Aeronautical Research Institute, University of Tokyo
Report No. 331, June 1958

An Example of Unsteady Laminar Boundary Layer Flow

By

Itiro TANI

Summary. The flow in two-dimensional incompressible laminar boundary layer is discussed when the velocity U outside the boundary layer is given by the form $U=V-x/(T-t)$, where t is the time, T a constant time, V a constant velocity, and x the distance parallel to the wall. A solution is obtained in the form of a power series in $\xi=8x/V(T-t)$ whose coefficients are functions of $\eta=(y/2)(V/\nu x)^{\frac{1}{2}}$, ν being the kinematic viscosity and y the distance normal to the wall. Six of the coefficients have been obtained by integrating the differential equations. Unfortunately the series converges so slowly that the coefficients obtained are not sufficient and an approximate method of continuation is required to carry out the solution to the point of separation. The method of continuation leads to the result that the separation occurs when $\xi=1.20$. The solution of the problem may be interpreted to provide some informations for the unsteady flow associated with a diffusor or an airfoil in which the angle of divergence or angle of attack varies with time.

INTRODUCTION

Most of the existing analyses of unsteady laminar boundary-layer flows are concerned with the earliest phase of the motion starting from rest. It is assumed in these analyses that there is initially a boundary layer of zero thickness. Unsteady flow in boundary layer having initial thickness seems to deserve consideration, although very little has been done on this type of the problem. Shibuya [*1*] first attempted to calculate the response of the incompressible laminar boundary layer to an impulsive change in outer flow. In order to apply Laplace transformation, it was required to assume a certain linearization of boundary-layer equations, because of which, however, the accuracy of the analysis has been questioned. Moore [*2*] considered the compressible laminar boundary layer over a heat-insulated flat plate with a time-dependent outer flow. The main object of the study seems to be in the search for the parameters, which govern the nature of the unsteady flow. As a matter of fact, the analysis is confined to the case in which the magnitude of the parameters is sufficiently small.

In the present paper, the flow in two-dimensional incompressible laminar boundary layer with a time-dependent pressure gradient is analysed; the pressure gradient is considered to vary with time, not arbitrarily, but in a specified manner so that the three independent variables, time and space coordinates, are grouped together into two. The solution to this problem may be interpreted to provide some informations for the unsteady flow associated with a diffusor or an airfoil in which the angle of divergence or angle of attack varies with time. The investigation was accomplished in 1955, when the writer was with the Institute of Science and Technology, University of Tokyo. The writer wishes to express his thanks to Mr. Shiro Fukui and Mrs. Chiyoko Asano for carrying out the numerical calculation.

Problem To Be Solved

We consider the solution of two-dimensional incompressible laminar boundary layer, when the velocity U outside the boundary layer is given by the form

$$U = V - x\Phi(t), \tag{1}$$

where x is the distance measured parallel to the wall, V a constant velocity, and $\Phi(t)$ an undetermined function of the time t. The case when Φ is constant is reduced to the steady decelerating flow, for which Howarth [3] has found the solution in a series of powers of $x^* = x\Phi/V$. On the other hand, the case when U depends only on t is that considered by Moore, who has shown that the governing parameters are $\zeta_n = (x^{n+1}/U^{n+2})(d^{n+1}U/dt^{n+1})$, $n=0, 1, 2, \cdots$. Tsuji [4] then demonstrated that, under certain circumstances which are usually met, ζ_1, $\zeta_2, \cdots$ can be expressed in terms of ζ_0. Since in the present problem

$$\zeta_0 = -\frac{x^2}{(V-x\Phi)^2}\frac{d\Phi}{dt} = -\frac{x^{*2}}{(1-x^*)^2}\frac{1}{\Phi^2}\frac{d\Phi}{dt}, \tag{2}$$

it is required to restrict ourselves to the case when $d\Phi/dt = \Phi^2$, or

$$\Phi(t) = \frac{1}{T-t}, \tag{3}$$

where T is a constant, in order that the three independent variables, x, y and t, are grouped together into two variables. We therefore consider the solution for the velocity distribution

