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Detection of Echo by Linear Prediction

By

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Summary: A new method of echo detection is proposed which is based on the principle that an echo appears as a delta function in the impulse response function of the optimum predictor for the signal containing the echo. The delta function showing the existence of the echo is scarcely overlooked because of its sharpness and easily detectable even if the band width of the time series is not wide enough and if the delay time of the echo is small. Experiments of sound propagation and flexural wave propagation show the effectiveness of this method. Comparison between this method and correlation method is also shown.

1. INTRODUCTION

In many different fields of research, there are many problems naturally arising as, or easily convertible into, questions of the existence and timing of echoes in time series. The use of correlation technique or Cepstrum analysis has proved powerful to solve these problems. If a time series is expressed by the sum of an original signal and its echo which is that delayed by a time difference σ seconds, the echo shows up as a peak at σ sec lag in the auto-correlation function. However, the correlation method is likely to fail to detect it if the spectrum of the original signal is complex enough to conceal the echo in the auto-correlation function. In such cases, Cepstrum analysis is very powerful [*1*]. A time series produced by adding an echo to the original series has a nearly cosinusoidal ripple in the power spectrum, whose "quefrency" is just equal to the time difference between the original signal and its echo. Cepstrum analysis is a technique suited for determining the quefrency. Nevertheless it is difficult to detect the echo under such conditions as the band width of the time series is not wide enough and the delay time is small. This paper presents a new method which is more useful even under these conditions. Applications to sound propagation and flexural wave propagation are also illustrated.

2. THEORY

In the simplest echo, values of a time series $y(t)$ are multiplied by a constant a, delayed by a time difference σ, and added to the original series to give a new series

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$$x(t)=y(t)+ay(t-\sigma) \tag{1}$$

The parameter a is the attenuation constant and its absolute value is supposed to be smaller than 1.0.

The intuitive understanding of the theory of the echo detection method is as follows. From Eq. (1), the echo in $x(t)$ is expressed by the infinite series $-\sum_{n=1}^{\infty}(-a)^n x(t-n\sigma)$. This expression holds for all t, and then the echo in the future value $x(t+\alpha)$ is given, if $0<\alpha<\sigma$, as a linear combination of the past of $x(t)$, which has already observed, as follows

$$ay(t+\alpha-\sigma)=-\sum_{n=1}^{\infty}(-a)^n x(t+\alpha-n\sigma) \tag{2}$$

This equation shows the existence of a realizable system whose output for the input $x(t)$ is $ay(t+\alpha-\sigma)$. The impulse response function of the system, denoted by $u(\tau)$, is given as

$$u(\tau)=-\sum_{n=1}^{\infty}(-a)^n \delta(\tau+\alpha-n\sigma) \tag{3}$$

where $\delta(\tau)$ is the Dirac delta function. As the echo in $x(t+\alpha)$ can be expressed using the past values of $x(t)$, we may be able to find a predicting filter for $x(t+\alpha)$ whose prediction is perfect with respect to the echo in it. The weighting function of such a predicting filter must have sharp pulses corresponding to the right hand side of Eq. (3). Their positions and heights will give us the information about the echo.

The optimum predicting filter in the sense of least mean-square error gives the perfect prediction for the echo. It is proved as follows. A time series $x(t)$ is assumed to be a sample function from real-valued wide-sense stationary random processes. The weighting function $h(\tau)$ of the realizable filter which gives a minimum mean-square error prediction for $x(t+\alpha)$ must satisfy the next integral equation [2].

$$\phi_x(\tau+\alpha)=\int_0^{\infty} h(\mu)\phi_x(\tau-\mu)d\mu, \quad \tau\geqq 0 \tag{4}$$

where α is a prediction time span, which should be selected so as to be smaller than σ, and $\phi_x(\tau)$ is the auto-correlation function of $x(t)$. From Eq. (4), the transfer function $H(j\omega)$ of the optimum predicting filter is given

$$H(j\omega)=\frac{1}{2\pi G(j\omega)}\int_0^{\infty} e^{-j\omega\tau}d\tau \int_{-\infty}^{\infty} e^{j\omega'(\tau+\alpha)}G(j\omega')d\omega' \tag{5}$$

where $G(j\omega)=G(p)$ is a function with all its poles and zeros in the left half p plane and also satisfies

$$G(j\omega)\cdot G^*(j\omega)=\Phi_x(j\omega) \tag{6}$$

where $G^*(j\omega)$ is the complex conjugate of $G(j\omega)$ and $\Phi_x(j\omega)$ is the power spectral

