Aeronautical Research Institute, University of Tokyo
Report No. 333, June 1958

Deep Drawing of Cylindrical Shell according to the So-called Hydroform Method

By

Shinji FUKUI, Kiyota YOSHIDA* and Kunio ABE

Summary. A mathematical analysis of deep drawing work in accordance with the hydroform method was made on the basis of the total strain theory and an investigation made on the general characteristics developing from the use of fluid pressure as a die.

The results of analyses and investigations disclose that insofar as the hydroform method of forming is concerned, the influence of friction at the forming radius portion can be neglected due to very little frictional resistance there and the radial tensile stress is reduced by the action of a radial compressive stress at the periphery of a flange. Moreover, the friction force developed between the side wall of a punch and the portion of the formed surface in contact prevents concentration of the forming force on the punch head. For these and other reasons, the hydroform method provides many beneficial features.

On the other hand, the detrimental effect of the friction arising between the flange portion and the blank holder plate is greater than when employing a solid metal die which is a great deficiency associated with the hydroform method. At any rate, it is the combination of these advantages and deficiencies which make up the characteristics of hydroforming. The characteristics associated with hydroforming were also investigated experimentally by forming of a cylinder of 30 mm in diameter, employing a fluid pressure chamber of 100 mm in diameter and under the condition of a maximum forming pressure of 1000 kg/cm^2.

FOREWORD

Various forms of forming methods employing a resilient material such as a liquid or rubber and other similar materials for either a punch or die, exist in conjunction with the forming of metal components. An example of one of these forms is the Guerin method. As these forming methods offer many advantages due to the fact of the resilient object being either an universal punch or die, many variations of these forming methods utilizing these advantages more fully have been introduced since about 1950. The hydroform method wherein hydraulic pressure is employed as an universal die is of special interest to our aircraft industries and in a country like Japan where rapid production is demanded in the production of multi varieties of products.

This method, considered from the standpoint of forming, possess many interest-

* Scientific Reserch Institute, Ltd.

ing features and affords a means for forming more complicated shapes and many actual cases exist which indicate a marked extension in the limit of forming [*1*], as compared to the metal tool method incorporating the use of a metal punch and die. Moreover, many experiments in respect to improvement and application of this method have been undertaken in recent times, wherefrom characteristics relating to forming are gradually becoming more clear [*2*], [*3*]. On the other hand, very little basic research has been conducted on the aspect of forming as well as in clarifying its inherent features and shortcomings and wherefore there is a definite lack of ample data to contribute to the wider use and development of the method.

Among the various kinds of forming work such as shearing, bending and deep drawing accomplishable by the method, the deep drawing operation was selected as most characteristic for study and experiments of deep drawing were performed on a cylindrical shell by means of a small and simple forming apparatus with the objective of investigating its basic characteristics and for mathematical analysis of the process of forming.

Moreover, the general characteristics derived from the experiments were compared with the characteristics of deep drawing according to the metal tool method.

The experiment apparatus consisted of a fluid pressure chamber of 100 mm in diameter and a punch 30 mm in diameter. The plate thickness ranged form 0.4 to 1.0 mm and the ratio of plate thickness to the diameter of the blank was maintained within a small range. Moreover, two of the authors are performing various experiments with a practical forming machine of their own specification [*4*] designed to accomodate blanks up to a maximum diameter of 300 mm and for a maximum forming pressure of 1000 kg/cm^2. The results of these experiments to data show that characteristics slightly different from those given in this report, will be found when the ratio of plate thickness to blank diameter increases substantially.

PART 1. STRESS AND STRAIN ANALYSES

1. *Basic Equation and Its Solution*

The mathematical analysis for deep drawing work by the hydroform method basically does not differ from the case for the metal tool method, except for the consideration of the effect of the forming pressure. Therefore, the method [*5*] based on the total strain theory, under which two of the authors had previously made analyses on deep drawing work based on the metal tool method, can be applied to the hydroform method. But the forming pressure acts entirely on the blank holder plate through the blank and the frictional resistance is created at the surface of contact between the blank holder plate and the flange portion of blank (Fig. 1 A–B) in the hydroforming. On the other hand, the face of blank adjacent to the fluid pressure chamber side is in contact with a rubber plates substantially more resilient than the blank holder plate and the rubber piece deformed and displaced together with the blank. The deformation resistance of the rubber piece is substantially small and as friction does not exist between the fluid and rubber piece, the subject covering the frictional resistance at the contacting face is disregarded.

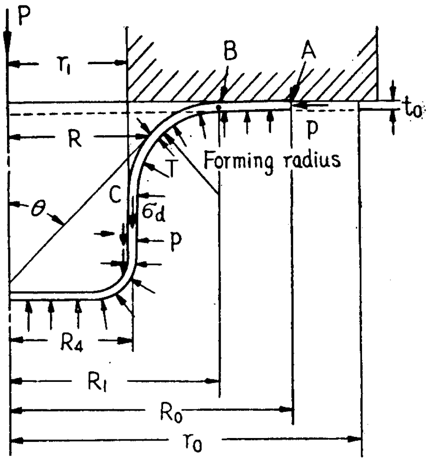


FIGURE 1. Dimensional reference diagram.

Now, in the case where the blank holder plate is a plane normal to the punch axis, the equations for radial stress and strain are expressed as follows:—

$$\frac{d(\sigma_1 \cdot R \cdot T)}{dR} = \sigma_3 T + \mu p R, \qquad (1)$$

$$\frac{d\varepsilon_3}{dR} = \frac{1}{R}(1 - e^{\varepsilon_3 - \varepsilon_1}). \qquad (2)$$

In the case for the metal tool method, the second term in the right hand side in equation (1) becomes zero since $p=0$. Similarly, the equation for radial stress and strain applicable to the portion of forming radius (B–C) are given as

$$\frac{d(\sigma_1 \cdot R \cdot T)}{dR} = \sigma_3 T, \qquad (3)$$

$$\frac{d\varepsilon_3}{dR} = \frac{1}{R}\left(1 - \frac{1}{\sin\theta} e^{\varepsilon_3 - \varepsilon_1}\right). \qquad (4)$$

In the case of the metal tool method a term caused by friction would be appended to the right hand side in equation (3) for the portion of die profile radius, whereas in the hydroform method, the equation becomes simple since the factor of friction between the blank and rubber skin, like the flange portion, can be neglected and which represents a feature of the hydroform method.

In the preceding equations, true stress is indicated as σ and logarithmic strain by ε. The numerical subscripts 1, 2 and 3 represent the radial, thicknesswise and circumferential components respectively. The tensile stress is designated by a plus (+) sign and minus (−) for compression.

The general relation between stress and strain in the plastic range can be found by the following equations, if the relation of equivalent stress $\bar{\sigma}$ to equivalent strain $\bar{\varepsilon}$ is approximated in accordance with the relation $\bar{\sigma} = c(\bar{\varepsilon})^n$.

$$\left.\begin{aligned} \varepsilon_1&=K(\bar{\sigma})^m\{\sigma_1-(\sigma_2+\sigma_3)/2\}, \\ \varepsilon_2&=K(\bar{\sigma})^m\{\sigma_2-(\sigma_1+\sigma_3)/2\}, \\ \varepsilon_3&=K(\bar{\sigma})^m\{\sigma_3-(\sigma_1+\sigma_2)/2\}. \end{aligned}\right\} \qquad (5)$$

In the above equations, $K=(1/c)^{1/n}$ and $m=(1-n)/n$ are inherent constants of a material determinable from tensile tests.

