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The Inertia Forces and Couples and their Balancing of the Star Type Engine.

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Introduction.

The star type engines are characterised by having the cylinders disposed at equal intervals around a complete circle. The number of cylinders is usually seven or nine. From their mechanism they are classified into two kinds :—radial and rotary engines. The former has stationary cylinders and a revolving crankshaft similar as the ordinary type of engines, and the latter has revolving cylinders and a fixed crankshaft. In this case the propeller hub is attached to the revolving crankcase.

In these engines a connecting rod containing the crank pin bearings acts as a main connecting rod, that is to say, it goes through a series of definite angular positions thereby determining the angular movements of the remaining rods which are pin-jointed to it. These latters may, for purpose of reference, be called auxiliary connecting rods. The piston attached to the main rod makes the usual harmonic motion of the ordinary type of engines, i. e., the motion of single obliquity. Those attached to the auxiliary rods*, however, produce a more or less distorted movements, i. e., the motions of double obliquity.

Usually the dynamics of these engines are approximately treated assuming that they have single obliquity, in other words, their all connecting rods are directly attached to one common crank pin. And the object of the present paper is to study the inertia forces and couples

* For simplicity we will call hereafter main connecting rod as main rod and auxiliary connecting rods as auxiliary rods.

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and their balancing of the star type engines taking it into account that they have double obliquity. This investigation consists of three parts; Part I the radial engine, Part II the rotary engine and Part III the comparisons with other types of engnies.

The writer expresses his hearty thanks to Prof. T. Suhara for the initiative suggestions on this subject and for many valuable aid and encouragement throughout this investigation.

Part I Radial engine.

(1) The linear and angular accelerations.

(*a*) Acceleration of piston which has double obliquity.

To find in the first place, the acceleration of piston which has double obliquity, let us consider for the sake of simplicity the case of the V type engine. The general arrangement of the V type engine having an auxiliary rod is shown in Fig. 1 and the following notations are used throughout the investigation.

OX_0 OX_1....cylinder axes.

$OB=r$ ......crank radius.

$AB=l$ ......main rod.

$CD=l'$ ......auxiliary rod.

$BC=a$ ......wrist pin radius.

$\angle X_0OX_1=\varphi$ ...included angle between two cylinder axes.

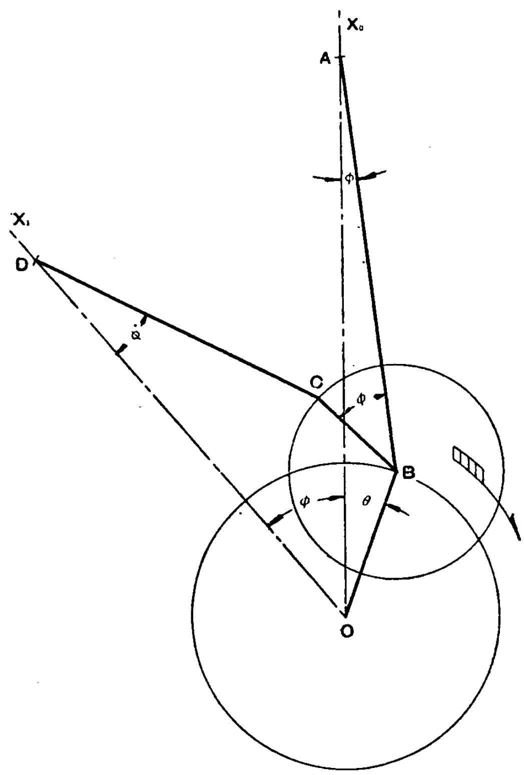

Fig. 1

$\angle ABC=\varphi$....angle between wrist radius and main rod axis.

$\angle AOB=\theta$....rotaion of crank from main rod cylinder axis at any instant.

$\angle OAB=\phi$....angularity of main rod at the same instant.

$\angle ODC=\phi'$....angularity of auxiliary rod at the same instant.

$$\frac{r}{l}=q \qquad \frac{r}{l'}=p \qquad \frac{a}{l'}=s$$

The auxiliary rod CD is jointed to the main rod ABC with wrist pin C which has a fixed position in the latter with a fixed angle $\angle ABC$ and radius BC, therefore its angular movement is controlled by the latter. Hence the motion of the auxiliary rod piston differs from and more complicated than that of main rod piston, and is called, as stated above, the motion of double obliquity.

In the above, we put the included angle $\angle XOX_1$ to be equal to the wrist angle $\angle ABC$ to make their strokes equal.*

Now let x be the displacement of the auxiliary rod piston from the centre of crankshaft O, then

$$x=OD$$

$$=r\cos(\theta+\varphi)+a\cos\phi+l'\cos\phi' \qquad \text{.....................(1)}$$

$$=r\cos(\theta+\varphi)+a\sqrt{1-q^2\sin^2\theta}+l'\sqrt{1-k^2\sin^2(\theta+\alpha)} \qquad \text{....(2)}$$

$$\text{where} \quad k^2=p^2+s^2q^2-2psq\cos\varphi \qquad \text{.....................(3)}$$

$$\alpha=\tan^{-1}\frac{r\sin\varphi}{r\cos\varphi-aq} \qquad \text{.....................(4)}$$

$$=r\cos(\theta+\varphi)+a\sum_{n=0}^{\infty}A_{2n}\cos 2n\theta+l'\sum_{n=0}^{\infty}A'_{2n}\cos 2n(\theta+\alpha) \qquad \text{..(5)}$$

$$\text{where}\left\{\begin{array}{l} A_0=1-\frac{1}{4}q^2-\frac{3}{64}q^4-\frac{5}{256}q^6-\dots \\ A_2=\frac{1}{4}q^2+\frac{1}{16}q^4+\frac{15}{215}q^6+\dots \\ A_4=-\frac{1}{64}q^4-\frac{3}{256}q^6-\dots \\ A_6=\frac{1}{512}q^6+\dots \\ \dots\dots\dots\dots\dots\dots \end{array}\right.$$

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* René Devillers: Le moteur à explosions. Tome I p. 346.

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$$\left\{\begin{aligned} A_0' &= 1 - \frac{1}{4}k^2 - \frac{3}{64}k^4 - \frac{5}{256}k^6 - \dots \\ A_2' &= \quad \frac{1}{4}k^2 + \frac{1}{16}k^4 + \frac{15}{512}k^6 + \dots \\ A_4' &= \qquad\qquad -\frac{1}{64}k^4 - \frac{3}{256}k^6 - \dots \\ A_6' &= \qquad\qquad\qquad\qquad \frac{1}{512}k^6 + \dots \\ &\dots\dots\dots\dots\dots\dots\dots\dots \end{aligned}\right.$$

Assuming that the crankshaft revolves with constant angular velocity ω, as is the case of high speed multi-cylindered aero engine, we get the acceleration as follows :

$$\frac{d^2x}{dt^2} = -\omega^2\Big[r\cos(\theta+\varphi) + a\sum_{n=1}^{\infty} B_{2n}\cos 2n\theta + l'\sum_{n=1}^{\infty} B'_{2n}\cos 2n(\theta+\alpha)\Big] \dots\dots\dots (6)$$

where

$$\left\{\begin{aligned} B_2 &= q^2 + \frac{1}{4}q^4 + \frac{15}{128}q^6 + \dots \\ B_4 &= \quad -\frac{1}{4}q^4 - \frac{3}{16}q^6 - \dots \\ B_6 &= \qquad\qquad \frac{9}{128}q^6 + \dots \\ &\dots\dots\dots\dots\dots\dots\dots\dots \end{aligned}\right.$$

$$\left\{\begin{aligned} B_2' &= k_2 + \frac{1}{4}k^4 + \frac{15}{128}k^6 + \dots \\ B_4' &\doteq \quad -\frac{1}{4}k^4 - \frac{3}{16}k^6 - \dots \\ B_6' &= \qquad\qquad \frac{9}{128}k^6 + \dots \\ &\dots\dots\dots\dots\dots\dots\dots\dots \end{aligned}\right.$$

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This is the expression of the acceleration of piston which has double obliquity and if the relation $l=l'+a$ exists, the expression (6) will also be applicable to that of the main rod piston. And in the actual case, the above relation practically holds, the result of negligence of the necessary correction upon it being almost of no effect, as will be stated later; and we may apply the expression (6) not only for the auxiliary rod piston but also for the main rod piston.

(*b*) Angular acceleration of auxiliary rod.

We obtain the following expression in transforming (1) into (2) in the preceding paragraph.

$$\sin\phi'=k\sin(\theta+\alpha)$$

Therefore

$$\phi'=\sum_{n=0}^{\infty}C'_{2n+1}\sin(2n+1)(\theta+\alpha)\quad\ldots\ldots(7)$$

where
$$\begin{cases} C'_1=k+\dfrac{1}{8}k^3+\dfrac{3}{64}k^5+\ldots\\ C'_3=\quad-\dfrac{1}{24}k^3-\dfrac{3}{128}k^5-\ldots\\ C'_5=\qquad\qquad\dfrac{3}{640}k^5+\ldots\\ \ldots\ldots\ldots\ldots \end{cases}$$

Assuming that the crankshaft revolves with constant angular velocity ω as before, we get

$$\frac{d^2\phi'}{dt^2}=-\omega^2\sum_{n=0}^{\infty}D'_{2n+1}\sin(2n+1)(\theta+\alpha)\quad\ldots\ldots(8)$$

where
$$\begin{cases} D'_1=k+\dfrac{1}{8}k^3+\dfrac{3}{64}k^5+\ldots\\ D'_3=\quad-\dfrac{3}{8}k^3-\dfrac{27}{128}k^5-\ldots\\ D'_5=\qquad\qquad\dfrac{15}{128}k^5+\ldots\\ \ldots\ldots\ldots\ldots \end{cases}$$

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This is the expression of the angular acceleration of auxiliary rod and is also applicable to that of the main rod as in the case of expression (6).

As shown in the expressions (3) and (4), k and α are the function of φ, which is the included angle between the main and an auxiliary rod cylinder axes. Therefore they have different values according to the position of its cylinder. As an actual example, these values from the 320 *HP. A, B, C.* "Dragonfly" engine, whose chief dimensions are given in the appendix I, are shown in Table I and plotted in Fig. 1 in the appendix.

(2) The inertia force and couple of the radial engine.

The inertia force and couple of the radial engine may be divided in two parts as follow :

(i) Inertia force of reciprocating parts.

(ii) Inertia force and couple of revoloring parts.

Piston and its accessories, gudgeon pin and one part of connecting rod etc. belong to the former and another part of connecting rod, wrist pin and two ball bearings etc. belong to the latter.

In assuming the inertia of the connecting rod equivalent as those at the small and big ends D and C in Fig. 1, when the mass of the rod is supposed to be concentrated at the two ends with the inverse ratio of their length from its centre of gravity, to its dynamical equivalence we must consider a correcting couple as the correction.*

In this study of the inertia force and couple of the engine we may consider the main rod as an auxiliary rod.

(3) The unbalanced inertia force of the reciprocating parts.

The inertia force of the reciprocating parts is given immediately by multiplying the expression of acceleration (6) by its mass and changing the sign, i. e.,

* Lorenz : Technische Mechanik starrer Systeme. s. 339—345.

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$$F=-\frac{W}{g}\frac{d^2x}{dt^2}$$

$$=+\frac{W}{g}\omega^2\left[r\cos(\theta+\varphi)+a\sum_{n=1}^{\infty}B'_{2n}\cos 2n\theta\right.$$

$$\left.+l'\sum_{n=1}^{\infty}B'_{2n}\cos 2n(\theta+\alpha)\right] \quad \ldots\ldots(9)$$

where $\dfrac{W}{g}$ = mass of the reciprocating parts.

This is the expression of the reciprocating inertia force and as the values of k and α differ from the value of φ as stated above, the inertia force of each cylinder also changes its value according to the position of the corresponding cylinder, and as an actual example the difference of its value of the main ($\varphi=0°$) and the No. 4 ($\varphi=160°$) cylinders of the "Dragonfly" engine is shown in Fig. 2 in the appendix.

To obtain the resultant unbalanced inertic force of the total cylinders h, project the inertia force F on the fixed axes OX_0 and OY_0, and let the projections be X_m and Y_m, then

$$\left.\begin{aligned}
X_m&=F\cos\varphi_m\\
&=+\frac{W}{g}\omega^2\Big\{\frac{1}{2}r\Big[\cos(\theta+2\varphi_m)+\cos\theta\\
&\quad+a\cos\varphi_m\sum_{n=1}^{\infty}B_{2n}\cos 2n\theta+l'\cos\varphi_m\sum_{n=1}^{\infty}B'_{2n}\cos 2n(\theta+\alpha_m)\Big\}\\
\text{and}\quad Y_m&=+\frac{W}{g}\omega^2\Big\{\frac{1}{2}r\Big[\sin(\theta+2\varphi_m)-\sin\theta\Big]\\
&\quad+a\sin\varphi_m\sum_{n=1}^{\infty}B_{2n}\cos 2n\theta+l'\sin\varphi_m\sum_{n=1}^{\infty}B'_{2n}\sin 2n(\theta+\alpha_m)\Big\}
\end{aligned}\right\}$$

Hence the resultant projections of those of the total cylinders are given as follow :

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$$\left.\begin{aligned}
\sum_{m=1}^{h} X_m &= +\frac{W}{g}\omega^2\Big\{\frac{1}{2}r\sum_{m=1}^{h}\Big[\cos\,(\theta+2\varphi_m)+\cos\,\theta\Big] \\
&\quad +a\sum_{m=1}^{h}\cos\,\varphi_m\cdot\sum_{n=1}^{\infty}B_{2n}\,\cos\,2n\theta \\
&\quad +l'\sum_{m=1}^{h}\Big[\cos\varphi_m\cdot\sum_{m=1}^{\infty}B'_{2n}\,\cos\,2n(\theta+\alpha_m)\Big]\Big\} \\
\sum_{m=1}^{h} Y_m &= +\frac{W}{g}\,\omega^2\,\Big\{\frac{1}{2}r\sum_{m=1}^{h}\Big[\sin\,(\theta+2\varphi_m)-\sin\,\theta\Big] \\
&\quad +a\sum_{m=1}^{h}\sin\,\varphi_m\cdot\sum_{n=1}^{\infty}B_{2n}\,\cos\,2n\theta \\
&\quad +l'\sum_{m=1}^{h}\Big[\sin\,\varphi_m\cdot\sum_{n=1}^{\infty}B'_{2n}\,\cos\,2n\,(\theta+\alpha_m)\Big]\Big\}
\end{aligned}\right\}$$

and as $\varphi_1=\frac{1}{2}\varphi_2=\ldots\ldots=\frac{1}{h}\varphi_n=\frac{2\pi}{h}$, *i. e.*, as each included angle between adjacent two cylinders is equal (see Fig. 2),

$$\sum_{m=1}^{h}\cos\,(n\theta+p\varphi_m)=0,\qquad \sum_{m=1}^{h}\sin\,(n\theta+p\varphi_m)=0$$

where n and p are any positive integers except when p is the multiple of h.

