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Critical Survey of Published Theories on the Mechanism of Leading-Edge Stall

By

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Summary. Critical survey is made of the published theories on the mechanism of breakdown of laminar separation bubble, which causes the leading-edge stall of moderately thin airfoil sections. The bubble is formed by the separation of laminar boundary layer with subsequent reattachment, which is considered to be effectuated by the entrainment process of turbulence produced in the separated flow. Analysis of available experimental data suggests that the bubble formation is possible provided that the boundary-layer Reynolds number at separation is greater than a certain critical value (Tani, Owen and Klanfer criterion), and that the pressure recovered in the reattachment process is less than a certain critical value (Crabtree criterion). The circumstance just prior to leading-edge stall is such that the first criterion is fulfilled while the second criterion is about to be violated. The first criterion is for the assurance of transition to turbulence in the separated flow, while the second criterion is connected with the existence of a maximum possible value of shear stress which is set up in the turbulent entrainment region to counteract the pressure difference. Based on this interpretation, a fairly consistent picture is gained for the breakdown of laminar separation bubble, and hence the mechanism of leading-edge stall of airfoil sections.

INTRODUCTION

It is well known that the stalling characteristics of airfoil sections at low subsonic speeds are classified into three basic types. According to McCullough and Gault [*14*], the three types are: *trailing-edge stall,* preceded by the movement of the separation point of turbulent boundary layer forward from the trailing edge with increasing angle of attack; *leading-edge stall,* caused by an abrupt separation of the laminar boundary layer near the leading edge without subsequent reattachment; and *thin-airfoil stall,* preceded by the laminar separation near the leading edge with turbulent reattachment at a point which moves progressively rearward with increasing angle of attack. Typical examples of these types of stall are provided by the experimental data for the NACA 63_3–018, 63_1–012 and 64A006 airfoil sections, respectively, obtained at a Reynolds number of $5{\cdot}8\times10^6$ [*14*]. As reproduced in Figure 1, the lift variation with angle of attack is gradual and continuous for the NACA 63_3–018 section (trailing-edge stall), while it shows an abrupt discontnuity at the stall for the NACA 63_1–012 section (leading-edge stall). The lift curve of the NACA 64A006 section (thin-airfoil stall) is characterized by a rounded peak, preceded by a slight discontinuity at an angle of attack of 5°.

Detailed observations on NACA 63_1–012 section show that separation of the laminar boundary layer takes place near the leading edge prior to the stall, and that the separated flow reattaches to the surface within a short distance as a turbulent boundary layer. The region underlying the separated flow, limited between the points of separation and reattachment, and set into circulating motion, is commonly referred to as a *bubble*, or a *short bubble* for reasons which will appear hereafter. The length or the streamwise extent of the bubble is of the order of one per cent of the airfoil chord, so that its presence has no significant effect on the pressure distribution. With increasing angle of attack the bubble contracts in length until it suddenly *breaks down* or *bursts*, thereby causing a flow separation without subsequent reattachment. This gives rise to an abrupt loss of lift at the stall.

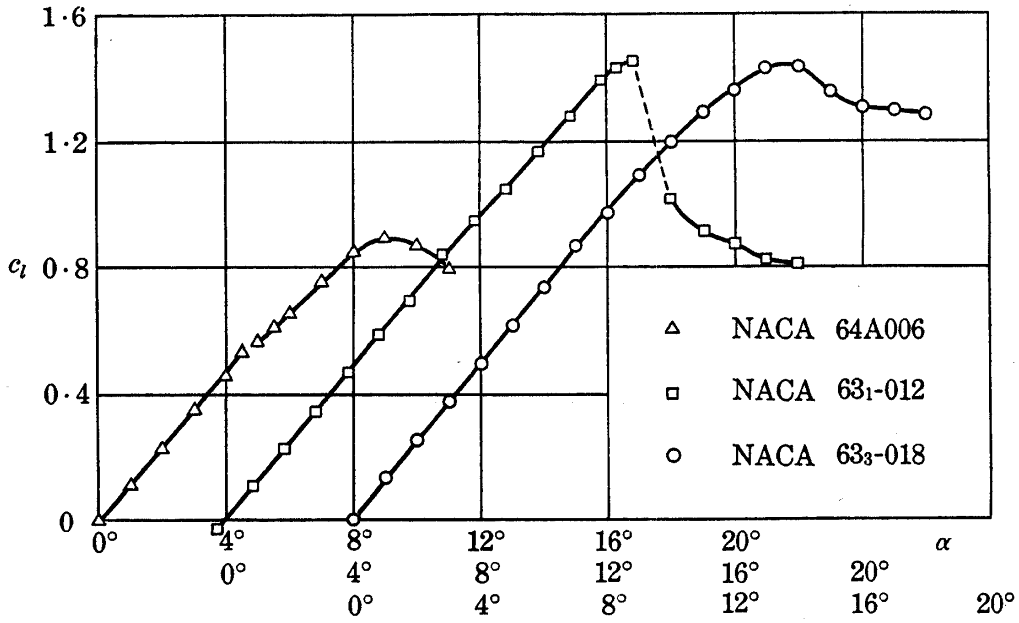

FIGURE 1. Variation of section lift coefficient with angle of attack for NACA 64A006, 63_1–012 and 63_3–018 airfoil sections.

However, the formation of a bubble is not necessarily restricted to the airfoil sections exhibiting leading-edge stall. Detailed observations reveal the presence of a short bubble on the NACA 64A006 section for angles of attack up to 4·5°. Beyond this angle, the bubble suddenly breaks down, causing the above-mentioned slight discontinuity in lift curve. However, the breakdown does not lead to a complete separation of flow; instead, the separated flow passes above the airfoil surface and reattaches farther downstream. With increase in angle of attack the reattachment point moves progressively rearward until it reaches the trailing edge, at which stage the maximum lift is attained.

The region underlying the separated flow is referred to as a *long bubble*. It has a length of several per cent of the airfoil chord upon formation at low angles of attack, and grows rapidly with increasing angle of attack until it extends over the entire chord. The distinction between long and short bubbles is made not only by their different lengths but also by their different effects on the pressure distribution. The presence of a long bubble makes the pressure distribution radically different from that in inviscid flow, the peak suction near the leading edge being destroyed. With increasing angle of attack the suction over the leading edge is reduced. On the contrary, the presence of a short bubble maintains the peak suction, which continues to increase as the angle of attack is increased up to the stall.

