Institute of Space and Aeronautical Science, University of Tokyo
Report No. 444, February 1970

A Real-Time Signal Processing System for Correlation and Spectrum Analysis

By

Yasushi ISHII

Summary: This paper reports a digital correlator operating in real-time and a spectrum analyzer which converts the output of the correlator into the power spectrum. The correlator employs three magnetostrictive delay lines as its main storage units and computes the values of auto- or cross-correlation function of the analog input signals at one hundred equally spaced points on the time delay axis simultaneously. The computed results are DA converted and displayed on a cathode-ray tube in the form of a bar-graph. The measured correlation function is always up-to-date and automatically follows the change of the input signals. With a special method of sampling of the input signals, the increment of the time delay can be made as small as 200 μs, which enable us, for instance, to make a real-time analysis of spoken voices. The spectrum analyzer receives the digital output of the correlator and multiplies it by an internally generated sinusoidal function. This Fourier transformation is performed at one hundred different freqnencies and the final output is displayed on a cathode-ray tube at every 200 ms, giving the entire form of the power spectrum within a specified frequency range. All the computations are performed digitally except the modification of output by the hamming window. Some applications of this signal processing system are also included in this paper.

1. INTRODUCTION

With the increasing requirements of the real-time analysis of random signals, several special purpose computers for correlation and spectrum analysis have been developed in the past few years and some of them are now commercially available. These are; real-time spectrum analyzers using a time compression technique [*1*][*2*], a high-speed digital signal processor based on Fast Fourier Transform [*3*][*4*] and real-time correlators using a switching multiplication [*5*][*6*].

This paper is also concerned to a real-time signal processor for correlation and spectrum analysis, but it is different from those mentioned above. It is essentially a combination of a digital real-time correlator and a spectrum analyzer which converts the output of the correlator into the power spectrum.

The correlator computes the values of auto- or cross-correlation function of the analog input signals at 100 equally spaced points on the time delay axis simultaneously. The computed results are DA converted and displayed on a cathod-ray oscilloscope in the form of a bar-graph. The spectrum analyzer performs the Fourier cosine transformation of the correlation function at 100 different frequen-

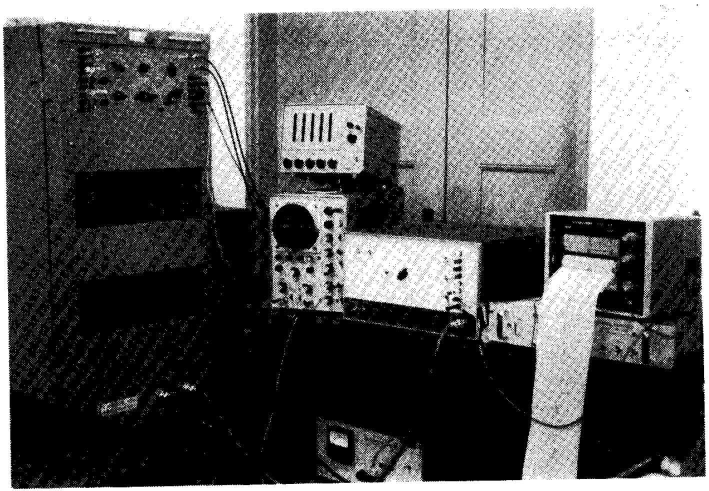


PHOTO. 1. Photograph of the real-time signal processing system. From left to right; real-time correlator, preset counter, displaying oscilloscope, spectrum analyzer, pen-writing recorder and digital printer.

cies, and the results are also displayed on a cathode-ray oscilloscope at every 200 ms. The operating principles and construction of this signal processor as well as several applications are described in the following sections.

2. CORRELATOR

The correlator is mainly composed of Resistor-Transistor-Logic circuits and contains 800 transistors and 3000 diodes. As shown in the block diagram of **Fig. 1**, two input signals, $x(t)$ and $y(t)$, which are to be correlated, are fed to the analog-to-digital converters and converted into 4-bits binary numbers at regular sampling intervals. The main storage of the correlator consists of three magnetostrictive delay lines. The cycle time of the storage units is 2 ms and the clock rate is 1 MHz. The numbers representing the sampled values of $x(t)$ are stored in the shift register SR_2 at first, and then introduced into the storage loop L_1, which conveys the past

