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An Approximate Treatment of Laminar Heat Transfer Coefficient and its Application to the Calculation of Temperature of Super-sonic Vehicles†

By

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Summary. An approximate treatment of expression for the heat transfer coefficient in compressible flow and its appliction to calculation of the skin temperature of the super-sonic vehicle are presented. The paper includes the following four items. First, the weak dependence of laminar heat transfer coefficient in compressible flow upon the reference temperature at which the values of physical properties included in the Reynolds number and the thermal conductivity of the air should be taken, is shown by inspecting the total effect of those changes in physical properties with the reference temperature on the heat transfer coefficient by the use of the power law expressions for the dynamic viscosity, specific weight, and thermal conductivity of the air with temperature. Secondly, some characteristics of flight trajectory of rocket are inspected with a view of approximating them with some typical flight paterns which can easily be expressed by simple mathematical forms such as constantly accelerated (or decelerated) ascent (or descent), and these approximations are proved to be satisfactory for the purpose of calculating the temperature of the vehicle. Thirdly, with a good use of the two facts mentioned above, a simple method for calculating the skin temperature of the vehicle in super-sonic flight is proposed. Finally, the result of the flight investigation performed is presented and the measured temperature of the vehicle is compared with the calculated one obtained by the use of weak dependence of laminar heat transfer coefficient upon the reference temperature presented in the first item of the paper.

DEPENDENCE OF THE LAMINAR HEAT TRANSFER COEFFICIENT IN COMPRESSIBLE FLOW ON THE REFERENCE TEMPERATURE

It has been shown that the heat transfer, q, from or to a laminar boundary layer on a smooth flat plate can be expressed by the following relation

$$q(x)=N_{u_x}\frac{\lambda^*}{x}(T_r-T_s), \qquad (1)$$

where x is the distance measured from the leading edge of the plate, N_{u_x} is the Nusselt number, λ^* is the thermal conductivity of the air, T_r is the wall tempe-

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rature (recovery temperature) when the wall is insulated (zero heat transfer to or from the boundary layer), and T_s is the skin temperature.

The recovery temperature can be computed from

$$T_r = T_\infty\left(1 + P_r^{\frac{1}{2}}\frac{\gamma-1}{2}M_\infty^2\right), \tag{2}$$

when the specific heat at constant pressure and the Prandtl number are constant.

The Nusselt number for the case of compressible flow can be expressed by the following relation which is the same equation for the case of incompressible flow,

$$N_{u_x} = 0.332\, R_{e_x}^{\frac{1}{2}} P_r^{\frac{1}{3}}, \tag{3}$$

provided that the values of the Reynolds number and the thermal conductivity of the air in Eq. (1) should be taken at the reference temperature T^*. The reference temperature, for example, is given by the following formula.

$$T^* = 0.5(T_s + T_\infty) + 0.22 P_r^{\frac{1}{2}}\frac{\gamma-1}{2}M_\infty^2 T_\infty. \tag{4}$$

Using these relations, we can obtain the following expression for the heat transfer coefficient.

$$h = 0.332\frac{\lambda^*}{x} R_{e_x}^{\frac{1}{2}} P_r^{\frac{1}{3}} \tag{5}$$

or

$$h = 0.332\lambda^*\left(\frac{u_\infty \gamma^*}{x g \mu^*}\right)^{\frac{1}{2}} P_r^{\frac{1}{3}}, \tag{6}$$

where γ^* and μ^* are the specific weight and the dynamic viscosity of the air, respectively.

Now, let us consider the variation of such physical properties of the air as λ^*, γ^*, and μ^* due to the change in the reference temperature. The dynamic viscosity μ^* is related to the temperature by the well-known Sutherland formula as follows:

$$\mu^* \infty \frac{\sqrt{T^*}}{(1 + cT^*)}. \tag{7}$$

where c is a constant. But the use of power law is more convenient for analysis and the following expression was used, for example, by von Kármán and Tsien.

$$\mu^* \infty T^{*^{0.76}}. \tag{8}$$

The thermal conductivity of the air can also be approximated by the power law formula as follows:

$$\lambda^* \propto T^{*^{0.76}}. \tag{9}$$

The assumption that the Prandtl number $P_r = c_p\mu^*/\lambda^*$ and the specific heat at constant pressure are constant, is used in deriving the above formula. Since the specific weight γ^* of the air is inversely proportional to the temperature,

$$\gamma^* \propto T^{*^{-1}}. \tag{10}$$

These formulae (8), (9) and (10) can be expressed by the following equations by the use of proportional constants,

$$\mu^* = \bar{\mu} T^{*0.76} \qquad (\bar{\mu} = \mu_\infty T_\infty^{-0.76}), \tag{11}$$

$$\lambda^* = \bar{\lambda} T^{*0.76} \qquad (\bar{\lambda} = \lambda_\infty T_\infty^{-0.76}), \tag{12}$$

$$\gamma^* = \bar{\gamma} T^{*-1} \qquad (\bar{\gamma} = \gamma_\infty T_\infty). \tag{13}$$

Substituting Eqs. (11), (12), and (13) into Eq. (6), we obtain the following expression for the heat transfer coefficient.

$$h = 0.332 \bar{\lambda} \left(\frac{u_\infty \bar{\gamma}}{x g \bar{\mu}} \right)^{\frac{1}{2}} P_r^{\frac{1}{3}} T^{*-0.12} \tag{14}$$

It should be noted that the variation of h due to the change in the reference temperature is rather small.