Therefore, many terms being balanced out they become as follow:

$$\left.\begin{aligned}
\Sigma X &= +\frac{W}{g}\omega^2 rh\Big\{\frac{1}{2}\,\cos\,\theta-sq\Big[\{1+\frac{3}{8}(p^2+s^2q^2)\} \\
&\quad +\frac{1}{8}s^2q^2+\ldots\Big]\cos\,2\theta+\frac{1}{2}s^3q^3\Big[1+\ldots\Big]\cos 4\theta+\ldots\Big\} \\
\Sigma Y &= -\frac{W}{g}\omega^2 rh\Big\{\frac{1}{2}\,\sin\,\theta-sq\Big[\{1+\frac{3}{8}(p^2+s^2q^2)\} \\
&\quad -\frac{1}{8}s^2q^2+\ldots\Big]\sin\,2\theta+\frac{1}{2}s^3q^3\Big[1+\ldots\Big]\,\text{sis}\,4\theta+\ldots\Big\}
\end{aligned}\right\}\ldots(10)$$

These are the unbalauced inertia forces of the radial engine and if we calculate these values by the "Dragonfly" engine, we obtain

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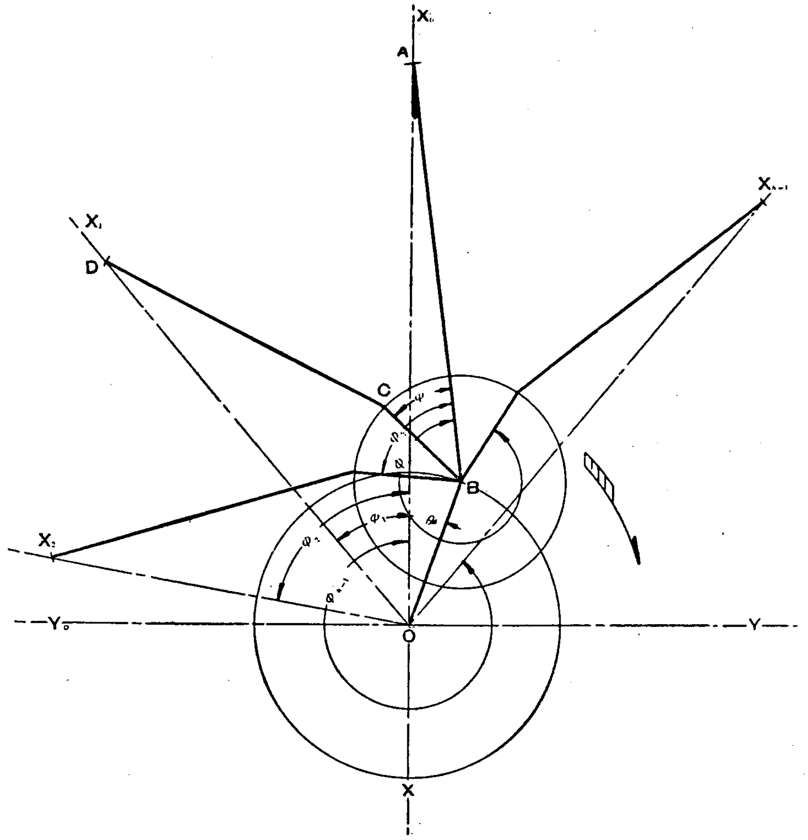

Fig. 2.

$$\left.\begin{aligned}\Sigma X &= +1{,}999.0 \cos\theta - [250.1 + .1 + ..] \cos 2\theta \\ &\quad +[0.4 + ..] \cos 4\theta + \ldots\ldots\ldots\ldots\ldots\ldots \\ \Sigma Y &= -1{,}999.0 \sin\theta + [250.1 - .1 + ..] \sin 2\theta \\ &\quad -[0.4 + ..] \sin 4\theta + \ldots\ldots\ldots\ldots\ldots\ldots\end{aligned}\right\} \text{in kg.}$$

In the expression (10) the first, second, third...... terms are the primary, secondary, quaternary .... unbalanced inertia forces respectively.

In Fig. 3 let OB be the crank position at any instant, then the primary appears at the same angular position as OB at the instant, *i. e.*, this becomes an unbalanced revolving inertia force revolving with the

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crank. Hence the primary can be balanced completely by a counter weight attached at the opposite side of the crank.

The second harmonics is divided into two parts as apparent in the above expression and appears at the position OB_2 and OB'_2; the former becomes an unbalanced revolving inertia force always making advance to the crank by 180° and revolving in the same direction with twice the angular velocity of the engine, and the latter also becomes an unbalanced inertia force always making advance to the crank by 180° but revolving in the reverse direction with twice the angular velocity of the engine. The latter is much smaller than the former and may by neglected in the practical case as is apparent in the above example.

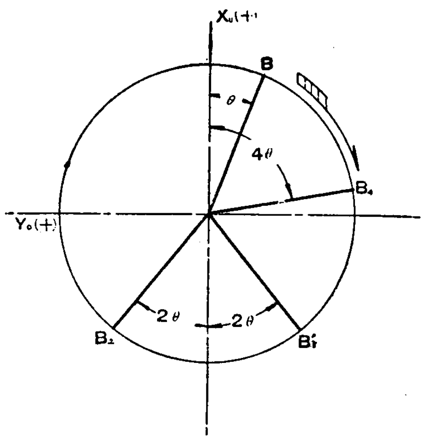

Fig. 3.

This second harmonics makes up almost the entire part of the unbalanced force of the radial engine.

The fourth harmonics will also be divided into two parts if we take its higher order. At the present, however, its first part appears only in the above expression. In Fig. 3 this appears at the position OB_4 and becomes an unbalanced revolving inertia force revolving in the same direction with the angular velocity four times as the crank. This being much smaller than the former of the second harmonics may be negligible in the practical case.

(4) The inertia force and couple of the revolving parts.

(*a*) Inertia force of revolving parts.

Let $\frac{Ws}{g}$ be the total mass of the revolving parts at the crank pin B, then it gives rise to a centrifugal force as follows :

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$$F_B = -\frac{Ws}{g}\omega^2 r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

This revolving inertia force is balanced completely by a counter weight attached to the opposite side of the crank, as in the case of the primary inertia force of the reciprocating parts.

(*b*) Inertia couple of revolving parts.

In addition to the above (*a*) there is an inertia couple acting about the crank pin *B*, which is produced by the revolving mass rotating relatively to the pin about its centre of gravity *B*. Let this inertia couple be C_B, then

$$C_B = -\left[I_B + \frac{w}{g}a^2 h\right]\frac{d^2\phi}{dt^2}$$

$$= +\left[I_B + \frac{w}{g}a^2 h\right]\omega^2 \sum_{n=0}^{\infty} D_{2n+1} \sin(2n+1)\,\theta \ldots\ldots\ldots\ldots (12)$$

where I_B....moment of inertia of the revolving part of the main rod and of the two ball bearings rotating relatively to the pin.

$\frac{w}{g}$....crank side mass of the auxiliary rod.

$$\begin{cases} D_1 = q + \frac{1}{8}q^3 + \frac{3}{64}q^5 + \ldots \\ D_3 = \quad -\frac{3}{8}q^3 - \frac{27}{128}q^5 + \ldots \\ D_5 = \qquad\qquad \frac{15}{128}q^5 + \ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{cases}$$

(*c*) Correcting couple.

The correcting couple that has been stated in § (2) is given by the following expression.*

$$\Phi_1 = -\frac{w'_1}{g}\omega^2 b\,(l' - c)\,\frac{d^2\phi'}{dt^2}$$

* René Derillers: Le moteur à explosions. Tome I p. 184-189.

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$$=\frac{w_1}{g}\omega^2 b\ (l'-c)\sum_{n=0}^{\infty}D'_{2n+1}\sin(2n+1)(\theta+\alpha)\ \ldots\ldots\ldots(13)$$

where $\frac{w_1}{g}$....mass of the connecting rod.

b....distance between the gudgeon pin centre and centre of gravity of the rod.

c....length of the equivalent simple pendulum.

The correcting couple change its value according to the position of its cylinder, and its unbalanced resultant couple will be shown later as its resultant couple given on the engine frame.

(5) The unbalanced force due to unequalities of the moving mass and wrist radius of each cylinder.

(*a*) for the unequality of moving mass of each cylinder.

In the above investigation the moving mass of each cylinder has been assumed to be equal. In actual case, however, each mass is unequal and differ from its mean value about one percent. Especially in the star type engine, this unequality is predominant due to the effect of main rod mass*, and may result greater unbalanced force accordingly. Now let us study this unbalenced force.

(i) Primary inertia force.

From the expresson (9) § (3) the primary inertia force is given as follows:

$$F_1=+\frac{W}{g}\omega^2 r\ \text{ces}\ (\theta+n\,\varphi)$$

and $\frac{W}{g}=M+m$

where M....mean mass.

m....deviation form mean mass.

then $\Sigma m=0$

* See Table III of appendix.

hence $\Sigma X_1 = +\frac{1}{2} M\omega^2 rh \cos\theta + \frac{1}{2}\omega^2 r \sum_{n=0}^{h-1} m_n \cos(\theta + 2n\varphi)$

in which the first term is the unbalanced force due to the mean mass and is the same as in the expression (10) § (3). Hence the unbalanced force due to the unequality of mass $\Sigma X_1'$ is given by the following expressin.

$$\left.\begin{aligned} \Sigma X_1' &= \frac{1}{2}\omega^2 r \sum_{n=0}^{h-1} m_n \cos(\theta + 2n\varphi) \\ \text{Similarly} \quad \Sigma Y_1' &= \frac{1}{2}\omega^2 r \sum_{n=0}^{h-1} m_n \sin(\theta + 2n\varphi) \end{aligned}\right\} \cdots\cdots\cdots\cdots (14)_1$$

This is the unbalanced primary force and if we calculate this value from our example,

$$\left.\begin{aligned} \Sigma X_1' &= +19.6\cos(\theta - 17^\circ) \\ \Sigma Y_1' &= +19.6\sin(\theta - 17^\circ) \end{aligned}\right\} \text{in kg.}$$

which corresponds to 8.2% of the unbalanced second harmonics § (3).

(ii) Second harmonics.

Similarly from the expression (9) the second harmonics is given as follows:

$$F_2 = (M+m)\,\omega^2\,[rp\cos(2\theta+2\varphi) - 2rsq\cos(2\theta+\varphi) + aq^2(1+s)\cos 2\theta]$$

and let X_2' and Y_2' express that due to the unequality of mass, then

$$\left.\begin{aligned} \Sigma X_2 = &+\frac{1}{2}\omega^2 rp\left[\sum_{n=0}^{h-1} m_n\cos(2\theta+3n\varphi) + \sum_{n=0}^{h-1} m_n\cos(2\theta+n\varphi)\right] \\ &-\omega^2 rsq\sum_{n=0}^{h-1} m_n\cos(2\theta+2n\varphi) + \omega^2 aq^2(1+s)\cos 2\theta\sum_{n=0}^{h-1} m_n\cos n\varphi \\ \Sigma Y_2' = &+\frac{1}{2}\omega^2 rp\left[\sum_{n=0}^{h-1} m_n\sin(2\theta+3n\varphi) - \sum_{n=0}^{h-1} m_n\sin(2\theta+n\varphi)\right] \\ &-\omega^2 rsq\sum_{n=0}^{h-1} m_n\sin(2\theta+2n\varphi) + \omega^2 aq^2(1+s)\sin 2\theta\sum_{n=0}^{h-1} m_n\sin n\varphi \end{aligned}\right\} (14)_2$$

This is the unbalanced second harmonics and if we calculate from our example we obtain

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$$\left.\begin{array}{l}\Sigma X_2' = +8.6 \cos(2\theta + 17^\circ) \\ \Sigma Y_2' = -4.9 \sin(2\theta - 20^\circ)\end{array}\right\} \text{ in kg.}$$

$\Sigma X_2'$ corresponds to 3.6% and $\Sigma Y_2'$ to 2.0% of the unbalanced second harmonics § (3).

(*b*) for the unequality of wrist radius of each cylinder.

To keep the compression ratio of each cylinder equal, we must correct the value of a or s according to the position of its cylinder* and the purpose of this paragraph is to study the unbalanced force due to this correction.

In the primary inertia force the term including a or s does not appear and therefore the effect of this correction begins to appear with the secondary.

$$F_2 = \frac{W}{g}\omega^2\Big[rp\cos(2\theta + 2\varphi) - 2rsq\cos(2\theta + \varphi) + aq^2(1+s)\cos 2\theta\Big]$$

and $\quad s = S + s$

where S....mean value.

s ....deviation from mean value.

then $\quad \Sigma s = o$

and the square of s may be neglected. If we express this effect by X_2'' and Y_2'' we get

$$\left.\begin{array}{l}\Sigma X_2'' = -\dfrac{W}{g}\,\omega^2\, rq \displaystyle\sum_{n=0}^{h-1} s_n \cos(2\theta + 2n\,\varphi) \\ \qquad + \dfrac{W}{g}\,\omega^2\, l'q^2(2S+1)\cos 2\theta \displaystyle\sum_{n=0}^{h-1} s_n \cos n\varphi \\ \Sigma Y_2'' = -\dfrac{W}{g}\,\omega^2\, rq \displaystyle\sum_{n=0}^{h-1} s_n \sin(2\theta + 2n\,\varphi) \\ \qquad + \dfrac{W}{g}\,\omega^2\, l'q^2(2S+1)\sin 2\theta \displaystyle\sum_{n=0}^{h-1} s_n \sin n\varphi \end{array}\right\} \quad \ldots.(15)$$

This is the unbalanced second harmonics due to this effect and if we calculate in our engine we get

* A paper on this subject was presented to the Japanese Society of Mechanical Engineers by K. Nakagawa **54** (No. 70), and also refer to Table II of appendix.

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$$\left.\begin{aligned}\Sigma X_2'' &= +3.2 \cos 2\theta \\ \Sigma Y_2'' &= +2.5 \sin 2\theta\end{aligned}\right\} \text{ in kg.}$$

$\Sigma X_2''$ corresponds to 1.3% and $\Sigma Y_2''$ to 1.0% of the unbalanced second harmonics § (3).

$\Sigma X_1'$ $\Sigma X_2'$ and $\Sigma X_2''$ of our "Dragonfly" are plotted on Fig. 3 and the effect of these resultant on the unbalanced inertia force § (3) on Fig. 4 in the appendix. Namely the sum $\Sigma X_1' + \Sigma X_2 + \Sigma X_2''$ corresponds to 10.0% of the unbalanced second harmonics § (3). The projection on the y-axis is omitted, since it will have a similar effect as that of the x-axis.

(6) The crank turning moment due to the inertia force and couple.

(*a*) due to inertia force F.

Now let us consider the crank turning moment due to the inertia forces and couples and its effect upon the engine torque curve. The crank turning moment due to the inertia force F is given from energy equation as follows, neglecting the frictional loss.*

$$\begin{aligned}
M_I &= F \frac{dx}{dt} \frac{1}{\omega} \\
&= -\frac{W}{g} \frac{d^2x}{dt^2} \cdot \frac{dx}{dt} \frac{1}{\omega} \\
&= -\frac{W}{g} \omega^2 \left\{ r \cos(\theta+\varphi) + a \sum_{n=1}^{\infty} B_{2n} \cos 2n\theta + l' \sum_{n=1}^{\infty} B'_{2n} \cos 2n(\theta+\alpha) \right\} \\
&\qquad \times \left\{ r \sin(\theta+\varphi) + a \sum_{n=0}^{\infty} 2n\, A_{2n} \sin 2n\theta \right. \\
&\qquad\qquad \left. + l' \sum_{n=0}^{\infty} 2n\, A'_{2n} \sin 2n(\theta+\alpha) \right\} \cdots\cdots (16)
\end{aligned}$$

And the resultant of the total cylinders h is given by the following expression.

* Other method of finding it from the condition of equilibrium of the main rod is given in the Automobile Engineer by T. L. Sherman. **5** (No 125) p. 105.

