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Sound Proapgation and Radiation through a Vaneless Diffuser

By

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Abstract: A simple analysis has been carried out to examine acoustic characteristics of a vaneless diffuser through which incident plane waves propagate and radiate out circumferentially. This involves the evaluation of the radiation impedance of circumferential openings as well as the longitudinal wave motion in which the sound speed varies locally. An effect of the through flow deceleration is also examined. The results are shown in the form of a reflection coefficient which expresses the ratio of the incident and reflected waves at the diffuser inlet.

1. INTRODUCTION

A particular problem posed in the study of noise generation in axial fan and/or compressor is that the presence of duct walls and intake bells or outlet diffusers will influence the spatial distribution of the acoustic energy travelling away from the fan, so the overall radiated noise levels.

A complete study on that subject has been carried out by Tyler and Sofrin (1962), Morfey (1964), and several fundamental facts, such as spinning acoustic modes and the corresponding duct cut-off phenomenon, have been correctly pointed out. In his paper (1964, 1969) Morfey also calculated the modal acoustic radiation impedance of a flanged annular opening, the concept of which had been developed by Rayleigh (1945) and Morse (1948) in the classical acoustic theory. A general analysis for the radiated noise level dependence upon the rotor speed, including the calculation of the acoustic impedance of the duct open end, has been done by Kaji and Okazaki (1972) who obtained excellent results based on a simple piston model replacing the actual sound source.

The cases so far handled are for the annular openings with baffle plates, which therefore send sounds more or less axially. In practice we encounter a situation in which the sounds propagating axially are often bent and radiated out through a diffuser towards the radial direction. The effects of this will be twofold, since firstly the acoustic impedance of circumferential openings might be different from that of annular open ends, and secondly the propagating surface area changes as the sound waves go along the diffuser passage. Further there is a question of the sound transmission around the annulus corner bend.

In this paper we therefore examine acoustic characteristics of a vaneless diffuser through which incident plane waves propagate and radiate out circumferentially. An effect of the through flow which yields some wave convection will be also examined.

For the sake of simplicity, the following assumptions are made;

(i) The perturbations are small in amplitude so that the first order approximation is valid.

(ii) A mean flow is present, but its circumferential velocity component is zero; i. e., no swirling flow is considered.

(iii) The flow is isentropic and the fluid is ideal.

(iv) The area change in the diffuser is only gradual and the wave length is large compared with the passage height.

The last assumption will be a weak point of the present analysis, since the corner bend is sometimes sharp enough to invalidate the approximation of quasi-one dimensional flow. The proper handling of this region, however, will involve much numerical work employing some three dimensional flow calculation methods, which has been avoided here. The effect of the annular corner bend upon the sound transmission therefore remains to be solved.

2. Formulation

Suppose that the incident sound waves A^+ propagate towards the longitudinal direction along the passage and radiate out through the diffuser into the semi-infinite open space bounded by a baffle plate. (Fig. 1)

Now the questions are the followings;

(i) How the sound waves propagate through the diffuser?

(ii) How much reflection occurs at the diffuser outlet?

(iii) What is the through flow effect on the diffuser acoustic characteristics?

We start from the calculation of radiation impedance at the diffuser outlet.

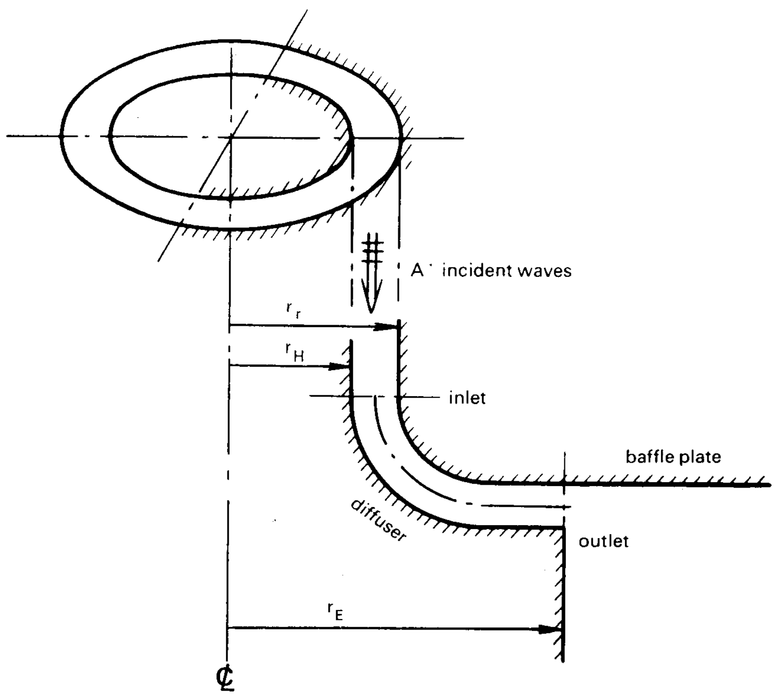

Fig. 1. Analysis model

2. 1. *Radiation impedance of radial diffuser outlet*

Firstly we remark that the presence of a baffle plate imposes the vanishing normal velocity there. This condition can be easily satisfied by considering the image of discharging flow against the baffle plate. (Fig. 2)

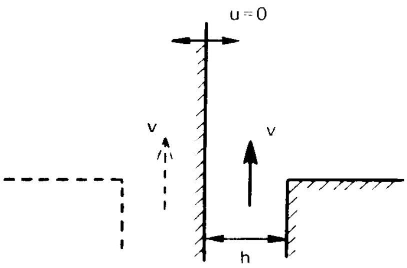

FIG. 2. Boundary condition.