It is interesting to note that certain selections of power formulae for the approximations of the dynamic viscosity and the thermal conductivity can eliminate entirely the influence of the reference temperature on the heat transfer coefficient in the compressible flow. One of these combinations is

$$\mu^* = \bar{\bar{\mu}} T^{*\frac{5}{6}} \qquad (\bar{\bar{\mu}} = \mu_\infty T_\infty^{-\frac{5}{6}}), \tag{15}$$

$$\lambda^* = \bar{\bar{\lambda}} T^{*\frac{2}{3}} \qquad (\bar{\bar{\lambda}} = \lambda_\infty T_\infty^{-\frac{2}{3}}). \tag{16}$$

The above approximations are shown in Figs. 1 and 2 to have a good accuracy

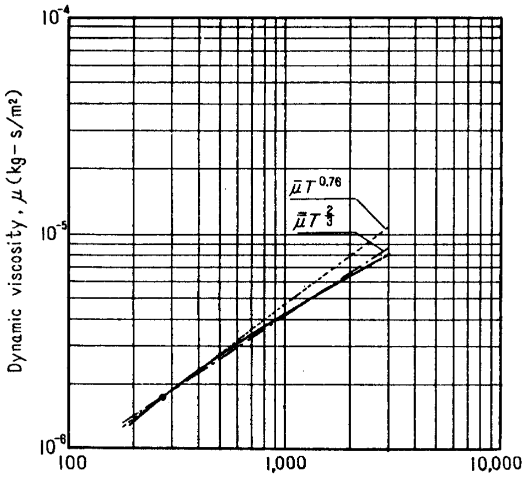

FIGURE 1. Approximation of dynamic viscosity of air as a function of temperature by power formulas.

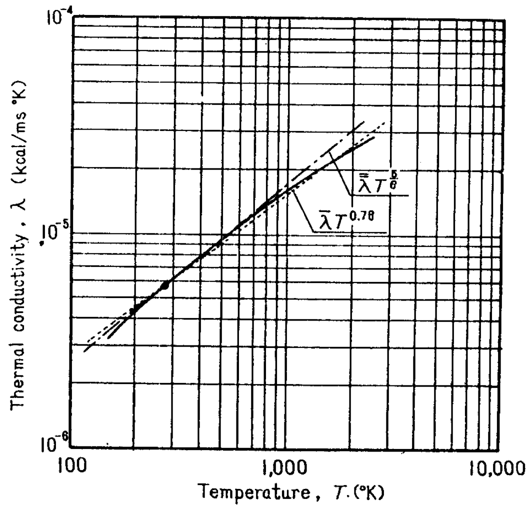

FIGURE 2. Approximation of thermal conductivity of air as a function of temperature by power formulas.

over a wide range of the reference temperature. Substituting Eqs. (10), (15), and (16) into Eq. (6) we obtain the following expression for the heat transfer coefficient which is independent of the reference temperature.

$$h = 0.332 \bar{\bar{\lambda}} \left(\frac{u_\infty \bar{\gamma}}{x g \bar{\bar{\mu}}} \right)^{\frac{1}{2}} P_r^{\frac{1}{3}} T^{*0}. \tag{17}$$

This formula can be written in terms of physical properties at the outer edge of the boundary layer by the use of relations in the parentheses in Eqs (13), (15), and (16).

$$h=0.332\lambda_\infty\left(\frac{u_\infty\gamma_\infty}{xg\mu_\infty}\right)^{\frac{1}{2}}P_r^{\frac{1}{3}}. \tag{18}$$

It should be noted that this conclusion about the weak dependence of heat transfer coefficient on the reference temperature is valid also in cases of the other definitions of the reference temperatures as well as the one given in Eq. (4). As a matter of course, the selections of power law expressions such as Eqs. (11), (12), (15), and (16) may have a relation to the analysis itself used in the derivation of the Nusselt number. But after all these considerations will not change the scope of the matter so far as the analysis uses the concept of the reference temperature at which the physical properties of the air are taken.

This characteristic of the heat transfer coefficient has been used by the author since 1957 at the preliminary design of the sounding rocket for the use in I.G.Y., and presented at the Committee for Aeroplane Structures, Japan Society of Aeronautical Engineering in July, 1958. The similar approach was made by R. R. Gold [1], who concluded the weak dependence of laminar heat transfer coefficient on the temperature from numerical calculation by using standard gas tables.

Some Characteristics of Flight Trajectories of Solid-Propellant Rockets

Only a few approaches have ever been made to obtain the compact formula for the skin temperature of supersonic vehicles, giving consideration to the particular flight trajectories of the vehicle. It is not merely because the temperature of the skin of the vehicle is a function of the heat transfer coefficient which, in turn, is a complex function of the temperature of the skin, but also because the flight trajectory defining the velocity and the altitude, at which the physical properties of the air are given, can not be given in a compact formula. Thus, in almost every case the calculation of the temperature usually relies on the step-by-step iteration procedure which is cumbersome as well as tedious.

Considering the independence (weak dependence) of the heat transfer coefficient on the reference temperature as shown in the previous section, we can remove the former complexity and then we shall search for the possibility of removing the latter cause of complexity due to the flight trajectory.

In Fig. 3 the velocity diagrams of two two-step solid-propellant rocket vehicles, namely, Nike-Deacon and Kappa are shown in order to inspect the general characteristics of flight missions of such vehicles. Apparently those missions for two sorts of solid propellant rockets can be featured approximately by the constantly accelerated flight and constantly decelerated flight. Theoretically, if the burning rate is constant during the accelerated stage, the amount of deviation from the constant acceleration is, in the main, due to the reduction of total mass of the vehicle by propellant consumption and to the increasing drag of the air. While in the stage of decelerated stage (coasting) the deviation is due to the decreasing drag of the air.

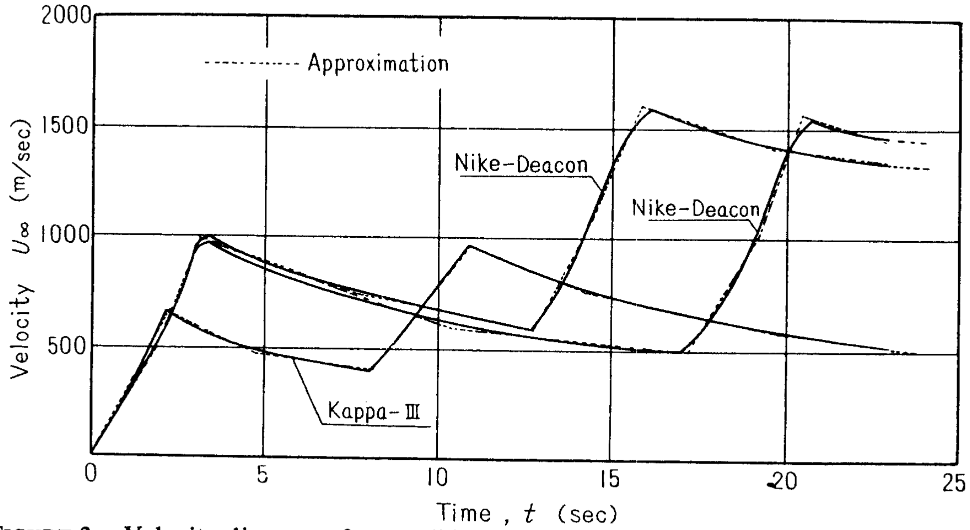

FIGURE 3. Velocity diagram of two solid-propellant rocket vehicles and their approximation by several number of constantly accelerated (or decelerated) flight paterns.

