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# Fundamental Formulation of Airship Performance and Flight Dynamics

By

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*Summary:* The fundamental analyses on the airship performance and flight dynamics and their formulation are presented. From the performance calculations of four exemplified airships, it can be concluded that only large airships the gross weight of which is more than about 500 ton have good vehicle efficiency comparing with other vehicles such as liner boat, hydrofoil ship, motor truck, rail way vehicle in the operational speed range of 50 to 200 km/hr. The handling qualities of the airships are, however, very poor specifically for larger sizes because of small control powers against huge virtual mass and moment of inertia of the hull. Thus, it seems to be very difficult to maneuver the large airships at and near the hovering flight.

## 1. INTRODUCTION

It is expected in modern traffic and transportation systems that a large number of human beings and necessary materials must be moved quickly, regularly, comfortably, safely and cheaply, without giving unfavourable affairs such as noise, pollution, and any other hazardous injury to themselves and others. Each transportation system should be designed to operate optimally in a given environmental condition by satifying some or all of the above requirements.

Table 1 shows an evaluation of expected transportation characteristics of various types of vehicles used in the current and future transportation system. The table seems to be a possibility or chance for revival of airship by comparing with other transportation systems in each item:

( i ) Since the airship is easy to extend to big size because of distributed loading and exhibits economical superiority in large size as will be described later, the airship can be a mass transportation vehicle being enquivalent to ship.

( ii ) Since the airship is uneconomical in high speed as will be precisely described later the speed is within similar values as those of ground vehicles which are yet faster than the ships.

( iii ) The scheduled flight is possible but strongly affected by weather specifically for long distance flight.

( iv ) The riding quality is the most excellent characteristic of the airship. Since the installed power can be reduced and there is no positive lifting

TABLE 1 Evaluation of Expected characteristics

| Items | Ships | | Ground vehicles | |
|---|---|---|---|---|
| | On surface | Submersible | On surface | Under ground |
| Large transport capacity | ○ As seen in huge tanker and freight boat. | × Impossible until completion of huge submersible. | ○ Individual car does not have big capacity but a composition of train in dense cycles does. | △ Limited by construction of subway. |
| Swiftness | × In addition to low speed, a lot of time for loading and unloading. | × Impossible until completion of high speed submersible. | △ Better for short haul due to near door-to door operation. | △ Only for short haul. |
| Regularity | △ Except bad weather. | ○ Unaffected by weather. | ○ Less affected by weather. | ○ Unaffected by weather. |
| Riding quality | △ Having enough space but strongly affected by waves in rough sea. | ○ Calm. | × Very bad for wheeled vehicles but hopefull for air or magnetic suspended vehicles. | × Similar to the left column. |
| Safety | △ Weak for bad weather. | △ Need collision avoidance system. | ○ In preparation of cubic crossing and collision avoidance system. | △ Might be suffered from disaster such as flood. |
| Noise and Pollution | ○ Should be careful for oil and waste disposal. | ○ Must be carefull for radio active waste disposal. | × Impossible to eliminate severe vibration and noise. | × Similar to the left column. |
| Price | ○ The cheapest one for weight and volume. | △ Expensive for ensuring safety. | △ Expensive for securing traffic line. | × Very expensive for constructing and securing traffic line. |
| Notes | Expected to development of hydrofoil boats and ground effect machines. | Waited for development of big and high speed submersible. | Waited for development of wheelless vehicles such as air or magnetic suspended vehicles. | |

of current and future transportation systems ○ Fair △ Marginal × Bad

| Aircraft | | | |
|---|---|---|---|
| In air | | | In space |
| Airplain | Helicopter | Airship | Space Vehicle |
| △ Jambo jet planes have capacity but limited in operation (area and cycles) by noise and pollution. | × Difficult to make large helicopter. | ○ Easy to extend to big size. | × Impossible by limited size and frequency. |
| △ Better for long distance. | △ Better for short distance. | △ Uneconomical in high speed. | △ Only for global operation. |
| △ Affected by weather. | △ Similar to the left column. | × Strongly affected by weather. | × Strongly affected by weather and maintenance. |
| △ Good only for short duration. | × Very bad due to noise and vibration. | ○ The most excellent one. | ○ Possibly good but narrow space. |
| △ Required to have all weather capacity and collision avoidance system. | △ Similar to the left column. | ○ Must be equipped with collision avoidance system and effective navigation system. | × Currently dangerous. |
| △ Required to develop the noiseless and pollutionless aircraft. | △ Similar to the left column. | ○ Noiseless and pollutionless due to small power. | ○ Outside of noise and pollution. |
| × Too much expensive for ensuring safety | × Similar to the left column. | △ Construction price of hull is similar to the airplane. | × The most expensive one. |
| Waited for development of collision avoidance system. | Waited for large helicopter. | Required redevelopment and redsign by advanced technology | Hopless in journey on earth. |

surface, the noise and the vibration due to gust can be very small comparing with those of airplane and helicopter. The space for comfortable journey is fully available due to the enough size of the airship.

( v ) Although the flight hour is longer than the jet airplane and the flight course is strongly influenced by the weather the fatal disaster can not be considered if the airship has a collision avoidance system and an effective navigation system which can take necessary local meteorological data in its computer unit in real time.

( vi ) Small power has less problems on noise and pollution.

(vii) The construction price of the airship hull must be similar to the airplane but the total operating cost can be reduced because the slower speed will reduce the price of instrumentations and facilities for safety.

In the past the hydrogen gas was mainly used for creating the static lifting force of airships. After meeting with several accidents of hydrogen filled airship and a tragic end of the Hindenburg at Lakehurst, N. J., U. S. A., on May 6, 1937, while completing its 37th commercial ocean crossing, the airships were forsaken and replaced with fixed wing and rotary wing aircraft in the transportation system.

Recently, new technologies made the helium available in low cost and promoted to develop helium filled airships as a new air transportation system although the helium has about 7 percent less lifting power than the hydrogen. Advanced methods and techniques developed in space and aeronautical engineerings will be helpful to introduce the following outcomes in new design of modern airships:

( i ) High strength materials with fatigue and corrosion resistance characteristics.

( ii ) Computer-aided-design technology.

( iii ) Precise calculations in the aerodynamic, structural and flight dynamic analyses.

( iv ) Low noise and low polluted power plants.

( v ) Ingenious automatic control system.

( vi ) New guidance and navigation system accompanied with forcasting and utilizing the weather condition. By installing the inertial navigation system the aireship can detect the local wind velocity along his flight course. Collecting the current information of weather from individual airship operating in many places will be helpful to make a weather map over the globe.

(vii) New and wide communication system.

(viii) Safety and survaivability technique.

( ix ) Systematized and computerized quality control and maintainance technique.

In this paper the author wants to describe the analytical expressions of the performance of conventional and hybrid airships and to investigate the possibility of the said expectation imposed on new airship developments.

## 2. Parametric Study of Performance

*Buoyant Force and Static Ceiling*

Since an airship has gas containers which are very flexible and are filled with either hydrogen or helium under the same or a little larger pressure than that of the ambient atmosphere, it can be seen that the buoyant force or "gross lift" of the airship flying in air is given by the difference of specific weight between the surrounding air and the contained gas times gas volume, $V_G$, and is directed vertically upward,

$$F_B=(\rho_a-\rho_g)gV_G=\rho_{a_0}gk_BV_G \tag{1}$$

where $k_B$ is "buoyant coefficient" given by

$$k_B=(\rho_a-\rho_g)/\rho_{a_0}=\sigma_a-\sigma_g=\sigma_a(1-s) \tag{2}$$

and where

$$\left.\begin{aligned}&s=\sigma_g/\sigma_a\\&\sigma_a=(\rho_a/\rho_{a_0})=(T_i/T_a)(p_a/p_0)\{1-0.379\phi(p_{T_c}/p_a)\}\\&\sigma_g=(\rho_g/\rho_{g_0})(\rho_{g_0}/\rho_{a_0})=(T_i/T_a)(p_g/p_0)(\rho_{g_0}/\rho_{a_0})(T_a/T_g).\end{aligned}\right\} \tag{3}$$

Other parameters are given by

| | |
|---|---|
| $\rho_{a_0}g=1.293\,\mathrm{kg/m^3}$: | specific weight of the dry air at 0°C and $p_a=760\,\mathrm{mmHg}$, |
| $1.225\,\mathrm{kg/m^3}$: | specific weight of the dry air at 15°C and $p_a=760\,\mathrm{mmHg}$. |
| $T_a=t_a+237.15$: | absolute temperature of the air at $t_a$°C and $p_a$ |
| $T_g=t_g+273.15$: | absolute temperature of the gas at $t_g$°C and $p_g$ |
| $T_i=273.15$°K: | absolute temperature at 0°C |
| $s_0=\rho_{g_0}/\rho_{a_0}$: | density ratio of gas with respect to the dry air at 0°C and $p_a=760\,\mathrm{mmHg}$ |
| $p_{T_c}$: | steam pressure of the air at $t_a$°C and $p_a$. |
| $\phi$: | relative humidity of the air. |

If the temperature and the pressure of the gas are equal to those of ambient atmosphere, then the density ratio of gas and air, $s=\sigma_g/\sigma_a$, becomes

$$s=\rho_g/\rho_a=(\rho_g/\rho_{g_0})(\rho_{g_0}/\rho_{a_0})/(\rho_a/\rho_{a_0})=s_0/\{1-0.379\phi(p_{T_c}/p_a)\}. \tag{4}$$

For helium and hydrogen with respect to the dry air this becomes[1]

$$\left.\begin{aligned}s=\rho_{g_0}/p_{a_0}=s_0&=0.1368 \text{ for helium}\\&=0.06965 \text{ for hydrogen}\end{aligned}\right\} \tag{5}$$

and the the buoyant coefficient is given by

$$\left.\begin{aligned}k_B=\sigma_a(1-s)&=0.8632\sigma_a \text{ for helium}\\&=0.93035\sigma_a \text{ for hydrogen}\end{aligned}\right\} \tag{6}$$

Usually the airship will leave the ground with very flexible gas bags pertly filled. Then the pressure of the gas is kept equal to the ambient pressure in ascending until the airship reaches to a "pressure height" at which the gas bags are completely distended, or $V_G = V_{G\max}$. As the airship continues to climb beyond the pressure height, if the gas is considered to be exhausted to keep the condition of null pressure difference between the gas inside the bag and the ambient air, then equations (4) through (6) can also be applied to such conditions.

However, if the gas is not exhausted but either titely sealed in the bags such that the pressure difference will rise-up with altitude after the gas bags have been inflated tautly or condensed to a liquid instead of exhausting in order to keep the null pressure difference, then the density ratio may be considered as

$$\left.\begin{aligned} s &= \rho_g/\rho_a = (\rho_{g_0}/\rho_a)(V_{G_0}/V_{G\max})(V_{G\max}/V_G) = s_0(k_0/k)/\sigma_a \\ &= \begin{cases} 0.1368(k_0/k)/\sigma_a & \text{for helium with respect to dry air} \\ 0.06965(k_0/k)/\sigma_a & \text{for hydrogen with respect to dry air} \end{cases} \end{aligned}\right\} \quad (7)$$

and

$$\left.\begin{aligned} k_B &= \sigma_a(1-s) \\ &= \begin{cases} \sigma_a - 0.1368(k_0/k) & \text{for helium with respect to dry air} \\ \sigma_a - 0.06965(k_0/k) & \text{for hydrogen with respect to dry air} \end{cases} \end{aligned}\right\} \quad (8)$$

where $k_0$ and $k$ show volume ratios of the gas bag,

$$k_0 = V_{G_0}/V_{G\max} \quad \text{and} \quad k = V_G/V_{G\max}. \quad (9)$$

Under the pressure altitude, since the relation of $(k_0/k)/\sigma_a = 1$ is established from the law of mass conservation and the equal pressure and temperature condition, equations (5) and (6) coincide with equation (7) and (8) respectively. Beyond the pressure altitude, however, $k$ becomes 1 and therefore both the density ratio and the buoyant coefficient are different from those given by equations (5) and (6). Hereafter, the maximum gas volume, $V_{G\max}$, will be written by $V_G$ for simplicity.