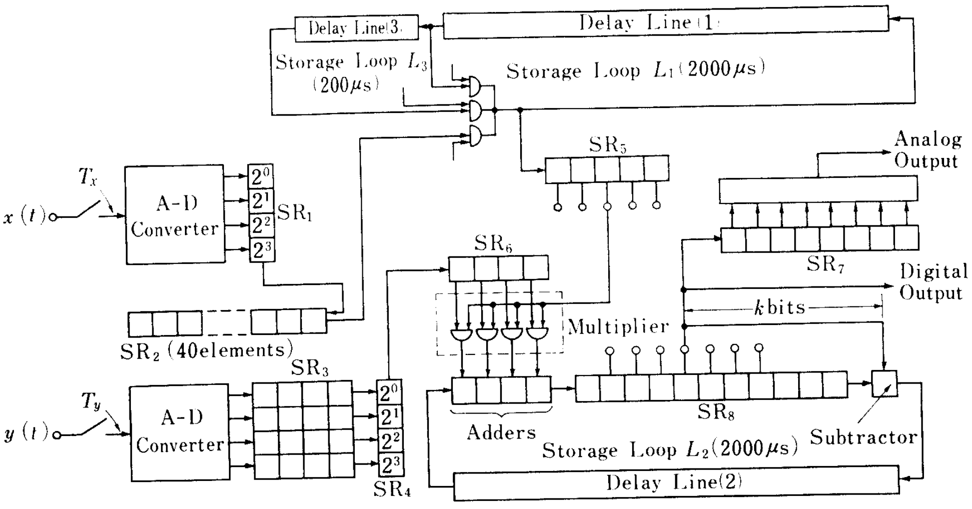


FIG. 1. Block diagram of the real-time correlator.

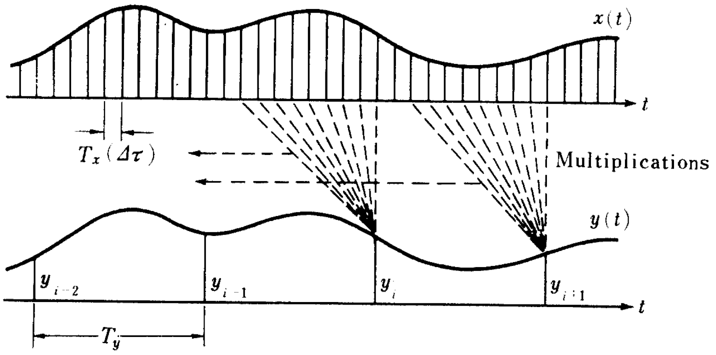


FIG. 2. Sampling and multiplication of the input signals.

100 sampled data of $x(t)$. These data are set on the shift register SR$_5$ in turn and multiplied by the number representing the newest sampled value of $y(t)$. The products are accumulated in the storage loop L$_2$. Thus, the correlator computes the values of a correlation function at 100 equally spaced points on the time delay (τ) axis simultaneously. (More accurately, 95 points on the τ-axis. The remaining five represent the mean and mean-squared values of the input signals. See Fig. 3 and Fig. 4.)

The sampling periods of the input signals are different in $x(t)$ and $y(t)$ as shown in **Fig. 2.** The sampling period of $y(t)$, T_y, is an integral multiple of the cycle time of storage units while the sampling period of $x(t)$, T_x, is one-tenth of T_y. That is, 10 data of $x(t)$ are sampled and stored in SR$_2$ while $y(t)$ is sampled once. The data in the storage loop L$_1$ are renewed ten by ten at each sampling instant of $y(t)$.

With such a method of sampling the increment of time delay or $\Delta\tau$, which is equal to T_x, can be made as small as 200 μs, one-tenth of the storage cycle time, at the sacrifice of the number of data to be integrated. However, it should be noted that the statistical accuracy of a measured correlation function is determined mainly by the data length over which the integration is taken and not by the number of data. The value of $\Delta\tau$ may be chosen between 200 μs and 20 s in 1-2-4 steps.

The storage loop L$_2$ accumurates the output of the multiplier. This loop includes a subtractor as well as adders and performs the following arithmetic operation [7][8] at each accumulation of the products;

$$I_{\text{new}} = I_{\text{old}} + z - (I_{\text{old}} + z)/2^k \qquad (1)$$

where z is the output of the multiplier or a product of $x(t-\tau)$ and $y(t)$, I_{old} and I_{new} are the accumulated values of the product before and after the arithmetic operation is made respectively and k is a positive integer. The division in the above equation is performed by just shifting the binary number of $(I_{\text{old}}+z)$ by k bits. This method of integration is equivalent to the averaging of an analog signal by an R-C filter whose time constant is $2^k T_y$. Therefore the accumulated products follow automatically the change of the stochastic nature of the input signals. This feature is very useful for the real-time processing of nonstationary signals. The averaging constant, 2^k, can be changed from 8 to 4096 in binary steps. Averaging of the products by the pure integration is also possible by inhibiting the operation of the subtractor. In this mode of averaging, a preset counter determines the number of data to be integrated.

