

## FUNDAMENTALS OF ACOUSTICAL SILENCERS

### (I) Theory and experiment of acoustic low-pass filters

*By*

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*Summary* Attenuation characteristics of several acoustical low-pass filters have been studied theoretically and experimentally. A method of electrical equivalent network was applied to the acoustical elements, such as cavities, resonators, internal tube type cavities and cavities with absorbing material. Each element was represented by four terminal matrix, and attenuation of the system was calculated from their products.

By an automatic recording system, frequency characteristics of various acoustical elements and their combination were measured. They show good agreement with calculation.

#### I. INTRODUCTION

Various sorts of acoustic filters have been used as silencing devices. They consist of several types of fundamental elements; expansion chamber (cavity), resonator and their deformed types. Theoretically, as one dimensional problem, they have been investigated fairly well [1] [2] [3]. In the present work, instead of equations of motion and continuity, an equivalent electric circuit has been considered. From the equivalent circuit, transmission characteristics can be calculated easily. Products of four terminal matrices of each section of muffler represent the relation between the transmission velocity and pressure at the considered point before and after the muffler is inserted, from which the attenuation can be estimated. In principle, it is possible to calculate the attenuation of the acoustic filter as a two-dimensional problem, if the equivalent network of the acoustic system is obtained. But it is fairly laborious work, and the calculation can be pursued by an electric simulator. The method and results of simulator will be reported in the following paper as Part (IV). Acoustical properties of the fundamental filter elements or their combination have been investigated experimentally. Frequency response of attenuation characteristics is recorded automatically by the high speed level recorder, for the case with input and output ends are terminated by non-reflective material. Incident acoustic pressure is controlled at a constant value for frequencies from 100 c/s to 3,000 c/s, then the recorded curve shows attenuation characteristics. Various types of muffler elements or their combination can be realized

by the cylinders, pistons and tubes of different sizes. Attenuation curves obtained for some types of resonators and the cavities of different sizes showed some interesting results. Particularly, the resonance frequencies of the resonators with holes at different places in the cavity and the attenuation characteristics of the cavity with internal tubes suggest the practical design of the silencing devices. The attenuation characteristics for complicated structure such as existing automobile mufflers have been obtained by the same methods. Absorbing materials such as glass-fiber, rock-wool and steel-wool in the cavity, though they have several troubles in the actual design, showed good attenuation characteristics, especially at high frequencies. Construction of the absorbing materials was studied also as muffler elements. In practical use, one section of the absorbing material in the muffler structure is remarkably more effective than any other constructions. But attention should be paid to the effects of the temperature and velocity of the exhaust gas on such materials.

## II. THEORY

An acoustical system of muffler is shown in a simplified form in Fig. 1, two straight long pipes connected to the muffler, are terminated at both ends by non-reflective material. An electrical equivalent circuit is as shown in Fig. 2, in which



FIGURE 1. A model of expansion chamber (cavity).

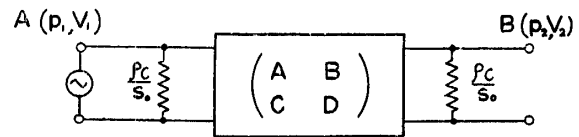


FIGURE 2. Equivalent circuit of a cavity element.

muffler element is expressed by four terminal constants. For a constant velocity sound source at A, sound pressure and volume velocity at  $A(p_1, V_1)$  and  $B(p_2, V_2)$  are, (B is an observe point and considered to be high impedance)

$$\begin{pmatrix} p_1 \\ V_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ S_0/\rho c & 1 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} p_2 \\ V_2 \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} \begin{pmatrix} p_2 \\ V_2 \end{pmatrix},$$

where  $S_0$  is a cross section of a pipe,  $c$  is sound velocity and  $\rho$  is density of air. Apparently in Fig. 2,  $V_2=0$ , then  $V_1=C'p_2$ ,  $C'=AS_0/\rho c + C + BS_0/\rho c + DS_0/\rho c$ .

In the simplest case, the muffler element is a cylindrical cavity (expansion chamber), of cross sectional area  $S$  and a length  $l$ , then

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos kl & (j\rho c/S) \sin kl \\ (jS/\rho c) \sin kl & \cos kl \end{pmatrix} \quad (1)$$

and

$$\begin{aligned} C' &= (2S_0/\rho c) \cos kl + j(S/\rho c) \sin kl + j(S_0^2/S\rho c) \sin kl \\ &= (1/\rho c) \{ 2S_0 \cos kl + j(S + S_0^2/S) \sin kl \}. \end{aligned}$$

When a straight tube of cross section  $S_0$  and length  $l$  is connected instead of a cavity, four terminal matrices of this system are expressed as,

$$\begin{pmatrix} A'' & B'' \\ C'' & D'' \end{pmatrix}$$

then

$$V_1' = C'' p_2', \text{ and } C'' = (2S_0/\rho c) \{ \cos kl + j \sin kl \}. \quad (2)$$

As  $V_1 = V_1'$  approximately, the attenuation of muffler is calculated as follows:

$$\begin{aligned} A &= 20 \log |p_2'|/|p_2| = 20 \log |C''|/|C''| \\ &= 10 \log \{ 1 + (1/4)(m - 1/m) \sin^2 kl \}, \end{aligned} \quad (3)$$

where

$$m = S/S_0, \quad k = \omega/c.$$

This is the same result as Davis' [3].

When two cavity elements, the lengths and cross sections of which are  $l_1, l_3$  and  $S_1, S_3$  are connected to a tube of length  $l_2$ , and cross section  $S_2$  (Fig. 3) then the attenuation of this system is calculated as follows:

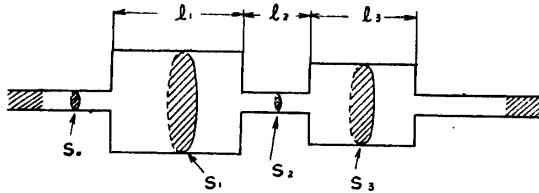


FIGURE 3. Series connection of cavities.

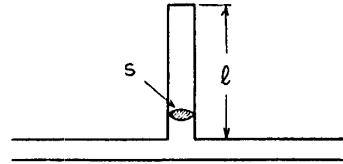


FIGURE 4. A side branch element.

$$\begin{pmatrix} p_1 \\ V_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ S_0/\rho c & 1 \end{pmatrix} \begin{pmatrix} \cos kl_1 & (j\rho c/S_1) \sin kl_1 \\ (jS_1/\rho c) \sin kl_1 & \cos kl_1 \end{pmatrix} \begin{pmatrix} \text{suffix 2} \\ \text{suffix 3} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ S_0/\rho c & 1 \end{pmatrix} \begin{pmatrix} p_2 \\ V_2 \end{pmatrix},$$

The attenuation can be calculated as (3)

$$\begin{aligned} A &= 10 \log (1/4) \{ \cos kl_1 \cos kl_2 \cos kl_3 \}^2 [ \{ 2 - (S_2/S_1 + S_1/S_2) \tan kl_1 \tan kl_2 \\ &\quad - (\text{suffix 2, 3}) - (\text{suffix 3, 1}) \}^2 + \{ (S_1/S_0 + S_0/S_1) \tan kl_1 + (\text{suffix 2}) \\ &\quad + (\text{suffix 3}) - (S_2S_0/S_1S_3 + S_3S_1/S_0S_2) \tan kl_1 \tan kl_2 \tan kl_3 \}^2 ] . \end{aligned} \quad (4)$$

Series connection of three or more cavity elements can be calculated similarly.

The attenuation of the side branch type (Fig. 4) is calculated from the equivalent circuit Fig. 5

$$\begin{pmatrix} p_1 \\ V_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ S_0/\rho c & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/Z & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ S_0/\rho c & 1 \end{pmatrix} \begin{pmatrix} p_2 \\ V_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} p_2 \\ V_2 \end{pmatrix},$$

$$A = 20 \log (|C|/|C''|),$$

where  $C''$  is a same expression of (2),  $Z = -(j\rho c/S) \cot kl$ ,  $l$ : length of a side branch,  $S$ : cross sectional area of a side branch.

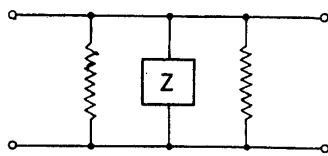


FIGURE 5. Equivalent circuit of a side branch element.