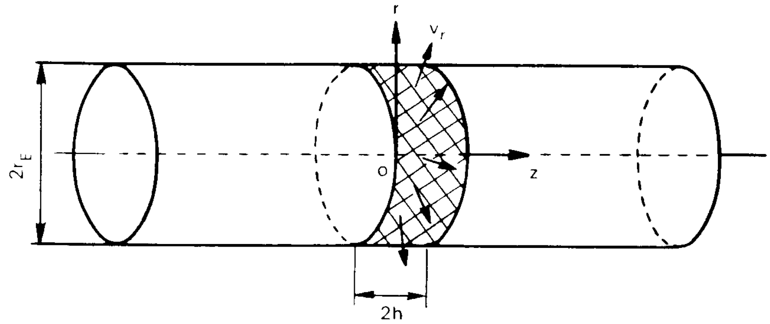

FIG. 3. A strip of equivalent acoustic source

Therefore we can simplify the problem, so that a strip of the cylindrical boundary surface ($-h \leqq z \leqq h$, $r = r_E$) fluctuates with the particle velocity v_r which is symmetrical against $z=0$. (Fig. 3)

Here we neglect the uniform discharging flow velocity V_r.

Equivalent simple source strength q may be expressed as follows, using a vector notation $\boldsymbol{r}$ for the coordinates (r, θ, z) in the open space;

$$q = q_E(\boldsymbol{r}) \cdot e^{i\omega t} \equiv \rho_0 \cdot v_{r_E}(r, \theta, z) \cdot \frac{1}{r} \cdot \delta(r - r_E) \cdot e^{i\omega t} \tag{2.1}$$

The corresponding basic equations valid in the open space are

$$\begin{aligned} &\frac{\partial \rho}{\partial t} + \rho_0 \nabla \cdot \boldsymbol{v} = q \\ &\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{1}{\rho_0} \nabla p \\ &p = c_0^2 \cdot \rho;\ c_0^2 \equiv \gamma \frac{p_0}{\rho_0} \end{aligned} \tag{2.2}$$

where

p_0, ρ_0, c_0 are the pressure, density and sound speed in the uniform open space, while p, ρ, $\boldsymbol{v}$ are the respective perturbations.

Equation (2.2) yields the wave equation written as

$$\square \cdot p(r, \theta, z; t) = -\frac{\partial q}{\partial t};\ \square \equiv \triangle - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \tag{2.3}$$

where $\triangle$ is the Laplacian.

This should be solved under the following boundary conditions;

(i) $\partial p / \partial r = 0$ at $r = r_E$, since the normal velocity at the cylinder surface vanishes.

(ii) p takes the form of outgoing waves as $r \to \infty$.

The Green function of (2.3) satisfying the boundary conditions can be obtained and the solution is written as

$$p(r,\theta,z;t)=\mathrm{e}^{i\omega t}\int_{-\infty}^{\infty}\int_{0}^{2\pi}\int_{1}^{\infty}(-)i\omega\cdot q_E(\boldsymbol{r}_0)\cdot G(\boldsymbol{r}/\boldsymbol{r}_0)\cdot r_0dr_0d\theta_0dz_0 \tag{2.4}$$

where all the length parameters are nondimensionalized by the opening radius r_E, and

$$G(\boldsymbol{r}/\boldsymbol{r}_0)=\frac{(-)}{4\pi}\sum_{m=-\infty}^{\infty}\mathrm{e}^{-im(\theta-\theta_0)}\int_0^\infty\frac{R_m(\mu r)\cdot R_m(\mu r_0)}{J_m'^2(\mu)+N_m'^2(\mu)}\cdot\mathrm{e}^{-\sqrt{\mu^2-k^2}|z-z_0|}\cdot\frac{\mu d\mu}{\sqrt{\mu^2-k^2}} \tag{2.5}$$

$$k=\omega r_E/c_0$$

$$R_m(\mu r)=J_m(\mu r)\cdot N_m'(\mu)-J_m'(\mu)\cdot N_m(\mu r)$$

where J_m, N_m are Bessel and Neumann functions of order m and prime implies the differentiation with respect to the argument.

In the case of circumferentially uniform discharge,

$$q_E(\boldsymbol{r}_0)=\rho_0\cdot v_r(z_0)\cdot\frac{1}{r_0}\delta(r_0-1) \tag{2.6}$$

and we reduce (2.4) to the following form,

$$p(r,z;t)=\mathrm{e}^{i\omega t}\int_{-\infty}^{\infty}\rho_0\cdot c_0\cdot v_r(z_0)\cdot dz_0\cdot\zeta(r,z-z_0;k) \tag{2.7}$$

where

$$\zeta(r,z-z_0,k)=\frac{ik}{2}\int_0^\infty\frac{R_0(\mu r)\cdot R_0(\mu)}{J_1^2(\mu)+N_1^2(\mu)}\cdot\mathrm{e}^{-\sqrt{\mu^2-k^2}|z-z_0|}\cdot\frac{\mu d\mu}{\sqrt{\mu^2-k^2}} \tag{2.8}$$

If $v_r(z_0)$ is given for $0\leqq z_0\leqq h$, we write

$$v_r(z_0)=\sum_{n=0}^{\infty}v_n\cdot\cos\frac{\pi n}{h}z_0;\quad -h\leqq z_0\leqq h \tag{2.9}$$

where

$$v_n=\frac{\varepsilon_n}{2h}\cdot\int_{-h}^{h}v_r(z_0)\cdot\cos\frac{\pi n}{h}z_0\cdot dz_0;\quad \varepsilon_n=\begin{cases}2\ ;\ n\neq0\\1\ ;\ n=0\end{cases}$$

Correspondingly the pressure fluctuation at the diffuser outlet is expressed as a Fourier cosine series,

$$p(1,z;t)=\mathrm{e}^{i\omega t}\sum_{\nu=0}^{\infty}p_\nu\cdot\cos\frac{\pi\nu}{h}z;\quad -h\leqq z\leqq h \tag{2.10}$$

Then, (2.7) and (2.9) yield,

$$\begin{aligned}p_\nu&=\sum_{n=0}^{\infty}\rho_0\cdot c_0\cdot v_n\cdot\frac{\varepsilon_\nu}{2h}\int_{-h}^{h}\zeta(1,z-z_0;k)\cdot\cos\frac{\pi n}{h}z_0\cdot\cos\frac{\pi\nu}{h}z\cdot dz_0\cdot dz\\&\equiv\sum_{n=0}^{\infty}\rho_0\cdot c_0\cdot v_n\cdot\zeta_{\nu n}(k)\end{aligned} \tag{2.11}$$

The modal radiation impedance may be defined by $\zeta_{\nu n}$, since it gives the contribution of n -mode particle velocity perturbation at the diffuser outlet to ν -mode sound pressure level there.