$$U = V - \frac{x}{T-t} = V\left(1-\frac{\xi}{8}\right), \tag{4}$$

where

$$\xi = \frac{8x}{V(T-t)}. \tag{5}$$

Method of Solution

The method of solution is similar to that put forward by Howarth for the steady decelerating flow. We assume an expansion of the form

$$\psi = \sqrt{\nu x V}\,[f_0(\eta) - \xi f_1(\eta) + \xi^2 f_2(\eta) - \cdots] \tag{6}$$

for the stream function ψ, where

$$\eta=\frac{y}{2}\sqrt{\frac{V}{\nu x}}, \tag{7}$$

and ν is the kinematic viscosity, y is the distance measured normal to the wall, and $f_0, f_1, f_2, \cdots$ are the functions of η to be determined. Substituting this form in the momentum equation of unsteady boundary layer

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}, \tag{8}$$

where u and v are the velocity components in the directions x and y, respectively, and equating coefficients of the various powers of ξ on the two sides of the equation, we obtain a series of differential equations

$$f_0'''+f_0f_0''=0, \tag{9}$$

$$f_1'''+f_0f_1''-2f_0'f_1'+3f_0''f_1=-1, \tag{10}$$

$$f_2'''+f_0f_2''-4f_0'f_2'+5f_0''f_2=-\tfrac{1}{2}f_1'+2f_1'^2-3f_1f_1'', \tag{11}$$

$$f_3'''+f_0f_3''-6f_0'f_3'+7f_0''f_3=-f_2'+6f_1'f_2'-3f_1f_2''-5f_1''f_2, \tag{12}$$

$$\begin{aligned}f_4'''+f_0f_4''-8f_0'f_4'+9f_0''f_4=&-\tfrac{3}{2}f_3'+8f_1'f_3'-3f_1f_3''-7f_1''f_3\\&+4f_2'^2-5f_2f_2'',\end{aligned} \tag{13}$$

$$\begin{aligned}f_5'''+f_0f_5''-10f_0'f_5'+11f_0''f_5=&-2f_4'+10f_1'f_4'-3f_1f_4''-9f_1''f_4\\&+10f_2'f_3'-5f_2f_3''-7f_2''f_3,\end{aligned} \tag{14}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

where dashes denote differentiations with respect to η. The boundary conditions are (since $u=v=0$ at $y=0$)

$$f_r(0)=f_r'(0)=0 \quad \text{for all values of } r, \tag{15}$$

and (since $u=U$ at $y=\infty$)

$$f_0'(\infty)=2, \quad f_1'(\infty)=\frac{1}{4}, \quad f_r'(\infty)=0 \quad \text{for } r\geqq 2. \tag{16}$$

The solutions of the first two equations, (9) and (10), are available. Indeed f_0 is the Blasius function for steady uniform flow over a flat plate, and f_1 is the second-order function of Howarth series solution for steady decelerating flow; both of them are tabulated by Howarth in [*3*]. Equations (11) to (14) subject to boundary conditions (15) and (16) have been solved by numerical integration to obtain the functions f_2 to f_5. The method used is described in appendix. The second derivatives of the functions at $\eta=0$ are found as follows:

$$f_0''(0)=1.328242, \quad f_1''(0)=\ \ 1.02054, \quad f_2''(0)=-0.04140,$$
$$f_3''(0)=0.01512, \quad f_4''(0)=-0.00501, \quad f_5''(0)=\ \ 0.00187.$$

