Institute of Space and Aeronautical Science, University of Tokyo
Report No. 461, March 1971

Discontinuity in Buckle Pattern Tessellation of Cylindrical Shells of Variable Curvature

By

Koryo MIURA and Masamori SAKAMAKI

Summary: This paper presents an experimental evidence that the discontinuity of buckle pattern tessellation exists in the elastic postbuckling configuration of cylindrical shells of variable curvature. This raises the question on the validity of some assumptions generally prevail in analytical studies about the problem.

It seems that an unproved assumption about the buckling and postbuckling deformations of cylindrical shells of variable curvature has been unconsciously used in the past literatures. The assumption is that the axial wavelength is constant about the buckles which are in a row along the circumference. No reasoning of it has ever been shown. Since this assumption is inevitably involved in the more general assumption on the continuity of buckle pattern, it is important to investigate its validity. This paper presents an experimental result which could clarify the question.

The knowledge of the buckling and postbuckling problems of cylindrical shells of variable curvature subject to axial compression has been deepened by the works of Marguerre [*1*], Kempner and Chen [*2*], Hutchinson [*3*], and others. The inextensional case has been treated by Miura [*4*]. As the problem has naturally some similarities to the constant curvature case, that is, the circular cylindrical shells, so the assumptions as well as the analytical procedures in those literatures inherit those used in the circular case. Thus the presumed buckled deformation is of the fully developed diamond pattern structure and is characterized by the quantities, the circumferential and the axial wavelengths of buckles.

Marguerre assumed that the stresses and displacements in the classical buckling regime are purely periodical in the axial coordinate x, and are directly proportional to $\sin(2\pi x/\lambda_x)$, with the wavelength λ_x, which is small compared to the shell length L_1. Thus the deflection function is assumed in the following form,

$$w = \sin(2\pi x/\lambda_x) \sum_{n=1}^{\infty} a_n \sin(n\pi s/L_2) \tag{1}$$

where s is the circumferential coordinate, a_n are coefficients, and L_2 is the total circumference of the shell.

Kempner and Chen, considering an expression applied in the analysis of axially compressed circular cylinders, suggests the selection of the following deflection function for the problem of the oval cylindrical shell.

$$w/r_0=e+A\cos(2x/r_0\alpha)+C\cos(4x/r_0\alpha)$$
$$+D\cos(2x/r_0\alpha)\cos(2ps/r_0)+\cos(x/r_0\alpha)\sum_{n=0}^{N}B_n\cos(ns/r_0) \quad (2)$$

In this equation, e, A, C, D, B_n and p are undetermined displacement parameters, r_0 is the average radius of curvature which is the radius of a circle whose circumference is equal to that of the oval, n is a positive integer and $\alpha=\lambda_x/L_2$ is the non-dimensional axial wavelength, where λ_x is the fundamental axial wavelength of buckles.

Hutchinson, in the study of the initial postbuckling of elliptical cylindrical shells, assumes the deformation function which is similar to the form used by Marguerre.

Apparently, in any of these analyses, the deformation function is constructed upon the basic assumption on the constancy of the fundamental axial wavelength with regard to the circumferential coordinate.

On the other hand, it will also be acceptable to consider the similarity of elastic behaviours between a certain small portion of a variable curvature shell and a