It is reasonable to assume that when these flight missions are approximated by several number of constantly accelerated or decelerated flights as shown by the dotted lines in Fig. 3, these deviations from the real trajectories may not produce substantial errors in the calculated temperatures of the skin of vehicles, as the integral operation is performed in the calculation.

Further it will be well admitted that as the flight path angle of the vehicle dose not change substantially during the short time considered and it has almost no influence upon the temperature of the skin, it is assumed to be constant during the time.

Conclusively, the flight missions of such solid-propellant rockets can be divided into several number of the constantly accelerated flight and constantly decelerated flight for the purpose of approximation in the calculation of temperature.

In the following section we shall analyse the aerodynamic heating of such vehicles featured by above-mentioned flight missions by making a good use of an approximate treatment of the laminar heat transfer coefficient in compressible flow developed in the preceding section of this paper.

A METHOD FOR CALCULATING THE TEMPERATURE OF THE VEHICLE IN SUPER-SONIC FLIGHT

Let us consider the aerodynamic heating of the super-sonic vehicle which is constantly accelerated (or decelerated). This trajectory can be expressed by the following formulae,

$$u_\infty = u_{\infty_i} + \dot{u}_\infty t, \tag{19}$$

$$\frac{dz}{dt} = u_\infty \sin\theta, \tag{20}$$

where u_∞ is the velocity, u_{∞_i} is the initial velocity, $\dot{u}_\infty$ is the acceleration (constant), z is the altitude, and θ is the flight path angle relative to the level (constant).

In the calculation of the temperature of the skin of a high-speed vehicle, the following assumptions are usually made: (1) the skin is so thin that the temperature gradient in the skin normal to the surface is negligible; (2) heat conduction along the skin is negligible; (3) no heat transfer takes place to or from other parts of the vehicle; and (4) no heat transfer by radiation takes place to or from the surface of the vehicle.

Accordingly, the differential equation for skin temperature is

$$c_s \gamma_s \delta_s \frac{dT_s}{dt} = h(T_r - T_s), \tag{21}$$

where c_s, γ_s, and δ_s are the specific heat, specific weight, and thickness of the skin, respectively. Subscript s refers to the skin. If heat transfer to other parts of the vehicles is considered, then that rate must be subtracted from the right hand side of Eq. (21) (or added if heat is transfered from other parts to the skin). The heat transfer described in items (2), (3), and (4) are usually small compared with the quantity due to the heat convection from or to the boundary layer.

It must be noted that Eq. (21) is a non-linear equation, since the heat transfer coefficient in the right hand side of the equation is a complex function of the reference temperature which, in turn, is a function of skin temperature.

The author, however, showed in the preceding section of the paper that the dependence of laminar heat transfer coefficient in the compressible flow on the reference temperature is rather weak and that the values of physical properties in the boundary layer such as μ, λ, and γ included in the heat transfer coefficient can be replaced with those values at the outer edge of the boundary layer. Only if that conclusion is accepted, Eq. (21) can be linearized. And this is one of the chief points the author intends to show to the reader.

Accepting the author's conclusion about the heat transfer coefficient, we can easily obtain the solution of Eq. (21). The procedure for solving Eq. (21) will now been shown in the following.

The heat transfer coefficient, therefore, can be expressed as a function of the velocity and the physical properties of the air at the outer edge of the boundary layer. Usually the flight trajectory of the vehicle includes the ascending or descending stages, then the changes in the properties of the air due to the change in the altitude must be considered in the calculation.

The change in the velocity of sound as well as that of the temperature of the air should also be considered, as they influence the recovery temperature in the right hand side of Eq. (21).

After due consideration about the quantities of changes in these physical properties of the air during a restricted short time as shown in the approximation of flight trajectory by a constantly accelerated (or decelerated) flight, it may be concluded that the only and most predominent effect on Eq. (21) is the change in the specific weight of the air which varies in a wide range during such a short time and that the changes in the other physical properties have only small effects on Eq. (21) and can be neglected.

The specific weight of the air can be expressed by the generally accepted formula

$$\gamma_\infty=\gamma_0 \exp(-\beta z), \tag{22}$$

where γ_0, and β are the constants which can be selected properly at each altitude for the purpose of obtaining better approximation, but the value at the level is generally used.

Conclusively, the heat transfer coefficient can be expressed as a function of the velocity of the vehicle and the altitude by the following equation

$$h=\bar{h}u_\infty^{\frac{1}{2}}e^{-\frac{1}{2}\beta z}, \tag{23}$$

where $\bar{h}$ is

$$\bar{h}=0.332\lambda_\infty\left(\frac{\gamma_0}{xg\mu_\infty}\right)^{\frac{1}{2}}P_r^{\frac{1}{3}}. \tag{24}$$

Substituting Eqs. (2) and (23) into Eq. (21) we obtain the following ordinary linear differential equation for the skin temperature:

$$c_s\gamma_s\delta_s\frac{dT_s}{dt}=\bar{h}u_\infty^{\frac{1}{2}}e^{-\frac{1}{2}\beta z}\left\{T_\infty\left(1+P_r^{\frac{1}{2}}\frac{\gamma-1}{2}M_\infty^2\right)-T_s\right\}. \tag{25}$$