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$$\Sigma M_I=\frac{W}{g}\,al'g\,\omega^2 h\Big[-\frac{1}{2}p^2[1+\frac{3}{8}p^2]+..\Big]\sin\theta+\frac{1}{32}sq^5[1+..]\sin 2\theta$$

$$-\frac{9}{16}p^2s^2q^2[1+..]\sin 3\theta+\frac{1}{4}sq^3[1+..]\sin 4\theta+..\Big\}\ldots\ldots(17)$$

This is the resultant crank turning moment due to the inertia force of the radial engine and if we calculate this value from the present example, we get

$$\Sigma M_I=-[11.38+..]\sin\theta+[.02+..]\sin 3\theta$$
$$-[.47+..]\sin 4\theta+\ldots.\text{in m.kg.}$$

(*b*) due to inertia couple C_B.

In addition to the above, another crank turning moment is produced by the inertia couple C_B and may be given as the following expression similar as the expression (16).

$$M_L=-C_i\frac{d\phi}{dt}\frac{1}{\omega}$$
$$=\Big[I_B+\frac{w}{g}a^2h\Big]\frac{d^2\phi}{dt^2}\frac{d\phi}{dt}\frac{1}{\omega}$$
$$=-\Big[I_B+\frac{w}{g}a^2h\Big]\omega^2\sum_{n=1}^{\infty}E_{2n}\sin 2\,n\theta\ \ldots\ldots\ldots\ldots\ldots(18)$$

where
$$\begin{cases}E_2=\frac{1}{2}q^2+o\quad-\frac{1}{32}q^6+\ldots.\\E_4=\quad-\frac{1}{4}q^4-\frac{1}{8}q^6-\ldots.\\E_6=\quad\frac{27}{128}q^6+\ldots.\\\ldots\ldots\ldots\ldots\ldots\ldots\end{cases}$$

and in our example we have

$$M_L=-[3.83+..]\sin 2\theta+[.11+..]\sin 4\theta+\ldots\ldots\text{in m.kg.}$$

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In this expression as the moment of inertia I_B is obtained by approximation*, this is but approximate value. We may, however, interpret the extent of its effect by the above expression.

If we add these two moments, we have

$$\Sigma M_I + M_L = [11.38 + ..]\sin\theta - [3.73 + ..]\sin 2\theta + [.02 + ..]\sin 3\theta - [.36 + ..]\sin 4\theta + \ldots\ldots \text{ in m. kg.}$$

This is the resultant crank turning moment due to the inertia force and couple of our "Dragonfly" engine and is plotted in Fig. (6) in the appendix.

(*c*) Correction due to correcting couple Φ_1.

The crank turning moment due to the correcting couple Φ_1 is given as follows :

$$\begin{aligned} M_\Phi &= -\Phi \frac{d\phi'}{dt}\frac{1}{\omega} \\ &= \frac{w}{g} b(l'-c) \frac{d^2\phi'}{dt}\frac{d\phi'}{dt}\frac{1}{\omega} \\ &= -\frac{w}{g} b(l'-c)\,\omega^2 \sum_{n=1}^{\infty} E'_{2n} \sin 2n(\theta+\alpha) \quad \ldots\ldots\ldots\ldots(19) \end{aligned}$$

where
$$\begin{cases} E'_2 = \frac{1}{2}k^2 + o \quad -\frac{1}{32}k^6 + \ldots\ldots \\ E'_4 = \quad -\frac{1}{4}k^4 - \frac{1}{8}k^6 - \ldots\ldots \\ E'_6 = \quad \frac{27}{128}k^6 + \ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{cases}$$

And the resultant of the total cylinders h is given as follows ;

$$\Sigma M_{\Phi_1} = -\frac{w}{g} b(l'-c)\,\omega^2 h \left\{ \frac{1}{2} s^2 q^2 \left[1 + 2p^2 + .. \right] \sin 2\theta - \left[\frac{1}{4} s^4 q^6 + .. \right] \sin 4\theta + \right\} \ldots\ldots\ldots\ldots\ldots\ldots(20)$$

* See appendix I c) N. B.

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If we calculate from our example we have

$$\Sigma M_{\Phi_1} = -[.58 + ..] \sin 2\theta + ...\text{in m. kg.}$$

which corresponds to 4.4% of the above resultant of our "Dragonfly" engine.

(7) The engine torque.

The mean value of engine torque may be calculated from the *HP*. and its *R. P. M.* The variation of its torque with the crankshaft rotation, *i. e.*, the torque curve, however, must be deduced from the indicator card. And as it is generally difficult in a high speed aero engine to draw the actual indicator card we must resort to the theoretical indicator card under the appropriate assumption.

Now, let P be the gas pressure acting on the piston, which is obtained from the indicator card, then the engine torque Mg is given similarly as the expression (16) § (6) as follows:

$$Mg = P\frac{dx}{dt}\,\frac{1}{\omega}$$

$$= P\Big[r\cos(\theta+\varphi) + a\sum_{n=1}^{\infty} 2nA_{2n}\sin 2n\theta$$

$$+ l'\sum_{n=0}^{\infty} 2nA_{2n}'\sin 2n(\theta+\alpha)\Big] \quad ....(21)$$

In this expression, as the pressure P is given from the card as a function of piston displacement x, the engine torque is the function of x and $\frac{dx}{dt}$. Therefore if the card of each cylinder is assumed to be equal, its mean torque will also be equal. Its torque curve, however, will be different in its form according to its cylinder position.

The resultant torque curve of all cylinders can be obtained by superposing each torque curve upon one another. But the change of its form with the respective cylinders is not notable and it is trouble-

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some and moreover not being our object, to find the torque curve of each cylinder, we assumed approximately that each torque curve is equal to that of main cylinder and the resultant was obtained upon this assumption. It is shown in Fig. 5 in the appendix.

The resultant torque curve thus obtained is affected by the crank turning moments due to the inertia force and couple which was given in § (6). This effect in our engine is shown in Fig. 6 in the appendix. Though the resultant torque curve is not acculate as stated above, and the effect of the inertia is thus obtained approximately, we can interpret the extent of its effect by Fig. 6. It points out us that the effect is not notable and if we compare with their max. value the effect comes out to be 6.7 % of the resaltant torque and if we compare with the mean value of the resultant torque the effect comes out to be 8.6 % of it.

(8) The couples on the engine frame due to the inertia force and couple.

(*a*) due to inertia force F and couple C_B.

The couple on the engine frame due to the inertia force F is equal in magnitude and opposite in direction to its crank turning moment. Namely

$$C_I = -M_I$$

$$= \frac{W}{g} \frac{d^2x}{dt} \frac{dx}{dt} \frac{1}{\omega} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

and

$$\Sigma C_I = \frac{W}{g} al' q\omega^2 h \Big\{ \frac{1}{2} p^2 \Big[1 + \frac{3}{8} p^2 + \ldots \Big] \sin \theta$$

$$- \frac{1}{32} s q^5 [1 + \ldots] \sin 2\theta + \frac{9}{16} p^2 s^2 q^2 [1 + \ldots] \sin 3\theta$$

$$- \frac{1}{4} s q^3 [1 + \ldots] \sin 4\theta + \ldots \Big\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

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The couple on the engine frame due to the inertia couple C_B, however, is unequal to its crank turning moment and is give as follows :

$$C_L=C_i\left[1+\frac{d\phi}{dt}\ \frac{1}{\omega}\right]$$

$$=\left[I_B+\frac{W}{g}a^2h\right]\omega^2\left\{\sum_{n=0}^{\infty}D_{2n+1}\sin(2n+1)\theta\right.$$

$$\left.+\sum_{n=0}^{\infty}E_{2n}\sin 2n\,\theta\right\}\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(24)$$

in which the second term is the reverse of its crank turning moment and the first term is equal to the inertia couple C_B.

If we calculate its value from our example, we get

$$C_L=[28.54+\ldots\ldots]\sin\theta+[3.73+\ldots]\sin 2\theta$$

$$-[.75+\ldots]\sin 3\theta-[.11+\ldots]\sin 4\,\theta+\ldots\ldots\ldots\text{in m. kg.}$$

and the sum of C_L and ΣC_I is as follows :

$$\Sigma C_I+C_L=[39.92+\ldots]\sin\theta+[3.72+\ldots]\sin 2\theta$$

$$-[.77+\ldots]\sin 3\theta+[.36+\ldots]\sin 4\,\theta+\ldots\text{in m. kg.}$$

As apparent in the above example the couple on the engine frame due to the inertia force F and couple C_B is much greater then their crank turning moment and comes out to be 20.6 % of the max. value and 26.6 % of the mean value of the couple on the engine frame due to gas pressure. These are shown in Fig 7 of appendix.

(*b*) Correction due to correcting couple Φ_1.

The couple on the engine frame due to corrceting couple Φ_1 is given similarly as the expression (24) as follows :

$$C_{\Phi_1}=\Phi\left[1+\frac{d\phi'}{dt}\ \frac{1}{\omega}\right]$$

$$=\frac{W}{g}b\,(l'-c)\,\omega^2\left[\sum_{n=0}^{\infty}D'_{2n+1}\sin(2n+1)(\theta+\alpha)\right.$$

$$\left.+\sum_{n=0}^{\infty}E'_{2n}\sin\ 2n\ (\theta+\alpha)\right]\ \cdots\cdots\cdots\cdots\cdots(25)$$

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and the resultant of the total cylinders h is given as follows:

$$\Sigma C_{\Phi_1}=\frac{W}{g}\,\omega^2 b\,(l'-c)\,h\Big\{\,sq\left[1+\frac{1}{4}p^2+\,..\right]\sin\theta$$

$$+\frac{1}{2}s^2q^2\left[1+2p^2+\,..\right]\sin 2\theta$$

$$-\frac{3}{8}s^3q^3[1+\,..]\sin 3\theta+\,..\Big\}\,\dots\dots\dots\dots\dots\dots\dots(26)$$

From our example we have

$$\Sigma C_{\Phi_2}=[9.61+\,..]\sin\theta+[.58+\,..]\sin 2\theta$$

$$-[.01+\,..]\sin 3\theta+\,\dots\dots\text{ in m. kg.}$$

which corresponds to 23.5 % of the above resultant couple of our "Dragonfly."

Part II Rotary engine.

(1) The radial and tangential accelerations.

The principal parts of rotary engine are shown in Fig. 4 and its nomenclatures are the same as given in § (1) Part I. In this case the cylinders rotate about the crankshaft O as a centre and the pistons and connecting rods about the crank pin B as a centre, the crank OB being fixed on the engine frame.

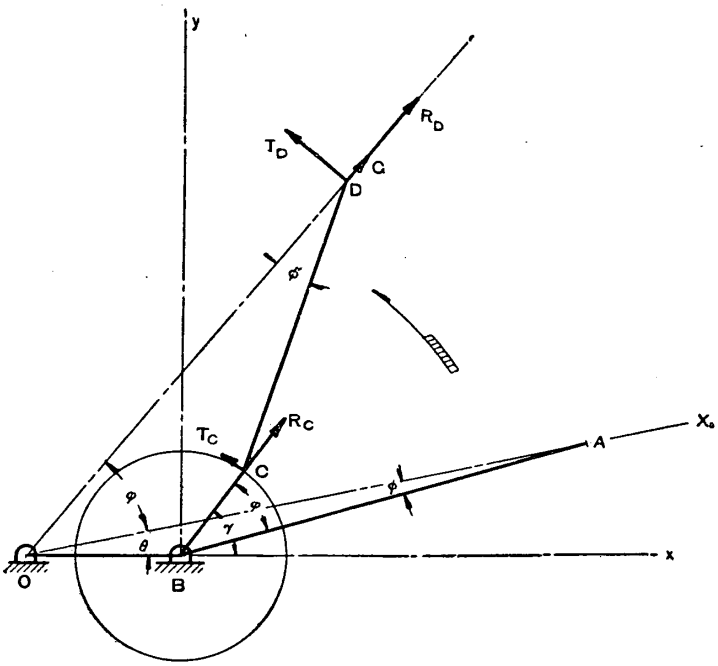

Fig. 4.

Now let x be the displacement of piston from the centre of crankshaft O as before and the angle which the wrist radius BC makes with the crank be γ. Then the radial and tengential accelerations at C and D are obtained by the following expressions, assuming that the engine revolves with constant angular velocity ω as before.

$$\left.\begin{array}{l}\text{Radial acceleration at } C=-a\left(\dfrac{d\gamma}{dt}\right)^2 \\ \text{Tangential acceleration at } C=a\,\dfrac{d^2\gamma}{dt^2}\end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots (27)$$

$$\left.\begin{array}{l}\text{Radial acceleration at } D=\dfrac{d^2x}{dt}-x\,\omega^2 \\ \text{Tangential acceleration at } D=2\dfrac{dx}{dt}\omega\end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots (28)$$

In Fig. 4 $\qquad \gamma=\varphi+\phi+\theta$

where φ is constant and $\dfrac{d\theta}{dt}$ is the constant angular velocity of engine ω.

Therefore $\qquad \dfrac{d\gamma}{dt}=\omega+\dfrac{d\phi}{dt}, \qquad \dfrac{d^2\gamma}{dt^2}=\dfrac{d^2\phi}{dt^2}$

and as $\dfrac{d\phi}{dt}$ and $\dfrac{d^2\phi}{dt^2}$ have been required in § (4) Part I, $\left(\dfrac{d\gamma}{dt}\right)^2$ and $\dfrac{d^2\gamma}{dt^2}$ are given as follow :

$$\left(\frac{d\gamma}{dt}\right)^2=\omega^2\sum_{n=0}^{\infty}F_n\cos n\,\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

$$\frac{d^2\gamma}{dt^2}=-\omega^2\sum_{n=0}^{\infty}D_{2n+1}\sin(2n+1)\,\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

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where

$$\begin{cases} F_0 = 1 + \frac{1}{2}q^2 + \frac{1}{4}q^4 + \frac{1}{16}q^6 + \dots \\ F_1 = 2q + \frac{1}{4}q^3 + \frac{3}{64}q^5 + \dots \\ F_2 = \frac{1}{2}q^2 + 0 - \frac{1}{32}q^6 + \dots \\ F_3 = -\frac{1}{4}q^3 - \frac{9}{64}q^5 - \dots \\ F_4 = -\frac{1}{4}q^4 - \frac{1}{16}q^6 + \dots \\ F_5 = \frac{3}{64}q^5 \dots\dots \\ F_6 = \frac{1}{32}q^6 + \dots \\ \dots\dots\dots \end{cases}$$

x, $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$ have been obtained in § (1) Part I and have the following expressions.

$$x = r\cos(\theta+\varphi) + a\sum_{n=0}^{\infty} A_{2n}\cos 2n\,\theta + l'\sum_{n=0}^{\infty} A'_{2n}\sin(\theta+\alpha) \Big\} \quad \dots\dots(5)$$

$$\frac{dx}{dt} = -\omega\Big\{ r\sin(\theta+\varphi) + a\sum_{n=0}^{\infty} 2n\,A_{2n}\sin 2n\,\theta + l'\sum_{n=0}^{\infty} 2n\,A'_{2n}\sin 2n(\theta+\alpha)\Big\} \quad ..(31)$$

$$\frac{d^2x}{dt^2} = -\omega^2\Big\{ r\cos(\theta+\varphi) + a\sum_{n=1}^{\infty} B_{2n}\cos 2n\,\theta + l'\sum_{n=0}^{8} B'_{2n}\cos 2n(\theta+\alpha)\Big\} \quad \dots\dots(6)$$

Consequently we can calculate the above radial and tangential accelerations at C and D given by the expression, (27) and (28).