For standard atmosphere at sea level a "specific lift", $\rho_{a_0} g k_B$, is $1.058\,\mathrm{kg/m^3} = 1.058\,\mathrm{ton}/1,000\,\mathrm{m^3}$ or $66\,l\mathrm{b}/1,000\,\mathrm{ft^3}$ for helium and $1.14\,\mathrm{ton}/1,000\,\mathrm{m^3}$ or $71\,l\mathrm{b}/1,000\,\mathrm{ft^3}$ for hydrogen. This means that pure helium has 7 percent less lifting power than the pure hydrogen. In practice neither gas is used at 100 percent purity so that a practical figure is 12 percent lower specific lift for helium, i.e., $62\,l\mathrm{b}$ as against $70\,l\mathrm{b}$ lift per $1.000\,\mathrm{ft^3}$ or $0.99$ ton as against $1.12$ ton lift per $1,000\,\mathrm{m^3}$* (See Fig. 1). The above difference will bring about 70 to 80 percent reduction in the cruising range of the helium filled airship comparing to the hydrogen filled airship. Considered in another

* $1\,l\mathrm{b/ft^3} = 16.018\,\mathrm{kg/m^3}$
$1\,l\mathrm{b}/1,000\,\mathrm{ft^3} = 0.016018\,\mathrm{kg/m^3} = 16.018\,\mathrm{kg}/1,000\,\mathrm{m^3}$

way, if it is desired to have two airships to carry the same payload and have the same cruising range, the helium filled airship would need to be some 15 to 25 percent larger than the hydrogen filled airship.[2)]

Fig. 2 shows the density ratio of dry air and the bouyant coefficients of helium and hydrogen against the air as a function of the altitude. It will be seen that the difference of the buoyant coefficients between unsealed and sealed gas bags perfectly inflated at the ground, $k_0=k=1.0$, or the difference between equations (6) and (8) is appreciable in higher altitude.

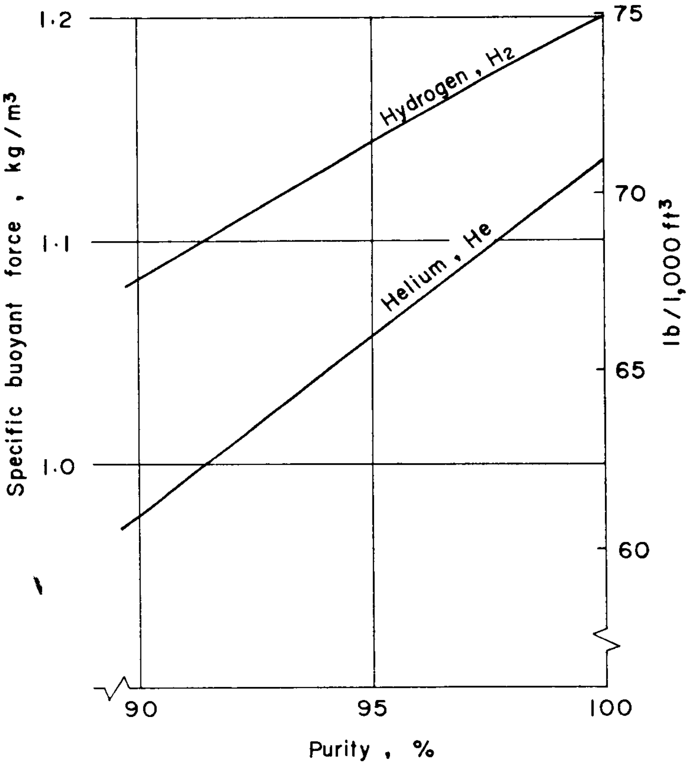



FIG. 1. Specific buoyant force for hydrogen and helium.

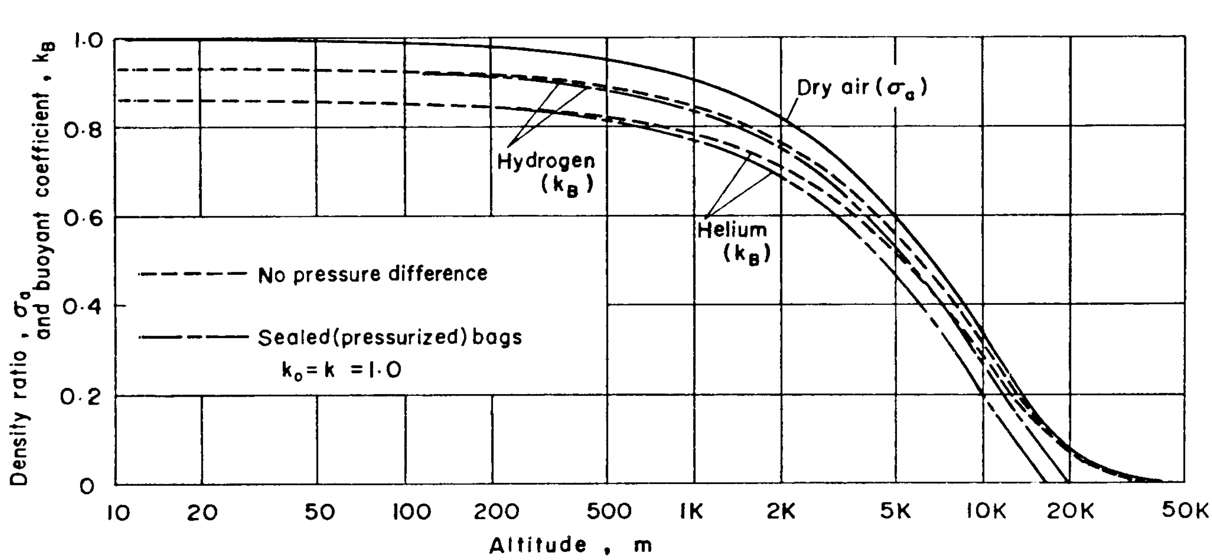



FIG. 2. Density ratio of dry air and buoyant coefficients for helium and hydrogen.

The moment created by the buoyant force is given as a vector product of the bouyant force and the position vector of center of gas volume with respect to the center of gravity. For a symmetric body with respect to the vertical plane the position vector of the center of gas volume is given by a height from the center of gravity so that no moment is created in the steady flight other than when the body has rolling angle.

As stated before, when an airship leaves a mooring tower or takeoffs from the ground by only buoyant force with partially filled gas the volume of which is given by a fill-up rate of $kV_G(k\leq 1.0)$ it will continue to ascend untile it reaches to a "density height" at which the average density of the airship is equal to that of the surrounding air. If theairship is lower than the pressure altitude and therefore the gas is not exhausted the specific weight of the airship, $\rho_b g$, can be given by

$$\left.\begin{aligned}\rho_b g &\equiv W/kV_G=(W/k_0V_G)(k_0/k)=(W/k_0V_G)(\rho_g/\rho_{g_0})\\ &=(W/k_0V_G)(p_g/p_{g_0})(T_{g_0}/T_g)=(W/k_0V_G)(p_a/p_{g_0})(T_{g_0}/T_g)\\ &=(W/k_0V_G)(T_a/T_g)\sigma_a\end{aligned}\right\}\tag{10a}$$

where subscript 0 means the values at the ground and where an assumption that the temperature of the gas and air at the ground was same has been introduced.

If, further, the temperature of the gas is equal to that of the ambient atmosphere at any height, then the above equation can be given by

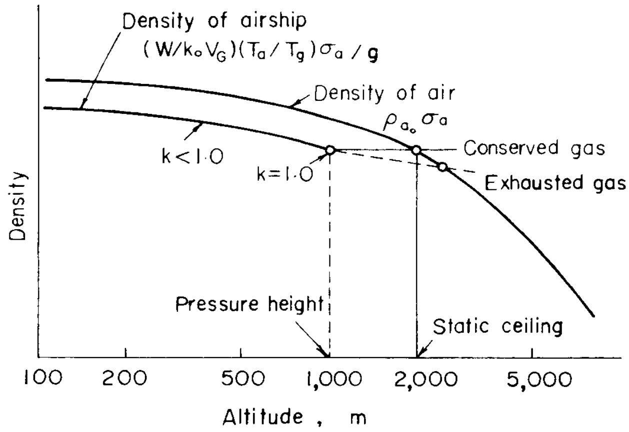
$$\rho_b g\equiv W/kV_G=(W/k_0V_G)(p_a/p_{a_0})(T_{a_0}/T_a)=(W/k_0V_g)\sigma_a. \tag{10b}$$

It is obvious that the parameter, $k$, is inversely proportional to the density ratio of air, $\sigma_a$, and therefore increases toward one as the altitude of the airship increases.

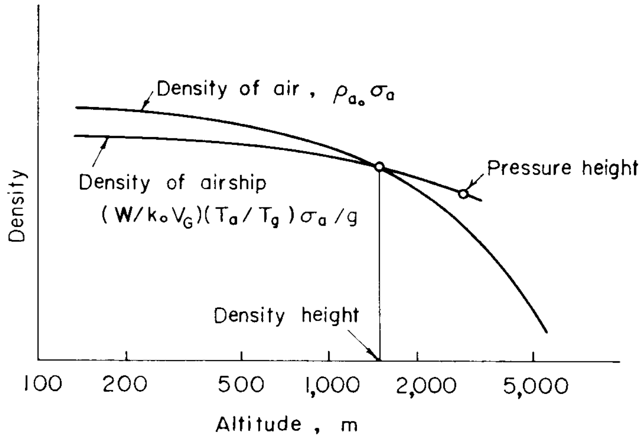
It will, thus, be apparent that when (10b) is valid the specific weight or density of the airship in ascending will never equal to or become greater than that of the surrounding air until altitude of the airship reaches to the pressure height at which $k=1.0$ as shown in Fig. 3a. After passing the pressure height, the gas volume is constant at the maximum value, $V_g$, and the density of the airship will be kept constant for conserved gas system and be reduced slightly for exausted gas system. Thus the density of the airship will cross the density of the air and reaches to an equilibrium at the height of crossing as shown in Fig. 3(a). Then the altitude is called "static ceiling." That is to say, the static ceiling of an airship in any given condition of loading is the altitude at which the gas is 100 percent full with the ship in static equilibrium.

In the nonrigid airship the gas volume of $k_0V_G$ and the ballonet volume $(1-k_0)V_G$ at the ground will respectively change to $V_G$ and 0 at the static ceiling.

When the airship is cruising in strong sunshine the temperature of the gas will be higher than that of the ambient atmosphere and therefor equation




(a) Flight at static ceiling
($T_a \leqq T_g$)




(b) Flight at density height
($T_a > T_g$)

FIG. 3. Pressure height and density height.

(10a) must be valid. The density of the airship is, then, smaller than that given by equation (10b). In this case, the situation of ascending airship is very similar to that of the preceding one, except that much higher static ceiling can be obtained than that of the equal temperature.

On the other hand, when the airship penetrates into a warm atmosphere and density of the airship will relatively rise up, it rarely happens that the pressure height is heigher than the density height as shown in Fig. 3 (b) and therefore the airship begins to descend. Then the airship will be in equilibrium at the density altitude with gas bags in partially inflated condition. This equilibrium state is, however, temporary phenomenon and finally settles to the true equilibrium at the static ceiling after the temperature equilization has been finished.

