Aeronautical Research Institute, University of Tokyo
Report No. 340, December 1958

Automatic Curve Follower Using Polarized Light Beam Chopper

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Summary. This paper describes a new automatic curve follower, which can follow a graph given in a thin black line on a white paper or a curve photographically recorded as an oscillogram. The detector consists of two illuminators of the line to be followed, a microscope objective lens by which the image of the line is formed, a polaroid disc which is fixed at the image plane and is divided into two equal parts having their principal planes perpendicular to each other, a rotating polaroid disc driven by a synchronous motor, and a phototube. The beam of light passed successively through the two polaroid disc flickers alternately at the two parts of the field. But the total flux, which the phototube receives, remains constant as far as the curve runs through the center of the field. When the curve deviates from the position, the deviation is picked up by the phototube as an alternating change of the total light flux having an amplitude proportional to the deviation and a reverse phase corresponding to its direction. This detector constitutes a servomechanism in conjunction with a voltage amplifier, a thyratron circuit and a split field motor, which drives the lead screw of the follower. An appropriate masking of the field much improved the performance of the detector.

1. INTRODUCTION

A differential analyser is usually equipped with an input table, through which an arbitrary function contained in the differential equation to be solved is put into the machine. It may be a function of the independent variable x in the equation, or a function of the dependent variable y or its derivatives dy/dx, d^2y/dx^2, ... and so on. In any case, these functions are usually introduced to the machine by a curve, which represents the function with respect to rectangular coordinates being traced manually or automatically. In a manually following method, an index point moving in the x direction is made to position in the y direction continually on the curve by rotating a handle. On the other hand, automatically following may be accomplished by using a phototube or the like to see whether a reference point is on the curve or not and controlling a servomotor continually so as to correct a deviation. Such servomechanisms were early developed by H. L. Hazen, J. J. Jaeger and G. S. Brown [1] in Massachusetts Institute of Technology and by P. M. S. Blackett and F. C. Williams [2] in Man-

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chester University. They were both those which follow the curve given as the boundary line between white and black parts. Also that reported by A. C. Cook, J. K. Kirchmayer and C. N. Weygandt [3], which was applied to the differential analyser at Moore School, was of the same type. For the purposes other than of the differential analyser, simpler methods have been also reported, such as following a line drawn in conducting ink [4], a formed wire pasted on a paper [5] or a line engraved on a carbon coated plate [6].

The curve follower described in this paper is that which follows a graph given in a thin black line on a white paper. So it is of the same type as that being used in the new differential analyser in M. I. T. [7] and that reported by C. S. French et al. [8] or that described recently by Y. Ishii [9]. It has, however, new features, in which it uses a polarized light beam chopper to detect the center line of the curve and is able to follow up even a curve photographically recorded as an oscillogram.

Although this device was originally intended to be used as a follower for our input table, it goes without saying that such automatic curve-followers may be applicable to many other analogue controlling or data reducing devices such as a program controller or an automatic correlator, because they can pick up again an analogue signal once left recorded as a curve for the purpose of being stored and reproduce it as a voltage or a shaft position. Thus its use is not only confined to differential analysers, but is opened to such an immense field, too.

2. Operating Principle

The principle of the arrangement which was made is illustrated in Figs. 1 and 2. As seen in the figures, two bright tungsten filament lamps L_1 and L_2 illuminate the curve given as a black line on a white paper from both sides of the objective lens tube O. The light reflected from the paper passing through the lens, the real image of the line is made in the plane of a fixed polaroid disc B. As this disc being divided into two equal parts B_1 and B_2 by the center line of the field and their principal planes being perpendicular to each other, the polarized light passed through the part B_1 has its plane of polarization perpendicular to that of the light passed through the part B_2. Just over this disc, there is another polaroid disc A, which is rotating at a speed of 25 rps., a rate corresponding to one half the frequency of line voltage, by being driven by a synchronous motor S through a gear train. The light fluxes passed successively through these two polaroid discs are received by a phototube P.