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(2) The inertia of the piston.

Generally the centre of gravity of piston is not situated at the small end D but a little higher position. In Fig. 4 let G be its centre of gravity and the length of GD be e. Take B as origin, OB as x-axis and its perpendicular line as y-axis. If the co-ordinates of the small end D be expressed by $x'' \, y''$ and the mass of piston by $\frac{w_2}{g}$, then the force of accelecation of the piston may by given as follows:

$$\left.\begin{aligned} \int d\left(\frac{w_2}{g}\right) \frac{d^2x}{dt^2} &= \frac{w_2}{g} \frac{d^2x''}{dt^2} - f \cos(\theta+\varphi) \\ \int d\left(\frac{w_2}{g}\right) \frac{d^2y}{dt^2} &= \frac{w_2}{g} \frac{d^2y''}{dt^2} - f \sin(\theta+\varphi) \end{aligned}\right\}$$

$$\text{where} \qquad f = \frac{w_2}{g} e \, \omega^2 \qquad \cdots\cdots\cdots(32)$$

and its moment about the origin B may be given by

$$\int d\left(\frac{w_2}{g}\right)\left[y\frac{d^2x}{dt^2} - x\frac{d^2y}{dt^2}\right] = \frac{w_2}{g}\left[y''\frac{d^2x''}{dt^2} - x''\frac{d^2y''}{dt^2}\right]$$
$$-f\left[y'' \cos(\theta+\varphi) - x'' \sin(\theta+\varphi)\right] - \Phi_2$$

$$\text{where} \qquad \Phi_2 = -\frac{w_2}{g} e\left[\frac{d^2x''}{dt^2} \sin(\theta+\varphi) - \frac{d^2y''}{dt^2} \cos(\theta+\varphi)\right]$$
$$= -2\frac{w_2}{g} e \frac{dx}{dt} \omega \qquad \cdots\cdots\cdots(33)$$

Therefore the inertia force of piston may be considered similar as the sum of the inertia force at the small end and the constant force f acting along the cylinder axis, when the whole mass of piston is assumed to be concentrated at this end. The latter force f is negligible as will be stated later.

In considering its moment, a correcting couple Φ_2 must be added to the moments due to the two forces above mentioned. This couple Φ_2 acting on the piston may be called the couple of busculement.

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(3) The inertia force and couple of the rotary engine.

(*a*) Inertia force of rotary engine.

As in the case of radial engine, the inertia of connecting rod may well be considered similar as those at the small and big ends D and C, when the whole mass of the rod is assumed to be concentrated at the two ends with the inverse ratio of their length from its centre of gravity. And by the preceding paragraph the inertia of piston may well be considered to be equal to that at the small end D when its whole mass is assumed to be concentrated at the end.

The effect due to the correcting couple Φ_1* Φ_2 and the constant force f may be considered separately later as the corrections.

Hence let the concentrated masses at the small and big ends be $\frac{W}{g}$ and $\frac{w}{g}$ respectively, then

$$\frac{W}{g}=\frac{l'-b}{l'}\,\frac{w_1}{g}+\frac{w_2}{g}, \qquad \frac{w}{g}=\frac{b}{l'}\,\frac{w_1}{g}$$

where $\frac{w_1}{g}$....mass of the connecting rod.

b....distance between small end and the centre of gravity of the rod.

Therefore the inertia force of rotary engine is given as the radial and tangential inertia forces at the two ends C and D as follow :

$$\left\{\begin{aligned} R_c&=\frac{w}{g}a\left(\frac{d\gamma}{dt}\right)^2\\ &=\frac{w}{g}\omega^2a\sum_{n=0}^{\infty}F_n\cos n\theta \quad\cdots\cdots(34)\\ T_c&=-\frac{w}{g}a\frac{d^2\gamma}{dt^2}\end{aligned}\right.$$

$$=\frac{w}{g}\omega^2a\sum_{n=0}^{\infty}D_{2n+1}\sin(2n+1)\theta \quad\cdots\cdots(35)$$

* Refer to the expression (13) § (4) Part I.

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$$\left\{\begin{aligned}R_D&=-\frac{W}{g}\left[\frac{d^2x}{dt^2}-x\omega^2\right]\\&=\frac{W}{g}\omega^2\left\{2r\cos(\theta+\varphi)\right.\\&\quad+a\left[\sum_{n=0}^{\infty}A_{2n}\cos 2n\,\theta+\sum_{n=1}^{\infty}B_{2n}\cos 2n\,\theta\right]\\&\quad\left.+l'\left[\sum_{n=0}^{\infty}A'\cos 2n(\theta+\alpha)+\sum_{n=1}^{\infty}B'_{2n}\cos 2n(\theta+\alpha)\right]\right\}\qquad\ldots(36)\\T_D&=-2\frac{W}{g}\frac{dx}{dt}\omega\end{aligned}\right.$$

$$=2\frac{W}{g}\omega^2\left\{r\sin(\theta+\varphi)+a\sum_{n=0}^{\infty}2n\,A_{2n}\sin 2n\,\theta\right.$$

$$\left.+l'\sum_{n=0}^{\infty}2n\,A'_{2n}\sin 2n(\theta+\alpha)\right\}\ldots\ldots\ldots\ldots(37)$$

In the case of radial engine its inertia force has been given by the expression (9) § (3) Part I, *i, e.*, $F=-\frac{W}{g}\frac{d^2x}{dt^2}$ which is equal to the first term of R_D. Namely in rotary engine the second term of R_D and the expression T_D are introduced in addition to that in radial engine.

The inertia force R_C and T_C may rather be considered as the couple T_Ca rotating the main rod about the crank pin B, as will be stated latter.

(b) Inertia couple of rotary engine

In addition to the above there is inertia couple acting about the crank pin B, which is produced by the revolving mass rotating about its centre of gravity B. The revolving mass consists of one part of the main rod and the rotating part of the ball bearings attached to the rod. Let its moment of inertia be I_B, then the inertia couple C_B' is given as follows :

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$$C_B'=-I_B\frac{d^2\gamma}{dt^2}$$

$$=+I_B\omega^2\sum_{n=0}^{\infty}D_{2n+1}\sin(2n+1)\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots(38)$$

which is equal to the first term of the expression (12) § (4) Part I.

(4) The unbalanced inertia force of the rotary engine.

(a) Composition of inertia forces R_D and T_D.

The inertia force R_D causes an equal force R_D on the crankshaft bearings O, together with a couple $R_D\frac{dx}{dt}\,\frac{1}{\omega}$ on the cylinder and a couple $-R_D\frac{dx}{dt}\,\frac{1}{\omega}$ on the engine frame. Similarly the inertia force T_D causes an equal force T_D on the crankshaft bearings O, together with a couple $T_D\,x$ on the cylinder.

Project these forces R_D and T_D at the crankshaft bearings O on the revolving axes OX_D and OY_D in Fig. 4, then

$$\left.\begin{aligned}X&=R_D\cos\varphi-T_D\sin\varphi\\ Y&=R_D\sin\varphi+T_D\cos\varphi\end{aligned}\right\}$$

or

$$\begin{aligned}X=\frac{W}{g}\omega^2\Bigg\{&2r\cos(\theta+2\varphi)+a\cos\varphi\Bigg[\sum_{n=1}^{\infty}A_{2n}\cos 2n\theta\\ &+\sum_{n=1}^{\infty}B_{2n}\cos 2n\theta\Bigg]-2a\sin\varphi\sum_{n=0}^{\infty}2n\,A_{2n}\sin 2n\theta\\ &+l'\cos\varphi\Bigg[\sum_{n=0}^{\infty}A'_{2n}\cos 2n(\theta+\alpha)+\sum_{n=1}^{\infty}B'_{2n}\cos 2n(\theta+\alpha)\Bigg]\\ &-2l'\sin\varphi\sum_{n=0}^{\infty}2n\,A'_{2n}\sin 2n(\theta+\alpha)\Bigg\}.\end{aligned}$$

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$$\left.\begin{aligned} Y=\frac{W}{g}\omega^2\Big\{&2r\sin(\theta+2\varphi)+a\sin\varphi\Big[\sum_{n=0}^{\infty}A_{2n}\cos\theta\\ &+\sum_{n=1}^{\infty}B_{2n}\cos 2n\theta\Big]+2a\cos\varphi\sum_{n=0}^{\infty}2n\,A_{2n}\sin 2n\theta\\ &+l'\sin\varphi\Big[\sum_{n=0}^{\infty}A'_{2n}\cos 2n(\theta+\alpha)+\sum_{n=1}^{\infty}B'_{2n}\cos 2n(\theta+\alpha)\Big]\\ &+2l'\cos\varphi\sum_{n=0}^{\infty}2n\,A'_{2n}\sin 2n(\theta+\alpha)\Big\} \end{aligned}\right\}\cdots(39)$$

and if we require the resultant projections of the total cylinders h, many terms are balanced out and the following remain as the resultant unbalanced force.

$$\left.\begin{aligned} \Sigma X=\frac{1}{4}\frac{W}{g}\omega^2 rsqh\Big\{&\Big[1+\frac{3}{8}(p^2+s^2q^2)+\ldots\Big]\\ &-\Big[\Big\{1+\frac{3}{8}(p^2+s^2q^2)\Big\}+\frac{9}{8}s^2q^2+\ldots\Big]\cos 2\theta\\ &+\frac{9}{8}s^2q^2[1+\ldots]\cos 4\theta+\ldots\ldots\Big\}\\ \Sigma Y=-\frac{1}{4}\frac{W}{g}\omega^2 rsqh\Big\{&0\\ &-\Big[\Big\{1+\frac{3}{8}(p^2+s^2p^2)-\frac{9}{8}s^2q^2+\ldots\Big]\sin 2\theta\\ &+\frac{9}{8}s^2q^2[1+\ldots]\sin 4\theta+\ldots\ldots\Big\} \end{aligned}\right\}\cdots\cdots(40)$$

In Fig. 5 let OB be the fixed crank and OX_0 and OY_0 be the revolving axes at any instant, then the first term of this expression (40) appears at the angular position OE_0, the former and the latter parts of the second term at OE_2 and OE'_2 and the third term at OE_4. OE_0 is the unbalanced zero harmonics, OE_2 and OE'_2 the second harmonics and OE_4 the fourth harmonics. The fourth harmonics will also be divided into two parts as in the case of the second if we take its higher order. The primary inertia force is balanced by itself.

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Now let us project the above unbalanced force (40) upon fixed axes, taking the crank OB as x-axis and its parpendicular as y-axis, then

$$\left.\begin{aligned}&\Sigma x=0\\&\Sigma y=\frac{1}{2}\,\frac{W}{g}\omega^2 rsqh\left\{\left[1+\frac{3}{8}(p^2+s^2q^2)+\,..\right]\sin\,\theta\right.\\&\qquad\qquad\left.-\frac{9}{8}s^2q^2[1+\,..]\,\sin\,3\theta+\,..\right\}\end{aligned}\right\}\quad....(41)$$

and in Fig. 5 this first term appears at Oe_1 and the second term at Oe_3.

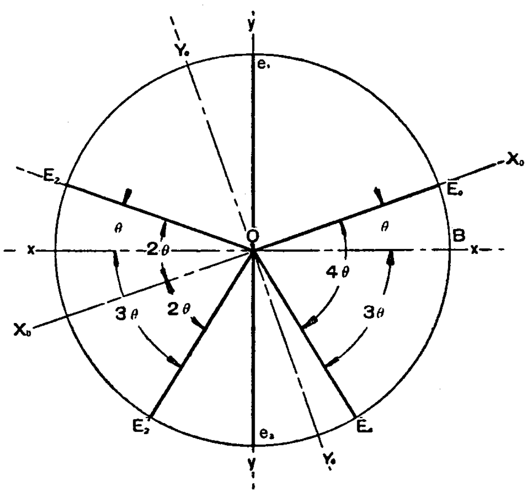

Fig. 5.

Consequently the resultant unbalanced inertia force of rotary engine acts always perpendicular to the fixed crank. Referring to those of radial engine, Oe_1 and Oe_3 correspond to their second and fourth harmonics, and in the magnitude Oe_1 corresponds to about one half of and Oe_3 is nearly equal to them respectively. In radial engine its unbalanced primary force has been balanced by a counter weight attached at the opposite side of crank. In rotary engine, however, its primary force is self-balanced as stated above. In short the unbalanced inertia force of rotary engine is practically equal to one half of that of radial engine and acts always perpendicular to the fixed crank.

If we calculate the above expression (41) by an actual example of 130 *HP* Clerget engine, whose chief dimensions are given in the appendix II, we get

$$\left.\begin{aligned}&\Sigma x=0\\&\Sigma y=[73.1+\,..]\,\sin\,\theta-[.2+\,..]\,\sin\,3\,\theta+\,..\end{aligned}\right\}......\text{in kg.}$$

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(*b*) Compositions of inertia forces R_C and T_C

The inertia forces R_C and T_C give a couple $T_C\ a$ on the main rod and act as eqnal forces R_C and T_C on the erank pin B. Their resultants of the total cylinders h are apparently balanced out.

Let the above couple be C_B'' then

$$C''_B = T_C \times a \times h$$

$$= \frac{w}{g}\omega^2 a^2 h \sum_{n=0}^{\infty} D_{2n+1} \sin(2n+1)\theta \quad \dots\dots\dots\dots\dots\dots (42)$$

Referring to the expression (38) § (3)

$$C_B = C'_B + C''_B$$

$$= \left[I_B + \frac{w}{g}\omega^2 h\right] a^2 \sum_{n=0}^{\infty} D_{2n+1} \sin(2n+1)\theta \quad \dots\dots\dots\dots (43)$$

which is equal to the expression (12) § (4) Part I.

(*c*) Correction due to inertia force f.

The inertia force f which has been given by the expression (32) causes an equal force f on the crankshaft bearings O, together with a couple $f\frac{dx}{dt}\ \frac{1}{\omega}$ on the cylinder and a couple $-f\frac{dx}{dt}\ \frac{1}{\omega}$ on the engine frame. This resultant of the total cylinders h is evidently balanced out.

The effects due to unequalities of the moving mass and wrist radius of each cylinder on the unbalanced inertia force, which have been treated in § (5) Part I in radial engine are omitted in our rotary engine.

(5) The engine turning moment due to the inertia force and couple.

(*a*) due to inertia forces R_D and T_D.