The characteristics of the NACA 64A006 section at a Reynolds number $5 \cdot 8 \times 10^6$ provides an example in which the breakdown of a short bubble at an angle of attack of 5° gives rise to the formation of a long bubble. An increase in either the airfoil thickness ratio or the Reynolds number increases the critical angle of attack until, for the thicker sections at least, maximum lift is attained before the flow change occurs [*19*]. This remark affords a unified interpretation of leading-edge and thin-airfoil stalls.

The formation of a short bubble is also detected at moderate angles of attack on thick airfoils such as the NACA 63_3–018 section which exhibits trailing-edge stall. With increasing angle of attack the bubble contracts but never breaks down before the attainment of maximum lift, which is caused by the forward movement of the separation point of turbulent boundary layer. Here the role of the bubble is to make the downstream boundary layer turbulent and affect the stalling characteristics through the initial thickness of the turbulent boundary layer [*9*].

Summarizing, a fairly satisfactory explanation of the stalling characteristics is now available. However, there still remain obscure points concerning the bubble flows, in particular the breakdown of a short bubble, an important factor not only in directly determining the leading-edge stall but also in indirectly affecting the other types of stall. This paper attempts to make a critical examination of various theories which have been postulated as the mechanism of bubble breakdown with a view to obtaining a consistent picture of bubble flows. It is pertinent, therefore, to outline these theories in some detail before presenting the critical discussion.

Theories of Bubble Breakdown

It seems expedient first to consider briefly the conditions under which the formation of a bubble† is possible. An obvious condition is the existence of an adverse pressure gradient sufficiently steep to cause boundary layer separation. But this does not necessarily mean that a bubble will actually be formed. If the Reynolds number is sufficiently high, transition from laminar to turbulent flow will take place ahead of the laminar separation point, namely that point at which separation would have occurred if the boundary layer had remained laminar. Under these circumstances, the bubble formation is precluded. On the other hand, if the Reynolds number is sufficiently low, the separated flow will not reattach to the

† For the remainder of the paper the simple term "bubble" is used by omitting the adjective "short".

surface and no bubble will be formed. Therefore, the bubble formation appears to be possible only for a certain range of Reynolds numbers, which undoubtedly will depend on the pressure distribution, the surface curvature, the surface irregularities, and the turbulence of the free stream.

The first postulate concerning the bubble formation was probably that put forward by von Doenhoff [6] on the ground of his measurements on a flat plate in a stream with an adverse pressure gradient. Von Doenhoff speculated that the separated flow proceeds in the direction along the tangent to the surface at the separation point, and that the transition to turbulence takes place in the separated flow so that the subsequent turbulent entrainment causes the flow to return to the surface. He assumed that the length of separated laminar flow can be defined by

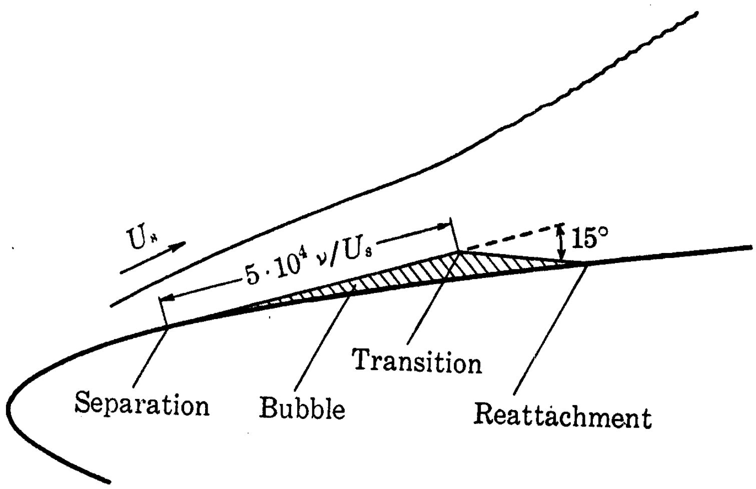

FIGURE 2. Von Doenhoff's concept of bubble formation.

a constant value of the Reynolds number based on the velocity U_s outside of the boundary layer at separation and the distance between the points of separation and transition. He suggested a value of 5×10^4 for this Reynolds number and a value of 15° for the deflection angle of the inner boundary of the separated flow. The bubble is then represented by a flat triangular region sketched in Figure 2, and the condition under which flow reattachment will not take place is determined graphically. This simple concept has proved most helpful to explain qualitatively many of the effects of Reynolds number on the stalling characteristics of airfoil sections [13] [14]. However, it is now generally regarded as not accurate enough to furnish more quantitative informations.

Experimental evidence of the failure of flow reattachment at sufficiently low Reynolds numbers led the present author [7] to seek for a criterion for the occurrence of reattachment. He postulated from dimensional consideration that the boundary-layer Reynolds number at separation must exceed a certain critical value for the separated flow to reattach to the surface. He suggested a value of 210 for the Reynolds number based on the outside velocity U_s and the boundary-layer momentum thickness θ_s at the separation point on the basis of the analysis of a few available experimental data on circular and elliptic cylinders as well as spheres. If the displacement thickness at separation δ_s^* is used instead of θ_s and the ratio $\delta_s^*/\theta_s=3{\cdot}7$ is assumed, the critical value of the Reynolds number based on U_s and δ_s^* becomes 780. As will be discussed later, this critical value is now considered too high.

Maekawa and Atsumi [*11*] carried out measurements on the region of separated flow downstream of the intersection of two plane surfaces and observed that the reverse flow forms a circulating motion which attracts the separated flow toward the surface. Flow reattachment will not occur unless sufficient energy is supplied to maintain the circulating motion against dissipation. On the basis of this reasoning Maekawa and Atsumi suggested that the Reynolds number based on the total extent of separated flow must be less than a certain critical value. They also suggested that the boundary-layer Reynolds number at separation must be greater than a certain critical value for the separated flow to become unstable so that the circulating motion is energized. The latter condition is the same as the criterion proposed by Tani [*7*]. However, a considerable doubt has been raised as to whether the flow separation forced at the intersection of two plane surfaces is exactly equivalent to that which produces a short bubble on the airfoil surface.