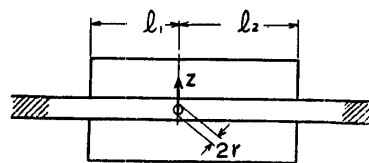


FIGURE 6. A resonator.

A resonator type muffler such as shown in Fig. 6 is that of frequently used in practice. The equivalent circuit is the same expression as shown in Fig. 5, if we choose proper impedance  $Z$ .

$$Z = -j(\rho\omega/c_0 + X) = jX',$$

where  $c_0 = n\pi r^2/(t + \beta r)$ ,  $t$ : thickness of a tube,  $n$ : number of holes,  $r$ : radius of holes,  $\beta \approx 1.6$ .

The impedance  $X$  is that of parallel connection of tubes of lengths  $l_1, l_2$ .

Then the attenuation of this system is calculated similarly as above example.

$$A = 10 \log (1 + Z_0^2/4X'^2) \\ = 10 \log [1 + \{m/(2kS'/c_0 - \cos kl - \cos k\Delta l/\sin kl)\}^2], \quad (5)$$

where  $Z_0 = \rho c/S_0$ ,  $l = l_1 + l_2$ ,  $\Delta l = l_1 - l_2$ ,  $S' = S - S_0$ ,

if  $\Delta l = l$  or  $l = l_1, l_2 = 0$  (holes are at the end of the tube)

$$A = 10 \log [1 + (1/4)\{m/(kS'/c_0 - \cos kl)\}^2], \quad (5')$$

$\Delta l = 0$ , or  $l_1 = l_2 = l/2$  (holes are the center),

$$A = 10 \log [1 + \{2m/(2kS'/c_0 - \cot kl/2)\}^2]. \quad (5'')$$

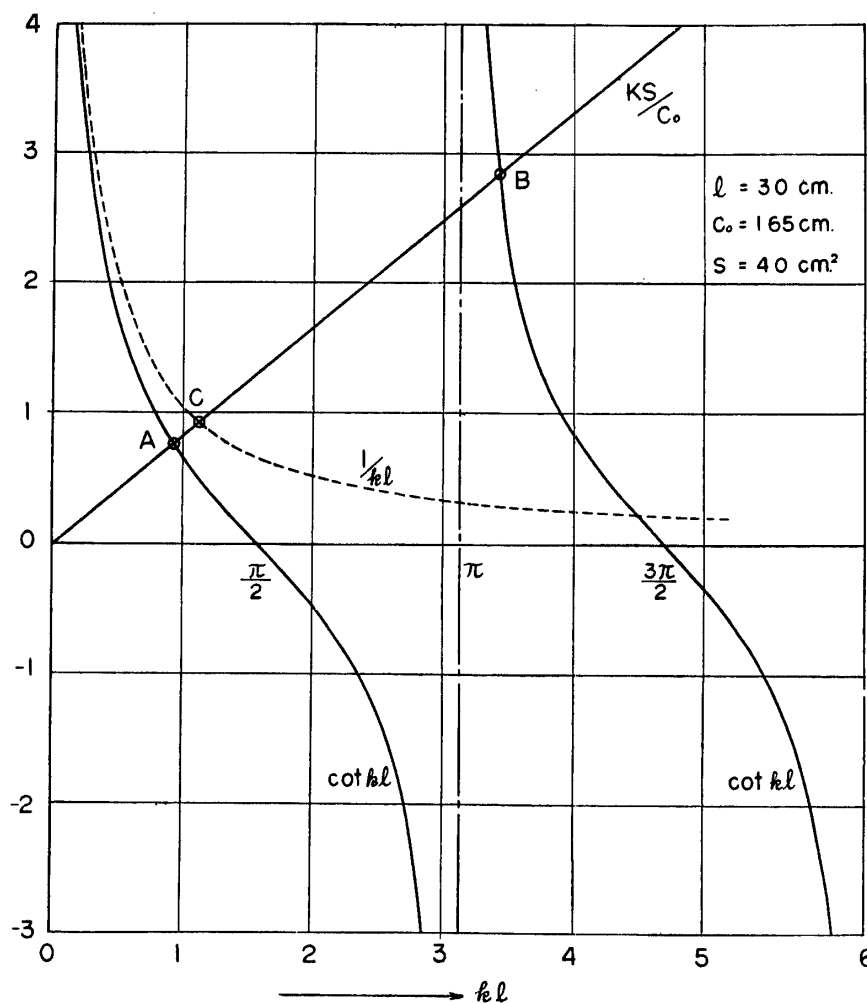


FIGURE 7. Graphical representation of resonance frequencies.

Davis derived these special cases in his report [3].

The difference of resonance frequencies calculated by (5') and an ordinary formula,

$$f_r = (c/2\pi) \sqrt{c_0/V} \quad (6)$$

can be explained by Fig. 7. In (5') the resonance frequency is determined from,  $kS'/c_0 = \cot kl$ , and (6) can be written in another form  $kS'/c_0 = 1/kl$ , because  $2\pi f/c = k$  and  $V = S'l$ .

$kS'/c_0$  is a linear function of  $k$ , and  $\cot kl$  is periodic function of  $k$ ,  $l$  as a parameter; then cross points (A), and (B) are resonance frequencies in the formula (5'), while the resonance frequency determined from (6) is a cross point  $kS'/c_0$  and an inverse curve of  $k$ ,  $l$  as a parameter, which is shown in the figure as (C).

The shift of the resonance frequency and the higher order resonance (B) can be explained by the wave phenomena in the cavity.

The attenuation of cavity with internal tubes as shown in Fig. 8 (a) is also calculated easily from the equivalent circuit of Fig. 8 (b).

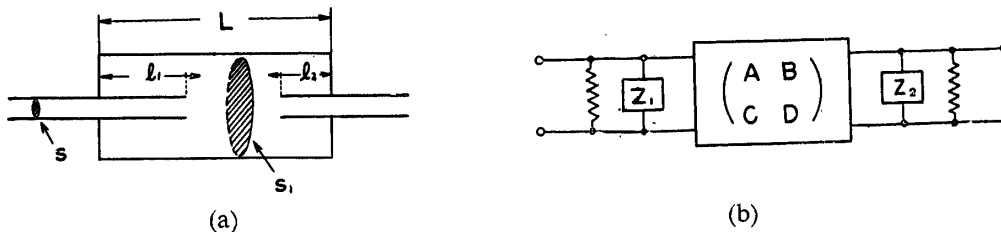


FIGURE 8. A internal tube type cavity element and its equivalent circuit.

$$\begin{pmatrix} p_1 \\ V_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ S_0/\rho c & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/Z_1 & 1 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ S_0/\rho c & 1 \end{pmatrix} \begin{pmatrix} p_2 \\ V_2 \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} \begin{pmatrix} p_2 \\ V_2 \end{pmatrix}$$

If  $L = 4l$ ,  $l_1 = 2l$ ,  $l_2 = l$

then  $Z_1 = -j(\rho c/S') \cot 2kl$ ,  $Z_2 = -j(\rho c/S') \cot kl$ ,  $S' = S - S_0$ ,

$$|C'| = 1/\rho c \left[ \{2S_0 \cos kl - (S_0 S'/S) (\tan 2kl + \tan kl) \sin kl\}^2 + \{S' \tan 2kl \cos kl + S \sin kl - (S'^2/S) \tan 2kl \tan kl \sin kl + S' \sin kl + (S_0^2/S) \sin kl\}^2 \right]^{1/2},$$

$$A = 20 \log |C'| / (2S_0/\rho c).$$

If an effective length of a tube (length in the cavity plus end correction) is equal to  $L/2$ , the first and the third pass bands of the cavity (axial length  $L$ ) will disappear, moreover, if an effective length of the other tube is equal to  $L/4$ , then the second and sixth pass bands of the cavity will be expected to disappear.

### III. EXPERIMENTAL PROCEDURE

Brass pipes (ac)(bd), 2.5 meters long each are shown in Fig. 9, the inside diameter of which is 3.5 cm. At the ends c, d of pipes non-reflective material such as glass-fiber of fifty centimeters long were packed loosely. A muffler element to be measured is inserted between a and b. Sound is conducted into the pipe through a small hole by a driver unit and picked up by a probe microphone.