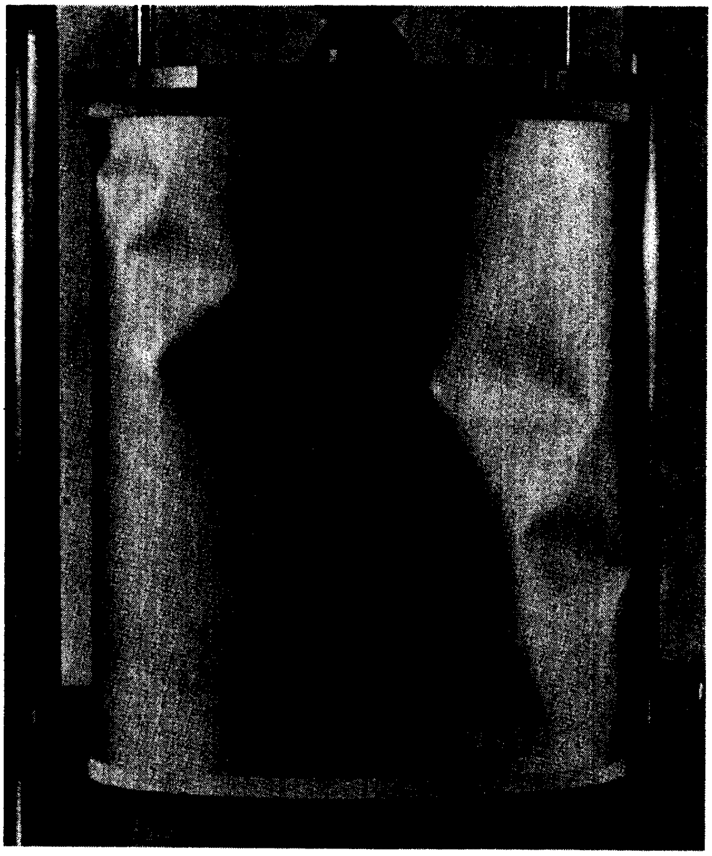


Fig. 1. **Discontinuity of spiral mode buckle pattern observed in an elliptical cylindrical shell subject to axial static loading, $a/b=2$, $r_0/t=770$.**

circular cylindrical shell having the identical curvature to the former. Thus the study of the circular cylindrical shell suggests the tendency of smaller axial wavelength for larger curvature, and larger axial wavelength for smaller curvature, if other conditions are equal. This is certainly conflicting with the assumption mentioned before. In order to clarify the ambiguity of the assumption, the present authors conducted an experimental study.

The postbuckling experiment was carried out by a small scale rigid type machine on several elliptical cylindrical shells. Models are made from Myler film of thickness t=0.1 mm, have dimensions of L_1=250 mm, and L_2=484 mm, and have different ratio of major to minor axis a/b. A typical result is shown in Fig. 1. The spiral mode buckle pattern is always observed for elliptical cylinders. Except in case of the circular cylinder, no one exhibits a uniformly and fully developed buckle pattern. It seems probable that there is a strong tendency of the appearance of spiral mode in case of the elliptical cylinder.

As our present purpose is to observe the constancy of the axial wavelength λ_x along the circumferential coordinate, at first sight the spiral mode buckle pattern will not offer any meaningful information. The reasoning of the relation between this phenomenon and the question about the constancy of the axial wavelength can be explained as follows.

In fact, in case of the long cylinder, the constancy of the axial wavelength along