When an airship is equilibrium at the static ceiling where $k=1.0$, and $p_a=p_g$ the bouyant force at that altitude is equal to the weight of the airship, i.e. from equation (1).

$$F_B=\rho_{a_0}g V_G(T_i/T_a)(p_a/p_0)\{1-0.379\phi(p_{T_c}/p_a)-s_0(T_a/T_g)\}=W. \quad (11)$$

On the other hand, for the standard atmosphere in troposphere the following relation between pressure and altitude can be obtained.[3)]

$$p_a/p_0=\{1-(aH/T_0)\}^n \tag{12}$$

where

$$n=5.2561 \quad \text{and} \quad a=0.0065^0K/m.$$

By substituting this into equation (11), the static ceiling can be obtained as follows:

$$H=(T_0/a)[1-\{W/\rho_{a_0}g V_G(T_i/T_a)(1-0.379\phi(p_{T_c}/p_a)-s_0(T_a/T_g))\}^{1/n}]. \tag{13}$$

If the gas is partially filled ($k<1.0$) just as the weight of airship is almost equal to the buoyant force at the ground or

$$W/\rho_{a_0}gk_0V_G\{1-0.379\phi(p_{T_c}/p_a)-s_0\}=1 \qquad \text{at } H=0, \tag{14}$$

then the static ceilin g($k=1$) for dry air can be obtained as

$$H=(T_0/a)[1-\{k_0(1-s_0)(T_a/T_i)/(1-s_0(T_a/T_g))\}^{1/n}]. \tag{15}$$

In a temperature equilibtium condition this becomes

$$H=(T_0/a)[1-\{k_0(T_a/T_i)\}^{1/n}]. \tag{16}$$

The height change results the temperature change of the standard atmosphere in troposphere as follows[3)]

$$\left.\begin{aligned} T_a/T_i&=T_0/T_i-(a/T_i)H \\ &=(T_0/T_i)\{1-(a/T_0)H\}\cong 1.055(1-2.26\times 10^{-5}H). \end{aligned}\right\} \tag{17}$$

Thus equation (16) yields the following simple expressien of the static ceiling:

$$\left.\begin{aligned} H&=(T_0/a)[1-\{k_0/T_i)\}^{1/n}]/[1-(1/n)\{k_0(T_0/T_i)\}^{1/n}] \\ &\cong 44,300\{(1-k_0^{1/n})/(1-0.2k_0^{1/n})\}. \quad \text{for } H<4,000\,\text{m} \end{aligned}\right\} \tag{18}$$

If the gas is in superheated condition the increment of the buoyant force and the increment of the static ceiling for dry air are respectively given by the following equations derived from equations (10a), (11) and (15):

$$\left.\begin{aligned} &\Delta F_b/F_b=\Delta\rho_b/\rho_b=-(T_a/T_g)(\Delta T_g/T_g)\cong-\Delta T_g/T_g \\ &\qquad \text{lower than the prssure altitude.} \end{aligned}\right\} \tag{19a}$$

$$\left.\begin{aligned} &\Delta F_b/F_b=[s_0(T_a/T_g)/\{1-s_0(T_a/T_g)\}](\Delta T_g/T_g) \\ &\cong\left\{\begin{aligned} &0.159(\Delta T_g/T_g) \quad \text{for helium} \\ &0.075(\Delta T_g/T_g) \quad \text{for hydrogen} \end{aligned}\right\} \text{higher than the pressure altitude.} \end{aligned}\right\} \tag{19b}$$

$$\left.\begin{aligned} \Delta H=&(T_0/na)[k_0(1-s_0)(T_a/T_i)/\{1-s_0(T_a/T_g)\}]^{(1-n)/n}. \\ &[k_0(1-s_0)s_0(T_0/T_i)(T_a/T_g)/\{1-s_0(T_a/T_g)\}^2](\Delta T_g/T_g) \end{aligned}\right\} \tag{20}$$

or

$$\Delta H=\begin{Bmatrix}1,340\\630\end{Bmatrix}\cdot\{k_0(T_a/T_i)\}^{1/n}\cdot(\Delta T_g/T_g)\begin{Bmatrix}\text{for helium}\\\text{for hydrogen}\end{Bmatrix}. \tag{21}$$

A super heat of 10°C brings the lift increment of almost 4 percent when the airship is lower than the pressure altitude, and of almost 0.6 percent for helium and of almost 0.3 percent for hydrogen when the airship is higher than the pressure altitude. The same temperature rise also results the height increment of 46m for helium and 22m for hydrogen when $k_0(T_a/T_i)\cong 1.0$.

When the temperature and pressure equilibrium between the gas and the surrounding air has been established equation (11) yields approximately

$$\left.\begin{aligned}\Delta F_B&=\rho_{a_0}g V_G\Delta[(T_i/T_a)(p_a/p_0)\{1-0.379\phi(p_{Tc}/p_a)-s_0(T_a/T_g)\}]\\&\cong\rho_{a_0}g V_G(T_i/T_a)\Delta\{(T_0/T_a)(p_a/p_0)\}\\&=-\rho_{a_0}g V_G(T_i/T_0)(n-1)\{1-(aH_0/T_0)\}^{n-2}(a/T_0)\Delta H\\&=-(n-1)F_B(a/T_a)\Delta H.\end{aligned}\right\} \tag{22}$$

From the force equilibrium at the pressure height the weight change results

$$\Delta F_B/F_B=\Delta W/W \tag{23}$$

and thus the following relation can be established :

$$\left.\begin{aligned}\Delta H&=-\{T_a/(n-1)a\}(\Delta W/W)\\&\cong-1.04\times 10^4(T_a/T_0)(\Delta W/W).\end{aligned}\right\} \tag{24}$$

The above equation tells that one percent of ballast abandonment brings height increment of about 100m in usual operational range of altitude. Further, by combining with equations (12) and (17) the density change due to one percent of ballast change becomes

$$\left.\begin{aligned}\Delta\rho_g/\rho_g&=\Delta p/p-\Delta T/T=(a/T_0)[1-n\{1-(a/T_0)H\}^{n-1}]\Delta H\cong 9.6\times 10^{-5}\Delta H\\&=-(T_a/T_0)[\{1-n(1-aH/T_0)^{n-1}\}/(n-1)](\Delta W/W)\cong(T_a/T_0)(\Delta W/W)\end{aligned}\right\} \tag{25}$$

This means that one percent of ballast abandonment brings one percent gas loss or roughly 10 percent loss for every 1,000m altitude rise if the gas is not conserved.

The maximum static ceiling can be attained by completely discharging ballast while in full of gas. If the ratio of dischargeable weight to the gross lift or gross weight is given by $W_D/G.W.$, then, by using equation (1), the maximum static ceiling is given by the altitude at which the air density ratio, $\sigma_a$, is equal to the following ratio:

$$\sigma_a=\rho_a/\rho_{a_0}=1-(W_D/G.W.). \tag{25}$$

This can be performed by using Fig. 2.

Nonrigid airships must have sufficient ballonet capacity to permit the descent from the maximum ceiling without loss of the internal pressure upon which maintenance of the form depends. The ratio of the ballonet volume to the total gas volume should be a little larger than the $W_D/W$ in order to permit of safe descent from an altitude reached dynamically above the static ceiling[4].

After passing some flight hour, the gross weight will be reduced because of fuel burning and nonrecovered. To avoid losing the gas for keeping the flight altitude the airship must take additional weight, for examples, by recovering water from the exhaust gas or picking up water from the sea, lake and river or by refueling from other aircraft in flight. The loss of gas means loss of lift when the airship returns below the greatest height reached. Thus it is usually important to operate the airship as low down as possible from the performance consideration.

It is, of course, possible to make the running takeoff by using aerodynamic lift acting on the hull. Then an additional loading capacity can be available more than that specified by the buoyant force only.

*Performance in Forward Flight*

Hereafter, it may be assumed that an airship is so called "hybrid" type and therefore has a body composed of a streamlined fuselage, four tail surfaces, two thrusters, a cabin and a wing as shown in Fig. 4 and, inside of the fuselage, gas containers filled with either hydrogen or helium gas. The wing is used to augment the lift during the cruising flight of the airship so that when the wing is removed the hybrid airship can be converted conceptually to a plain airship.

The drag of the airship in horizontal flight with velocity $U$ may be split into two parts : the drag of the body without wing, $D_F$, and the drag of the wing, $D_W$, and can, therefore, be expressed by

$$D = D_F + D_W = (1/2)\rho U^2 V_B^{2/3} C_D \tag{26}$$

where $C_D$ is a total drag coefficient of the airship based on a reference area defined by two-thirds of the displacement volume of the airship, $V_B$. The

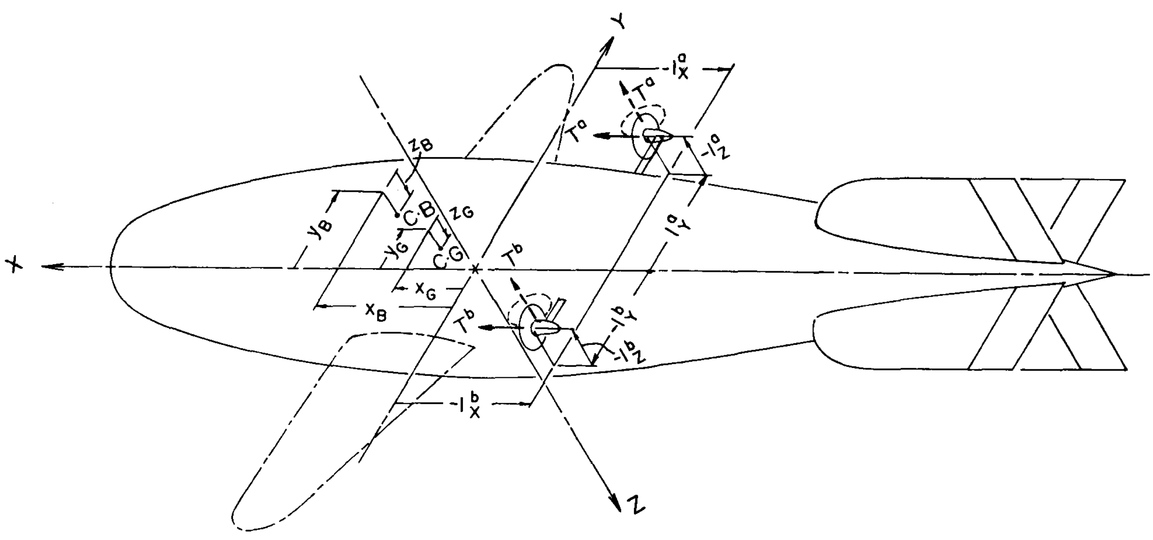



FIG. 4. General configuration of airship.

use of displacement volume has an advantage that it is a fixed quantity instead of a variable like the gas volume and is directly related to the aerodynamic quantities.

The above two drag components can respectively be given by

$$\left.\begin{aligned} D_F &= (1/2)\rho_{a_0}\sigma_a U^2 V_B^{2/3} C_{D_F} \\ D_W &= (1/2)\rho_{a_0}\sigma_a U^2 S_W C_{D_W} = (1/2)\rho_{a_0}\sigma_a U^2 S_W (C_{D_0} + C_{L_W}{}^2/\pi AR), \end{aligned}\right\} \quad (27)$$

where $C_{D_F}$ is the drag coefficient of the body except wing and is assumed constant through all flight phases and where $C_{D_W}$ is the drag coefficient of wing based on the wing area, $S_W$, and is comprised of form drag coefficient $C_{D_0}$ and the induced drag coefficient $C_{L_W}{}^2/\pi AR$. Thus the total drag coefficient can be given by

$$C_D = C_{D_F} + (S_W/V_B^{2/3})(C_{D_0} + C_{L_W}{}^2/\pi AR). \quad (28)$$

The horse power, HP, required to make a steady horizontal flight is

$$\mathrm{HP} = DU/75 = (1/2)\rho_{a_0}\sigma_a U^3 C_D V_B^{2/3}/75 \quad (29)$$

where the factor 75 has been resulted from the conversion of power in metric cystem. The necessary horse power, $\mathrm{HP}_N$, for the power plant is, therefore, given by

$$\mathrm{HP}_N = \mathrm{HP}/\eta = (1/2)\rho_{a_0}\sigma_a U^3 C_D V_B^{2/3}/75\eta \quad (30)$$

where $\eta$ is the total efficiency of a drive system consists of propellers, gear trains coupled between propeller and output shaft of the individual engine having same performance characteristics, and accessaries. When the airships have other auxiliary power plants for supplying necessary energy of crew's living and/or of loading preservation, the $\eta$ must be counted to include these penalties.

Then the fuel consumption FC can be given from the specific fuel consumption, SFC, as follows:

$$\mathrm{FC} = \mathrm{SFC} \cdot \mathrm{HP}_N. \quad (31)$$

By assuming that the necessary power is kept constant the range of the flight, $R$, can be given by

$$R = U \cdot E \quad (32)$$

where $E$ is the endurance or flight time and is given by dividing weight of fuel, $W_f$, with the fuel consumption,

$$E = W_f/\mathrm{FC}. \quad (33)$$

Combining this with equations (5~7) yields

$$\left.\begin{aligned} R &= U W_f/\mathrm{FC} = (W_f/W)(W/\mathrm{HP}_N)(U/\mathrm{SFC}) \\ &= 75\eta (W_f/W) W / \{(1/2)\rho_{a_0}\sigma_a U^2 C_D V_B^{2/3}\} (\mathrm{SFC}) \end{aligned}\right\} \quad (34)$$

$$E=R/U=75\eta(W_f/W)W/\{((1/2)\rho_{a_0}\sigma_a U^3 C_D V_B^{2/3})(\mathrm{SFC})\} \tag{35}$$

where $W_f/W$ and $W/\mathrm{HP}_N$ are "fuel loading ratio" and "power loading" respectively.