The velocity distribution being given by

$$u=\frac{V}{2}[f_0'(\eta)-\xi f_1'(\eta)+\xi^2 f_2'(\eta)-\cdots], \tag{17}$$

values of the functions f_2' to f_5' are presented in table 1. The condition for separation $(\partial u/\partial y)_{y=0}=0$ leads to

$$f_0''(0)-\xi f_1''(0)+\xi^2 f_2''(0)-\cdots=0. \tag{18}$$

Method of Continuation

Unfortunately, however, the series converges so slowly that the functions so far evaluated are not sufficient to give a reasonably accurate representation unless ξ is small. Probably at least four more functions would be required in order to determine the value of ξ for separation. But we can obtain an approximate answer in the following way. First we plot $\log_{10}|f_r''(0)|$ as a function of r for $r=2$, 3, 4 and 5 and extrapolate the curve to larger values of r as shown dotted in figure 1. Assuming

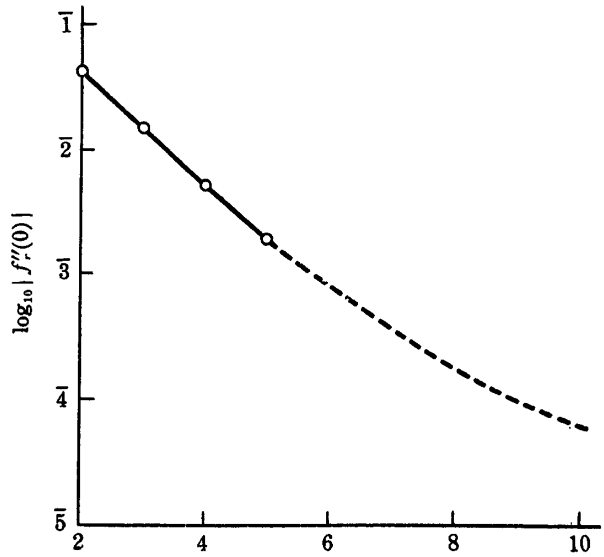

Figure 1

the values of $f_r''(0)$ continue to alternate in sign as r increases, we may estimate

$$f_6''(0)=-0.00080,\quad f_7''(0)=\ \ 0.00036,\quad f_8''(0)=-0.00017,$$
$$f_9''(0)=\ \ 0.00010,\quad f_{10}''(0)=-0.00007.$$

With these values we find from (18) that separation occurs when $\xi=1.20$.

An alternative and more reliable estimate is obtained by applying an approximate method of procedure, which is originally due to Howarth, and based on the assumption that the functions f_r' for $r\geqq 6$ are expressible in the form $K_rG(\eta)$ where K_r are constants. A fairly good estimate for the function $G(\eta)$ may be obtained as shown in figure 2 from the knowledge of $|f_r'|/|f_r'|_{\max}$ plotted as functions of η for $r=2$, 3, 4 and 5. The numerical values of $G(\eta)$ are presented in table 1. The velocity distribution may then be written

$$u=\frac{V}{2}[f_0'(\eta)-\xi f_1'(\eta)+\cdots-\xi^5 f_5'(\eta)-F(\xi)G(\eta)], \tag{19}$$

where $F(\xi)$ is a function to be determined.

Substituting the expression (19) in the momentum integral of unsteady boundary layer

$$\left(\frac{\partial}{\partial t}-U\frac{\partial}{\partial x}\right)\int_0^\infty (U-u)dy+\frac{\partial}{\partial x}\int_0^\infty (U^2-u^2)dy=\nu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \tag{20}$$

which is obtained by integrating the momentum equation (8) with respect to y through the boundary layer, we have the differential equation