Manipulation shows,

$$\zeta_{\nu n}(k)=i\frac{4k}{\pi^2}\int_0^\infty \frac{1}{J_1^2(\mu)+N_1^2(\mu)}\cdot\left[\frac{\delta_{n\nu}}{\mu^2-k^2+\left(\frac{\pi n}{h}\right)^2}\right.$$

$$\left.-\frac{(-)^{\nu-n}\cdot\varepsilon_\nu\cdot(\mu^2-k^2)}{\left\{\mu^2-k^2+\left(\frac{\pi n}{h}\right)^2\right\}\left\{\mu^2-k^2+\left(\frac{\pi\nu}{h}\right)^2\right\}}\cdot\frac{1-e^{-2h\sqrt{\mu^2-k^2}}}{2h\sqrt{\mu^2-k^2}}\right]\cdot\frac{d\mu}{\mu} \tag{2.12}$$

where

$$\delta_{n\nu}=\begin{cases}1 & ;\ n=\nu\\ 0 & ;\ n\neq\nu\end{cases}$$

2. 2. Sound propagation through the diffuser

Within the diffuser region it is convenient to take the coordinate ξ along the passage. (Fig. 4) The basic equations of mass continuity, momentum and isentropy may be written as follows;

$$\frac{d}{d\xi}(R\cdot S\cdot W)=0$$

$$R\cdot S\cdot\frac{D}{Dt}\left(\frac{\rho}{R}\right)+\frac{\partial}{\partial\xi}(R\cdot S\cdot w)$$

$$+\frac{1}{r}\frac{\partial}{\partial\theta}(R\cdot S\cdot v_\theta)=0 \tag{2.13}$$

$$W\frac{dW}{d\xi}=-\frac{1}{R}\frac{dP}{d\xi}$$

$$\frac{D}{Dt}(R\cdot S\cdot w)+2R\cdot S\cdot w\cdot\frac{dW}{d\xi}=-S\cdot\frac{\partial p}{\partial\xi}$$

$$-\frac{\rho}{R}\cdot R\cdot S\cdot W\frac{dW}{d\xi} \tag{2.14}$$

$$\frac{D}{Dt}v_\theta=-\frac{1}{R}\cdot\frac{1}{r}\frac{\partial p}{\partial\theta} \tag{2.15}$$

$$\frac{P}{R^\gamma}=\text{const}$$

$$\frac{p}{P}=\gamma\frac{\rho}{R};\ c^2\equiv\gamma\frac{P}{R} \tag{2.16}$$

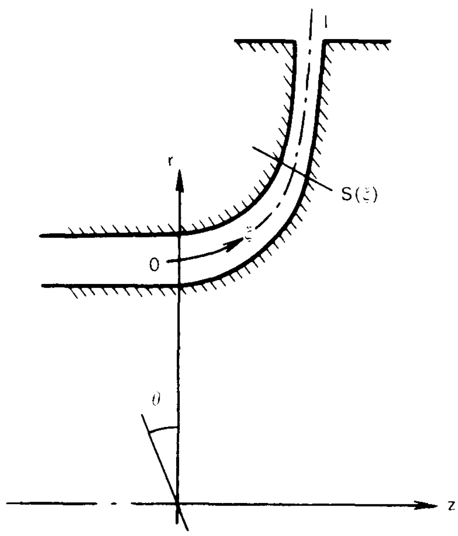

FIG. 4. ξ-coordinate along diffuser passage

where

$S(\xi)$, $r(\xi)$ are the cross section area and the radius height, respectively. Flow parameters are $P+p$, $R+\rho$, $(W+w)e_\xi+v_\theta\cdot e_\theta$, in which P, R, W are pressure, density and flow speed of the main stream, while p, ρ, w, v_θ are the corresponding perturbations taking the harmonic wave form of $\exp(i\omega t+im\theta)$ and e_ξ, e_θ are the unit vectors towards ξ, θ direction. Thus,

$$\frac{1}{r}\frac{\partial}{\partial\theta}\equiv\frac{im}{r},\quad \frac{D}{Dt}\equiv i\omega+W\frac{d}{d\xi}$$

Note that the sound speed C depends upon ξ.

They can be arranged into more appropriate forms to yield the wave equation, e. g.

$$\left(i\omega+2\frac{dW}{d\xi}\right)\frac{w}{W}=\frac{c^2}{\gamma W}\left[\left\{i\omega+(\gamma-1)\frac{dW}{d\xi}\right\}\frac{W}{c^2}-\left(1-\frac{W^2}{c^2}\right)\frac{\partial}{\partial\xi}\right]\frac{p}{P}+W\cdot\frac{im}{r}\frac{v_0}{W} \tag{2.17}$$

$$L_\xi\left[\frac{p}{P}\right]=\frac{\gamma}{c^2}W\cdot\left\{2\frac{dW}{d\xi}-\frac{W}{r}\frac{dr}{d\xi}-\frac{2W\frac{d^2W}{d\xi^2}}{i\omega+2\frac{dW}{d\xi}}\right\}\cdot\frac{im}{r}\frac{v_\theta}{W} \tag{2.18}$$

$$W\frac{\partial}{d\xi}\left(\frac{im}{r}\cdot\frac{v_0}{W}\right)=\frac{c^2}{\gamma W}\cdot\frac{m^2}{r^2}\cdot\frac{p}{P}-\left\{i\omega+\frac{1}{r}\frac{d}{d\xi}(rW)\right\}\cdot\frac{im}{r}\frac{v_\theta}{W} \tag{2.19}$$