If the independent variable t is transfered to z by using the following relation

$$\frac{dT_s}{dt}=\frac{dT_s}{dz}\cdot u_\infty \sin\theta \tag{26}$$

besides, the use of relation

$$u_\infty^2-u_{\infty_i}^2=2\dot{u}_\infty(z-z_i)/\sin\theta, \tag{27}$$

then Eq. (25) can be written as follows:

$$\frac{dT_s}{dz}=\frac{\bar{h}\exp(-\beta z/2)}{c_s\gamma_s\delta_s\sin\theta}\left\{\frac{2\dot{u}_\infty}{\sin\theta}\left(z-z_i+\frac{u_{\infty_i}^2\sin\theta}{2\dot{u}_\infty}\right)\right\}^{-\frac{1}{4}}$$
$$\times\left[T_\infty\left\{1+P_r^{\frac{1}{2}}\frac{\gamma-1}{2}\cdot\frac{1}{a_\infty^2}\cdot\frac{2\dot{u}_\infty}{\sin\theta}\left(z-z_i+\frac{u_{\infty_i}^2\sin\theta}{2\dot{u}_\infty}\right)\right\}-T_s\right]. \tag{28}$$

The subscript i refers to the initial conditions of the variables. Introducing the new independent variable ζ in place of z,

$$\left.\begin{aligned}\zeta&=z-z_i+u_{\infty_i}^2\sin\theta/2\dot{u}_\infty,\\ \zeta_i&=u_{\infty_i}^2\sin\theta/2\dot{u}_\infty,\\ \zeta_0&=-z_i+u_{\infty_i}^2\sin\theta/2\dot{u}_\infty.\end{aligned}\right\} \tag{29}$$

Eq. (28) can be written in terms of the new variable in case of accelerated flight ($\dot{u}_\infty>0$) as follows:

$$\frac{dT_s}{d\zeta}=\kappa\zeta^{-\frac{1}{4}}e^{-\frac{1}{2}\beta\zeta}\{T_\infty(1+\varepsilon\zeta)-T_s\}, \tag{30}$$

where κ and ε are

$$\left.\begin{aligned}\kappa&=\frac{\bar{h}\exp(\beta\zeta_0/2)}{c_s\gamma_s\delta_s(\sin\theta)^{3/4}|2\dot{u}_\infty|^{1/4}},\\ \varepsilon&=P_r^{\frac{1}{2}}\frac{(\gamma-1)\dot{u}_\infty}{a_\infty^2\sin\theta}.\end{aligned}\right\} \tag{31}$$

Thus the solution for T_s is

$$T_s(\zeta)=\exp\left\{-\kappa\int_{\zeta_i}^{\zeta}\zeta^{-\frac{1}{4}}e^{-\frac{1}{2}\beta\zeta}d\zeta\right\}$$
$$\times\left[\int_{\zeta_i}^{\zeta}\kappa\zeta^{-\frac{1}{4}}e^{-\frac{1}{2}\beta\zeta}T_\infty(1+\varepsilon\zeta)\exp\left\{\kappa\int_{\zeta_i}^{\zeta}\zeta^{-\frac{1}{4}}e^{-\frac{1}{2}\beta\zeta}d\zeta\right\}d\zeta+T_{s_i}\right]. \quad (32)$$

Defining new functions $\phi(\zeta)$ and $\Phi(\zeta)$ by the following equations,

$$\left.\begin{aligned}\phi(\zeta)&=\zeta^{-\frac{1}{4}}e^{-\frac{1}{2}\beta\zeta},\\ \Phi(\zeta)&=\int_0^{\zeta}\phi(\zeta)d\zeta,\end{aligned}\right\}. \quad (33)$$

we can express Eq. (32) by the use of these functions as follows:

$$T_s(\zeta)=\exp[-\kappa\{\Phi(\zeta)-\Phi(\zeta_i)\}]$$
$$\times\left[\int_{\zeta_i}^{\zeta}\kappa\phi(\zeta)T_\infty(1+\varepsilon\zeta)\exp[\kappa\{\Phi(\zeta)-\Phi(\zeta_i)\}]d\zeta+T_{si}\right]. \quad (34)$$

In the case of decelerated flight ($\dot{u}_\infty<0$), the equation corresponding to Eq. (30) is

$$\frac{dT_s}{d\zeta}=\kappa(-\zeta)^{-\frac{1}{4}}e^{-\frac{1}{2}\beta\zeta}\{T_\infty(1+\varepsilon\zeta)-T_t\}. \quad (35)$$

In exactly a similar manner as in the case of accelerated flight, let us define new functions $\phi_d(\zeta)$ and $\Phi_d(\zeta)$ by the following equations.

$$\left.\begin{aligned}\phi_d(\zeta)&=(-\zeta)^{-\frac{1}{4}}e^{-\frac{1}{2}\beta\zeta},\\ \Phi_d(\zeta)&=\int_0^{\zeta}\phi_d(\zeta)d\zeta.\end{aligned}\right\} \quad (36)$$

Then we have the solution for the case of decelerated flight as follows:

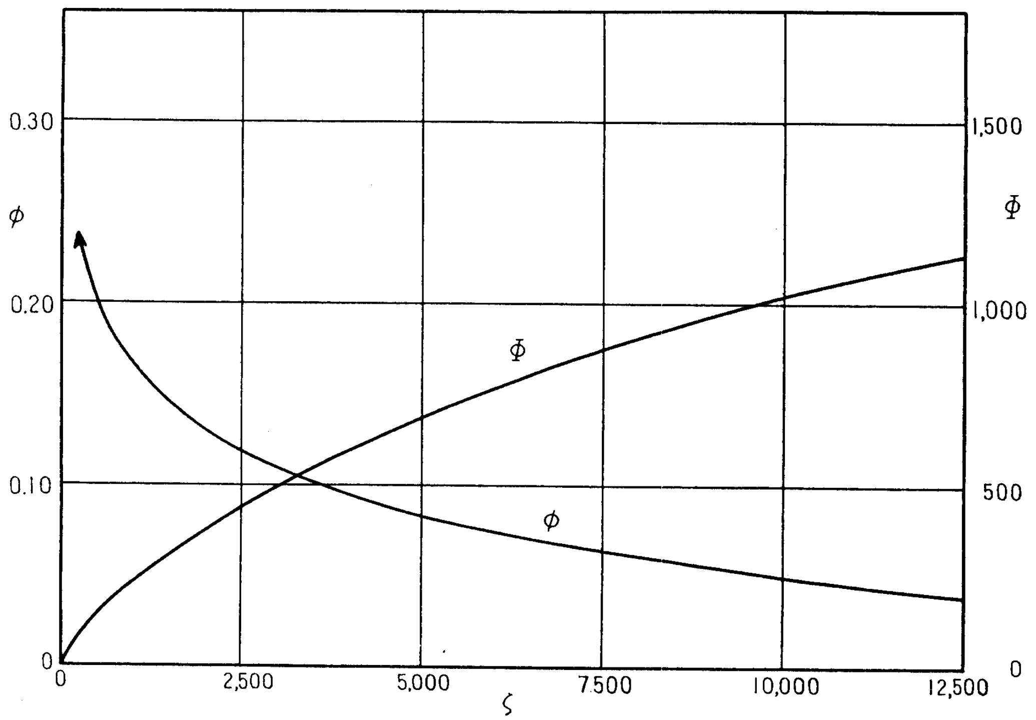

FIGURE 4-1. Functions $\phi(\zeta)$ and $\Phi(\zeta)$.

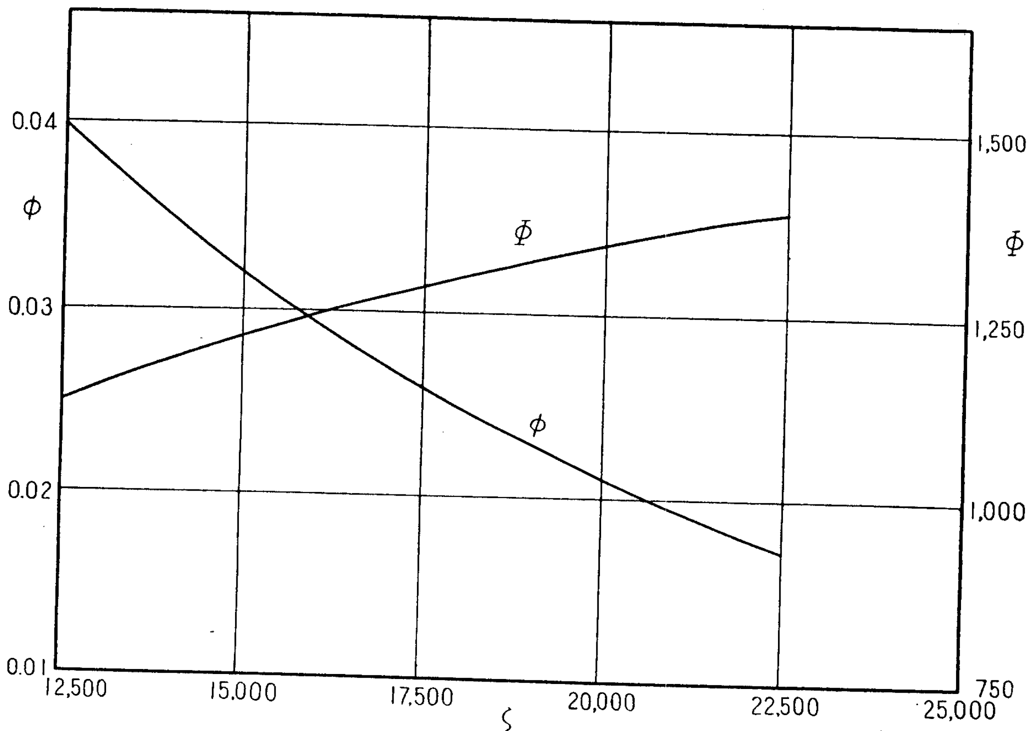

FIGURE 4-2. Functions $\phi(\zeta)$ and $\Phi(\zeta)$.

$$T_s(\zeta)=\exp[-\kappa\{\Phi_d(\zeta)-\Phi_d(\zeta_i)\}]$$
$$\times\left[\int_{\zeta_i}^{\zeta}\kappa\phi_d(\zeta)T_\infty(1+\varepsilon\zeta)\exp[\kappa\{\Phi_d(\zeta)-\Phi_d(\zeta_i)\}]d\zeta+T_{wi}\right]. \tag{37}$$

Values of functions ϕ, ϕ_d, Φ, and Φ_d were calculated and are presented in Figs. 4 and 5. The temperature of the skin of the vehicle can easily be obtained by per-

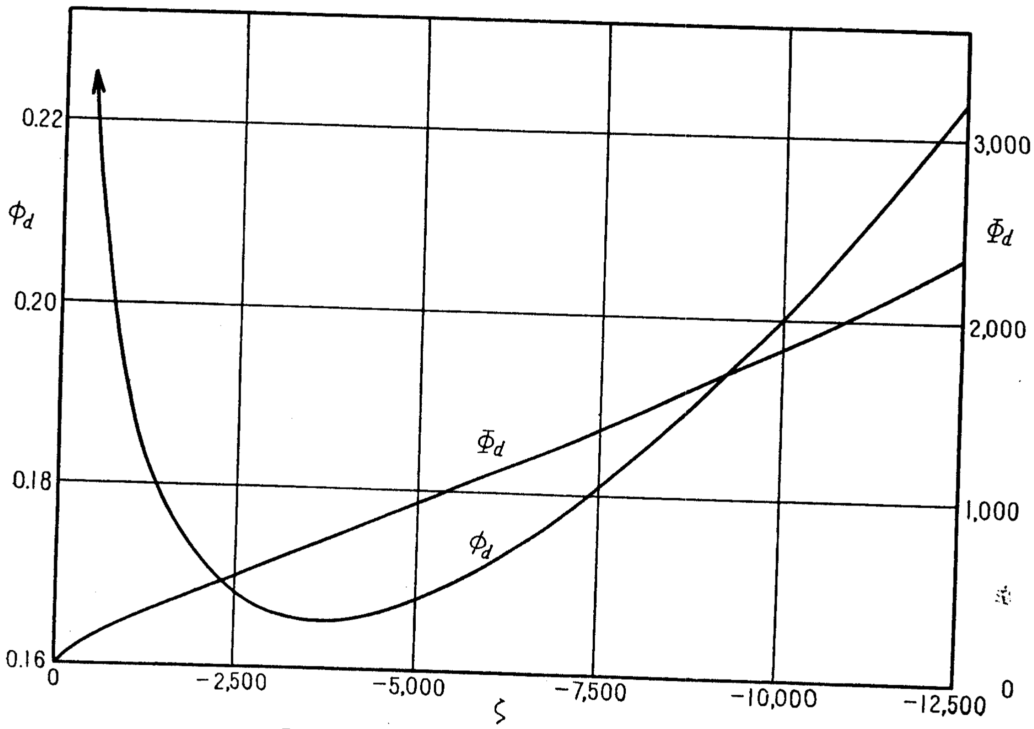

FIGURE 5-1. Functions $\phi_d(\zeta)$ and $\Phi_d(\zeta)$.