Now, as seen in the lower left side of Fig. 1, the angle the principal plane of the rotating polaroid disc A makes with that of B_1 at a certain instant is called θ. Then the light flux passed through the part B_1 of the fixed disc and subsequently through the rotating disc A is

$$\Phi_1 = \Phi_{10} \cos^2 \theta ,$$

because the flux is proportional to the square of a wave amplitude. On the other hand, the flux passed through the part B_2 is

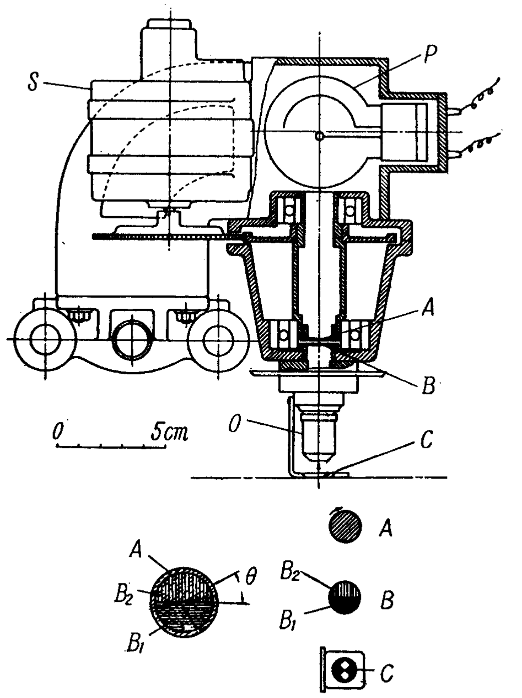


FIGURE 1. Side view of the detector head showing also its operating principle.
A; Rotating polaroid disc.
B_1, B_2; Two parts of the fixed polaroid disc having their principal planes perpendicular to each other.
C; Mask of the field.
O; Microscopic objective lens.
P; Phototube.
S; Synchronous motor.

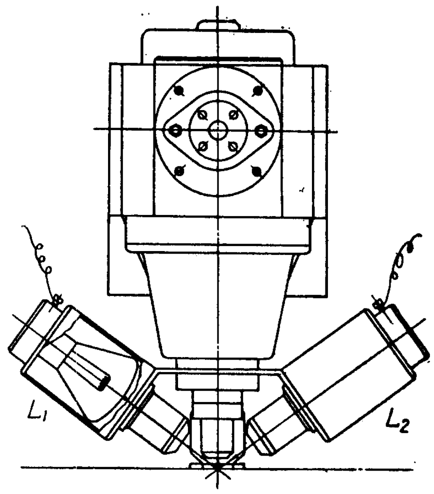


FIGURE 2. Front view of the detector head.
L_1, L_2; Intense illuminators.

$$\Phi_2 = \Phi_{20} \sin^2 \theta .$$

Then the total flux Φ, the sum of Φ_1 and Φ_2, reaching the phototube becomes

$$\Phi = \Phi_{10} \cos^2 \theta + \Phi_{20} \sin^2 \theta$$

or

$$\Phi = \frac{\Phi_{10} + \Phi_{20}}{2} + \frac{\Phi_{10} - \Phi_{20}}{2} \cos 2\theta .$$

Now, if the speed of rotation of the disc A is such that θ increases by 2π, which is equivalent to one revolution of the disc, every 2 cycles change of the line voltage,

$$\theta = 2\pi(f/2)t = \pi f t ,$$

where f denotes the frequecy. Substituting this value in the above equation, we obtain

$$\Phi = \frac{\Phi_{10} + \Phi_{20}}{2} + \frac{\Phi_{10} - \Phi_{20}}{2} \cos 2\pi f t .$$

Thus the phototube receives a total light flux changing alternately at the same frequency as that of the line voltage with an amplitude proportional to the maximum difference between the light fluxes passed through the two parts of the field. Then, when the illuminated curve to be followed under the objective lens crosses the field and there runs through the center, the light fluxes passed through each part of the field will be equal, and consequently no alternating change of the photoelectric current will occur. But when there is any deviation of the line from the position, it is picked up as an alternating change of photoelectric current at the same frequeocy as that of the line voltage with an amplitude proportional to the deviation and with a phase reversed corresponding to whether the curve deviates to one part or to the other.

Stating in other words, the beams passed successively through the two polaroid discs B and A flicker alternately at the two parts of the field. But as their fluxes are proportional to $\cos^2\theta$ and $\sin^2\theta$ respectively, their sum remains constant as far as Φ_{10} and Φ_{20} are equal. When the curve deviates from the position, however, a modulated component appears in the photoelectric current and the arrangement serves as a light beam chopper, capable of picking up the deviation of curve from the center of the field.

Thus detected deviation is corrected by a servomechanism, so as the light fluxes reaching the phototube from the two parts of the field to be continually equal in the average. Fig. 3 shows the general view of the follower mounted on the input table of our differential analyser.