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density of $x(t)$. From Eq. (1),

$$\Phi_x(j\omega)=\Phi_y(j\omega)(1+ae^{-j\omega\sigma})(1+ae^{j\omega\sigma}) \tag{7}$$

where $\Phi_y(j\omega)$ is the power spectral density of $y(t)$. Let $K(j\omega)=K(p)$ be a function with all its poles and zeros in the left half p plane and satisfy

$$K(j\omega)K^*(j\omega)=\Phi_y(j\omega) \tag{8}$$

Then $G(j\omega)$ is given as

$$G(j\omega)=K(j\omega)(1+ae^{-j\omega\sigma}) \tag{9}$$

Substituting $G(j\omega)$ as given by Eq. (9) into Eq. (5) yields

$$H(j\omega)=R(j\omega)[P_0(j\omega)+ae^{-j\omega(\sigma-\alpha)}] \tag{10}$$

where

$$R(j\omega)=(1+ae^{-j\omega\sigma})^{-1} \tag{11}$$

and

$$P_0(j\omega)=\frac{1}{2\pi K(j\omega)}\int_0^\infty e^{-j\omega\tau}d\tau\int_{-\infty}^\infty K(j\omega')e^{j\omega'(\tau+\alpha)}d\omega' \tag{12}$$

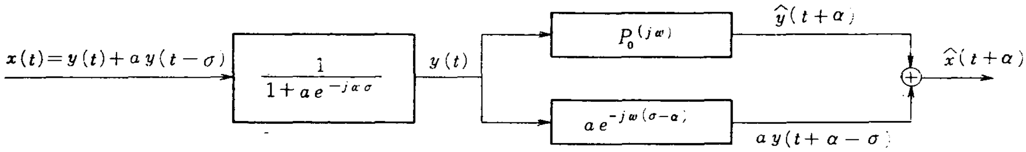

FIG. 1. The transfer characteristics of the optimum predictor.

The putout of the system with transfer characteristics $R(j\omega)$ is $y(t)$ if the input is the sum of $y(t)$ and $ay(t-\sigma)$. $P_0(j\omega)$ is the transfer function of the optimum predicting filter for the original signal, which gives the optimum prediction $\hat{y}(t+\alpha)$ for $y(t+\alpha)$ if the input is $y(t)$. The transfer characteristics $ae^{-j\omega(\sigma-\alpha)}$ is that of a pure gain element with the time delay of $(\sigma-\alpha)$ seconds. Then, the output of the optimum predicting filter $H(j\omega)$ for the input $x(t)$ is, as shown in Fig. 1, the sum of the prediction $\hat{y}(t+\alpha)$ for $y(t+\alpha)$ and the perfect prediction for the echo $ay(t+\alpha-\sigma)$.

The impulse response function $h(\tau)$ of the predicting filter is

$$\begin{aligned} h(\tau)&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty H(j\omega)e^{j\omega\tau}d\omega \\ &=\sum_{n=0}^\infty(-a)^n\delta(\tau-n\sigma)*\{p_0(\tau)+a\delta(\tau+\alpha-\sigma)\} \\ &=-\sum_{n=1}^\infty(-a)^n\delta(\tau+\alpha-n\sigma)+\sum_{n=0}^\infty(-a)^np_0(\tau-n\sigma) \end{aligned} \tag{13}$$

where

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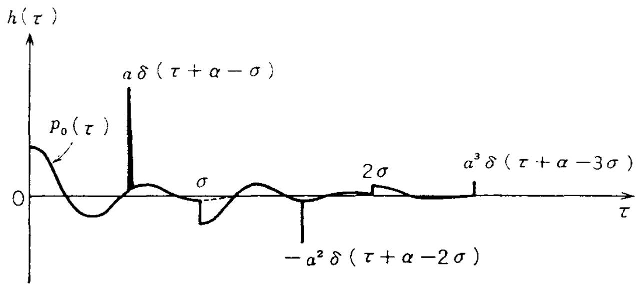