$$\frac{dF}{d\xi}=\frac{A+BF+g_2F^2}{C-4g_2\xi F}, \tag{21}$$

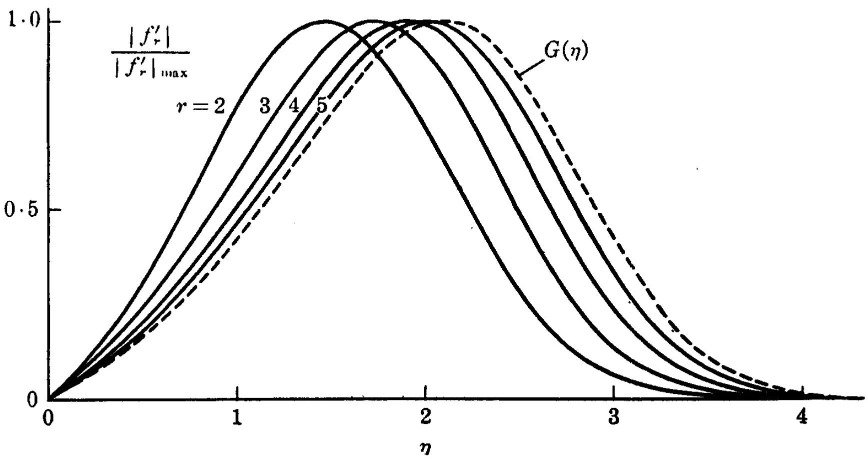

FIGURE 2

where

$$A=2\left(1-\frac{\xi}{8}\right)Q+(4\xi-\xi^2)\frac{dQ}{d\xi}-R-2\xi\frac{dR}{d\xi}+S, \tag{22}$$

$$B=-g_0+2\left(1-\frac{\xi}{8}\right)g_1-P-2\xi\frac{dP}{d\xi}, \tag{23}$$

$$C=-(4\xi-\xi^2)g_1+2P\xi, \tag{24}$$

$$P=2\sum_{r=0}^{5}(-\xi)^r\int_0^\infty f_r'Gd\eta, \tag{25}$$

$$Q=\lim_{\eta\to\infty}(2\eta-f_0)+\xi\lim_{\eta\to\infty}\left(f_1-\frac{1}{4}\eta\right)-\sum_{r=2}^{5}(-\xi)^r f_r(\infty), \tag{26}$$

$$R=\int_0^\infty(4-f_0'^2)d\eta+\xi\int_0^\infty(2f_0'f_1'-1)d\eta+\xi^2\int_0^\infty\left(\frac{1}{16}-f_1'^2-2f_0'f_2'\right)d\eta$$
$$-\sum_{r=3}^{5}(-\xi)^r\sum_{m=0}^{r}\int_0^\infty f_m'f_{r-m}'d\eta-\sum_{s=1}^{5}(-\xi)^{5+s}\sum_{n=s}^{5}\int_0^\infty f_n'f_{5+s-n}'d\eta, \tag{27}$$

$$S=\sum_{r=0}^{5}(-\xi)^r f_r''(0), \tag{28}$$

$$g_0=G'(0),\quad g_1=\int_0^\infty Gd\eta,\quad g_2=\int_0^\infty G^2d\eta. \tag{29}$$

Starting from the value of $F(\xi)=0$ at $\xi=0.4$ for which u is given by $(V/2)(f_0'-\xi f_1'+\cdots-\xi^5 f_5')$ with sufficient accuracy, the equation (21) may be integrated graphically for $F(\xi)$. The condition for separation is

$$g_0F(\xi)=S(\xi). \tag{30}$$

The integral curve together with the curve given by (30) is shown in figure 3. It

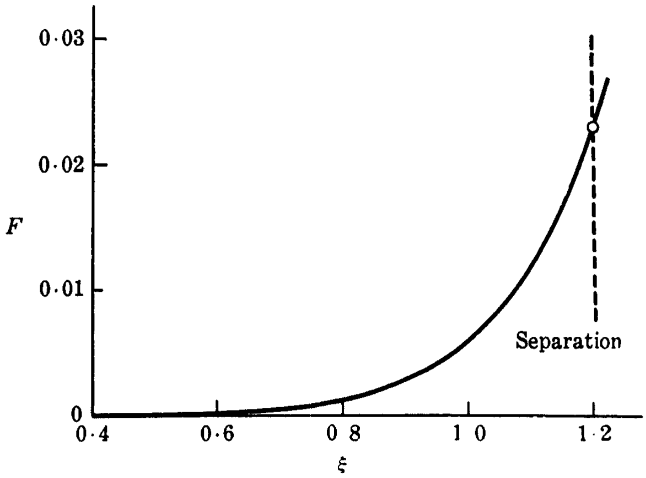

FIGURE 3

will be noticed that the two curves intersect at $\xi=1.197$.

