

Zeta-Core Sandwich—Its Concept and Realization

By

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Summary: A new form of sandwich core of high strength created by a topological transformation of a single plane is proposed herein. The resultant core is to be manufactured from a single sheet through a plastic forming. The purpose of this proposition is to present a sandwich core which has following principal characteristics; simplicity of form, applicability to both flat and curved sandwiches, possibility of circulating fluid between facings, the easiness of manufacture, the adaptability to large as well as small scale core, and the isotropy or controllable orthotropy of shear modulus. This core, designated as zeta-core by this author, is a sophisticated shell structure whose mid-surface is characterized by a doubly corrugated surface combining a couple of facings. An elementary analysis proves that the directional mean of shear modulus of zeta-core can compete with that of honeycomb core of identical apparent density, and that it is possible to design zeta-core of any orthotropy including, of course, isotropy in shear modulus. As for the realization aspect, no essential difficulty was encountered in trial manufacture of zeta-core from aluminum, plastics, and G.R.P. sheet materials. Furthermore, it can be predicted that the cost of production of zeta-core is relatively low in comparison with honeycomb core. It seems probable that the introduction of zeta-core of high shear modulus which can be made of various engineering materials will open up fresh possibilities for structural sandwich construction.

1. INTRODUCTION

The purpose of this study is to create a new structural form of core of sandwich construction which can compete with the honeycomb core in rigidity-to-weight ratio basis. Prospective characteristics of such a core are; the simple and continuous geometric form, the applicability to both flat and curved sandwiches, the possibility of circulating fluid between facings, the adaptability to large as well as small scale core, the easiness of manufacture, and the isotropy or the controllable orthotropy of shear modulus.

This paper comprises of three sequential steps of the research on the subject, and these are the proposition of a candidate form, analysis, and realization. The first step is to select a best prospective candidate of core form which is seemed to have necessary properties mentioned above. Since at this step, the detailed properties are, of course, not known yet, this process largely depends on the general knowledge about core geometry. The second step is to estimate analytically the elastic properties of the candidate; these are shear modulus, shear and compression strengths and others. The last step is to discuss the problems such as manufacturing, applications, and future study.

2. CORE FORM CREATED BY TOPOLOGICAL TRANSFORMATION OF A PLANE

The geometric form of the core of sandwich construction can be grouped into three classes, such as the cellular form, the grid form, and the single surface form. Typical examples which represent these classes are, in the same order, a foamed plastic core, a honeycomb core, and a corrugated core. Due to the necessity of circulation of fluid within the core, we have to aim at the core form of the last category.

As is well-known, the corrugated core is usually made of flat sheet materials by the corrugation machine through only bending deformation without substantial amount of in-plane stretching deformation. Thus this process is essentially the isometric transformation of a plane to a corrugated surface. In other words, such core forms created by the isometric transformation of a plane can be characterized by being developable surfaces.

The core form created by a single surface is not restricted to the developable surface, because the present technology in sheet forming allows a rather large amount of in-plane deformation. In fact, we have already some core forms created through non-isometric transformation of a plane. Hence, it might be said that whether the transformation is isometric or not will become less important in future.

A further fundamental nature relating to the transformation of a single plane to a core form is whether it is the topological transformation or not. The topological transformation (mapping) means the transformation of a original surface to an image surface without changing the topological property. It is called these two surfaces are in homeomorphic relation. For instance, making cuts, holes, and contacts with itself, does change the homeomorphic relation. Being abandoned its mathematical exactness, the topological transformation can also well describe a kind of transformation of a surface in engineering sense. Strictly speaking, pricking a single pin-hole to a continuous surface changes the homeomorphic relation. While in engineering sense of topology, we assume the understanding that pricking a considerable number of pin-holes to such an extent that the macroscopic elastic property is hardly effected is considered to keep the homeomorphism.

It can be deduced from experience that the core structure having cuts on purpose of obtaining versatility in form is generally unstable due to the free boundaries, and that the core structure having contacts with itself may gain the added rigidity but loses simplicity of the structure. These core structures are clearly aside from the purpose of obtaining the core of high strength with the simplest possible form. Therefore, it is most desirable that the core form is created through the topological transformation of a plane. Further necessary geometric natures are; the image surface is bounded between a couple of parallel surfaces, and it is to be expressed by a single-valued function referred to curvilinear coordinates on the reference surface set midway between the parallel surfaces (Fig. 1). These points being settled, we can now proceed with our problem of studying possible geometric forms created by such topological transformation.

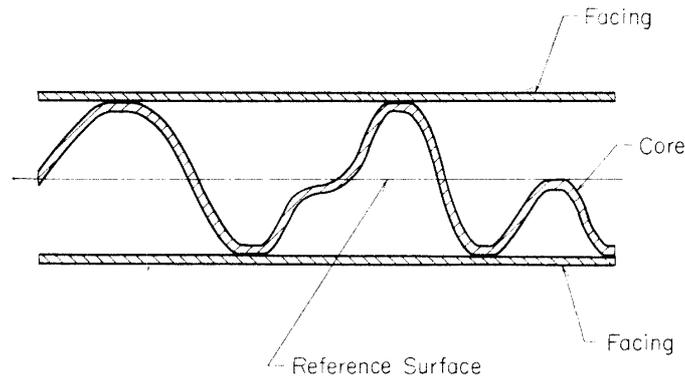


FIG. 1. Core Form Created by Topological Transformation of a Plane (Section)

Now, we denote the two-dimensional universal domains of a couple of facing surfaces and the core surface by I, II, and III, respectively. The relations existing between these domains are studied first in the following.

The sum of domains of facing surface I which are in contact with the core surface III is called I' , and since this is the common set of I and III

$$I' = I \cap III \quad (1)$$

In the same manner as above, the sum of domains of facing surface II which are in contact with the core surface III is called II' , and this is the common set of II and III

$$II' = II \cap III \quad (2)$$

The sum of domains of core surface III which are not in contact with facing surfaces is called III' , and since this is the complementary set of the sum of I' and II' in III

$$III' = C_{III}(I' \cup II') \quad (3)$$

The universal domain I is divided into I' and its complementary set $C_I(I')$ as follows

$$I = I' \cup C_I(I') \quad (4)$$

Similarly, II is divided into II' and $C_{II}(II')$ as follows

$$II = II' \cup C_{II}(II') \quad (5)$$

The universal domain III, that is, the core surface is divided into three parts as follows

$$III = I' + II' + III' \quad (6)$$

In domain III, it goes without saying that I' and II' not only satisfy the following relation

$$I' \cap II' = \emptyset \quad (7)$$

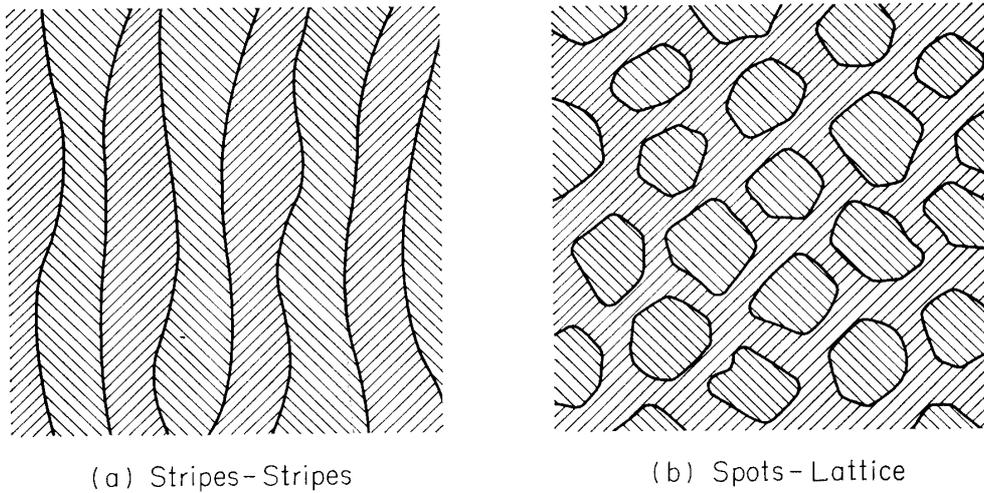


FIG. 2. Fundamental Division of a Two-Dimensional Space into Two Kinds of Domain

Pattern of Domain I' \ Pattern of Domain II'	Spots	Stripes	Lattice
Spots	Type A	Type B	Type C
Stripes	Type B	Type D	Impossible
Lattice	Type C	Impossible	Impossible

Domain III (Core Surface)		: Domain I' (Contact Area with Facing Surface I)
		: Domain II' (Contact Area with Facing Surface II)
		: Domain III' (Free Area)

FIG. 3. Fundamental Core Forms Obtainable from Topological Transformation of a Plane

but also they cannot be adjacent domains.

Using these relations, we can furnish important information concerning methods of dividing domain III and thereby exhausts every possible forms of core surface. Obviously I' and $C_r(I')$ must be the division, in a topological sense, of two-dimensional space into two kinds of domains. Since I' corresponds to the bonding region of the facings of sandwich construction, such a division should be done by the repetition of a finite pattern. This is somewhat similar to the tessellation of a plane, but it should be noted that the concepts of length and angle exist no longer in the topology. The most fundamental methods of such a division may be those illustrated in Venn-diagrams of Fig. 2.

Fig. 2a shows the division of the universal domain into one-dimensional groups, that is, the stripes. In this case, the complementary domain is also the stripes. Fig. 2b shows the division of the universal domain into two-dimensional groups composed of scattering circular domains and the matrix domain. These domains are termed the spots and the lattice, respectively. Both are complementary with each other. The more complicated division can be obtained by using multiply-connected domains in place of circular domains, but such a division is not attractive to the present purpose. Resultantly, the form of I' should necessarily be either of stripes, spots, and lattice pattern. The same can be said about the domain II' .