Now, since the gross weight of the airship is suspended by the lift comprised of both aerostatic and aerodynamic forces, or buoyant force $F_B$ and dynamic lift $F_L$, the following equilibrium equation in vertical direction can be obtained:

$$\left.\begin{aligned} W&=L=F_B+F_L \\ &=\rho_{a_0}gk_B V_G+(1/2)\rho_a U^2(S_W C_{Lw}+V_B^{2/3}C_{LF}) \end{aligned}\right\} \tag{36}$$

where $C_{Lw}$ is based on wing area, $S_W$, while $C_{LF}$ is based on the volume area, $V_B^{2/3}$. By introducing a new lift coefficient defined by

$$C_L=F_L/(1/2)\rho_a U^2 V_B^{2/2}=(S_W/V_B^{2/3})C_{Lw}+C_{LF} \tag{37}$$

the above equation becomes

$$W=\rho_{a_0}\sigma_a g V_G[(1-s)+\{(1/2)U^2 C_L(V_B/V_G)^{2/3}/V_G^{1/3}g\}]. \tag{38}$$

The weight of the airship is, in somecase, kept almost constant during the flight by compensating the weight reduction due to the fuel consumption by feeding, for an example, water.

The first bracket shows static lift which is proportional to the gas volume and the second bracket shows dynamic lift which is created by fuselage, tail surfaces and wing if installed and is proportional to lifting surfaces times dynamic pressure.

An available gas volume is practically smaller than the displacement volume of the airship and it about 70~75 percent at the ground and almost 98 percent in fully inflated condition at any altitude higher than the pressure altitude for nonrigid airship and is about 90 through 96 percent for the rigid airship so that the volume ratio takes values of $V_B/V_G\cong 1.04 \sim 1.11$. Thus it follows that with increasing size of airships it becomes increasingly difficult to use dynamic lift to compensate for disturbances of the static equilibrium in a constant flight speed.

In plain airship, the dynamic lift created by fuselage and tail surfaces can share about 10 percent of the gross lift if the airship flies at about 8 through 10 degree incidence. The dynamic lift can be used to compensate the change of buoyant force due to the temperature change in flight. Actually, the airship is used to cruise in low altitude and is, therefore, affected favorably by the cloud which shades the sunshine and protects to rise-up the hull temperature.

The lift and drag ratio, $L/D$, can be given by

$$\left.\begin{aligned} L/D&=(F_B+F_L)/D \\ &=2(V_G/V_B)^{2/3}(1-s)(gV_G^{1/3}/C_D U^2)+(C_L/C_D). \end{aligned}\right\} \tag{39}$$

It is, thus, appreciated that the overall lift drag ratio increases in proportion to the linear dimension of airship size, $V_G^{1/3}$, and in inversely proportion to the square of speed, $U^2$. The first term degenerates from infinity at $U=0$ to smaller values at higher speeds, whereas the second term contributes appreciable in high speed for winged airship.

By using again the assumption of constant necessary power which is equivalent to assume that the dynamic lift is kept constant through the flight even if the weight compensation is abandoned, and by substituting again equations (36) and (39), equation (34) yields

$$R=\{75\eta(W_f/W)/(\mathrm{SFC})\}[(1-s)gV_G^{1/3}/\{(1/2)U^2C_D(V_B/V_G)^{2/3}\}+(C_L/C_D)]. \tag{40}$$

It can be seen immediately from the above equation that good propeller efficiency, large fuel loading ratio, small drag coefficient with aerodynamically refined body shape and efficient power plant having small specific fuel consumption are, like any other flight vehicle, primary factors to increase the flight range of the airship, and that the large gas volume and small flight speed are very favorable to increase the range of plain airship. For the hybrid airship the large wing and good lift-drag ratio is also important factor for the large range.

Since the endurance, $E$, can be obtained by $R/U$ the smaller speed increases the endurance unlimitedly.

A ratio of payload with respect to the gross weight, $W_P/W$, and a "vehicle efficiency" of the airship, $P$, which is measure of efficiency on the transportation are respectively given by

$$W_P/W=1-(W_F/W)-(W_W/W)-(W_f/W) \tag{41}$$

$$P=W_PU/75\mathrm{HP}_N=(1/75)(W_P/W)U(W/\mathrm{HP}_N) \tag{42}$$

where $W_F$ and $W_W$ are weight of the body other than wing and fuel and weight of the wing respectively. By substituting equations (30), (40) and (41), equation (42) yields

$$\begin{aligned} P&=\eta\{1-(W_F/W)-(W_W/W)\}[(1-s)gV_G^{1/3}/\{(1/2)U^2C_D(V_B/V_G)^{2/3}\\ &\quad+(C_L/C_D)]-(1/75)\cdot R\cdot(\mathrm{SFC})\\ &=\eta\{1-(W_F/W)-(W_W/W)\}(L/D)-(1/75)\cdot R\cdot((\mathrm{SFC}). \end{aligned} \tag{43}$$

It will, thus, be appreciated that in order to increase the vehicle efficiency of the plain airship the following items must be considered:

(1) Increase the propeller efficiency.

(2) Reduce the weight of body structures with respect to the gross weight, $W_F/W$ and $W_W/W$.

(3) Increase the gas volume and therefore increase the size of airship and approaches the ratio $V_B/V_G$, which is larger than 1, to one.

(4) Reduce the flight speed.

(5) Reduce the range of flight.

(6) Use the good engine for specific fuel consumption.

By taking partial dirivative of $P$ with respect to the lift coefficient, $C_L$, the optimal lift-drag ratio for a given $U$ can be obtained as follows:

$$C_L/C_D=(\pi AR/2C_{Lw})-(1-s)g V_G^{1/3}/\{(1/2)U^2(V_B/V_G)^{2/3}C_D\}. \quad (44)$$

Thus, the vehicle efficiency of the hybrid airship can be increased additionally as follows:

(7) Select the lift coefficient to be optimal as satisfying equation (44). The optimal lift coefficient, $C_{L_{opt}}$, takes larger larger value as the airship speed becomes faster.

It is interesting to find that the vehicle efficiency given by equation (43) is not explicitely dependent on the air density or flight altitude of the airship. However, as seen from equation (38), since the gross weight of the plain airship, $W$, is nearly equal to the buoyant force which is directly related to the air density or flight altitude, the vehicle efficiency decreases as the proposed altitude increases.

For the winged airship, by combining equations (38) and (43) and eliminating $U$ the vehicle efficiency can be further modified as follows:

$$P=\eta[\{1-(W_F/W)-(W_W/W)\}(C_L/C_D)/\{1-(F_B/W)\}]-R\cdot(\mathrm{SFC})/75. \quad (45)$$

The above equation shows that as the flight altitude increases the buoyant force, $F_B=\rho_{a_0}g(\sigma_a-\sigma_g)V_G$, decreases so that the vehicle efficiency of the winged airship also decreases. Thus it can be said:

(8) Take the flight course in low altitude and increase the buoyant force.

The above statement is, however, equivalent to the item (4).

It can also be concluded from equation (45) that:

(9) Take the maximum lift-drag ratio, $(C_L/C_D)_{max}$.

The above statement does not conflict with the item (7) because in item (7) the flight speed, $U$, can be selected arbitrary but here the $U$ must be decided as a function of $C_L$ and $\rho_a$.

It must be mentioned that the lift coefficient, $C_L$, is constrained within a stall limit which is the function of wing planform as well as airfoil section. Thus, for a given wing area the flght speed becomes very important factor to obtain the optimal vehicle efficiency.

Instead of the vehicle efficiency $P$ a very similar parameter, which may be called "transportation efficiency" and expressed by $\eta_T$, can be introduced. The parameter is a ratio of useful transportation energy represented by payload times flight range, $W_P\cdot R$, to the consumed energy due to the fuel burning, given by fuel weight times specific fuel energy (available energy of unit weight of fuel), $W_f\cdot(\mathrm{SFE})$, i.e.,

$$\eta_T=W_PR/W_f\cdot(\mathrm{SFE}). \quad (46)$$

Substituting equations (40) and (41) into the above equation yields

$$\eta_T=\{75/(\mathrm{SFE})(\mathrm{SFC})\}P. \quad (47)$$

Thus, the previousely obtained results are also true for the consideration on $\eta_T$.

Now let us compare the vehicle efficiency of a winged airship, $P_W$, and that of a wingless or plain airship, $P$, under the following assumption:

( i ) Both ships have same gross weight.

( ii ) The plain airship can take-off vertically but winged needs a runway for take-off in order to sustain the gross weight by the dynamic lift plus buoyant force.

( iii ) The flight range in consideration is same for both airships.

( iv ) The installed engine characteristics are same for both airships.

( v ) The buoyant force of the winged airship can be reduced by the amount just equal to the dynamic lift created by the installed wing,

$$\left.\begin{array}{l}(1/2)\rho_a U^2 S_W C_L = \rho_a(1-s)g(V_G - V_{G,W}) \\ \text{or} \\ V_{G,W} = V_G - (1/2)U^2(S_W/V_B{}^{2/3})V_B{}^{2/3}C_L/(1-s)g\end{array}\right\} \quad (48)$$

( vi ) From the fact of item (v), the fuselage of the winged airship can be made more slender than that of the plain airship so that the drag area of the fuselage can be reduced. Thus the drag coefficient of fuselage may be considered as a function of the slenderness ratio, $C_{D_F} = C_{D_F}(l/d)$.

The total drag coefficients of winged airship is given by

$$(C_D)_W = (C_{D_F})_W + (S_W/V^{2/3}{}_{B,W})(C_{D_0} + C_{L_W}{}^2/\pi AR). \quad (49)$$

(vii) The weight penalty of wing is equal to a summation of weight reduction of the fuselage resulting from the gas volume reduction and of weight increment due to the reinforcement of the fuselage for wing installation,

$$W_F/W = (W_F/W)_W + (W_W/W)_W. \quad (50)$$

Then, a difference of vehicle efficiency between both airships, $P_W - P$, for a given range can be given by

$$\left.\begin{array}{l}(P_W - P)/\eta = \{1-(W_F/W)\}[(1-s)g\{(V_G/C_D V_B{}^{2/3})_W \\ \qquad -(V_G/C_D V_B{}^{2/3})\}\Big/\left(\dfrac{1}{2}\right)U^2 + (C_L/C_D)_W - (C_L/C_D)]\end{array}\right\} \quad (51)$$

where $(\ )_W$ shows the quantity $(\ )$ of winged airship.

(viii) The gas volume ratio is constant for both airships,

$$(V_G/V_B)_W = (V_G/V_B). \quad (52)$$

Thus the winged airship has good efficiency over the plain airship if the speed $U$ is larger than a " critical speed ", $U^*$, given by

$$U^* = \left[2(1-s)g(V_G/V_B)^{2/3}V_G^{1/3}\{(1/C_D)-(1/C_D)_W,\ (V_{B,W}/V_B)^{1/3}\}/ \{(C_L/C_D)_W-(C_L/C_D)\}\right]^{1/2} \tag{53}$$

Then the following statements can be obtained from equations (50) through (53):

(1) As lift drag ratio or $(C_L/C_D)_W$ increases the critical speed, $U^*$, beyond which superiority of the winged airship over the plain airship is established, becomes small.

(2) As the size of airship increases the critical speed also increases approximately in proportion to the root of linear dimension of the ship.