In the storage loop L_2, 20 bits are assigned to each accumulated product and the first 8 bits of the most significance are taken by the register SR_7 to be converted into a voltage. This voltage is fed to a oscilloscope and displayed on the cathode-ray tube in a bar-graph form at the every circulation time, namely, at every 2 ms. This voltage is also fed to the intensity modulation circuit of a cathode-ray oscilloscope to achieve a 3-dimension display of the computed result. In this form of display, or in "correlatogram" [9], the shutter of the camera which takes the picture of the correlatogram is kept open and the spot on the cathode-ray tube is swept vertically with the repetition rate of the correlator output while scanning horizontally in real-time. The value of the correlation function is represented by the intensity of brightness as shown in **Fig. 3** and **Fig. 4.** Output terminals for digital readout

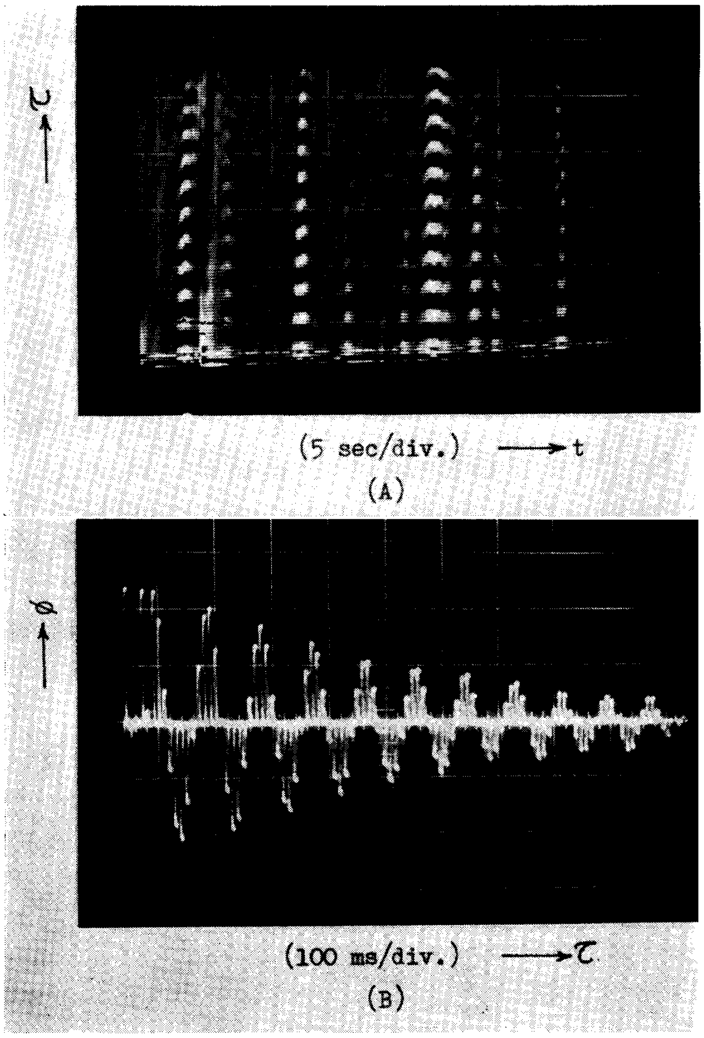
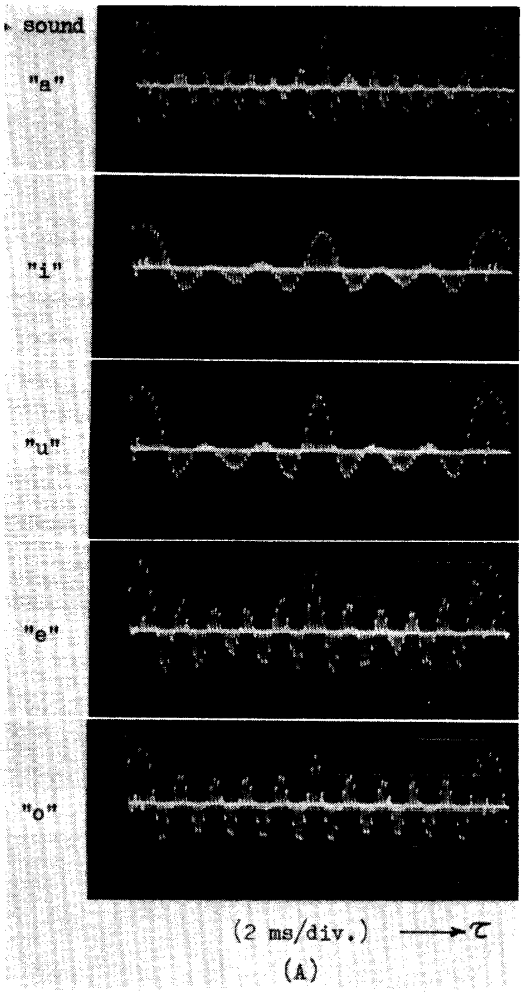
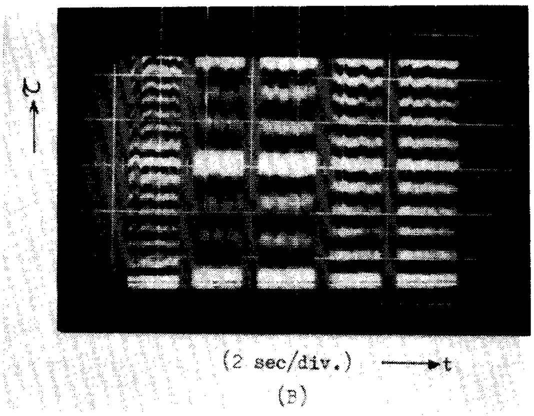


FIG. 3. A correlatogram of brain wave showing the occurence of the α-wave (A) and the auto-correlation function of the α-wave (B). The correlation function is displayed from left to right in the order of;

$$\overline{x^2}, \bar{x}, -\bar{x}, \overline{y^2}, \bar{y}, \phi(0), \phi(1), \phi(2), \cdots, \phi(94).$$



(A)



(B)

FIG. 4. Measured auto-correlation functions of human voice (A) and their correlatogram (B).

of the contents of L_2 are also provided. The digital output is used for printing out the results or further processing of the data by the spectrum analyzer.