Both I' and II' being determined in domain III, it is easy to reach the conclusion on the basis of Eqs. 6 and 7 that III' should necessarily be the matrix filling the rest of the domain III. In addition, as I' and II' can not be adjacent domains, the matrix should enclose every elements of I' and II' . Eventually, the possible divisions of core surface are schematically shown in Fig. 3. It is shown that there are four types of core surfaces obtainable from the topological transformation of a plane.

A plastic core which belongs to the type A configuration is shown in Fig. 4.

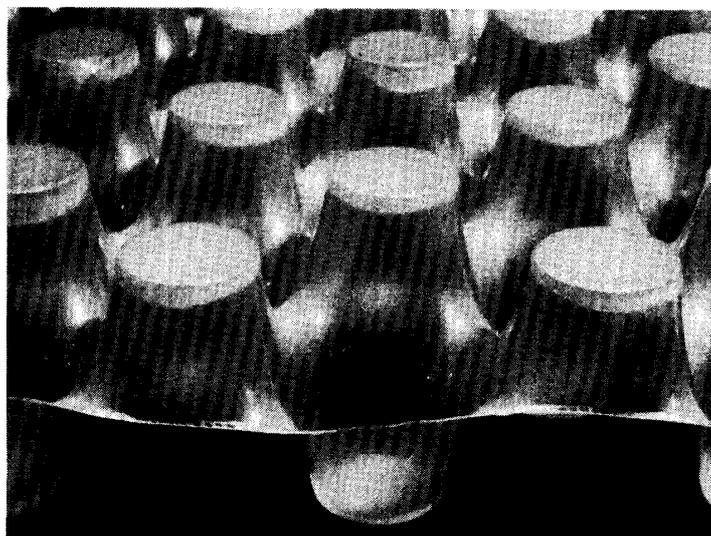


FIG. 4. Plastic Core of Type A Configuration

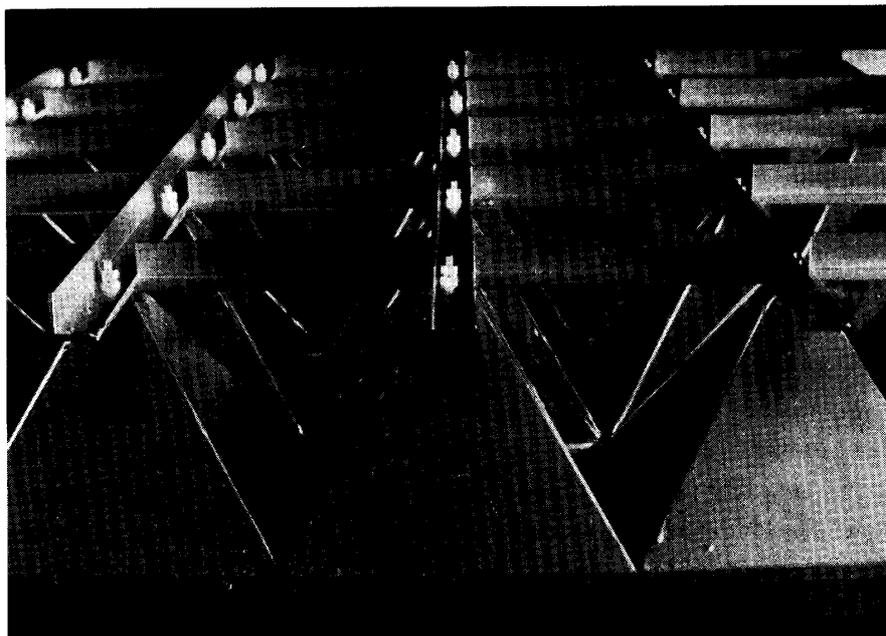


FIG. 5. Double Layered Stressed-Skin Space Frame Structure using Pyramidal Core which is considered to be Type C Configuration

This is made of thermoplastic sheets through a special forming technique. As for type B configuration, the present author has not ever known the realized one. The type C configuration has been used for the core of double layered stressed-skin space frames which is considered to be a certain variation of sandwich construction (Fig. 5). It is needless to say about the type D configuration.

3. PROPOSITION OF ZETA-CORE CONCEPT

With due consideration of the shear strength, manufacturing, and bonding, it seems that the type D configuration may be the first choice for structural sandwich used in high stress level. The corrugated core, a representative of type D, is characterized really by high shear rigidity, but its orthotropic behavior is almost inevitable. Then the idea of superposing two corrugations in mutually orthogonal directions might be a natural approach to the problem of obtaining an isotropic core.

Now it is assumed that f and g represent certain continuous, single-valued, bounded, and periodic functions defined in orthogonal coordinates x , y , z .

$$z = f_{xz}(x), \quad (x-z \text{ plane}) \quad (8)$$

$$z = g_{yz}(y), \quad (y-z \text{ plane}) \quad (9)$$

The product of these two functions

$$z = f_{xz}(x) \cdot g_{yz}(y) \quad (10)$$

is really a doubly periodic function that is periodic in two mutually orthogonal directions. However, the resulting surface described by this function is not a form of type D as expected vaguely, and instead, it corresponds to the type A (Fig. 6a). The instance of this form was introduced previously in Fig. 4. Because of its intermittent contact with facings and low shear modulus, this core can not be used at high stress level even if it may have attained the isotropy in shear rigidity. Thus, this kind of approach to the problem proves to be misleading.

In fact, there is another method of superposing two corrugations in mutually orthogonal directions [1] [2]. Now, a periodic function g is considered on y - z plane as before. But, as another periodic function, the function h is considered on x - y plane instead of x - z plane.

$$y = h_{xy}(x), \quad (x-y \text{ plane}) \quad (11)$$

The synthesized function

$$z = g_{yz}[y - h_{xy}(x)] \quad (12)$$

is a locus of the periodic curve $g_{yz}(y)$ translating parallel along another periodic curve $h_{xy}(x)$. This function really represents a surface, that belongs to the type D configuratoin, having periodicity in two mutually orthogonal planes. Further, if the amplitude of the function $g_{yz}(y)$ is taken to be constant, it figures a surface that inscribes the two parallel planes (Fig. 6b). It seems there is no difficulty about the generalization of this idea to the synthesis of similar surfaces in general