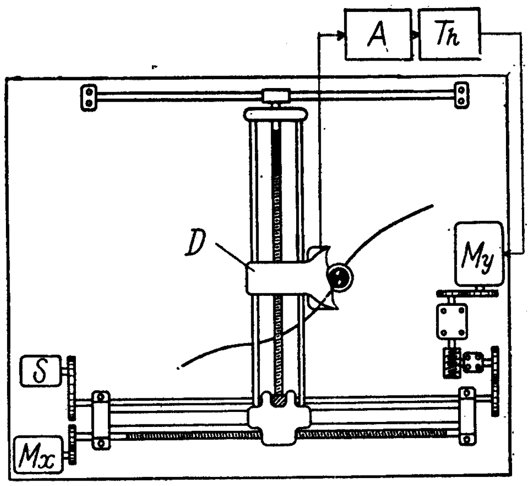


FIGURE 3. General view of the curve follower mounted on a input table.
M_x ; Motor driving x-coordinate lead screw.
M_y ; Servo-motor driving y-coordinate lead screw.
S ; Selsyn transmitting data to other devices.
A ; Voltage amplifier.
T_h ; Thyratron circuit.
D ; Detector head.

In a course of testing, we tried to place a mask C as shown in Fig. 1 in the field of view of the head, and found that, by doing so, the performance of the follower was greatly improved. This mask just above the paper has two square openings across the point which is to be in coincidence with the curve, and puts the other part out of sight. By this masking, the following advantages may be obtained: the center of the field as a reference point becomes positively determined, the sensitivity to the deviation becomes almost independent of an inclination of the curve to the divided line of the fixed polaroid disc and the effect of non-uniform illumination diminishes. When changing the form of the openings, the characteristic of sensitivity vs. curve inclination may be varied. Therefore, an appropriate masking may be adopted according to purpose.

3. Detailed Constructions

(a) *Detector head.* Its side and front views have been shown already in Figs. 1 and 2 respectively, and a picture of the head is shown in Fig. 4. In construction, the fixed polarizing disc *B* is made of a polaroid plate about 0.2 mm thick. Of

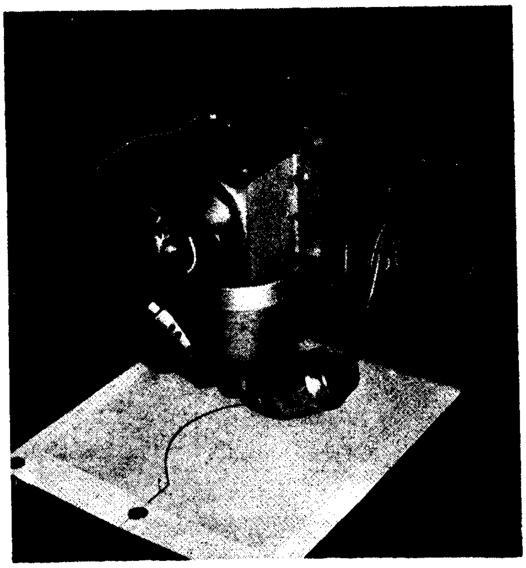


FIGURE 4. View of the detector head.

the plate, two semicircular forms, 10 mm in diameter, are cut out, so as the principal plane of the one will be perpendicular to that of the other, and they are put together side by side and sandwiched between two organic glass plates cut in circular form also 0.2 mm thick. The composite disc made in this way is fixed to the head body, so as the divided line to be in parallel to the x axis of the input table. The rotating disc also made of the same polaroid plate, is mounted in a hollow cylinder of brass which, in turn, is supported by a large ball bearing and driven at a speed of 1500 rpm by a synchronous motor of 5 Watt-rating through a gear coupling of the ratio 1 to 1.

As the light sources to illuminate the field from both sides of the objective lens tube, two automobile head light lamps of 6 V, 35 candle powers are used. The respective condenser lenses concentrate the lights to the field. As an electric supply to the lamps, a battery is used instead of a.c. source to prevent the brightness from fluctuating at the frequency twice as high as that of the source as has been found to occur.

(b) *Phototube and voltage amplifier.* All the circuits operate on 50 cps frequency of the line voltage. The phototube used is of gas-filled PG-50 G type, having a high sensitivity at the sacrifice of linearity which may be not so important for an unbalance detector. The circuit connection of an amplifier for an alternating change of photoelectric current is shown in Fig. 5. It is a common RC coupled 3-stage amplifier having a gain of about 125 db.