The pipes (ac)(bd) are buried in sand to avoid the vibration of pipes. The

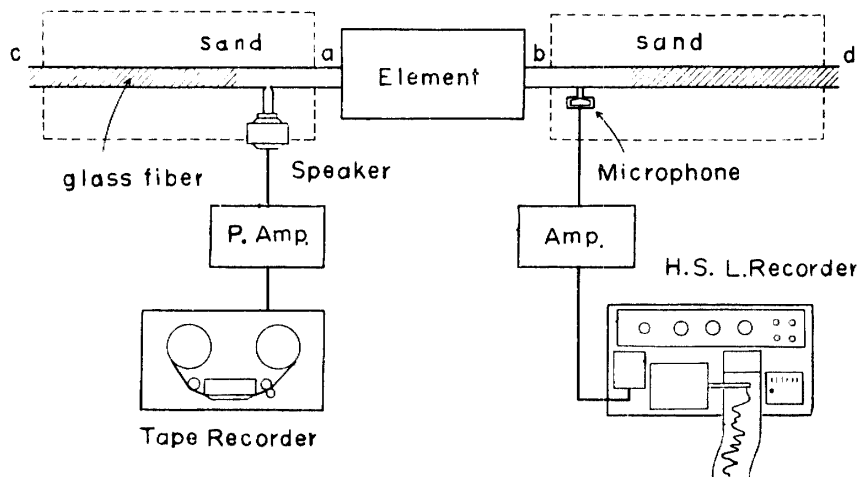
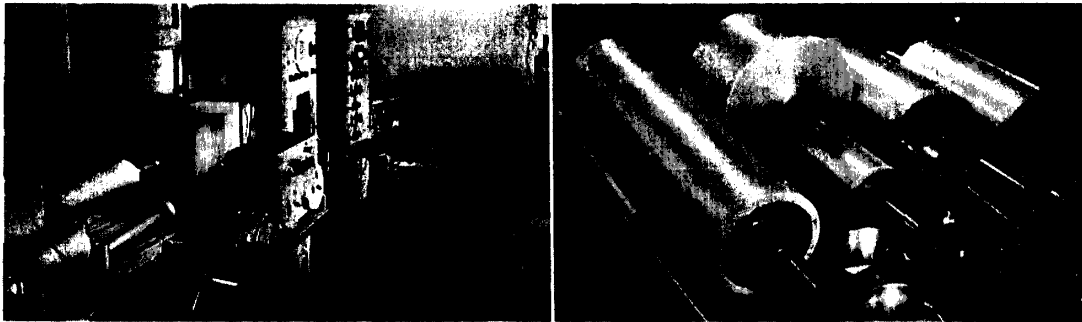


FIGURE 9. Scheme of apparatus for the measurement of attenuation. An element to be measured is inserted between a and b. The controlled electric signal recorded on the magnetic tape is supplied to a speaker. Conducting pipes ac and bd are packed with glass-fiber and buried in sand.



(a) Measuring equipment.

(b) Various elements.

FIGURE 10. A photograph of a measuring apparatus.

source side pipe (ac) can be moved on the rail to be able to connect the muffler element of arbitrary length (Fig. 10).

At the beginning of experiments, muffler element is substituted by a straight pipe and input voltage to the speaker is controlled by the output voltage of the microphone, then the sound pressure at the microphone position is held constant for all frequencies.

The oscillator is of electronically sweep type one. The controlled electric voltage was recorded on the magnetic tape from 100c/s to 3,000c/s, which has been the source of sound through the experiment.

The controlled sound pressure at the output of the microphone is shown in Fig. 11, thus the intensity of the incident wave to the muffler is held constant for all frequencies. When the muffler element is inserted between a and b then the measured curve presents the attenuation directly.

As muffler elements, aluminium cylinders whose axial lengths 50cm, 30cm, diameters 20cm, 12cm, 8cm each were prepared. The thickness of their walls were 1.0cm. By sliding pistons and conducting tubes, various types of muffler elements and their combination could be constructed easily.

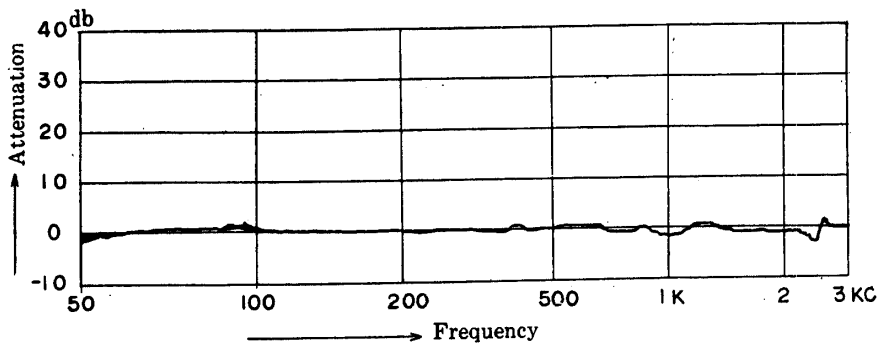


FIGURE 11. Controlled out put sound pressure.

## IV. EXPERIMENTAL RESULTS OF SEVERAL UNIT ELEMENTS

(1) *Cavity type.* Attenuation of a cylindrical cavity was measured by the method mentioned above.

The recorded attenuation curves are shown in Fig. 12, the axial length of the cavity was varied by a piston. Theoretical values calculated from formula (3) agree fairly well with experimental curves (Fig. 13), but when the length of the cylinder is below 3 cm, namely it is smaller than the radius, there appears particular attenuation (Fig. 14), which is a two-dimensional phenomenon.

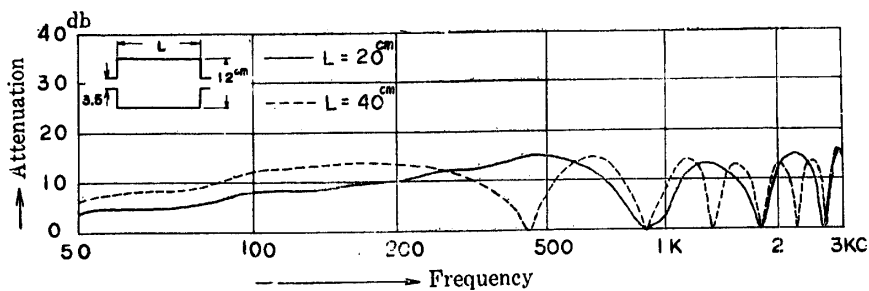


FIGURE 12. Attenuation characteristics of cavity elements.

Full line: axial length of a cavity, 20 cm. Dotted line: axial length, 40 cm.

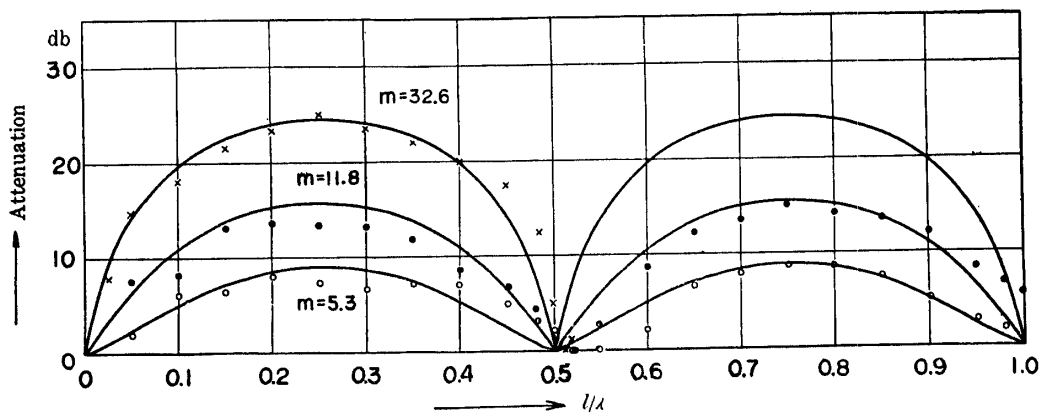


FIGURE 13. Attenuation characteristics of cavity elements for various ratios of cross sectional areas. Circles are measured points, full lines are theoretical ones.

Even in the case where an axial length is long, the attenuation at high frequencies depends also on the wave phenomenon in it, and the effects of the modes are different according to the positions of the inlet and outlet of the cavity (Fig. 15).