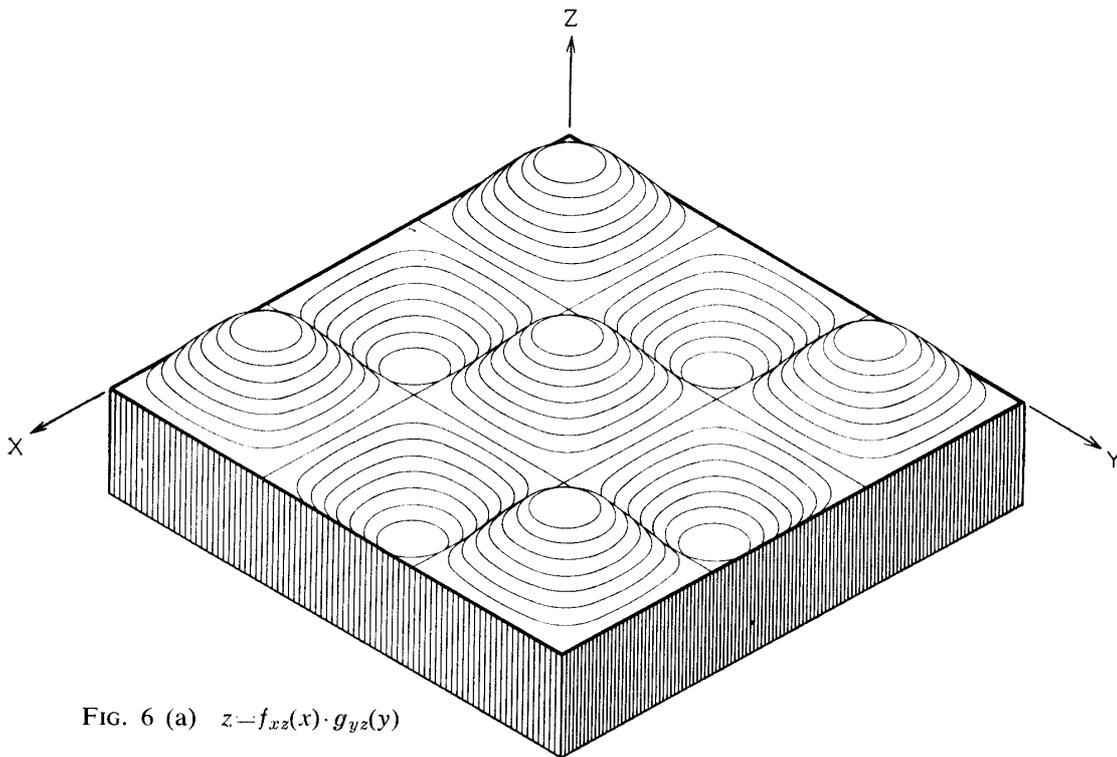


FIG. 6 (a) $z = f_{xz}(x) \cdot g_{yz}(y)$

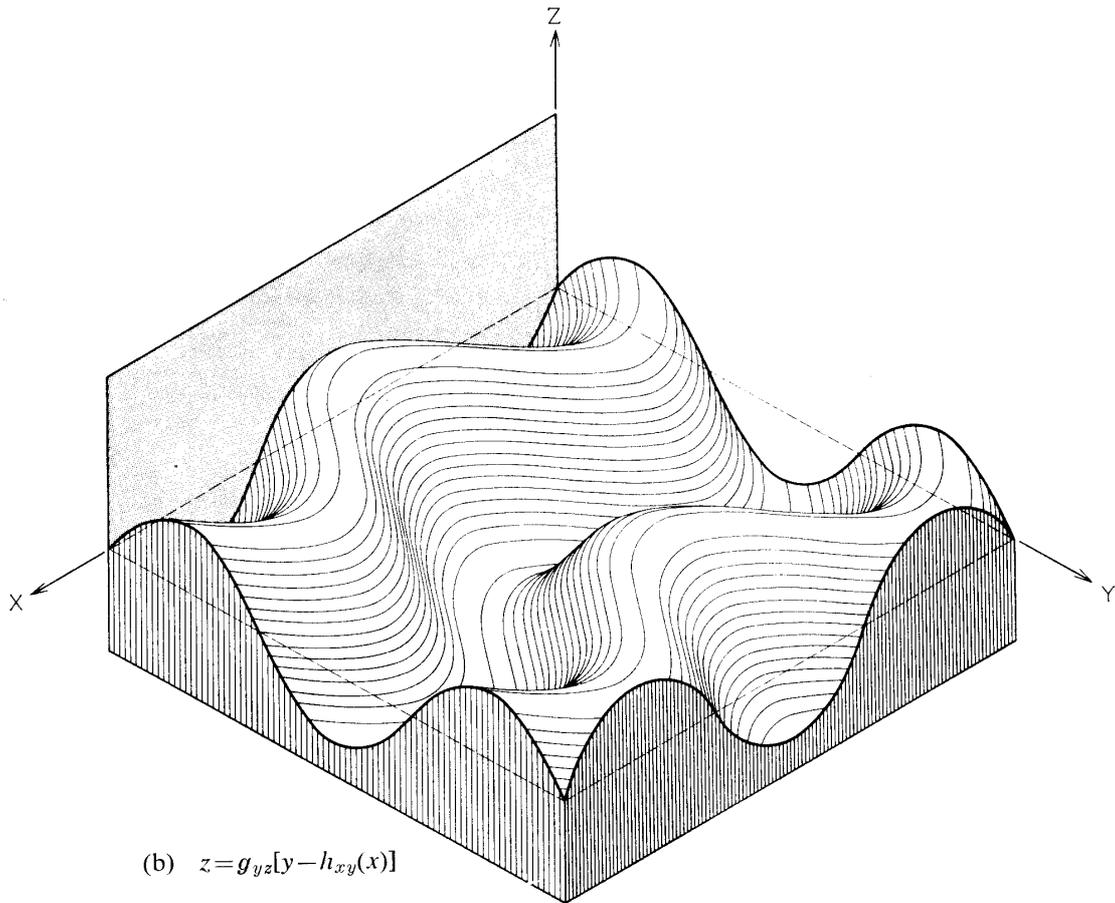


FIG. 6. Synthesis of Two Corrugations in Mutually Orthogonal Directions

curvilinear coordinates. There the facing surfaces are represented by a couple of coordinational surfaces. Tentatively, these core surfaces are called by the double corrugation surface.

Now we must examine whether the core in the form of double corrugation surface can really attain the isotropic behavior. Also other important requirements as the core have to be checked. These are now roughly considered about a typical core whose midsurface is represented by using a truncated zigzag function $g_{yz}(y)$ and a zigzag function $h_{xy}(x)$ as shown in Fig. 7. The following Table 1 shows the itemized comparison of this core with the single corrugation core.

So far as this table is concerned, this core looks very promising and is considered to be worthy of studying further. It should, however, be noted that the truly useful form of core will be limited to specified classes in double corrugation surfaces. In general, the function $g_{yz}(y)$ has more important role on rigidity and strength than the function $h_{xy}(x)$. In the example shown previously in Fig. 7, the truncated zigzag function $g_{yz}(y)$ is used together with a zigzag function $h_{xy}(x)$. The resulting domain III' of the surface, or the side of the double corrugation surface, becomes a folded plate surface similar to accordion pleats. If it is used together with a curved wavy function $h_{xy}(x)$, a wavy cylindrical surface will be

TABLE 1. Comparison of the core in the form of double corrugation surface with the single corrugation core

Items of comparison	Superiority or inferiority	Comments
Shear rigidity	Same	
Compressive rigidity	Same	
Shear strength	Superior	Good local stability
Compressive strength	Superior	Good local stability
Isotropy	Superior	Controllable orthotropy
Bonding	Superior	Zigzag contact surface
High temp. application	Same	
Arbitrary formability	Inferior	Geometric restriction
Manufacture	Inferior	Need large extensional deformation

resulted as illustrated in Fig. 8. In both cases, the side part of the core composes a general cylindrical shell whose generators are inclined at a certain angle to facings. Therefore, these will give very stable and rigid structures against loads applied through facings. While, if the curved wavy function $g_{yz}(y)$ is used, the resulting side part becomes either a cylindrical surface whose generators are parallel to facing or a surface of double curvature depending on function $h_{xy}(x)$. In these cases, the side part is generally subject to considerable bending stress, which is undesirable for thin core structures.

One of the important roles of the function $h_{xy}(x)$ is that this is deeply relating to the orthotropic characteristic of the core. In other words, the possibility of obtaining an isotropic core will primarily depends on this function.

Now, we have attained the stage of giving a geometric definition to this category of cores which hold common characteristics. Such a definition can be given to

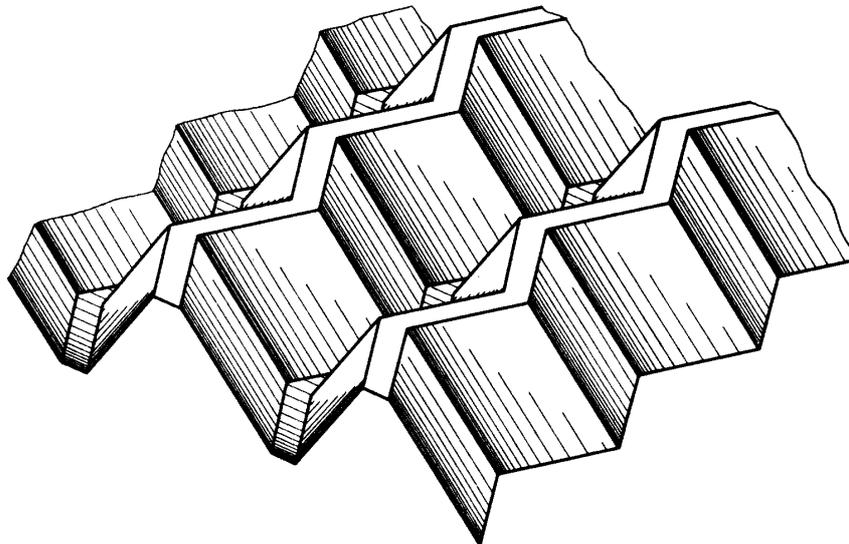


FIG. 7. Relief of a Zeta-Core Form of Folded Plate Type

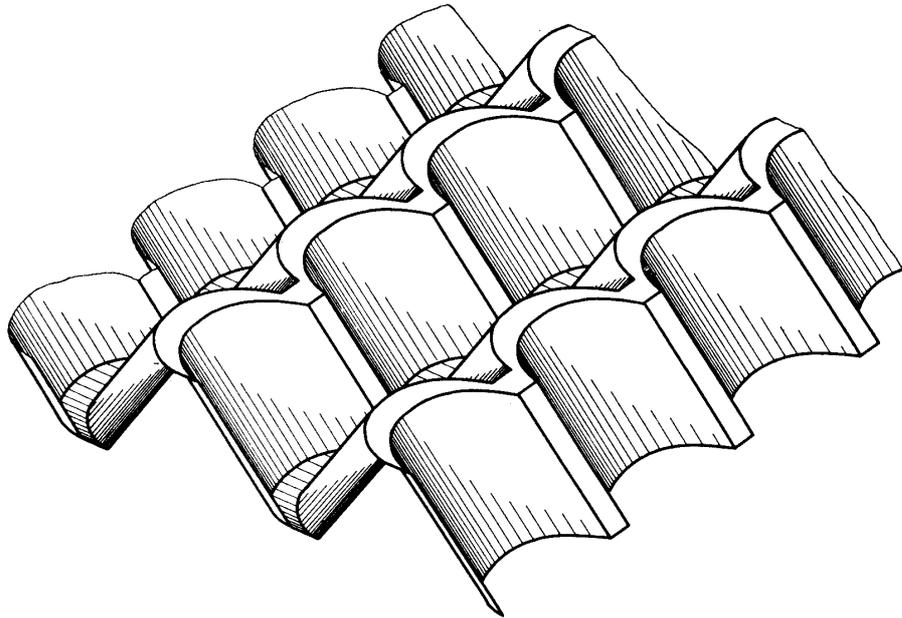


FIG. 8. Relief of a Zeta-Core Form of Cylindrical Type

the reference surface, the top (bottom), and the side part of the core surface on referring to the relief of Fig. 9. The reference surface of the sandwich using this core can be arbitrary and it includes free formed surfaces. Each periodic curve, on facing surface, which determines the borders of top (bottom) part of the core surface has an approximately identical phases with regard to the curvilinear coordinates on the reference surface. The side part of the core surface is defined by a ruled surface formed by translating a generating line intersecting two adjacent periodic curves, each one on each facing surface. The ruled surface includes folded plate surfaces, cylindrical surfaces, conical surfaces, and hyperbolic para-

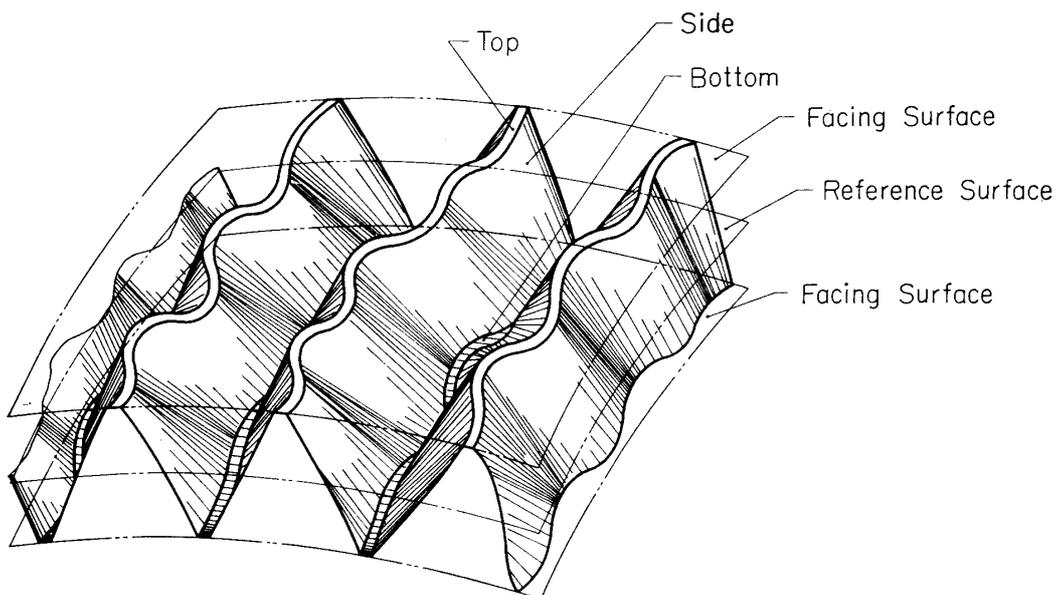


FIG. 9. Relief of general Zeta-Core Form

boloids.