where

$$L_\xi\equiv\frac{1}{SP}\frac{\partial}{\partial\xi}\left(SP\frac{\partial}{\partial\xi}\right)-\frac{1}{c^2}\frac{D^2}{Dt^2}-\frac{m^2}{r^2}-\frac{i\omega}{c^2}\left(2\frac{dW}{d\xi}+\frac{\gamma-3}{i\omega+2\frac{dW}{d\xi}}\cdot W\frac{d^2W}{d\xi^2}\right)$$

$$+\left\{\frac{\gamma+1}{c^2}W\frac{dW}{d\xi}+\left(1-\frac{W^2}{c^2}\right)\frac{2\frac{d^2W}{d\xi^2}}{i\omega+2\frac{dW}{d\xi}}\right\}\cdot\frac{d}{d\xi}$$

The solution can be easily recognized for the following extreme cases;

(i) Constant area annular duct, in which S_0. P_0, R_0, W_0 are all constants and r means the radial distance up to the mean passage surface. (Fig. 5) (2.18) becomes

$$\left[\frac{\partial^2}{\partial\xi^2}-\frac{1}{c_0^2}\frac{D^2}{Dt^2}-\frac{m^2}{r^2}\right]\frac{p}{P_0}=0$$

whence

$$\frac{p}{P_0}\sim\exp\left\{i\omega t+im\theta+\frac{i\frac{\omega}{c_0}M_0\pm\sqrt{(1-M_0^2)\frac{m^2}{r^2}-\left(\frac{\omega}{c_0}\right)^2}}{1-M_0^2}\cdot\xi\right\}$$

This is well known as the solution for an annulus duct of high hub/tip ratio. The cut-off frequency is given by

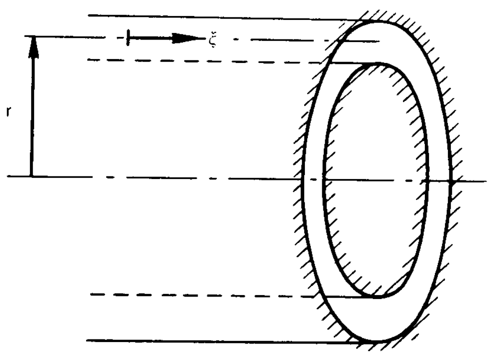

FIG. 5. Annular duct of large boss ratio

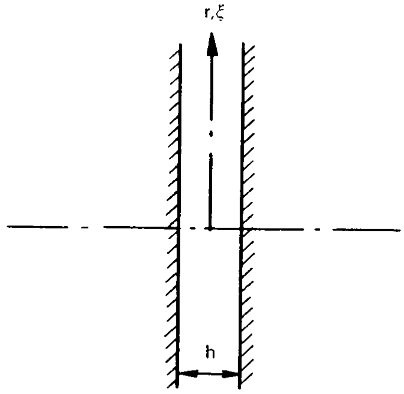

FIG. 6. Coaxial parellel disks

$$\frac{\omega}{c_0} \leqq \frac{m}{r}\sqrt{1-M_0^2}$$

(ii) Coaxial parallel disks, in which no through flow is present. (Fig. 6) ξ can be identified as r and $S=\pi rh$. (2.18) yields

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{\omega^2}{c_0^2}-\frac{m^2}{r^2}\right]\frac{p}{P_0}=0$$

whence

$$\frac{p}{P_0}\sim e^{i\omega t+im\theta}\cdot\begin{bmatrix} H_m^{(1)}\left(\frac{\omega}{c_0}r\right) \\ H_m^{(2)}\left(\frac{\omega}{c_0}r\right)\end{bmatrix}$$

In the present analysis the attention will be restricted to the problem of circumferentially uniform cases, so the value of m is chosen to be zero or $(\partial/\partial\theta)=0$, hereafter.
Then (2.15) yields, directly

$$\frac{r\cdot v_\theta}{r\cdot v_\theta|_{\xi=0}}=\exp\left[-i\omega\int_0^\xi\frac{d\xi}{W}\right] \tag{2.20}$$

which shows the conservation of angular momentum.
In the inlet section of the diffuser ($\xi\leqq 0$), (2.18) becomes a familiar plane wave equation, since S_0, P_0. R_0. W_0 are all constants, i. e.

$$\left[\frac{\partial^2}{\partial\xi^2}-\frac{1}{c_0^2}\frac{D^2}{Dt^2}\right]\frac{p}{P_0}=0;\quad c_0^2\equiv\gamma\frac{P_0}{R_0}$$

whence we obtain

$$\frac{p}{P_0}=e^{i\omega t}\cdot\left(A^-\cdot\exp\left[i\frac{\omega/c_0}{1-M_0}\cdot\xi\right]+A^+\cdot\exp\left[-i\frac{\omega/c_0}{1+M_0}\cdot\xi\right]\right)$$

$$\frac{w}{W_0}=e^{i\omega t}\cdot\frac{1}{\gamma M_0}\left(-A^-\cdot\exp\left[i\frac{\omega/c_0}{1-M_0}\cdot\xi\right]+A^+\cdot\exp\left[-i\frac{\omega/c_0}{1+M_0}\cdot\xi\right]\right) \tag{2.21}$$

where M_0 is the Mach number of the inlet flow and A^+, A^- are the amplitude of the incident and reflected waves, respectively.

On the other hand, the solution of (2.18) will have to be found numerically for the diffuser section ($0\leqq\xi\leqq l$). In general it consists of two oppositely going waves, which we express as

$$\frac{p}{P}=e^{i\omega t}\cdot(B^-\cdot e^{i\lambda^-\cdot\xi}+B^+\cdot e^{i\lambda^+\cdot\xi}) \tag{2.22}$$

where $B^\pm$, $\lambda^\pm$ are the amplitude and the wave number of corresponding waves and are functions of such parameters as ξ, c, W, $(dW/d\xi)$, $(d^2W/d\xi^2)$, ω.