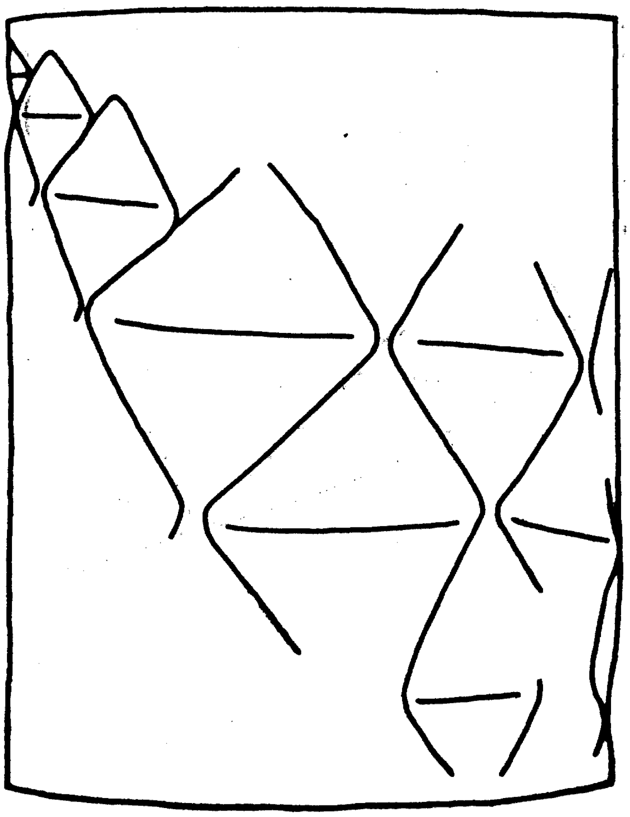


Fig. 2. A freehand sketch of discontinuous spiral mode pattern of the shell in Fig. 1.

the circumference is exactly synonymous with the constancy of the axial wave number. If the axial wave number were variable along the circumference, we certainly do not know how to tessellate the diamond pattern regularly as wll as continuously on the surface with zero Gaussian curvature [5]. The constancy of λ_x, therefore, reduces to the 'continuity' in the buckle pattern tessellation in the end. Furthermore, the geometry of diamond pattern tessellation of this type involves necessarily the continuity of buckle patterns, that is, the smooth connection of adjacent patterns, along the helical direction as well as axial and circumferential directions. Therefore, if the continuity of the buckle pattern along any of those directions is not supported by the experimental fact, the continuity in the buckle pattern tessellation has to be questioned. In fact, in this experiment we could see only the evidence against the continuity.

Fig. 2 shows a freehand sketch of discontinuous spiral mode pattern of the shell in Fig. 1, while Fig. 3 shows one of continuous spiral mode pattern predicted in inextensional case. The clear distinction is observed between Fig. 2 and Fig. 3. Another evidence is observed by the collapsing pattern of paper models subject to dynamic axial loading, which are shown in Fig. 4. In these fully developed patterns, the continuity in the buckle pattern tessellation no more exists. It is, however, interesting to note that there is still a certain regularity in these discontinuous