It may be convenient to have an adequate designation for the core thus defined. The “double corrugation core” may be a candidate, but it is too lengthy to pronounce. Since both capital and small letters of Greek alphabet Z and ζ remind us of periodic functions g and h or an overall impression about the core, the English pronunciation of this letter, zeta, is adopted here like as “zeta-core”.

4. PREDICTION OF ELASTIC PROPERTIES OF ZETA-CORE

4-1 Shear Modulus

The principal quantities relating to the elastic properties of core are the shear modulus and strength, and the flatwise compression modulus and strength. Above all, the shear modulus G_c , or more specifically, the effective modulus of rigidity of core in planes including normals to facings is an important as well as only representative quantity. In this section, the shear modulus for the core whose mid-surface is defined by a truncated zigzag function g_{yz} and a zigzag function h_{xy} , that is,

$$z = g_{yz}[y - h_{xy}(x)]$$

whose configuration is shown Fig. 7, is obtained analytically.

The fundamental region of this surface is composed of four congruent rhomboids and two congruent chevron patterns, when the symmetry in both functions g_{yz} and h_{xy} is presumed, as shown in Fig. 10. However, this is not necessarily be required in the analysis. The whole surface is constructed by two independent parallel transfers of this fundamental region in mutually orthogonal directions.

For simplicity purpose, it is assumed here that the facings are infinitely rigid as for both in-plane and bending deformations. The rectangular coordinates x , y ,

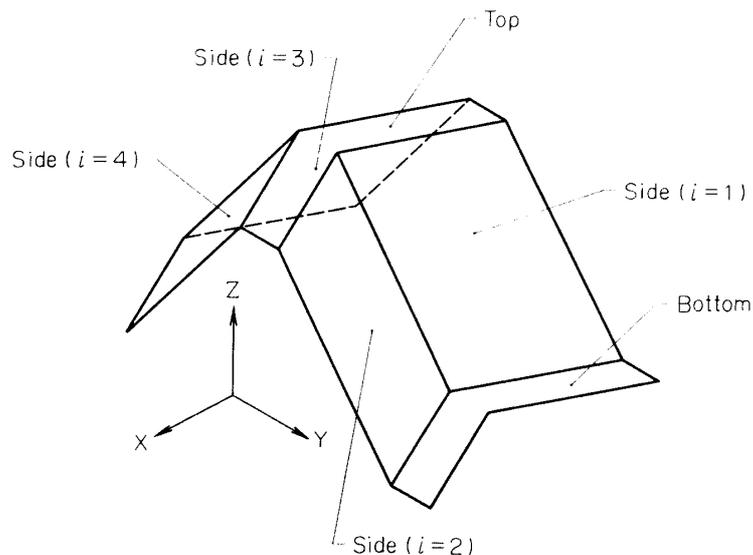


FIG. 10. Fundamental Region of Zeta-Core Form of Fig. 7

z are taken so that x and y axes may lie on the facing surface II. Further, the direction ϕ is defined which is ϕ radian counter-clockwise from x axis on $x-y$ plane (Fig. 11).

The strain of the core is now considered when the sandwich structure is subject to a shear deformation $\gamma_{\phi z}$ in $\phi-z$ plane. Because the deformation in the top of core is that of the rigid motion of the facing, the strain in this region is zero. The stress and strain distribution in the inclined side wall elements of rhomboidal form is undoubtedly rather complicated. Because the present aim is to obtain a macroscopic elastic quantity with regards to shear property, and not to get detailed local stress distribution within core elements, some adequate approximation can be made without impairing an essence of the problem.

The approximation of membrane stress state may be acceptable since the thickness of core element is usually very small compared with other dimensions. For an arbitrary element i , the deformation applied externally from facings is the relative parallel transfer of a pair of edges on surfaces I and II. Therefore, the first approximation on the strain distribution, which can be compatible with the displacements at facings, is the homogeneity of strain within the element i . If these assumptions are accepted, the resulting strains in each element are equivalent to the strains of the identical position in a hypothetical core made of a homogeneous continuum subjected to the said external displacements [2]. In addition, these adopted assumptions are equal to those used for the elementary analysis of honeycomb core frequently referred. In subsequent μ value, for instance, theo-

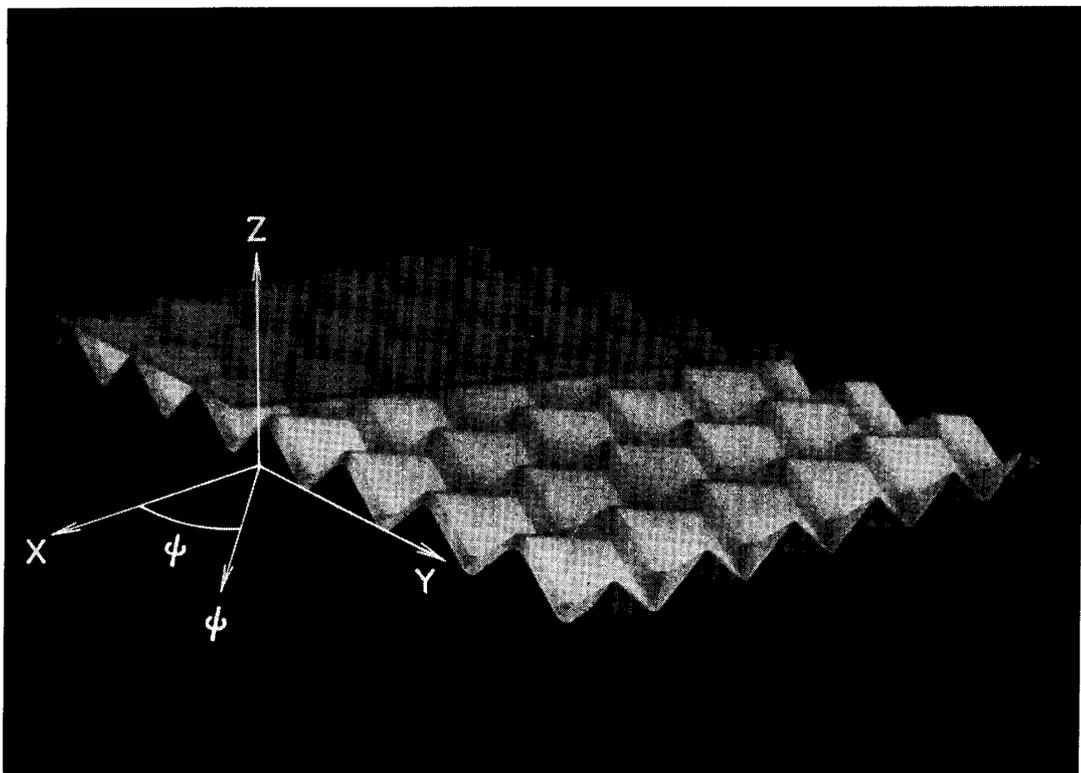


FIG. 11. A Zeta-Core Sandwich and Coordinational Axes

retical and tested values are 0.625 and 0.7, respectively, for the ribbon direction of honeycomb (see Appendix).

Now, a normal vector R_{0i} (l_i, m_i, n_i) is considered about the i th rhomboidal element as shown in Fig. 12. Further, a couple of vectors R_{1i} and R_{2i} are considered, R_{1i} on the intersection of the $x-y$ plane and the i th element, and R_{2i} vertical to R_{1i} on i th element. The direction cosines of these vectors are summarized in the following.

$$\begin{aligned} R_{0i} & [\quad \quad \quad l_i, \quad \quad \quad m_i, \quad \quad \quad n_i] \\ R_{1i} & [\quad m_i/(1-n_i^2)^{1/2}, \quad -l_i/(1-n_i^2)^{1/2}, \quad 0] \\ R_{2i} & [-l_i n_i/(1-n_i^2)^{1/2}, \quad -m_i n_i/(1-n_i^2)^{1/2}, \quad (1-n_i^2)/(1-n_i^2)^{1/2}] \end{aligned}$$

The shear deformation applied to the sandwich structure in the plane including z and ϕ directions is denoted by $\gamma_{\phi z}$. Thus, the shear strains γ_{xz} and γ_{yz} of the hypothetical core are related to $\gamma_{\phi z}$ by the following formulas.

$$\gamma_{xz} = \gamma_{\phi z} \cos \phi \quad (13)$$

$$\gamma_{yz} = \gamma_{\phi z} \sin \phi \quad (14)$$

The strain components of the rhomboidal element i due to $\gamma_{\phi z}$ can be calculated as the deformations of two vectors R_{1i} and R_{2i} .