If now, equations (1) & (2) and (3) & (4) are each respectively combined by employing the relations in (5) above, the following simultaneous equations can be established.

$$\frac{d\sigma_1}{dR}=\frac{B'C-BC'}{AB'-A'B},$$

$$\frac{d\sigma_3}{dR}=\frac{AC'-A'C}{AB'-A'B}.$$

For the flange portion, however, we will have from equations (1) and (2),

$$A=R+R\sigma_1\left\{\frac{1}{4}Km(\bar{\sigma})^{m-2}(2\sigma_2-\sigma_1-\sigma_3)(2\sigma_1-\sigma_3-\sigma_2)-\frac{1}{2}K(\bar{\sigma})^m\right\},$$

$$B=R\sigma_1\left\{\frac{1}{4}Km(\bar{\sigma})^{m-2}(2\sigma_2-\sigma_1-\sigma_3)(2\sigma_3-\sigma_1-\sigma_2)-\frac{1}{2}K(\bar{\sigma})^m\right\},$$

$$C=\sigma_3-\sigma_1+\mu p\frac{R}{T},$$

$$A'=\frac{1}{4}Km(\bar{\sigma})^{m-2}(2\sigma_1-\sigma_3-\sigma_2)(2\sigma_3-\sigma_1-\sigma_2)-\frac{1}{2}K(\bar{\sigma})^m,$$

$$B'=\frac{1}{4}Km(\bar{\sigma})^{m-2}(2\sigma_3-\sigma_1-\sigma_2)^2+K(\bar{\sigma})^m,$$

$$C'=\frac{1}{R}\{1-e^{(3/2)K(\bar{\sigma})^m(\sigma_3-\sigma_1)}\}.$$

For the forming radius portion and from equations (3) & (4), we have

$$A=R+R\sigma_1\left\{\frac{1}{4}Km(\bar{\sigma})^{m-2}(2\sigma_2-\sigma_1-\sigma_3)(2\sigma_1-\sigma_3-\sigma_2)-\frac{1}{2}K(\bar{\sigma})^m\right\},$$

$$B=R\sigma_1\left\{\frac{1}{4}Km(\bar{\sigma})^{m-2}(2\sigma_2-\sigma_1-\sigma_3)(2\sigma_3-\sigma_1-\sigma_2)-\frac{1}{2}K(\bar{\sigma})^m\right\},$$

$$C=\sigma_3-\sigma_1,$$

$$A'=\frac{1}{4}Km(\bar{\sigma})^{m-2}(2\sigma_1-\sigma_3-\sigma_2)(2\sigma_3-\sigma_1-\sigma_2)-\frac{1}{2}K(\bar{\sigma})^m,$$

$$B'=\frac{1}{4}Km(\bar{\sigma})^{m-2}(2\sigma_3-\sigma_1-\sigma_2)^2+K(\bar{\sigma})^m,$$

$$C'=\frac{1}{R}\left\{1-\frac{1}{\sin\theta}e^{(3/2)K(\bar{\sigma})^m(\sigma_3-\sigma_1)}\right\}.$$

The stress and strain distributions in the deformed part as well as the forming force may be determined from the above relations, but since these determinations cannot be solved analytically, the determinations are made numerically by first finding initial values from given boundary conditions for forming i.e., the initial values σ_{10}, σ_{20} and σ_{30} for the flange portion can be found from the following.

For $\qquad R=R_0, \qquad \sigma_{10}=p=\sigma_{20}.$

Therefore and from equation (5) we have

$$\varepsilon_{30}=\ln\frac{R_0}{r_0}-K(\sigma_{10}-\sigma_{30})^{m+1}.$$

In the calculations related to the portion of forming radius, the initial values of σ_1 and σ_3 at the point B (Fig. 1) on the inside edge of the flange are used, where for the position of the point B must be established, since the point B will vary according to the applied forming pressure and the value of the radial stress created in the blank piece. This point was determined by utilizing the following relation. In other words, if we take the punch and the blank portion R_1, as shown in Fig. 1 as an integral unit and take into account the axial equlibrium, we will have

$$P=\pi R_1^2 p.$$

In carrying out the calculations, the punch force P and forming pressure p found from experiments were used in determining the value of R_1, while simultaneously profile photographs of the formed test piece were taken to ascertain its position. Moreover, the relation of R and θ in equation (4) was also found.

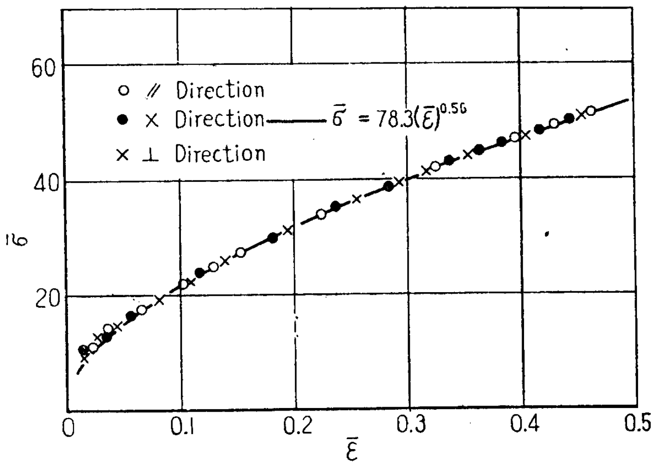


FIGURE 2. Stress-strain curve for 7/3 brass used for calculation.

The plastic curve of the material is shown in Fig. 2 wherein the solid line representing the curve corresponding to $\bar{\sigma}=78.3(\bar{\varepsilon})^{0.56}$ kg/mm^2 coincides very closely with the experiment points for large strains.

2. *Calculated Results and Observations*

The test pieces considered for calculations consisted of brass plates 66.7 mm in diameter and 1.0 mm and 0.4 mm in thickness hydroformed with a punch 30 mm in diameter provided with a head profile radius of 4 mm and 1 mm. Caluclations were made on four typical examples in lieu of calculations for the entire working process. The final forming pressure, punch force and given values for each of the test specimens are indicated in Table 1, in other words, cases were taken for ex-

TABLE 1. CALCULATED VALUES FOR SPECIMENS

Specimen code	Blank radius r_0 mm	Blank thickness t_0 mm	Punch stroke S mm	Forming pressure p kg/mm^2	Punch force P kg	ε_{30}	R_1
A	33.35	1.02	9.8	1.0	2100	−0.05	25.8
B	33.35	1.02	19.7	4.0	5520	−0.26	21.0
C	33.35	1.02	10.0	4.0	5740	−0.06	21.4
D	33.35	0.43	9.5	4.0	4680	−0.07	19.3

ample using a forming pressure p of 1.0 kg/mm^2 and 4 kg/mm^2 for t_0=1.0 mm for a punch stroke S of approximately 10 mm. Furthermore, a forming pressure of 4 kg/mm^2 for 1.0 mm corresponding nearly to the maximum punch force P at a stroke S of 19.7 mm was adopted. The stress distribution for each of the specimens was calculated using a friction coefficient μ=0.2 and the strain distributions determined therefrom were plotted as shown in Figs. 3∼6. The values of strains determined from experiments were also plotted wherefrom, and it will be noted that the strain distributions for specimens A, B and C of 1.0 mm thickness coincide very

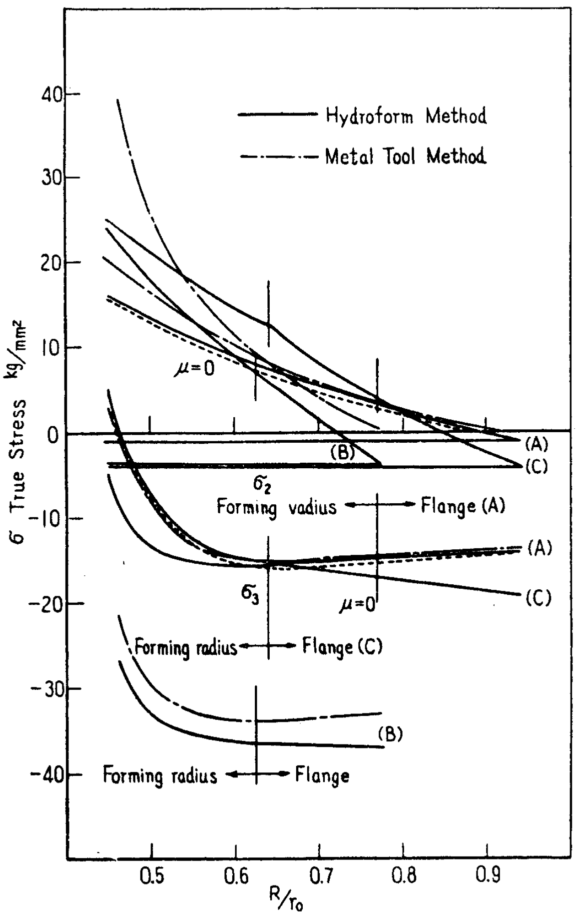


FIGURE 3. Stress distribution in specimen A, B, C for t_0=1.0 mm according to hydroform and metal tool methods.