Now let us find the engine turning moment due to the inertia forc e

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and couple. The engine turning moment due to the inertia forces R_D and T_D is given as follows :

$$M_I = R_D \frac{dx}{dt} \frac{1}{\omega} + T_D x$$

$$= -\frac{W}{g} \frac{dx}{dt} \left[\frac{d^2x}{dt^2} \frac{1}{\omega} + x\omega \right]$$

$$= -\frac{W}{g}\omega^2 \left\{ r \sin(\theta+\varphi) + a \sum_{n=0}^{\infty} 2n A_{2n} \sin 2n\theta \right.$$

$$+ l' \sum_{n=0}^{\infty} 2n A'_{2n} \sin 2n(\theta+\alpha) \Big\} \times \left\{ a \left[\sum_{n=0}^{\infty} B_{2n} \cos 2n\theta \right. \right.$$

$$- \left. \sum_{n=0}^{\infty} A_{2n} \cos 2n\theta \right] + l' \left[\sum_{n=1}^{\infty} B'_{2n} \cos 2n(\theta+\alpha) \right.$$

$$\left. \left. - \sum_{n=0}^{\infty} A'_{2n} \cos 2n(\theta+\alpha) \right] \right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(44)$$

In which the first term $-\frac{W}{g} \frac{d^2x}{dt^2} \frac{dx}{dt} \frac{1}{\omega}$ is equal to that in radial engine, and in our case there appears another additional term $-\frac{W}{g} \frac{dx}{dt} x\omega$. If we require the resultant of the total cylinders h we get as follows :

$$\Sigma M_I = -\frac{W}{g} a l' q \omega^2 h \left\{ \frac{1}{2} p_2 \left[1 + \frac{3}{8} p^2 + \ldots \right] \sin\theta \right.$$

$$- \frac{1}{2} q \left[(1+2s+s^2) - \frac{1}{4} s p^2 (2+s) + \ldots \right] \sin 2\theta$$

$$- \frac{9}{16} q^2 s^2 p^2 [1 + \ldots] \sin 3\theta$$

$$\left. + \frac{1}{16} q^3 [1 + 4s + \ldots] \sin 4\theta + \ldots \right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(48)$$

and from our example we get

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$$\Sigma M_I = -[5.54+..]\sin\theta + [17.73+..]\sin 2\theta$$
$$-[.25+..]\sin 4\theta + \ldots\ldots\ldots\ldots\ldots \text{in m. kg.}$$

Referring to the expression (17) § (6) Part I, the first term of the above (45) is equal to that of the radial engine. The second term, however, is unequal and much greater, and in our example the second term corresponds to 3.2 times of the first. Therefore the engine turning moment due to the inertia force is much greater than that in the radial engine.

(*b*) due to inertia couple C_B.

In addition to the above, another engine turning moment is produced by the inertia couple C_B and it is given as follows:

$$M_L = +C_B\left\{1+\frac{d\phi}{dt}\,\frac{1}{\omega}\right\}$$
$$=\left[I_B+\frac{w}{g}a^2h\right]\omega^2\left\{\sum_{n=0}^{\infty}D_{2n+1}\sin(2n+1)\theta\right.$$
$$\left.+\sum_{n=0}^{\infty}E_{2n}\sin 2n\,\theta\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

This is equal to the couple on the engine frame due to this inertia couple C_B in the radial engine, and is much greater than its crank turning moment. If we calculate the above value from our example, we get

$$M_L = [8.61+..]\sin\theta + [1.27+..]\sin 2\theta$$
$$-[.28+..]\sin 3\theta - [.04+..]\sin 4\theta + \ldots\ldots \text{in m. kg.}$$

In this expession as the moment of inertia I_B was obtained with approximation this is but an approximate value. We may, however, interpret the extent of its effect by the above expression.

Adding these two moments, we obtain

$$\Sigma M_I + M_L = [3.07+..]\sin\theta + [19.00+..]\sin 2\theta$$
$$-[.28+..]\sin 3\theta - [.21+..]\sin 4\theta + \ldots\ldots \text{in m. kg.}$$

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In our example the mean torque of engine becomes 87.6 *m. kg.* taking its mechanical efficienty to be 85 % and the greatest fluctuation due to these moment corresponds to 24.2 % of the mean torque. Therefore the fluctuation of the torque curve due to these moment has much greater values compared to that of the radial engine.

(*c*) Corrections due to correcting couples Φ_1, Φ_2 and inertia force *f*.

(i) due to correctieg couple Φ_1.

$$M_{\Phi_1}=\Phi_1\left\{1+\frac{1}{\omega}\ \frac{d\phi'}{dt}\right\}$$

$$=-\frac{w_1}{g}b\ (l'-c)\ \omega^2\left[\sum_{n=0}^{\infty}D'_{2n+1}\sin\ (2n+1)\ (\theta+\alpha)\right.$$

$$\left.+\sum_{n=0}^{\infty}E'_{2n}\sin\ 2n\ (\theta+\alpha)\right]\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(47)$$

and
$$\Sigma M_{\Phi_1}=\frac{w_1}{g}\ b\ (l'-c)\ \omega^2 h\left\{s\,q\left[1+\frac{1}{4}p^2+\ldots\right]\sin\theta\right.$$

$$+\frac{1}{2}\ s_2 q^2\left[1+2p^2+\ldots\right]\sin 2\theta$$

$$\left.-\frac{3}{8}\ s^2q^3\left[1+\ldots\right]\sin 3\theta+\ldots\right\}\ \ldots\ldots\ldots\ldots\ldots\ldots(48)$$

(ii) due to correcting couple Φ_2.

$$M_{\Phi_2}=\Phi_2=-2\frac{w_2}{g}e\ \omega\ \frac{dx}{dt}$$

$$=2\frac{w_2}{g}e\ \omega^2\left[r\sin\ (\theta+\varphi)+a\sum_{n=0}^{\infty}2n\ A_{2n}\sin\ 2n\theta\right.$$

$$\left.+l'\sum_{n=0}^{\infty}2n\ A'_{2n}\sin\ 2n\ (\theta+\alpha)\right]\ \ldots\ldots\ldots\ldots\ldots\ldots(49)$$

and
$$\Sigma M_{\Phi_2}=\frac{w_2}{g}\ ea\omega^2 h\left\{q^2\left[1+s+\frac{1}{4}q^2+\ldots\right]\sin 2\theta\right.$$

$$\left.-\frac{1}{8}q^4\left[1+\ldots\right]\sin 4\theta+\ldots\right\}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(50)$$

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(iii) due to inertia force f.

$$M_f = -f\frac{dx}{dt}\;\frac{1}{\omega} = +\frac{w_2}{\omega}e\omega\frac{dx}{dt} = -\frac{1}{2}M_{\Phi_2} \quad\ldots\ldots\ldots\ldots(51)$$

and $$\Sigma M_f = -\frac{1}{2}\;\frac{w_2}{g}ea\omega^2 h\Big\{q^2\left[1+s+\frac{1}{4}\;q^2+\ldots\right]\sin 2\theta$$

$$-\frac{1}{8}\;q^4\left[1+\ldots\right]\sin 4\theta+\ldots\Big\}\quad\ldots\ldots\ldots\ldots\ldots\ldots(52)$$

If we calculate the sum of these corrections from our example, we get

$$\Sigma M_{\Phi_1}+\Sigma M_{\Phi_2}+\Sigma M_f = [1.97+\ldots]\sin\theta+[.67+\ldots]\sin 2\theta$$

$$-[.03+\ldots]\sin 3\theta-[.01+\ldots]\sin 4\theta+\ldots\ldots\text{in m. kg.}$$

and the effect due to these corrections corresponds to 9·8 % of the resultant moment due to the inertia force and couple $\Sigma M_I + M_L$.

(6) The couple on the engine frame due to the inertia force and couple.

(*a*) due to intertia forces R_D and T_D.

The couple on the engine frame due to the inertia forces R_D and T_D is given as follows :

$$C_I = -R_D\frac{dx}{dt}\;\frac{1}{\omega}$$

$$=\frac{W}{g}\;\frac{dx}{dt}\left[\frac{d^2x}{dt^2}\;\frac{1}{\omega}-x\omega\right]$$

$$=\frac{W}{g}\omega^2\Big\{r\sin(\theta+\varphi)+a\sum_{n=0}^{\infty}2n\,A_{2n}\sin 2n\theta$$

$$+l'\sum_{n=0}^{\infty}2n\,A'_{2n}\sin 2n\,(\theta+\alpha)\Big\}\times\Big\{2r\cos(\theta+\varphi)$$

$$+a\left[\sum_{n=0}^{\infty}B_{2n}\cos 2n\theta+\sum_{n=0}^{\infty}A_{2n}\cos 2n\theta\right]$$

$$+l'\left[\sum_{n=1}^{\infty}B'^{2n}\cos 2n\,(\theta+\alpha)+\sum_{n=0}^{\infty}A'_{2n}\cos 2n\,(\theta+\alpha)\right]\Big\}\quad\ldots(53)$$

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in which the first term $\frac{W}{g}\;\frac{d^2x}{dt^2}\;\frac{dx}{dt}\;\frac{1}{\omega}$ is equal to that in the radial engine, and in our case there appears another additional term $-\frac{W}{g}\;\frac{dx}{dt}x\omega$. If we compare with its crank turning moment (44), the first term is reverse and the second term is equal to that of the expression (44) respectively. And the resultant of the total cylinders h becomes as follows :

$$\Sigma C_I=\frac{W}{g}al'q\omega^2h\left\{\frac{1}{2}p^2\left[1+\frac{3}{8}p^2+\ldots\right]\sin\theta\right.$$

$$+\frac{1}{2}q\left[(1+2s+s^2)-\frac{1}{4}sq^2(2+s+\ldots)\right]\sin 2\theta$$

$$-\frac{9}{16}p^2s^2q^2\left[1+\ldots\right]\sin 3\theta$$

$$\left.-\frac{1}{16}q^3\left[1+4s+\ldots\right]\sin 4\theta+\ldots\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(54)$$

which is equal to the expression (45) in the preceding paragraph except that the sign of its first and third terms are opposite.

(*b*) due to inertia couple C_B.

The couple on the engine frame due to inertia couple C_B is given as follows :

$$C_L=-C_B\frac{d\phi}{dt}\;\frac{1}{\omega}$$

$$=-\left[I_B+\frac{w}{g}a^2h\right]\omega^2\sum_{n=0}^{\infty}E_{2n}\sin 2n\theta\ldots\ldots\ldots\ldots\ldots\ldots\ldots(55)$$

This is equal to its crank turning moment in the radial engine and much less than its crank turning moment in our case. And from our example we get

$$C_L=-[1.27+\ldots]\sin 2\theta+[.04+\ldots]\sin 4\theta+\ldots\ldots\ldots\text{in m. kg.}$$

Adding these two couples in our example, we obtain

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$$\Sigma C_I + C_L = [5.54 + ..] \sin\theta + [16.46 + ..] \sin 2\theta$$

$$-[.21 + ..] \sin 4\theta + \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{in m. kg.}$$

Notwithstanding the smaller value of C_L compared with M_L, the sum $\Sigma C_I + C_L$ has as nearly equal effect on the engine frame as the sum $\Sigma M_I + M_L$ has on the engine torque, and its maximum fluctuation corresponds to 23.3 % of the mean couple due to gas pressure, which is nearly equal to that in the case of radial engine.

(*c*) Corrections due to correcting couples Φ_1, Φ_2 and inertia force *f*.

(i) due to correcting couple Φ_1.

$$C_{\Phi_1} = -\Phi_1 \frac{d\phi'}{dt} \frac{1}{\omega}$$

$$= \frac{w_1}{g} b\,(l'-c)\,\omega^2 \sum_{n=0}^{\infty} E'_{2n} \sin 2n\,(\theta+\alpha) \ldots\ldots\ldots\ldots\ldots (56)$$

and
$$\Sigma C_{\Phi_1} = \frac{w_1}{g} b\,(l'-c)\,\omega^2 \left\{ \frac{1}{2} s^2 q^2\,[1 + 2p^2 ..] \sin 2\theta + .. \right\} \quad \ldots (57)$$

(ii) due to correcting couple Φ_2.

$$C_{\Phi_2} = 0$$

(iii) due to inertia force *f*.

$$C_f = f \frac{dx}{dt} \frac{1}{\omega}$$

$$= \frac{w_2}{g} e\omega^2 \left\{ r \sin(\theta+\varphi) + a \sum_{n=0}^{\infty} 2n\, A_{2n} \sin 2n\,\theta \right.$$

$$\left. + l' \sum_{n=0}^{\infty} 2n\, A'_{2n} \sin 2n\,(\theta+\alpha) \right\} \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

and
$$\Sigma C_f = \frac{1}{2} \frac{w_2}{g} e\,\omega^2 \left\{ q^2 [1 + s + q^2 + ..] \sin 2\theta \right.$$

$$\left. - \frac{1}{8} q^4\,[1 + ..] \sin 4\theta + .. \right\} \ldots\ldots\ldots\ldots\ldots\ldots (59)$$

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Calculating these values for our example we get

$$\Sigma C_{\Phi}+\Sigma C_{2f}=[.71+..]\sin 2\theta-[.01+..]\sin 4\theta+....in\ m.\ kg.$$

Hence the effect due to these corrcetions corresponds to 3.3 % of the couple on the engine frame due to the inertia force and couple ΣC_I+C_L.

Part III Comparison with other types of engine.

We have thus obtained the unbalanced inertia forces and couples of the star type engines. Now let us compare these unbalanced forces with those of types which are commonly used in the aeroplane engine.

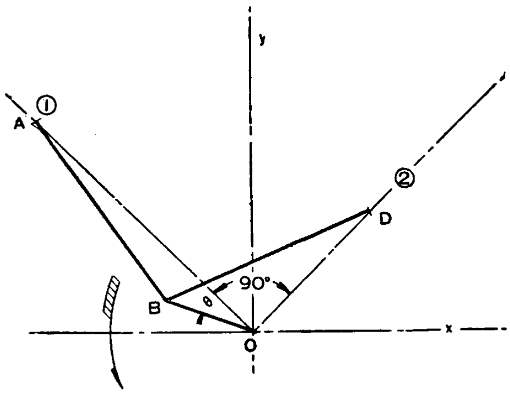

Fig. 6. (a)

(1) 8 cylinder 90° V type.

(*a*) Single obliquity.

In the first place let us consider the 8 cylinder 90° V type engine. In Fig. 6 (*a*), let OB be the crank position, then the unbalanced inertia forces of two rows, each of which has 4 cylinders having the crank angle 180°, are given as follows :

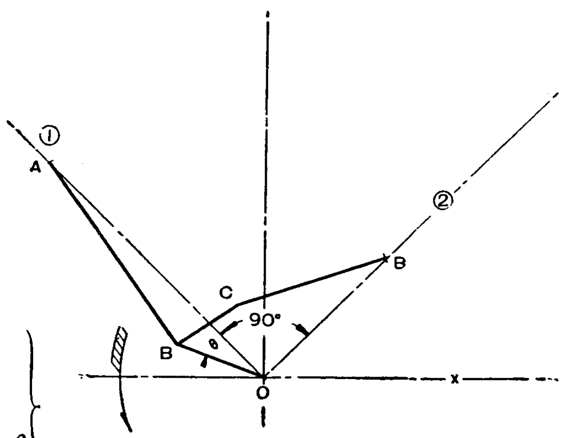

Fig. 6. (b)

$$\left.\begin{aligned}\Sigma F_1&=\frac{W}{g}\omega^2 l\,4\sum_{n=1}^{\infty}B_{2n}\cos 2n\theta\\ \Sigma F_1&=\frac{W}{g}\omega^2 l4\sum_{n=1}^{\infty}(-1)^n B_{2n}\cos 2n\theta\end{aligned}\right\}$$

where B_{2n} has been given in § (1) Part I. And the resultant projections on the horizontal and vertical axes are expressed as follows :

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$$\left.\begin{aligned}\Sigma X&=\frac{W}{g}\omega^2 r\,8.\frac{1}{\sqrt{2}}\Big\{-q\Big[1+\frac{1}{4}q^2+\ldots\Big]\cos 2\theta\\&\quad-\frac{9}{128}q^5[1+\ldots]\cos 6\theta+\ldots\Big\}\\\Sigma Y&=\frac{W}{g}\omega^2 r\,8.\frac{1}{\sqrt{2}}\Big\{-\frac{1}{4}q^3[1+\ldots]\cos 4\theta+\ldots\Big\}\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots(60)$$

(*b*) Double obliquity.

