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FUNDAMENTALS OF ACOUSTICAL SILENCERS*

(II) Determination of four terminal constants of acoustical elements

By

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Summary. The measurement of attenuation of an acoustic element can be realized as was reported in the previous paper (Part I). On the other hand, the attenuation characteristics can be calculated from the equivalent circuit of an acoustic system, if the four terminal constants of each section are known.

Present work has been devoted to the determination of the four terminal constants, A, B, C and D of fundamental elements of acoustic filters [1]. The method and some results were reported. They are in considerable agreement with the theory. Attenuation of a combination filter was calculated also from measured A, B, C and D.

I. INTRODUCTION

In Part I, the attenuation characteristics of various acoustic filters were calculated theoretically from the four terminal constants of each section and the measured results agreed fairly well when the system was considered to be one dimensional. However, when the radius of the cylinder was large compared with the wave length, obtained results showed particular attenuation, which could not be expected from the one dimensional theory. If we can measure the four terminal constants of any elements, they imply complicated wave phenomena in the elements, and calculated attenuation from the measured four terminal constants will be an exact one, even in the case where the two or three dimensional phenomena can not be neglected.

The method of determination of four terminal constants was studied, and for the simple elements, the obtained results were compared with the theory.

II. FOUR TERMINAL CONSTANTS OF VARIOUS ELEMENTS

(A) *A cavtiy type element.*

Cavity type element with conducting tubes at both sides is shown in Fig. 1(a) and it can be replaced by the equivalent electrical circuit, Fig. 1(b). Four termi-

* Parts of this work are taken from a thesis submitted in partial fulfilment of the requirement for a master's degree at the University of Tokyo by T. Miwa.

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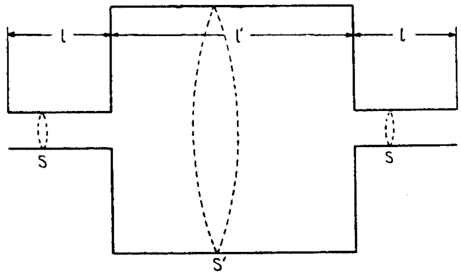


FIGURE 1(a). Cavity type element with short tubes at both sides.

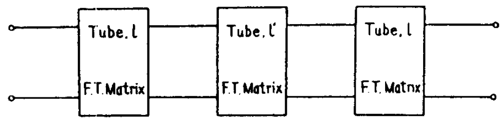


FIGURE 1(b). Equivalent circunit of the cavity type element.

nal constants of a cavity should be determined without conducting tubes. But at the discontinuity of cross sections, phase of sound would be anomalous, then the short tubes were connected at both sides of an element. Another elements in this report have such conducting tubes, and calculating A, B, C and D contain the effects of definite length of conducting tubes at both sides.

The matrix can be written as follows:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{S}{\rho c}\sin kl & \cos kl \end{pmatrix} \begin{pmatrix} \cos kl' & j\frac{c\rho}{S'}\sin kl' \\ j\frac{S'}{\rho c}\sin kl' & \cos kl' \end{pmatrix}$$

$$\times \begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{S}{\rho c}\sin kl & \cos kl \end{pmatrix},$$

$$\left.\begin{aligned} A &= \cos kl'(\cos^2 kl - \sin^2 kl) - \cos kl \sin kl \sin kl'\left(\frac{S'}{S} + \frac{S}{S'}\right) = D, \\ \left(B \Big/ j\frac{\rho c}{S}\right) &= \sin kl'\left(\frac{S}{S'}\cos^2 kl - \frac{S'}{S}\sin^2 kl\right) + 2\sin kl \cos kl \cos kl', \\ \left(C \Big/ j\frac{S}{\rho c}\right) &= \left(B \Big/ j\frac{\rho c}{S}\right) + \left(\frac{S'}{S} - \frac{S}{S'}\right)\sin kl'. \end{aligned}\right\} \quad (1)$$

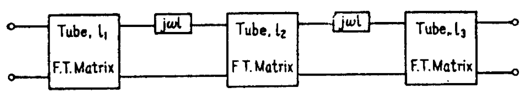


FIGURE 2(a). Equivalent circuit for the Karal's correction of the cavity type.

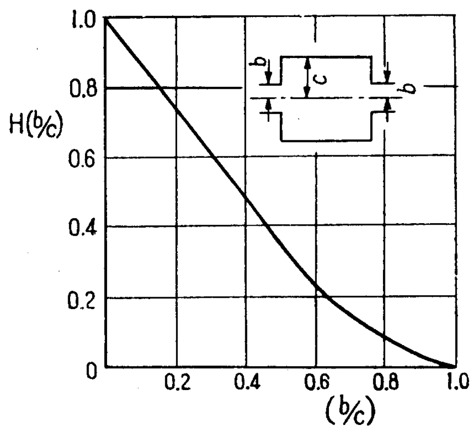


FIGURE 2(b). Karal's correction function. $L=(8\rho/3\pi^2 b)H(b/c)$

When the dimension of the cross section S' is large compared with the wave lengh, the equation (1) is no longer valid. As to the correction for the discontinuity at the cavity, Karal [2] introduced series connection of inductances, as shown in Fig. 2(a). The function, $H(b/c)$ is presented in Fig. 2(b).

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{S}{\rho c}\sin kl & \cos kl \end{pmatrix} \begin{pmatrix} 1 & j\omega L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos kl' & j\frac{\rho c}{S'}\sin kl' \\ j\frac{S'}{\rho c}\sin kl' & \cos kl' \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & j\omega L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{S}{\rho c}\sin kl & \cos kl \end{pmatrix}, \tag{2}$$

$$L = \frac{8\rho}{3\pi^2 b} H\left(\frac{b}{c}\right).$$

(B) *Side branch type element.*

The four terminal constants of the side branch type, (Fig. 3(a)) are calculated from the equivalent circuit, Fig. 3(b).

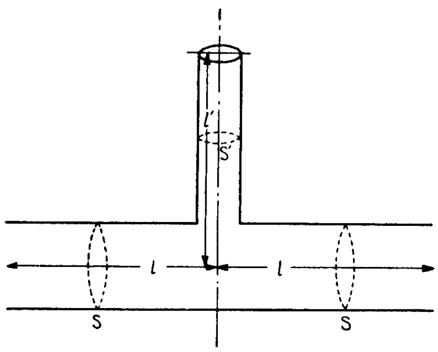


FIGURE 3(a). Side branch type element.

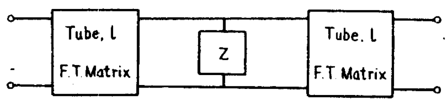


FIGURE 3(b). Equivalent circuit of the side branch type element.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{S}{\rho c}\sin kl & \cos kl \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/z & 1 \end{pmatrix} \begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{S}{\rho c}\sin kl & \cos kl \end{pmatrix};$$

$$z = -j\frac{\rho c}{S'}\cot kl'$$

$$\left.\begin{aligned} A = D &= (\cos^2 kl - \sin^2 kl) - \frac{S'}{S}\sin kl\cos kl\tan kl', \\ \left(B\Big/j\frac{\rho c}{S}\right) &= 2\sin kl\cos kl - \frac{S'}{S}\tan kl'\sin^2 kl, \\ \left(C\Big/j\frac{S}{\rho c}\right) &= \left(B\Big/j\frac{\rho c}{S}\right) + \frac{S'}{S}\tan kl'. \end{aligned}\right\} \tag{3}$$

It must be remembered that if the cross section of the side tube is large in comparison with a wave length, the phase of sound over this cross section is not the same value and the so-called impedance Z can not be defined.