Later, Owen and Klanfer [*15*] put forward independently a similar criterion in terms of the boundary-layer Reynolds number at separation. From a large number of available experimental data on bubbles, they calculated the boundary-layer Reynolds number at the separation point, $R_{\delta_s^*}=U_s\delta_s^*/\nu$, and concluded that the bubble is short or long according as $R_{\delta_s^*}$ is greater or less than a critical value

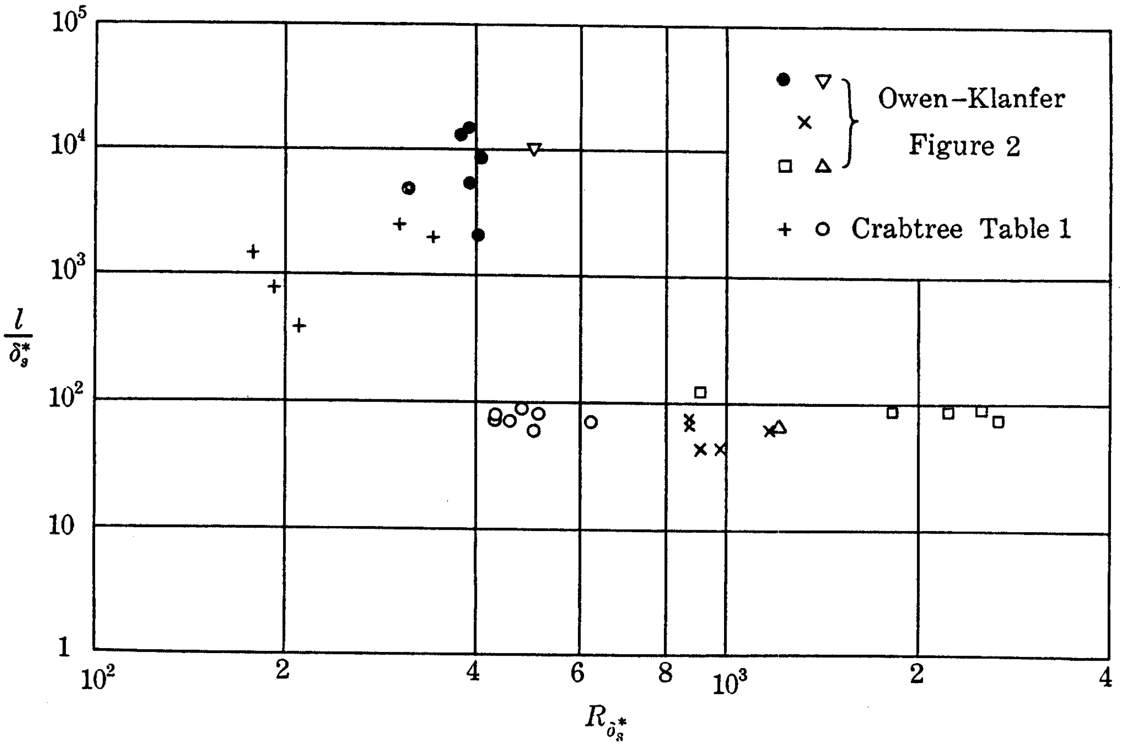

FIGURE 3. Variation of bubble length with boundary-layer Reynolds number at separation. Analyses by Owen and Klanfer, and Crabtree.

in the neighborhood of 400 to 500. This is illustrated with striking clearness in Figure 3†, in which the bubble length l in terms of δ_s^* is plotted against the Reynolds number $R_{\delta_s^*}$.

The criterion successfully distinguishes between short and long bubbles of separated flow, but as pointed out by Gault [*18*], there appears to be no universal critical value of $R_{\delta_s^*}$ for the breakdown of a short bubble. As soon as a short bubble has broken down the value of $R_{\delta_s^*}$ falls to below about 500, although immediately prior to breakdown $R_{\delta_s^*}$ may be appreciably greater than 500. This is not altogether surprising since the breakdown brings about a radical change in pressure distribution. From this argument it is clear that the criterion based on $R_{\delta_s^*}$ is not the criterion for the breakdown of a short bubble. It is meant only for the statement that $R_{\delta_s^*}$ is always greater than about 500 whenever a short bubble is present. In this sense it is essentially the same as the criterion advanced by Tani [*7*], although Tani's suggested value for $R_{\delta_s^*}$ appears to be too high.

In seeking for a criterion for bubble breakdown, Crabtree [*22*] made the plausible suggestion that the breakdown occurs because of the existence of a maximum possible value of pressure that can be recovered in the turbulent entrainment process causing flow reattachment. He indicated by analysis of the available experimental results that the value of the pressure recovery coefficient, $\sigma=\Delta p/\frac{1}{2}\rho U_s^2$, gradually increases either with the increase in angle of attack at constant Reynolds number (Figure 4), or with the decrease in Reynolds number at constant angle of attack (Figure 5), where Δp is the pressure rise between the points of

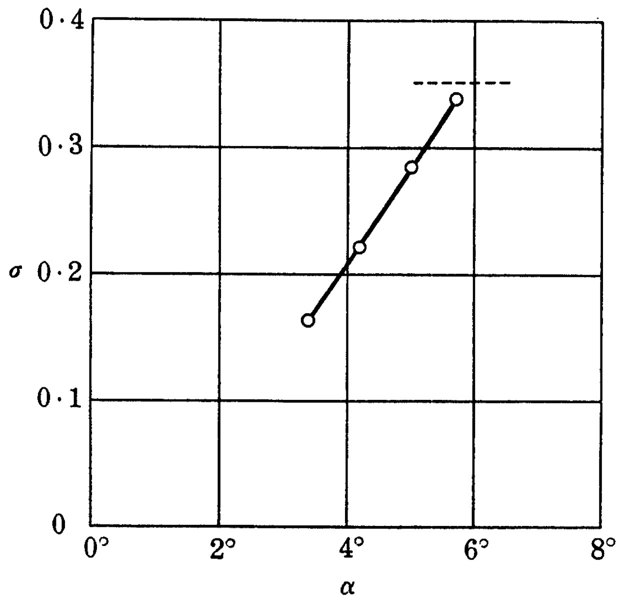

FIGURE 4. Variation of pressure recovery coefficient with angle of attack at a Reynolds number of $1{\cdot}7\times10^6$. Crabtree's analysis of McGregor's experimental data on a Piercy airfoil.

† The points in this figure are those replotted from Figure 2 of Owen and Klanfer's paper [*15*] by omitting the data obtained from measurements in supersonic flows as well as those produced by Crabtree's experiments [*17*] which were carried out to reconfirm the criterion.