3. SPECTRUM ANALYZER

The spectrum analyzer accepts digital data of an auto-correlation function from the real-time correlator and multiplies them by internally generated sinusoidal functions of various frequencies to obtain the power spectrum. The analyzer consists of about 250 IC packages and all the computations are performed digitally except the modification of the analog output by the hamming window. Controlling signals necessary for the operation as well as the clock pulse are also supplied by the correlator.

As shown in the block diagram of **Fig. 5,** the analyzer has a 94-stages shift register controlled by the gated oscillator. All of the stages except one are in "0" state. The position of "1" state in the shift register corresponds to the angle of the sinusoidal function. When the gated oscillator delivers a burst of shift pulses, the "1" state moves toward right and stays at a new position. Thus, this "1" circulates intermittently around the shift register. The number of stages passed by one movement determines the frequency of the sinusoidal function.

The function generator is a wire-memoried digital circuit. When one of the inputs is energized, the function generator delivers a binary number representing the value of the sinusoidal function at the angle corresponding to the energized input. Although the value of correlation function is stored in the correlator as a 20-bits binary number, only the 8-bits of the most significance are taken by the spectrum analyzer and multiplied by the number of the sinusoidal function. The products are integrated by the 20-bits accumulator.

The output of the correlator recurs at every 2 ms. The frequency of the sinusoidal function is increased step by step when the integration is repeated at this every recurrence. The Fourier transformation is done at 100 frequencies. Therefore, 200 ms is spent on performing entire computation.

The computation described above is presented mathematically as the following;

$$P_0(r)=\phi(0)/2+\sum_{i=1}^{h}\phi(i)\cos(2\pi ri/2h) \qquad (2)$$

$$r=-1, 0, 1, 2\cdots\cdots, 98$$

where ϕ is the correlation functions and h is the number of the increments of time delay of the correlation functions or 94 in this case. $P_0(-1)$ is necessary for the modification of the output by the hamming window. For an exact transformation,

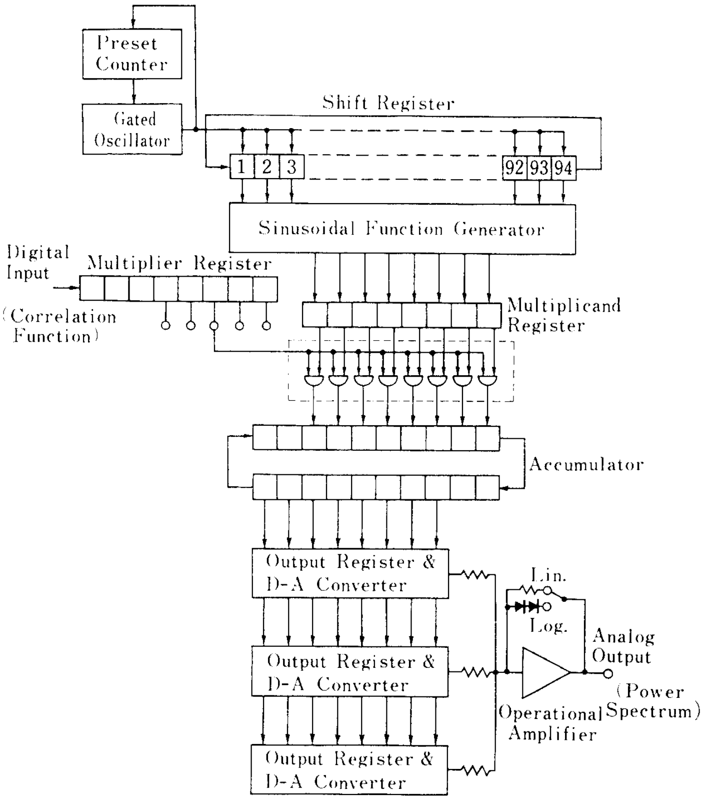


FIG. 5. Block diagram of the spectrum analyzer.

$\phi(94)$ in the above equation should also be halved but this was ignored for the simplicity of the circuits. The frequency range analyzed is determined by the increment of time delay of the correlator. The folding frequency, $1/2\Delta\tau$, corresponds to $r=94$ in Eq. (2) and the Fourier transformation is performed beyond this to $r=98$. (See **Fig. 6**)

The most significant 8 bits of the integrated value in the accumulator are transfered to the first output register at the end of the integration. Three successive integrated values are stored in the output registers and each is converted to an analog voltage. According to the hamming window [*10*], the weighted sum of these three voltages is obtained by an operational amplifier, namely;

$$P_m(r)=0.23\,P_0(r-1)+0.54\,P_0(r)+0.23\,P_0(r+1) \qquad (3)$$

where P_m represents the final output of the spectrum analyzer.