(C) *Resonator type element.*

The resonator type elements are devided into three cases, (i) the side branch with a closed cavity as shown in Fig. 4, (ii) the cavity penetrated by the tube with one hole as shown in Fig. 5(a), (iii) a special case of (ii), an axial length of a cavity being smaller than radius, as shown in Fig. 6.

(i) The equivalent circuit of Fig. 4 is similar to the side branch type. However, the impedance looking into the side branch is not the same form as the side branch type. The four terminal constants are as follows:

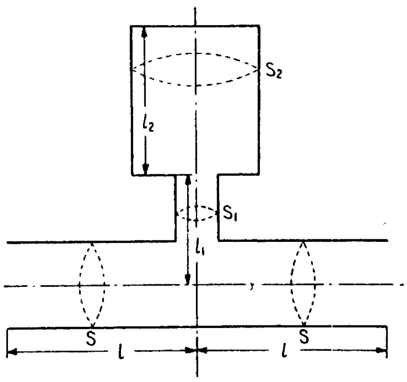


FIGURE 4. Resonator type element (i).

$$\left.\begin{aligned}
A=D&=(\cos^2 kl-\sin^2 kl)-\frac{1}{S}\frac{S_1\sin kl_1\cos kl_2+S_2\cos kl_1\sin kl_2}{\cos kl_1\cos kl_2-\dfrac{S_2}{S_1}\sin kl_1\sin kl_2}\sin kl\cos kl\,,\\
\left(B\Big/j\frac{\rho c}{S}\right)&=2\sin kl\cos kl-\frac{1}{S}(\sin^2 kl)\frac{S_1\sin kl_1\cos kl_2+S_2\cos kl_1\sin kl_2}{\cos kl_1\cos kl_2-\dfrac{S_2}{S_1}\sin kl_1\sin kl_2}\,,\\
\left(C\Big/j\frac{S}{\rho c}\right)&=2\sin kl\cos kl+\frac{1}{S}(\cos^2 kl)\frac{S_1\sin kl_1\cos kl_2+S_2\cos kl_1\sin kl_2}{\cos kl_1\cos kl_2-\dfrac{S_2}{S_1}\sin kl_1\sin kl_2}\,.
\end{aligned}\right\}\quad(4)$$

(ii) The equivalent circuit of Fig. 5(a) is similar to that of the side branch type, but the impedance Z should be determined from the equivalent circuit Fig. 5(b) where Z_1 and Z_2 are the impedances looking from $A\,A'$ section of this cavity

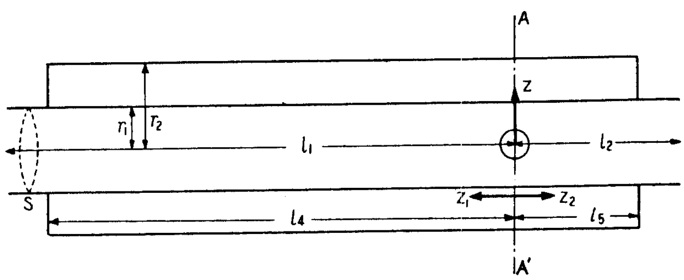


FIGURE 5(a). Resonator type element (ii).

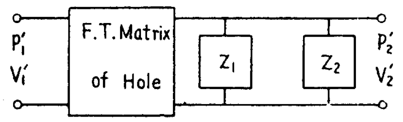


FIGURE 5(b). Equivalent circuit of hole and cavity in Fig. 5(a).

toward the closed ends respectively.

$$\frac{1}{Z_1}=j\frac{S_2}{\rho c}\tan kl_4\,,\quad \frac{1}{Z_2}=j\frac{S_2}{\rho c}\tan kl_5\,,\quad S_2=\pi(r_2^2-r_1^2)\,.$$

If the thickness of the tube is very thin,

$$\cos kl_3 \doteqdot 1 \qquad \sin kl_3 \doteqdot kl_3$$

$$\begin{pmatrix} p_1' \\ V_1' \end{pmatrix}=\begin{pmatrix} 1 & j\frac{\rho c}{S_3}kl_3 \\ j\frac{S_3}{\rho c}kl_3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{1}{Z_1}+\frac{1}{Z_2} & 1 \end{pmatrix}\begin{pmatrix} p_2' \\ 0 \end{pmatrix},\quad Z=\frac{p_1'}{V_1'}\,,$$

$$\left(\frac{\rho c}{S}\frac{1}{Z}\right)=j\frac{\frac{S_3}{S}kl_3+\frac{S_2}{S}(\tan kl_4+\tan kl_5)}{1-\frac{S_2}{S_3}kl_3(\tan kl_4+\tan kl_5)}\,.$$

The four terminal constants of this system are as follows:

$$\left.\begin{aligned} A&=\cos kl_1\cos kl_2-\sin kl_1\sin kl_2+j\left(\frac{\rho c}{S}\frac{1}{Z}\right)\sin kl_1\cos kl_2\,,\\ \left(B\Big/j\frac{\rho c}{S}\right)&=\sin kl_2\left(\cos kl_1+j\frac{\rho c}{S}\frac{1}{Z}\sin kl_1\right)+\sin kl_1\cos kl_2\,,\\ \left(C\Big/j\frac{S}{\rho c}\right)&=\cos kl_2\left(\sin kl_1-j\frac{\rho c}{S}\frac{1}{Z}\cos kl_1\right)+\cos kl_1\sin kl_2\,,\end{aligned}\right\}\qquad (5)$$

(iii) As shown in Fig. 6 the axial length of a cylindrical cavity is smaller than its radius and the tube with one hole penetrates at its center. The cross section

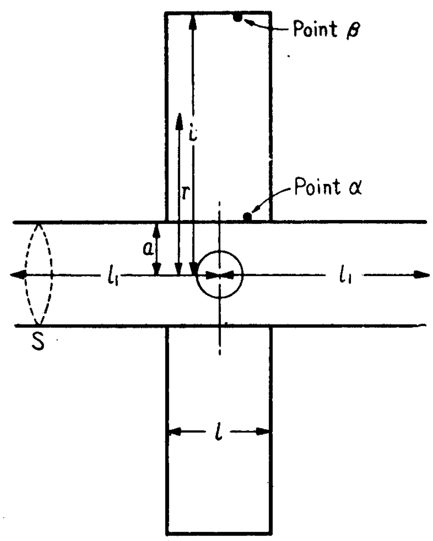


FIGURE 6. Resonator type element (iii).

and the thickness of the tube wall are S' and l' respectively. For analyzing the problem, only the radial direction is considered, and the θ direction in the cylindrical coordinate is neglected in accordance with the symmetry.