We thus arrive at the result that separation occurs when $\xi=1.20$.

RESULTS OF CALCULATION

We now calculate the displacement thickness $\delta^*=\int_0^\infty(1-u/U)dy$, the momentum thickness $\theta=\int_0^\infty(1-u/U)(u/U))dy$ and the wall velocity gradient $(\partial u/\partial y)_{y=0}$ by the following formulas:

$$D(\xi)\equiv\sqrt{\frac{8}{\xi}}\frac{\delta^*}{\sqrt{\nu(T-t)}}=\frac{Q+g_1F}{1-\frac{\xi}{8}}, \tag{31}$$

$$\Theta(\xi)\equiv\sqrt{\frac{8}{\xi}}\frac{\theta}{\sqrt{\nu(T-t)}}=\frac{R+PF-g_2F^2}{2\left(1-\frac{\xi}{8}\right)^2}-\frac{Q+g_1F}{1-\frac{\xi}{8}}, \tag{32}$$

$$H(\xi)\equiv\sqrt{2\xi}\sqrt{\nu(T-t)}\frac{1}{V}\left(\frac{\partial u}{\partial y}\right)_{y=0}=S-g_0F. \tag{33}$$

In table 2 the values of $F(\xi)$, $D(\xi)$, $\Theta(\xi)$ and $H(\xi)$ are tabulated against ξ over the entire range of the solution.

APPLICATION

The solution so far obtained may be interpreted to provide some informations for the unsteady flow associated with a diffusor or an airfoil in which the angle of divergence or angle of attack varies with time. Strictly speaking, however, the solution of the problem can only be realized in the following way.

Consider a two-dimensional flow through a channel as shown in figure 4. The

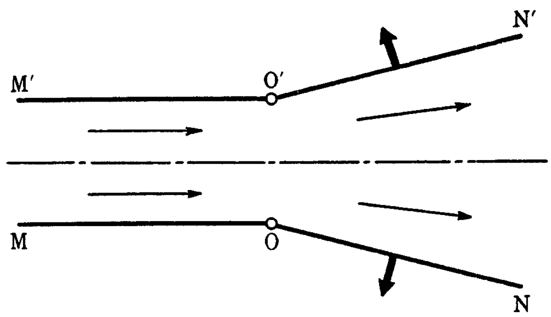

FIGURE 4

upstream walls MO and M'O' are fixed and nearly parallel, while the downstream walls ON and O'N' are moving outward in such a way that the velocity outside the boundary layer is given by the form (4), where x is the distance measured downstream from O along the wall ON and V is the value of U at $x=0$. It is further required that the suction is applied to the upstream boundary layer so that a new boundary layer is developed along the wall ON with zero initial thickness at $x=0$. The preceding solution can immediately be adapted to this unsteady channel flow, in which the separation is predicted to occur when $8x/V(T-t)=1.20$.

Next, consider the case when the boundary layer has a nonzero momentum thickness $\theta(0,t)$ at the commencement of the region for which the solution is applied. The distance X is measured downstream from this point, and related with x by $x=X+X_0$, where X_0 is a constant length; $\theta(0,t)$ denotes the value of $\theta(X,t)$ at $X=0$. Knowing the initial momentum thickness $\theta(0,0)$ the corresponding value of $\xi(0,0)$ is determined from (32), namely from

$$\Theta[\xi(0,0)]\sqrt{\xi(0,0)}=\sqrt{\frac{8}{\nu T}}\,\theta(0,0). \tag{34}$$