(3) The hydrogen filled airship has higher critical speed than that of the helium-filled airshi.p

(4) As the speed inceases beyond the critical speed the vehicle efficiency increases to the largest value given by

$$(P_W-P)_{\lim U\to\infty} = \{1-(W_F/W)\}\ \{(C_L/C_D)_W-(C_L/C_D)\} \tag{54}$$

(5) As the " wing area ratio ", $S_W/V_B^{2/3}$, increases the larger vehicle ef-

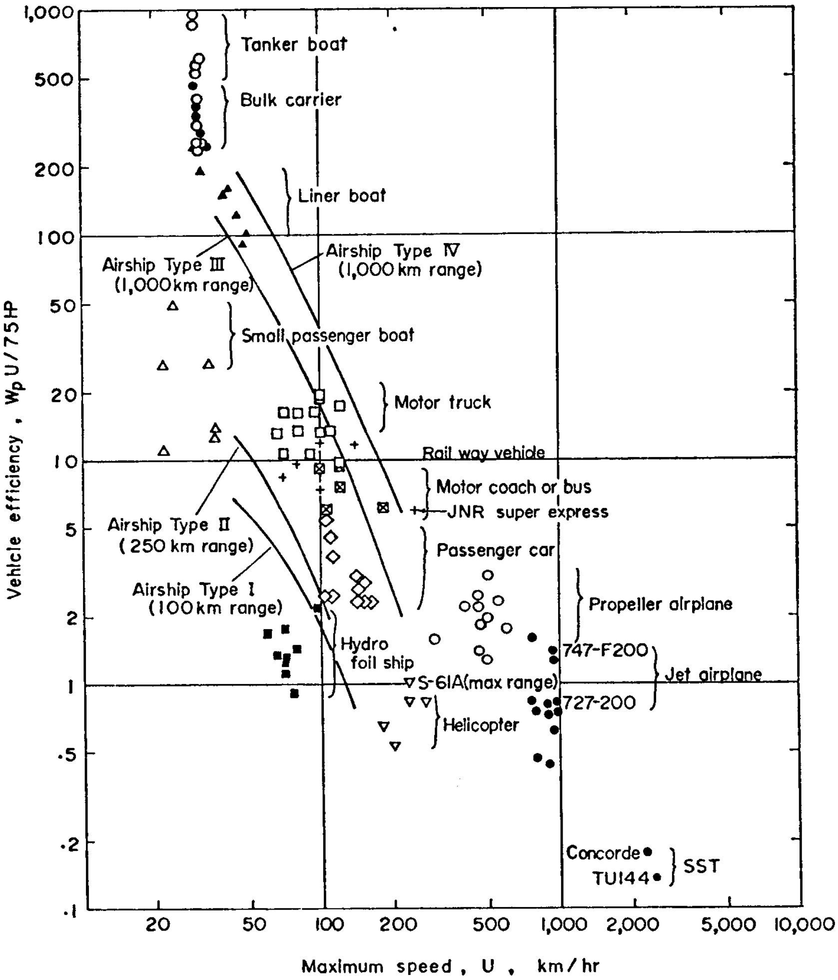



FIG. 5. Comparison of vehicle efficiency among various vehicles*

* The author is indebted to Mr. Y. Oka, Mitsubishi Heavy Industries, LTD. for drawing up figure.

TABLE 2. Dimensions and data of exemplified airshps

| Item | | Type I | Type II | Type III | Type IV |
|---|---|---|---|---|---|
| Length (m) | | 60 | 120 | 240 | 550 |
| Diameter (m) | | 12 | 24 | 48 | 110 |
| Displacement Volume, $V_B$ ($m^3$) | | $3.99\times10^3$ | $3.19\times10^4$ | $2.22\times10^5$ | $2.55\times10^6$ |
| Gas Volume, $V_G$ | | $3.90\times10^3$ | $3.10\times14^4$ | $2.15\times10^5$ | $2.45\times10^6$ |
| Number of Crew | | 2 | 4 | 4 | 4 |
| Effective Tail Fin Area, Vertical or Horizontal ($m^2$) | | 66 | 260 | 950 | 4,890 |
| $\frac{\text{(Virtual Mass)}}{\rho V_B}$ | Longitudinal | 1.06 | 1.06 | 1.06 | 1.06 |
| | Vertical or Lateral | 2.01 | 2.01 | 2.01 | 2.01 |
| $\frac{\text{(Vertural Moment of Inertia)}}{\rho V_B^{5/3}}$ | Pitching or Yawing | 1.55 | 1.55 | 1.52 | 1.52 |
| | Rolling | 0.113 | 0.113 | 0.0694 | 0.0694 |
| Empty Weight (ton) | | 2.60 | 19.8 | 111 | 1,163 |
| Gross Weight (ton) | | 3.9 | 31 | 215 | 2,450 |
| Power Plant | Type | Reciprocating | Reciprocating | Gas Turbine | Turbo-Prop |
| | House Power (HP) | 200 | 1,300 | 6,660 | 51,000 |
| | Specific Fuel Consumption (kg/HP·hr) | 0.2 | 0.2 | 0.2 | 0.3 |
| Maximum Speed (km/hr) | | 80 | 94 | 140 | 160 |
| Linear Acceleration (G) | Longitudinal | 0.04 | 0.03 | 0.02 | 0.01 |
| | Vertical | 0.02 | 0.015 | 0.01 | 0.006 |
| Angular Acceleration ($sec^{-2}$) | Pitching | No positive control moment because two propellers arranged side-by-side. | | $3.32\times10^{-3}$ | $12.9\times10^{-5}$ |
| | Yawing | $1.23\times10^{-2}$ | $4.67\times10^{-3}$ | $1.52\times10^{-3}$ | $5.33\times10^{-5}$ |

ficency difference can be obtained in higher speed.

(6) The aspect ratio of the wing also improves the maximum vehicle efficiency difference.

Figure 5 shows an example of calculation of the vehicle efficiency for two types of nonrigid airship and two types of rigid airship comparing with other transportation vehicles. The detailed dimensions of the above exemplified airships are listed in Table 2. It can be seen that the larger airships like Type III and IV have better efficiency than other vehicles within the speed range of 50 through 200 km/hr.

## 3. Flight Dynamics

The most important imperfection of the airship comparing with the airplane is to have unsatisfactory characteristics in the flight dynamics of the airship such as either poor stable or unstable in rectlinear flight and lack of control power and damping in or near hovering flight. The drawback will be more stressed in larger sized airships which have big virtual mass and huge virtual moment of inertia in any direction. In is, therefore, important to conquer the above defficiency of control power and damping in the handling quality of the airships by introducing a sophisticated automatic stability equipment.

*Fquations of Motion*

In very usual configurations of airship as shown in Fig. 4 the following nondimensional equations of motion for a small perturbed or controlled motion can be obtained:

$$\left.\begin{aligned}&(X_{U_{X'}}\bar{D}+X_{U_X})\Delta\bar{U}_X+(X_{D_{Z'}}\bar{D}+X_{U_Z})\Delta\bar{U}_Z+X_{\theta''}\bar{D}^2+X_{\theta'}\bar{D}+X_\theta)\Delta\theta\\&=X_\delta(\Delta\delta_Y+\Delta\delta_Z)+X\delta_X{}^a\Delta\delta_X{}^a+X_{\delta_X}{}^b\Delta\delta_X{}^b+X_M\Delta\bar{M}\end{aligned}\right\}\quad(55a)$$

$$\left.\begin{aligned}&(Z_{U_{X'}}\bar{D}+Z_{U_X})\Delta\bar{U}_X+(Z_{U_{Z'}}\bar{D}+Z_{U_Z})\Delta\bar{U}_Z+Z_{\theta''}\bar{D}^2+Z_{\theta'}\bar{D}+Z_\theta)\Delta\theta\\&=Z_\delta\Delta\delta_Z+Z_{\delta_Z{}^a}\Delta\delta_Z{}^a+\Delta_{\delta_Z{}^b}\Delta\delta_Z{}^b+\Delta\bar{M}\end{aligned}\right\}\quad(55b)^*$$

$$\left.\begin{aligned}&(M_{U_{X'}}\bar{D}+M_{U_X})\Delta\bar{U}_X+(M_{U_{Z'}}\bar{D}+M_{U_Z})\Delta\bar{U}_Z\\&+(M_{\theta''}\bar{D}^2+M_{\theta'}\bar{D}+M_\theta)\Delta\theta\\&=M_\delta\delta_Z+M_{\delta_Z{}^a}\Delta\delta_Z{}^a+M\delta_Z{}^b\Delta\delta_Z{}^b+M_{\delta_X{}^a}\Delta\delta_X{}^a+M\delta_X{}^b\Delta\delta_X{}^b\\&+M_M\Delta\bar{M}+M_{x_G}\Delta\bar{x}_G+M_{x_B}\Delta\bar{x}_B+M_{Z_G}\Delta\bar{z}_G+Mz_B\Delta\bar{z}_B\end{aligned}\right\}\quad(55c)$$

$$\left.\begin{aligned}&(Y_{U_{Y'}}\bar{D}+Y_{U_Y})\Delta\bar{U}_Y+(Y_{\psi''}\bar{D}^2+Y_{\psi'}\bar{D}+Y_\psi)\Delta\psi+(Y_{\phi''}\bar{D}^2+Y_{\phi'}\bar{D}+Y_\phi)\Delta\phi\\&=Y_\delta\Delta\delta_Y\end{aligned}\right\}\quad(56a)$$

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* The effect of the altitude change on the buoyant force has been neglected in this small perturbed motion because the restoring force due to the altitude change is so small that the period of vertical motion around a trimmed altitude is quitely larger than other oscillations given by equations (55). See Appendix.

$$\left.\begin{aligned}&(L_{U_Y'}\bar{D}+L_{U_Y})\Delta\bar{U}_Y+(L_{\psi''}\bar{D}^2+L_{\psi'}\bar{D}+L_{\psi})\Delta\psi\\&\quad+(L_{\phi''},\bar{D}^2+L_{\phi'},\bar{D}+L_{\phi})\Delta\phi\\&\quad=L_M\Delta\bar{M}+L_{\delta z^a}\Delta\delta_Z{}^a+L_{\delta z^b}\Delta\delta_Z{}^b+L_{y_G}\Delta\bar{y}_G\end{aligned}\right\}\quad(56\text{b})$$

$$\left.\begin{aligned}&(N_{U_Y'}\bar{D}+N_{U_Y})\Delta U_Y+(N_{\psi''}\bar{D}^2+N_{\psi'}\bar{D}+N_{\psi})\Delta\psi\\&\quad+(N_{\phi''}\bar{D}^2+N_{\phi'}\bar{D}+N_{\phi})\Delta\phi\\&\quad=N_{\delta}\Delta\delta_Y+N\delta_{X^a}\Delta\delta_X{}^a+N_{\delta X^b}\Delta\delta_X{}^b+N_M\Delta\bar{M}++N_{y_G}\Delta\bar{y}_G\end{aligned}\right\}\quad(56\text{c})$$

where

$$\left.\begin{aligned}&X_{U_X'}=\bar{M}_0+\bar{A}_{11}\\&X_{U_X}=-(C_{FX})_0\bar{U}_{X,0}-P_x(C_{TX,\lambda}{}^a+C_{TX,\lambda}{}^b)/(\overline{R\Omega})_X\\&X_{U_Z'}=\bar{A}_{13}\\&X_{U_Z}=-(C_{FX})_0\bar{U}_{Z,0}-F_{r,0}(C_{FX,\alpha}/\bar{U}_{R,0})\\&X_{\theta''}=\bar{M}_0\bar{z}_{G,0}+\bar{A}_{15}\\&X_{\theta'}=(\bar{M}_0+\bar{A}_{33})\bar{U}_{Z,0}+\bar{A}_{13}\bar{U}_{X,0}-P_X(C_{TX,\lambda}{}^a\bar{l}_Z{}^a+C_{TX,\lambda}{}^b\bar{l}_Z{}^b)/(R\Omega)_X\\&X_{\theta}=-(1-\bar{M}_0)\cos\theta_0\\&X_{\delta}=F_{r,0}C_{FX,\delta}\\&X_{\delta X^a}=P_XC_{TX,\delta}{}^a;\quad X_{\delta Y^b}=P_XC_{TX,\delta}{}^b\\&X_M=-\sin\theta_0\end{aligned}\right\}\quad(57)$$

$$\left.\begin{aligned}&Z_{U_X'}=\bar{A}_{13}\\&Z_{U_X}=-(C_{FZ})_0\bar{U}_{X,0}\\&Z_{U_Z'}=\bar{M}_0+\bar{A}_{33}-F_{r,0}(C_{FZ,\alpha'}/\bar{U}_{R,0})\\&Z_{U_Z}=-(C_{FZ})_0\bar{U}_{Z,0}-F_{r,0}(C_{FZ,\alpha}+C_{FZ,\alpha^2}|\alpha_0|)/\bar{U}_{R,0}\\&\qquad-P_Z(C_{TZ,\lambda}{}^a+C_{TZ,\lambda}{}^b)/(\overline{R\Omega})_Z\\&Z_{\theta''}=-\bar{M}_0\bar{x}_{G,0}+\bar{A}_{35}\\&Z_{\theta'}=-(\bar{M}_0+\bar{A}_{11})\bar{U}_{X,0}-\bar{A}_{13}\bar{U}_{Z,0}-F_{r,0}C_{FZ,\theta'}\\&\qquad+P_Z(C_{TZ,\lambda}{}^a\bar{l}_X{}^a+C_{TZ,\lambda}{}^b\bar{l}_X{}^b)/(\overline{R\Omega})_Z\\&Z_{\theta}=-(1-\bar{M}_0)\sin\theta_0\\&Z_{\delta}=F_{r,0}C_{FZ,\delta}\\&Z_{\delta Z^a}=P_ZC_{TZ,\delta}{}^a;\quad Z_{\delta Z^b}=P_ZC_{TZ,\delta}{}^b\end{aligned}\right\}\quad(58)$$