The elongation ϵ' of any linear element r' of direction cosines l', m', n' through a point of a deformed three-dimensional body can be given by

$$\epsilon' = l'^2 \epsilon_x + m'^2 \epsilon_y + n'^2 \epsilon_z + l' m' \gamma_{xy} + m' n' \gamma_{yz} + n' l' \gamma_{zx} \quad (15)$$

where $\epsilon_x, \epsilon_y, \epsilon_z$ are unit elongations in the x, y, z directions and $\gamma_{yz}, \gamma_{zx}, \gamma_{xy}$ the three unit shear strains related to the same directions [4]. The shear strain $\gamma_{r'r''}$ between linear elements r' and r'' (l'', m'', n'') which are perpendicular to each other is also given by

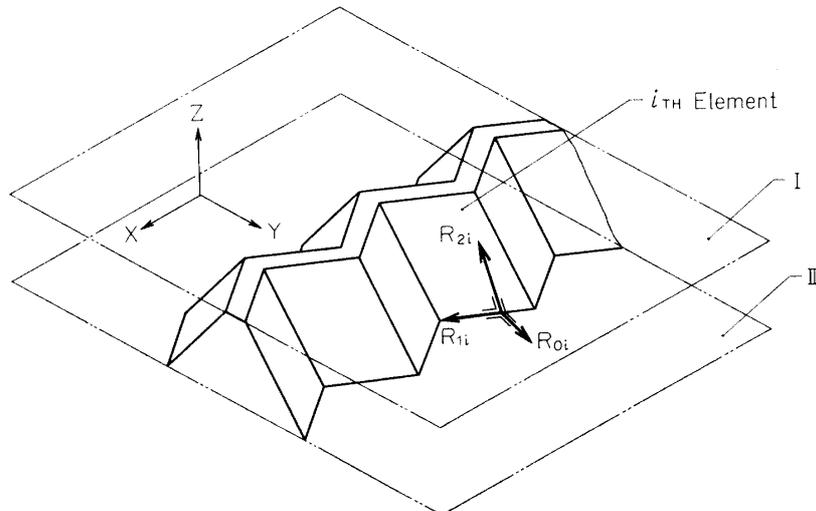


FIG. 12. Normal Vectors of the i th Rhomboidal Element of a Zeta-Core Surface

$$\begin{aligned}\gamma_{r'r''} = & 2(\varepsilon_x l'l'' + \varepsilon_y m'm'' + \varepsilon_z n'n'') \\ & + \gamma_{xy}(l'm'' + l''m') + \gamma_{yz}(m'n'' + m''n') + \gamma_{zx}(n'l'' + n''l')\end{aligned}\quad (16)$$

The shear strain γ_{12i} due to $\gamma_{\phi z}$ can be calculated as the shear deformation of two vectors R_{1i} and R_{2i} , which were originally perpendicular to each other before deformation, by using Eqs. 13, 14, and 16 as follows

$$\gamma_{12i} = \gamma_{\phi z}(-l_i \sin \phi + m_i \cos \phi) \quad (17)$$

The normal strains of the element i are represented by the extensional strains ε_{1i} and ε_{2i} of the vectors R_{1i} and R_{2i} , respectively. Therefore,

$$\varepsilon_{1i} = 0 \quad (18)$$

$$\varepsilon_{2i} = \gamma_{\phi z}(-m_i n_i \sin \phi - l_i n_i \cos \phi) \quad (19)$$

The strain energy of the element i in plane stress state is expressed by the following formula.

$$U_i = a_i t_i \left[\frac{E}{2(1-\nu^2)} (\varepsilon_{1i}^2 + \varepsilon_{2i}^2 + 2\nu \varepsilon_{1i} \varepsilon_{2i}) + \frac{G}{2} \gamma_{12i}^2 \right] \quad (20)$$

where a_i and t_i represents the area and the constant thickness of the element, respectively. It can be written in terms of direction cosines of normal vectors, the direction ϕ , and the applied shear strain $\gamma_{\phi z}$ as follows.

$$\begin{aligned}U_i = & \frac{1}{2} a_i t_i \gamma_{\phi z}^2 \left[G(-l_i \sin \phi + m_i \cos \phi)^2 \right. \\ & \left. + \frac{E}{1-\nu^2} (m_i n_i \sin \phi + l_i n_i \cos \phi)^2 \right]\end{aligned}\quad (21)$$

The total strain energy of the core is a simple summation of the partial strain energy for each element and is given by

$$\begin{aligned}U = & \frac{1}{2} \gamma_{\phi z}^2 \sum_i a_i t_i \left[G(-l_i \sin \phi + m_i \cos \phi)^2 \right. \\ & \left. + \frac{E}{1-\nu^2} (m_i n_i \sin \phi + l_i n_i \cos \phi)^2 \right]\end{aligned}\quad (22)$$

In the following, the calculation is made about the case where the fundamental region can be defined. Since the whole surface is constructed by the repetition of this fundamental region with the identical orientation, it is sufficient to consider only this region in order to calculate the macroscopic elastic quantity of the core structure. In this sense, the summation in above formula should be done only in the fundamental region. Now the gross volume between facings filled by a fundamental region of zeta-core is denoted by v . The strain energy of the hypothetical continuum of the same gross volume v and with the shear modulus G_c , which is subject to a uniform shear strain $\gamma_{\phi z}$, is expressed by

$$U = \frac{1}{2} \gamma_{\phi z}^2 \nu G_c \quad (23)$$

Therefore, the effective shear modulus of the zeta-core is given by the following formula

$$G_c = \frac{1}{\nu} \sum_i^{\text{f.r.}} a_i t_i \left[G(-l_i \sin \phi + m_i \cos \phi)^2 + \frac{E}{1-\nu^2} (m_i n_i \sin \phi + l_i n_i \cos \phi)^2 \right] \quad (24)$$

where the summation is done within the fundamental region.

Hereupon we introduce the concept of form efficiency of core that is a dimensionless form of shear modulus defined by

$$\mu = \frac{G_c}{G\alpha} \quad (25)$$

where G is the shear modulus of the material of core, α is the spatial filling factor of core [1] [2]. The physical meaning of the form efficiency is explained in detail in the Appendix, where typical μ values for several cores are also shown. In addition, the following reduction ratio is defined in order to exclude the top (bottom) part of the core from the computation, because this dead volume does not contribute to the strain energy.

$$\beta = \frac{(\text{net volume}) - (\text{volume of top \& bottom})}{(\text{net volume})} \quad (26)$$

From Eqs. 24, 25, and 26, the form efficiency of the zeta-core is given by the following formula.

$$\mu(\phi) = \frac{\beta}{\sum_i^{\text{f.r.}} a_i t_i} \sum_i^{\text{f.r.}} a_i t_i \left[G(-l_i \sin \phi + m_i \cos \phi)^2 + \frac{E}{1-\nu^2} (m_i n_i \sin \phi + l_i n_i \cos \phi)^2 \right] \quad (27)$$

As an illustrating example, the computation of form efficiency is carried out about a zeta-core whose fundamental region is composed of four congruent rhomboids and two congruent chevron pattern as shown in Fig. 10. If an appropriate rectangular coordinates x, y, z are chosen like as Fig. 10, the direction cosines of normal vectors of four rhomboidal elements composing a fundamental region can be expressed solely by a set of l^*, m^*, n^* , which are assigned to positive, as follows.

$$\begin{aligned} R_{01} & (l^*, m^*, n^*) \\ R_{02} & (-l^*, m^*, n^*) \\ R_{03} & (-l^*, -m^*, n^*) \\ R_{04} & (l^*, -m^*, n^*) \end{aligned}$$

Substituting these data into Eqs. 24 and 27, both the shear modulus and the form efficiency of this zeta-core are easily obtained and the latter is shown in the following.

$$\mu^*(\phi) = \beta \left[\left(l^{*2} + \frac{2m^{*2}n^{*2}}{1-\nu} \right) \sin^2 \phi + \left(m^{*2} + \frac{2l^{*2}n^{*2}}{1-\nu} \right) \cos^2 \phi \right] \quad (28)$$

For $\nu=1/3$, it can be written in even simpler form as follows.

$$\mu^*(\phi)|_{\nu=1/3} = \beta [(l^{*2} + 3m^{*2}n^{*2}) \sin^2 \phi + (m^{*2} + 3l^{*2}n^{*2}) \cos^2 \phi] \quad (29)$$

For coordinatinal directions, the values of form efficiency are given as follows.

$$\mu^*(0) = \mu_x^* = \beta \left[m^{*2} + \frac{2l^{*2}n^{*2}}{1-\nu} \right] \quad (30)$$

$$\mu^*\left(\frac{\pi}{2}\right) = \mu_y^* = \beta \left[l^{*2} + \frac{2m^{*2}n^{*2}}{1-\nu} \right] \quad (31)$$

4-2 Isotropy Condition and Maximum Form Efficiency

Since the design of sandwich construction is primarily effected by the elastic property of the weakest direction, the isotropy of the core property is generally desirable. In case of the previous example, the circular isotropy condition is realizable when the following relation is satisfied between direction cosines of normals of rhomboidal elements.

$$l^{*2} + \frac{2m^{*2}n^{*2}}{1-\nu} = m^{*2} + \frac{2l^{*2}n^{*2}}{1-\nu} \quad (32)$$

There are two cases that satisfy above relation and these are

$$\text{Case 1.} \quad l^* = m^* \quad (33)$$

$$\text{Case 2.} \quad n^* = \left(\frac{1-\nu}{2} \right)^{1/2} \quad (34)$$

These cases are discussed in the following separately.

Case 1.

Substituting the isotropy condition into Eq. 28, the form efficiency is obtained as a function of direction cosine n .

$$\mu_{\text{iso}}^* = \beta \left[\frac{1}{2} + \frac{(1+\nu)n^{*2}}{2(1-\nu)} - \frac{n^{*4}}{1-\nu} \right] \quad (35)$$

If $\nu=1/3$,

$$\mu_{\text{iso}}^* = \beta \left[\frac{1}{2} + n^{*2} - \frac{3n^{*4}}{2} \right] \quad (36)$$

The dependency of the form efficiency μ_{iso}^* on the angle of inclination of rhomboidal

elements, $\cos^{-1} n^*$, is shown in the curves of Fig. 13.