With this value for $\xi(0,0)$ we obtain from (5)

$$\xi(X,0)=\xi(0,0)+\frac{8X}{VT}=\frac{8(X+X_0)}{VT}, \tag{35}$$

which leads to

$$X_0=\frac{VT}{8}\xi(0,0). \tag{36}$$

We therefore find

$$\xi(X,t)=\xi(0,0)\frac{T}{T-t}+\frac{8X}{V(T-t)}, \tag{37}$$

$$U(X,t)=V\left[1-\frac{\xi(0,0)}{8}\frac{T}{T-t}\right]-\frac{X}{T-t}. \tag{38}$$

Thus, the outside velocity distribution should be of the form (38) in order that the preceding solution can be adapted. The curves of velocity distribution are shown graphically in figure 5. The velocity distribution for the region of negative X is to some extent arbitrary, provided that the momentum thickness at $X=0$ satisfies the

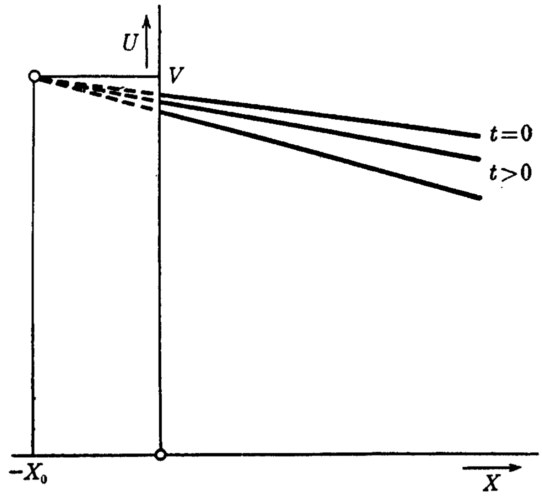

FIGURE 5

condition

$$\theta(0,t)=\theta(0,0)\frac{\Theta[\xi(0,t)]}{\Theta[\xi(0,0)]}. \tag{39}$$

The separation is found when $\xi(X,t)$ attains the value 1.20.

Department of Aerodynamics
Aeronautical Research Institute
University of Tokyo, Tokyo
May 21, 1958

REFERENCES

[1] Shibuya, I.: Theory of the Laminar Boundary Layer for Sudden Change of Flow and its Application (in Japanese). Jour. Japan Soc. App. Mech., vol. 1 (1948), pp. 94–100.
[2] Moore, F. K.: Unsteady Laminar Boundary-Layer Flow. NACA Tech. Note 2471 (1951).
[3] Howarth, L.: On the Solution of the Laminar Boundary Layer Equations. Proc. Roy. Soc., A, vol. 164 (1938), pp. 547–579.
[4] Tsuji, H.: Note on the Solution of the Unsteady Laminar Boundary-Layer Equations. Jour. Aero. Sci., vol. 20 (1953), pp. 295–296.

APPENDIX

NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS

The differential equations (11) to (14) can be written in the form

$$f_r''' + f_0 f_r'' - 2r f_0' f_r' + (2r+1) f_0'' f_r = P_r , \tag{A1}$$

where $r=2, 3, 4$ and 5, P_r is the function of $f_0, f_1, \cdots, f_{r-1}$, and dashes denote differentiations with respect to η. The boundary conditions satisfied are

$$f_r = f_r' = 0 \qquad \text{at} \quad \eta = 0 , \tag{A2}$$

$$f_r' = 0 \qquad \text{at} \quad \eta = \infty . \tag{A3}$$

Since the equations are linear and the boundary conditions are given at two points, the most obvious method of integration is to write the solution in the form

$$f_r = \lambda C + I , \tag{A4}$$

where C is a complementary function satisfying the boundary conditions

$$C = C' = 0 , \quad C'' = 1 , \quad \text{at } \eta = 0 , \tag{A5}$$

and I is a particular integral satisfying the boundary conditions

$$I = I' = 0 , \quad I'' = 1 , \quad \text{at } \eta = 0 . \tag{A6}$$

After numerical integrations have been carried out for C and I, the constant λ is determined by applying the condition at infinity (A3). Thus, for each of the equations (A1), at least two numerical integrations to infinity are required.

