Institute of Space and Aeronautical Science, University of Tokyo
Report No. 493, May 1973

An Application of Cross-Spectral Analysis to the Measurement of Contribution of Noise Sources

By

Yasushi ISHII and Masahiro ISHIDA

Summary : The measurement of the contributions of noise sources is to determine the amount of acoustic power contributed to the field at a given point by each of several sources. The signal from the microphone placed at an observation point is cross-correlated with the signal from the second microphone placed sufficiently close to a noise source. The measured cross-correlation function is converted into the cross-spectral density through digital computations and then divided by the power-spectrum of the signal from the second microphone, giving the frequency transfer function between the noise source and the observation point. Using the computed transfer function, we can separate the component due to the source in question from the total sound power-spectrum at the observation point. Some preliminary experiments including the separation of exhaust noise of an automobile were made, and the results verified the effectiveness of this cross-spectral method.

1. INTRODUCTION

The measurement of the contributions of noise sources is the problem of determining how much of the acoustic power at a given point is contributed by each of several noise sources. An example of this type of problem would be the determination of which machine in a large factory area should be quieted first in order to reduce the noise level at a given point. If we could turn all the machines off and then turn on one at a time, we could measure the contribution of each source directly and the problem would be solved. However, it is not always possible to have such complete control over the sources as well as the extraneous noise.

K. W. Goff [*1*] presented an approach to this measurement problem that, by use of the correlation function for random signals, yields the desired results without requiring control of the sources involved. In his method, the output signal $y(t)$ of the microphone placed at a given observation point is cross-correlated with the reference signal $x(t)$ obtained by the second microphone as shown in **Fig. 1**. The second microphone is placed sufficiently close to the source of interest to pick up essentially nothing but the sound produced by that source. Then the contribution of the source to the total acoustic power at the observation point is given as

$$\beta=\frac{\sum_{m} \phi_{xy}^{2}(\tau_m)}{\phi_{xx}(0)\phi_{yy}(0)}, \qquad (1)$$

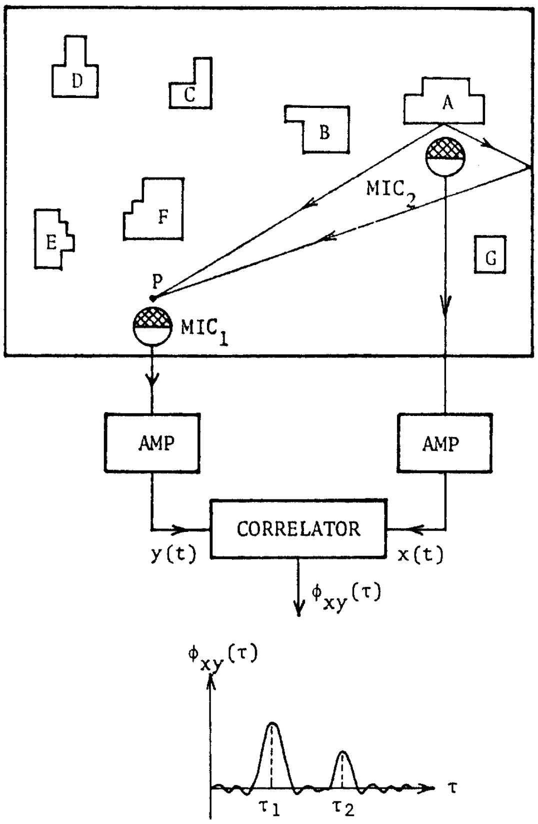

FIG. 1. Block diagram of setup for measuring contributions of noise sources by a correlation technique (K. W. Goff, 1955).

where τ_m is the time delay for which the cross-correlation function $\phi_{xy}(\tau)$ takes a maximum.

In practice, however, the application of this correlaiton technique is limited by the restrictive conditions imposed. These are:

(1) The noise produced by the source of interest must have a broad frequency band and not contain any periodic component.

(2) The acoustic transmission characteristics from the source to the observation point must be composed of pure delays.

(3) The transmission time difference of any two acoustic paths from the source to the observation point must be sufficiently large so that the corresponding peaks of the cross-correlation function are separable.

This paper proposes a new approach to the problem using the cross-spectral density function for the acoustic signals. This method of analysis in the frequency domain eliminates those restrictions inherent to the correlation technique. The results of preliminary experiments in an anechoic room as well as an experiment of the separation of exhaust noise of an automobile verify the effectiveness of this cross-spectral method.