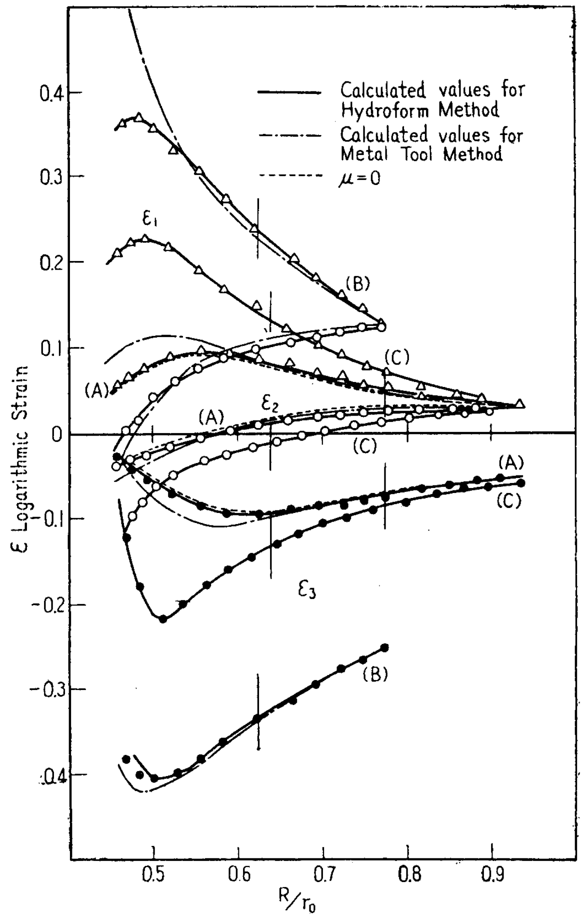


FIGURE 4. Strain distribution for specimen A, B, C for t_0=1.0 mm according to hydroform and metal tool methods.

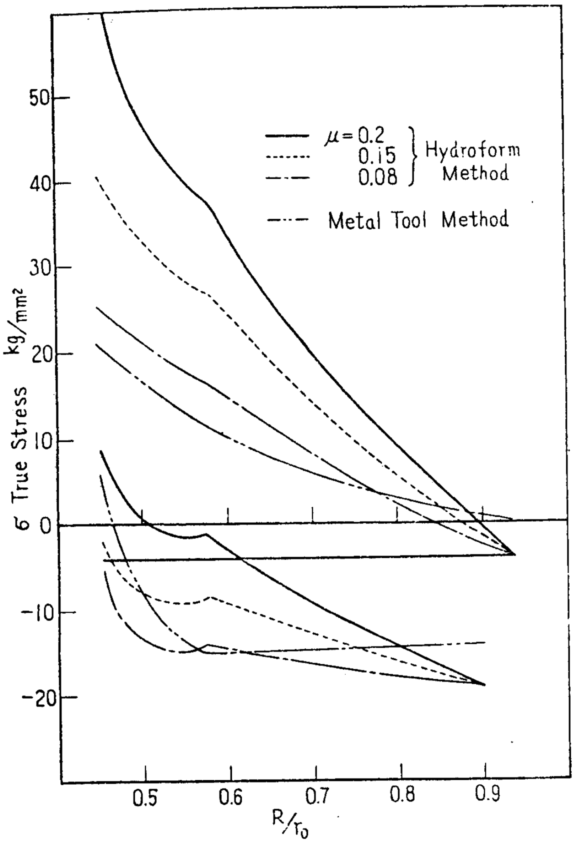


FIGURE 5. Variation of stress distribution for specimen D (t_0=0.4 mm) for various values of friction coefficient.

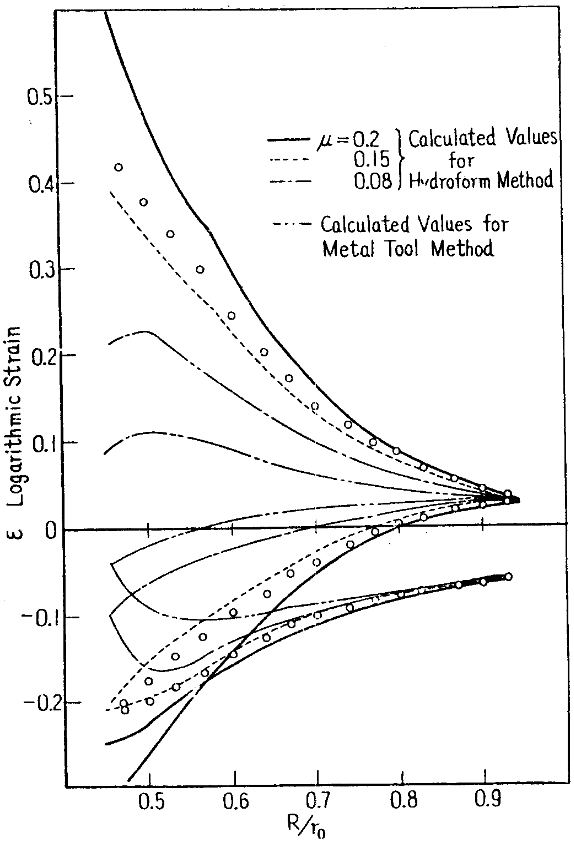


FIGURE 6. Strain distribution in specimen D (t_0=0.4 mm) for various values of friction coefficient.

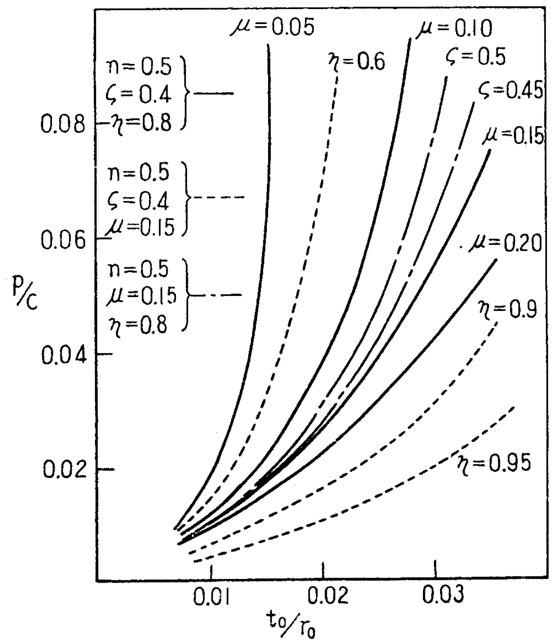


FIGURE 7. Influence of friction coeficient μ, peripheral deformation ratio η, and drawing coefficient ζ, to t_0/r_0 vs. p/c.

closely with calculated results. The results of calculations for stress distribution and strain distribution for the specimen D of thickness 0.4 mm for friction coefficients 0.2, 0.15 and 0.08, respectively, are plotted as shown in Fig. 5 and Fig. 6 respectively. Although a substantial deviation is noted, we could expect a fairly good coincidence, if the friction coefficient μ were to be held about 0.17. The value of the friction coefficient found in accordance with a separately conducted experiment coincides very close with the above noted value.