FIG. 2. The impulse response function of the optimum predictor.

$$p_0(\tau)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}P_0(j\omega)e^{j\omega\tau}d\omega \tag{14}$$

and the symbol $*$ means convolution integral. Then the impulse response $h(\tau)$ can be illustrated as shown in Fig. 2, from which it is clear that

$$h(\tau)=a\delta(\tau+\alpha-\sigma)+p_0(\tau), \qquad 0\leq\tau<\sigma \tag{15}$$

This delta function is the biggest in the impulse response function and then it is easy to detect it. It is this delta function that gives us the information about the echo. Namely, its position and height correspond to the timing $(\sigma-\alpha)$ seconds and the coefficient a respectively. To obtain such information, we must detect the delta function under the disturbance of $p_0(\tau)$. The advantage of this prediction method is attributable to the sharpness of the delta function. If the original signal is a white noise, which is unpredictable, $p_0(\tau)$ vanishes and the delta function $a\delta(\tau+\alpha-\sigma)$ is easily detectable. If $y(t)$ is a coloured noise, $p_0(\tau)$ does not vanish, but in many cases of physical phenomena it seems to be a smooth function and the delta function is too sharp to be covered up by $p_0(\tau)$ and also easily detectable.

If the echo travels by a different path from the direct signal, there is no reason for the relative intensity transmitted or the relative time delay to be independent of frequency. If the intensity and the time delay of the echo depend on frequency, the sum of the original series and its distorted echo can be represented by the Fourier transform of

$$K(j\omega)\{1+D(j\omega)e^{-j\omega\rho}\} \tag{16}$$

where $D(j\omega)e^{-j\omega\rho}$ shows the distortion of the echo and the magnitude of the linear phase lag ρ is selected so that the phase of $D(j\omega)$ does not become positive. All poles and zeros of the function (16) are in the left half p plane if

$$|D(p)|<1 \qquad \text{where} \qquad Re(p)>0 \tag{17}$$

Then the transfer function of the optimum predicting filter is given as

$$H(j\omega)=\frac{1}{1+D(j\omega)e^{-j\omega\rho}}[P_0(j\omega)+D(j\omega)e^{-j\omega(\rho-\alpha)}], \quad \alpha<\rho \tag{18}$$

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and the impulse response function $h(\tau)$ is

$$h(\tau)=p_0(\tau)+d(\tau+\alpha-\rho)+\text{higher order terms} \tag{19}$$

where $d(\tau)$ is the inverse Fourier transform of $D(j\omega)$. If the frequency dependence of the echo is not strong, the function $d(\tau)$ is a pulse-like function and no difficulty will arise in detecting the echo. In this case, we can know the mean delay time of the distorted echo, which we denote by $\bar{\sigma}$. The function $d(\tau+\alpha-\rho)$ reaches its maximum at $(\bar{\sigma}-\alpha)$ sec lag.

3. Experiments

3.1 Calculation of the impulse response $h(\tau)$

In this chapter, considerations are restricted to systems with discrete time for the convenience of computer application. The impulse response of a predictor $q(\tau)$ is approximated by a finite set of values

$$q(0), q(1), q(2), \cdots, q(N-1).$$

where $q(i)$ is the ith sampled impulse response of the predictor, N is so chosen that $N\Delta t$ covers the significant duration of the impulse response and Δt is the sampling period. Let the input sequence be

$$x(1), x(2), x(3), \cdots$$

and then the output of the predictor $\bar{x}(n)$ at the nth sampling instant is given by