The output voltage of the operational amplifier is fed to the vertical input terminal of an oscilloscope to visualize the result in a bar-graph form or fed to the intensity modulation terminal to display the results in a sonagram just like to the

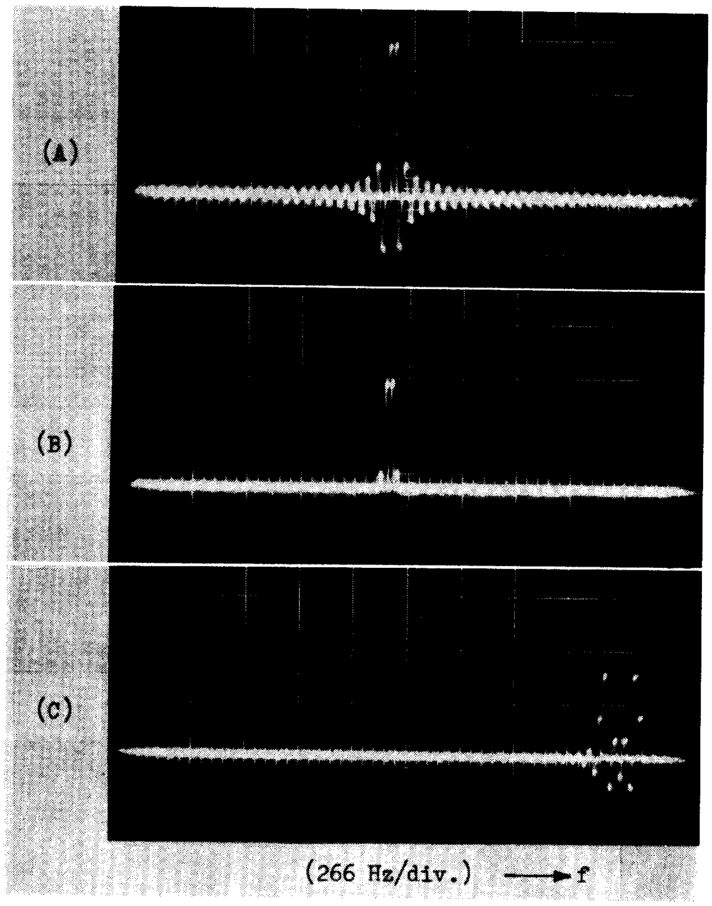


FIG. 6. Power spectra of a sinusoidal signal measured with the real-time spectrum analyzing system using the do-nothing window (A) and the hamming window (B). The measured power spectrum when the frequency of the signal is near to the folding frequency shows the aliased spectrum (C).

correlatogram. Or it can be recorded by a pen-writing servo-recorder in the slow display mode. A logarithmic output voltage is also obtainable by interchanging the feedback resistor of the operational amplifier with semi-conductor diodes. The voltage-current characteristics of the diodes in forward direction are utilized in this conversion.

Fig. 7 and **Fig. 8** are the measured spectra corresponding to the correlation functions of Fig. 3 and Fig. 4 respectively.

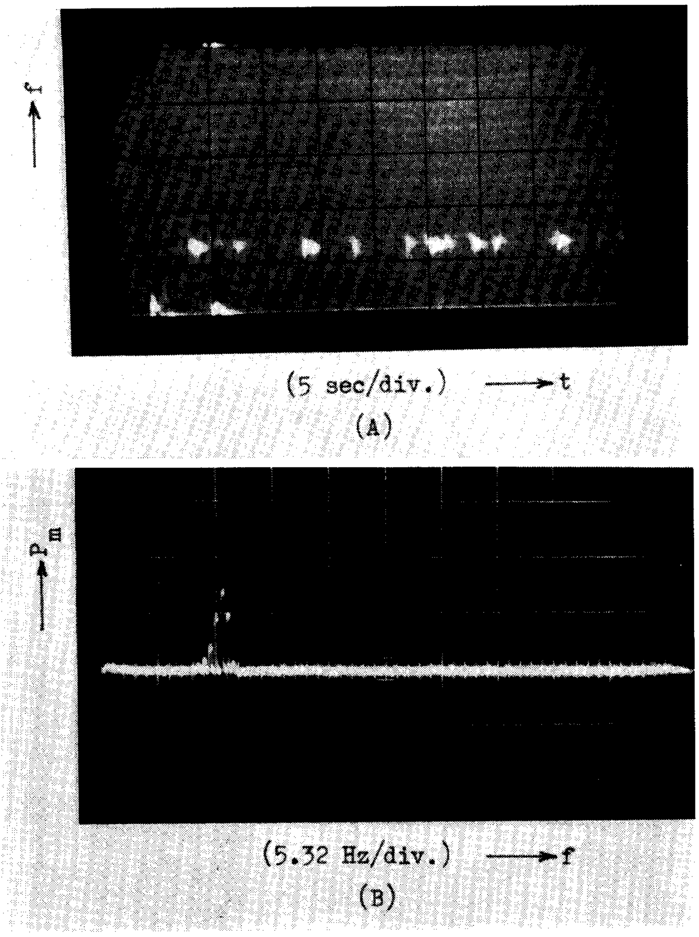


FIG. 7. A sonagram of brain wave (A) and the power spectrum of the α-wave (B).