Then, (2.17) allows us to evaluate w, thereby the acoustic impedance of the propagating waves. Since the radiation impedance at the diffuser outlet $\xi=l$ is known, we can proceed the calculation to obtain the impedance at the diffuser inlet $\xi=0$ by firstly assuming the value of B^- (B^+ being fixed to be 1.0) and solving (2.18) and (2.17) employing, for instance, R K G method to check whether the impedance thus calculated agrees with the known value at the diffuser outlet.

When the frequency is relatively high and the cross section area varies so that linear deceleration should hold, we can obtain an approximate solution in a closed analytical form, e. g.
putting

$$\frac{dW}{d\xi}=\frac{W_l-W_0}{l}=-\varepsilon\frac{W_0}{l},\ \varepsilon \text{ deceleration rate (constant)}>0 \tag{2.23}$$

or

$$W=W_0\left(1-\varepsilon\frac{\xi}{l}\right)$$

we have

$$\left.\begin{aligned}
&\frac{S}{S_0}=\frac{M_0}{M}\left(\frac{1+\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M_0^2}\right)^{\frac{1}{2}\frac{\gamma+1}{\gamma-1}}\\
&\frac{c}{c_0}=\sqrt{\frac{1+\frac{\gamma-1}{2}M_0^2}{1+\frac{\gamma-1}{2}M^2}}\\
&\frac{M}{M_0}\sqrt{\frac{1+\frac{\gamma-1}{2}M_0^2}{1+\frac{\gamma-1}{2}M^2}}=1-\varepsilon\cdot\frac{\xi}{l}
\end{aligned}\right\}\ ;\quad \begin{aligned}M&=\frac{W}{c}\\ M_0&=\frac{W_0}{c_0}\end{aligned} \tag{2.24}$$

Relationships (2.24) describe the geometry and flow condition along the diffuser passage. Now, introducing nondimensional frequency parameter $\bar{\omega}$ by

$$\bar{\omega}=\omega\bigg/\frac{dW}{d\xi} \tag{2.25}$$

equations (2.17) and (2.18) yield

$$(i\bar{\omega}+2)\frac{w}{W}=\frac{1}{\gamma}\left[i\bar{\omega}+\gamma-1-(1-M^2)\cdot\frac{c^2}{W\cdot\frac{dW}{d\xi}}\cdot\frac{d}{d\xi}\right]\frac{p}{P} \tag{2.26}$$

$$\begin{aligned}&\left[(1-M^2)\frac{d^2}{d\xi^2}-2i\bar{\omega}\cdot M\cdot\frac{1}{c}\frac{dW}{d\xi}\left(1+\frac{\gamma+1}{i\bar{\omega}}+\frac{1-M^2}{2i\bar{\omega}M^2}\right)\frac{d}{d\xi}\right.\\
&\left.\quad+\frac{\bar{\omega}^2}{c^2}\cdot\left(\frac{dW}{d\xi}\right)^2\left(1-\frac{2i}{\bar{\omega}}\right)\right]\cdot\frac{p}{P}=0\end{aligned} \tag{2.27}$$

Retaining the terms of the highest order of $\bar{\omega}$, WKB method gives the following solution;

$$\begin{aligned}\frac{p}{P}=e^{i\omega t}\cdot\Big\{&B^-\cdot\exp\left[-i\bar{\omega}\varepsilon M_0\cdot\int_0^{\xi}\frac{1}{1-M}\cdot\frac{c_0}{c}\cdot d\xi\right]\\
&+B^+\cdot\exp\left[i\bar{\omega}\varepsilon M_0\cdot\int_0^{\xi}\frac{1}{1+M}\cdot\frac{c_0}{c}\cdot d\xi\right]\Big\}\end{aligned} \tag{2.28}$$

$$\frac{w}{W}=\frac{e^{i\omega t}}{\gamma\cdot M}\cdot\left\{-B^{-}\cdot\exp\left[-i\bar{\omega}\varepsilon M_0\cdot\int_0^{\bar{\xi}}\frac{1}{1-M}\cdot\frac{c_0}{c}\cdot d\bar{\xi}\right]\right.$$
$$\left.+B^{+}\cdot\exp\left[i\bar{\omega}\varepsilon M_0\cdot\int_0^{\bar{\xi}}\frac{1}{1+M}\cdot\frac{c_0}{c}\cdot d\bar{\xi}\right]\right\} \tag{2.28}$$

where $B^{\pm}$ are constants and $\bar{\xi}=\xi/l$.
The solution (2.28) physically indicates that the waves are convected by the local mean flow stream. We can show that it reduces to the form (2.21) when the main flow velocity remains to be constant. ($\varepsilon=0$, $W=W_0\neq 0$, $S=S_0$).

In particular the limiting case of no wind can be considered, in which equation (2.18) takes a very simple form; i. e.

$$\left[\frac{1}{S}\frac{\partial}{\partial\xi}\left(S\cdot\frac{\partial}{\partial\xi}\right)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right]\frac{p}{P_0}=0;\ c_0^2=\gamma\cdot\frac{P_0}{R_0} \tag{2.29}$$

where $W_0=0$ and P_0, R_0 are constant.
Discussions are focussed on a diffuser having similar geometry. Equation (2.24) yields the following variation of the cross section area as $M(=M_0)$ becomes zero;

$$\frac{S}{S_0}=\frac{M_0}{M}\left(\frac{1+\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M_0^2}\right)^{\frac{1}{2}\frac{\gamma+1}{\gamma-1}}=\frac{1}{1-\varepsilon\cdot\bar{\xi}}\sqrt{\frac{1+\frac{\gamma-1}{2}M_0^2}{1+\frac{\gamma-1}{2}M^2}}\left(\frac{1+\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M_0^2}\right)^{\frac{1}{2}\frac{\gamma+1}{\gamma-1}}$$
$$\to\frac{1}{1-\varepsilon\cdot\bar{\xi}} \tag{2.30}$$