In the case of double obliquity of the second row (Fig. 6 (*b*)), we get

$$\left.\begin{aligned}\Sigma F_1&=\frac{W}{g}\omega^2 l\,4\sum_{n=1}^{\infty}B_{2n}\cos 2n\,\theta\\\Sigma F_2&=\frac{W}{g}\omega^2 4\Big\{a\sum_{n=1}^{\infty}B_{2n}\cos 2n\,\theta\\&\quad+l'\sum_{n=1}^{\infty}B'_{2n}\cos 2n\,(\theta+\alpha)\Big\}\end{aligned}\right\}$$

where B'_{2n} and α have been given in § (1) Part I.

And

$$\left.\begin{aligned}\Sigma X&=\frac{W}{g}\omega^2 r\,8.\frac{1}{\sqrt{2}}\Big\{-q\Big[1+\frac{1}{4}q^2(1+s+2s^2)+\ldots\Big]\cos 2\theta\\&\quad+sq\Big[1+\frac{1}{4}q^2(1+2s+2s^2)+\ldots\Big]\sin 2\theta-\frac{1}{2}sq^3[1\\&\quad-s-s^2+\ldots]\cos 4\theta+\frac{1}{2}sq^3[1+2s+\ldots]\sin 4\theta+\ldots\Big\}\\\Sigma Y&=\frac{W}{g}\omega^2 r\,8.\frac{1}{\sqrt{2}}\Big\{-\frac{1}{4}sq^3[1+2s+\ldots]\cos 2\theta\\&\quad+sq\Big[1+\frac{1}{4}q^2\,(1+2s+2s^2)+\ldots\Big]\sin 2\theta\\&\quad-\frac{1}{4}q^3[1+2s\,(1-s-s^2)+\ldots]\cos 4\theta\\&\quad+\frac{1}{2}sq^3\,[1+2s+\ldots]\sin 4\theta+\ldots\Big\}\end{aligned}\right\}\ldots(61)$$

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(2) 12 cylinder 60° W type.

(*a*) Single obliquity.

In Fig. 7 (*a*) let *OB* be the crank position, then the unbalanced inertia forces of three rows, each of which has 4-cylinders similarly as (1), are given as follow :

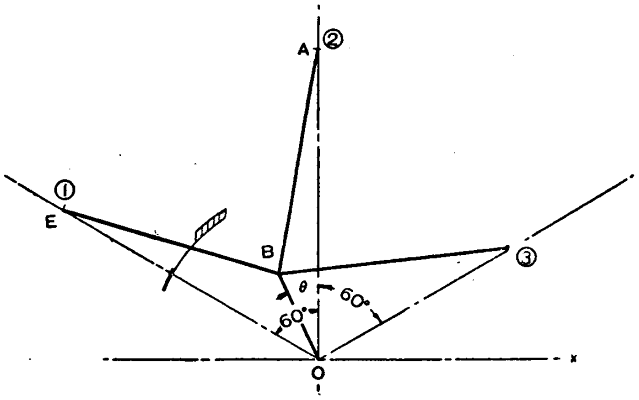

Fig. 7. (a)

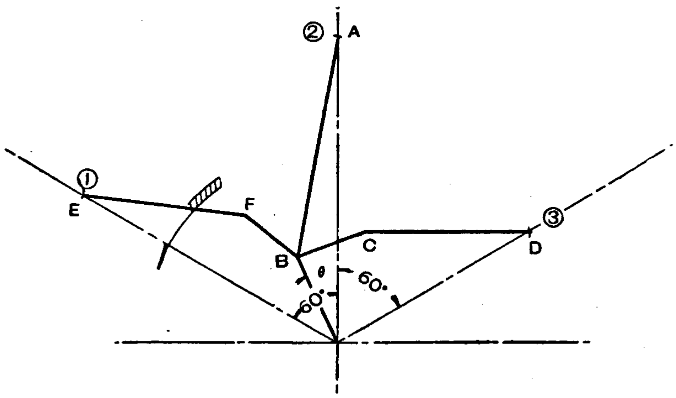

Fig. 7. (b)

$$\left.\begin{aligned}
\Sigma F_1 &= \frac{W}{g}\omega^2 l 4 \sum_{n=1}^{\infty} B_{2n} \cos 2n\,(\theta - 60^\circ) \\
\Sigma F_2 &= \frac{W}{g}\omega^2 l\, 4 \sum_{n=1}^{\infty} B_{2n} \cos 2n\,\theta \\
\Sigma F_3 &= \frac{W}{g}\omega^2 l 4 \sum_{n=1}^{\infty} B_{2n} \cos 2n\,(\theta + 60^\circ)
\end{aligned}\right\}$$

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and

$$\left.\begin{aligned}\Sigma X&=\frac{W}{g}\omega^2 r\ 12.\frac{1}{2}\Big\{-q\Big[1+\frac{1}{4}q^2+..\Big]\sin 2\theta\\&\quad-\frac{1}{4}q^3[1+..]\sin 4\theta+..\Big\}\\\Sigma Y&=\frac{W}{g}\omega^2 r\ 12.\frac{1}{2}\Big\{\ q\Big[1+\frac{1}{4}q^2+..\Big]\cos 2\theta\\&\quad-\frac{1}{4}q^3[1+..]\cos 4\theta+..\Big\}\end{aligned}\right\}\quad........(62)$$

(*b*) Double obliquity.

In this case let the middle row be the row of the main rod cylinder, then :—

$$\left.\begin{aligned}\Sigma F_1&=\frac{W}{g}\omega^2 4\Big\{a\sum_{n=1}^{\infty}B_{2n}\cos 2n\,\theta+l'\sum_{n=1}^{\infty}B'_{2n}\cos 2n\,(\theta-\alpha)\Big\}\\\Sigma F_2&=\frac{W}{g}\omega^2 4l\sum_{n=1}^{\infty}B_{2n}\cos 2n\,\theta\\\Sigma F_3&=\frac{W}{g}\omega^2 4\Big\{a\sum_{n=1}^{\infty}B_{2n}\cos 2n\,\theta+l'\sum_{n=1}^{\infty}B'_{2n}\cos 2n\,(\theta+\alpha)\Big\}\end{aligned}\right\}$$

and

$$\left.\begin{aligned}\Sigma X&=\frac{W}{g}\omega^2 r\ 12.\frac{1}{2}\Big\{-q\Big[(1-s)+\frac{1}{4}q^2(1-s^3)+..\Big]\sin 2\theta\\&\quad-\frac{1}{4}q^3\Big[1+s(2-4s+s^2)+..\Big]\sin 4\theta+..\Big\}\\\Sigma Y&=\frac{W}{g}\omega^2 r\ 12.\frac{1}{2}\Big\{-q\Big[(1-s)+\frac{1}{4}q^2\Big\{1-s\,(2+4s+s^2)\\&\quad+..\Big\}\Big]\cos 2\theta-\frac{1}{4}q^3\Big[1-s(7-7s+s^2)\\&\quad+..\Big]\cos 4\theta+..\Big\}\end{aligned}\right\}..(63)$$

(3) 6 cylinder vertical type.

In this case all cylinders are arranged on the vertical axis having

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the crank angle 120°. Hence the unbalanced inertia force appears only on the y-axis. Namely

$$\left.\begin{aligned} &\Sigma X=0 \\ &\Sigma Y=\frac{W}{g}\omega^2 r\, 6.\left\{\frac{9}{128}q^5[1+\ldots]\cos 6\theta+\ldots\right\} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots (64)$$

(4) 12 cylinder 60° V type.

(*a*) Single obliquity.

This type has two rows of the above 6 cylinders having the crank angle 120° aud being arranged in 60° V. Hence

$$\Sigma F_1=\frac{W}{g}\omega^2 l\, 6 \sum_{n=1}^{\infty} B_{6n}\cos 6n\,\theta$$

$$\Sigma F_2=\frac{W}{g}\omega^2 l\, 6 \sum_{n=1}^{\infty} B_{6n}\cos 6n\,\theta$$

and

$$\left.\begin{aligned} &\Sigma X=0 \\ &\Sigma Y=\frac{W}{g}\omega^2 r\; 12.\frac{\sqrt{3}}{2}\left\{\frac{9}{128}q^5[1+\ldots]\cos 6\theta+\ldots\right\} \end{aligned}\right\} \quad \cdots\cdots\cdots (65)$$

(*b*) Double obliquity.

In this case if we take the second row to be double obliquity, we get

$$\left.\begin{aligned} &\Sigma F_1=\frac{W}{g}\omega^2 l\, 6\sum_{n=1}^{\infty} B_{6n}\cos 6n\,\theta \\ &\Sigma F_2=\frac{W}{g}\omega^2 6\left\{a\sum_{n=1}^{\infty} B_{6n}\cos 6n\,\theta\right. \\ &\qquad\left.+l'\sum_{n=1}^{\infty} B'_{6n}\cos 6n\,(\theta+\alpha)\right\} \end{aligned}\right\}$$

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and
$$\left.\begin{aligned}
\Sigma X &= \frac{W}{g}\omega^2 12.\frac{1}{2}\Big\{\frac{9}{128}aq^6\Big[3-\frac{15}{2}s-20s^2+\,..\Big]\cos 6\theta \\
&\quad -\frac{9\sqrt{3}}{128}aq^6\Big[3+\frac{15}{2}s+\,..\Big]\sin 6\theta+\,..\Big\} \\
\Sigma Y &= \frac{W}{g}\omega^2 12.\frac{\sqrt{3}}{2}\Big\{\frac{9}{128}q^5\Big[r\,(1+\,..)+aq\,(3-\frac{15}{2}s \\
&\quad -20s^2+\,..)\Big]\cos 6\theta \\
&\quad -\frac{9\sqrt{3}}{128}aq^5\Big[3+\frac{15}{2}s+\,..\Big]\cos 6\theta+\,..\Big\}
\end{aligned}\right\}\,..(66)$$

(5) Star type.

(*a*) Radial engine.

In this case we have obtained in § (3) Part I, taking its double obliquity into account, as follows: (Fig. 8 (a))

$$\left.\begin{aligned}
\Sigma X &= \frac{W}{g}\omega^2 r\,h\Big\{sq\Big[1+\frac{3}{8}(p^2+s^2q^2)+\frac{1}{8}s^2q^2\,..+\Big]\cos 2\theta \\
&\quad -\frac{1}{2}s^3q^3\,[1+\,..]\cos 4\theta+\,..\Big\} \\
\Sigma Y &= \frac{W}{g}\omega^2 r\,h\Big\{-sq\Big[1+\frac{3}{8}(p^2+s^2q^2)-\frac{1}{8}s^2q^2+\,..\Big]\sin 2\theta \\
&\quad +\frac{1}{2}s^3q^3\,[1+\,..]\sin 4\theta+\,..\Big\}
\end{aligned}\right\}\,..(10)'$$

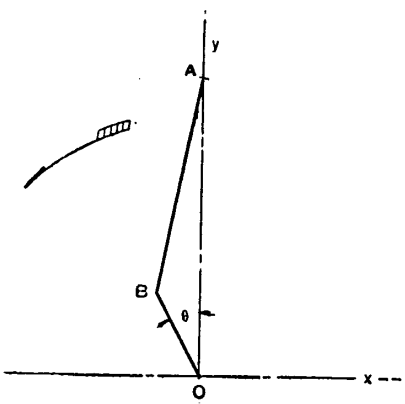

Fig. 8. (a)

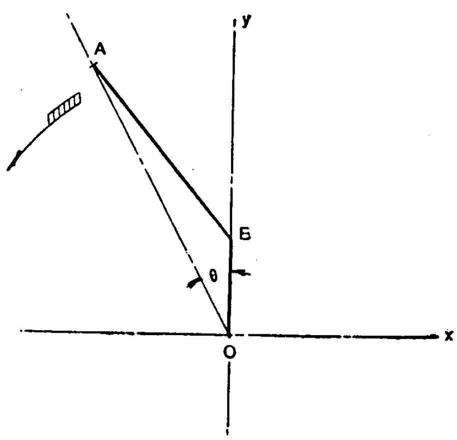

Fig. 8. (b)

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In this engine if we assume it to be of single obliquity, the resultant may be balanced until $(h-2)^{\text{th}}$ harmonics.

(*b*) Rotary engine.

In the rotary engine we have obtained in § (4) Part II as follows:

$$\left.\begin{aligned}\Sigma X=&\frac{W}{g}\omega^2 rh\Big\{-\frac{1}{2}sq\Big[1+\frac{3}{8}(p^2+s^2q^2)+\ldots\Big]\sin\theta\\&+\frac{9}{16}s^3q^3[1+\ldots]\sin 3\theta+\ldots\Big\}\\\Sigma Y=&0\end{aligned}\right\}\ldots\ldots(41)'$$

In this case the resultant may be balanced until $(h-2)^{\text{th}}$ harmonies if we assume it to be single obliquity. (similarly as in the radial eugine)

Now let us compare these resultant unbalanced inertia forces of all types by an actual example. Let $q=.297$ $s=.217$ anp $p=.361$ taking the example of 120 *HP* Clerget rotary engine, then we have

$$(1)\ (a)\quad \left.\begin{aligned}\Sigma X=&\frac{W}{g}\omega^2 r8\Big\{-.215\cos 2\theta-.000\cos 6\theta+\ldots\Big\}\\\Sigma Y=&\quad,,\quad\Big\{-.005\cos 4\theta+\ \ldots\ldots\ldots\ldots\Big\}\end{aligned}\right\}$$

$$(b)\quad \left.\begin{aligned}\Sigma X=&\frac{W}{g}\omega^2 r8\Big\{-.216\cos 2\theta+.047\sin 2\theta\\&\qquad-.002\cos 4\theta+.003\sin 4\theta+\ldots\Big\}\\\Sigma Y=&\quad,,\quad\Big\{-.001\cos 2\theta+.047\sin 2\theta\\&\qquad-.006\cos 4\theta+.003\sin 4\theta+\ldots\Big\}\end{aligned}\right\}$$

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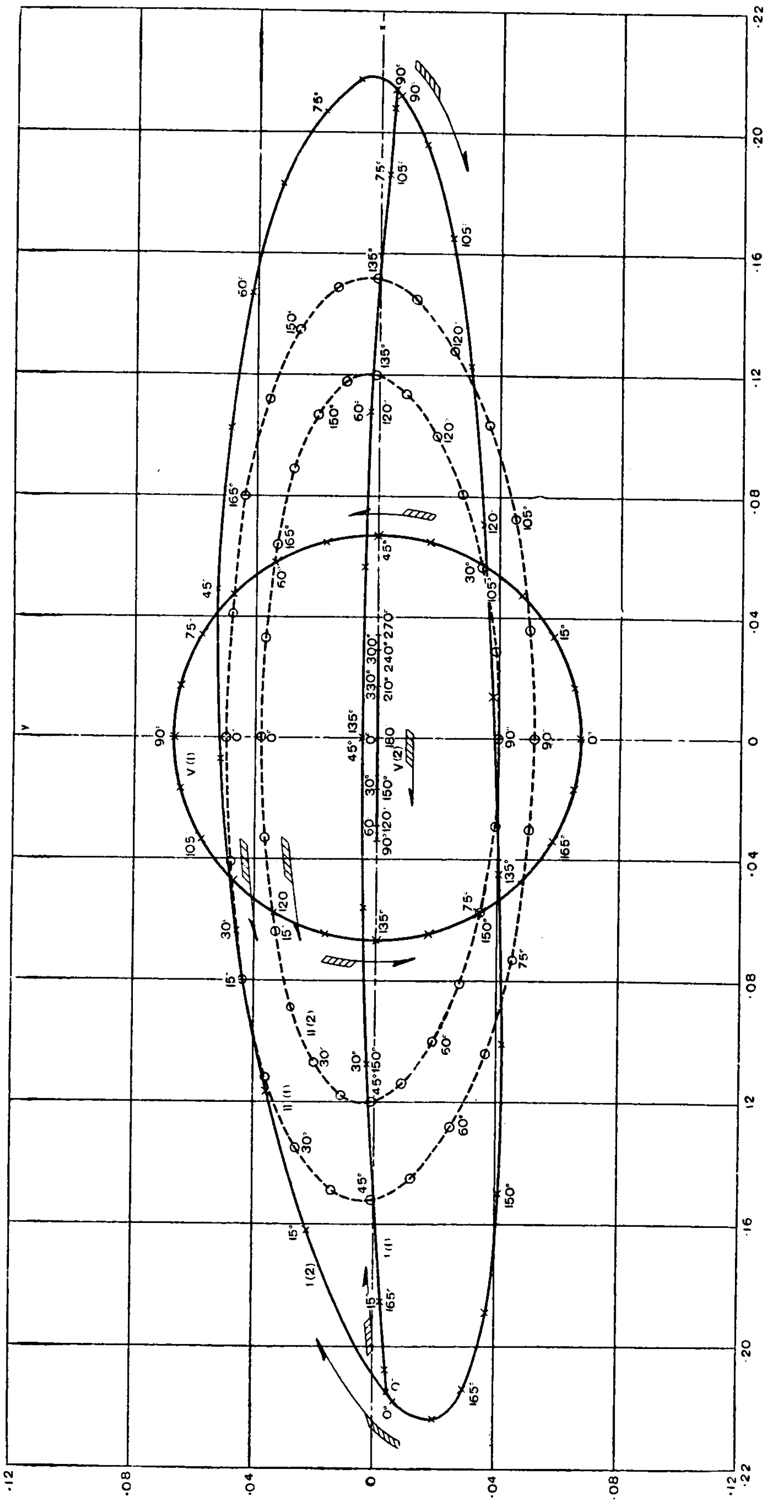

Fig. 9.