$$\begin{aligned}&M_{U_X'}=\bar{M}_0\bar{z}_{G,0}+\bar{A}_{15}\\&M_{U_X}=-2\bar{A}_{13}\bar{U}_{X,0}+(\bar{A}_{33}-\bar{A}_{11})\bar{U}_{Z,0}-(C_{MY})_0\bar{U}_{X,0}\\&\qquad-P_X(\bar{l}_Z{}^aC_{TX,\lambda}{}^a+\bar{l}_Z{}^bC_{TX,\lambda}{}^b)/(R\Omega)_X\end{aligned}$$

$$
\left.
\begin{aligned}
&M_{U_Z'}=-\overline{M}_0\bar{x}_{G,0}+\bar{A}_{35}-F_{r,0}(C_{MY,\alpha'}/\overline{U}_{R,0})\\
&M_{U_Z}=2\bar{A}_{18}\overline{U}_{Z,0}+(\bar{A}_{33}-\bar{A}_{11})\overline{U}_{X,0}-F_{r,0}C_{MY,\alpha}/\overline{U}_{R,0}-(C_{MY})_0\overline{U}_{Z,0}\\
&\qquad+P_Z(C_{TZ,\lambda}{}^{a}\bar{l}_X{}^a+C_{TZ,\lambda}{}^{b}\bar{l}_X{}^b)/(\overline{R\Omega})_Z\\
&M_{\theta''}=\bar{I}_{Y,0}+\bar{A}_{55}\\
&M_{\theta'}=(\overline{M}_0\bar{z}_{G,0}+\bar{A}_{15})\overline{U}_{Z,0}+(\overline{M}_0\bar{x}_{G,0}-\bar{A}_{35})\overline{U}_{X,0}-F_{r,0}C_{MY,\theta'}\\
&\qquad-P_Z(C_{TZ,\lambda}{}^{a}\bar{l}_X{}^a+C_{TZ,\lambda}{}^{b}\bar{l}_X{}^b)/(\overline{R\Omega})_Z\\
&M_\theta=(\overline{M}_0\bar{z}_{G,0}-\bar{z}_{B,0})\cos\theta_0-(\overline{M}_0\bar{x}_{G,0}-\bar{x}_{B,0})\sin\theta_0\\
&M_\delta=F_{r,0}C_{MY,\delta}\\
&M\delta_Z{}^a=-P_ZC_{TZ,\delta}{}^a\bar{l}_X{}^a;\quad M\delta_Z{}^b=-P_ZC_{TZ,\delta}{}^b\bar{l}_X{}^b\\
&M_M=-\bar{x}_{G,0}\cos\theta_0-\bar{z}_{G,0}\sin\theta_0\\
&M_{x_G}=-\overline{M}_0\cos\theta_0\\
&M_{x_B}=\cos\theta_0\\
&M_{z_G}=-\overline{M}_0\sin\theta_0\\
&M_{z_B}=\sin\theta_0\\
&M\delta_{X^a}=P_XC_{TX,\delta}{}^a;\quad M\delta_{X^b}=P_XC_{TX,\delta}{}^b
\end{aligned}
\right\}
\tag{59}
$$

$$
\left.
\begin{aligned}
&Y_{U_Y'}=\overline{M}_0+\bar{A}_{22}-F_{r,0}(C_{FY,\beta'}/\overline{U}_{R,0})\\
&Y_{U_Y}=-F_{r,0}(C_{FY,\beta}/\overline{U}_{R,0})\\
&Y_{\psi''}=(\overline{M}_0\bar{x}_{G,0}+\bar{A}_{26})\cos\theta_0+(\overline{M}_0\bar{z}_{G,0}-\bar{A}_{24})\sin\theta_0\\
&Y_{\psi'}=(\overline{M}_0+\bar{A}_{11})\overline{U}_{X,0}\cos\theta_0+(\overline{M}_0+\bar{A}_{33})\overline{U}_{Z,0}\sin\theta_0\\
&\qquad+\bar{A}_{13}(\overline{U}_{Z,0}\cos\theta_0+\overline{U}_{X,0}\sin\theta_0)-F_{r,0}C_{FY,\psi'}\\
&Y_\psi=0\\
&Y_{\phi''}=-\overline{M}_0\bar{z}_{G,0}+\bar{A}_{24}\\
&Y_{\phi'}=-(\overline{M}_0+\bar{A}_{33})\overline{U}_{Z,0}-\bar{A}_{13}\overline{U}_{X,0}+F_{r,0}(-C_{FY,\phi'})\\
&Y_\phi=(1-\overline{M}_0)\cos\theta_0\\
&Y_\delta=F_{r,0}C_{FY,\delta}
\end{aligned}
\right\}
\tag{60}
$$

$$
\begin{aligned}
&L_{U_Y'}=-\overline{M}_0\bar{z}_{G,0}+\bar{A}_{24}-F_{r,0}(C_{MX,\beta'}/\overline{U}_{R,0})\\
&L_{U_Y}=\bar{A}_{13}\overline{U}_{X,0}+(\bar{A}_{22}-\bar{A}_{33})\overline{U}_{Z,0}-F_{r,0}C_{MX,\beta}/\overline{U}_{R,0}\\
&L_{\psi''}=(-\bar{J}_{XZ,0}+\bar{A}_{46})\cos\theta_0-(I_{X,0}+\bar{A}_{44})\sin\theta_0\\
&L_{\psi'}=-(\overline{M}_0\bar{z}_{G,0}+\bar{A}_{15})\overline{U}_{X,0}\cos\theta_0-(\overline{M}_0\bar{z}_{G,0}-\bar{A}_{24})\overline{U}_{Z,0}\sin\theta_0\\
&\qquad+(\bar{A}_{26}+\bar{A}_{35})\overline{U}_{Z,0}\cos\theta_0-F_{r,0}C_{MX,\psi'}
\end{aligned}
$$

$$
\left.
\begin{aligned}
&L_{\psi}=0\\
&L_{\phi''}=\bar{I}_{X,0}+\bar{A}_{44}\\
&L_{\phi'}=(\overline{M}_0\bar{z}_{G,0}-\bar{A}_{24})\overline{U}_{Z,0}-F_{r,0}C_{Mx,\phi'}\\
&L_{\phi}=(\overline{M}_0\bar{z}_{G,0}-\bar{z}_{B,0})\cos\theta_0\\
&L_{\delta z^a}=-P_Z\bar{l}_Y{}^aC_{Tz,\delta}{}^a;\quad L_{\delta z}{}^b=-P_Z\bar{l}_Y{}^bC_{Tz,\delta}{}^b\\
&L_M=\bar{y}_{G,0}\cos\theta_0\\
&L_{yG}=\overline{M}_0\cos\theta_0
\end{aligned}
\right\}\qquad (61)
$$

$$
\left.
\begin{aligned}
&N_{UY'}=\overline{M}_0\bar{x}_{G,0}+\bar{A}_{26}-F_{r,0}C_{Mz,\beta'}/\overline{U}_{R,0}\\
&N_{UY}=-\bar{A}_{13}\overline{U}_{Z,0}+(\bar{A}_{11}-\bar{A}_{22})\overline{U}_{X,0}-(C_{Mz})_0\overline{U}_{Y,0}-F_{r,0}C_{Mz,\beta}/\overline{U}_{R,0}\\
&N_{\phi''}=(\bar{I}_{Z,0}+\bar{A}_{66})\cos\theta_0+(\bar{J}_{XZ,0}-\bar{A}_{46})\sin\theta_0\\
&N_{\phi'}=(\overline{M}_0x_{G,0}+\bar{A}_{26})U_{X,0}\cos\theta_0+(\overline{M}_0\bar{z}_{G,0}-\bar{A}_{35})\overline{U}_{Z,0}\sin\theta_0\\
&\qquad-(\bar{A}_{15}+\bar{A}_{24})\overline{U}_{X,0}\sin\theta_0-F_{r,0}C_{Mz,\psi'}\\
&\qquad-P_X\{C_{Tx,\lambda}{}^a(\bar{l}_Y{}^a)^2+C_{Tx,\lambda}{}^b(\bar{l}_Y{}^b)^2\}/(\overline{R\Omega})_X\\
&N_{\psi}=0\\
&N_{\phi''}=-\bar{J}_{XZ,0}+\bar{A}_{46}\\
&N_{\phi'}=(\bar{A}_{15}+\bar{A}_{24})\overline{U}_{X,0}-F_{r,0}C_{Mz,\phi'}\\
&N_{\phi}=-(\overline{M}_0\bar{x}_{G,0}-\bar{x}_{B,0})\cos\theta_0\\
&N_{\delta}=F_{r,0}C_{Mz,\delta}\\
&N_{\delta x^a}=-P_XC_{Tx,\delta}{}^a\bar{l}_Y{}^a;\quad N_{\delta x^b}=-P_XC_{Tx,\delta}{}^b\bar{l}_Y{}^b\\
&N_M=-\bar{y}_{G,0}\sin\theta_0\\
&N_{yG}=M_0\sin\theta_0
\end{aligned}
\right\}\qquad (62)
$$

$\overline{M}=M/\rho_aV_B=W/\rho_agV_B$ ; nondimensional mass

$(\bar{I}_X, \bar{I}_Y, \bar{I}_Z)=(I_X, I_Y, I_Z)/\rho V_B{}^{5/3}$ ;
nondimentional moments of inertia about the origin of the $(X, Y, Z)$ coordinate system

$\bar{t}=t/\sqrt{V_B{}^{1/3}/g}$ ; nondimensional time

$\overline{D}=d/d\bar{t}$ ;
differential operator with respect to nondimensional time

$(\overline{U}_X, \overline{U}_Y, \overline{U}_Z)=(U_X, U_Y, U_Z)/\sqrt{V_B{}^{1/3}g}$ ;
nondimentional velocities

$$\left.\begin{array}{l}
(\bar{\omega}_X, \bar{\omega}, \bar{\omega}_Z)=(\omega_X, \omega_Y, \omega_Z)\cdot\sqrt{V_B^{1/3}/g}\ ; \\
\qquad \text{nondimentional angular velocities} \\
(\bar{F}_X, \bar{F}_Y, \bar{F}_Z)=(F_X, F_Z, F_Y)/\rho g V_B\ ; \quad \text{nondimentional forces} \\
(\bar{M}_X, \bar{M}_Y, \bar{M}_Z)=(M_X, M_Y, M_Z)/\rho g V_B^{4/3}\ ; \\
\qquad \text{nondimentional moments about the origin of the } (X,Y,Z) \text{ coordinate system} \\
(\bar{x}_G, \bar{y}_G, \bar{z}_G)=(x_G, y_G, z_G)/V_B^{1/3}\ ; \\
\qquad \text{nondimentional distance of center of gravity} \\
(\bar{x}_B, \bar{y}_B, \bar{z}_B)=(x_B, y_B, z_B)/V_B^{1/3}\ ; \\
\qquad \text{nondimentional distance of center of buoyancy} \\
\left.\begin{array}{ll}
\overline{A_{ij}}=A_{ij}/\rho V_B & \text{for } i, j\leq 3 \\
\overline{A_{ij}}=A_{ij}/\rho V_B^{4/3} & \text{for } i\leq 3 \text{ and } j\geq 4 \text{ or} \\
 & \text{for } j\leq 3 \text{ and } i\geq 4 \\
\overline{A_{ij}}=A_{ij}/\rho V_B^{5/3} & \text{for } i, j\geq 4
\end{array}\right\} ;\ \text{nondimentional added masses}
\end{array}\right\} \quad (63)$$

$$\left.\begin{array}{ll}
P_{X,Z}=\rho S_{X,Z}(R\Omega)_{X,Z}^2/\rho_a g V_B\ ; & \text{Thrust parameters} \\
C_{TX,Z}^{a,b}=T_{X,Y,Z}^{a,b}/\rho_a S_{X,Z}(R\Omega)_{X,Z}^2\ ; & \\
\qquad \text{Thrust coefficient for propellers directed } (X,Y,Z) \text{ axes respectively} & \\
F_r=(1/2)\rho_a U_R^2/\partial_a g V_B^{1/3} & ;\ \text{Fround number} \\
U_R=\sqrt{U_X^2+U_Y^2+U_Z^2} & ;\ \text{Absolute velocity} \\
C_{TX,Z,\delta}=(\partial C_{TX,Z}/\partial\delta\theta)\ ; & \\
\qquad \text{Collective pitch derivative of thrust coefficient} & \\
\delta_{X,Z}^{a,b} & ;\ \text{Collective pitch} \\
C_{TX,Z,\lambda}=(\partial C_{TX,Z}/\partial\lambda)\ ; & \\
\qquad \text{Inflow derivative of thrust coefficient} & \\
\lambda_{X,Z}=\{(v+U)/R\Omega\}_{X,Z} & ;\ \text{Inflow ratio} \\
v=R\Omega C_T/2\lambda \text{ for } U>v \text{ and } =R\Omega\sqrt{C_T/2} \text{ for } U=0 & ;\ \text{Induced flow} \\
\overline{R\Omega}=R\Omega\sqrt{V_B^{1/3}g} & \\
\delta_{Y,Z} & ;\ \text{Rudder or elevator angle}
\end{array}\right\} \quad (64)$$

*Stability*

The stability of pitching and rolling motions in hovering flight can be treated as a single-degree-of-freedom motion and be expressed by equations

(55c) and (56b) respectivectively in which only the coupling moment created by gravity and buoyant forces is vital.