The apparent deceleration rate ε_0 in the case of no through flow is therefore given by $\varepsilon_0=1-1/(S_l/S_0)$, S_l/S_0 is the diffuser outlet/inlet area ratio.
The solution of (2.29) is obtained, i. e.

$$\frac{p}{P_0}=(1-\varepsilon_0\cdot\bar{\xi})\cdot\left\{B_0^{-}\cdot H_1^{(2)}\left[\frac{\omega l/c_0}{\varepsilon_0}(1-\varepsilon_0\cdot\bar{\xi})\right]+B_0^{+}\cdot H_1^{(1)}\left[\frac{\omega l/c_0}{\varepsilon_0}(1-\varepsilon_0\cdot\bar{\xi})\right]\right\} \tag{2.31}$$

which represents the superposition of outgoing and incoming waves. $B_0^{\pm}$ are constant and $H_1^{(1)}$, $H_1^{(2)}$ are Hankel functions of 1st and 2nd kind, order 1. Corresponding velocity perturbation can be found by (2.17), e. g.

$$i\omega w=-\frac{c_0^2}{\gamma}\frac{d}{d\xi}\frac{p}{P_0}$$

whence

$$\frac{w}{c_0}=\frac{1}{i\gamma}(1-\varepsilon_0\cdot\bar{\xi})\cdot\left\{B_0^{-}\cdot H_0^{(2)}\left[\frac{\omega l/c_0}{\varepsilon_0}(1-\varepsilon_0\cdot\bar{\xi})\right]+B_0^{+}\cdot H_0^{(1)}\left[\frac{\omega l/c_0}{\varepsilon_0}(1-\varepsilon_0\cdot\bar{\xi})\right]\right\} \tag{2.32}$$

2. 3. *Reflection coefficient*

The presence of the diffuser results in a change in the reflected waves or in the acoustic impedance at the duct opening, as viewed from the otherwise terminated upstream duct in which the incident waves originate. The reflection coefficient χ is defined by A^{-}/A^{+} which is equal to B^{-}/B^{+} at the diffuser inlet $\bar{\xi}=0$.

According to the approximate solutions of (2.28), that can be expressed by

$$\chi=\frac{\zeta_l-1}{\zeta_l+1}\cdot\exp\left(i\bar{\omega}\varepsilon M_0\cdot\int_0^1\frac{2}{1-M^2}\cdot\frac{c_0}{c}\cdot d\bar{\xi}\right)$$

where ζ_l is the radiation impedance at the diffuser outlet $\xi=l$, and is given by putting $n=\nu=0$ in (2.12).

The integral appearing in the argument of the exponent can be performed by substituting the relationship (2.24), e. g.

$$\tau(M_0, M_l)\equiv\varepsilon M_0\cdot\int_0^1\frac{2}{1-M^2}\cdot\frac{c_0}{c}\cdot d\bar{\xi}=\varepsilon M_0\cdot\int_{M_0}^{M_l}\frac{2}{1-M^2}\cdot\frac{c_0}{c}\cdot\frac{d\bar{\xi}}{dM}\cdot dM$$

$$=\ln\frac{1-M_l}{1-M_0}\cdot\frac{1+M_0}{1+M_l}\cdot\left(\frac{1-M_0^2}{1-M_l^2}\cdot\frac{1+\frac{\gamma-1}{2}M_l^2}{1+\frac{\gamma-1}{2}M_0^2}\right)^{\frac{1}{\gamma+1}} \tag{2.33}$$

Thus, the reflection coefficient is finally given by

$$\chi=\frac{\zeta_l-1}{\zeta_l+1}\cdot e^{i\bar{\omega}\tau}=\frac{\zeta_l-1}{\zeta_l+1}\cdot\exp\left[-i\frac{\tau}{\varepsilon M_0}\cdot\frac{\omega l}{c_0}\right] \tag{2.34}$$

In terms of the acoustic impedance at the diffuser inlet $\xi=0$, viewed from the upstream annular duct, we obtain

$$\zeta_0=\frac{p_0/w_0}{R_0\cdot c_0}=\frac{1}{\gamma M_0}\cdot\frac{p_0/P_0}{w_0/W_0}=\frac{1+\chi}{1-\chi} \tag{2.35}$$

As the deceleration rate ε approaches zero, in other words, the diffuser becomes nothing but the extension of an another annular duct, (2.34) takes the following form,

$$\chi=\frac{\zeta_l^*-1}{\zeta_0^*+1}\cdot\exp\left[-i\frac{2}{1-M_0^2}\cdot\frac{\omega l}{c_0}\right] \tag{2.36}$$

in which ζ_l has been changed into ζ_l^* i. e. the corresponding radiation impedance of a flanged duct opening, since the coefficient now applies for obtaining the acoustic impedance in a constant area duct at the station a distance 1 upstream from the opening. In the limit of no flow, equations (2.31) and (2.32) yield

$$\chi=-\frac{H_1^{(1)}\left[\frac{1-\varepsilon_0}{\varepsilon_0}\cdot\frac{\omega l}{c_0}\right]+i\cdot\zeta_l\cdot H_0^{(1)}\left[\frac{1-\varepsilon_0}{\varepsilon_0}\cdot\frac{\omega l}{c_0}\right]}{H_1^{(2)}\left[\frac{1-\varepsilon_0}{\varepsilon_0}\cdot\frac{\omega l}{c_0}\right]+i\cdot\zeta_l\cdot H_0^{(2)}\left[\frac{1-\varepsilon_0}{\varepsilon_0}\cdot\frac{\omega l}{c_0}\right]}\cdot\frac{H_1^{(2)}\left[\frac{1}{\varepsilon_0}\cdot\frac{\omega l}{c_0}\right]}{H_1^{(1)}\left[\frac{1}{\varepsilon_0}\cdot\frac{\omega l}{c_0}\right]} \tag{2.37}$$

In this way, once the radiation impedance ζ_l at the diffuser outlet is known, we can easily estimate the diffuser effect upon the reflection coefficient.