As predicted from the formula (3) the maximum attenuation of the cylindrical cavity depends on the ratio of the cross sectional areas of pipe and cylinder. Experimentally, it was verified as presented in Fig. 16, the measured values of the

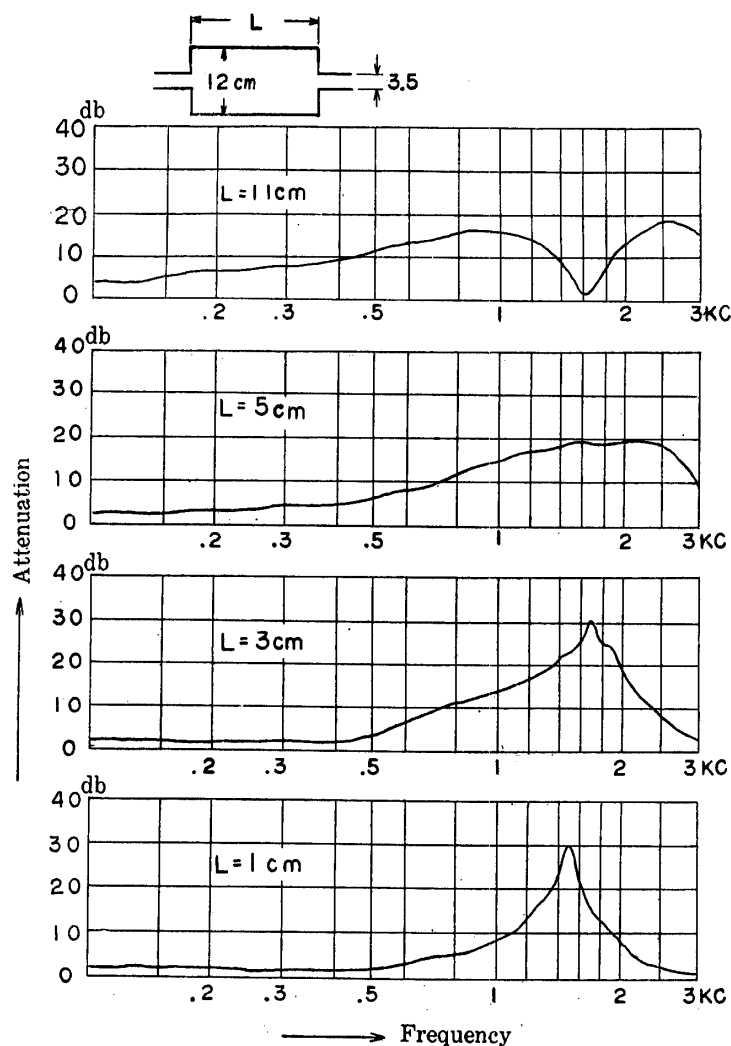


FIGURE 14. Effect of radial modes on attenuation of cavities having small axial lengths.

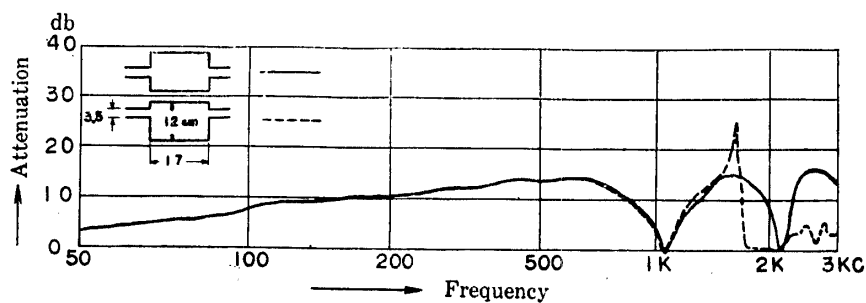


FIGURE 15. Effect of positions of input and output tubes.

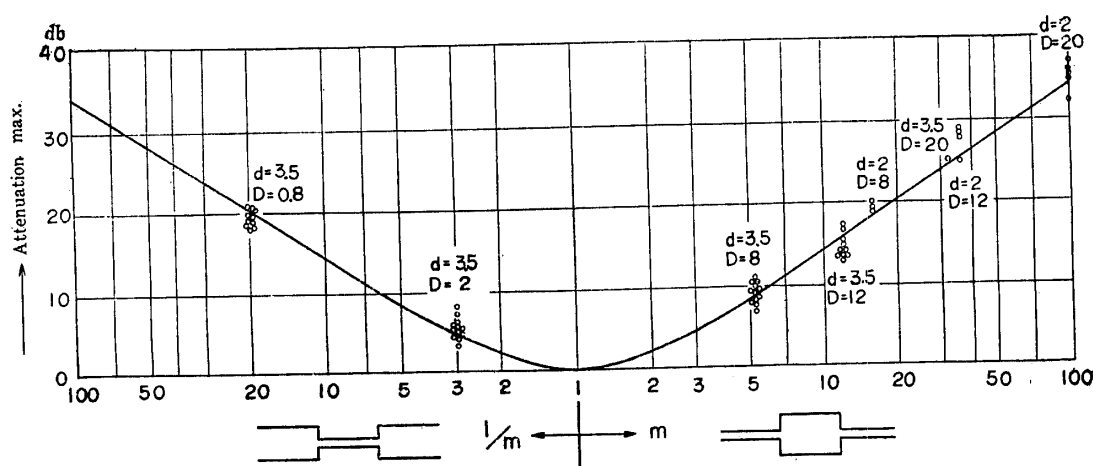


FIGURE 16. Attenuation maxima for various ratios of cross sectional areas.

maximum attenuation are plotted for the various ratios of cross sections. They are zero at  $m=1$ , and become larger when  $m$  becomes larger or smaller.

The experimentally observed values correspond fairly well to the theoretical values up to  $m=100$ . This chart is very useful for practical designing of the cavity type muffler.

(2) *Resonator type.* In actual design the resonator is in most cases the combination of a cylinder and a tube with holes.

The guide tube with two holes of diameter 10mm passes through the cavity. The results of the attenuation measurements for various position of holes are illustrated in Fig. 17. The resonance frequency of Fig. 17(b) agrees well to that calculated from (6) when the holes are located at the center of the cavity, and the length of a cylinder is short compared with wave length. The resonance frequency thus calculated from (6) is 199 c/s. But when the holes are at the ends of the cavity (a), the resonance frequency observed is 175 c/s, this shift of the frequency can be explained by the equation (5') and Fig. 7; namely the location of holes must be taken into account in this case. Moreover, in Fig. 17(a) the second resonance appears in the attenuation curve, which is explained also by Fig. 7. The calculated values in these cases from (5') or (5'') agree fairly well to the experimental results as presented in the figures. Another figures (c), (d) in Fig. 17 are the measured results of the elements whose holes are positioned separately. They can be explained by parallel connection of four terminal matrices, which will be reported in Part III. Fig. 18 is the relation between the attenuation maxima and the area of holes when resonance frequency is held constant. The diameter of the cylinder is 12cm and that of the guide pipe is 3.5cm, numbers marked in the figure are the number of holes of 3.0 mm diameter. Approximately, the attenuation increases 6db when the area of holes doubled.

(3) *Cavity with internal tubes* Cavity type elements present periodic attenuation characteristics as shown in Fig. 13, for  $kl=\pi, 2\pi, \dots$ , attenuation is zero, namely acoustic waves pass through freely. But when a tube is inserted in the cylinder,