In the case of the specimen of 1.0 mm thickness, and as shown in Fig. 4, the deduction is made that we would obtain results coinciding approximately with experiment values even though a friction coefficient $\mu=0.17$ were to be used since there would be very little difference in calculated values as compared with those for a specimen of thickness 0.4 mm, even if calculations were to be made using a friction coefficient $\mu=0$. According to the results therefore, the mathematical treatment expounded herein can be adjudged at least as being appropriate. Furthermore, and for the purpose of comparison, the calculated values of the stress and strain distributions in shells assumed to have been formed by a metal die having a die profile radius approximately equivalent to the forming radius of the various specimens and subjected to the same degree of deformation, are plotted in the respective figures.

According to the hydroform method, a negative radial stress as deduceable from the figures is created at the periphery of the blank due to the forming pressure. On the other hand, however, the radial stress increment due to the friction resistance between the blank and blank holder plate is greater than in the case of the metal tool method which is a detracting factor for forming. Moreover, the increment of stress σ_1 in the portion of forming radius is smaller than in the case of the metal tool method due to the absence of the factor of friction which in effect is an advantageous condition for forming.

Now, the drawing stress, the value of the radial stress at the point of contact between the blank and punch according to $(\sigma_1)_{R=R_4}=\sigma_d$ is taken as a summarization of all these factors, which are compared with each other for both the hydroform and metal tool methods, and are as shown in Table 2.

TABLE 2. DRAWING STRESS IN HYDROFORM AND METAL TOOL METHOD

Specimen code	Drawing stress in hydroform method $(\sigma_1)_{R=R4}$ (α) kg/mm^2	Drawing stress in metal tool method $(\sigma_1)_{R=R4}$ (β) kg/mm^2	$\frac{\alpha}{\beta}$
A	15.1	20.4	0.74
B	23.4	39.4	0.59
C	24.6	(20.4)	1.21
D	57.6	(20.4)	2.82

The drawing stress in specimens A and B of thickness 1.0 mm formed at a slightly higher pressure than the critical blank holding pressure as hereinafter denoted is found lower by 25–40% than that according to the metal tool method.

However, we will find that even though the blank may be of similar thickness (1.0 mm) such as in the case of specimen C, a higher stress would be generated by resorting to the hydroform method, if the specimen is subjected to a substantially high forming pressure in the initial stage of working. Moreover, and although an extremely high drawing stress is found to develop in the blank 0.4 mm in thickness, no fracture develops in the blank. These various features differing from those for the metal tool method are the characteristics developed by the combination of the action of the forming pressure on the total area of the blank during the process of forming and the friction between the tool and blank. In a further qualitative analysis of these chracteristics, the variation of thickness in equation (1) was neglected and with the assumption of uniform distribution of the maximum principal shear stress ($\sigma_1-\sigma_3=k$) over the total flange area, it is found that the drawing stress can be expressed simply as follows.

$$\sigma=k\ln\left(\frac{R_0}{R_4}\right)+\frac{\mu p}{t_0}(R_0-R_1)-p\,. \qquad (6)$$

The first term in the right-hand side of the above equation represents the force necessary for the deforming of blank, the second term being the frictional effect of the flange created by the forming pressure and the third term indicates the effect resulting from the application of a radial compressive force to the flange by the forming pressure.

The second and third terms contribute an opposing effect to the drawing stress. Moreover, the magnitude of the second and third terms will vary in relation to the magnitude of the friction coefficient, thickness, and size of blank and forming stroke. So that there will be occassions when the forming pressure will act advantageously as well as disadvantageously and will become an element in contributing to dimensional effect.

Now, in order to investigate the qualitative tendency of the relations in the aforenoted factors, the relation of

$$\eta-\frac{R_1}{r_0}\gtrless\frac{\alpha}{\mu} \qquad (7)$$

in which

$\eta=R_0/r_0$ peripheral deformation ratio,
$\alpha=t_0/r_0$ thickness ratio,

in the second and third terms in equation (6), is graphically plotted as shown in Fig. 7. However, from the equilibrium of force in the forming radius portion in the direction of the punch axis and from equation (6), we have

$$\frac{R_1}{r_0}=\mu\zeta\pm\sqrt{(\mu\zeta)^2+\zeta^2+\left\{\frac{k}{p}\ln\frac{\eta}{\zeta}+\frac{\mu\eta}{\alpha}-1\right\}2\zeta\alpha}\,,$$

where $\zeta=(r_1+t_0)/r_0\doteqdot R_4/r_0$ drawing coefficient.

The upper portion segregated by each curve represents the range described for $\eta-R_1/r_0>\alpha/\mu$ whereas the lower portion indicates the range for $\eta-R_1/r_0<\alpha/\mu$.

Among these curves, special note should be taken of the major effect of the friction between the blank and blank holder plate, which indicates the necessity for thorough consideration of lubrication of the flange in conjunction with forming work. Moreover, the increase in effectiveness of the third term in equation (6), as the forming work progresses, may in some cases be undesirable in relation to the development of wrinkles in the flange at the completion of the forming cycle. The existence of these characteristics at the flange and no friction (normally of small magnitude even when present) acts at the forming radius portion, are factors affecting the drawing stress. For instance, in the example of specimens A and B formed from a plate of 1.0 mm thickness under a relatively low pressure as indicated in Figs. 3 and 4, it will be seen that although the rate of radial stress increment in the flat portion of the flange is greater in the case of the hydroform method due to the friction effect, its effect is cancelled out due to the condition of $\eta - R_1/r_0 < \alpha/\mu$. Moreover, a smaller drawing stress would be produced than in the case of metal tool method because of the absence of friction at the forming radius portion. On the other hand, when $\eta - R_1/r_0 > \alpha/\mu$ the advantageous feature of the absence of friction at the forming radius portion disappears, whereby it will be found that a greater drawing stress would result in the case of hydroform method. The case for this condition is depicted by specimen C and D.

In analyzing the forming limit, it is necessary to consider not only the drawing stress, but also the magnitude of the fracture resistance as well at the same time. In the metal tool method, the punch force is concentrated at the punch head corner and the forming limit is established by the fracture of the part. The apparent fracture resistance is very neary equal to the tensile strength. In the hydroform method, however, a friction force is generated between the side wall of the punch and the side wall of the formed shell by virtue of the forming pressure, which in effect prevents the concentration of the punch force at the punch head corner. The friction force may support practically all of the forming force under adequate combination of forming pressure, length of side wall of the shell and plate thickness. Consequently, we can expect fracture to occur not only at the point of contact of the punch head corner, but also along the side wall of the formed shell. Therefore, the fractures to be expected may be classified into three types and the apparent fracture resistance to vary widely.

1) Fracture at the point of contact with the corner of the punch head.

This type of fracture with the exception of a few special cases generally controls the drawing limit for deep drawing according to the metal tool method. This type of fracture will also occur in the hydroform method, if the pressure applied in the initial stage of working is too high because of an increase in friction force between the flange and blank holder plate and the addition of a bending stress accompanying the reduction of forming radius. If the blank is excessively large, a fracture likened to a shear fracture would occur in the early stage of forming.

2) Fracture along the side wall of the formed shell.

This type of fracture may occur after the forming progresses to a certain degree without fracturing as described in the preceding parapraph (1), when the forming operation reaches a point, where the formed part is compressed against the side wall of the punch under the forming pressure and such so that the created friction force is made to take up most of the punch force.