2. THEORY

The problem of measuring the contribution of a noise source is represented basically by the block diagram of **Fig. 2**. In the figure, $x(t)$ and $y(t)$ are acoustic signals obtained at the source and the observation point respectively. $G(f)$ is assumed to be the frequency transfer function between these two points. Not only pure delay but any linear transmission characteristics can be assumed as $G(f)$. All the sounds which come from the other noise sources to the observation point are regarded as the disturbances to this acoustic system and are represented together by signal $n(t)$. Signal $z(t)$ is the component of $y(t)$ which is due to the source in question.

Although signal $z(t)$ itself is not directly observable, it is possible to estimate the power-spectrum of $z(t)$ using the spectra of the other signals. Namely,

$$\Phi_{zz}(f)=|G(f)|^2\Phi_{xx}(f)=\left|\frac{\Phi_{xy}(f)}{\Phi_{xx}(f)}\right|^2\Phi_{xx}(f)=\frac{|\Phi_{xy}(f)|^2}{\Phi_{xx}(f)}, \qquad (2)$$

where $\Phi(f)$'s stand for the spectra of the signals designated by the respective sub-indices. Then, the power contribution desired is simply given by

$$\beta=\frac{\int_{-\infty}^{\infty}\Phi_{zz}(f)df}{\int_{-\infty}^{\infty}\Phi_{yy}(f)df}=\frac{\int_{-\infty}^{\infty}\frac{|\Phi_{xy}(f)|^2}{\Phi_{xx}(f)}df}{\int_{-\infty}^{\infty}\Phi_{yy}(f)df}. \qquad (3)$$

It should be noted that the microphone sensitivities as well as the gains of the amplifiers used can not be included in the above expression. Furthermore, it is shown that the frequency response of the microphone placed at the noise source has, at least theoretically, no influence to the measured contribution. **Fig. 3** is the block diagram of the acoustic system including the transfer function of the microphone as $H(f)$. The output of the microphone is represented by $x'(t)$. In this instance, $\Phi_{x'y}(f)$ and $\Phi_{x'x'}(f)$ are used in the places of $\Phi_{xy}(f)$ and $\Phi_{xx}(f)$ respectively, and the expression for the contribution becomes

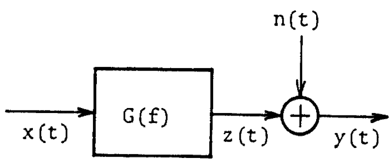

FIG. 2. Basic block diagram of the acoustic system of the measurement.

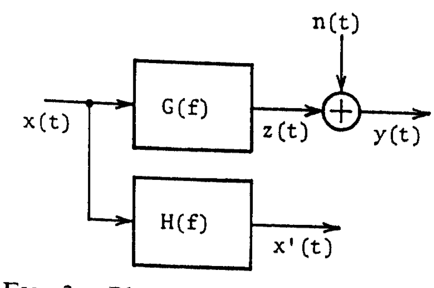

FIG. 3. Block diagram of the acoustic system including the transfer function of a microphone.

$$\beta = \frac{\displaystyle\int_{-\infty}^{\infty} \frac{|\Phi_{x'y}(f)|^2}{\Phi_{x'x'}(f)} df}{\displaystyle\int_{-\infty}^{\infty} \Phi_{yy}(f) df}. \tag{4}$$

For those spectra, however, the following relations hold:

$$\left.\begin{aligned} \Phi_{x'y}(f) &= H^*(f)\Phi_{xy}(f) \\ \Phi_{x'x'}(f) &= |H(f)|^2 \Phi_{xx}(f), \end{aligned}\right\} \tag{5}$$

where $H^*(f)$ is the complex conjugate of $H(f)$. Substitution of the above relations into Eq. (4) cancels out $H(f)$.

On the other hand, the frequency response of the microphone placed at the observation point directly affects the final result. Therefore it should be flat within