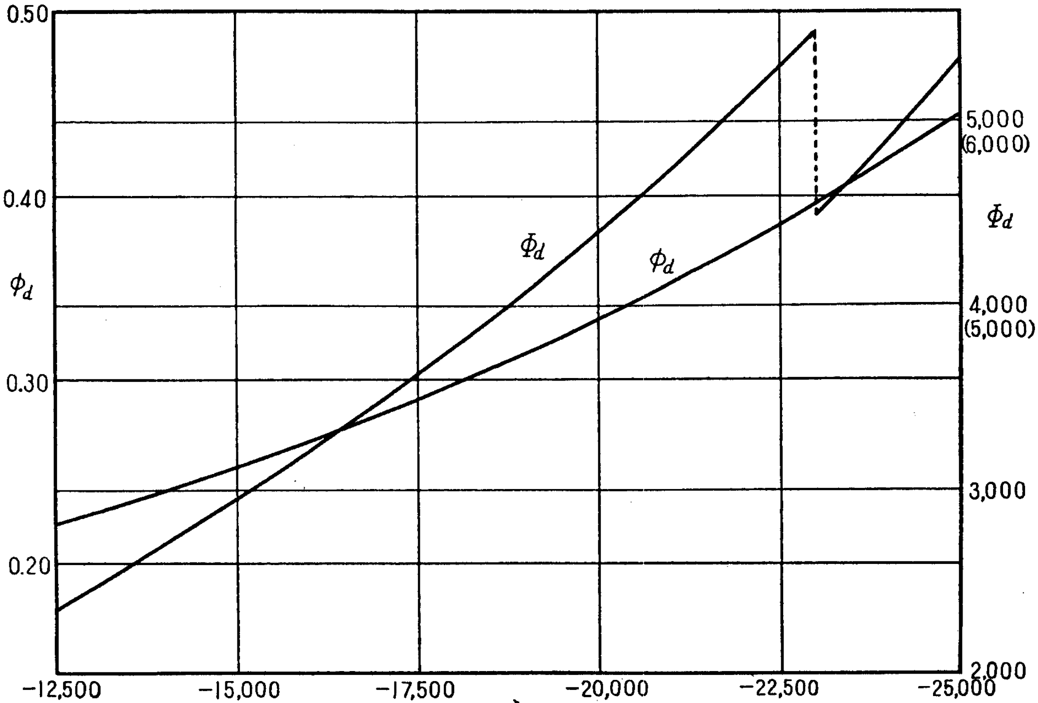

FIGURE 5-2. Functions $\phi_d(\zeta)$ and $\Phi_d(\zeta)$.

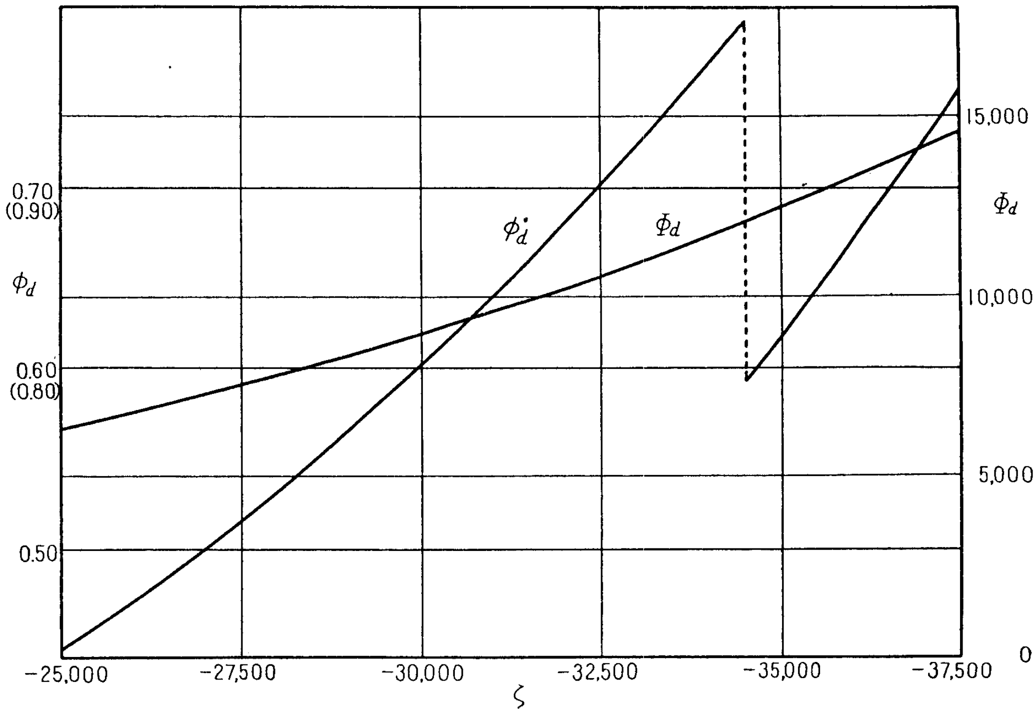

FIGURE 5-3. Functions $\phi_d(\zeta)$ and $\Phi_d(\zeta)$.

forming numerical integrations in Eqs. (34) and (37) by the aid of these functions.