3) Fracture at the forming radius of a formed shell.

This type of fracture will also occur when the forming has progressed to some degree. The origin and cause is considered to be attributable to the combination of a decrease in plate thickness as may be due to a bending deformation resulting from a substantial decrease in the forming radius due to an extremely high forming pressure and the increase in drawing force as well.

PART 2. EXPERIMENT FINDINGS

1. *Experiment Apparatus and Specimen Materials*

The apparatus employed for hydroform experiments is shown in Fig. 8. A thin rubber bag (rubber bag for gas analysis) of shore durometer hardness 40 is contained within the semi-spherical fluid pressure chamber of 100 mm in diameter.

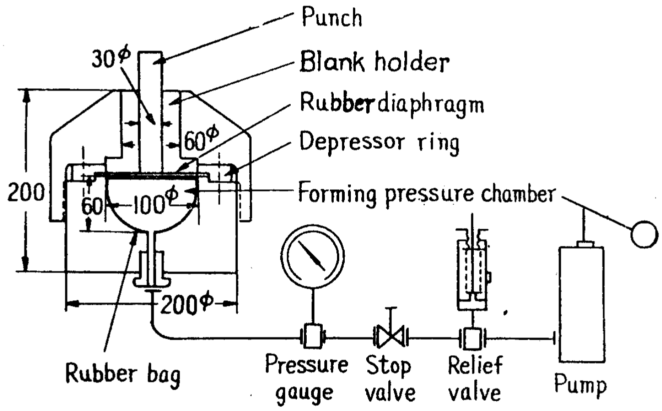


FIGURE 8. Experiment apparatus for hydroform method.

The line interconnecting this bag to a manually operated high pressure pump of 1000 kg/cm^2 capacity is provided with a pressure gage of maximum working capacity of 1000 kg/cm^2, a relief valve and a stop valve. The upper portion of the rubber bag is provided with a protective sheet of about 3 mm in thickness of laminated rubber with a shore durometer hardness 50 and secured to the main body by means of a retaining ring. The cover fitted with the blank holder serving as a guide for the punch and screwed-on directly to the exterior of the fluid chamber, serves to take up the reactive force applied to the cover. The punch force and forming stroke were measured by means of a 10-ton compression tester and the forming pressure appropriate to the punch stroke regulated by means of the manually operated high pressure pump and relief valve and the pressure determined from the pressure gage.

A standard punch with a body diameter of 30 mm provided with a head profile radius of 4 mm, was employed. In addition, two other types were used, one with a profile radius of 1.0 mm and the other with a semi-spherical head in order to investigate the effect of the shape of a punch head.*

Three kinds of rolled plates were used for blanks–aluminium, mild steel and 7/3 brass. The plate thickness and mechanical properties of each of the materials are given in Table 3.

TABLE 3. MECHANICAL PROPERTIES OF SPECIMENS

Material	Hardness V.H.N.	0.2% residual strain yield stress kg/mm²	Tensile strength kg/mm²	Elongation %	Annealing temperature °C	Plate thickness mm
Al	22.8	2.7	8.3	41.7	360°C×1 hr	1.0
7/3 Brass	67.1	11.2	32.6	74.3	500°C×1 hr	1.0, 0.6, 0.4
Mild steel	120.0	27.0	35.4	35.7	——	0.6

Circular blanks were cut to the required size by means of a lathe from the plates. The aluminium and brass plates were annealed at the temperature and time specified in the Table and furnace cooled. Mild steel plates used were the commercial sheets.

The lubricant used was soybean oil identical to the oil used in the experiments for deep drawing according to the metal tool method previously investigated by the authors.

2. *Minimum Required Forming Pressure*

In deep drawing work according to the metal tool method, the geometrical shape of the blank during deformation is unconditionally controlled mainly by the contour and shape of the die used. In the hydroform method, however, wherein the forming pressure serves as a die, the shape features a wide range of variation depending on the magnitude of the applied forming pressure.

For example, as the forming radius deforms dimensionally in balance with the forming pressure, the radius will decrease as the forming pressure rises. Moreover, this radius in combination with the material properties, plate thickness and product shape, will assume complicated values and thereby produces a variety of drawing characteristics. Therefore, in adopting the hydroform method for deep drawing work, the manner in which the forming pressure in proceeding with the forming work is to be varied, will be an important factor in establishing the outcome of of the work.

One factor that can be cited as governing the appropriateness of deep drawing work according to the metal tool method would be that the development of wrinkles in the flange and body. In the case of deep drawing of a cylinder, these wrinkles can be attributable principally to a deficiency in blank holding force and also to too great a profile radius in the die shoulder. If the dimensional size of the profile

* Material for fluid pressure chamber made by Japan Special Steel Co. specification SNCM-2, Brinnel hardness, 300. High pressure pump and valves made by Matsuura High Pressure Machinery Co.

radius of the die is appropriate, the only problem concerned would be that of the development of wrinkles in the flange. The blank holding force necessary to prevent this can be decreased to a desirable degree near the closing stage of the work. In the hydroform method also, the development of wrinkles becomes a factor that governs the forming pressure, since in essence, the deformation of the blank is regarded as being similar to that in the case for the metal tool method. Moreover, as the forming radius will vary in accordance with this method, we can expect the development of wrinkles not only in the flange, but also in the body proper as well.

Fig. 9 indicates the relation obtained experimentally of the minimum required

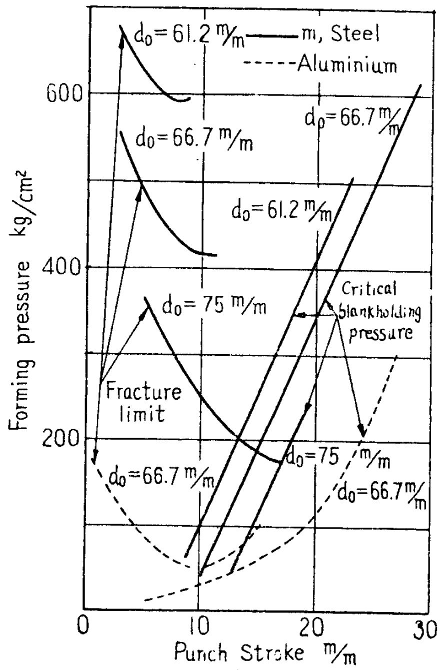


FIGURE 9. Variation of forming pressure according to material and blank diameter.

forming pressure (critical wrinkle developing pressure) to the punch stroke, whereby forming can be performed without causing the development of wrinkles for the forming of a mild steel plate and aluminium plate employing a punch with a body diameter of 30 mm and a head provided with a 4 mm profile radius. The experiments were conducted by subjecting several specimens to forming under different conditions of pressure and readings taken of the forming pressure at the point of wrinkle development and the relative punch stroke. It was found in this instance that as wrinkles started to form due to buckling of lower order, a decrease in the punch force would occur and be registered by the load gage on the tester. Otherwise, specimens were removed from the apparatus and inspected macroscopically for wrinkles.

It was found that insofar as the blanks of the sizes and thicknesses treated in this experiment were concerned, the forming radius would become large leaving practically no flange area, if the forming pressure is too low in the final stage of forming. As a result, several body wrinkles will develop at the upper edge of the product due to buckling of lower order. The result shown in Fig. 9 includes the pressure necessary to prevent the development of this kind of wrinkle and represents the necessary pressure to serve a function differing from the blank holding pressure in the metal tool method. It is also attributed to a rapid increase in the critical pressure after a certain time in the course of forming work. The drawing also indicates that on occassion, the pressure for preventing flange wrinkles may be greater than the pressure for preventing body wrinkles at a certain forming stroke depending on the combination of blank size and plate thickness.