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+k^2\phi=0\,,$$

$$\phi=AJ_0(kr)+BY_0(kr)\,.$$

A and B are the arbitrary constants, J_0 and Y_0 are the Bessel function. From the relations

$$\frac{dJ_0(x)}{dx}=-J_1(x)\,,\quad \frac{dY_0(x)}{dx}=-Y_1(x)\,,\quad S=2\pi rl\,,$$

following equations are obtained:

$$p=\rho\dot{\phi}=\rho j\omega[AJ_0(kr)+BY_0(kr)]\,.$$

$$V=-S\frac{\partial\phi}{\partial r}=-ASkJ_1(kr)-BSkY_1(kr)\,,$$

At the point α

$$p_1=A\rho j\omega J_0(ka)+B\rho j\omega Y_0(ka)\,,$$
$$V_1=-AS_1kJ_1(ka)-BS_1kY_1(ka)\,.$$

At the point β

$$p_2=A\rho j\omega J_0(kb)+B\rho j\omega Y_0(kb)\,,$$
$$V_2=-AS_2kJ_1(kb)-BS_2kY_1(kb)\,.$$

Provided that $\qquad S_2=2\pi bl\,,\quad S_1=2\pi al\,.$

The four terminal constants of the r direction are obtained.

$$\begin{pmatrix}p_1\\V_1\end{pmatrix}=\frac{1}{\begin{vmatrix}J_0(kb)&Y_0(kb)\\J_1(kb)&Y_1(kb)\end{vmatrix}}\begin{pmatrix}\begin{vmatrix}J_0(ka)&Y_0(ka)\\J_1(kb)&Y_1(kb)\end{vmatrix} & j\dfrac{\rho c}{S_2}\begin{vmatrix}J_0(ka)&Y_0(ka)\\J_0(kb)&Y_0(kb)\end{vmatrix}\\ j\dfrac{S_1}{\rho c}\begin{vmatrix}J_1(ka)&Y_1(ka)\\J_1(kb)&Y_1(kb)\end{vmatrix} & \dfrac{S_1}{S_2}\begin{vmatrix}J_0(kb)&Y_0(kb)\\J_1(ka)&Y_1(ka)\end{vmatrix}\end{pmatrix}\begin{pmatrix}p_2\\V_2\end{pmatrix}.$$

The impedance looking toward the cavity from the hole can be obtained by the following method. Pressure and volume velocity at the hole and at the point β are p_1', V_1', p_2' and V_2' respectively. $V_2'=0$, because it is the volume velocity at the closed end.

$$Z=\frac{p_1'}{V_1'}\,,\quad \frac{\rho c}{S}\frac{1}{Z}=j\frac{\dfrac{S'}{S}kl\begin{vmatrix}J_0(ka)&Y_0(ka)\\J_1(kb)&Y_1(kb)\end{vmatrix}+\dfrac{S_1}{S}\begin{vmatrix}J_1(ka)&Y_1(ka)\\J_1(kb)&Y_1(kb)\end{vmatrix}}{\begin{vmatrix}J_0(ka)&Y_0(ka)\\J_1(kb)&Y_1(kb)\end{vmatrix}-\dfrac{S_1}{S'}kl\begin{vmatrix}J_1(ka)&Y_1(ka)\\J_1(kb)&Y_1(kb)\end{vmatrix}}\,,$$

$$\left.\begin{aligned}A&=(\cos kl_1)^2-(\sin kl_1)^2+j\left(\frac{\rho c}{S}\frac{1}{Z}\right)\sin kl_1\cos kl_1\,,\\ \left(B\Big/j\frac{\rho c}{S}\right)&=2\sin kl_1\cos kl_1+j\left(\frac{\rho c}{S}\frac{1}{Z}\right)\sin^2 kl_1\,,\\ \left(C\Big/j\frac{S}{\rho c}\right)&=2\sin kl_1\cos kl_1-j\left(\frac{\rho c}{S}\frac{1}{Z}\right)(\cos kl_1)^2\,.\end{aligned}\right\}\qquad(6)$$

(D) *Cavity type with internal tubes.*

From the acoustical system of Fig. 7(a), an equivalent circuit shown in Fig. 7(b) is derived, where

$$S_3=S_2-S\,,\quad \frac{1}{Z_1}=j\frac{S_3}{\rho c}\tan kl_4\,,\quad \frac{1}{Z_2}=j\frac{S_3}{\rho c}\tan kl_5\,.$$

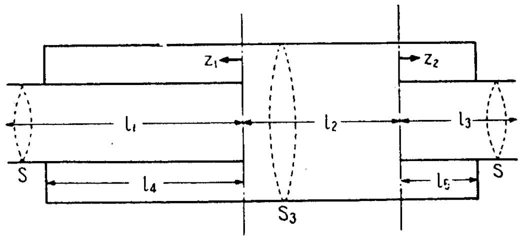


FIGURE 7(a). Cavity type with internal tube.

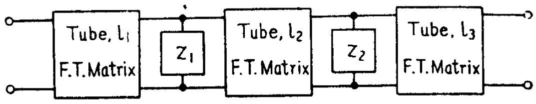


FIGURE 7(b). Equivalent circuit of Fig. 7(a).

The four terminal constants are calculated as follows:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} \cos kl_1 & j\dfrac{\rho c}{S}\sin kl_1 \\ j\dfrac{S}{\rho c}\sin kl_1 & \cos kl_1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \dfrac{1}{Z_1} & 1 \end{pmatrix}\begin{pmatrix} \cos kl_2 & j\dfrac{\rho c}{S_2}\sin kl_2 \\ j\dfrac{S_2}{\rho c}\sin kl_2 & \cos kl_2 \end{pmatrix}$$

$$\times\begin{pmatrix} 1 & 0 \\ \dfrac{1}{Z_2} & 1 \end{pmatrix}\begin{pmatrix} \cos kl_3 & j\dfrac{\rho c}{S}\sin kl_3 \\ j\dfrac{S}{\rho c}\sin kl_3 & \cos kl_3 \end{pmatrix}.$$

For special case where

$$l_1=l_3\,,\quad l_4=l_5\,,\quad \text{then}\quad \frac{1}{Z_1}=\frac{1}{Z_2}=\frac{1}{Z}\,,$$

the four terminal constants can be shown as,

$$\left.\begin{aligned}
A=&\{\cos^2 kl_1-\sin^2 kl_1\}\cos kl_2+j2\frac{\rho c}{S}\frac{1}{Z}\sin kl_1\cos kl_1\cos kl_2\\
&-\left\{\frac{S_3}{S}+\frac{S}{S_2}+\frac{S}{S_3}\left(\frac{\rho c}{S}\frac{1}{Z}\right)^2\right\}\sin kl_1\cos kl_1\sin kl_2\\
&+j\frac{S}{S_3}\left(\frac{\rho c}{S}\frac{1}{Z}\right)\cos^2 kl_1\sin kl_2-j\frac{S}{S_3}\left(\frac{\rho c}{S}\frac{1}{Z}\right)\sin^2 kl_1\sin kl_2\,,\\
\left(B\Big/j\frac{\rho c}{S}\right)=&\,2\sin kl_1\cos kl_1\cos kl_2+j2\left(\frac{\rho c}{S}\frac{1}{Z}\right)\sin^2 kl_1\cos kl_2\\
&+\frac{S}{S_2}\cos^2 kl_1\sin kl_2-\left\{\frac{S_2}{S}+\frac{S}{S_2}\left(\frac{\rho c}{S}\frac{1}{Z}\right)^2\right\}\sin^2 kl_1\sin kl_2\\
&+j2\frac{S}{S_2}\left(\frac{\rho c}{S}\frac{1}{Z}\right)\sin kl_1\cos kl_1\sin kl_2\,,\\
\left(C\Big/j\frac{S}{\rho c}\right)=&\,2\sin kl_1\cos kl_1\cos kl_2+\left\{\frac{S_2}{S}+\frac{S}{S_2}\left(\frac{\rho c}{S}\frac{1}{Z}\right)^2\right\}\cos^2 kl_1\sin kl_2
\end{aligned}\right\}\quad (7)$$

$$-j2\left(\frac{\rho c}{S}\frac{1}{Z}\right)\cos^2 kl_1 \cos kl_2 + j2\frac{S}{S_2}\left(\frac{\rho c}{S}\frac{1}{Z}\right)\sin kl_1 \cos kl_1 \sin kl_2$$
$$-\frac{S}{S_2}\sin^2 kl_1 \sin kl_2 .$$

(E) *Quincke tube type element.*

In Quincke type element, attenuation is caused by the interference phenomenon of sound waves. In Fig. 8(a), the cavity is passed througn by the tube with two holes, and to simplify the theory, hole are placed at both ends of the cavity. This is similar to a resonator type, however, the equivalent circuit is considered to be the Quincke type Fig. 8(b). The cross section of hole is S and the thickness of the wall l_3.