3. Example

A case study has been made on a diffuser having an outlet/inlet area ratio S_l/S_0 of 5.99 and a circumferential opening width/radius ratio h/r_E of 0.1 under the assumption of linear deceleration.

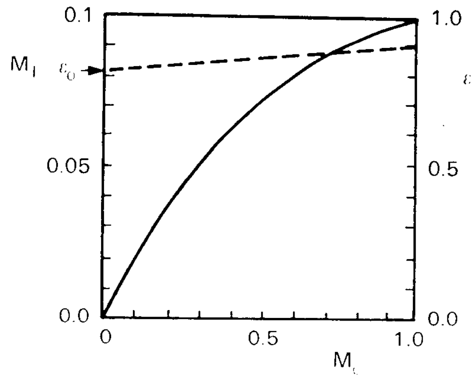

FIG. 7. Outlet Mach number M_l and deceleration rate ε versus inlet Mach number M_0. ——M_l, ------ε.

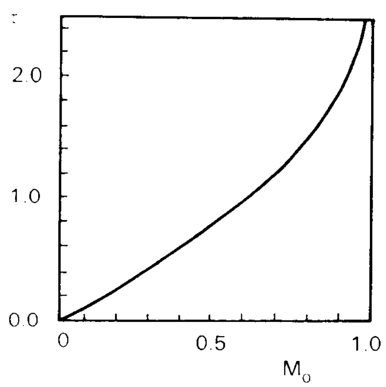

FIG. 8. τ-function versus inlet Mach number M_0.

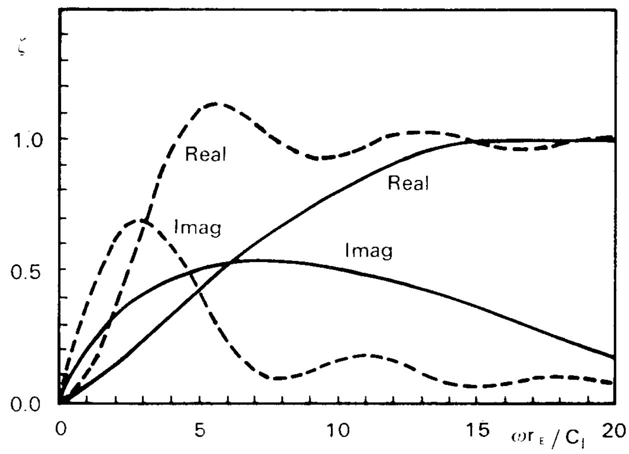

FIG. 9. Radiation impedance of plane wave mode. —— ζ_l circumferential opening, ------ ζ_l* flanged duct opening.

The relationship between the inlet and outlet Mach number and the deceleration rate ε are given in Fig. 7. The value of ε is close to that of the apparent deceleration rate ε_0 for a wide range of M_0. The corresponding τ function of equation (2.33) is shown in Fig. 8. Fig. 9 compares the impedance of an incident plane wave as radiated from the diffuser outlet with that for a flanged duct opening. The corresponding radius a of the latter may be obtained by equating the radiation area at both types of opening, e. g.

$$S_l = 2\pi r_E \cdot h = \pi a^2 \quad \text{or} \quad a/r_F = \sqrt{2 \cdot h/r_E} \tag{3.1}$$

Morse (1948) gives the formula to calculate the impedance of such a duct opening,

$$\zeta_l^* = 1 - \frac{2 \cdot J_1(k^*)}{k^*} + i \cdot \frac{4}{\pi} \cdot \int_0^{\pi/2} \sin(k^* \cdot \cos\psi) \cdot \sin^2\psi \cdot d\psi \tag{3.2}$$

where

$$k^* = 2\omega a/c_l = \frac{\omega r_E}{c_l} \cdot \frac{2a}{r_E}$$

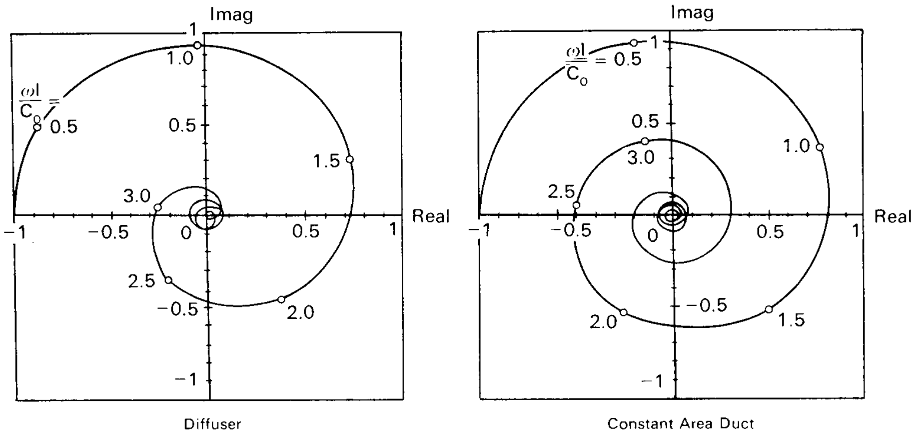

FIG. 10. Reflection Coefficient χ

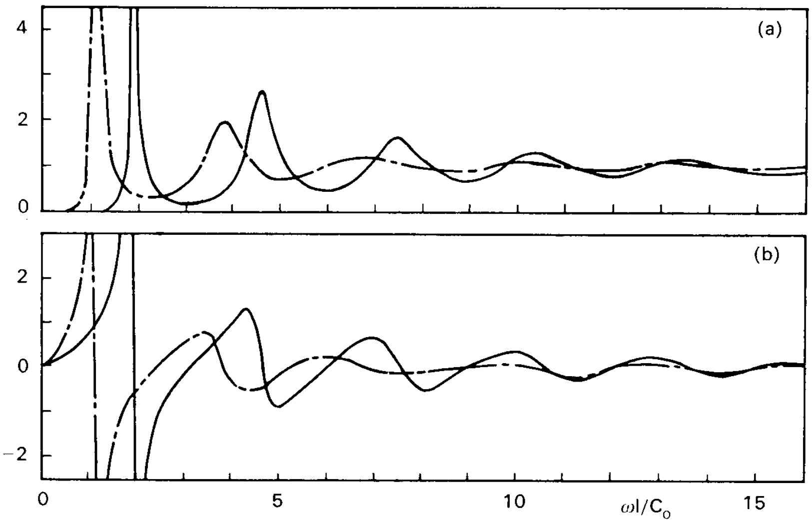

FIG. 11. Acoustic impedance at diffuser inlet. (a) Real part, (b) Imaginary part.
—— diffuser, - - - constant area duct.