In order to prevent the development of body wrinkles, it will be necessary to vary the forming pressure so that a certain width of the flange on the blank will always rest on the blank holder plate and moreover, deform with a forming radius of an appropriate dimension. Therefore, it will generally be found necessary to increase the forming pressure so that the forming radius may become small as the periphery of the blank shrinks with progress of forming.

From the above, the observation is made that insofar as the hydroform method is adopted in the forming of a cylindrical shell, the minimum necessary pressure would be governed by the following three conditions.

1. That the outermost periphery of the blank is at least in contact with the surface of the blank holder. If a gap were to exist between the outermost periphery of the blank and the surface of the blank holder, the intrusion of the rubber film in this gap and its sandwiching between the punch sidewall and blank would make it difficult to obtain a satisfactorily formed object.

2. That the pressure be of a value sufficient to prevent the development of wrinkles in the flange. In this experiment where blanks of selected dimensions were worked, no wrinkles were found developing in the flange. However, the development of this kind of wrinkle is often found in cases where the ratio of blank diameter to thickness is greater than those covered in this experiment or depending on the anisotropic qualities and shape of the blank [*4*].

3. That the pressure be of a value sufficient to prevent the development of body wrinkles. In the case of forming a cylinder, body wrinkles can be prevented by adopting a pressure that would produce an appropriately sized forming radius. Generally however, it will be found in many cases that the required pressure would be substantially influenced by such factors as the product shape, blank dimensions and anisotropic qualities in the plate.

Examples of the relations found experimentally of the critical blank holding pressure to the material properties, blank diameter and plate thickness satisfying the condition in the preceding are presented in Figs. 9 and 10. It will be noted that the critical blank holding pressure will gradually decrease as the blank diameter is increased. Since any wrinkles as would be found in the preceeding case would be body wrinkles, it is thought that a smaller forming radius would be required for a blank of smaller diameter than for a larger diameter for the same punch stroke insatitisfying the condition indicated in (1) above. Moreover and upon the basis of results as noted hereinafter, the fact of non-correspondence of the relation between punch stroke and the amount of deformation of the blank size differs, might also be considered as a cause for body wrinkles. If the forming radius r_f at which body wrinkles form can be established as being several times the plate thickness, we would have the following approximate relation $r_f/t_0 \approx \sigma_1/p$. Therefore, if blanks are of the same material and identical in diameter, the critical blank holding pressure should not vary too greatly even with changes in plate thickness. This is depicted in the results shown in Fig. 10 and the fact of the extremely small difference in relation to plate thickness can be given as a major reason that supports the above mentioned reasoning. Of course, there are cases where a like reasoning cannot be given so simply particulary as in the case of deep drawing of a large cylinder where we will find that the pressure for suppressing flange wrinkles is greater than for body wrinkles. In situations of this kind, the effect of plate thickness might shot up more noticeably.

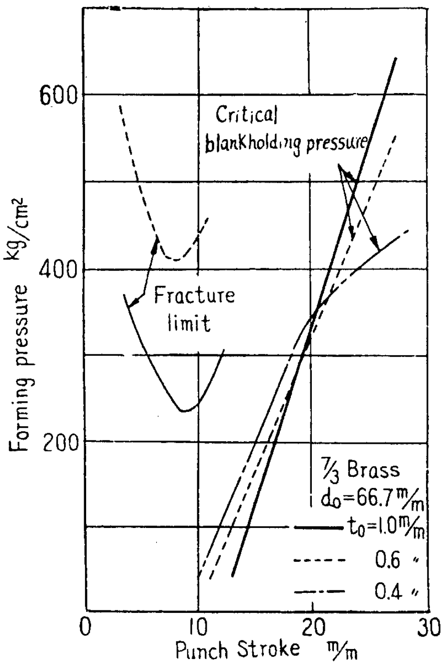


FIGURE 10. Variation of forming pressure according to plate thickness.

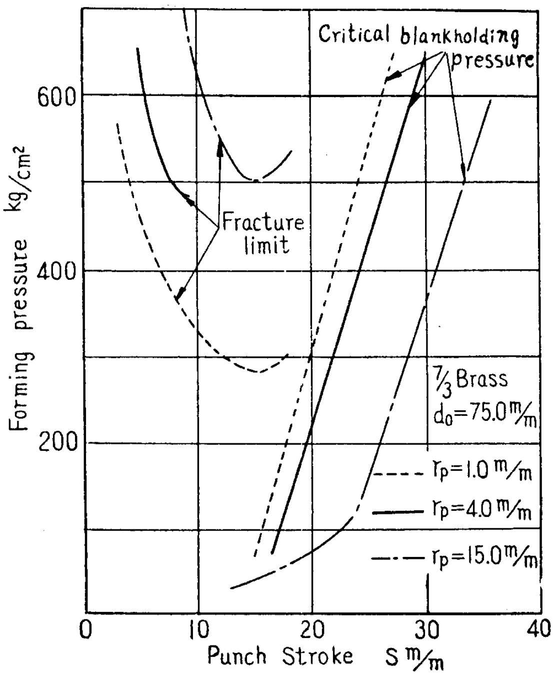


FIGURE 11. Variation of forming pressure according to punch shape.

Fig. 11 shows the results of experiments on brass plates of identical diameter and thickness (1 mm) for various punch head shapes plotted against punch stroke. It will be noted from the plotted curve that the wrinkle developing limit would be lower as the punch head profile radius becomes larger. However, it will found in this case that the average amount of deformation in the deformed part even for identical punch strokes will differ and the degree of deformation in the peripheral part of the flange for the same stroke will be small as the punch head profile radius becomes greater.

Fig. 12 is a graphical presentation of the critical pressure in terms of the peripheral deformation ratio R_0/r_0 (See Fig. 1) of the blank instead of the punch stroke; a blank of a different diameter is also taken into consideration. An inspection of the results shown in the graph discloses that if the conditions of material properties, plate thickness and the punch diameter remain the same, the critical blank holding pressure in relation to a fixed peripheral deformation ratio will remain about the same even though the punch head radius and blank diameter may vary.

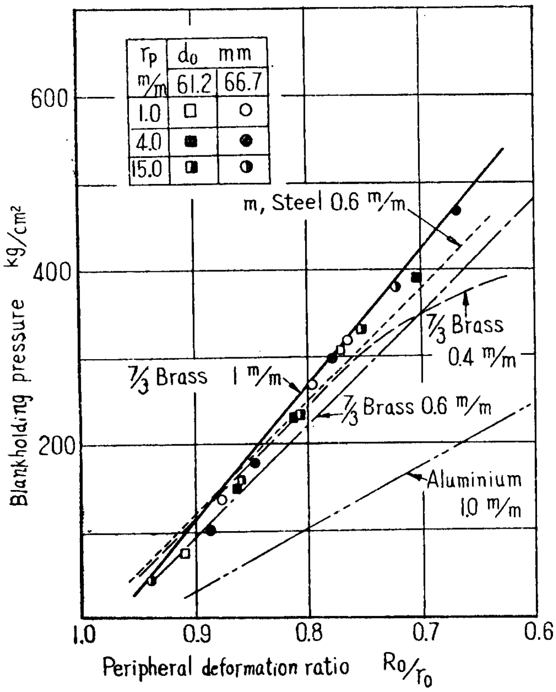


FIGURE 12. Relation of critical blank holding pressure to peripheral deformation ratio.

The variation of the wrinkle developing limit in relation to different material properties is also shown in Fig. 12. However, it has not been possible to deduce a simple relation between these two from the finding of experiments performed to date. Generally, however, a material possessing a high tensile strength seems to show a high wrinkle developing limit.