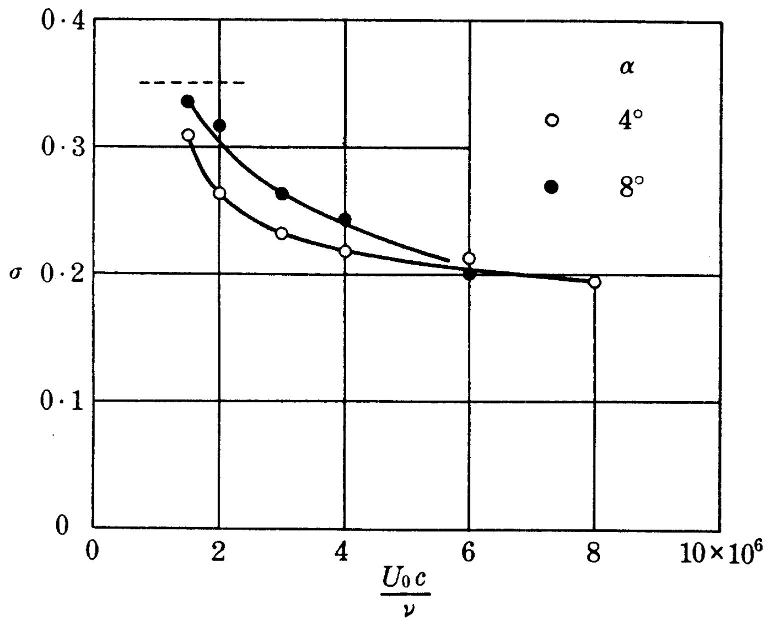

FIGURE 5. Variation of pressure recovery coefficient with Reynolds number at constant angle of attack. Crabtree's analysis of Gault's experimental data on a modified NACA 0010 airfoil.

separation and reattachment and U_s is the velocity outside of the boundary layer at separation. When σ attains a value of about 0·35, the bubble suddenly breaks down to give rise to a flow pattern associated with a long bubble†. It can therefore be concluded that a short bubble exists only when $R_{\delta_s^*}$ is greater than about 500 and σ is less than about 0·35.

In further corroboration of the existence of a maximum pressure recovery, Crabtree mentioned the flow through a pipe having a sudden enlargement, for which the maximum theoretically possible value of σ is 0·5 for an area ratio of 2. Crabtree [*23*] also made an analysis for the simplified model of a type of flow with a bubble as depicted in Figure 6. He assumed that the turbulent entrainment process and the corresponding pressure recovery occur only in the re-

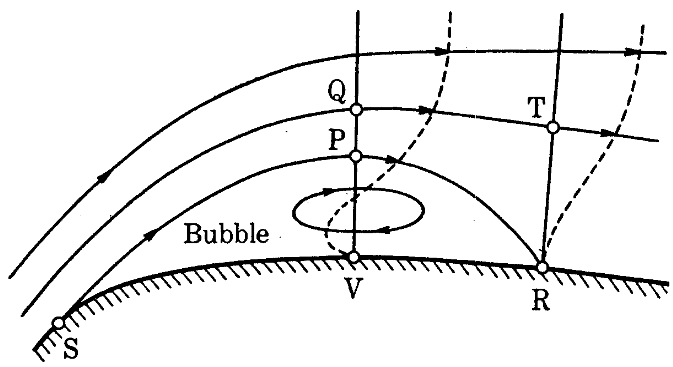

FIGURE 6. Crabtree's simplified model of flow pattern with a bubble.

† Further experimental confirmation was presented by Moore [*26*], who suggested a value of 0·36 for σ_{max}.

gion PQTR, that the fluid passing through PQ is diffused to form a boundary layer of thickness RT in such a way that the velocity along the streamline QT falls from U_1 to U_2 without loss of energy, that the velocity profile across PQ is given by $u=U_1[a+(1-a)(y/\delta_1)]$, where $\delta_1=\overline{PQ}$, while that across RT by $u=U_2(y/\delta_2)^n$, where $\delta_2=\overline{RT}$, that no fluid enters or leaves the region across VP on the average, and that the curvature of airfoil surface SVR is neglected and QT is parallel to VR. Application of equation of continuity and momentum theorem to the region VQTR yields an expression for the pressure recovery coefficient σ, which in this simplified model is given by $(p_2-p_1)/\frac{1}{2}\rho U_1^2$, where p_1 and p_2 are the pressure in the sections VPQ and RT, respectively, the variation of pressure in the direction normal to the surface being neglected. In the limiting case $n=1$, Crabtree obtained the value of σ varying from 0·37 to 0·45 as the constant a is varied from 1·0 to 0·5†.

On the other hand, Wallis [*16*] [*27*] viewed the problem from a different standpoint. Wallis and his coworkers carried out experimental observations on airfoil models provided either with air jets or with roughness strips with a view to simulating high Reynolds number flows, and found that the reattached turbulent boundary layer exhibits just downstream of the reattachment point a peak in the development of the shape parameter H, the ratio of displacement and momentum thicknesses. As the angle of attack is increased, the peak value of H approaches the value associated with turbulent separation. With the model smooth and air jet off, the variation of H was found almost constant. Wallis then made a suggestion that the separation, or more correctly, reseparation of the reattached turbulent boundary layer provides the mechanism of flow breakdown at least for moderate to high Reynolds numbers. On the other hand, however, Wallis postulated that the flow pattern involving a short bubble, a short extent of attached turbulent boundary layer and subsequent complete separation, is not stable, so that the flow leaves at the laminar separation point to form a long bubble.

Further evidence of turbulent reseparation was presented by Moore [*26*] from his measurements on an airfoil model without boundary-layer disturbing devices but equipped with an auxiliary airfoil. On the other hand, Evans and Mort [*25*] put forward an analysis from which it is inferred that there are two distinct mechanisms of sudden stall, the one presumed to be due to the breakdown of bubble and the other due to the reseparation of turbulent boundary layer. The data analysed are not the experimental results in which the pressure distribution was measured, but the standard force test data on two-dimensional airfoil models in the NACA low-turbulence pressure tunnel at a Reynolds number of 6×10^6. Only those data were used for which the stall is sudden, as selected by the requirement that the two points of lift curve defining stall should indicate a slope $\Delta c_l/\Delta\alpha$ of at least 0·1 per degree, with Δc_l itself at least 0·1. From the theoretical velocity distribution for inviscid flow just prior to stall, the boundary-layer calculation was made to obtain the value of the Reynolds number based on momentum thickness at separation, $R_{\theta_s}=U_s\theta_s/\nu$, the separation being assumed to occur where the

† See the first footnote on p. 186.