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$$(2)\ (a)\quad \left.\begin{aligned} \Sigma X &= \frac{W}{g}\omega^2 r\, 12\left\{-.152 \sin 2\theta - .004 \sin 4\theta + \ldots\right\} \\ \Sigma Y &= \quad ,, \quad \left\{+.051 \cos 2\theta - .001 \cos 4\theta + \ldots\right\} \end{aligned}\right\}$$

$$(b)\quad \left.\begin{aligned} \Sigma X &= \frac{W}{g}\omega^2 r\, 12\left\{-.120 \sin 2\theta - .004 \sin 4\theta + \ldots\right\} \\ \Sigma Y &= \quad ,, \quad \left\{+.039 \cos 2\theta - .001 \cos 4\theta + \ldots\right\} \end{aligned}\right\}$$

$$(3)\quad \left.\begin{aligned} \Sigma X &= 0 \\ \Sigma Y &= \frac{W}{g}\omega^2 r\ 6\left\{.000 \cos 6\theta + \ldots\ldots\ldots\ldots\right\} \end{aligned}\right\}$$

$$(4)\ (a)\quad \left.\begin{aligned} \Sigma X &= 0 \\ \Sigma Y &= \frac{W}{g}\omega^2 r\, 12\left\{.000 \cos 6\theta + \ldots\ldots\ldots\ldots\right\} \end{aligned}\right\}$$

$$(b)\quad \left.\begin{aligned} \Sigma X &= \frac{W}{g}\omega^2 r\, 12\left\{.000 \cos 6\theta + \ldots\right\} \\ \Sigma Y &= \quad ,, \quad \left\{.000 \cos 6\theta + \ldots\right\} \end{aligned}\right\}$$

$$(5)\ (a)\quad \left.\begin{aligned} \Sigma X &= \frac{W}{g}\omega^2 r\ h\left\{+.067 \cos 2\theta - .000 \cos 4\theta + \ldots\right\} \\ \Sigma Y &= \quad ,, \quad \left\{-.067 \sin 2\theta + .000 \cos 4\theta + \ldots\right\} \end{aligned}\right\}$$

$$(b)\quad \left.\begin{aligned} \Sigma X &= \frac{W}{g}\omega^2 r\ h\left\{-.034 \sin \theta + .000 \sin 3\theta + \ldots\right\} \\ \Sigma Y &= 0 \end{aligned}\right\}$$

In these expressions, taking the mass, angular velocity ω and crank radius r to be eqal, the resultant unbalanced inertia force per cylinder of each type is plotted in Fig. 9 with no dimension, in which the above (3), (4)(a), and (4)(b) do not appear as their resultant unbalanced forces are much less than those of other types.

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In 8 cylinder V type the resultant appears practically on the horizontal axis only and its magnitude is much greater than any other type. With its double obliquity the resulant has vertical component corresponding to 22 % of its horizontal. Thus it has nearly a elliptic form having the major axis in the horizontal direction.

In 12 cylinder W type it has more regular elliptic form also having its major axis in the horizontal direction and in its maximum it corresponds to 71 % of that of the 8 cylinder V type. In its double obliquity its magnitude is less than that of single obliquity having the similar elliptic form and corresponds to 56 % of that of the 8 cylinder V type. This is remarkable and we may be able to minimize it by adoption of the double obliquity or offsetting the auxiliary rod cylinder axes.

In the radial engine it assumes practically circular form and corresponds to 31 % of that of the 8 cylinder V type and in the rotary engine it appears only on the horizontal axis and corresponds to 16 % of that of the 8 cylinder V type in our example.

Summary.

The above is one of the methods of treating the inertia forces and couples and their balancings of the star type engines and from the investigation we may arrive at the following remarks :

(1) If we assume that they have single obliquity, the resultant inertia forces will be balanced out until $(h-2)^{th}$ harmonics, *i. e*, they will be balanced out almost completely in both cases. But as the effect of their double obliquity there remains such unbalanced inertia forces as shown in the expression $(10)'$ in the radial engine and as shown in the expression $(41)'$ in the rotary engine. The former draws a circular locus as shown in Fig. 9 and the latter appears only on the horizontal axis and in its magnitude corresponds practically to one half of the former. These resultants per cylinder are smaller compared with those of 8 cylinder 90° V type and 12 cylinder 60°

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W type engines, but much greater than those of 6 cylinder vertical and 12 cylinder 60° V type engines. The method of reducing these unbalanced forces is to take the value of s or a as small as their main rod construction will permit, or in other words to bring the double obliquity as near as possible to the single obliquity.

(2) The effect on the unbalanced inertia force due to the unequalities of mass and s is considerable as shown in § (5) Part I in the case of the radial engine. In the former it is chiefly caused by the unequalness of the reciprocating mass of main rod to that of auxiliay rod. Hence it is necessary to equalize these masses as much as the strength of the main rod will permit. In the latter the unequality of s will be unavoidable and its effect will be smaller than that of the former.

(3) The resultant engine turning moments due to the inertia forces and couples will also be balanced out almost completely if we assume them to be single obliquity. But in consequence of their double obliquity there remains such resultant moments as shown in the expressions (17) and (18) in the radial engine and (45) and (46) in the rotary engine. In the latter this resultant moment gives much greater effect on the engine torque curve than in the former and in its magnitude may correspond to three times of the former. In the latter, however, the revolving cylinders and crankcase act the part of flywheel which is dispenced with in the radial engine and may reduce its greater fluctuation of the torque curve.

(4) The resultant couples on the engine frame are given in the expressions (23) and (24) in the radial engine and (54) and (55) in the rotary engine, which will also be balanced out if assumed to be single obliquity. The couples have almost equal effects on the engine frame torque curves in both cases and may be comparable in their magnitude to that of the engine turning moment in the rotary engine.

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Appendix I Radial engine.

(*a*) Chief dimensions of 320 *HP A,B,C.* "Dragonfly" aircooled engine.

No. of cylinders h	9
Rated HP.	320
Normal $R. P. M.$	1650
Compression ratio	4.42 : 1
Bore × Stroke	$5\frac{1}{2}$ in × $6\frac{1}{2}$ in

(*b*) Lengths of its principal parts.

Crank radius r	82.5 mm.
Main rod l	310.7 mm.
Auxiliary rod l'	254.5 mm.
Wrist pin radius a (mean value)	57.5 mm.
$p=r/l'$	.324
$q=r/l$	.266
$s=a/l'$	.226
Length between the small end and the centre of gravity of rod b	107.1 mm.
Length of equivalent simple pendulum of rod c	195.2 mm.

(*c*) Weights of its principal parts.

Weight of reciprocating part of one cylinder W (mean)	1,770.4 gr.
Weight of one auxiliary rod w_1 (mean)	894.3 gr.
Weight concentrated at the wrist pin C w (mean)	506.9 gr.
Total weight of revolving parts W_B	10,956.2 gr.
Total weight of revolving parts rotating relatively to the crank pin (approximate)	6,530.2 gr.
Moment of inertia of revolving parts I_B (approximate)	2,027.4 mm. gr.

N. B. The revolving parts rotating relatively to the crank pin consists of the crank side of main rod and one part of the two ball bearings

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and their accessories, and the moment of inertia I_B is obtained assuming that the revolving mass has the forms of several concentric rings having their centres at the crank pin.

(d) Values of k and α.

Table I

φ	k	k'	α	α'	$\alpha-\varphi$	$\alpha'-\varphi$
0°	.264	.265	0	0	0	0
20°	.268	.268	24° 25′	24° 19′	4° 25′	4° 19′
40°	.281	.282	47° 56′	47° 53′	7° 56′	7° 53′
60°	.299	.299	70° 2′	70° 9′	10° 2′	10° 9′
80°	.319	.319	90° 40′	90° 58′	10° 40′	10° 58′
100°	.340	.340	110° 2′	110° 17′	10° 2′	10° 17′
120°	.358	.358	128° 22′	128° 26′	8° 22′	8° 26′
140°	.372	.372	145° 58′	145° 55′	5° 58′	5° 55′
160°	.381	.380	163° 6′	163° 1′	3° 6′	3° 1′
180°	.384	.383	180° 0′	180° 0′	0°	0°

The values of k' and α' are those when a *or* s are corrected to keep the compression ratio of each cylinder equal.

(e) Measurement and calculation of the corrected values of a and s.

Table II

Cylinder No.	a. Measured	a. Calculated	$s=\frac{a}{l'}$	Deviation from mean value	% Deviation
Main	56.2mm.	56.2mm.	.221	−.0053	−2.35
1 or 8	57.2	57.2	.225	−.0013	−0.58
2 „ 7	59.0	59.0	.232	+.0057	+2.51
3 „ 6	58.1	58.2	.229	+.0026	+1.15
4 „ 5	56.3	56.4	.222	−.0043	−1.90
Mean value		57.5	.226		

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(*f*) Measurement of the reciprocating weight of each cylinder.

Table III

Cylinder No.	Piston with its accesories	Gudgeon pin	Piston side of con. rod.	Sum	Deviation from mean value	% Deviation
Main	1,217.6gr.	236.3gr.	423.4gr.	1,867.3gr.	+96.9gr.	+5.48
1	1,246.2	226.8	302.6	1,775.6	+ 5.2	+ .29
2	1,212.4	226.6	298.5	1,737.5	+32.9	−1.86
3	1,241.4	226.7	298.1	1,766.2	− 4.2	− .24
4	1,241.3	225.6	300.6	1,767.5	− 2.9	− .16
5	1,244.7	220.9	291.9	1,757.5	−12.9	− .71
6	1,209.7	226.0	292.2	1,727.9	−42.5	−2.40
7	1,236.3	225.1	299.4	1,760.8	− 9.6	− .54
8	1,248.1	225.6	299.2	1,772.9	+ 2.6	+ .14
Mean.	1,233.1	225.5	311.8	1,770.4		

(*g*) Figures.

Fig. 1

Values of $k=\sqrt{p^2+s^2q^2-2psq\cos\varphi}$ and $\alpha-\varphi\ \tan^{-1}\dfrac{a\sin\varphi}{l-a\cos\varphi}$.

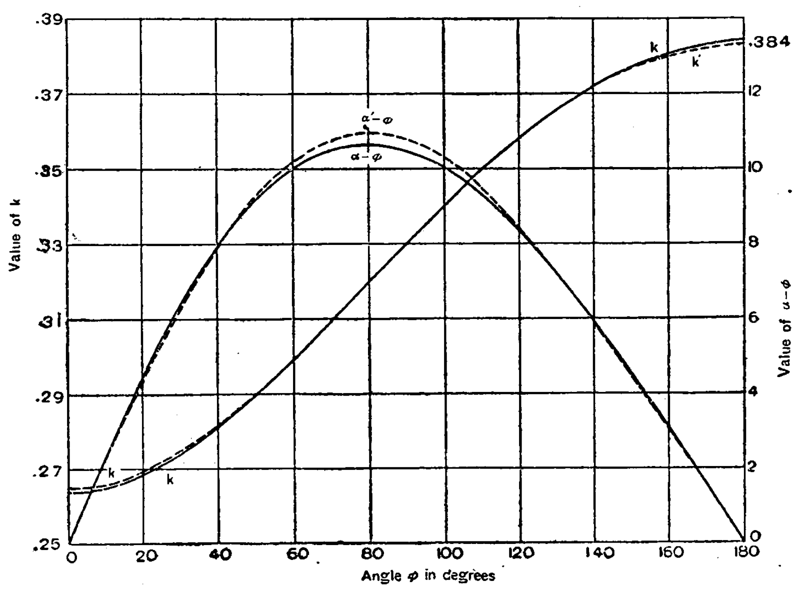

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Fig. 2

Inertia forces of main and No. 4 cylinders.

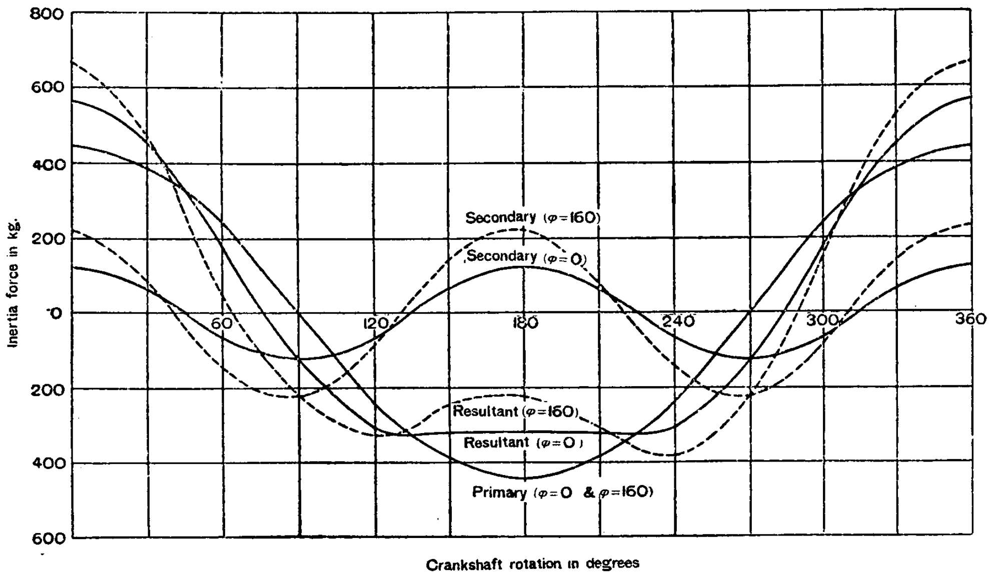

Fig. 3

Unbalanced inertia forces due to unequalilies of mass and s projected on x-axis.