In can be noticed from the figure that the present diffuser is a less efficient radiator than the corresponding flanged duct.

In the calculation of the radiation impedance the presence of the main through flow has been neglected. When this is the case, the reflection coefficient χ for the plane waves is calculated according to equation (2.37). Fig. 10 shows the result of the calculation for $l=r_E$. The result given by equation (2.36) for a constant area duct is also shown in the figure.

The corresponding acoustic impedance at the diffuser inlet is calculated from equation (2.35) and is shown in Fig. 11. Comparison shows that resonance peaks are much more marked for the diffuser and shifted to the

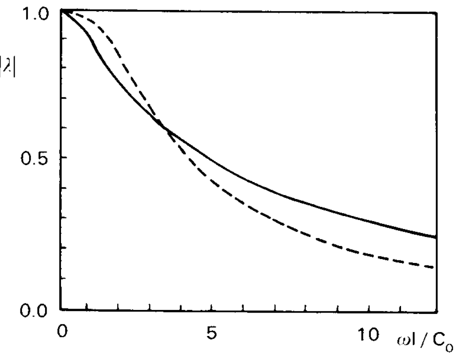

FIG. 12. Convection effect upon reflection coefficient χ. —— $M_0 \neq 0$, ------ $M_0=0$.

higher frequencies.

In the present analysis the through flow effect appears only in the wave propagation within the diffuser passage. This can be allowed, since the outlet Mach number becomes very low due to the deceleration. Along the passage the main flow is decelerated, which causes the change in the wave convection. τ function (2.33) includes this effect. The solution (2.34) yields the shift in the phase, but not in the modulus, of the reflection coefficient. Figure 12 compares the modulus of the solution (2.34) with that of the exact limiting solution of no wind (2.37). We remind that the present approximation is valid for relatively high frequencies. The figure indicates at high frequencies the larger coefficient values in the presence of a through flow. Further comparison between (2.34) and (2.37) shows that the phase of the coefficient is less affected in a diffuser than in a duct of constant cross section by a change of the inlet Mach number, because of the large deceleration in the passage.

4. Conclusion

Quasiplane wave propagation within the diffuser passage and the reflection at the outlet opening have been examined in detail based upon a simple one-dimensional analysis. A case study for a proposed diffuser which has an outlet/inlet area ratio 5.99 and an outlet opening width/radius ratio 0.1 showed that the wave reflection, thus the longitudinal resonance peaks built up within the diffuser passage is more marked than that of the corresponding flanged duct. The presence of a through flow causes wave convection, and hence a change in the wave pitch. Compared with that of a duct of constant cross section, the acoustic impedance of a diffuser will be less affected by a change of the inlet Mach number, because of the large deceleration in the passage. The results are available in terms of a reflection coefficient which expresses the ratio between the incident and reflected waves at the diffuser inlet.

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NOTATION

A, B	amplitude of waves
a	radius of circular duct opening
c	sound speed
$(\boldsymbol{e}_\xi, \boldsymbol{e}_\theta)$	unit vectors of (ξ, θ) coordinates
G	Green function of equation 2.3
h	circumferential opening width
$H_m^{(1)}, H_m^{(2)}$	Hankel function of 1st and 2nd kind, order m
i	imaginary unit
J_1	Bessel function of 1st order
k	wave number factor of diffuser opening $\omega r_E/c_l$
k^*	wave number factor of duct opening $2\omega a/c_l$
l	diffuser length
M	Mach number W/c
m	circumferential mode order
n	transverse mode order
N_1	Neumann function of 1st order
P, p	pressure of the main flow and its perturbation
q	simple acoustic source strength
R	density of the main flow
r	radius (function of ξ)
$\boldsymbol{r}$	vector representation of coordinate (r, θ, z)
r_E	radial distance of diffuser outlet
R_m	function defined by equation 2.5
S	cross section area of diffuser passage
t	time
$\boldsymbol{v}$	perturbation velocity
V_r, v_r	radial discharge velocity and its perturbation
v_θ	velocity perturbation in the θ direction
W, w	velocity of the main flow and its perturbation
γ	ratio of specific heats
ε	deceleration rate (equation 2.23)
ε_0	deceleration rate of no wind limit
ζ	radiation impedance
θ	circumferential coordinate
λ	wave number in the ξ direction
ν	transverse mode order
ξ	londitudinal coordinate along diffuser passage
$\bar{\xi}$	nondimensionalised ξ coordinate ξ/l
ρ	density perturbation
τ	function defined by equation 2.33
χ	reflection coefficient
ω	radian frequency
$\bar{\omega}$	nondimensionalised radian frequency $\omega/(dW/d\xi)$

Superscript

$+$	incident or transmitted wave
$-$	reflected wave
$*$	related to cylindrical duct opening

Subscript

0	inlet ($\xi=0$) or free space condition
l	outlet ($\xi=l$)
n	transverse mode order

ν transverse mode order

Operator

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + W \frac{\partial}{\partial \xi}$$

$$\Box \equiv \triangle - \frac{1}{c_0{}^2} \frac{\partial^2}{\partial t^2}$$

$\triangle$ the Laplacian