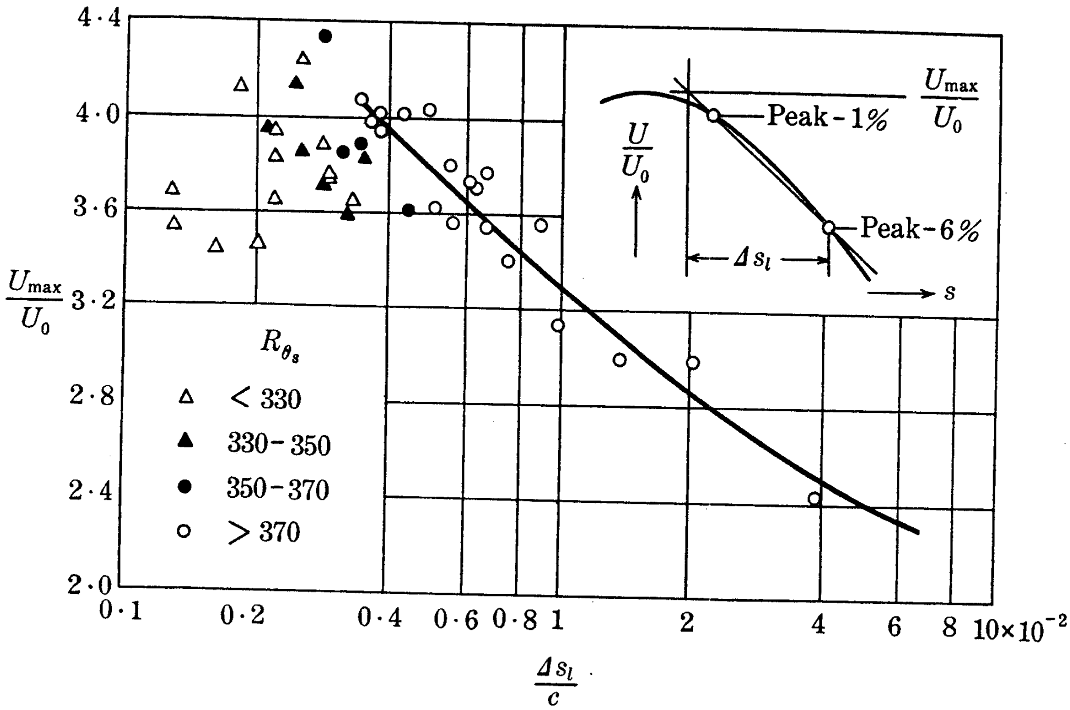

FIGURE 7. Correlation of peak velocity at sudden stall with adverse distance. Analysis by Evans and Mort.

velocity has fallen 6 per cent from its peak value U_{max}. For each velocity distribution, the initial adverse gradient was taken to be defined by the surface distance to the separation point, Δs_l, as illustrated in the inset of Figure 7. By plotting the peak velocity ratio U_{max}/U_0 against the adverse distance ratio $\Delta s_l/c$ as reproduced in Figure 7, where U_0 is the undisturbed velocity and c is the airfoil chord, Evans and Mort observed that a good correlation exists for those points for which R_{θ_s} is greater than about 350. A correlation between high peak velocity and short adverse distance is that to be expected from the assumption of stall ascribed to reseaparation mechanism, but not that ascribed to bubble breakdown mechanism. This argument led Evans and Mort to adopt the view that there is a critical value of R_{θ_s} above which sudden stalls are due to reseparation and below which they are due to bubble breakdown.

DISCUSSION

The formation of bubble is based on two phenomena: the laminar boundary layer must separate from the surface and the separated flow must subsequently reattach to the surface. As regards the second of these phenomena, von Doenhoff's conjecture [*6*] that the onset of turbulence in the separated flow effectuates the reattachment appears extremely plausible. Indication of its validity is obtained from the results of subsequent measurements that the fully developed turbulence is first observed in the separated flow fairly closely to that position where the flow begins to return to the surface and the corresponding pressure recovery occurs [*13*] [*18*]. Nevertheless, the schematic representation of a bubble shown in Figure 2 appears to oversimplify the process of reattachment. The experimental results of Gault [*18*] corroborate neither the invariance of the Reynolds number for the length of separated laminar flow, nor the existence of a unique value for the deflection angle. Moreover, it seems too geometrical to the present author to judge the failure of reattachment by the non-attainment of turbulent entrainment to the surface. A more satisfactory formulation may probably be obtained by requiring the turbulent stress set up in the entrainment region to be in equilibrium with the pressure acting on the bubble. This will be more fully discussed later.

The boundary-layer Reynolds number at the laminar separation point is always greater than a certain critical value whenever a short bubble is present. The significance of the Reynolds number in this criterion is now obvious†, since at high Reynolds numbers instability develops in the separated flow, eventually leading to turbulence, which is an essential prerequisite to flow reattachment toward the surface. The critical value first suggested by the present author [*7*] for the Reynolds number based on displacement thickness $R_{\delta_s^*}$ is 780, which is

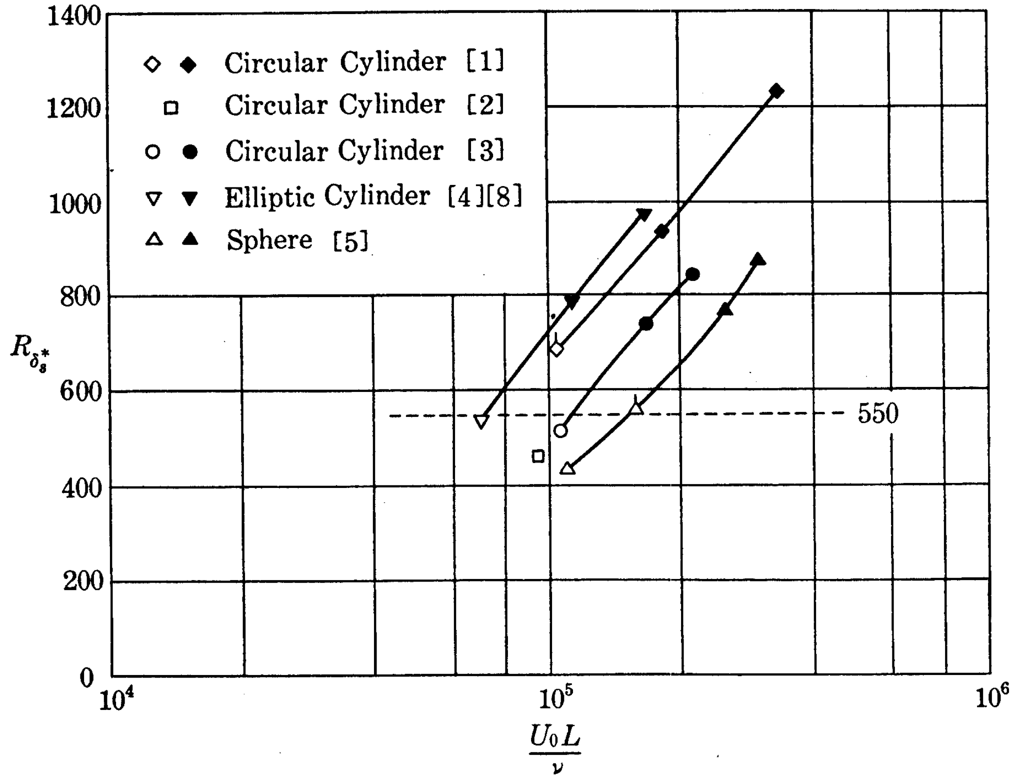

FIGURE 8. Variation of boundary-layer Reynolds number at separation with overall Reynolds number. Author's analysis.