$$\bar{x}(n)=\sum_{k=0}^{N-1} q(k)x(n-k-A) \tag{20}$$

where

$$A=\frac{\alpha}{\Delta t} \tag{21}$$

The calculation method is called "learning method for system identification," which is based on the error-correcting training procedure in learning machines [3]. Here we denote the ith sample of the impulse response function of the optimum predicting filter by $h(i)$. The adjustment procedure for $q(i)$ to converge into $h(i)$ is as follows. The correction $\Delta q(i)$ for $q(i)$ at the nth step is proportional to the magnitudes of both the prediction error and the component of the input corresponding to $q(i)$, that is,

$$\Delta q(i)=\beta\cdot\{x(n)-\bar{x}(n)\}\cdot\frac{x(n-i-A)}{\sum_{k=0}^{N-1} x^2(n-k-A)} \tag{22}$$

where β is an error-correcting coefficient and $0<\beta<2$. Then $\Delta q(i)$ is added to $q(i)$. By this training procedure,

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$$q(i) \xrightarrow{n\to\infty} h(i), \qquad (i=0, 1, 2, \cdots, N-1) \tag{23}$$

The proof of Eq. (23) is trivial.

The advantage of the use of this method is that it can be used to identify the optimum predictor for the signal in which the parameters of the echo, a and σ, vary slowly in comparison with the time required for identification.

3.2 Simulation

An analog noise generator and a delay element whose delay time was 35 msec were used. A noise generated by the generator was considered as a direct signal from a signal source. Its echo was obtained by supplying it to the delay element. By summing up these two signals with an analog adder, a time series containing an echo was obtained. It was supplied to A-D converter and stored in core memory of a mini-computer. Then the impulse response function of the optimum predicting filter was calculated.

Two kinds of signals, a wide band and a narrow band signals, were used as test signals. The results are shown in Figures 3 and 4. In each figure, the upper shows the correlation function and the lower the impulse response function of the predictor.

Fig. 3 shows one example of the case where the signal band width is wide enough. The direct signal is hardly predictable and it means that $p_0(\tau)$ is almost negligible. Then the impulse response function is composed of almost only delta functions. The pulse observed at 30 msec lag indicates the existence of the

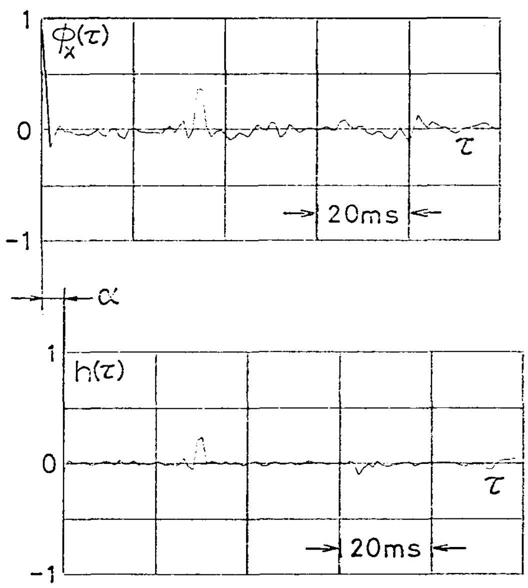

FIG. 3. Simulation results. The upper and the lower figures are the auto-correlation function and the impulse response function of the optimum predictor respectively. (frequency range of the signal: 0～355 Hz, Δt: 1 msec, α: 5 msec)

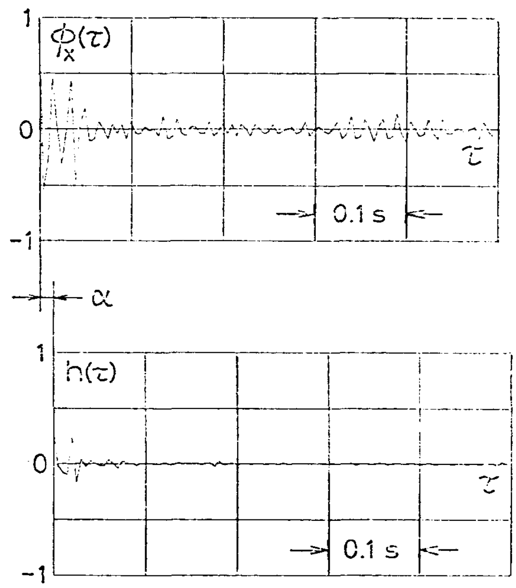

FIG. 4. Simulation results. The signal band width is narrower than that of Fig. 3 and the echo cannot be detected by correlation method. (frequency range of the signal: 56～90 Hz, Δt: 5 msec, α: 15 msec)

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echo. The negative pulse at 65 msec lag corresponds in the right hand side of Eq. (3) to the second order delta function $-a^2\delta(\tau+\alpha-2\sigma)$.