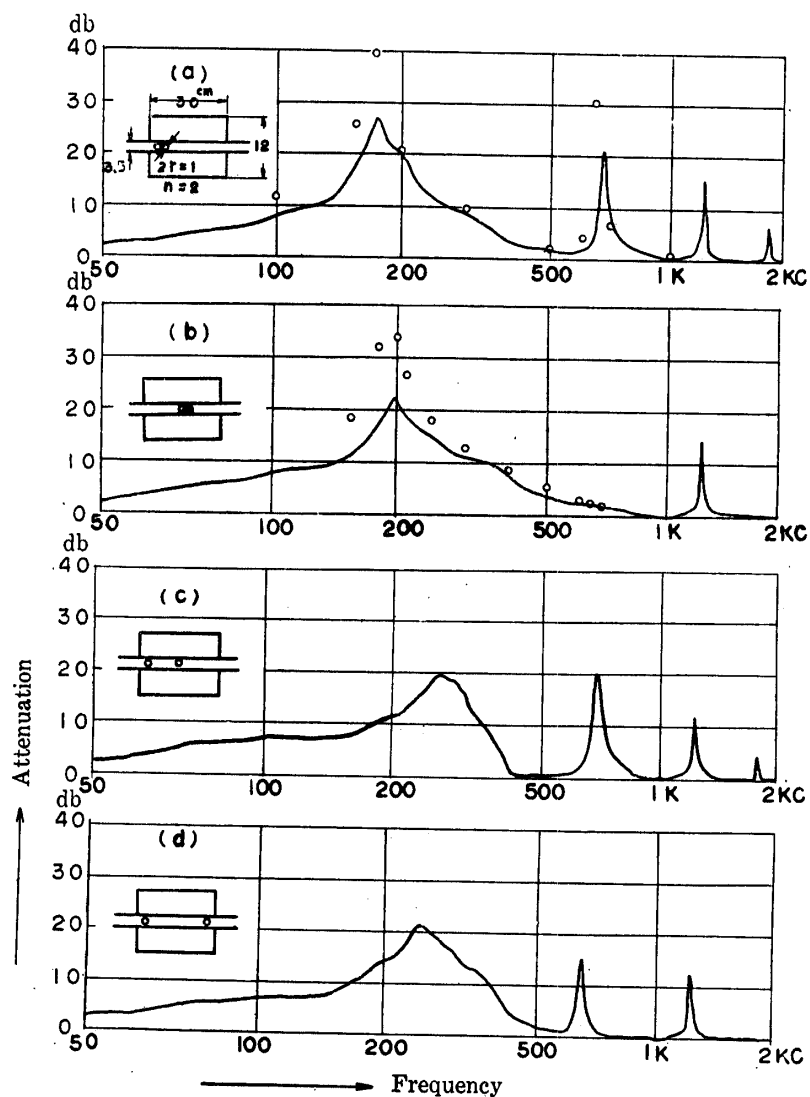


FIGURE 17. Attenuation characteristics of resonators.

- (a) Holes are at an end of a cavity. (b) Holes are at a center of a cavity.  
 (c) Holes are positioned separately, one is at a center and the other is at an end. (d) One hole is at an end and the other is at another end.

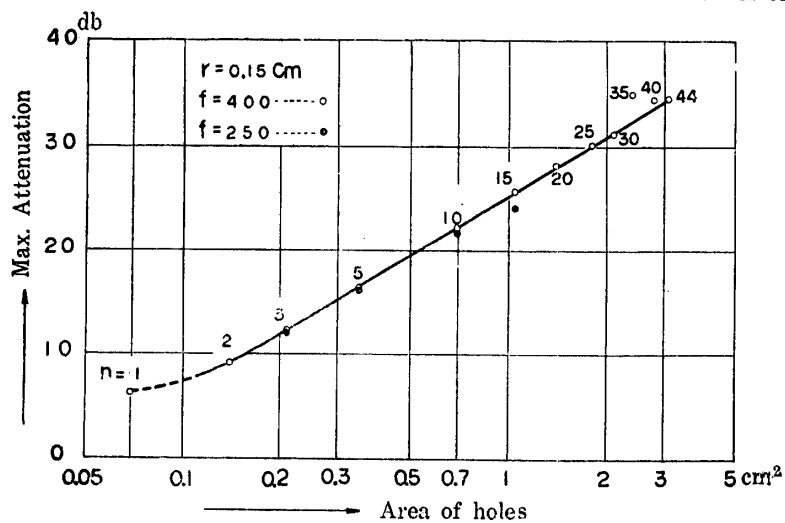


FIGURE 18. Attenuation maxima of resonators for various areas of holes.

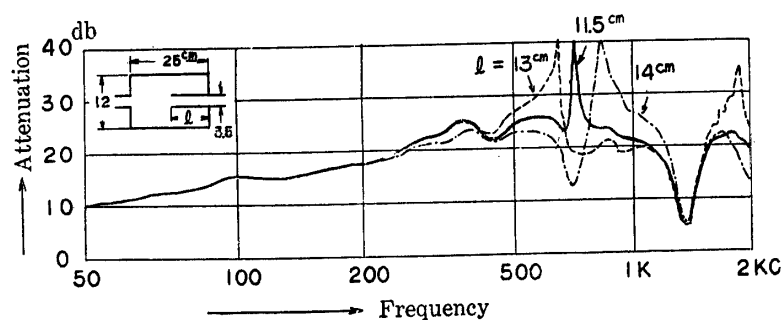


FIGURE 19. Attenuation characteristics of an internal tube type cavity.

additional attenuation is observed at the frequencies to which the effective length of the tube becomes odd multiple of a quarter wave length. In Fig. 19 attenuation curves for various tube lengths are illustrated. For the cylinder length  $L$ , an internal tube of  $L/2$  improves attenuation characteristics at  $f = c/2L, 3c/2L, \dots$ , and that of length  $L/4$  improves them at  $f = c/L, 3c/L, \dots$ , then by two internal tubes  $L/2$  and  $L/4$  at both sides of cylinder, the pass bands of cavity type element are perfectly improved except at  $f = 2c/L, 4c/L, \dots$ , and broad band low-pass filter can be obtained (Fig. 20). Exactly speaking, the tube lengths should be a little shorter than  $L/2$  and  $L/4$  by an amount of end correction of the tube.

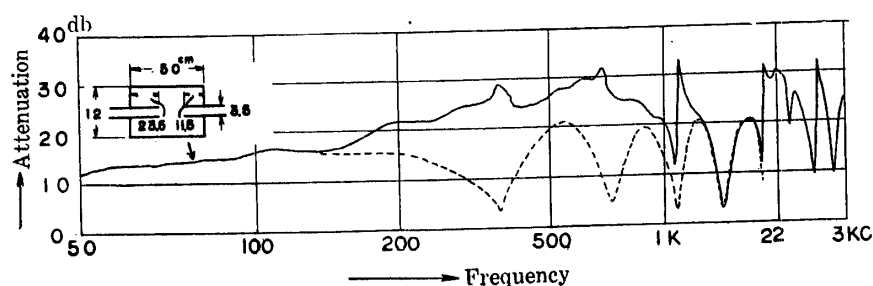


FIGURE 20. When effective lengths of internal tubes are a half and a quarter of cavity length, attenuation characteristics of a cavity are improved. For comparison, result of a cavity is illustrated.

(4) *Absorbing material.* In practical muffler, absorbing material such as glass-fiber, rock-wool, steel-wool has been frequently used. But the theory and systematic measurement have not ever been established. When one or both ends of the cylinder is covered by felt of 1 cm thick, the attenuation at the pass band increases greatly (Fig. 21), but at the attenuation band, little. The amount of increase at the pass bands are plotted against frequencies in Fig. 22 by changing the length of cavity.

Absorbing material at the circular wall of the cylinder is also effective for increasing the attenuation, and the pass bands at the low frequency shift to the lower parts. The amount of attenuation at the pass band is plotted for various lengths of absorbing material (Fig. 23). It depends on the frequencies of pass bands but not on the order of the pass bands. The third type is a most frequently used structure, the absorbing material fills up the cavity and the perforated tube penetrates through the cavity. When the absorbing material is removed, the attenuation characteristics depend on the total area of holes in the cavity. When

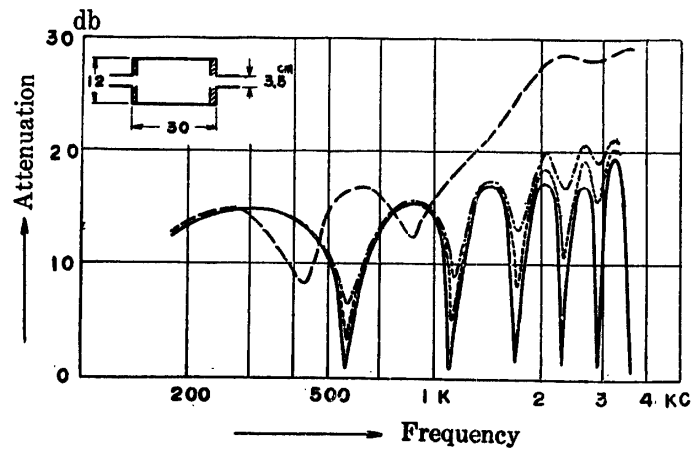


FIGURE 21. Attenuation characteristics of cavities with absorbing material (Felt 1 cm).  
Full line: empty, dotted line: material at one side, chain line: material at both sides, long chain: material covers circular wall.