It should be noted, however, that the function $\phi(\zeta)$ becomes infinite as ζ approches to zero. This condition appears in the case of launching from the level at an initial velocity of zero. But the heat transfer to the skin during the short time after launching by such a condition is negligible because of the low velocity

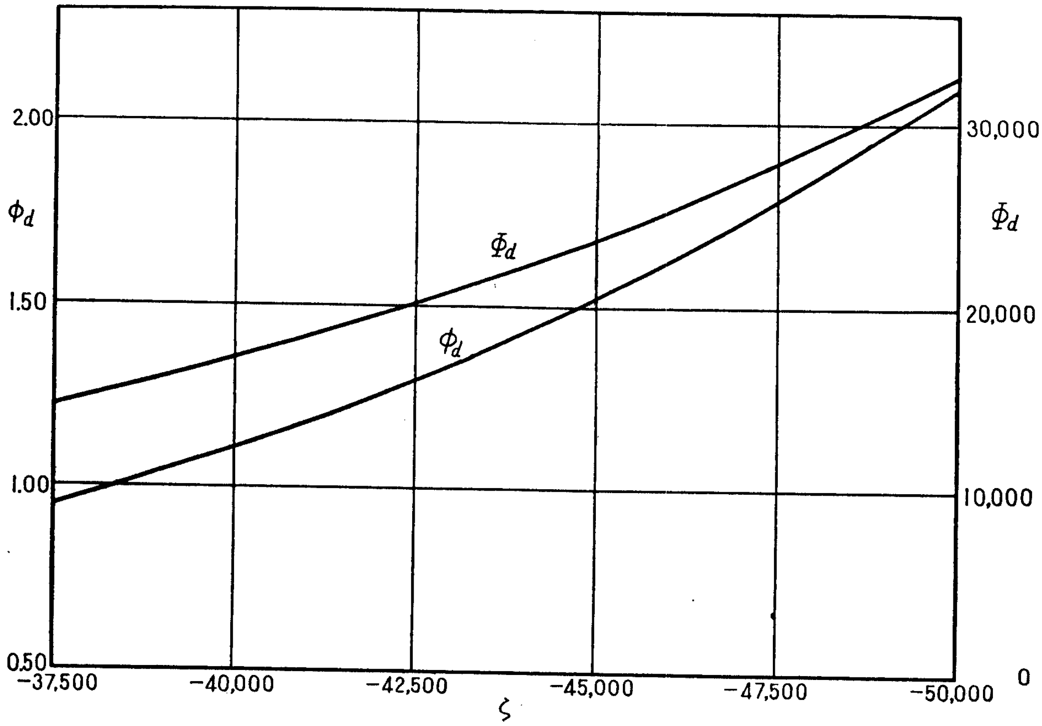

FIGURE 5-4. Functions $\phi_d(\zeta)$ and $\Phi_d(\zeta)$.

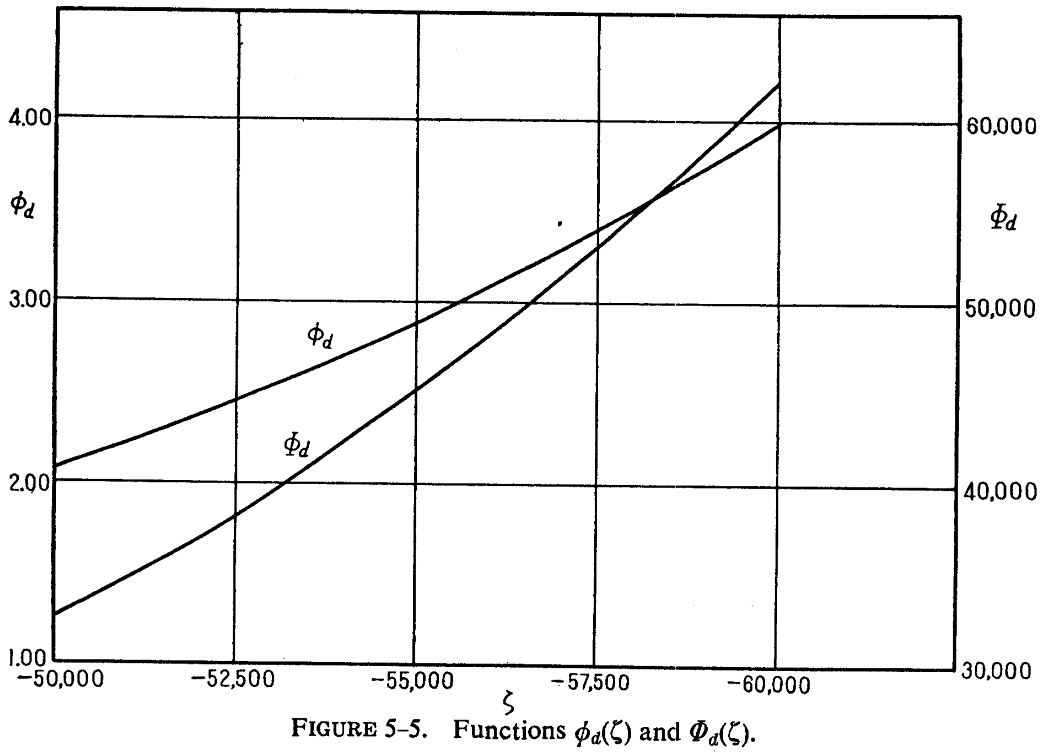

FIGURE 5-5. Functions $\phi_d(\zeta)$ and $\Phi_d(\zeta)$.

of the vehicle. Then the method taking the lower limit of the integration in Eq. (34) as such finite values as 250 m or 500 m, is to be advisable.

