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Propagation of Electron Waves in a Non-Maxwellian Plasma

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Summary: The dispersion relation of electron waves in a non-Maxwellian plasma have been investigated experimentally. It is observed that the dispersion relation of the Landau mode is modified and that a new wave mode with the dispersion relation $\omega/k \simeq v_c$ (v_c: truncated velocity) is observed for frequencies less than the electron Langmuir frequency. These experimental results are compared with a simple theoretical model.

I. INTRODUCTION

Propagations of electron waves in collisionless plasmas have been studied in detail both theoretically [1] and experimentally [2–8]. In those cases, the velocity distribution function for electrons is assumed to be Maxwellian. If the distribution function deviates from the Maxwellian distribution function, however, the propagation characteristics of the electron plasma wave (the Landau mode) [1] would be modified. In fact, there are a few number of high energy electrons, whose energy is about equal to discharge voltage, in the plasma produced by dc discharges so that we frequently encounter to cases that the distribution function in the discharge plasma deviates from the Maxwellian.

If the distribution function of the electron is composed of the Maxwellian distribution function and the water-bag one as shown in Fig. 1, we can obtain easily the following dispersion relation [9] for the electron plasma wave:

$$1=\frac{\omega_{pe}^2}{k^2 v_e^2} Z'(\omega/kv_e)+\frac{\omega_{pc}^2}{\omega^2 - v_c^2 k^2} \qquad (1)$$

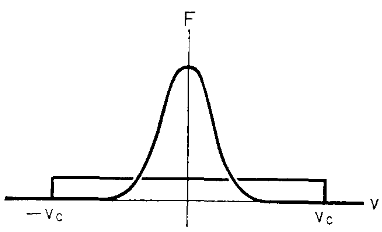

FIG. 1. Velocity distribution function.

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where ω_{pe} and ω_{pc} are the Langmuir frequencies of thermal electrons and water-bag electrons, respectively, v_e and v_c are the thermal velocity and truncated one of electrons, and Z' is derivative of the plasma dispersion function. If $v_c \gg v_e$, for $\omega \gg \omega_{pe}$ the dispersion relation of the Landau mode is modified such that $\omega \simeq kv_c$, for $\omega < \omega_{pe}$ the new dispersion relation $\omega \simeq kv_c$ is obtained, and for $\omega \sim \omega_{pe}$ the Landau mode and the new wave mode exist and interference of both waves would be observed.

On the other hand, for frequencies less than the electron Langmuir frequency, the free-streaming electron [*10–12*] also is excited when the electron wave is excited by grids or probes. Therefore, for $\omega < \omega_{pe}$, both the new wave mode and the free-streaming electron will be excited generally in experiments on the electron wave in a non-Maxwellian plasma. The new wave mode is, however, dominant compared with the free-streaming electron for the Maxwellian distribution function plus the water-bag one since the damping rate of the new wave mode is much smaller than that of the free streaming electron, as estimated from Eq. (1).

In this paper, we report experimental results that when the discharge voltage is varied by changing gas pressures, the dispersion relation of the electron wave is modified. In Secs. II and III the experimental arrangement and the experimental results are described, and we summarize them in Sec. IV.

II. Experimental Arrangement

The experiment was performed using the space chamber [*6*] at the Institute of Space and Aeronautical Science, University of Tokyo, as shown in Fig. 2. The chamber, which is grounded, is 2 m in diameter and 3 m in length. The plasma sources were set face to face, one at each end of the axis of the chamber. They consisted of a mesh anode and hot cathode (15 cm in length). The density and temperature of the plasma electrons were measured using the Langmuir probe and were found to be homogeneous over the chamber. The plasma density n_e was in the range of 10^6-10^7/cc, and the electron temperature T_e was in the range of 3–5 eV. The experiments were carried out under conditions of continuous pumping and the introduction of argon gas through a needle valve. The base pressure in the chamber was below 5×10^{-7} Torr and the working pressure was ranged from 10^{-5} to 10^{-3} Torr. The plasma potential was varied in a range from -30 to 30 V with respect to the ground potential by changing the bias potential of the mesh anodes, as shown in Fig. 2.

An electrostatic electron wave was excited by a transmitter which consisted of three mesh grids (17 cm in diameter) made of 0.03-mm diam. tungsten wires spaced 0.5 mm apart. The Faraday cup was used as a receiver; this was 10 cm in diameter and consisted of two mesh grids and a collector. The outer grids of the transmitter and of the receiver were grounded.