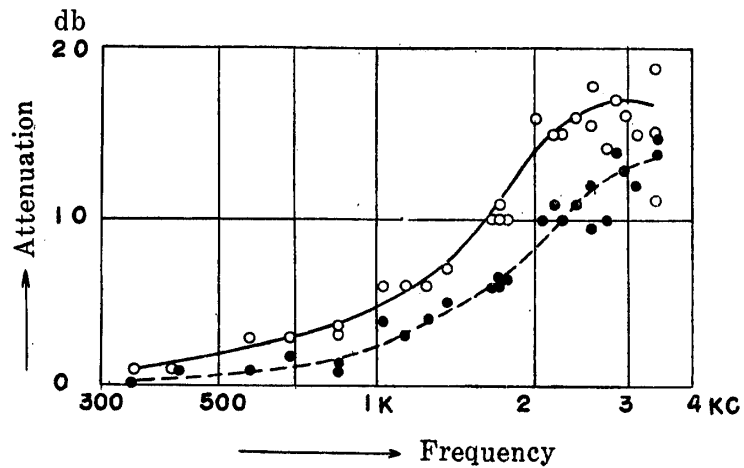


FIGURE 22. Measured attenuation at the loss free points of a cavity.  
Dotted line: material at one side, full line: material at both sides.

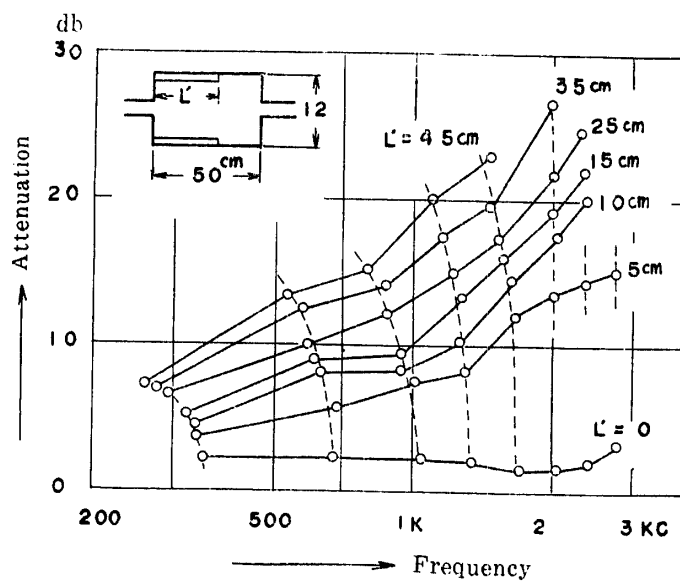


FIGURE 23. Increase of attenuation at loss free points for various length of felt attached at the circular wall.

the ratio of the holes to the area of the surface of the tube in the cavity is greater than 40 percent, the attenuation curve is similar to that of cavity itself. But when the ratio decreases, the attenuation curves become irregular as are shown in Fig. 24. Experiments were performed for various perforation ratios and some different density of material.

In Fig. 25 attenuation characteristics for the perforation ratio 45% to 2% are presented, when the density of the material held constant. Fig. 26 is the relation between the density of absorbing material and attenuation for perforation ratio 45 and 2 percent. When the ratio of perforation is large, attenuation increases as the density of material increases, but for small perforation, it decreases as the density increases. In the former case muffler element is considered to be a cavity type, but in the latter case that is a resonator type and the absorbing material in the cavity affects the attenuation characteristics inversely. For small perforation element absorbing material is not effective at all. Summing up Fig. 25 and

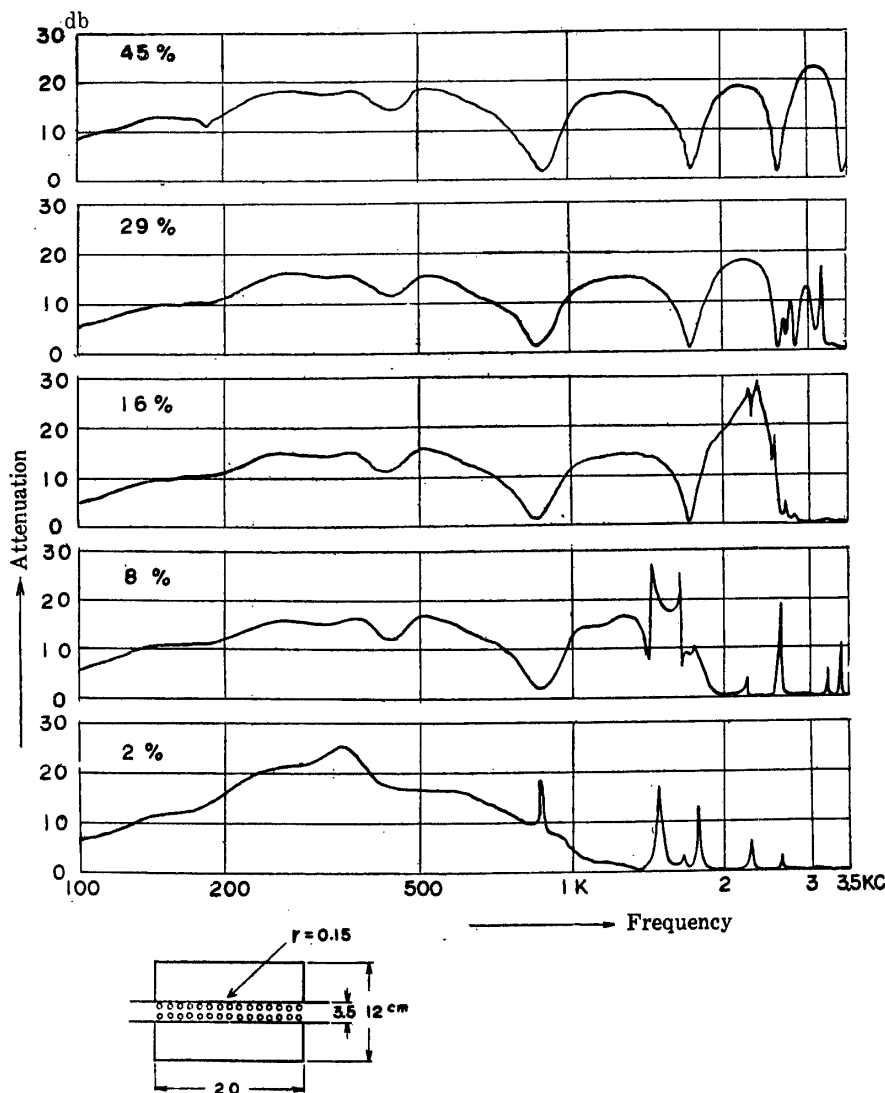


FIGURE 24. Attenuation characteristics of cavities with internal perforated tube (absorbing material is removed).

Fig. 26, in Fig. 27 it is presented the relation between the attenuation maximum and the perforation ratio, density of the material as a parameter.

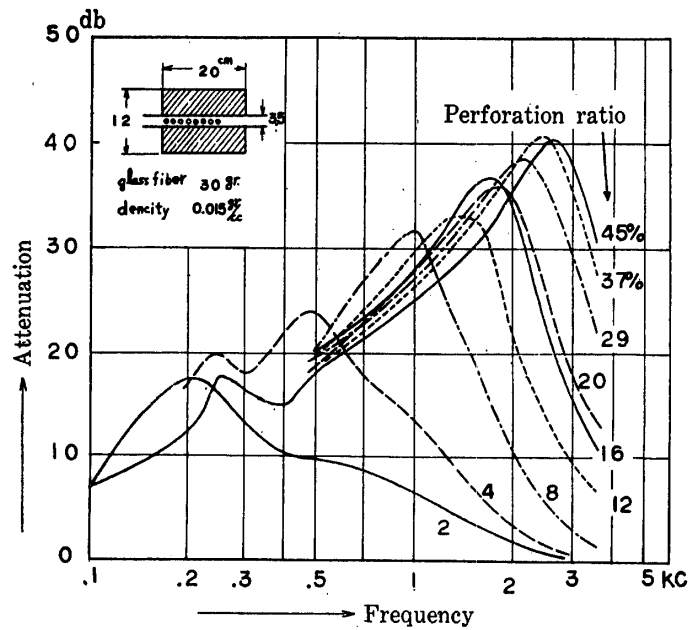


FIGURE 25. Attenuation characteristics for various perforation ratios of tube. Density of absorbing material is held constant.