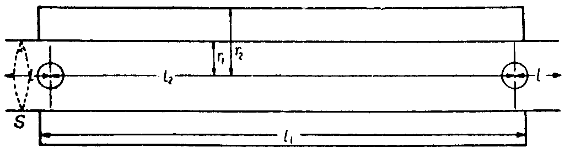


FIGURE 8(a). Quincke type elements.

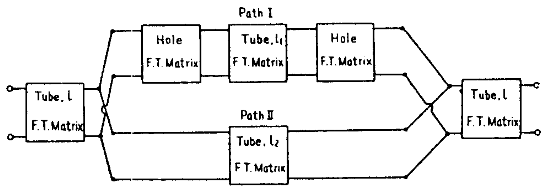


FIGURE 8(b). Equivalent circuit of Fig. 8(a).

From path (I) in Fig. 8(b)

$$\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} = \begin{pmatrix} 1 & j\frac{\rho c}{S_3}kl_3 \\ j\frac{S_3}{\rho c}kl_3 & 1 \end{pmatrix} \begin{pmatrix} \cos kl_1 & j\frac{\rho c}{S_1}\sin kl_1 \\ j\frac{S_1}{\rho c}\sin kl_1 & \cos kl_1 \end{pmatrix} \begin{pmatrix} 1 & j\frac{\rho c}{S_3}kl_3 \\ j\frac{S_3}{\rho c}kl_3 & 1 \end{pmatrix},$$

where

$$S_1 = \pi(r_2^2 - r_1^2) .$$

From path (II)

$$\begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} = \begin{pmatrix} \cos kl_2 & j\frac{\rho c}{S}\sin kl_2 \\ j\frac{S}{\rho c}\sin kl_2 & \cos kl_2 \end{pmatrix}.$$

The matrix of parallel connection of path (I) and path (II) is

$$\begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = \frac{1}{B_1+B_2} \begin{pmatrix} A_1B_2+B_1A_2 & B_1B_2 \\ -(A_1-A_2)^2+(B_1+B_2)(C_1+C_2) & A_1B_2+B_1A_2 \end{pmatrix},$$

$$\therefore \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{S}{\rho c}\sin kl & \cos kl \end{pmatrix} \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} \begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{S}{\rho c}\sin kl & \cos kl \end{pmatrix}.$$

The four terminal constants of this system are,

$$\left.\begin{aligned} A&=A'(\cos^2 kl-\sin^2 kl)+j\left(\frac{\rho c}{S}\sin kl\cos kl\right)C'+j\left(\frac{S}{\rho c}\sin kl\cos kl\right)B',\\ \left(B\Big/j\frac{\rho c}{S}\right)&=2A'\sin kl\cos kl-j\left(\frac{S}{\rho c}\cos^2 kl\right)B'+j\left(\frac{\rho c}{S}\sin^2 kl\right)C',\\ \left(C\Big/j\frac{S}{\rho c}\right)&=2A'\sin kl\cos kl+j\left(\frac{S}{\rho c}\sin^2 kl\right)B'-j\left(\frac{\rho c}{S}\cos^2 kl\right)C'. \end{aligned}\right\}\quad (8)$$

III. Principle of Measuring the Four Terminal Constants of Acoustic Filters

In an acoustic transmission line, pressure and volume velocity are complex quantities, and the pressure is described as $p=|p|\exp j\theta$, where $|p|$ is its absolute value and θ is its phase angle. A, B, C and D can be determined by measuring the absolute value of pressure $|p|$ and its phase angle θ. If the loss of energy is neglected, attenuation of sound in our acoustic system is caused by mismatching of impedance, so that it corresponds to reactance type four terminal networks of the electric system. Therefore, A and D are real, and B and C are imaginary.

(i) *Measurement of A, B and D.*

In Fig. 9(a), a sound source is a driver unit type speaker and the sound is introduced into the main tube through a wire-filled pipe, then the sound source can be considered as a constant current one. Both ends of the main tube were packed with glass-wool to avoid the reflection. The equivalent ciruit of this system is shown in Fig. 9(b). At the point of B, the pressure and volume velocity can be described,

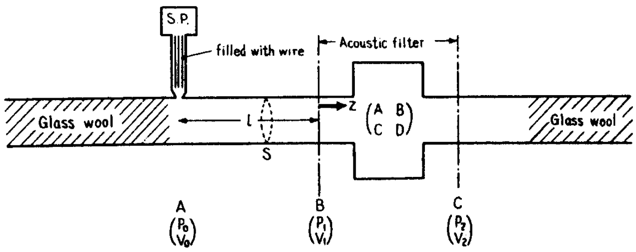


FIGURE 9(a). Arrangement for the measurement of A, B and D.

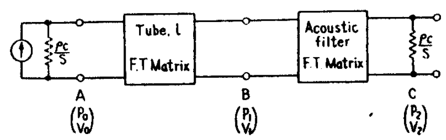


FIGURE 9(b). Equivalent circuit of Fig. 9(a).

$$\begin{pmatrix} p_0\\ V_0\end{pmatrix}=\begin{pmatrix} 1 & 0\\ \frac{S}{\rho c} & 1\end{pmatrix}\begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl\\ j\frac{S}{\rho c}\sin kl & \cos kl\end{pmatrix}\begin{pmatrix} p_1\\ V_1\end{pmatrix},\quad V_1=\frac{p_1}{Z},\quad Z=\frac{A+(S/\rho c)B}{C+(S/\rho c)D}.$$

At the point of C,

$$\begin{pmatrix} p_0\\ V_0\end{pmatrix}=\begin{pmatrix} 1 & 0\\ \frac{S}{\rho c} & 1\end{pmatrix}\begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl\\ j\frac{S}{\rho c}\sin kl & \cos kl\end{pmatrix}\begin{pmatrix} A & B\\ C & D\end{pmatrix}\begin{pmatrix} p_2\\ V_2\end{pmatrix},\quad V_2=\frac{p_2}{(\rho c/S)}.$$

By eliminating p in these equations,

$$\frac{p_1}{p_2} = A + B\Big/\left(\frac{\rho c}{S}\right). \tag{9}$$

As A is real and B is imaginary, $R(p_1/p_2)$, $I(p_1/p_2)$ correspond to A and $B/(\rho c/S)$ respectively, provided the acoustic filter is symmetrical one, $(A=D)$. But if it is not so, D can be obtained by interchanging the left and the right sides of this filter system for each other and closing the one end as is shown in Fig. 10.

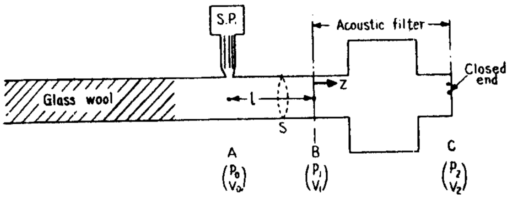


FIGURE 10. Arrangement for the measurement of C and D. The open end closed.