4. MEASUREMENT OF CEPSTRA AND AVERAGE RESPONSES

Fig. 9 shows the procedure for measuring a "cepstrum" [*11*] with this signal processing system. The repeating waveform of the logarithmic output of the spectrum analyzer is stored by a tape recorder for a few minutes. In this recording procedure, the output waveform is a staircase form instead of being the discontinuous bar-graph form. After disconnecting the original input from the correlator, the reproduced output of the tape recorder is fed to the correlator through an analog band-pass filter in order to determine the frequency of the ripple superimposed on the waveform of the logarithmic power spectrum. Thus we get a cepstrum show-

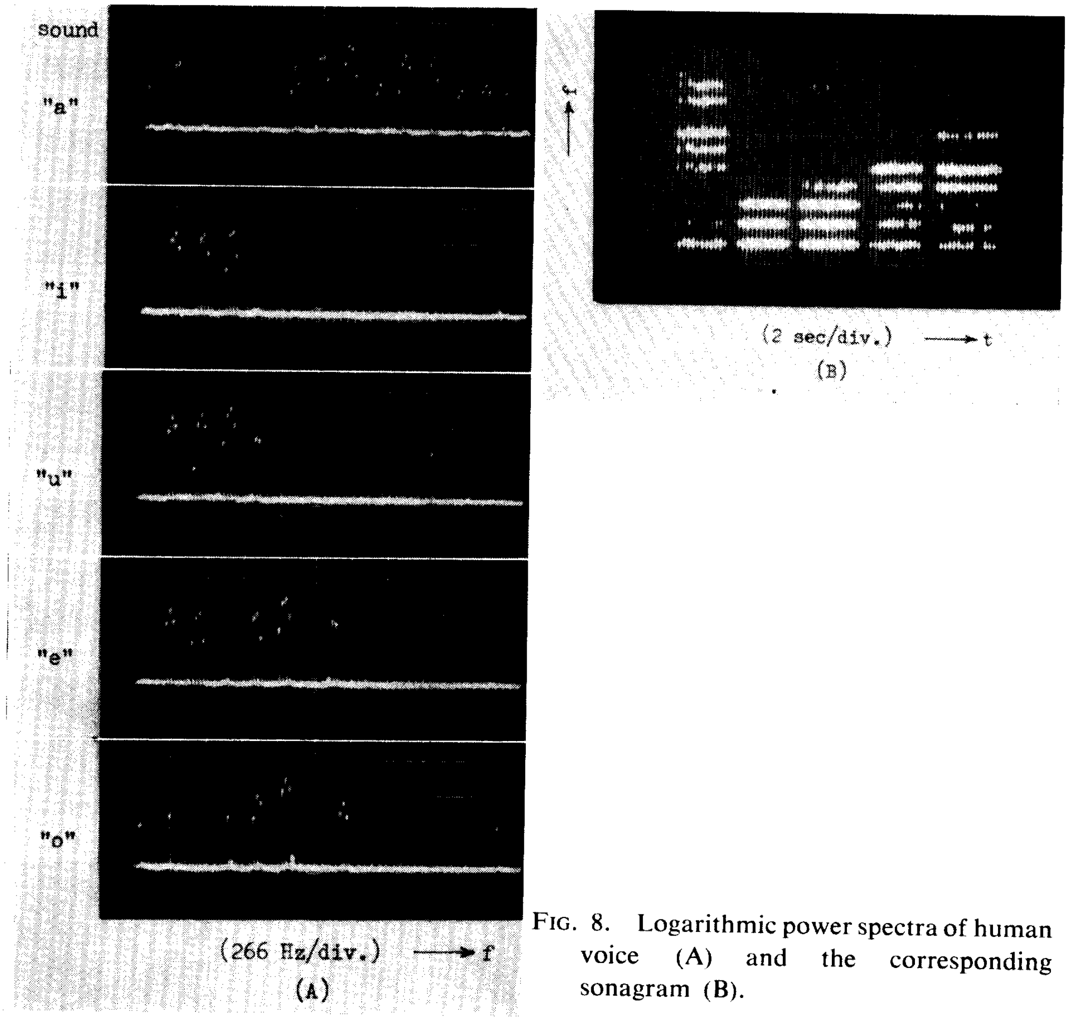


FIG. 8. Logarithmic power spectra of human voice (A) and the corresponding sonagram (B).

ing the ripple frequency which in turn shows the time delay of the reflected signal in the original input. An experiment was made on an acoustic system shown in Fig. 9 and the results are presented in **Fig. 10.**

The technique of average response is now widely used in many scientific fields to detect small stimulus-locked components buried in a noise. This technique, however, can be considered as a special case of the cross-correlation analysis in which one of the signals is a train of the stimulating pulses. Actually the first average response computer [*12*] stemmed from a correlator.