Fig. 4 shows one example of the case where the signal band width is narrow. The echo cannot be detected in the auto-correlation function. Nevertheless, the impulse response function shows its existence clearly at 35 msec lag. In this figure, the second order pulse cannot be observed. It is obtained empilically that

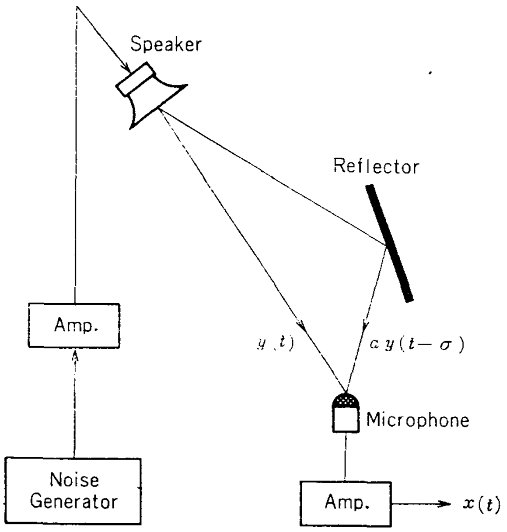
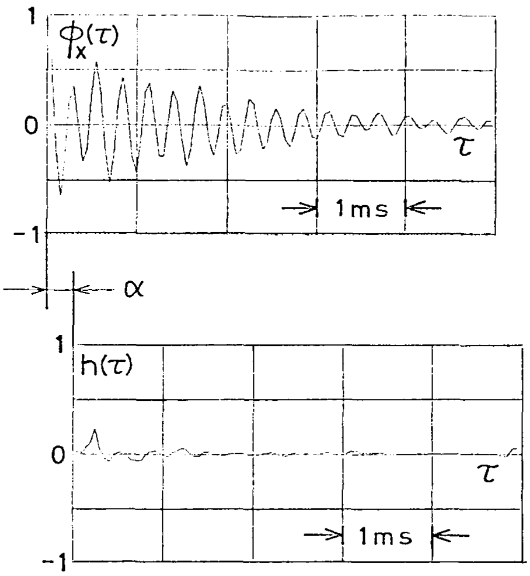
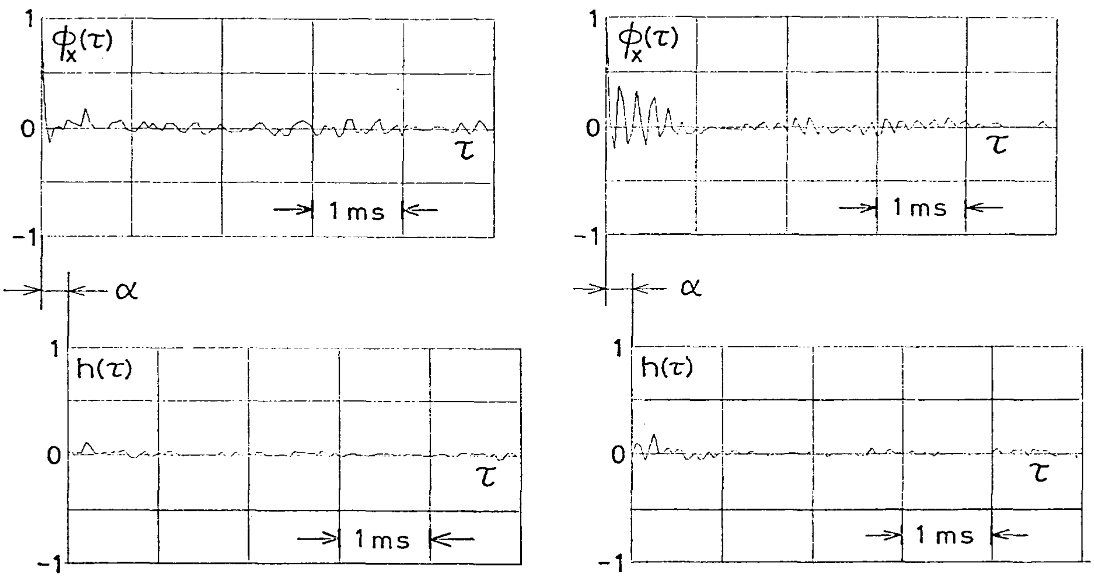