3. *Maximum Applicable Forming Pressure*

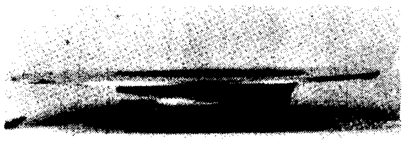
Fractures as described in Part 1 may be classified into three types and depending on the type of fracture, the maximum applicable pressure with the hydroform method would be limited.

The curves shown in Figs. 9, 10 and 11 indicating the fracture limit represent this maximum pressure or so-called critical fracture pressure derived from experiments. The variation of the limit line associated with each blank develops from the combination of the type of deformation to which the fractured part has been subjected to and the work hardening resulting therefrom (This also governs the point of fracture). This limit, like the wrinkle developing limit will be affected by the blank diameter, plate thickness and material properties.

For example, we will note from Fig. 9 that if the blank diameter is increased the fracture limit decreases, because the point of fracture will take place in the area of deformation suffering the most reduction in plate thickness because of the increase in the deformation area in the blank itself as well as of the increase in friction force in the flange. Thus the fracture limit would decrease substantially until it finally coincides with the wrinkle deveopting limit whereat the forming limit for the hydroform method would be established. Moreover, provided that the conditions of fracture remain generally the same, the effect of plate thickness on the fracture limit would, as shown in Fig. 10 and similarly as in the case of the metal tool method, be such that the fracture limit would vary approximately in proportion to the plate thickness.

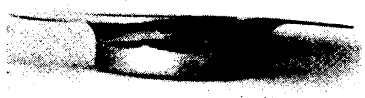
The variation in the fracture limit due to different material properties of the blank is found to be fairly great and in some instances, shown a difference greater than the difference in tensile strength of the material. This is interpreted as being caused principally by combination of conditions those of the differences in the degree of deformation suffered by the fractured part, place of fracture and effectiveness of friction.

Fig. 13a indicates the type of fractures discovered in blanks of 1 mm and 0.6 mm thickness subjected to forming pressures as adopted in this experiments. These



(a)

(a) Cup showing fracture in punch head 7/3 brass: $t_0=0.6$ mm, $d_0=66.7$ mm, $p=470$ kg/cm^2, $S=6.5$ mm.



(b)

(b) Cup showing fracture in sidewall 7/3 brass: $t_0=0.4$ mm, $d_0=66.7$ mm, $p=400$ kg/cm^2, $S=9.0$ mm.

FIGURE 13. Fracture in hydroformed blank.

were found to be of the type conforming to the category under No. 1 in the preceding paragraph. Fractures, moreover, are found to occur in a relatively early stage of working and although there does not seem to be much difference in the manner in which the blank works around the punch head as compared to the metal tool method, the friction between the blank and punch sidewall does not seem to be effectively utilized. On the other hand, when a blank of a greater ratio of diameter to thickness, i.e., 66.7 mm and 0.4 mm respectively, is subjected to forming with a punch head profile radius of 1.0 mm, a fracture of the No. 2 types as depicted in Fig. 13b would result. The relation of punch stroke to the critical fracture pressure for this case is shown in Fig. 14. It is to be noted that unless the blank comes into flush contact with the punch head in the initial stroke cycle, a fracture of the No. 1 type is liable to develop and the value of the critical fracture pressure gradually decrease as the punch stroke advances. As the blank comes into contact with the sidewall of the punch with the advance of the punch beyond a certain point, it will be found that a fracture of the second type would occur accompained again by a rise in the critical fracture pressure. The apparent drawing stress for the respective cases of fractures in the above are determined from

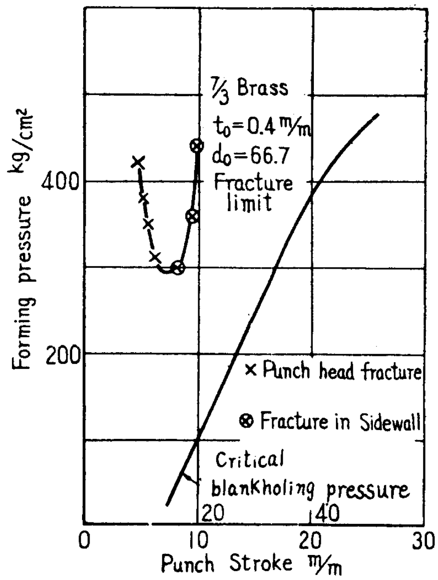


FIGURE 14. Punch stroke vs. forming pressure.

$$\sigma' = \frac{P'}{\pi(2r_1 + t_0)t_0},$$

$$P' = P - p\pi(r_1 + t_0)^2.$$

Table 4 shows the comparative relation of the apparent drawing stress to the tensile strength of the material from which we will note that when fractures of

TABLE 4. FRACTURE RESISTANCE IN HYDROFORM METHOD

	Punch stroke S mm	Forming pressure p kg/mm²	Punch force P kg	σ' kg/mm²	σ'/σ_B	Place of fracture
7/3 Brass	5	350	4180	28.7	0.88	Punch head
$t_0=0.4$	6	310	3640	34.4	1.06	Punch head
$d_0=66.7$	8	305	3840	40.6	1.24	Side wall
$d_1=30$	9	420	4930	46.5	1.43	Side wall
	9.5	460	5340	49.3	1.51	Side wall

the second type begin to develop, the area originally most subject to drawing stress shifts to the upper part of the formed blank due to the friction between the blank and punch side wall and also due to the work hardening of the part, small decreases in plate thickness as described hereafter causing the fracture resistance to increase.

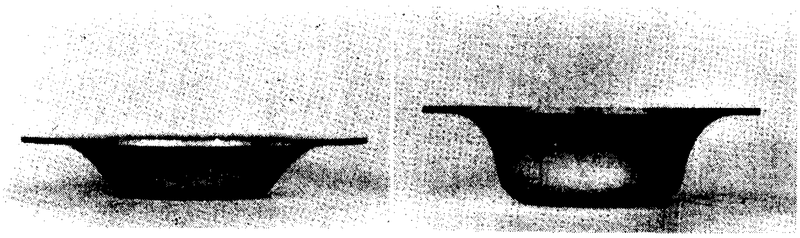
4. *Pressure Range for Forming and Deep Drawing Limit*

The forming pressure range, within which a blank can be formed without development of wrinkles or fracture in accordance with the hydroform method, embraces the area between the wrinkle developing limit and the fracture limit (See Figs. 9∼11). Therefore, as long as these limits are not infringed on, forming may be accomplished regardless of the manner of pressure application. However, any abrupt changes in pressure during the course of forming work would induce irregular variation in the forming radius and ultimately create undesirable deficiencies in appearance and accuracy in the product's sidewall.

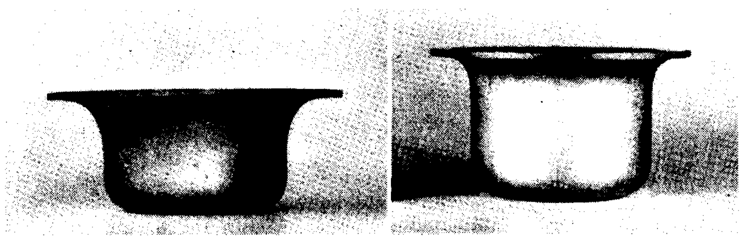
Pressure control is a matter of vital importance and should be considered seriously in designing the pressure control system of an apparatus.

Tentatively, it is deemed desirable to increase pressure monotonously as the forming work progresses.

Fig. 15 shows an example of the state deformation at various stages of a shell



$S=10$ mm, $p=100$ kg/cm^2 $S=15$ mm, $p=250$ kg/cm^2



$S=20$ mm, $p=400$ kg/cm^2 $S=24$ mm, $p=580$ kg/cm^2

FIGURE 15. Process of blank deformation found in hydroform method. (Material 7/3 brass, 1.0 mm thick., 66.7 mm in diameter)

formed within the abovenoted area from a brass plate of 1.0 mm thickness.