At the point of B,

$$\begin{pmatrix} p_0 \\ V_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{S}{\rho c} & 1 \end{pmatrix} \begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{S}{\rho c}\sin kl & \cos kl \end{pmatrix} \begin{pmatrix} p_1 \\ V_1 \end{pmatrix}, \quad \frac{p_1}{Z} = V_1, \quad Z = \frac{D}{C}.$$

At the point of C,

$$\begin{pmatrix} p_0 \\ V_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{S}{\rho c} & 1 \end{pmatrix} \begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{S}{\rho c}\sin kl & \cos kl \end{pmatrix} \begin{pmatrix} D & B \\ C & A \end{pmatrix} \begin{pmatrix} p_2 \\ V_2 \end{pmatrix}, \quad V_2 = 0,$$

from above equations

$$\left(\frac{p_1}{p_2}\right) = D. \tag{10}$$

As D is real, the phase difference between at the points B and C is 0 or π.

(ii) *Measurement of C.*

In a natural network C relates to other constants as $AD-BC=1$, then from A, B and D, C can be calculated. But C is able to be measured independently.

In Fig. 10, at the point of B,

$$\begin{pmatrix} p_0 \\ V_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{S}{\rho c} & 1 \end{pmatrix} \begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{S}{\rho c}\sin kl & \cos kl \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{pmatrix} \begin{pmatrix} p_1 \\ 0 \end{pmatrix}; \quad \frac{p_1}{Z} = V_1; \quad Z = \frac{A}{C}. \tag{11}$$

At the point of C,

$$\begin{pmatrix} p_0 \\ V_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{S}{\rho c} & 1 \end{pmatrix} \begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{S}{\rho c}\sin kl & \cos kl \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} p_2 \\ 0 \end{pmatrix}; \quad V_2 = 0. \tag{12}$$

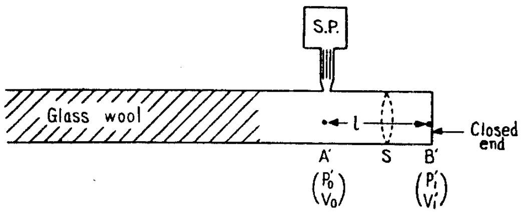
From the equation (11)

$$V_0=\left(\frac{S}{\rho c}+\frac{1}{Z}\right)e^{jkl}p_1\,. \tag{13}$$

From (11) and (12)

$$\frac{p_1}{p_2}=A\,.$$

When the filter is removed and the open end of the conducting tube is closed (Fig. 11).



FIFURE 11. The element is removed and the open end of the tube is closed.

$$\begin{pmatrix} p_0' \\ V_0 \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ \frac{S}{\rho c} & 1 \end{pmatrix}\begin{pmatrix} \cos kl & j\frac{\rho c}{S}\sin kl \\ j\frac{S}{\rho c}\sin kl & \cos kl \end{pmatrix}\begin{pmatrix} p_1' \\ 0 \end{pmatrix},$$

$$\therefore\quad V_0=\left(\frac{S}{\rho c}\right)e^{jkl}p_1'\,. \tag{14}$$

As the source is constant current one, from (13) and (14) V_0 can be eliminated, then

$$C\Big/\left(\frac{S}{\rho c}\right)=\left(\frac{p_1'}{p_2}-A\right). \tag{15}$$

As A is real, $I(p_1'/p_2)$ corresponds $C/(S/\rho c)$.

IV. Equipment of Measurement

Block diagram of measurement is shown in Fig. 12. To measure the phase, the

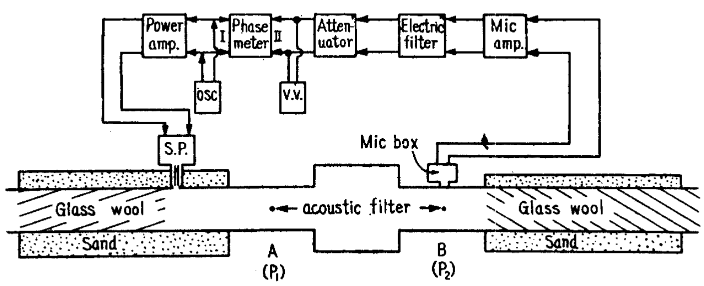
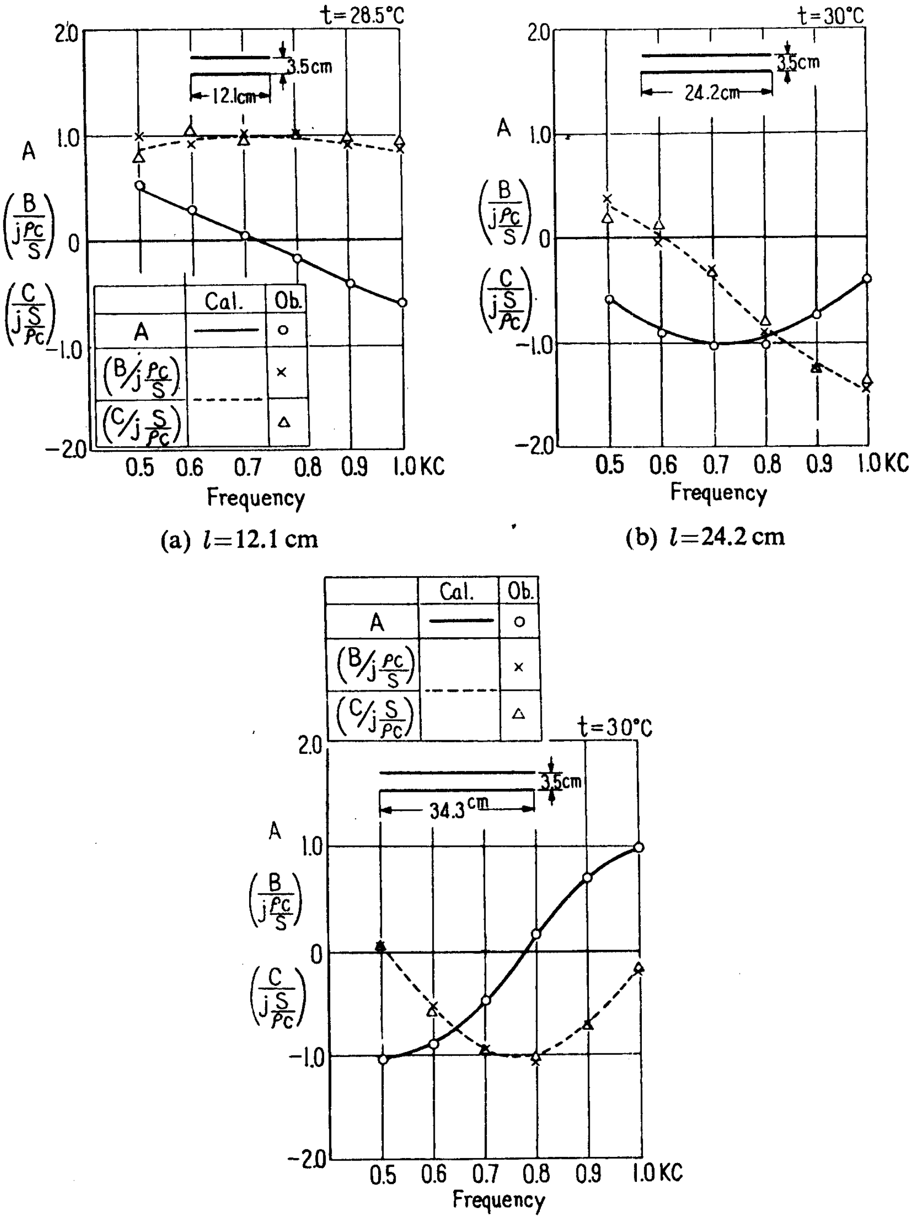


FIGURE 12. The block diagram of the measurement.
Channel I ; from oscillator to the phase meter.
Channel II; from oscillator, passing through the acoustic system and amplifier, electric filter and attenuator to the phase meter.