† As a possible explanation of the significance of the Reynolds number Crabtree [*22*] mentioned Schubaur and Klebanoff's experimental evidence (NACA Tech. Note No. 3489, 1955) that turbulent spots grow rapidly in the boundary layer on a flat plate when the Reynolds number based on displacement thickness is greater than about 450. However, the situation may be different for a separated layer away from the surface, and the coincidence of values of Reynolds numbers seems to be only fortuitous.

considerably higher than that proposed by Owen and Klanfer [*15*] on the basis of the analysis of a large number of experimental results. Reexamination of the original analysis has therefore been made to explain the discrepancy between the two suggested values.

Because of the scarcity of available data at that time, the author had to estimate the critical boundary-layer Reynolds number virtually from the pressure distribution measurements on a circular cylinder [*1*] [*2*] [*3*], an elliptic cylinder [*4*][*8*], and a sphere [5] for Reynolds numbers within the critical range over which the drag coefficient experiences a large fall. Figure 8 shows the calculated value of $R_{\delta_s^*}$ as a function of the overall Reynolds number based on the undisturbed stream velocity U_0 and the streamwise length of the body L. This is slightly different from the original presentation in Figure 1 of Reference [*7*], in that δ_s^* has been used instead of θ_s by assuming $\delta_s^* = 3{\cdot}7\theta_s$, and that the values have been recalculated by the same method as used by Owen and Klanfer. Open symbols denote the points for which the separated flow completely leaves the surface, while solid symbols those for which the separated flow reattaches to the surface to form a turbulent boundary layer. The flagged symbols denote the points which were originally considered to represent a complete separation, but are suspected of the presence of a faint kink in pressure distribution just downstream of the laminar separation point. If these points are taken as representing flow reattachment, a line demarcating the points can be drawn to give rise to a critical value of about 550 for $R_{\delta_s^*}$. This is in good agreement with the value suggested by Owen and Klanfer.

Wallis [*16*][*27*] postulates that the flow breakdown at moderate to high Reynolds numbers commences with the reseparation of turbulent boundary layer just downstream of the bubble reattachment point. However, there are some experimental results that do not support this postulate. For example, McCullough and Gault [*14*] state definitely that no region of incipient turbulent separation is revealed just downstream of a short bubble on the airfoil section NACA 63–009. Thus it appears difficult to deduce a generalization from the result observed by Wallis on one particular airfoil.

In the result discussed by Wallis, the reattached turbulent boundary layer exhibits a peak in the development of the shape parameter H, whose peak value increasing with increasing angle of attack. Upon closer examination it is found that the increase in peak value of H is mostly accounted for by the increase in initial value of H, namely, the value at the reattachment point. The latter value increases as the reattachment becomes more incomplete. This reasoning leads one to conclude that the flow breakdown caused by reseparation just downstream of reattachment amounts to the same thing as the bubble breakdown itself.

In Crabtree's analysis, the momentum theorem for the region VQTR is formulated by additionally assuming that no momentum enters or leaves the region across VP on the average and that the shear stress acting on the surface VR is negligible, although these assumptions are not explicitly stated in Reference [*23*]. Otherwise, the momentum theorem when applied to the region PQTR necessitates

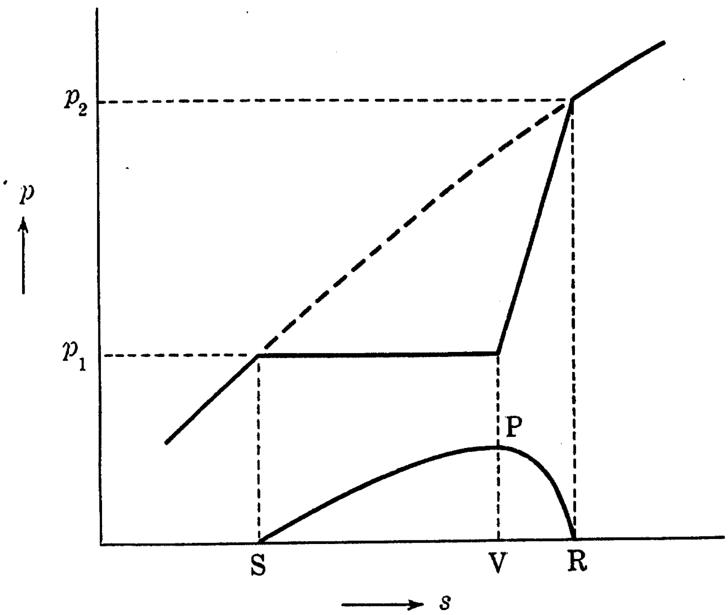

FIGURE 9. Simplified pressure distribution on the bubble.

the knowledge of distributions of pressure and shear stress along the bubble boundary PR†. It is rather remarkable that the analysis can be developed with a minimum of experimental evidence.