Table 5 gives the drawing limits for the hydroform and metal tool methods

TABLE 5. FORMING LIMITS FOR HYDROFORM AND METAL TOOL METHOD

Material	Drawing limit d_0/D_0 for hydroform method	Drawing limit d_0/D_0 for metal tool method	Plate thickness mm
Aluminium	0.43	0.48	1.0
7/3 Brass	0.38	0.44	1.0
Mild steel	0.42	0.45	0.6

respectively, from which it can be seen that the drawing limit under the hydroform method has been improved. The causes for this improvement in drawing limit can be attributed to the beneficial effect for forming such as the action of a radial compressive force resulting from the application of hydrostatic pressure on the flange and as described in Part 1, absence of frictional action at the forming radius portion and shifting of the fracture area to another part having a high fracture strength due to the effectiveness of the friction between the formed part and the punch sidewall.

The forming pressure applied to the flange of a blank in the hydroform method is much greater in magnitude than the blank holding force in the metal tool method and the resulting increase in drawing stress due to the friction is disadvantageous to forming work as described heretofor. Therefore, a further improvement in the forming limit could be expected, if only the forming pressure applied to the flange part could in some way be reduced to the order of magnitude of the blank holding force in the metal tool method.

5. *Strain Distribution*

The distribution of strain throughout the entire range of forming was measured and compared in order to ascertain more clearly the features of deep drawing according to the hydroform method and metal tool method.

Strain measurements were taken at three stages of forming of a brass blank of 66.7 mm in diameter and 1.0 mm in thickness. The results of measurements in the case of the hydroform method is shown in Fig. 16. The strain distribution for the case of a blank drawn to the proportion identical to the above with a punch 32.6 mm in diameter provided with a head profile radius of 4 mm and a flat metal die with a shoulder radius of 5.0 mm is shown in Fig. 17. Although punches of the same diameter were not used in both forming methods, it was believed, nevertheless, that differences in the general deformation process could be checked.

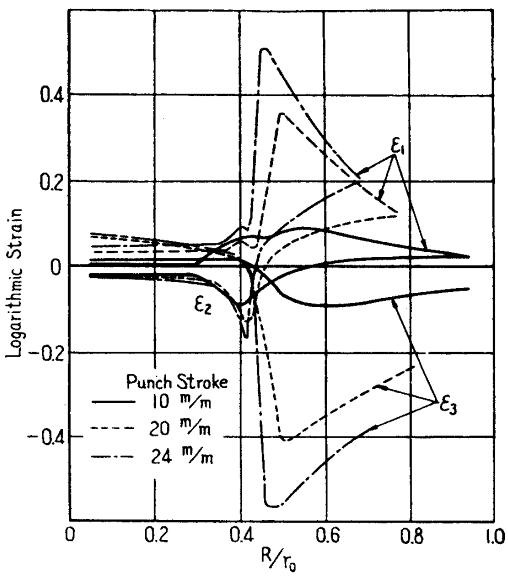


FIGURE 16. Strain distribution in cup according to hydroform method.
(Material 7/3 brass $t_0=1.0$, $d_0=66.7$, $d_1=30.0$, $r_p=4.0$)

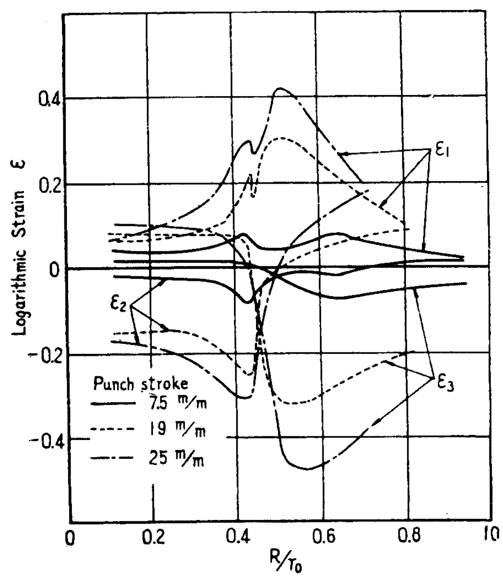


FIGURE 17. Strain distribution in cup according to metal tool method.
(Material 7/3 brass $t_0=1.0$, $d_0=72.5$, $d_1=32.6$, $r_p=4.0$)

The results of measurements show that no appreciable difference exists in the strain in the unformed peripheral part of the flange with either of the drawing methods. On the other hand, the variation in plate thickness in the formed part was found to be generally less with the hydroform method and particularly so in the area in contact with the punch head corner. In other words, since the blank envelops the punch head in the hydroform method, the thickness of the plate in contact with the head suffers practically no variation irrespective of the progress of forming. This means that a part or most part of the drawing force is countered by the friction existing between the punch side surface and blank. Therefore, in order to minimize the danger of fracture in the area adjacent to the punch head corner or to minimize irregularities in plate thickness in deep drawing work according to the hydroform method, it would be better to proceed with the forming work at a possibly lowest preassure until the blank completely envelops the punch head corner. The treatment with respect to pressure application, manner of control and other factors for controlling uniformity in plate thickness and for improvement of product accuracy, is omitted for lack of sufficient and conclusive data.

SUMMARY

Stress and strain distributions based on the analysis according to the total strain theory were found in order to determine basic data essential for deep drawing of a cylindrical shell according to the hydroform method and such data compared with those with respect to the metal tool method.

An apparatus consisting of an fluid pressure chamber of 100 mm in diameter and a punch of 30 mm in diameter was used in conjunction with experiments on various combinations of blank of different properties, thickness and shapes in determing characteristics associated with forming.

The following is a summary of the points clarified.

The minimum required forming pressure in the hydroform method is decided on the basis of formation of wrinkles in the flange and body and its maximum value governed according to three types of fractures. Moreover, the forming pressure not only functions to apply a radial compressive stress to the blank periphery, but also creates an increse in the frictional reisitance in the flange and contributes an opposing effect to the drawing force. The effect of this frictional resistance is espcially great so that due consideration must be given to lubtication of the flange.

Main causes contributing to the improvement of the forming limit as compared with the metal tool method are summarized as follows.

1. Development of a compressive stress at the outermost periphery of the blank due to fluid pressure.

2. Smallness of increment in drawing stress due to the absence of the effect of friction at the forming radius portion.

3. Bearing of the drawing force by the side surface of the punch due to the friction between the blank and punch sidewall and shifting of the cross section most susceptible to fracture to an area of greater fracture strength through the convenient process of work hardening and change in plate thickness. This improvement in forming limit, being a feature of this method besides requirement of

less number of punch and die, permits simplicity in die installation and adjustment and eliminates surface flaws in the formed article.

In closing, the authors wish to express their deep appreciation to the Yasukawa Electric Mfg. Co., Meiki Mfg. Works, Sumitomo Metal Industries, Japan Special Steel Company and Toyo Menka Company for the generous assistance and many conveniences accorded this research program.

Department of Materials
Aeronautical Research Institute
University of Tokyo, Tokyo,
June 2, 1958

REFERENCES

[1] C. B. Lewis, Machinery (A) Vol. 58, No. 2 (1951).
" , " (B) Vol. 80, June 24 (1952) p. 160–164.
[2] M. Masuda, T. Murota, Special lecture on plastic working, Nov. (1955).
[3] Panknin, V. D. I., (1957), p. 295–306.
[4] S. Fukui, K. Yoshida, M. Kunogi, Commemorial lecture, 60th Anniversary J.S.M.E. (1957).
[5] S. Fukui, Y. Yuri, K. Yoshida, Report of the Inst. Sci. Tech., Univ. of Tokyo, Vol. 8, No. 4 (1954) p. 179–196.