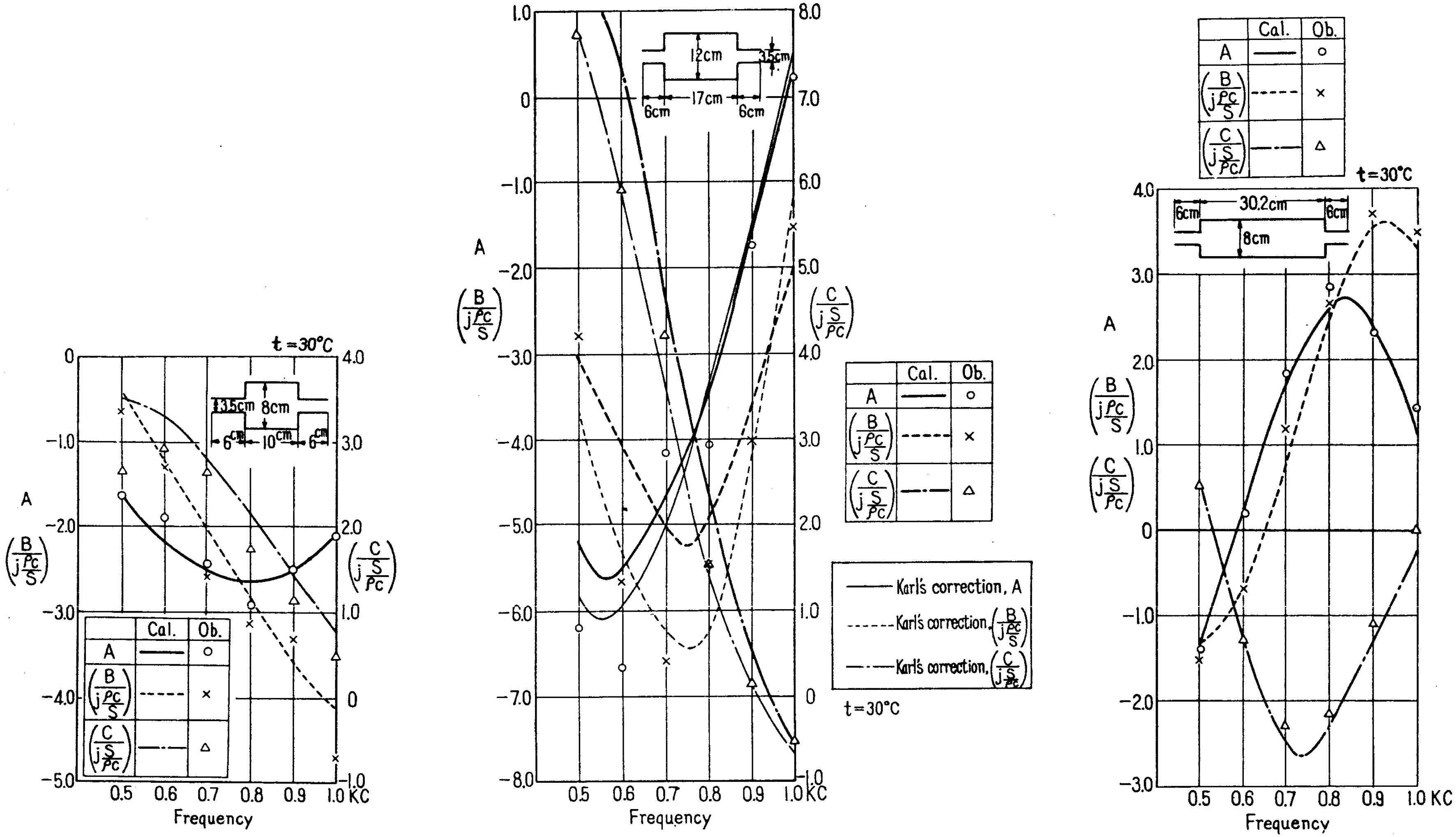
direct signal from oscillator (channel I) and the signal passing through the acoustic system (channel II) are applied to the phase meter, and the phase difference between these signals is measured. To measure the sound pressure, the input voltage of the phase meter from channel II, (microphone set at A) and the another input voltage of the channel I are adjusted at moderate values, then the microphone is replaced to B without changing the gain of the electric circuits. The attenuator is adjusted to be the same reading of the vacuum tube volt meter as before. Then the difference of the reading of the attenuator corresponds to the ratio of the sound pressure, $|p_1/p_2|$. Provided that p_1, p_2, θ_1 and θ_2 are the sound pressure, and phase angle at the points A and B respectively, then

$$p_1/p_2 = |p_1/p_2| \exp j(\theta_1 - \theta_2).$$



(c) l=34.3 cm

FIGURE 13. A, B and C of the straight tubes.



(a). Diameter and axial length are 8 and 10 cm respectively.

(b). Diameter 12 cm, axial length 17 cm. Calculated values obtained by Karal's correction are illustrated by dotted lines.

(c). Diameter 8 cm, axial length 30.2 cm.

FIGURE 14. *A*, *B* and *C* of the cavity type elements.

The brass tube 2 meters in length, 3.5 cm in diameter was filled with glass-wool at both ends and was buried in sand, to prevent vibration of the wall. The microphone was packed in a brass cylinder to keep off the external sound.

V. Discussion of Experimental Results

Frequency range of measurement was limited between 500 c/s and 1 kc, beyond this range owing to the attenuation it was difficult to measure the phase angle.

To examine the principle of the measurement mentioned above, A, B and C of the straight tube of various lengths were measured as shown in Figs. 13 (a), (b) and (c). Obtained results agree considerably well with the calculated ones.

Next, to simplify the numerical calculation, the symmetrical system such as the cavity type was measured. (Figs. 14 (a), (b) and (c)). The good agreement between the experimental and theoretical values in the Figures (a) and (c) is ascribed to the satisfaction of the one dimensional assumption, because the radius of the cavity is small compared with the wave length within this frequency range. On the other hand, when the change of cross section at the joint is large, the results obtained (Fig. 14 (b)) do not agree with the theory in the low frequency range. But by the Karal's correction for the discontinuity, these descrepancies can be explained fairly well, as are shown in the same Figure.

The result with the side branch type is shown in Fig. 15. The theoretical values agree condiderably well with the experiment, where length of a side branch is that measured from the center of the main tube to the closed end of the side branch tube.

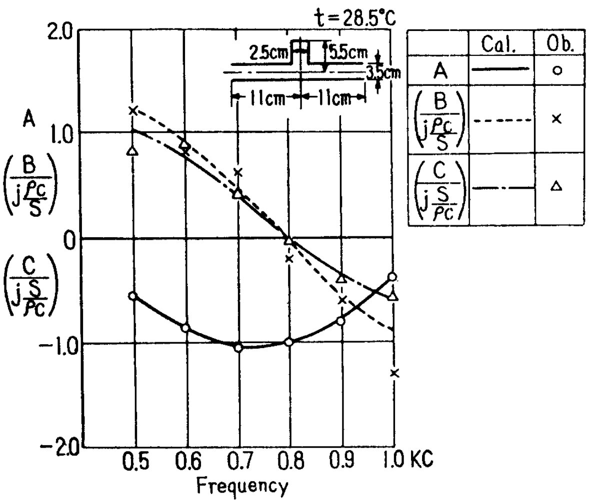


Figure 15. A, B and C of the side branch elements.