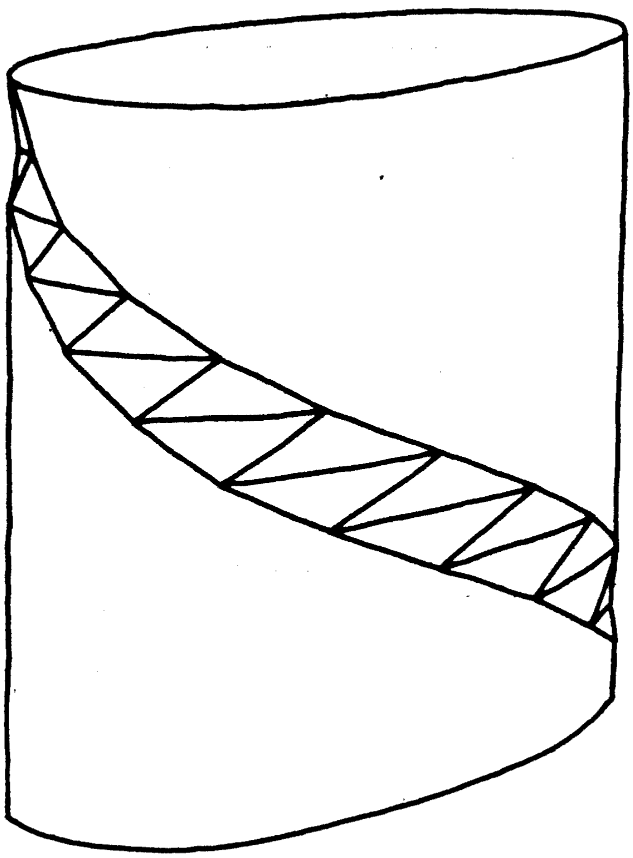
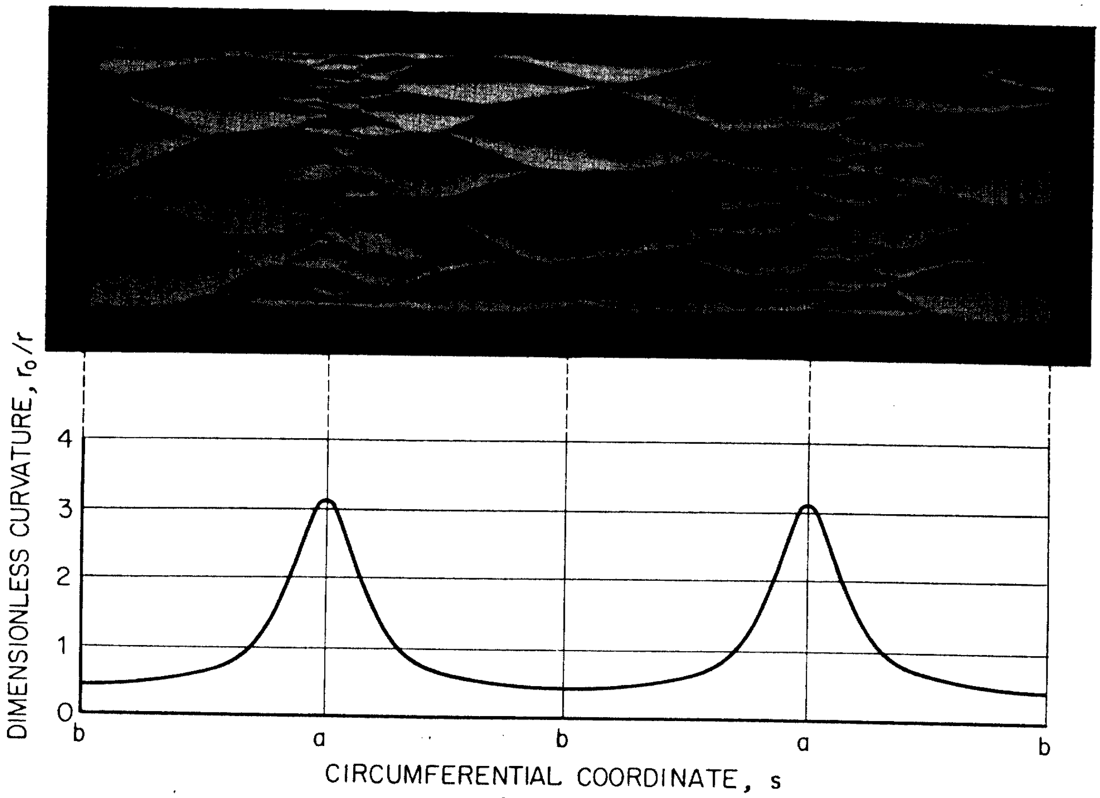
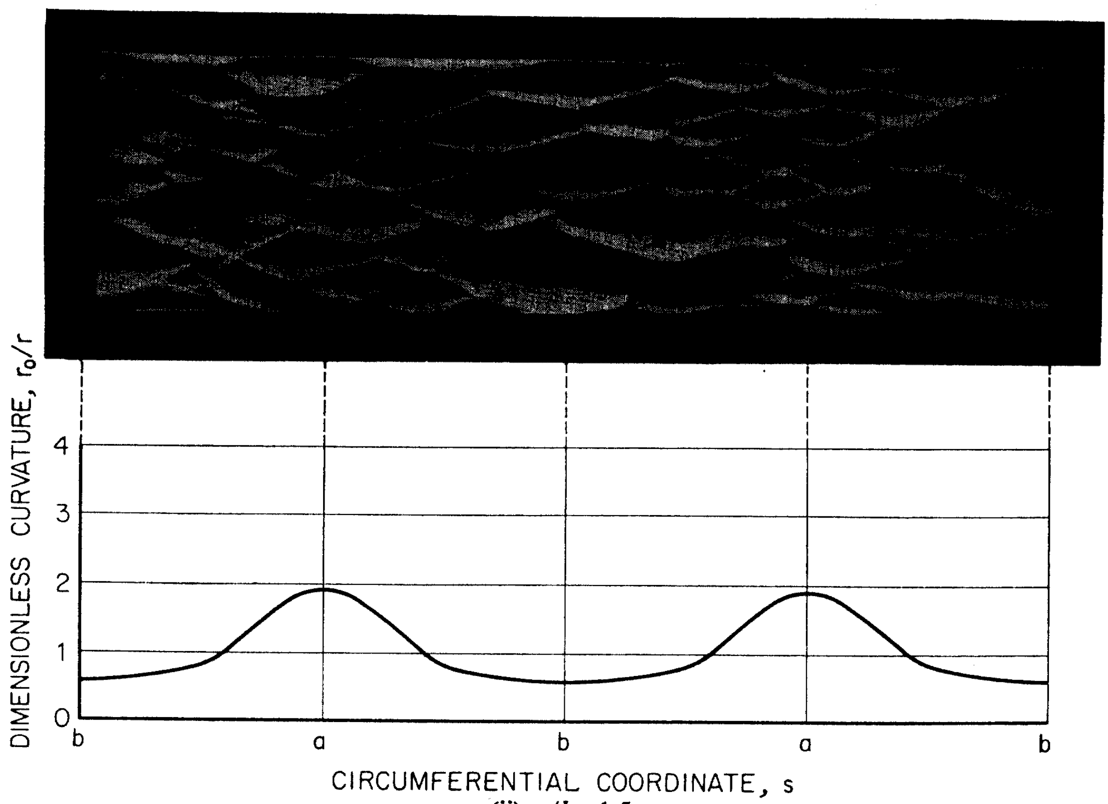