The signal, picked up by the Faraday cup and amplified with a wide band amplifier, was fed into a balanced mixer with a reference signal from the transmitter to form an interferometric system. The position of the exciter was indicated on

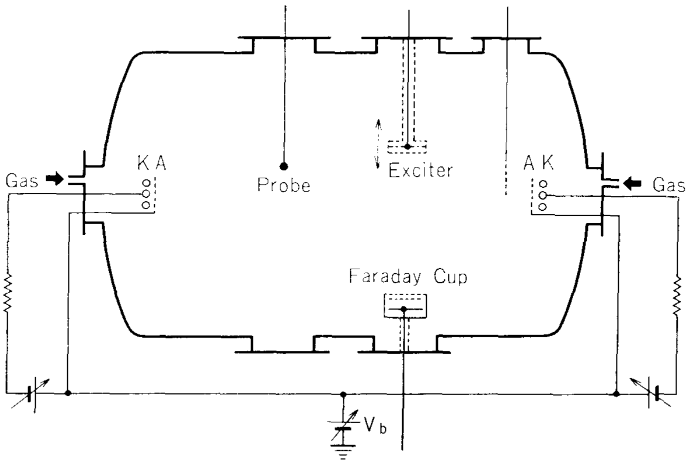

FIG. 2. Schematic diagram of the experimental apparatus.

the x axis of an $x-y$ recorder and the mixer output was applied to the y axis. The distance x between the transmitter and the receiver was varied from 5 to 85 cm by moving the position of the transmitter.

The excitation voltage applied on the transmitter grid ranged from 0.3–10 V peak to peak. The frequency of the applied sinusoidal signal could be changed from 1–60 MHz to cover the range of ω/ω_p from 0.1–2.5.

III. EXPERIMENTAL RESULTS

An electron wave was excited and the dispersion relation was obtained from raw data by normalizing the real and the imaginary parts of the wavenumber by the Debye wavenumber k_D for different ω/ω_p and was plotted as a parameter of the anode potential in Fig. 3, where k_D and ω_{pe} were calculated from the measured electron density and electron temperature, respectively. There are two types of propagation modes as shown in Fig. 3 and they interfere each other in a frequency range close to ω_{pe}. When the anode potential is at 70 V, experimental values are in good agreement with the theoretical ones [1], [10–12]. Thus, for the low anode potential, two kind of the waves observed are considered to be the Landau mode and the free-streaming electron, respectively, which has been investigated in detail in the previous report [10]. As the anode potential is increased, the real and imaginary parts of the wavenumber of the waves become smaller than those for the Maxwellian plasma. In particular, it should be noted that the frequency of the excited wave is proportional to the wavenumber except for frequencies close to the electron Langmuir frequency.

We studied the frequency dependence of the phase velocity (v_{ph}) in detail for $\omega < \omega_{pe}$. Figure 4 shows a typical example of this variation for the different anode potentials. It is found from Fig. 4 that (i) for lower anode potentials the phase velocity is proportional to $\omega^{1/3}$, and (ii) for large anode potentials the phase velocity is constant and the constant values increase with V_a.

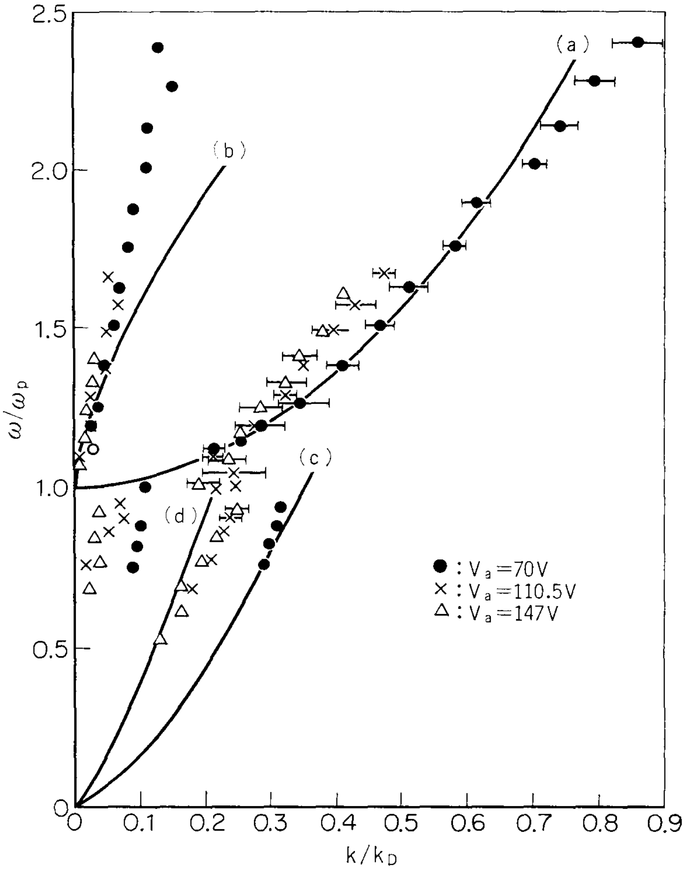

FIG. 3. Dispersion relation. Solid lines (a) and (b) are the real and imaginary parts of the wavenumber of the Landau mode, respectively. Solid lines (c) and (d) are the real and imaginary parts of the wavenumber of the free-streaming electron, respectively.