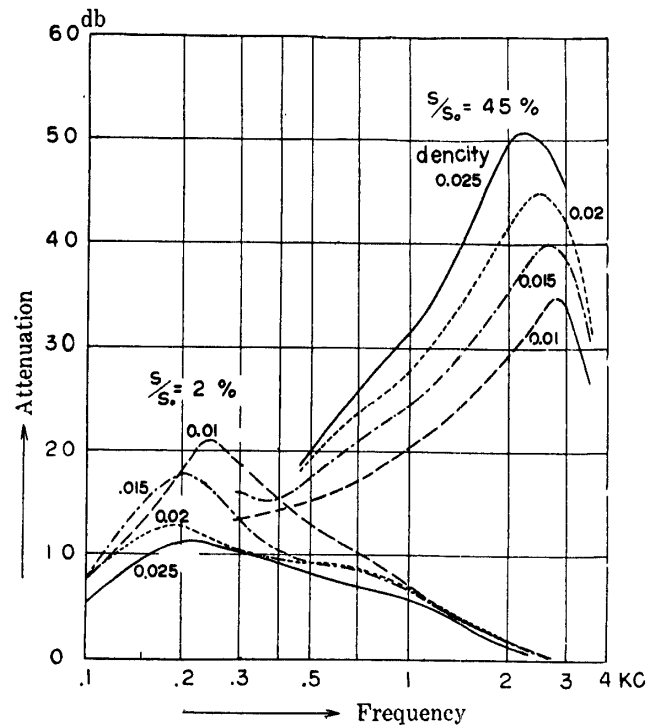


FIGURE 26. Attenuation characteristics for various density of absorbing material; perforation ratios are 45 and 2% respectively.

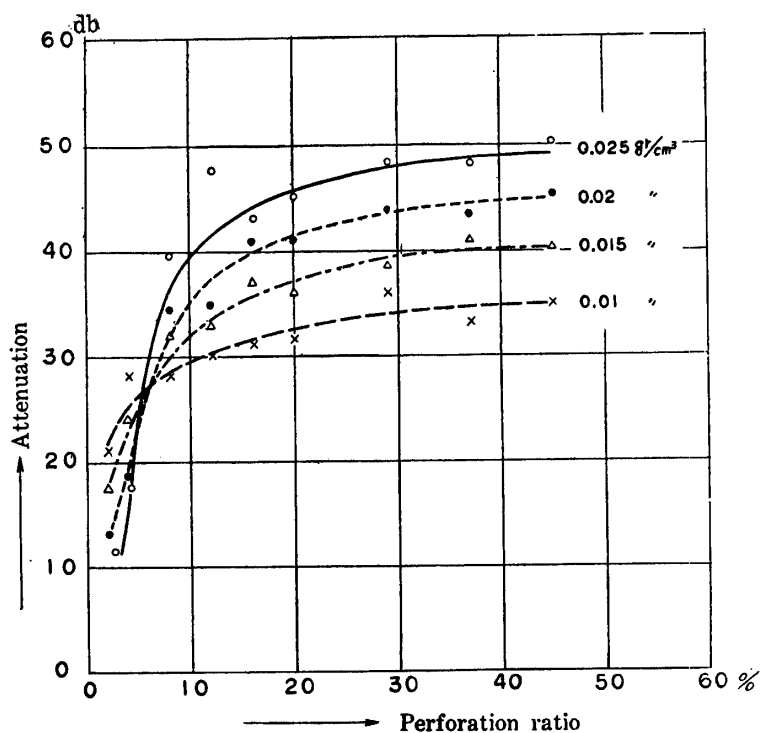


FIGURE 27. Relation between attenuation and perforation ratio; density of absorbing material as a parameter.

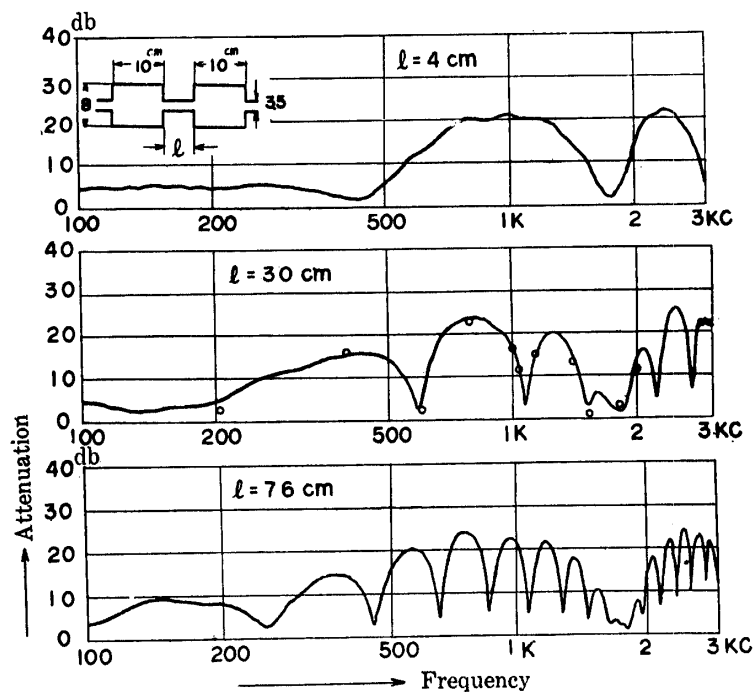


FIGURE 28. Series connection of cavities, for various lengths of connecting tube. Circles are calculated values.

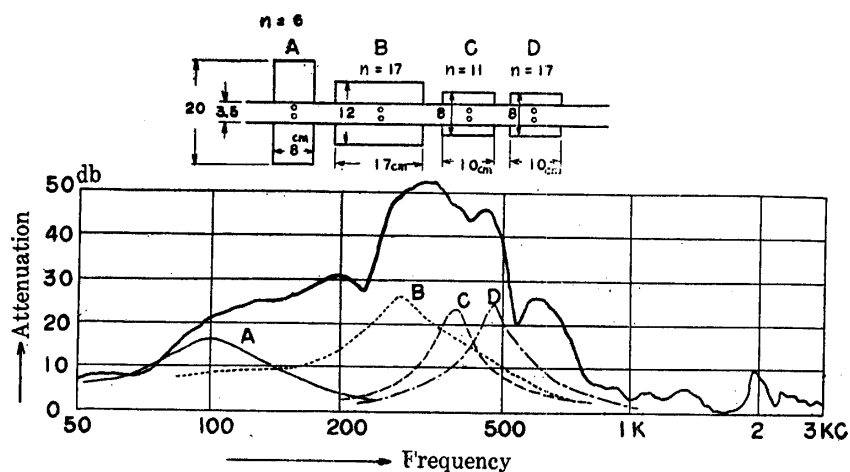


FIGURE 29. Series connection of resonators of different sizes. Individual characteristics are also shown.

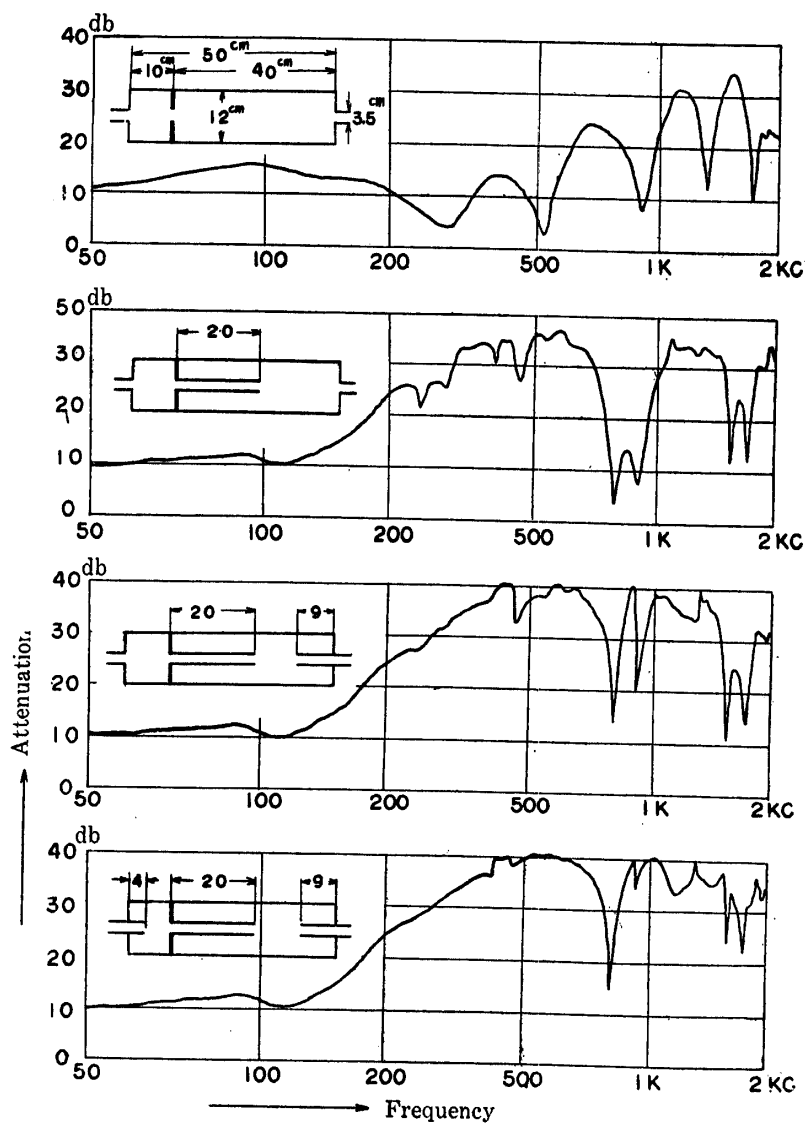


FIGURE 30. Series connection of cavities with and without internal tubes.