Practically speaking, however, there is a possibility that $C'(\infty)$ and $I'(\infty)$ become so large in numerical values that the solution f_r can only be found very roughly. Additional integrations to infinity then have to be made for I with different values of $I''(0)$. Several repetitions are necessary in most cases in order to determine the solution to fifth decimal places.

Another method of integration was therefore tried for determining the functions f_2 to f_5. The method is the iterative one and consists in writing the equation in the form

$$f_r''' + f_0 f_r'' = 2r f_0' f_r' - (2r+1) f_0'' f_r + P_r , \tag{A7}$$

in which f_r and f_r' on the right side are assumed known. Then the formal intergration gives

$$f_r'' = \alpha f_0'' + f_0'' \left[2r \int_0^\eta \frac{f_0'}{f_0''} f_r' d\eta - (2r+1) \int_0^\eta f_r d\eta + \int_0^\eta \frac{P_r}{f_0''} d\eta \right] , \tag{A8}$$

where α is an arbitrary constant. The solution is then assumed in the form $f_r' = \beta g$, where β is another arbitrary constant and g is a function of η to be suitably assumed which vanishes at $\eta=0$ and $\eta=\infty$. Substituting this form in (A8) and integrating with respect to η, we have

$$\beta g = \alpha f_0' + \beta h + k , \tag{A9}$$

where

$$h=2r\int_0^\eta\left[f_0''\int_0^\eta\frac{f_0'}{f_0''}g\,d\eta\right]d\eta-(2r+1)\int_0^\eta\left[f_0''\int_0^\eta\left(\int_0^\eta g\,d\eta\right)d\eta\right]d\eta, \tag{A10}$$

$$k=\int_0^\eta\left[f_0''\int_0^\eta\frac{P_r}{f_0''}d\eta\right]d\eta. \tag{A11}$$

The constants α and β are determined by satisfying (A9) at two points, $\eta=\eta_1$ and $\eta=\infty$. The choice of η_1 is arbitrary, but it seems convenient to take η_1 somewhere near the position of maximum g. In the present problem $\eta_1=1.8$ has been used irrespective of the value of r. With these constants α and β, the left and right sides of (A9) are made equal at $\eta=\eta_1$ and $\eta=\infty$, but not at other values of η. Interpolating between both sides we revise the function g to be used for the next iteration.

This method of iteration has the advantage that all the boundary conditions are satisfied in each stage of iteration. Three or four iterations have usually been required to determine the solution to fifth decimal places, but the numerical procedure has been found rather easier and quicker than the usual numerical integration mentioned before.