The maximum value of μ_{iso}^* is given by

$$\mu_{iso,max}^* = \beta \left[\frac{1}{2} + \frac{(1+\nu)^2}{16(1-\nu)} \right], \quad \text{at } n^* = \frac{(1+\nu)^{1/2}}{2} \quad (37)$$

If $\nu = 1/3$, it reduces to

$$\mu_{iso,max}^* = \beta \left[\frac{2}{3} \right], \quad \text{at } n^* = \frac{1}{3^{1/2}} \quad (38)$$

If a typical value of β is now assumed to 0.8, then $\mu_{iso,max}^* = 0.533$ for the inclination angle of $\cos^{-1} n^* = 53^\circ 44'$, when $\nu = 1/3$. This value is very close to the directional mean value of honeycomb core, that is, 0.5. It is also shown in this figure that, if the inclination angle is in between 45° and 60° , the form efficiency holds a sufficiently high value. This fact is important because the tentative form of zeta-core will probably fall under such a region of inclination angle.

Case 2.

The form efficiency corresponds to another isotropy condition is

$$\mu_{iso}^* = \beta \left(\frac{1+\nu}{2} \right) \quad (39)$$

If $\nu = 1/3$, it is

$$\mu_{iso}^* = \beta \left(\frac{2}{3} \right) \quad (40)$$

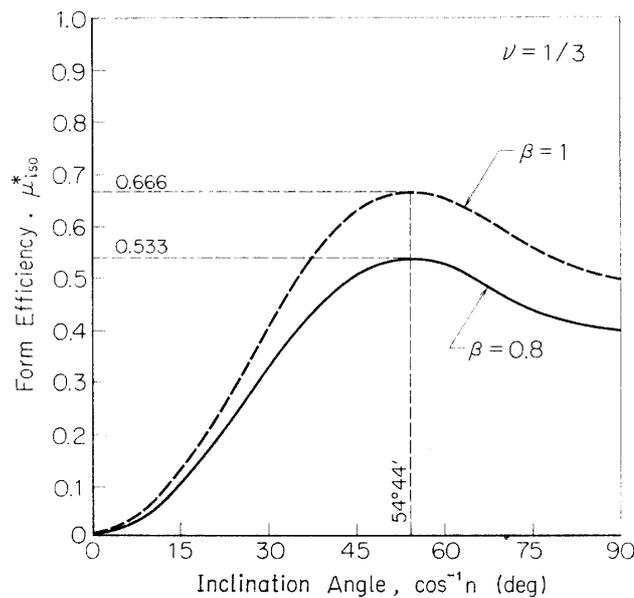


FIG. 13. Form Efficiency vs. Inclination Angle of an Isotropic Zeta-Core of Fig. 7

Apparently this condition seems to be independent of l and m . But if the value m/l differs greatly from 1, the fundamental assumptions of the analysis on which this condition is based become uncertain.

It should be noted that these conditions of isotropy are based on the approximate solution and that there is no guarantee these are the true isotropy conditions. The same can be said about the magnitude of form efficiency. However, it is quite sure that the true isotropy condition exists in the neighbourhood of predicted conditions. In this sense, the trial and the error approach of manufacturing is considered to be the best way to find the true isotropy condition.

It may be added that designing core of any orthotropy, if required, is possible by using Eq. 27.

4-3 Stress and Strength of Zeta-Core

There are several typical failure modes of sandwich construction which must be considered in relation to core rigidity and strength properties. As for flat sandwich panels, these failure modes include general buckling, shear crimping, face wrinkling, transverse failure, flexural crushing, local crushing, and intercell buckling. Besides shear and compression rigidity, the primary controlling factors affecting failures are the core shear and core flatwise compression strengths. It is, therefore, necessary to evaluate these strength properties of zeta-core before advancing from a conceptual study to a realization study.

First of all, it should be realized that the thickness of the core element is usually considered to be very small compared with other dimensions. Therefore, the core strength properties largely depend on the instability of the thin element composing the core and not on the material failure strength.

In order to check the instability of thin elements of core, the stress distribution within them has to be first investigated. For instance, in case of the zeta-core of Fig. 10, the normal stress in the direction of vector R_{2i} , and σ_{2i} , is given by the following formula.

$$\sigma_{2i} = \frac{E}{1-\nu^2} \gamma_{\phi z} (-m_i^* n_i^* \sin \phi - l_i^* n_i^* \cos \phi) \quad (41)$$

If we put $\phi=0$, $\pi/2$ into this equation, we have

$$\sigma_{2i} = \frac{E}{1-\nu^2} \gamma_{xz} (-l_i^* n_i^*), \quad (\phi=0) \quad (42)$$

$$\sigma_{2i} = \frac{E}{1-\nu^2} \gamma_{yz} (-m_i^* n_i^*), \quad (\phi=\pi/2) \quad (43)$$

In the first case, when $\phi=0$, whether stress is tensile or compressive can be judged by the following formulas, which are obtained by using the direction cosines of the normal vectors R_{0i} shown in page 151.

$$\begin{aligned} \sigma_{2i} &> 0, & i &= 2, 3 \\ \sigma_{2i} &< 0, & i &= 1, 4 \end{aligned} \quad (44)$$

In the second case, when $\phi = \pi/2$, we have

$$\begin{aligned} \sigma_{2i} > 0, & \quad i = 3, 4 \\ \sigma_{2i} < 0, & \quad i = 1, 2 \end{aligned} \quad (45)$$

It became clear that this sign of the normal stress σ_{2i} depends on the direction cosines of the normal vector R_{0i} and the direction ϕ of the applied shear deformation. It can be either positive or negative. The same can be said about the normal stress σ_{1i} , though it is generally smaller. Needless to say, the shear stress τ_{12i} is of primary importance and can easily be calculated in similar manner.

Such stress state, apparently due to the inclination of composing elements of this core, offers a marked contrast with that of the honeycomb core where the simple shear stress condition prevails within its vertical cell walls.

The problem of instability of core elements, therefore, reduces to that of a rhomboidal plate simply supported at four edges and subjected to the combined normal stresses and shear stress. This is undoubtedly a very difficult problem. These solution available at present are about the classical buckling loads in case of pure shear and that of uni-axial compression. This is an embarrassing situation.

Some insight into this problem can be obtained by comparing the zeta-core sandwich with the single corrugation type sandwich. Obviously, the buckling load for single corrugation core is that of an infinite long strip plate of a width loaded compression by two opposite edges which are simply supported, while the buckling load for zeta-core is that of a rhombic plate of equal width simply supported at all edges. Unless the rhombic plate has high aspect ratio, the buckling load for zeta-core is certainly four orders of magnitude higher than that of the single corrugation core.

The post-buckling behavior of a flat element must also be considered. It is most likely that the flat element may have an initial imperfection and it further reduces the rigidity of it to some extent. A buckled plate, however, when it is supported at all edges, can resist to further increase in load. In addition, since the reverse stress condition always exists in adjacent elements as indicated in Eqs. 44 and 45, the deformation can not easily advance further.

As far as the stability of core is concerned, the use of zeta-core having cylindrical side parts may be beneficial. In any case, the design of core form is especially effected by the stability consideration.

5. REALIZATION OF ZETA-CORE CONCEPT

5-1 Manufacturing of Zeta-Core

In the foregoing study on zeta-core concept, it has been shown that the core is worthy of further investigation on realization aspect. In this respect, the manufacturing problem of zeta-core will be discussed briefly in the following.

Consider a continuous surface formed by taking away the top and bottom parts from a zeta-core surface whose fundamental region is shown in Fig. 10. It yields

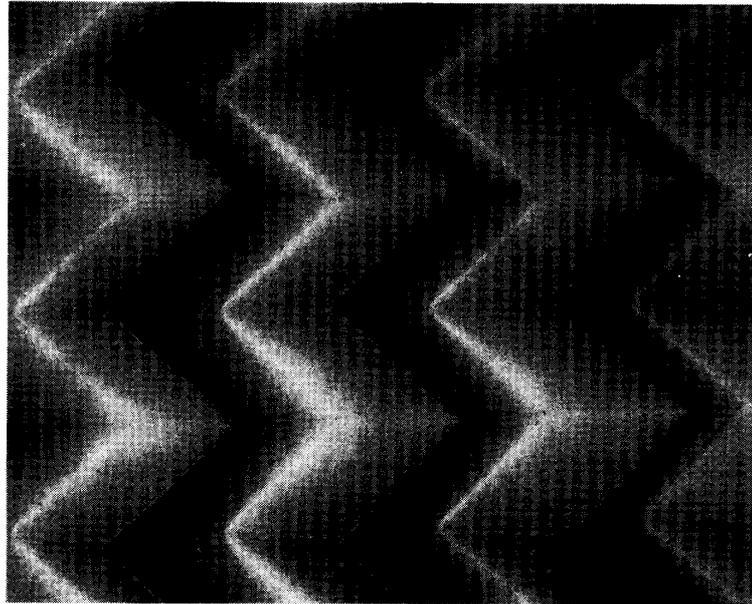


FIG. 14. A Developable Double Corrugation Surface
(Caution: Stabilize your vision so that the lighter zigzag
ridges may be looked valley ridges.)

a surface with really distinguished characteristics (Fig. 14). This surface, provisionally named by the author "a developable double corrugation surface," can be developed into a plane, because the sum of vertex angles around any vertex is always 2π radian. The particular geometry of this surface is given in some details in the references [1], [2], and [3].

This surface has in truth been known before to peoples who had an interest in paper folding art. This strange form might have attracted attention of structural engineers and it led to the invention of such core by Géwice in France and Rapp [5] in U.S.A. in the end of fifties. The foundation of the invention depends on the fact that such a core should theoretically be manufactured from a sheet material through an isometric transformation, that is, a bending working. It is true that a model of this type can be made of a paper by hand, however, developing an acceptable mass production method of the core can not be expected for a while. The main difficulty is due to the general discontinuity of fold lines, which is an intrinsic feature of the geometry of this surface. Another fault of this invention is the absence of the parallel surface with the facing surface which is used for bonding area.