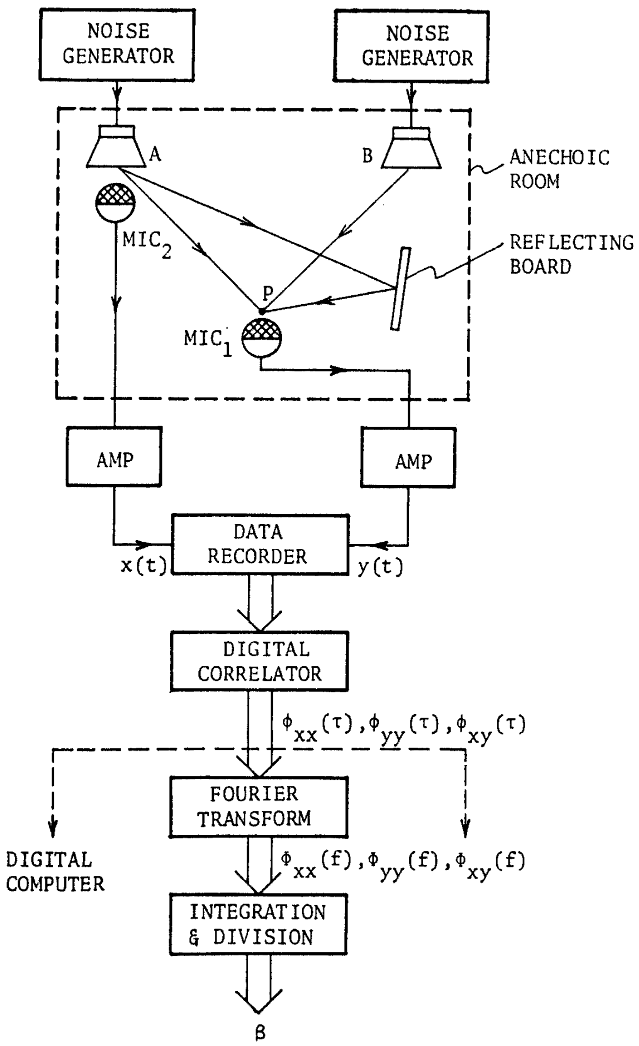

FIG. 4. Block diagram of setup for the preliminary experiments in an anechoic room.

the frequency range of interest. Or, it would be more practical to apply an appropriate weighting function such as A, B, or C curve for sound level meters to the output of the microphone.

3. Experiments

Fig. 4 shows the block diagram of setup for the preliminary experiments in an anechoic room. Two speakers, A and B, were placed several meters apart in the anechoic room. The noises produced by these two speakers were independent of each other. The first microphone was placed at P, the point several meters apart from both of the speakers. The second microphone was set sufficiently close to one of the speakers, say A. A sound reflecting board was also placed in the room to get a multipath transmission of the acoustic signals.