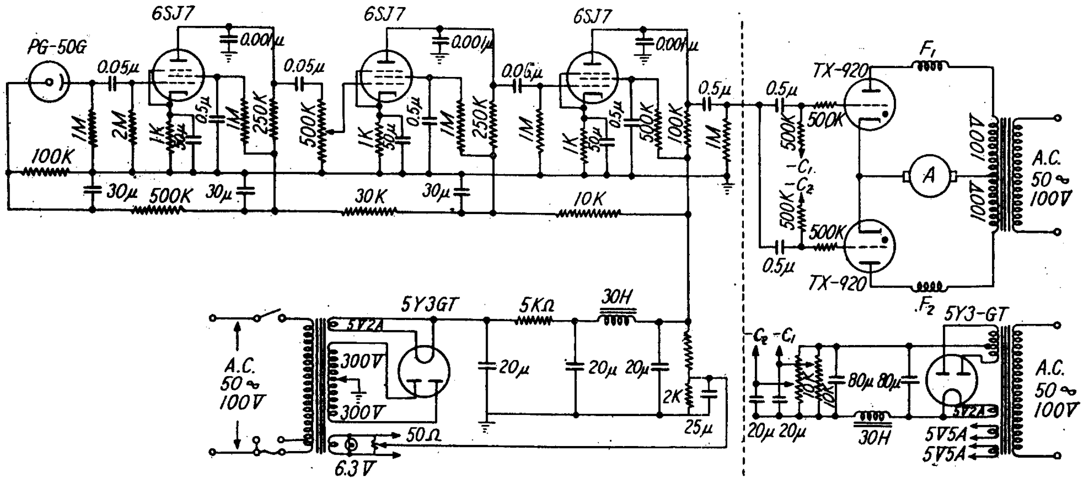


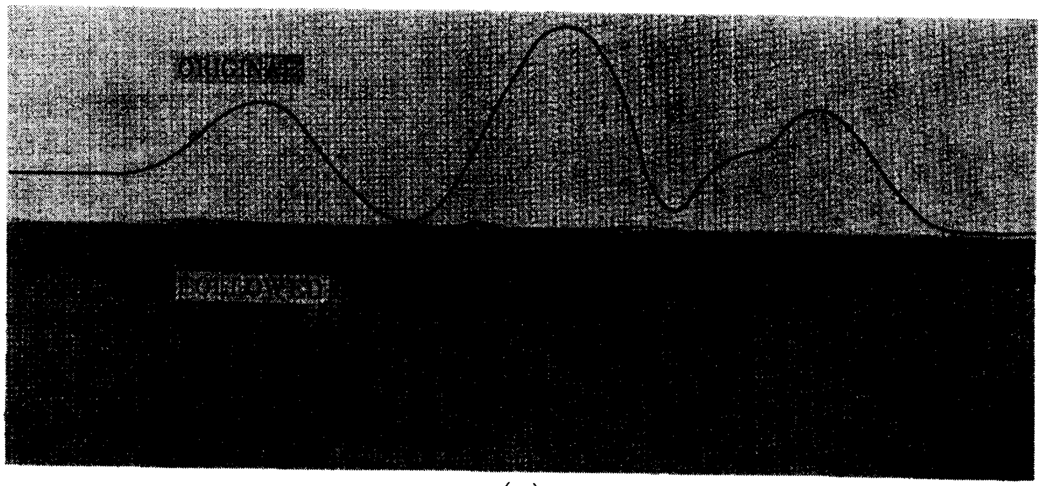
FIGURE 5. Electronic circuits.

(c) *Servo-amplifier and Servomotor.* As a servomotor, split field type, into which a 1/30 HP direct current series motor has been altered by dividing its field coil, is used. This motor is driven by thyratron plate currents flowing through its armature coil in common and through the two field coils F_1 and F_2 connected separately to the respective plates, as shown in Fig. 5. According to a phase of the photoelectric current amplifier output, either thyratron conducts and makes the motor rotate in a proper direction with the plate current flowing through the corresponding field coil. To the grids of the thyratrons, adjustable biases $-C_1$ and $-C_2$ are given.