FIG. 5. The experimental setup of sound propagation.



(b). Frequency range: 0～2,200 Hz.



(a). Frequency range: 0～10,000 Hz. (c). Frequency range: 7,100～9,000 Hz.

FIG. 6. Experimental results of sound propagation. The band width of the sound of Fig. (a) is wide enough and the echo is observed in both the auto-correlation function and the impulse response function. The proposed prediction method is, as shown in Figures (b) and (c), more effective than correlation method if the band width of the sound is narrow. (Δt: 50 μsec, α: 0.3 msec)

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the second order pulse and higher order pulses cannot be observed generally in the impulse response function if the signal band width is not wide.

3.3 Sound Propagation in an Anechoic Room

A speaker, a microphone and a reflecting board were placed in an anechoic room as shown in Fig. 5. The output signal of the microphone contained not only the direct sound but also the reflected sound. Its delay time calculated from geometrical arrangement was 0.5 msec.

Three kinds of signals were used. The results are shown in Fig. 6. The echo, which should appear as a peak at 0.5 msec lag in the auto-correlation function, can be observed only in the correlation function of Fig. 6(a). In the others, no typical peaks can be observed. It is because (1) the echo is energyless, (2) the band width of the sound is narrow and (3) the delay time is small. However each impulse response function shows the existence of the echo clearly in spite of these conditions.

3.4 Flexural Wave Propagation in a Steel Strip [*4*]

The propagation of the flexural wave in solid body, which is an example of dispersive wave, was measured. The measuring system is shown in Fig. 7. A long thin steel strip whose cross section was 0.5 × 38 mm was suspended by cotton string horizontally. A small metal bar (about the size of pencil) was used to strike the strip at the end to excite flexural wave. It was detected by a light weight (about 1 g) accelerometer attached at 60 cm from the other end. The signal obtained by the accelerometer was the mixture of the direct wave and

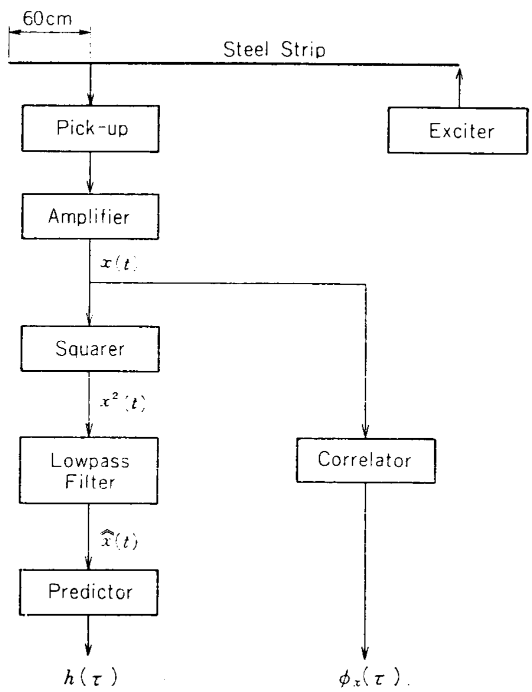

FIG. 7. The experimental setup of flexural wave propagation.