The signals from the microphones were once recorded on a magnetic tape by a

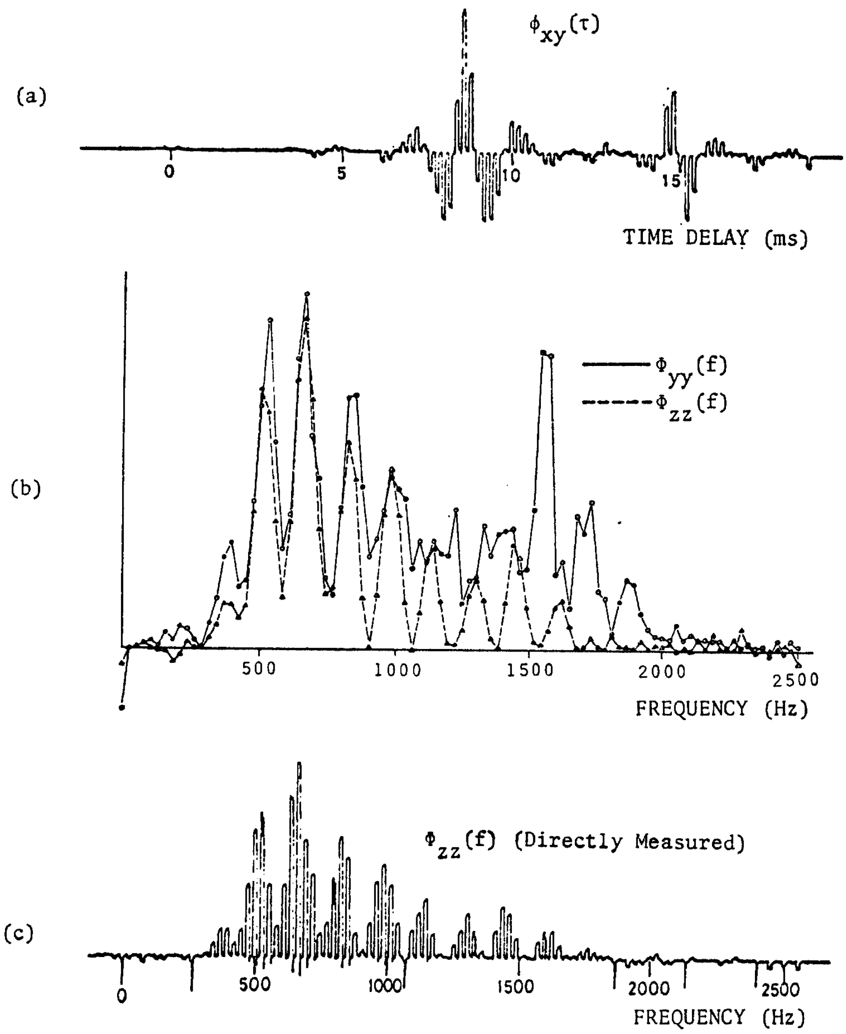

Fig. 5. An example of the preliminary experimental results.

multichannel tape recorder. The cross-correlation function, $\phi_{xy}(\tau)$, and the auto-correlation functions, $\phi_{xx}(\tau)$ and $\phi_{yy}(\tau)$, were measured on the reproduced outputs of the recorder with a real-time digital correlator [2]. In order to minimize the statistical error of the final result, these correlation functions were measured using the same portion of the magnetic tape. The punched data of the measured correlation functions were fed to a digital computer and converted into the spectra. The associated computations for getting the contribution such as integrations of the spectra were also performed by the computer.

Fig. 5 is one of the experimental results. (a) is the cross-correlation function $\phi_{xy}(\tau)$ showing the peaks corresponding to the signal directly transmitted and the signal reflected by the board. The solid line in (b) is the power spectrum of the total sound at the observation point P and the dotted line shows the estimated power spectrum of the component originating from speaker A. (c) is the directly measured spectrum of this component obtained by quieting the disturbing speaker B. This is shown for the comparison with the dotted curve in (b).

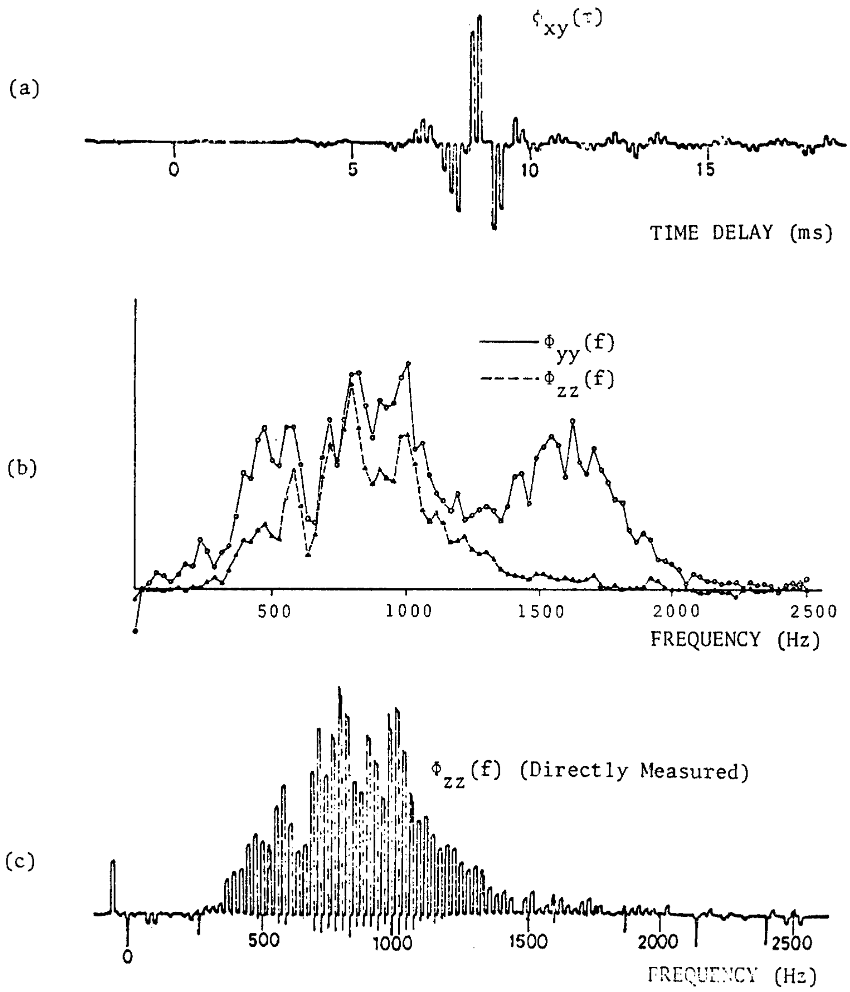

FIG. 6. An experimental result for a smaller transmission time difference between the direct and reflected acoustic paths.