Fig. 11 shows the setup for the measurement of average responses using the real-time correlator. The signal to be analyzed is introduced to the X input terminal and a constant non-zero voltage is given to the Y input terminal. This constant voltage makes the multiplications performed by the correlator into the additions of the sampled data of $x(t)$ with a constant coefficient. Besides these two inputs, the stimulating pulses delayed by a fixed time with the delaying sweep circuit of an oscilloscope are also fed to the correlator as a triggering input. The correlator performs one accumulation of the stored data of $x(t)$ at each reception of the triggering pulse. The fixed time delay of the stimulating pulse is necessary because the

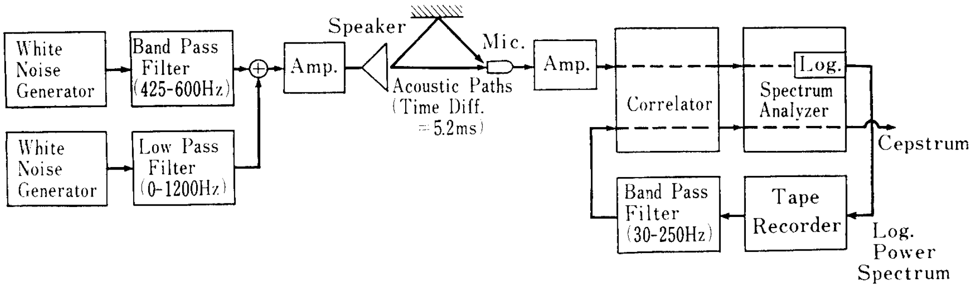


FIG. 9. Block diagram showing measurement of a cepstrum with the real-time signal processing system.

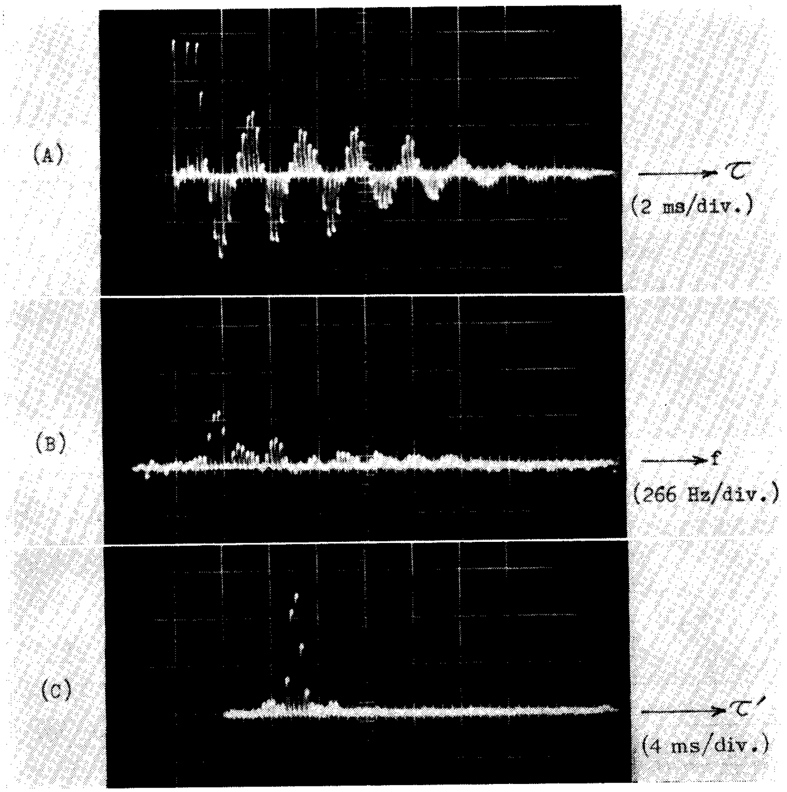


FIG. 10. Auto-correlation function (A), logarithmic power spectrum (B) and cepstrum (C) of the acoustic signal shown in Fig. 9.

correlator always stores the past 100 data of $x(t)$ while the response appears after the stimulating pulse. Therefore, the length of time delay must be such that the accumulation is performed just after the response has died out.

Fig. 12 is an example of the averaged evoked potentials measured with the real-time correlator. Since the stimulating pulse in this experiment is not synchronized to the correlator and the accumulation is not started until the data in the storage loops of the correlator come to a proper position after the reception of the triggering pulse, there is a timing error which uniformly distributes between 0 and $\Delta\tau$, By the same reason, in the externally synchronized mode, the minimum time resolution of this average response computer is limited to 2 ms, the cycle time of the storage units.