Since the areodynamic damping in rolling motion is very poor even in high forward speed the distance between the center of gravity and the center of buoyancy must be small adequately in order to get low undamped natural frequency and to avoid annoying motion for the airship occupants. A military specification for V/STOL aircraft, MIL-F-83300[5], will be helpful to evaluate the degree of stability because there is no adequate criterion for the airship stability.

Since the banking does not serve a useful purpose on the airship in control, no roll control was provided in the past airships. It is, however, possible to introduce the rolling control for keeping a " coordinated turn " in which there is no side slip by taking a differential control in left and right elevator or in upside and downside rudders.

The coupled pitching and heaving motion in vertical plane due to the mass dropping and mass transferring can be described, from equations (55b and c), as follows:

$$\left.\begin{aligned} &(Z_{Uz'}\bar{D}+Z_{Uz})\Delta\bar{U}_Z+Z_{\theta'}\bar{D}\Delta\theta=\Delta\bar{M} \\ &M_{Uz}\Delta\bar{U}_Z+(M_{\theta''}\bar{D}^2+M_{\theta'}\bar{D}+M_\theta)\Delta\theta=-(\bar{M}_0\Delta\bar{x}_G+\bar{x}_{G0}\Delta\bar{M}) \end{aligned}\right\} \quad (65)$$

where $Z_{\theta''}$, $Z_\theta$ and $M_{Uz'}$ have been neglected as small quantities. Then the characteristic equation is given by

$$\left.\begin{aligned} D=&Z_{Uz'}M_{\theta''}S^3+(Z_{Uz}M_{\theta''}+Z_{Uz'}M_{\theta'})S^2+(Z_{Uz}M_{\theta'}+Z_{Uz'}M_\theta \\ &-M_{Uz}Z_{\theta'})S+Z_{Uz}M_\theta=A_1S^3+B_1S^2+C_1S+D_1=0\,. \end{aligned}\right\} \quad (66)$$

For the stability, the contribution of $M_\theta$, which is mainly resulted from the distance between the center of gravity and the center of buoyancy, is very important. A pair of oscillatory motion can be approximately represented by

$$\left.\begin{aligned} &\omega_n=\{(Z_{Uz}M_{\theta'}+Z_{Uz'}M_\theta-M_{Uz}Z_{\theta'})/Z_{Uz'}M_{\theta''}\}^{1/2} \\ &2\zeta\omega_n=(Z_{Uz}M_{\theta''}+Z_{Uz'}M_{\theta'})/Z_{Uz'}M_{\theta''}\,. \end{aligned}\right\} \quad (67)$$

It can be observed that the period of oscillation is strongly related to the $M_\theta$ specifically in low forward speed. This is nearly equivalent to the short period oscillation in the airplane dynamics where the $M_{Uz}$ or $C_{M_{Y,\alpha}}$ is a key factor to decide the period of the oscillation instead of $M_\theta$. As the speed increases the nondimensional damping increases appreciably and the nondimensional frequency of oscillation also increases slightly but the time to damp to half amplitude and the period of oscillation decreases. Here also criteria[5~9] imposed on the V/STOL aircraft and helicopter will be helpful to evaluate the degree of stability of the airship. The real root is less meaning because of simplification of equation of motion in the present process.

So called phugoid oscillation in the airplane dynamics can not be observed in the airship motion. The reason is as follows: Under the assumption of no change in angle of attack, no damping, and no moment of inertia the equations of motion can be approximated by

$$\left.\begin{aligned} &(X_{U_{X'}}\bar{D}+X_{U_X})\Delta U_X+X_\theta\Delta\theta=X_\delta\Delta\delta_X \\ &Z_{U_X}\Delta U_X+(Z_{\theta'}\bar{D})\Delta\theta=\Delta\bar{M} \end{aligned}\right\} \quad (68)$$

where $X_\theta$ and $Z_{U_X}$ are very small comparing with those of the airplane because of the lack of the positive lifting surface in the airship. In this form the characteristic equation has two real roots, $S_{1,2}=-1/T_{1,2}$ having the following time constants

$$\left.\begin{aligned} &T_1=-X_{U_X}Z_{\theta'}/X_\theta Z_{U_X}\cong\{(C_{F_X})_0/(C_{F_Z})_0\}\ \{(\bar{M}_0+\bar{A}_{11})/(1-\bar{M}_0)\cos\theta_0\}\ \bar{U}_{X0} \\ &T_2=[(X_{U_X}/X_{U_{X'}})+(X_\theta Z_{U_X}/X_{U_X}Z_{\theta'})]^{-1}\cong[(C_{F_X})_0(\bar{U}_{X0})/(\bar{M}_0+\bar{A}_{11}) \\ &\qquad -\ \{(C_{F_Z})_0/(C_{F_X})_0\}\ \{(1-\bar{M}_0)\cos\theta_0/(\bar{M}_0+\bar{A}_{11})/\bar{U}_{X0}]^{-1}, \end{aligned}\right\} \quad (69)$$

where $T_1$ is very long, whereas $T_2$ is short.

The complete equations of motion in vertical plane have been given by

$$\left.\begin{aligned} &(X_{U_{X'}}\bar{D}+X_{U_X})\Delta\bar{U}_X+(X_{U_{Z'}}\bar{D}+X_{U_Z})\Delta\bar{U}_Z+(X_{\theta''}\bar{D}+X_{\theta'}\bar{D}+X_\theta)\Delta\theta \\ &\qquad\qquad =X_\delta\Delta\delta_X \\ &(Z_{U_{X'}}\bar{D}+Z_{U_X})\Delta\bar{U}_X+(Z_{U_{Z'}}\bar{D}+Z_{U_Z})\Delta\bar{U}_Z+(Z_{\theta''}\bar{D}^2+Z_{\theta'}\bar{D}+Z_\theta)\Delta\theta \\ &\qquad\qquad =Z_\delta\Delta\delta_Z+Z_\delta\Delta\delta_e \\ &(M_{U_{X'}}\bar{D}+M_{U_X})\Delta\bar{U}_X+(M_{U_{Z'}}\bar{D}+M_{U_Z})\Delta\bar{U}_Z+(M_{\theta''}\bar{D}^2 \\ &\qquad +M_{\theta'}\bar{D}+M_\theta)\Delta\theta=-(\bar{M}_0\Delta\bar{x}_G+\Delta\bar{M}\bar{x}_{G0})+\Delta\bar{x}_B+M_\delta\delta_e \end{aligned}\right\} \quad (70)$$

where $\Delta\delta_X$ and $\Delta\delta_Z$ are thrust change of installed propeller in logitudinal and vertical directions respectively and where $\Delta\delta_e$ is the deflection angle of elevator.

Crocco[10] gave approximate characteristic equation of the above system as follows:

$$(X_{U_{X'}}S+X_{U_X})\ \{(Z_{U_{Z'}}S+Z_{U_Z})(M_{\theta''}S^2+M_{\theta'}S+M_\theta)-(M_{U_Z}Z_{\theta'})S\}=0\,. \quad (71)$$

One root is nearly equal to that of equation (69). Another root can be found[11] between $-Z_{U_Z}/Z_{U_{Z'}}$ and $-(Z_{U_{Z'}}M_{\theta'}+Z_{U_Z}M_{\theta''})$.

The undamped natural frequency and the damping ratio of a pair of oscillatory roots have been approximated by equations (67).

A coupled lateral and directional motion can be characterized by equations (56a and c) by assuming $\Delta\phi=0$. By considering the nearly axisymmetrical form of the airship hull the characteristic equation can be given by

$$\left.\begin{aligned} D &= Y_{U_Y'}N_{\psi''}S^3+(Y_{U_Y}N_{\psi''}+Y_{U_Y'}N_{\psi'})S^2+(Y_{U_Y}N_{\psi'}-N_{U_Y}Y_{\psi'})S \\ &= A_2S^3+B_2S^2+C_2S=0 \end{aligned}\right\} \quad (72)$$

where $Y_{\psi''}$, $Y_\psi$ and $N_{U_Y'}$ have been negliected as small quantities and $N_\psi=0$. Assuming $\theta_0=0$ the above coefficients $A_2$ and $B_2$ are respectively equal to those given by equation (66) but $C_2$ is given by assuming $M_\theta=0$ in $C_1$ of equation (66).

Since the slender configuration has strong nonlinear characteristics in both side-force (or lift) and yawing (or pitching) moment coefficients the stability problem of the above equation is complex. In and near rectlinear flight the airship is sometimes unstable and characterized by two exponential modes represented by the following time constants:

$$\left.\begin{aligned} T_{1,2} = [&(Y_{U_Y}N_{\psi''}+Y_{U_Y'}N_{\psi'})/2Y_{U_Y'}N_{\psi''}\mp\{(Y_{U_Y}N_{\psi''}+Y_{U_Y'}N_{\psi'})^2/ \\ &4(Y_{Y'}N_{\psi''})^2-(Y_{U_Y}N_{\psi'}-N_{U_Y}Y_{\psi'})/Y_{U_Y'}N_{\psi''}\}^{1/2}]^{-1}. \end{aligned}\right\} \quad (73)$$

The unstable characteristic may not be refused because of quick response at the initiation of maneuver.

In a steady turn, however, the side force is generated to balance the centrifugal force by taking a yawed position across the flight path with a a nose-in attitude[7] and the motion becomes stable at some angle of side slip because of the large restoring moment at that angle. In such lateral motion the stability criterion can be given approximately as follows:[4,12]

$$Y_{U_Y}N_{\psi'}-Y_{\psi'}N_{U_Y}>0 \quad (74)$$

or

$$\bar{U}_{X,0}(C_{M_Z,\psi'}/C_{M_Z,\beta})>(R/V_B^{1/3})(\Delta U_Y/U_{X,0}) \quad (75)$$

where $R$ is the radius of turning and where both $(Y_{U_Y'}N_{\psi''}-Y_{\psi''}N_{U_Y'})$ and $(Y_{U_Y'}N_{\psi'}+Y_{U_Y}N_{\psi''}-Y_{\psi'}N_{U_Y'}-Y_{\psi''}N_{U_Y})/(Y_{U_Y'}N_{\psi''}-Y_{\psi''}N_{U_Y'})$ have been assumed positive. Since observational facts in the statistics meaning tell that

$$R(\Delta U_Y/U_{X,0})\propto(l_V-l_G)$$

where $l_V-l_G$ is the distance from the center of gravity to the aerodynamic center of vertical fins, equation (75) can be again approximated by[13]

$$\bar{U}_{X,0}(C_{M_Z,\psi'}/C_{M_Z,\beta})>\kappa(l_V-l_G)/V_B^{1/3} \quad (76)$$

where the proportional constant $\kappa$ is roughly given by $\kappa\cong0.9$ for conventional rigid airship.