Fig. 3. Continuous spiral mode pattern predicted in inextensional case, $a/b=2$.



(i) $a/b=2$



(ii) $a/b=1.5$

Fig. 4. Fully developed collapsing patterns of elliptical cylindrical shells subject to dynamic axial loading; these photos are obtained by deploying the collapsed shells; the diagrams show the curvature distribution, where r is the radius of curvature, and a and b indicate the major and the minor axis intersection, respectively.

patterns as if Nature knows how to manage an abrupt change of integer number series in a continuous field.

Conclusively, the discontinuity in the buckle pattern tessellation exists in the elastic postbuckling configuration of cylindrical shell of variable curvature, and therefore, the assumption based on the continuity of buckle pattern tessellation such as the constancy of the axial wavelength along the circumference, seems inadequate. This also raises the identical problem in the classical buckling regime.

Department of Aerodynamics and Structures
Institute of Space and Aeronautical Science
University of Tokyo, Tokyo
January 28, 1971

References

[1] Marguerre, K.: "Stability of the Cylindrical Shell of Variable Curvature," NACA TM 1302, July 1951.

[2] Kempner, J. and Chen, Y.-N.: "Large Deflections of an Axially Compressed Oval Cylindrical Shell," *Proceedings of the Eleventh International Congress of Applied Mechanics, Munich, Germany*, Springer-Verlag, Berlin, 1964, pp. 299–306.

[3] Hutchinson, J. W.: "Buckling and Initial Postbuckling Behavior of Oval Cylindrical Shells Under Axial Compression," Report SM-11, Dec. 1966, Harvard University.

[4] Miura, K.: "Inextensional Buckling Deformations of General Cylindrical Shells," AIAA Journal, Vol. 6, No. 5, May 1968, pp. 966–968.

[5] Miura, K.: "Proposition of Pseudo-Cylindrical Concave Polyhedral Shells," ISAS Report No. 442, Nov. 1969, Institute of Space and Aeronautical Science, University of Tokyo, Meguro, Tokyo, Japan.