The results of resonators (i) and (ii) are shown in the Figs. 16(a) and (b). The result of resonator type (iii) is shown in Fig. 16(c), which presents good agreement except the value of C at 800 c/s, which would be caused by small signal to noise ratio owing to a large attenuation. Cavity with internal tube is indicated in Fig. 17, and good agreement could not be obtained in this case. The difference is considered to be caused by the imperfection of the one dimensional theory.

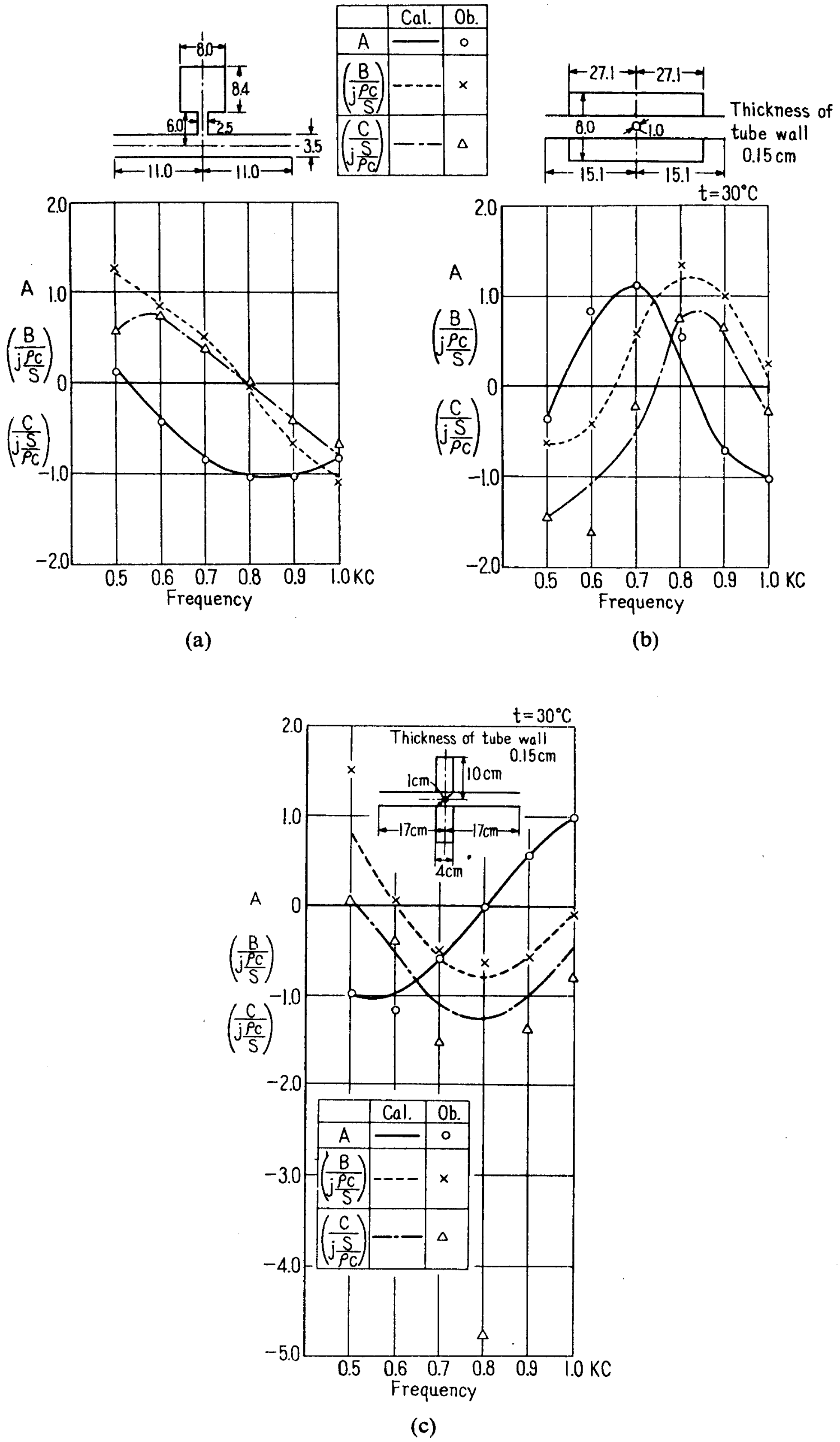


FIGURE 16. *A*, *B* and *C* of the resonator type elements.

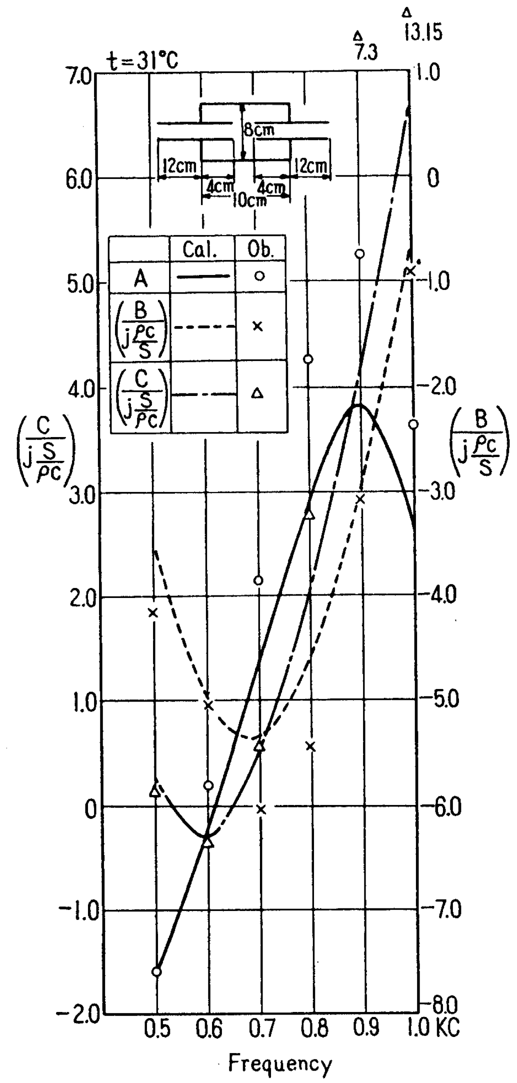


FIGURE 17. *A*, *B* and *C* of the cavity with internal tubes.