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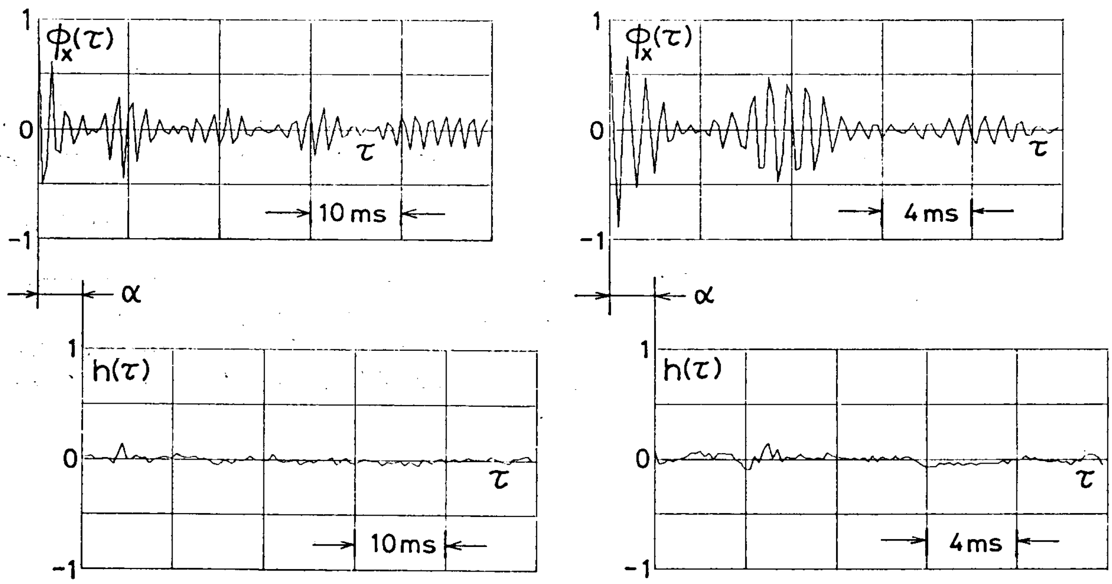
its reflection.

The flexural wave is dispersive and its group velocity depends on frequency. Then to measure the group delay time of the reflected wave, it is necessary to know the lag time of the peak of correlation envelope. However it does not necessarily agree with the group delay time precisely because it is disturbed by other terms than the cross-term between the direct signal and its reflection. On the other hand, it is known that the envelope of the flexural wave propagates with the group velocity. Then the group delay time can be measured by linear prediction of the envelope of the flexural wave. It is denoted by $\hat{\hat{x}}(t)$ in Fig. 7 and obtained by averaging the squared signal $x^2(t)$ with first order low-pass filter. $\hat{\hat{x}}(t)$ can be written as

$$\hat{\hat{x}}(t)=\hat{\hat{y}}(t)+a\hat{\hat{y}}(t-\sigma)+z(t) \tag{24}$$

where $\hat{\hat{y}}(t)$ is the envelope of the direct wave and $z(t)$ is the cross-term between the direct and the reflected waves. It is considered as a noise which disturbes the relation that the observed signal is the sum of the direct signal and its echo.

Two examples are shown in Figures 8(a) and (b). In each figure, the peak of correlation envelope is not easy to determine from the auto-correlation function of the flexural wave itself. Nevertheless the impulse response function shows the group delay time very sharply. The frequency dependence of the flexural wave velocity is observed by comparing these two examples.



(a) Frequency range: 560~900 Hz. (Δt: 500 μsec, α: 5 msec) (b) Frequency range: 1,120~1,800 Hz. (Δt: 200 μsec, α: 2 msec)

FIG. 8. Experimental results of flexural wave propagation.

4. CONCLUSION

A new method of echo detection was proposed. The method is based on the

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principle that an echo appears as a delta function in the impulse response function of the optimum predictor for the signal containing the echo. It has an enormous advantage in that the delta function can be scarcely overlooked because of its sharpness. Experiments of sound propagation show its effectiveness even under such conditions as:

(1) The signal band width is narrow.
(2) The delay time of the echo is small.
(3) The echo is not intense.

It is also shown that this method is effective for dispersive wave propagation. In many cases, velocity measurement problems are convertible into those of the delay time measurement and then this method may be effective in measuring the velocities of signal propagations using only one signal detector.

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