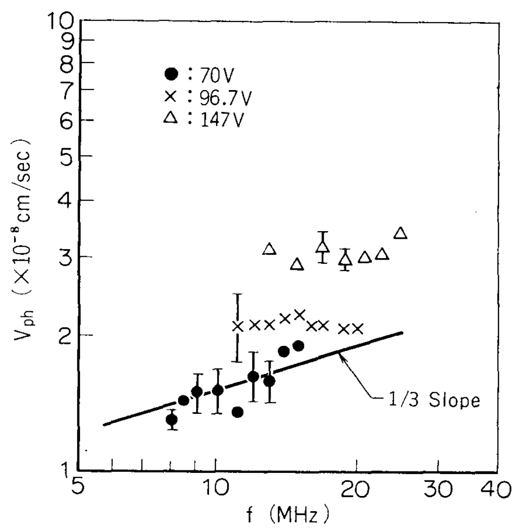

FIG. 4. Phase velocity variations of the waves as a function of frequencies, where the parameter is the anode potential.

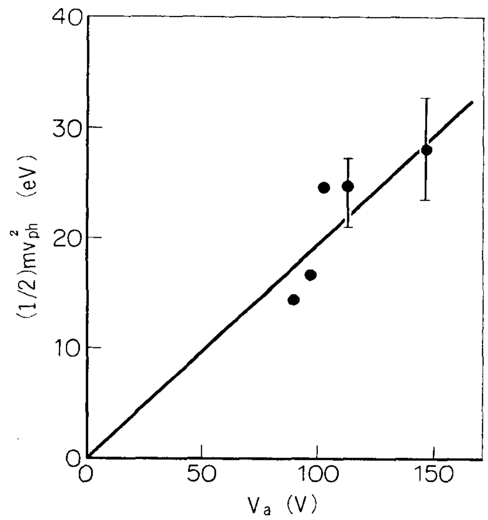

FIG. 5. The equivalent energy of the wave calculated from the phase velocity as a function of the anode potential.

We plotted the equivalent energy of the wave, $(1/2)mv_{ph}^2$, calculated from the phase velocity as a function of the anode potential in Fig. 5. This equivalent energy increases to be proportional to the anode potential. If the energy of the truncated electron with the velocity of v_c corresponds to the anode potential V_a, we can obtain the relation, $v_{ph} \simeq 1/2v_c$, from Fig. 5.

We measured the energy distribution function of the electron by the Faraday cup for different anode potential. The tail of the energy distribution function was extended with increasing the anode potential. We also found that the observed distribution function did not depend on the direction. Therefore, the energy distribution function observed in our experiment can be approximated to be composed of the Maxwellian plus the water-bag distribution function. Thus, the lower frequency mode observed for the large anode potential can be regarded as the new wave mode different from the free-straming electron, predicted on Sec. I.

IV. Discussions and Conclusions

For the case of the lower anode potential, the dispersion relations obtained for $\omega > \omega_{pe}$ and $\omega < \omega_{pe}$ are in good agreement with the theoretical ones of the Landau mode and the free-streaming electron, respectively, as seen from Fig. 3, which indicates the distribution function of the electron to be Maxwellian. Fig. 3 shows that the observed damping rate of the free-streaming electron is found to be smaller than the expected value. The discrepancy between them can be attributed [*13*] to the contribution of the higher-order Landau mode [*14*].

In the case of the Maxwellian plus the monoenergetic distribution function, the new dispersion relation, $\omega \simeq kv_b$ (v_b: beam velocity), is also obtained. The possibility that this mode may be observed in our case is negative since the distribution function is isotropic as described in Sec. III.

As the anode potential is increased, the new wave mode with the dispersion relation of $\omega/k \simeq v_c$ for $\omega < \omega_{pe}$ becomes to be excited, which means that the excitation coefficient [9] of the new wave mode is much larger than that of the free-streaming electron. Since the excitation coefficient of the new wave mode depends strongly on the density ratio $(\omega_{pc}/\omega_{pe})^2$, which was observed to be below few %, there may be the critical density in which the excitation coefficient of the new wave mode is equal to that of the free-streaming electron.

Henry and Treguier [*15*] have observed that the phase velocity of the new wave mode is equal to v_c. In the present experiment, however, the phase velocity of the new wave mode is small by a factor 1/2 compared with the case of the Maxwellian plus the water-bag distribution function as shown in Fig. 5. So the actual distribution function of the electron may not be described by a simple distribution function.

In summary, electron waves were excited by a three-mesh exciter in a large-volume plasma. It was found that (i) when the distribution function of electrons was Maxwellian, the Landau mode for $\omega > \omega_{pe}$ and the free-streaming electron for

$\omega < \omega_{pe}$ were observed, respectively, and (ii) when the velocity distribution function was approximated by the Maxwellian plus the water-bag distribution function, the Landau mode for $\omega > \omega_{pe}$ and the new wave mode for $\omega < \omega_{pe}$ were observed, respectively. The phase velocity of Landau mode and the new wave mode increased with the anode potential, while the damping rate of them decreased with the anode potential. These experimental results agree qualitatively with the expected results based on the assumption that the distribution function is composed of the Maxwellian distribution function plus the water-bag one.

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