In calculating the temperature by Eqs. (34) and (37), the numerical calculation of the integrals are necessay and may be still a tedious work. Fortunately those functions which must be integrated can be approximated by the second order polynominals with a good accuracy. The Sympson's rule for integration, therefore, can be used in the calculation and in practical cases only a few number of division are usually required for obtaining the result within 10 degree error. Using the Sympson's rule, simple formulae for calculating the temperature are presented in the following :

$(\dot{u}_\infty > 0)$

$$T_s(\zeta) = \exp[-\kappa\{\Phi(\zeta) - \Phi(\zeta_i)\}]$$
$$\times \Big[\frac{1}{6}(\zeta - \zeta_i)\kappa T_\infty[\phi(\zeta_i)(1+\varepsilon\zeta_i)$$
$$+ 4\phi(\zeta_m)(1+\varepsilon\zeta_m)\exp[\kappa\{\Phi(\zeta_m) - \Phi(\zeta_i)\}]$$
$$+ \phi(\zeta)(1+\varepsilon\zeta)\exp[\kappa\{\Phi(\zeta) - \Phi(\zeta_i)\}]] + T_{si}\Big], \tag{38}$$

$$T_s(\zeta_m) = \exp[-\kappa\{(\Phi(\zeta_m) - \Phi(\zeta_i)\}]$$
$$\times \Big[\frac{1}{6}(\zeta - \zeta_i)\kappa T_\infty[1.25\phi(\zeta_i)(1+\varepsilon\zeta_i)$$
$$+ 2\phi(\zeta_m)(1+\varepsilon\zeta_m)\exp[\kappa\{\Phi(\zeta_m) - \Phi(\zeta_i)\}]$$
$$- 0.25\phi(\zeta)(1+\varepsilon\zeta)\exp[\kappa\{(\Phi(\zeta) - \Phi(\zeta_i)\}]] + T_{si}\Big], \tag{39}$$

where ζ_m is defined by

$$\zeta_m = \frac{\zeta_i + \zeta}{2}, \tag{40}$$

$(\dot{u}_\infty < 0)$

In this case the functions ϕ_d and Φ_d should be used in place of ϕ and Φ, respectively, in Eqs. (38) and (39).

For an actual example, in the case of $z = 20{,}600$ m, $z_i = 13{,}000$ m, $\dot{u}_\infty = 168.5$ m/s^2, $u_{\infty_i} = 475$ m/s, and $\tau = 7.51$ s, the result obtained by dividing $z - z_i$ into 4 sections is 680 °K while the one for 2 sections is 685 °K. Then the error due to the use of Eqs. (38) and (39) is small and the accuracy proves to be considerably good. And the easiness of calculation allows us to estimate the temperature of the skin of the multi-step rocket only several times of manipulation.

Flight Investigation of the Skin Temperature of a Two-Stage Solide-Propellant Rocket

Kappa III two-stage solid-propellant rocket vehicle was flight-tested to evaluate its use as meteological sounding rocket. In order to evaluate the thermal property of the wing and the body, the temperature measuring instrument was carried by the vehicle and the temperature of the skin of the nose cone 300 mm after the nose was measured and transmitted by a telemeter.

The programmed velocity diagram of this vehicle was already shown in Fig. 3. It was reported that the real velocity was deviated from the programmed one to a certain extent. The nose cone is built up of 0.8 mm thick stainless steel and the platinum temperature gage was fitted to the inner surface of the skin.

The temperature history of the skin was calculated on the basis of the approximate treatment of heat transfer coefficient presented in the foregoing section of the paper by using step-by-step procedure, i. e.,

$$\Delta T_{s_{n+1}} = \frac{h(t_{n+1})}{c_s \gamma_s \delta_s} \{T_r(t_{n+1}) - T_{s_n}\} \Delta t_{n+1}, \tag{41}$$

$$T_{s_{n+1}} = T_{s_n} + \Delta T_{s_{n+1}}, \tag{42}$$

where $\Delta T_{s_{n+1}}$ is the increment of the temperature of the skin during a short time Δt_{n+1}, $T_r(t_{n+1})$ is the recovery temperature at the time t_{n+1}, and T_{s_n} is the temperature of the skin at the time t_n and so on. The programmed velocity diagram was used in the calculation. The measured and the calculated temperatures of the skin of the vehicle are shown in Fig. 6.

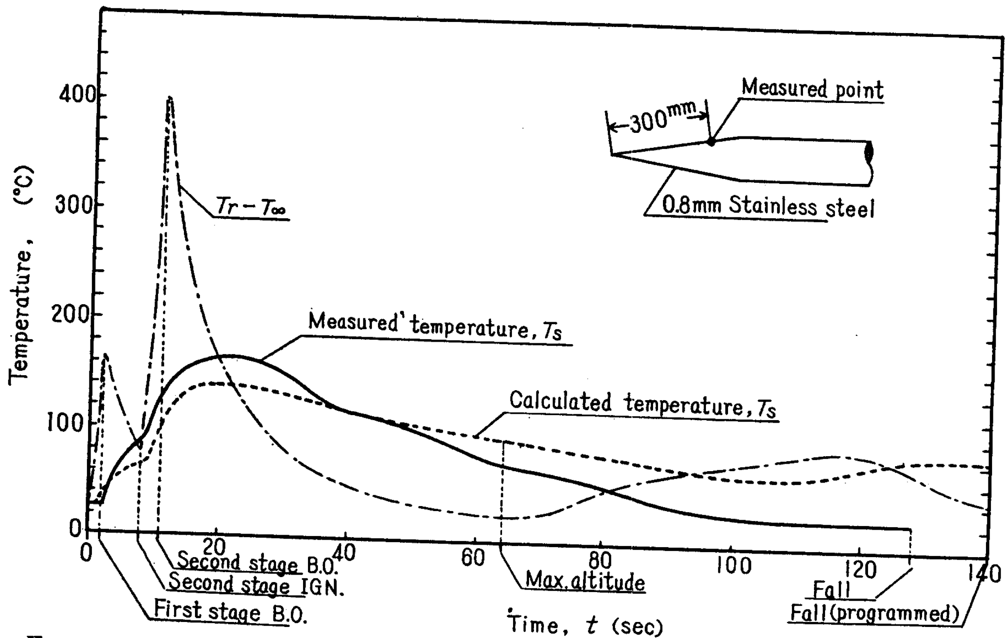

FIGURE 6. Measured and calculated skin temperature of Kappa-III rocket vehicle.

The overall tendencies of the two curves seemed to be similar. However, the quantitive comparison of the two proves that neither the former nor the latter is satisfactory. As very few flight test data on the skin temperature of supersonic vehicles are available now and, furthermore, the accuracy of instrument is not perfectly assured, accumulation of more data is desired.

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March 31, 1960.

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