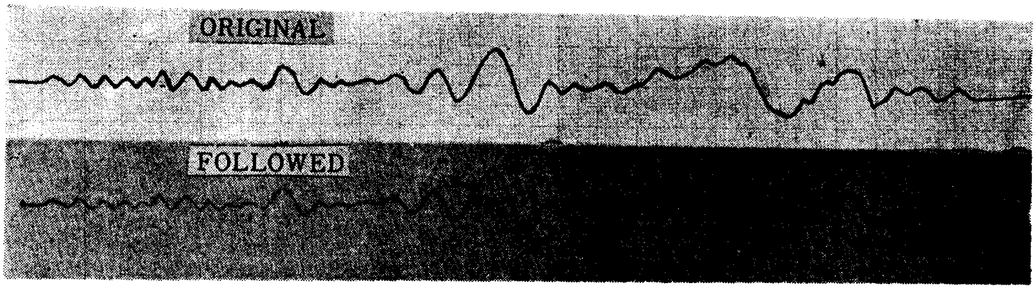
The rotation of this motor which is reduced to 1/60 by a worm gear, rotates a square-edged lead screw 2 mm in pitch and this screw, in turn, drives the carriage of the follower head. In passing, the rotor A of this servomotor has a moment of inertia of about 1 kg·cm^2.

4. PERFORMANCE

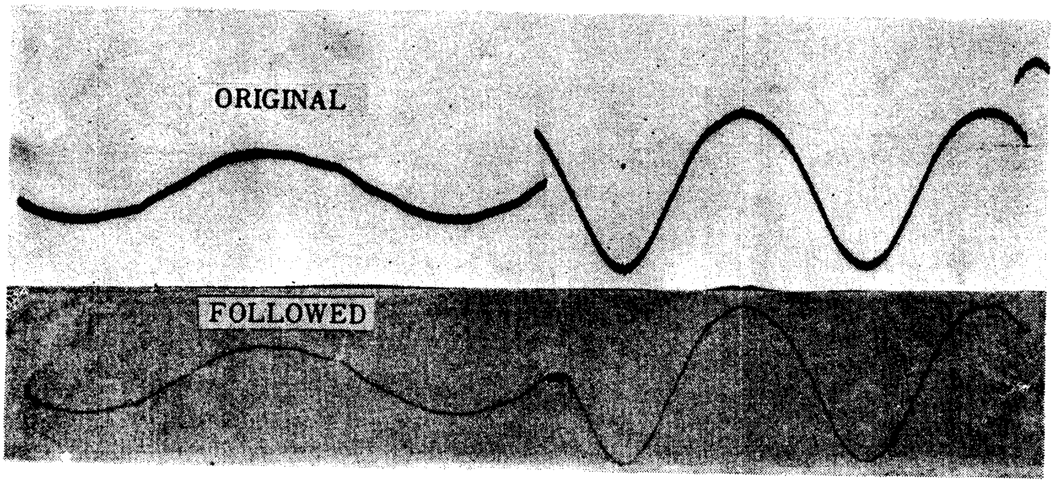
Three examples of actual following recorded by a pen attached to the head, are shown in Figs. 6 (a), (b) and (c). The original curve was drawn in black ink in (a), written in pencil in (b) and photographically recorded as an oscillogram in (c). The width of the original line in the cases of (a) and (b) were about 1 mm, but it has been found that thinner lines 0.15 or 0.2 mm wide can also be traced out. On the other hand, the figure (c) shows that the arrangement could follow up a wider line of about 2 mm of an oscillogram along the center of the line.



(a)



(b)



(c)

FIGURE 6. Record of curve following.
(a) The original curve is drawn in black ink.
(b) It is written in pencil.
(c) It is photographically recorded as an oscillogram.

For examining following up errors, the traced curves were put upon the original, and discrepancies between the two were examined. By this means, errors only as small as unappreciable by naked eyes were found, therefore they were estimated to be an order of about 0.1 or 0.2 mm. The carrying speed in the X direction in these tests was about 1 mm/sec.

When an inclination of the curve to be followed is too steep, the arrangement of course becomes unable to follow up. The limit of this angle in our present system is about 75 degrees under the condition of 1 mm/sec carrying speed in the x direction.

It seems to be appropriate here to place some discussions about this critical angle and the following up errors.