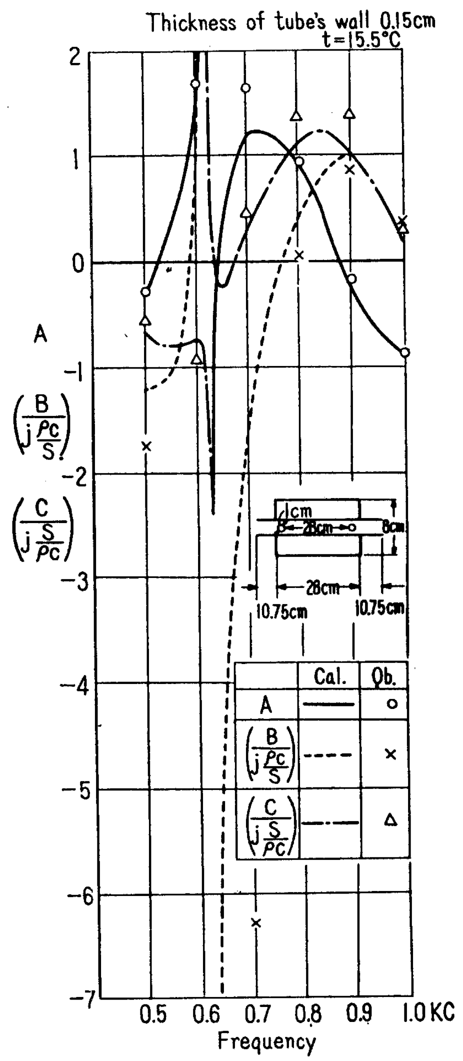


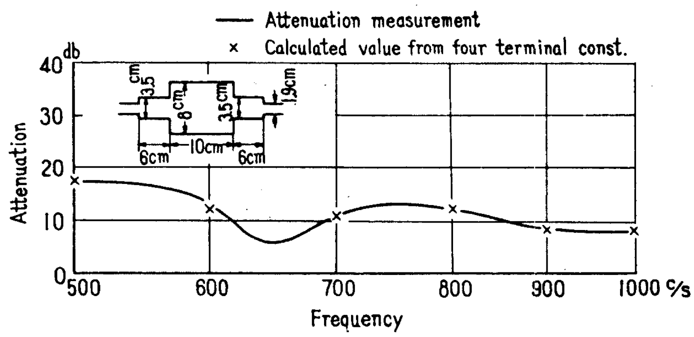
FIGURE 18. *A*, *B* and *C* of the Quincke type element.

In Fig. 18 Quincke type element is shown. The agreement is fairly satisfactory.

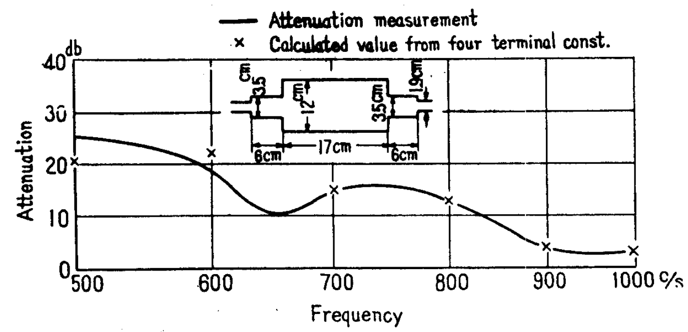
VI. Calculated Attenuation from the Four Terminal Constants

Attenuation characteristics of the filter elements can be obtained from the measured four terminal constants.

For several elements, they were calculated and were compared with those obtained by an acoustic method mentioned in previous paper (Part I). If the constants of each elements of the filter are obtained, the attenuation of the filter will be calculated from the cascade connection of each matrix. Examples are shown in Figs. 19(a) and (b), in which three cavities of different dimenstions were connected in cascade. They are in good agreement with the acoustically obtained values.



(a)



(b)

FIGURE 19. Comparison of the attenuation between calculated values from the measured four terminal constants and those of obtained by the acoustical method mentioned in previous paper.

VII. CONCLUSION

The method of measuring the four terminal constants is reported. The four terminal matrices of the acoustic filter elements are the very convenient conception for calculating the attenuattion of the acoustic filter, as mentioned in the previous paper. But if the structure of the element is complicated, the four terminal constants can not be estimated easily, because of the complex wave phenomena in it.

Even in the simplest cavity type element, the descrepancy between the measured four terminal constants and those estimated from one dimentsional theory was observed.

The measured constants imply the complex wave phenomena in the elements, then the calculated attenuation from these constants will show the actual attenuation. Some examples were shown.

APPENDIX I. BASIC CONCEPTION OF THE FOUR TERMINAL CONSTANTS

Neglecting the viscosity of air and the heat conduction, the following equation is obtained:

$$\ddot{\phi} = c^2 \Delta\phi$$

where ϕ is velocity potential. If the phenomenon is restricted only to x direction,

$$\ddot{\phi}=c^2\frac{\partial^2\phi}{\partial x^2}.$$

For a periodic motion, $\partial/\partial t$ can be replaced by $j\omega$, where ω is angular frequency, and c is sound velocity and $\omega/c=k$, $\omega=2\pi f$.

$$\frac{\partial^2\phi}{\partial x^2}=-k^2\phi.$$

The solution is

$$\phi=Ae^{jkx}+Be^{-jkx}.$$

A and B are arbitrary constants. Then,

$$p=\rho\dot{\phi}=\rho j\omega(Ae^{jkx}+Be^{-jkx}),$$

$$V=-S\frac{\partial\phi}{\partial x}=-Sjk(Ae^{jkx}-Be^{-jkx}),$$

where p is the pressure of sound and V is the volume velocity and S is the cross section of the tube.

At the point x_1

$$\left.\begin{aligned}p_1&=j\omega\rho(Ae^{jkx_1}+Be^{-jkx_1}),\\ V_1&=-Sjk(Ae^{jkx_1}-Be^{-jkx_1}).\end{aligned}\right\}\qquad(1)$$

At the point x_2

$$\left.\begin{aligned}p_2&=j\omega\rho(Ae^{jkx_2}+Be^{-jkx_2}),\\ V_2&=-Sjk(Ae^{jkx_2}-Be^{-jkx_2}).\end{aligned}\right\}\qquad(2)$$

From (1) (2) and $x_2-x_1=l$, the following equation can be obtained.

$$\begin{pmatrix}p_1\\V_1\end{pmatrix}=\begin{pmatrix}\cos kl & j\dfrac{\rho c}{S}\sin kl\\ j\dfrac{S}{\rho c}\sin kl & \cos kl\end{pmatrix}\begin{pmatrix}p_2\\V_2\end{pmatrix}.$$

This matrix corresponds with that of the electrical transmission line.

APPENDIX II

Various methods [3] have been used for measuring the phase. In our measurements, the phase meter was constructed, the block diagram and the ciruit of which are shown in Figs. 20 and 21. 6BN6 was used as the detector of the phase angle between the two signals [4].

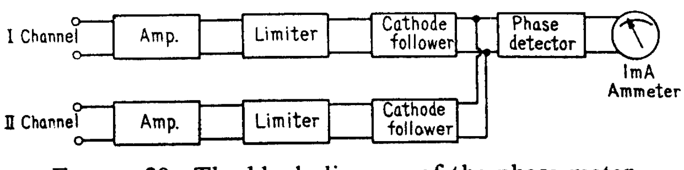


FIGURE 20. The block diagram of the phase meter.

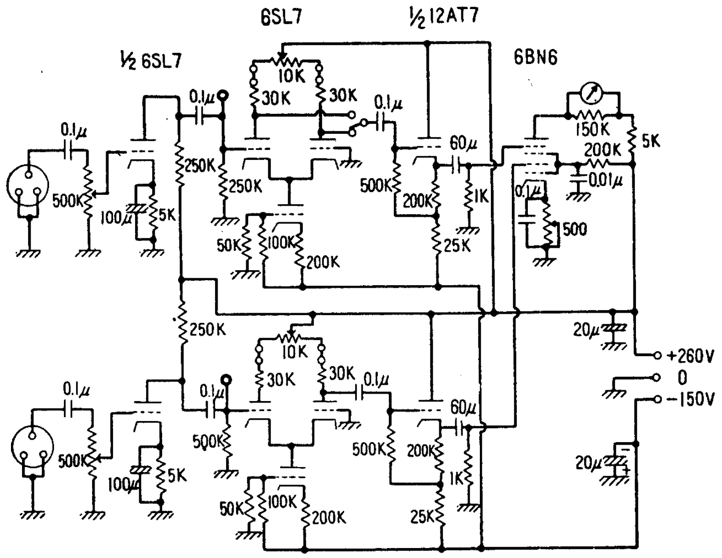


FIGURE 21. The electric circuit of the phase meter.

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