TABLE 1

η	f_2'	f_3'	f_4'	f_5'	G
0.0	−0.00000	0.00000	−0.00000	0.00000	0.000
0.1	−0.00422	0.00152	−0.00050	0.00019	0.029
0.2	−0.00887	0.00307	−0.00102	0.00039	0.060
0.3	−0.01424	0.00469	−0.00158	0.00061	0.094
0.4	−0.02050	0.00641	−0.00219	0.00085	0.131
0.5	−0.02767	0.00826	−0.00284	0.00111	0.171
0.6	−0.03567	0.01027	−0.00355	0.00139	0.215
0.7	−0.04429	0.01248	−0.00432	0.00169	0.261
0.8	−0.05323	0.01486	−0.00516	0.00202	0.312
0.9	−0.06212	0.01740	−0.00606	0.00238	0.368
1.0	−0.07053	0.02006	−0.00701	0.00276	0.426
1.1	−0.07801	0.02279	−0.00799	0.00317	0.490
1.2	−0.08414	0.02550	−0.00900	0.00360	0.558
1.3	−0.08855	0.02806	−0.01002	0.00403	0.627
1.4	−0.09096	0.03032	−0.01101	0.00444	0.696
1.5	−0.09121	0.03213	−0.01192	0.00484	0.763
1.6	−0.08929	0.03334	−0.01270	0.00522	0.827
1.7	−0.08537	0.03385	−0.01330	0.00555	0.885
1.8	−0.07973	0.03362	−0.01368	0.00580	0.934
1.9	−0.07272	0.03264	−0.01380	0.00596	0.972
2.0	−0.06476	0.03094	−0.01362	0.00602	0.994
2.1	−0.05634	0.02863	−0.01313	0.00597	1.000
2.2	−0.04790	0.02586	−0.01238	0.00579	0.987
2.3	−0.03980	0.02281	−0.01142	0.00549	0.958
2.4	−0.03232	0.01964	−0.01029	0.00509	0.906
2.5	−0.02565	0.01650	−0.00904	0.00462	0.840
2.6	−0.01989	0.01353	−0.00775	0.00410	0.763
2.7	−0.01508	0.01084	−0.00650	0.00355	0.678
2.8	−0.01118	0.00849	−0.00532	0.00301	0.586
2.9	−0.00811	0.00651	−0.00425	0.00250	0.498
3.0	−0.00576	0.00487	−0.00332	0.00202	0.413
3.1	−0.00400	0.00355	−0.00254	0.00159	0.335
3.2	−0.00271	0.00252	−0.00190	0.00122	0.264
3.3	−0.00180	0.00175	−0.00138	0.00092	0.203
3.4	−0.00117	0.00119	−0.00098	0.00068	0.154
3.5	−0.00074	0.00080	−0.00068	0.00049	0.114
3.6	−0.00046	0.00053	−0.00046	0.00035	0.082
3.7	−0.00029	0.00034	−0.00031	0.00024	0.057
3.8	−0.00018	0.00021	−0.00020	0.00016	0.040
3.9	−0.00011	0.00013	−0.00013	0.00010	0.026
4.0	−0.00006	0.00008	−0.00008	0.00006	0.015
4.1	−0.00003	0.00004	−0.00005	0.00003	0.008
4.2	−0.00001	0.00002	−0.00003	0.00002	0.004
4.3	−0.00000	0.00001	−0.00001	0.00001	0.002
4.4	−0.00000	0.00000	−0.00000	0.00000	0.001

TABLE 2

ξ	F	D	Θ	H
0.0	0.0000	1.721	0.664	1.328
0.1	0.0000	1.779	0.678	1.226
0.2	0.0000	1.842	0.693	1.122
0.3	0.0000	1.910	0.708	1.018
0.4	0.0000	1.985	0.723	0.912
0.5	0.0001	2.067	0.738	0.805
0.6	0.0002	2.158	0.754	0.697
0.7	0.0005	2.258	0.769	0.587
0.8	0.0013	2.371	0.783	0.475
0.9	0.0028	2.498	0.797	0.360
1.0	0.0059	2.643	0.810	0.243
1.1	0.0119	2.812	0.820	0.122
1.2	0.0230	3.013	0.825	0.000

概　　要

非定常な層流境界層の解の例

谷　一　郎

二次元の縮まない層流境界層の方程式を，境界層の外側の速度 U が $U=V-x/(T-t)$ の形で与えられる場合に解いたものである．t は時間，T は一定の時間，V は一定の速度，x は壁に平行に測つた距離である．なお ν を動粘性，y を壁に垂直に測つた距離として，解を $\xi=8x/V(T-t)$ のべき級数の形で表わし，その係数を $\eta=(y/2)(V/\nu x)^{\frac{1}{2}}$ の函数とする．この係数の 6 個を微分方程式を積分することによつて求めた．しかし級数の収束はあまり速かでないので，それだけの係数では十分でなく，剝離点まで解を進めて行くためには，近似的な接続法を必要とする．このようにして計算すると，剝離は $\xi=1.20$ で起ることが知られる．この問題の解は，拡散筒の開きまたは翼の迎角が時間的に変る場合の非定常な流れの理解に有用と考えられる．