The manufacturing of zeta-core is only possible through literally the topological transformation of a flat sheet, that is, the press forming of a sheet which necessarily includes both stretching in-plane and bending deformations. The amount of in-plane stretching deformation is a increasing function of the inclination of side element of core. The difficulty increases as this angle increases. Hence the angle near 55° which gives the optimum form efficiency is a goal. It should be noted that the flat top part of the core has an eminent alleviating effect on this difficulty. Without this flat part, occurring of either excessive thinning or fracture in the region

of sharp crest ridges of the core is almost inevitable. This is another reason why above-mentioned core having a developable mid-surface is unable to manufacture successfully even by a press forming.

The manufacture of zeta-core from thermo-plastic sheet was carried out with the cooperation of Mitsui Toatsu Chemicals, Inc. We found no difficulty in trial manufacture. The resulted zeta-core made of polystyrene sheet and its sandwich are shown in Fig. 15. The manufacture of zeta-core from aluminum alloy sheet was carried out with the cooperation of Sumitomo Light Metal Industries, Ltd. This case was naturally more difficult than the former, since the necessary deformation is close to the limit of the formability of aluminum alloy. The resulted zeta-core from aluminum alloy sheet and its sandwich are shown in Fig. 16.

Conclusively, it seems there is no essential difficulty which can not be solved in manufacturing zeta-core of this type from plastics, G.R.P., and metal sheets. Also it can be predicted that, due to the simplicity in forming process, the production cost of zeta-core is relatively low in comparison with honeycomb core. Much study should be done, however, before the effective manufacturing method of zeta-core is to be found.

5-2 Applications of Zeta-Core

If we sum up the principal features of zeta-core predictable at this stage of

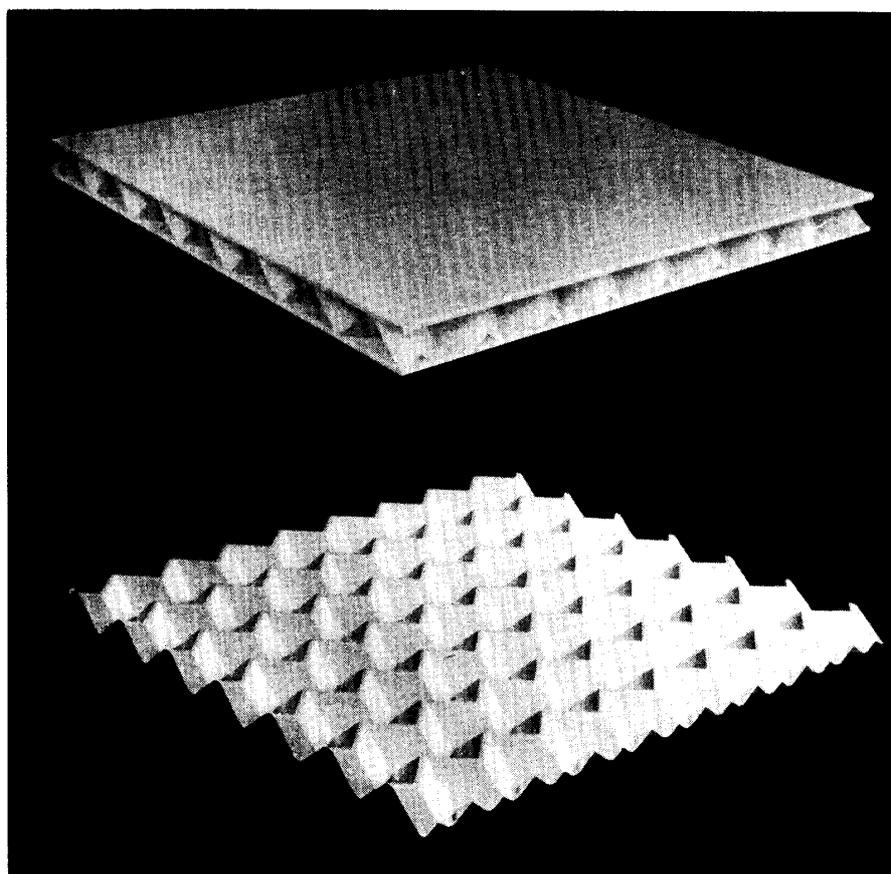


FIG. 15. Zeta-Core Made of Plastic Sheet

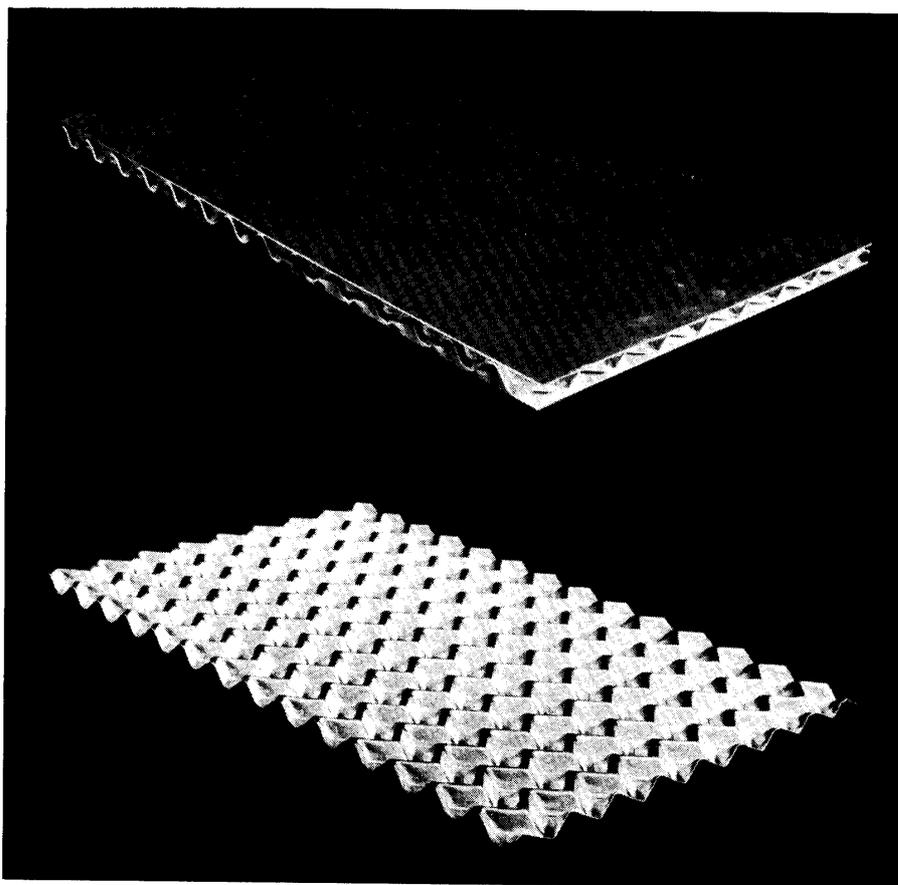


FIG. 16. Zeta-Core Made of Aluminum Alloy Sheet

study, these are as follows; the high shear modulus almost equivalent to that of honeycomb core, the isotropy or controllable orthotropy in shear property, simplicity of its monocoque structure with no bonding within itself, and low cost of manufacturing. Such characters are undoubtedly most desirable for general applications of sandwich constructions. Hence, the zeta-core concept may develop the possibility that the sandwich construction is accepted by much wider field of industries than before as the primary load carrying members to an extent which is not heretofore attained.

The possible applications of zeta-core which are deeply concerned with its particular properties are now discussed briefly in the following.

The honeycomb sandwich can not generally be used for high temperature structure because of its difficulty in brazing and welding to facings. For high speed air cruising vehicles the light weight stressed panels usable at an elevated temperature environment is needed. Zeta-core has some apparent merits in this respect. As for bonding, there is not any bonding within the core and also there is the flat contact surface purposely designed for bonding with facings. Therefore, either brazing or welding can be done firmly with ease. Another merit exists in the configuration of zeta-core sandwich which facilitates the circulation of a fluid within spaces between core and facings.

Another important feature of this core is the applicability to sandwich of free

formed surfaces. The necessity of free formed surface shell structures is growing in various fields and will be still increasing in future. In principle, the zeta-core can be designed to meet with any free formed surface. Moreover, the core of moderate single curvature can be made of a flat type zeta-core by bending.

The zeta-core can be used also as the stiffener for the stiffened panel structure. It provides stiffening effect in two mutually orthogonal directions which is similar of a waffle plate. The understanding of this interesting mechanism needs further investigation.

There are some possibilities of using zeta-core for non-structural purposes such as a shock energy absorbing structure and a heat exchanging element, but there consideration is outside the scope of this paper.

In any case, however, much study should be done for the purpose of developing the rational applications which take good advantage of particular properties of zeta-core.

6. CONCLUSIONS

1. The geometric forms of the core of sandwich construction can be grouped into three classes, such as the cellular form, the grid form, and the single surface form. The last form, which is created by a topological transformation of a plane, can further be grouped into four types.

2. With due consideration of the shear strength, isotropy, bonding, and manufacture, the core form which is composed of side parts in inclined ruled surface and top parts in zigzag form is considered to be the most promising one. The core is designated by "zeta-core."

3. It has been shown by an approximate analysis that the shear modulus of zeta-core can compete with that of honeycomb core of identical apparent density, and that it is possible to design zeta-core of any orthotropy including isotropy in shear modulus.

4. The trial manufacture of zeta-core from both plastics and aluminum alloys has been successful. It seems there is no essential difficulty in mass production of zeta-core from these materials. Hence, the concept of zeta-core is completely realizable.

5. Such features as simplicity, high shear modulus, isotropy, handling easiness, and low cost of zeta-core are most desirable for general applications of sandwich construction. Thus, the zeta-core concept may develop the possibility that the sandwich structure is accepted by much wider field of industries.

6. Particular applications of zeta-core includes elevated temperature applications, sandwich of free formed surfaces, and shock absorbing structures.