Nevertheless, the existence of a maximum pressure recovery might preferably be corroborated by realizing that the bubble is maintained by the turbulent shear stress set up in the entrainment region. For simplicity it is assumed that the pressure is constant and equal to p_1 from the separation point S to the bubble top P and then rises linearly with distance to attain a value p_2 at the reattachment point R (see Figure 9), the variation of pressure across the bubble being neglected‡. Then the resultant of the pressure acting on the boundary SPR is $\frac{1}{2}h(p_2-p_1)$, directed to the left, where $h=\overline{\mathrm{VP}}$. Transition is assumed to take place in the neighborhood of P, so that the resultant of shear stress acting on the boundary PR amounts to $l'\tau$, directed to the right, where $l'=\overline{\mathrm{VR}}$ and τ denotes the shear stress averaged over the boundary PR. Equilibrium of forces requires

$$-\frac{1}{2}h(p_2-p_1)+l'\tau=0\,,$$

whence the pressure recovery coefficient is

$$\sigma=\frac{2l'}{h}\frac{\tau}{\frac{1}{2}\rho U_1^2}\,.$$

Examination of experimental results presented in References [*9*][*13*][*14*][*18*] and [*22*] gives a value ranging from 15 to 40 for $2l'/h$. There exists no ex-

† Only a brief outline of analysis is given in Reference [*23*], the full account having been presented in the RAE Tech. Note Aero. 2352, 1955, which was not available to the author. Based on the assumptions stated above, however, the author could not reproduce the result obtained by Crabtree; in the case $n=1$, σ was found to vary from 0·37 to 0·15 as a was varied from 1·0 to 0·5.

‡ In this simplified model, the velocity U_1 outside of the boundary layer at the point P is equal to the outside velocity at separation, U_s.

perimental data for the shear stress acting on the bubble on an airfoil model, but a useful information may be obtained from the measurements on a half jet [*10*] and a separated flow over a step or a groove [*24*][*28*]. The maximum value of $\tau/\frac{1}{2}\rho U_1^2$ observed in these measurements ranges from 0·01 to 0·02. Multiplication of the two values gives a value ranging from 0·15 to 0·80 for σ, which is of the same order of magnitude as the maximum possible value 0·35. It seems therefore that the existence of a maximum pressure recovery is accounted for by the existence of a maximum possible shear stress in the turbulent entrainment.

Another corroboration† may be provided by making use of the postulate put forward by Korst [*20*] and Chapman, Kuehn and Larson [*21*] in the analysis of base pressure phenomena at high speeds. The postulate amounts to assuming no loss in total pressure along the boundary streamline PR, and brings about the relation

$$p_1+\frac{1}{2}\rho a^2 U_1^2=p_2\,,$$

where aU_1 is the velocity at the point P. The pressure recovery coefficient is then given by

$$\sigma = a^2\,.$$

Determination of the value of a is made by considering the constant-pressure laminar entrainment process along the boundary streamline SP. If the boundary-layer thickness at separation is zero, Chapman's calculation [*12*] applies directly and gives a constant value of 0·587 for the velocity ratio a, that is, the ratio of the velocity on the boundary streamline to that at the outside of the entrainment region. If the boundary-layer thickness at separation is sizable, the velocity ratio a is zero at the outset, and increases with the distance from separation, approaching asymptotically to a constant value of 0·587. The approach becomes more gradual as the boundary-layer thickness is increased. In view of these circumstances, the value of σ obtained by taking $a=0{\cdot}587$, that is, $\sigma=0{\cdot}587^2=0{\cdot}345$, may be considered to be an upper limit. Further, as the angle of attack is increased, the laminar separation point moves forward so that the boundary-layer thickness at separation is reduced. This accounts for the increase in pressure recovery coefficient with angle of attack as observed in Figure 4.

CONCLUSION

The foregoing arguments lead the author to take the following view on the breakdown of a laminar separation bubble on the airfoil surface:

The bubble is formed by the separation of laminar boundary layer with subsequent reattachment, which is considered to be effectuated by the entrainment process of turbulence produced in the separated flow. Analysis of available ex-

† NOTE ADDED IN PROOF: The author's attention was called to a recent Canadian paper, in which a similar attempt was made. See Savage, S.B.: An Approximate Analysis for Reattaching Turbulent Shear Layers in Two-Dimensional Incompressible Flow. Mech. Engng. Res. Lab., McGill Univ., Montreal, Rep. Ae 3, 1960.

perimental data suggests that the bubble formation is possible provided that the boundary-layer Reynolds number at separation is greater than a certain critical value (Tani, Owen and Klanfer criterion), and that the pressure recovered in the reattachment process is less than a certain critical value (Crabtree criterion). The circumstance just prior to leading-edge stall is such that the first criterion is fulfilled while the second criterion is about to be violated. The first criterion is for the assurance of transition to turbulence in the separated flow, while the second criterion is connected with the existence of a maximum possible value of shear stress which is set up in the turbulent entrainment region to counteract the pressure difference. Based on this interpretation, a fairly consistent picture is gained for the breakdown of laminar separation bubble, and hence the mechanism of leading-edge stall of moderately thin airfoil sections.

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SYMBOLS

a : ratio of velocity on boundary streamline to that at outside of boundary layer or entrainment region
c : airfoil chord
c_l : section lift coefficient
H : ratio of displacement and momentum thicknesses of boundary layer
h : height of bubble
L : streamwise representative length of body
l : length of bubble
l' : distance from transition to reattachment
n : exponent in the velocity profile at reattachment
p : pressure
p_1 : pressure at separation
p_2 : pressure at reattachment
Δp : pressure difference between separation and reattachment
$R_{\delta_s^*}=U_s\delta_s^*/\nu$: boundary-layer Reynolds number based on displacement thickness at separation
$R_{\theta_s}=U_s\theta_s/\nu$: boundary-layer Reynolds number based on momentum thickness at separation
s : distance parallel to airfoil surface
Δs_l : adverse distance to separation
U : velocity outside of boundary layer
U_0 : velocity of undisturbed stream
U_{max}: peak value of velocity outside of boundary layer
U_s : velocity outside of boundary layer at separation

U_1 : velocity outside of boundary layer at bubble top
U_2 : velocity outside of boundary layer at reattachment
u : velocity within boundary layer
y : distance normal to airfoil surface
α : angle of attack of airfoil section
δ_1 : distance between bubble top and edge of boundary layer
δ_2 : boundary-layer thickness at reattachment
δ_s^* : boundary-layer displacement thickness at separation
θ_s : boundary-layer momentum thickness at separation
ν : kinematic viscosity of fluid
ρ : density of fluid
σ : pressure recovery coefficient defined by $\Delta p/\frac{1}{2}\rho U_s^2$ or $(p_2-p_1)/\frac{1}{2}\rho U_1^2$
τ : average value of shear stress in turbulent entrainment

REFERENCES

[1] Fage, A.: *The Airflow around a Circular Cylinder in the Region where the Boundary Layer Separates from the Surface.* ARC R. & M. No. 1179, 1928.

[2] Green, J. J.: *Viscous Layer Associated with a Circular Cylinder.* ARC R. & M. No. 1313, 1930.

[3] Fage, A., and Falkner, V. M.: *Further Experiments on the Flow around a Circular Cylinder.* ARC R. & M. No. 1369, 1931.