## V. COMBINATION OF THE ELEMENTS

The attenuation of a single element is in most cases 10–20db at the maximum value. Series connection of these elements are expected to exhibit higher attenuation value. Some examples of cavities are presented in Fig. 28, maximum attenuation is nearly the sum of the single cavities but detail response depends on the length of the connecting tube. Theoretical values calculated from (4) agree with measured curve fairly well. Resonator type elements are frequently used to eliminate specific frequency band. In Fig. 29 attenuation curves of four resonators, adjusted to different frequencies are illustrated. Series connection of these elements show broad band attenuation characteristics.

Combination of a cavity and a cavity with internal tubes is also presented in Fig. 30.

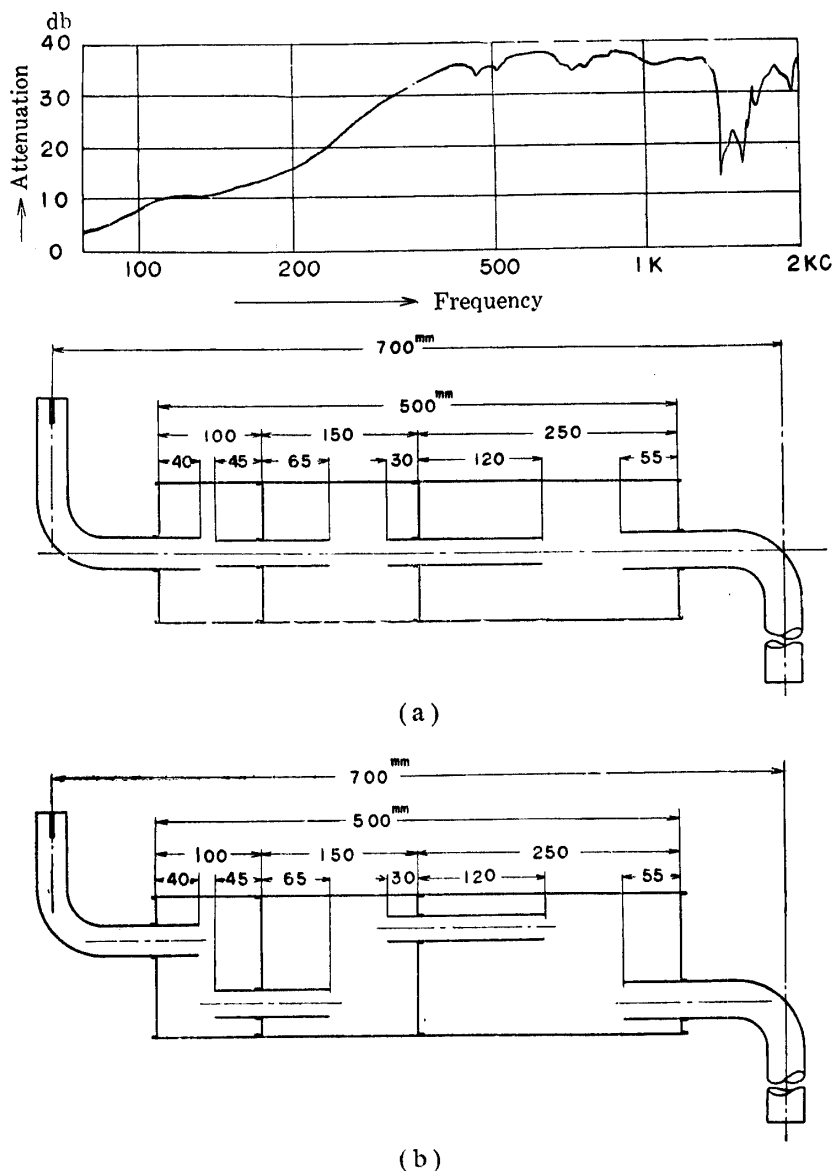


FIGURE 31. Series connection of three cavities with internal tubes.

Series connection of three cavities with internal tubes of different sizes have good attenuation characteristics as Fig. 31(a), and Fig. 31(b) present similar results except higher frequencies. This difference seems to be caused by the effects of radial modes. As an automobile engine muffler, however, (a) is not effective at all, owing to the generated noise by air flow, and (b) attenuates engine noise fairly well as illustrated in Fig. 32. Generated noise caused by high speed air flow will be discussed in Part (II).

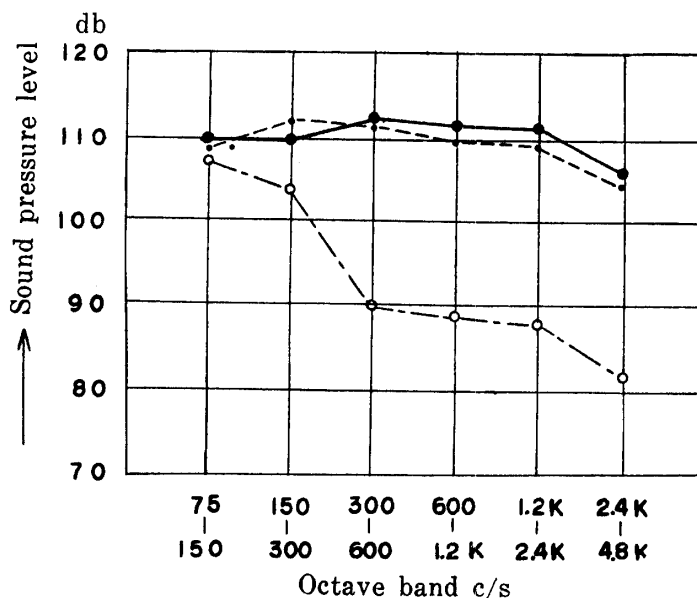


FIGURE 32. Attenuation characteristics of Fig. 31 (a) and (b) as practical mufflers of a automobile engine. Full line: muffler (a), chain line: muffler (b), dotted line: muffler removed.

## VI. CONCLUSION

Attenuation characteristics of acoustical elements such as expansion chamber, (cavity) resonator and deformed types of them have been studied theoretically and experimentally. They can be expressed by four terminal matrices in equivalent electric circuit. Sound pressure and volume velocity and the attenuation characteristics can be obtained without any difficulties, even for the case of several combination of these elements.

A method of measuring attenuation characteristics was developed, and frequency response of various elements was measured.

Results obtained are as follows:

(1) *Cavity type*. Maximum attenuation of this element depends on the ratio of cross sectional areas of cavity and connecting tubes as equation (3) indicated, observed values agree well up to the ratio  $m=100$ . Moreover, attenuation corresponding to  $1/m$  is equal to that for  $m$ .

When the radial length of the cavity becomes large compared with the wave length, one dimensional theory can not be applied.

Some examples of these cases were presented.

(2) *Cavity with internal tubes.* Loss free points of a cavity element can be improved by inserting tubes of proper lengths, one should be a half length of the axial length of the cavity and the other is a quarter of it.

(3) *Resonator type.* Resonance frequencies of calculated from equation (6) are valid only when the dimension of the cavity of the resonator is small compared with the wave length, otherwise, the wave phenomena in the cavity should be taken into account.

(4) *Absorbing material.* Absorbing material packed in the cavity as in Fig. 25 showed good attenuation characteristics, and when the areas of holes are large, attenuation increases as the density of the material increases, but when the areas of holes are small attenuation decreases as it density increases.

Concerning the acoustical silencing devices, the following papers will be published shortly.

Part II. Noise generated by high speed air flow in the acoustical elements.

Part III. Determination of four terminal constants of acoustical elements.

Part IV. Attenuation characteristics studied by an electric simulator.

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