7. Future study should include the effective production methods, the optimum design, and the development of rational applications.

APPENDIX

Definition of Form Efficiency of Core Form

For the structural evaluation and comparison of cores in different forms, we would first select a certain quantity which can reflect most of the important factors involved in each case and then calculate the quantity to decide the evaluation. Such quantity can be numerous and each of them has its own feature suitable for a particular purpose. It will be convenient, however, if we could find a single dimensionless quantity which can be used for the most general evaluation for the core form.

The present author proposes the use of the “form efficiency” of core defined by the following formula as such an evaluating quantity of most general nature.

$$\mu = \frac{G_e}{G\alpha} \quad (\text{A-1})$$

In this formula, G is the shear modulus of the core material, G_e is the effective shear modulus of the core (or simply the shear modulus of the core), and α is the filling factor of the space between facings which is given by the ratio of apparent density to material density of core.

Introduction of this concept is due to the following idea. If we select a single quantity which represents most important mechanical properties of core, we will undoubtedly take the shear modulus of core. This consideration leads the thought that the core efficiency should be defined as the one which is in proportion to the dimensionless form of the shear modulus of the core, that is, G_e/G .

Now we shall make clear what is considered to be a definition of “core form.” The filling factor α for the usual cores is relatively low and is in between 0.1 and 0.01. It follows from this fact that the microscopic form of the element of core structure can not be three dimensional, and instead, it must be at most two-dimensional. This reasoning is supported by the observation that the majority of cores we use are composed of shell elements, that is, the two-dimensional structural elements. Thus, the geometric form of a core can be expressed completely by the core surface and the thickness of it. For a certain core surface, the filling factor is logically in proportion to the thickness, or more specifically, the average value of the thickness.

For thin shells, the assumption of the state of membrane stress is an first approximation. If we assume it, the total strain energy stored in the core by a given deformation is in proportion to the average thickness, and thus, to the filling factor. Furthermore, the total strain energy is in proportion to the effective shear modulus due to its definition. Therefore, it can be concluded that the filling factor α is essentially in proportion to the effective shear modulus G_e . It should be noted, however, that this linear relation is valid for neither very low nor high values of α ; the reason of it is rather obvious.

If we take the view that the core form means the form of the core surface excluding the filling factor, and that we needs an evaluating quantity only for the

core surface, such a quantity must be independent of α .

Then, it will require a little ingenuity to arrive at the following dimensionless parameter proposed before.

$$\frac{G_c}{G\alpha}$$

This is really a dimensionless quantity which represents the mechanical property of a core and depends on solely the core surface and is independent of both material and filling factor. The higher value of the form efficiency is of course desirable.

A better understanding about the physical meaning of form efficiency can be obtained by considering μ values for a core form of Fig. 17, where cores of sequential decreasing of α are shown. Obviously μ equals to 1 when α equals to 1, and μ approaches 0.5 as α decreases. Also from the previous argument, μ is almost independent of α when $0.01 < \alpha < 0.1$. With these information on hand, we can compose a typical μ curve in α - μ coordinates as shown in Fig. 17.

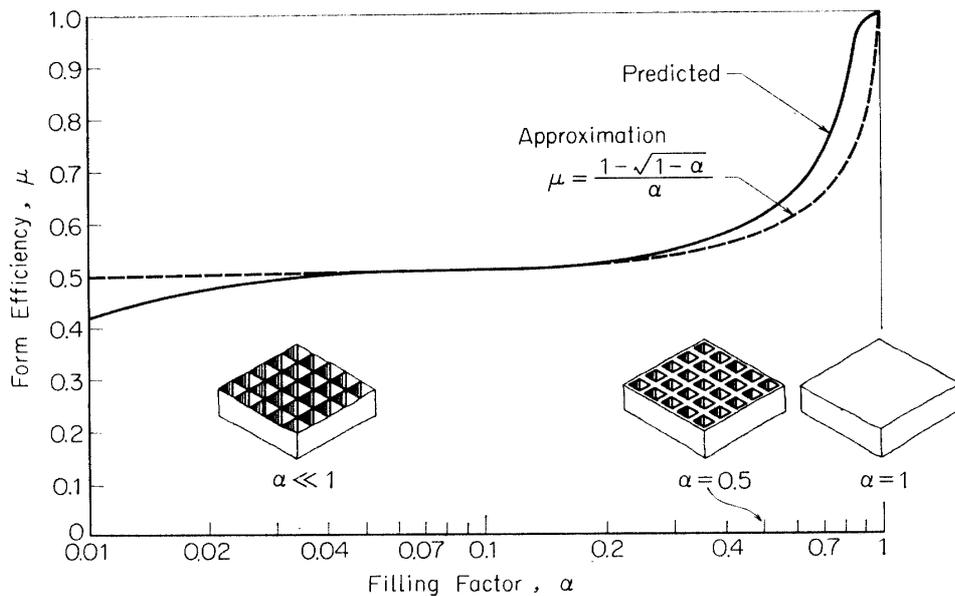


FIG. 17. Form Efficiency vs. Filling Factor for Grid Type Core

Now, let us calculate the μ value of honeycomb core from theory and also from the test result. The shear modulus of a honeycomb core of regular hexagonal type is given by the following formula [6], on the basis of equal assumptions to those used in the previous zeta-core analysis,

$$G_c = \frac{5}{3\sqrt{3}} \left(\frac{t}{b}\right) G, \quad (L \text{ direction}) \quad (\text{A-2})$$

$$G_c = \frac{1}{\sqrt{3}} \left(\frac{t}{b}\right) G, \quad (W \text{ direction}) \quad (\text{A-3})$$

where b is the length of a side of the regular hexagon and t is the thickness of cell wall. The filling factor α for hexagonal core is

$$\alpha = \frac{8}{3\sqrt{3}} \left(\frac{t}{b} \right) \quad (\text{A-4})$$

From these equations, we have finally

$$\mu = \frac{5}{8} = 0.625, \quad (L \text{ direction}) \quad (\text{A-5})$$

$$\mu = \frac{3}{8} = 0.375, \quad (W \text{ direction}) \quad (\text{A-6})$$

If we assume the mean values of μ_m as an arithmetic mean of values for L and W directions, we have

$$\mu_m = \frac{1}{2} = 0.5 \quad (\text{A-7})$$

It is clear from these formulas, the form efficiency of honeycomb core is independent of α and is inherent in the core form.

While, μ values calculated from the tested result on shear modulus by Hexcell catalogue [7] are plotted in Fig. 18. It is shown that except very low value of α , the mean value of μ is a very weak function of α and is close to 0.5. Thus it can be concluded from both theory and test the form efficiency is a constant which is intrinsic to honeycomb form. This fact provides an ample evidence that the concept of the form efficiency is just what we expected previously. Hence, the concept of the form efficiency is considered to be very useful for the evaluation of core forms.

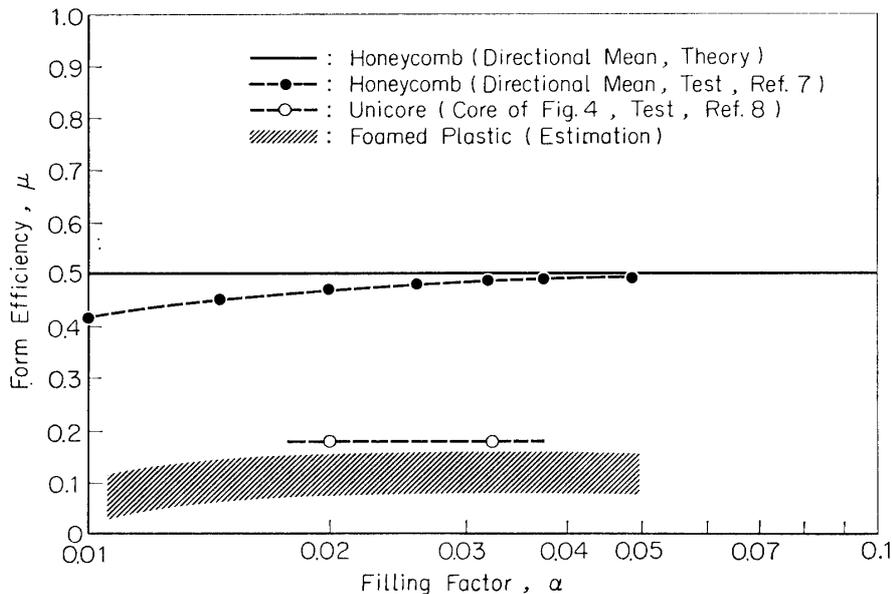


FIG. 18. Form Efficiency vs. Filling Factor for Several Core Forms

It should also be noted that the same relation but in the different arrange as shown below

$$G_c = \mu \alpha G \quad (\text{A-8})$$

can be used for the prediction of the shear modulus of a core of known form with different filling factor and material. For reference, μ values for several core forms are plotted in Fig. 18.

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SYMBOLS

Symbols for primary concept

a	area of a rhomboidal element
b	side length of regular hexagon
c	constant
f	periodic function
g	periodic function
h	periodic function
l, m, n	direction cosines
t	uniform thickness of a rhomboidal element or a honeycomb wall
v	gross volume between facings filled by a fundamental region of zeta-core
x, y, z	rectangular Cartesian coordinates
E	Young's modulus
G	Shear modulus
R	metric vector
U	strain energy
α	filling factor of core
β	reduction ratio (see Eq. 26)
γ	shear strain
ε	normal strain
μ	form efficiency of core (see Eqs. 25 and A-1)
σ	normal stress
ν	Poisson's ratio
τ	shear stress
ϕ	direction, ϕ radian counter-clockwise from x -axis on x - y plane
\emptyset	null set
I, II, ...	domain
f.r.	fundamental region

Subscript

c	core property
i	i th element
r	linear element

Superscript

*	quantity relating to a zeta-core whose fundamental region is composed of four congruent rhomboids and two congruent chevron patterns (see Fig. 10)
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