[4] Schubauer, G. B.: *Air Flow in a Separating Laminar Boundary Layer.* NACA Tech. Rep. No. 527, 1935.

[5] Fage, A.: *Experiments on a Sphere at Critical Reynolds Numbers.* ARC R. & M. No. 1766, 1937.

[6] von Doenhoff, A. E.: *A Preliminary Investigation of Boundary-Layer Transition along a Flat Plate with Adverse Pressure Gradient.* NACA Tech. Note No. 693, 1938.

[7] Tani, I.: *Note on the Interplay between the Laminar Separation and the Transition from Laminar to Turbulent of the Boundary Layer* (in Japanese). J. Soc. Aero. Sci. Japan, Vol. 6, pp. 122–134, 1939.

[8] Schubauer, G. B.: *Air Flow in the Boundary Layer of an Elliptic Cylinder.* NACA Tech. Rep. No. 652, 1939.

[9] von Doenhoff, A. E., and Tetervin, N.: *Investigation of the Variation of Lift Coefficient with Reynolds Number at a Moderate Angle of Attack on a Low-Drag Airfoil.* NACA Wartime Rep. L–661, 1942.

[10] Liepmann, H.W., and Laufer, J.: *Investigations of Free Turbulent Mixing.* NACA Tech. Note No. 1257, 1947.

[11] Maekawa, T., and Atsumi, S.: *Transition Caused by the Laminar Flow Separation* (in Japanese). J. Soc. App. Mech. Japan, Vol. 1, pp. 187–192, 1948. Translated as NACA Tech. Memo. No. 1352, 1952.

[12] Chapman, D. R.: *Laminar Mixing of a Compressible Fluid.* NACA Tech. Note No. 1800, 1949; superseded by NACA Tech. Rep. No. 958, 1950.

[13] Bursnall, W. J., and Loftin, L. K., Jr.: *Experimental Investigation of Localized Regions of Laminar-Boundary-Layer Separation.* NACA Tech. Note No. 2338, 1951.

[14] McCullough, G. B., and Gault, D. E.: *Examples of Three Representative Types of Airfoil-Section Stall at Low Speed.* NACA Tech. Note No. 2502, 1951.

[15] Owen, P. R., and Klanfer, L.: *On the Laminar Boundary Layer Separation from the Leading Edge of a Thin Aerofoil.* RAE Rep. Aero. 2508, 1953; reissued as C.P. No. 220, 1955.

[16] Wallis, R.A.: *Experiments with Air Jets to Control the Nose Stall on a 3 Ft. Chord NACA 64A006 Aerofoil.* Aero. Res. Lab. (Australia), Aero. Note No. 139, 1954.

[17] Crabtree L.F.: *The Formation of Regions of Separated Flow on Wing Surfaces. Part I. Low-Speed Tests on a Two-Dimensional Unswept Wing with a 10 Per Cent Thick RAE 101 Section.* RAE Rep. Aero. 2528, 1954; reissued as Part I of ARC R. & M. No. 3122, 1959.

[18] Gault, D.E.: *An Experimental Investigation of Regions of Separated Laminar Flow.* NACA Tech. Note No. 3505, 1955.

[19] McCullough, G.B.: *The Effect of Reynolds Number on the Stalling Characteristics and Pressure Distributions of Four Moderately Thin Airfoil Sections.* NACA Tech. Note No. 3524, 1955.

[20] Korst, H.H.: *A Theory for Base Pressures in Transonic and Supersonic Flow.* J. App. Mech., Vol. 23, pp. 593–600, 1956.

[21] Chapman, D.R., Kuehn, D.M., and Larson, H.K.: *Investigation of Separated Flows in Supersonic and Subsonic Streams with Emphasis on the Effect of Transition.* NACA Tech. Note No. 3869, 1957.

[22] Crabtree, L.F.: *The Formation of Regions of Separated Flow on Wing Surfaces. Part II. Laminar-Separation Bubbles and the Mechanism of the Leading-Edge Stall.* RAE Rep. Aero. 2578, 1957; reissued as Part II of ARC R. & M. No. 3122, 1959.

[23] Crabtree, L.F.: *Effects of Leading-Edge Separation on Thin Wings in Two-Dimensional Incompressible Flow.* J. Aero. Sci., Vol. 24, pp. 597–604, 1957.

[24] Tani, I.: *Experimental Investigation of Flow Separation over a Step.* Grenzschichtforschung, Symposium Freiburg i. Br., August 1957, pp. 377–386. Berlin-Göttingen-Heidelberg 1958.

[25] Evans, W.T., and Mort, K.W.: *Analysis of Computed Flow Parameters for a Set of Sudden Stalls in Low-Speed Two-Dimensional Flow.* NASA Tech. Note D-85, 1959.

[26] Moore, T.W.F.: *A Note on the Causes of Thin Aerofoil Stall.* J. Roy. Aero. Soc., Vol. 63, pp. 724–730, 1959.

[27] Wallis, R.A.: *Boundary Layer Transition at the Leading Edge of Thin Wings and its Effect on General Nose Separation.* Preprint, ICAS Second Congress, Zürich, Sept. 1960.

[28] Tani, I., Iuchi, M., and Komoda, H.: *Experimental Investigation of Flow Separation Associated with a Step or a Groove.* Aero. Res. Inst., Univ. Tokyo, Rep. No. 364, 1961.

概　　要

前縁失速の機構に関する理論の批判研究

谷　　一　　郎

この論文は，比較的に薄い翼断面の前縁失速の原因と考えられる層流剝離泡の破壊の機構について，今までに発表された理論の批判研究を行なったものである．泡は層流境界層が剝離して後，再び表面に付着することによって形成されるが，この再付着は，剝離流の中で発生する乱れの混合作用で遂行されるものと考えられる．今までの実験結果を解析して見ると，泡が形成されるためには，まず剝離点での境界層レイノルズ数がある臨界値より大きく（谷および Owen と Klanfer の判定），次に再付着によって回復される圧力がある臨界値より小さいこと（Crabtree の判定）が必要である．前縁失速が起る直前の状態は，第一の判定はみたされるが，第二の判定がみたされなくなるような場合である．第一の判定は，剝離流の中で乱流への遷移の起ることを保証するものであり，第二の判定は，圧力差と釣合うべき乱流混合による剪断応力の大きさに限界のあることと関連している．このような解釈を採ることにより，層流剝離泡の破壊の機構，従って翼断面の前縁失速の機構について，矛盾のない一貫した説明を与えることができる．