PHOTO. 1. Experiment for the separation of exhaust noise of an automobile.

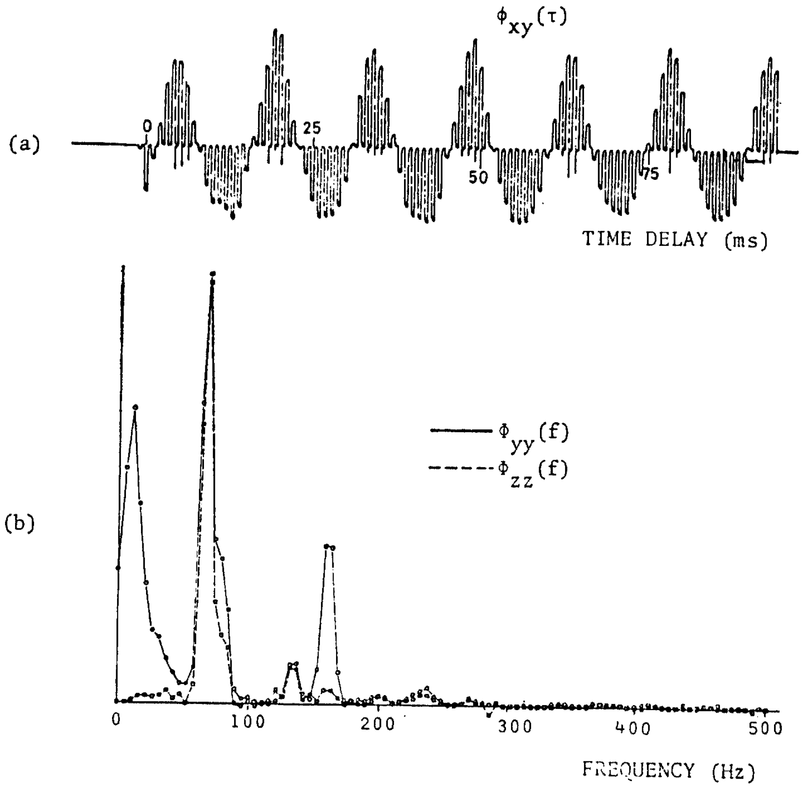

FIG. 7. Experimental result of the separation of exhaust noise.

The value of contribution obtained by the cross-spectral method using the curves in (b) becomes 55.1%, while the directly measured value was 52.2%. The computation of contribution is also done using peak heights of the cross-correlation function of (a), resulting the value of 46.0%.

Fig. 6 is another example of the experimental results. The conditions of the experiment are the same as those of the former case except for a smaller transmission time difference between the direct and reflected acoustic paths. The correlation technique fails to work in this case because the peaks of the cross-correlation function overlap together as shown in (a). However, the cross-spectral method is applied without any difficulties, yielding the measured contribution of 45.3%. This agrees to the value of 45.0% obtained by the direct measurement.

In order to verify the applicability of the method to highly periodic noises, an attempt was made to separate the exhaust noise of an automobile from the mechanical noise produced by the engine. The experiment was done outdoors as shown in **Photo. 1**. The microphone collecting the total noise $y(t)$ was set about 5 meters apart from a side of the car. The second microphone to pick up the reference signal $x(t)$ was placed by the exhaust outlet keeping out of the gas stream. The engine was running with no load at the speed of 1900 rpm.

Fig. 7 is the result of the experiment. The cross-correlation function is almost periodic as shown in (a). The curve of $\Phi_{yy}(f)$ in (b) shows a large peak in the frequency range less than 50 Hz. This is the power of the wind noise produced at the structure of the microphone. The peaks at 65 Hz and 130 Hz are mainly due to the exhaust noise, while the power of the engine noise, except the fan noise component of 160 Hz, spreads over to the higher frequencies.

Department of Instrument and Electronics
Institute of Space and Aeronautical Science
University of Tokyo
May 1, 1973

References

[1] K. W. Goff: The Application of Correlation Techniques to Some Acoustic Measurements, J. Acout. Soc. Amer., Vol. 27, No. 2, p. 236 (Mar. 1955).
[2] Y. Ishii: A Real-Time Signal Processing System for Correlation and Spectrum Analysis, Inst. Space and Aero. Science, Report No. 444 (Feb. 1970).