(1) *Limited speed of motor.* Let the speed of carriage, when following up a curve, be u and v in the x and the y direction respectively. Then the relation

$$v = u \tan \alpha ,$$

must be held, where α denotes the angle of inclination of the curve to the x axis. Accordingly, v must increase in proprotion to $\tan \alpha$ with α increasing, so far as u remains constant. That means v must be infinite when α becomes 90 degrees. Of course this will not be possible, because the speed of the motor, which drives the head in the y direction, is limited by its maximum output torque, and comes in balance with the load. Even under a small angle of inclination, on the other hand, a velocity error appears, since in this servo system the motor speed is controlled so as to be proportional to a deviation signal. And this error is easily seen to be also proportional to $\tan \alpha$. When α increases, this error grows, becoming to take a value beyond an admissible limit. From these discussions about the servo performance, it is easily understood that there is a limit of curve steepness the follower can follow up. Slowing down the carrying speed in the x direction seems to be one way to make this limit higher.

(2) *Deviation detecting ability limited.* Without masking and with the field of view circular, the deviation signal or the difference between the average light fluxes passed through the two parts of the field will be proportional to $\Delta y \cos \alpha$, where Δy is the deviation of the curve from the center in the y direction. Therefore, the detecting sensitivity is maximum, when the curve is parallel to the divided line of the fixed polarizing disc which halves the field. And when α becomes 90 degrees, the signal will, in principle, become zoro, therefore the deviation of the curve cannot be detected. Actually, under an angle much less than 90 degrees, the arrangement becomes impossible to follow up, because the S/N ratio of the amplifier becomes low.

It is in this connection that the masking stated above is effective for improvement of the detecting ability. If an appropriate masking is applied, it can be possible to make the sensitivity almost independent of the angle α up to a certain value. The mask now being used has two square openings arranged with corner to corner across the center line, as shown in Fig. 1, and by this means the servo is able to trace out until α becomes 75 degrees as stated above. If this form of the openings is made a little oblonger, the followable limit may be a little higher.

In order to be completely free from the above stated limitation about the angle of inclination of the curve, there would be no other ways than adopting another method such as by turning automatically the follower head itself so as the divided line to be continually parallel to the curve and detecting only the deviation in the normal to the curve.

(3) *Optical nonuniformities.* When the optical characteristics of the two semicircular parts of the fixed polaroid disc are not perfectly equal, or when the illumination over the field is not uniform, certain accompanying optical errors will occur, of course.

Fig. 7 shows a result of the dynamic behaviour-testing of this servo system by an indicial response method. The curve shown is the response of the head to a

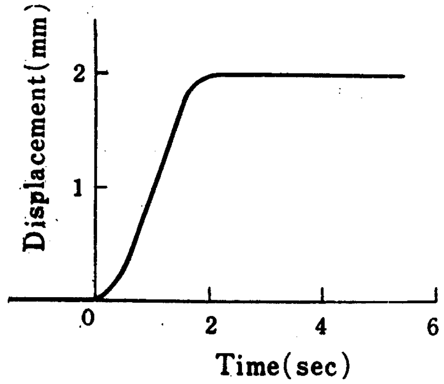


FIGURE 7. Indicial response of the follower.

sudden shift of the reference straight line parallel to the x axis in the y direction. From this curve the maximum follow up speed of the head is seen to be limited to about 1 mm/sec, but no overshootings and no hunting oscillations are seen.

5. Improvements Made on the Head

After the constructing and testing of the arrangement stated above were all finished, some improvements were noticed desirable to be made. Thus, a new improved head was worked out.

The principal improvement was to place the mask in the image plane inside the head tube. The mask itself of the new head was made by the aid of a photographic procedure. A large size figure of the openings composed of two squares similarly arranged to the old one, was drawn in black ink, and its photograph was taken. The negative film made thus was cut in a circular form and was put immediately under the fixed polaroid disc existing in the image plane, so as their centers be in exact coincidence with each other. With this construction, as it seems necessary to provide a means to be able to see at which point on the paper the head to aim, a celluloid pointer indexing the optical center of the field was extended from the lower end of the tube onto the paper. This pointer does not disturb the detection, because it covers only the portion of the field, which the mask puts out of sight. By these means, it has become much easier to set the head so as to aim at the curve correctly at the start of following.

Also to the new head, a lighter synchronous motor of 2-Watt output and a smaller size phototube were employed, by means of which the head body has become much lighter than the old one, enabling the servo performance to be improved.

6. Acknowledgement

The authors wish to express their hearty thanks to personnel of the Workshop Department of the Aeronautical Resarch Institute for their kind cooperation in constructing the arrangement.

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November 20, 1958

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