correlation method which was proposed by the authors before can be used in the case when the dispersive waves are considered.

The experiments of the velocity measurement of the flexural wave propagation in the steel strips are performed, and the results are shown to be good agreement with the theoretical values.

We owed much to Dr. Yasushi ISHII of our institute, who designed the electronic

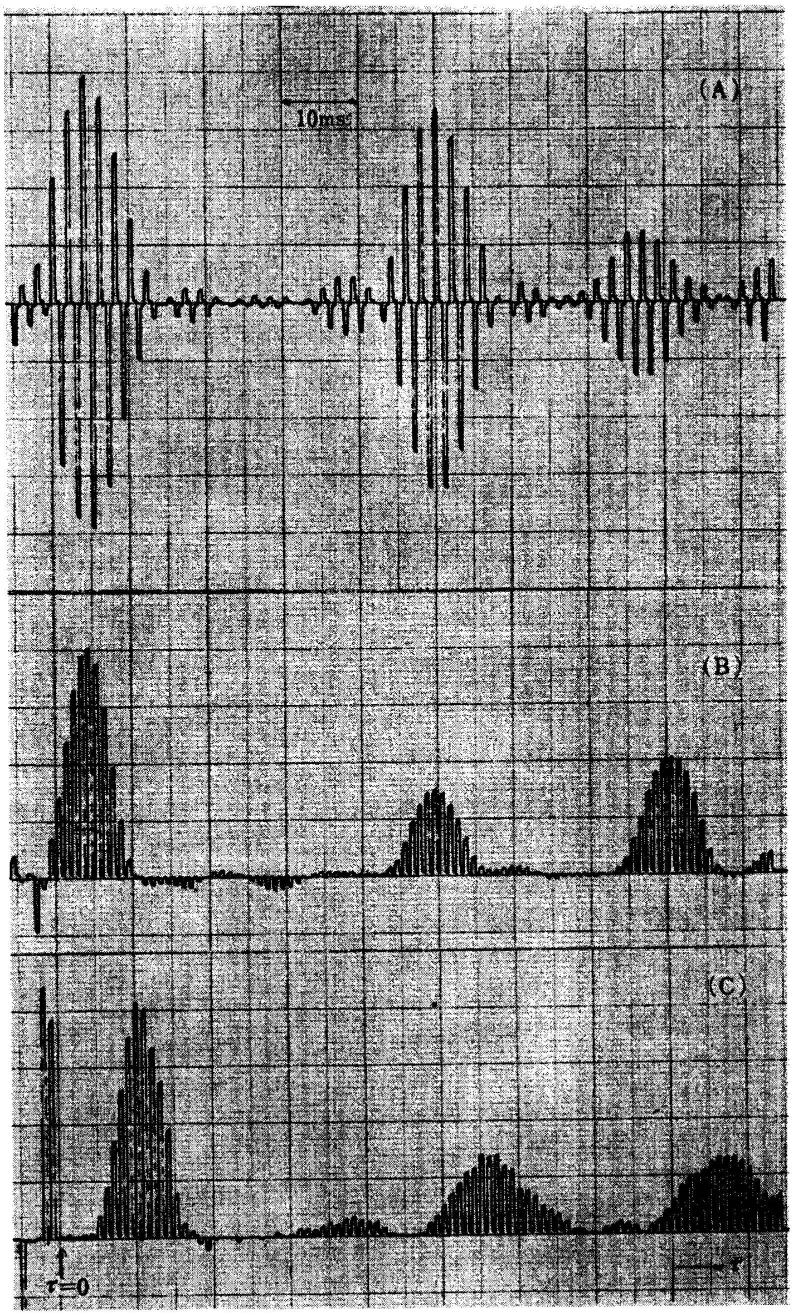


FIG. 11. Measurement of multiple wave paths by (A) ordinary correlation method, (B) squared signal correlation method and (C) M-sequence signal correlation method.

digital correlator. The authors express great thanks for his valuable advises and suggestions.

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January 18, 1969

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