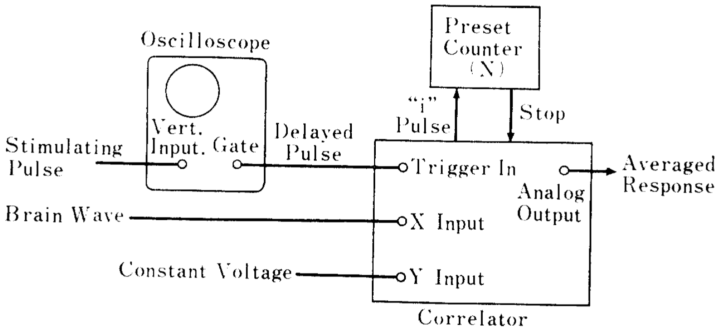


FIG. 11. Measurement of average responses with the real-time correlator.

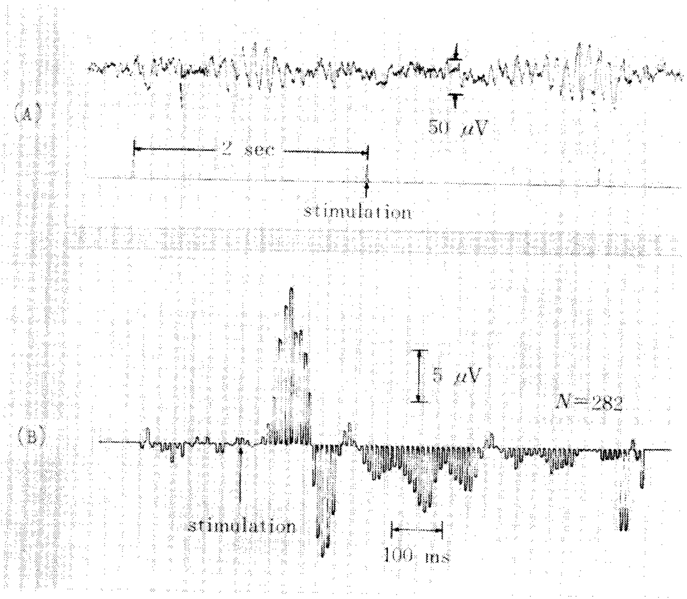


FIG. 12. An example of evoked potentials to visual stimulation buried in stationary brain wave (A) and the averaged evoked potential (B).

5. CONCLUSIONS

A real-time signal processing system for correlation and spectral analysis was presented. The application of this system covers a wide range of signal analysis including the measurement of the cepstrum and the average response. Although the processing speed of the present system is not so high, it is not so difficult to construct a system having the clock rate of 10 MHz or more using quartz delay lines, which will enable us to get the spectrum of, for instance, the actually spoken voices.

ACKNOWLEDGEMENT

The author wishes to express his thanks to Dr. T. Ueno, F. Hirai, M. Kimbara, R. Andō, S. Fujimori, S. Furui and R. Oka for their cooperation in constructing the signal processor. He also wishes to acknowledge Matsunaga Science Foundation by whom this research was partially supported.

Department of Instruments and Electronics
Institute of Space and Aeronautical Science
University of Tokyo, Tokyo
Jan. 9, 1970

REFERENCES

[*1*] J. S. Gill: A Versatile Method for Short-Term Spectrum Analysis in Real-Time, Nature, Vol. 189, No. 4759, pp. 117–119 (1961).

[*2*] T. Koshikawa, *et al.*: Real-Time Colored Display of Sound Spectrograms, Reports of 6th Inter. Cong. on Acoustics, Aug. 1968, Tokyo, pp. C65–C68.

[*3*] Special Issue on FFT, IEEE Trans., Vol. AU-15, No. 2 (1967).

[*4*] R. Klahn and R.R. Shively: FFT—Shortcut to Fourier Analysis, Electronics, Apr. 15, 1968, pp. 124–129.

[*5*] B. P. Th. Veltman and A. van den Bos: The Applicability of the Relay Correlator and Polarity Coincidence Correlator in Automatic Control, Automatic and Remote Control (Proc. 2nd IFAC Cong.), Theory, pp. 620–627, Butterworths (1964).

[*6*] B. LuBow: Correlation Entering New Fields with Real-Time Signal Analysis, Electronics, Oct. 31, 1966, pp. 75–81.

[*7*] T. Ueno and N. Mano: A Method for Real-Time Computation of Correlation Functions, Graduation Thesis, Dept. of Applied Physics, Univ. of Tokyo, Mar. 1963.

[*8*] Y. Lundh: A Digital Integrator for On-Line Signal Processing, IEEE Trans., Vol. EC-12, No. 1, pp. 26–28 (1963).

[*9*] W. R. Bennett: The Correlatograph—A Machine for Continuous Display of Short Term Correlation, Bell Sys. Tech. J., Vol. 32, No. 5, pp. 1173–1185 (1953).

[*10*] R. B. Blackman and J. W. Tukey: The Measurement of Power Spectra, Dover (1959).

[*11*] R. P. Bogert, M. J. Healy and J. W. Tukey: The Quefrency Alanysis of Time Series for Echoes, Time Series Analysis (ed. M. Rosenblatt), pp. 209–243, John Wiely (1963).

[*12*] W. A. Clark, *et al.*: The Average Response Computer, IRE Trans., Vol. BME-8, No. 1, pp. 46–51 (1961).