When the criterion is satisfied the motion is characterized by the following single oscillatiory mode around a trimmed state:

$$\left.\begin{aligned} \omega_n &= \{(Y_{U_Y}N_{\psi'}-N_{U_Y}Y_{\psi'})/Y_{U_Y'}N_{\psi''}\}^{1/2} \\ 2\zeta\omega_n &= (Y_{U_Y}N_{\psi'}+Y_{U_Y'}N_{\psi'})/Y_{U_Y'}N_{\psi''}. \end{aligned}\right\} \quad (77)$$

The damping of lateral motion in cruising flight is usually very good for small airship but needs high speed for large airship to satisfy the specification of level 1 of MILL-F-83300.[5]

*Handling Quality*

The " handling quality " of airship is essentially related to the pilot's feeling in controlling or maneuvering the airships. To obtain an airship with satisfactory handling qualty for safe and efficient operation, we shall have to study the maneuver stability and handling characteristics of the airship as well as the inherent stability in the sense of both open-loop control and closed-loop or feedback control for a combined man-machine system.

If the airship is required to operate as a VTOL aircraft such as helicopter the airship must be capable to make vertical take off and landing in a confined area and to make hovering flight over a fixed small area for the handling of cargo sling loads, the rescueing victims and so on. Then the requirements on the handling quality are similar to those of the helicopter.

For the requirements of desirable handling quality of the helicopter and V/STOL aircraft, the said MIL-F-83300[5] and MIL-H-8501A[6] have been well known. Here, the maneuver stability and the handling characteristics of the airships will be discussed in simplified methods.

Since a pilot controls an airship on which he is riding with his control forces through mechanical control systems the control systems must be movable by a moderate maneuvering force and in a proper range of control travel in order to attain precisely an intended flight condition without resulting pilot fatigue. Thus proper breakout forces including friction, preload, etc., should be mained within some limits which are comparable numbers to those of other aircraft. All unbalanced control forces, even those small magnitude, are objectionable during lengthy manual flight as well as instrument flight and means must be provided for training such forces to zero about all axes.

Similarly, the control force gradient must be kept within desirable limits and be harmonized in all axes. In cruising flight, however, the feeling on the steering will be similar to those of large ships or boats. The requirements related to the control travel may also be similar to those of ships.

The airship is, at all flight speeds, required to posses the positive control position stability as well as static longitudinal control force with respect to the speed. This trim stability is desirable to cover possibly the rearward flight if necessary and descending flights too.

The control sensitivity is determined by a " control power " which is the yawing (or pitching) moment per unit stick input and by a " damping " which is the resisting moment per unit yawing (or pitching) angular velocity of the airship. The criteria on the sensitivity of the V/STOL aircraft will be good guidance for determining the sensitivity of the airships.

The control moment about yawing axis of the exemplified airships can be obtained by taking rudder deflection and differential thrust change between right and left thrusters, whereas the control moment about pitching axis is

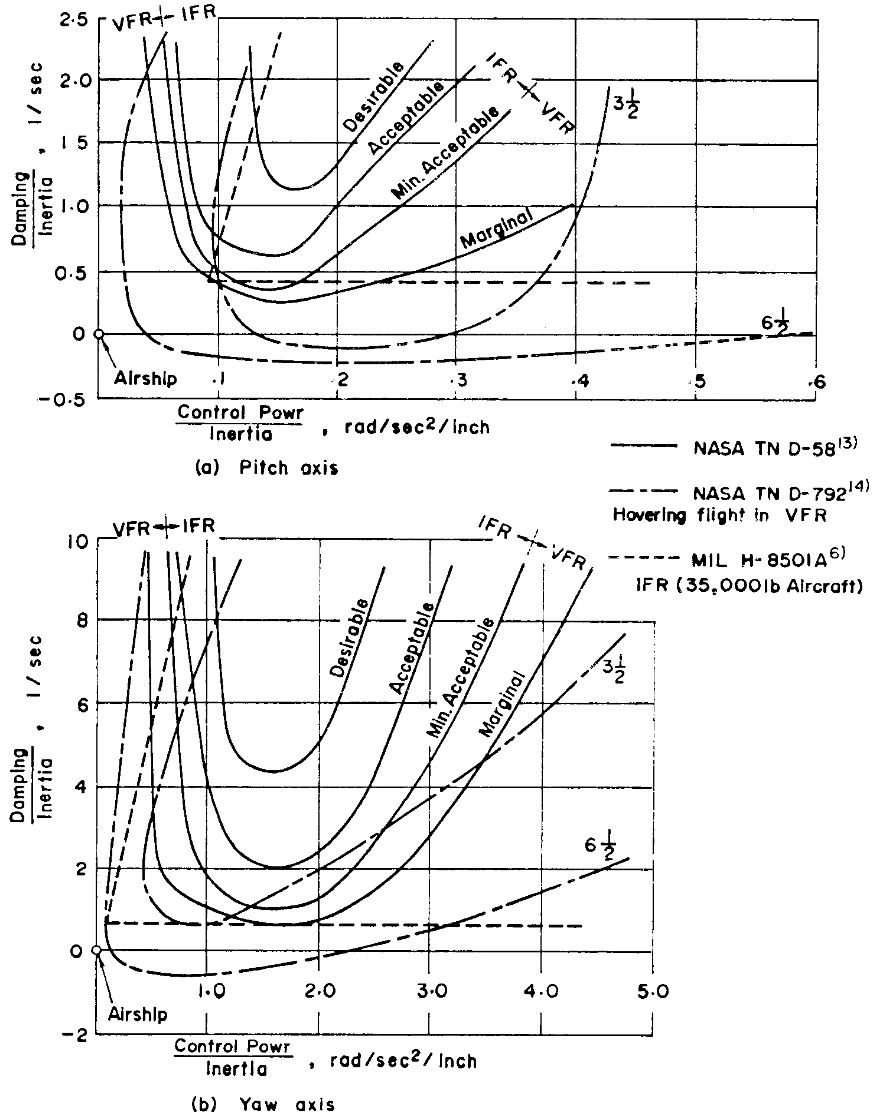


FIG. 6. Sensitivity requirements for helicopter at hovering flight.

obtained by taking elevator deflection and transfering the trim weight along longitudinal axis or by taking the differential thrust change of vertical thrusters if installed, between front and rear lifting thrusters in vertical direction. The dampings in yawing and pitching motions are considered to be resulted from only the aerodynamic damping moment created by the fuselage, tail surfaces, and a little gas motion in the envelope.

There is no positive control system on the rolling motion of airship because of the buoyant force creates inherent rolling stability by couping with the gravity force, and because the inclined attitude about rolling axis can not create any side force.

Typical requirements on the control power and damping of the helicopter about pitching and yawing axes are shown in Fig. 6. These sensitivity requirements of the helicopters are resulted from their tasks conducted at and near the hovering flight. The control moment and damping of the all four exemplified airships, which have been illustrated in the preceding section, at and near the hovering flight are very close to the origin of coordinates in this figure. Table 2 gives the angular acceleration as well as linear acceleration along respective axis of the exemplified airships.

It will, from Fig. 6 and Table 2, be well recognized that the airships in conventional configuration are great lack of control moment and of damping in yawing and pitching motion, specifically so for larger airships. This is due to the following reason: The moment of inertia plus added mass of moment or virtual moment of inertia increases with fifth power of its linear dimension, while the necessary thrust to overcome the drag of the airship is proportional to the square of the linear dimension.

In addition, as seen from the Table 2, the worse characteristics in the control sensitivity or fatal deficiencies in the control power and damping of the airships at hovering flight reach the extreme in the translational motion or longitudinal, lateral and vertical translations. Usually, in the longitudinal translation the linear acceleration resulted by the full thrust operation should be at least 0.1G for quick response in helicopter and V/STOL operation.

In the lateral motion of the helicopter, a tilt of the tip path plane, which is almost suddenly resulted by taking necessary cyclic pitch change of the rotor blades, produces not only the lateral force in the rotor tilted direction but also the moment about the center of gravity by which the additional side force is derived after the body has been followed to the control moment about the center of gravity. The resulted side force is the product of sine of the tilt angle of the rotor tip path plane with respect to the horizonal plane and the rotor thrust which is nearly equal to the helicopter weight.

In the lateral motion of the airship, on the other hand, there is no control system in rolling direction. Even if it is prepared, the rolling of the hull can not create any side force because the most of weight of the airship is sustained by the buoyant force and only the restoring rolling moment is produced.

Thus in the hovering flight without any wind the direct lateral translation is impossible for the airship. Only way to change the position in lateral direction is to change the azimuthal direction of the airship and then moves to that direction and must stop again over an aimed point. Even turning motion spends time of order of seconds.

For the control of translation in vertical direction, the airship must have a valve mechanism which can change the buoyant force by either inflating or deflating the gas bag filled with helium or hydrogen gas, or a dropping device which can cast the ballast. The control of the buoyant force is one of slow response and the casting of the ballast is accompanied with danger on the ground. In the helicopter, the time contant of the vertical motion is well in the order of a few seconds by taking collective pitch change of the rotor. If the propellers can be tilted to vertical direction to make the vertical motion of airships, then the maximum accelerations in the vertical direction are those given in Table 2. These facts will make the airship to be difficult to stay at a fixed area over the aimed point on the ground.

It will, therefore, be appreciated that if the pilot of airship is required to perform the same job as that of the helicopter he will have to feel slugish during his operation. The airship is much more closely akin to that of the ship than of the aircraft. In order to improve the situation the airship must provide with propellers directed to the sideways and vertically upways.

## Conclusion

The fundamental analyses on the airship performance and flight dynamics and their formutation have been presented. The vehicle efficiency which is a means of the transportation efficiency of the vehicle can be increased by increasing the propeller efficiency, gas volume or size of the airship, and by reducing the empty weight, flight speed, range of flight, the fuel consumption, and the flight altitude.

For winged or hybrid airship the critical speed has been introduced as an important factor to compare the vehicle efficiency with that of the plain airships. The critical speed increases by increasing the airship size and by reducing the lift-drag ratio. The good vehicle efficiency can be obtained by taking the maximum lift-drag ratio.

From the performance calculations of four exemplified airships the gross weight of which are more than about 500 ton have good vehicle efficiency comparing with other vehicles such as liner boat, hydrofoil ship, motor truck, and rail way vehicle in the operational speed range of 50 to 200 km/hr.

Nondimensional equations of motion of the airship have been established, by which unified treatment on the stability and control problems has been possible for any size of the airships. The handling qualities of airships are very poor specifically for larger sizes because of small control powers against huge virtual and moment of inertia of the hull. The linear and angular accelerations and dampings of the exemplified airships have been obtained and showed comparing with the sensitivity requirements of the helicopter. It will, thus, be very difficult for the large airships in size to make landing in bad weather, hovering flight over an aimed point on the ground, and so on.

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## Appendix A VERTICAL OSCILLATION AROUND A TRIMMED ALTITUDE*

When the buoyant force is considered to be a function of altitude the

* The author is indebted to Mr. N. Kawakami, Fuji Heavy Industries, LTD. for the development of anolysis.

equation of motion in a single degree-of-freedom for vertical direction can be written as

$$(M+A_{22})\ddot{H}=F_B-W-D\,. \tag{A-1}$$

If the drag force, D, may be neglected, then a perturbation equation for $H=H_0+\Delta H$ can be given as

$$\left.\begin{aligned}
(M+A_{22})\Delta\ddot{H}&=\Delta F_B\\
&=\rho_{a0}g\,V_G\Delta[(T_i/T_a)(P_a/P_0)\{1-0.379\phi(P_{Tc}/P_a)-s_0(T_a/T_g)\}]\\
&\cong\rho_{a0}g\,V_G(T_i/T_0)\Delta\{(T_0/T_a)(P_a/P_0)\}\\
&=-\rho_{a0}g\,V_G(T_i/T_0)(n-1)\{1-(aH_0/T_0)\}^{n-2}(a/T_0)\Delta H\\
&=-F_B(n-1)(a/T_a)\Delta H
\end{aligned}\right\} \tag{A-2}$$

Since $A_{22}$ can be approximated roughly by

$$A_{22}\cong M=W/g=F_B/g\,, \tag{A-3}$$

the above equation yields

$$\Delta\ddot{H}+(1/2)(n-1)g(aT_a)\Delta H=0\,. \tag{A-4}$$

Thus the period of this motion becomes

$$P=2\pi\sqrt{(1/2)g(n-1)(a/T_a)}\cong 300\text{ sec} \tag{A-1}$$

in usual operational range. It is very much interesting to say that the period of this oscillation around a trimmed altitude is independent of the size of the airship and is so long that the damping due to the vertical drag may not be neglected. Hence, equations (55b) and (65) are valid in the present approximation.

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