(1) Primary unbalanced inertia force due to unequality of the reciprocating mass.

(2) Secondary unbalanced inertra force due to unequality of the reiprocating mass.

(3) Unbalanced inertia force due to unequality of the length s.

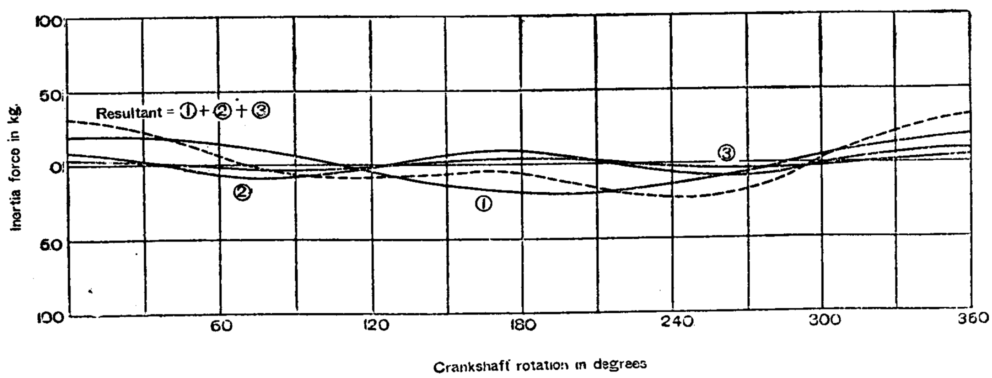

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Fig. 4

The effect on the usual unbalanced inertia force due to unequalities of mass and s projected on x-axis.

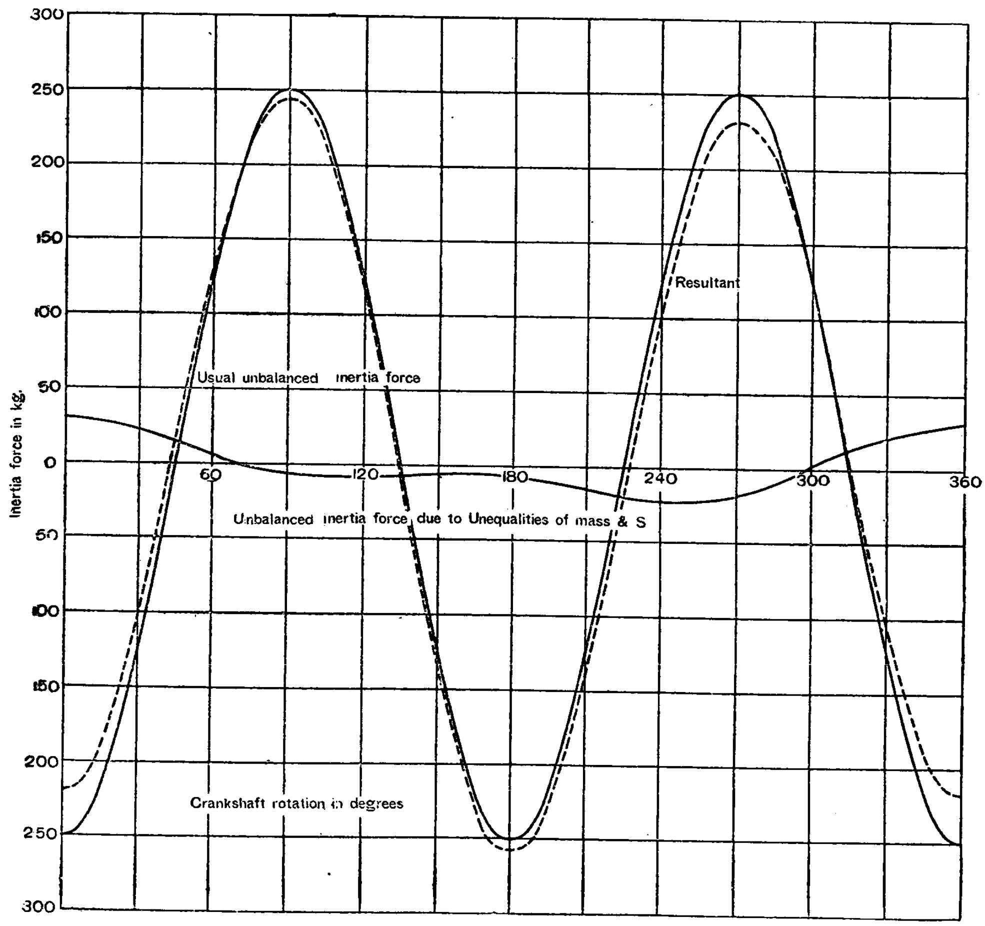

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Fig. 5

Torque Curve of *A*, *B*, *C*. "Dragonfly" engine (1).

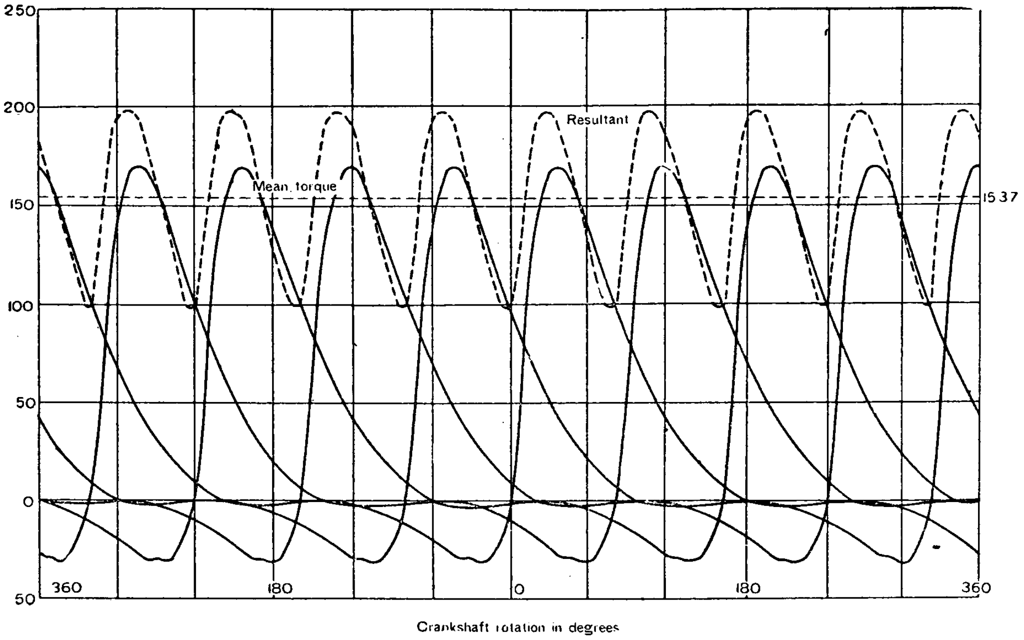

Fig. 6

Torque Curue of *A*, *B*, *C*. "Dragonfly" engine (2).

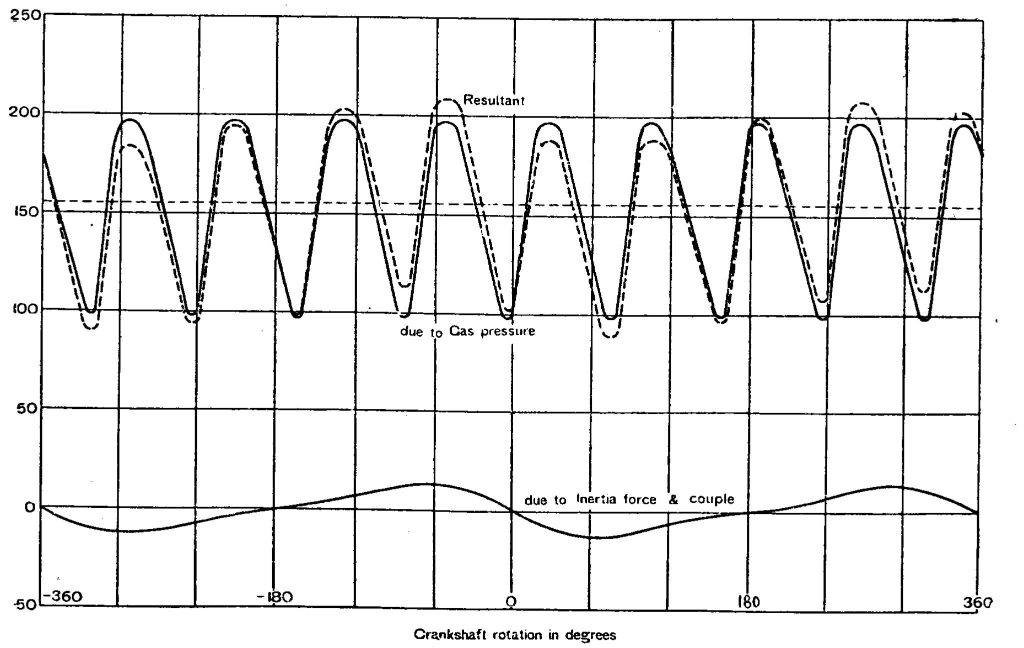

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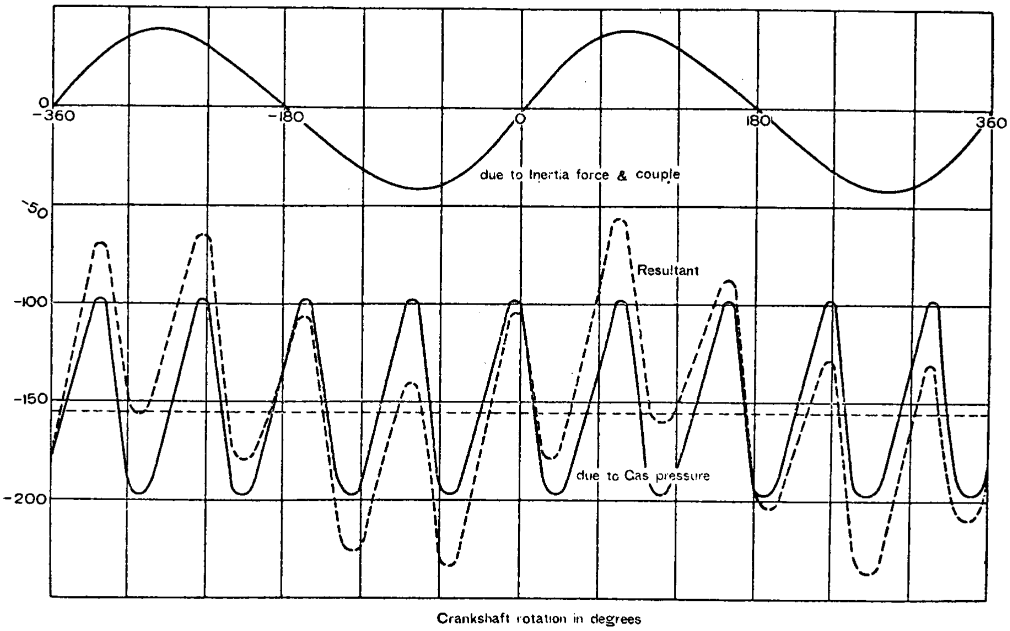
Fig. 7

Engine frame torque curve of *A, B, C.* "Dragonfly" engine.



Appendix II Rotary engine.

(*a*) Chief dimensions of 130 *HP.* Clerget rotary engine.

No. of cylinders h	9
Rated *HP*	130
Normal *R. P. M.*	1250
Compression ratio	4:1
Bore × Stroke	120 mm. × 160mm.

(*b*) Lengths of its principal parts.

Crank radius r	80.0 mm.
Main rod l	269.5 mm.
Auxiliary rod l'	221.5 mm.
Wrist radius a	48.0 mm.
$p=r/l'$	.361

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$q=r/l$	.297
$s=a/l'$	.217
Length DG e	14.4 mm.
Length between the small end and the centre of gravity of rod b	75.0 mm.
Length of equivalent simple pendulum of rod c	194.5 mm.

(*c*) Weights of its principal parts.

Weight of piston and its accessories w_2 (mean)	1,123.8 gr.
Weight concentrated at the small end D W (mean)	1,714.0 gr.
Weight of connecting rod and gudgeon pin w_1 (mean)	935.6 gr.
Weight concentrated at wrist pin C w (mean)	480.1 gr.
Weight of revolving mass concentrated at crank pin B (approximate)	2,558.7 gr.
Momont of inertia of the revolping mass I_B (approximate)	729.67 mm. gr.

N. B. The revolving part concentrated at the crank pin B consists of the crank side weight of main rod and 40% of two ball bearings' weight approximately, and its moment of inertia is obtained assuming that the revolving mass has the forms of two rings the inner and outer radius being 43 mm. and 61 mm. respectively and their thickness to be 28 mm.

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(d) Values of k and α.

φ	k	α	$\alpha-\varphi$
0	.297	0°	0°
20	.302	24° 5′	4° 5′
40	.315	47° 25′	7° 25′
60	.334	69° 30′	9° 30′
80	.356	90° 10′	10° 10′
100	.377	109° 35′	9° 35′
120	.397	128° 5′	8° 5′
140	.412	145° 45′	5° 45′
160	.422	163° 0′	3° 0′
180	.425	180°	0°

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第十號

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抄　　錄

星型發動機の不平衡力．不平衡偶力及びその平衡法．

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氣筩が放射狀に配列せる星型發動機に働く慣性．慣性偶力等を論ずるに當りてその固定式なると廻轉式なるとを問はず普通簡單にその構造を一般の發動機の如く單傾斜の構造と假定する．然れども星型發動機は多くの場合主連桿副連桿の構造を有して復傾斜となる．本論文はこの復傾斜を考慮してこの種の發動機に働く不平衡力．不平衡偶力等を求め且つそれ等の平衡法に就て多少論ぜしものにしてその概要は下の如し。

1. 單傾斜と假定せば發動機に働く慣性は h を氣筩數とせば $(h-2)$ 次まで．即ち殆んど完全に平衡の狀態となるものなるが復傾斜の結果固定式の場合は (10) 式廻轉式の場合は (41) 式で示せる不平衡力が殘る．之を圖示せば第九圖に於て前者は圓周の軌跡を畫き後者は水平軸の方向にのみ働きてその大きさは前者の 1/2 に等し。

2. 此の星型發動機の一氣筩當りの不平衡力は 8 氣筩V型12氣筩W型のものより小にして 6 氣筩垂直．12氣筩V型のものよりも遙に大なり．此の不平衡力を減少せしむる方法は主連桿の構造の許す限り肘桿栓の距離 a を小さくする事にして換言せば復傾斜を出來得る限り單傾斜に近ける事なり。

3. 發動機の運動部分の質量の不同及び肘桿栓の距離 a の不同が發動機の不平衡力に及ぼす影響は可なりのものにして固定式の場合に就て第一章第五節に論ぜり．質量の不同は主として主連桿と副連桿の質量不同に基くものなれば出來得る限りそれ等の質量を等くする事必要なり． a の不同は壓縮比を一定にする爲に避け難きものにしてその影響は前者よりも小なり。

4. 慣性．慣性偶力に依る發動機の廻轉能率も單傾斜と假定せば殆んど完全に平衡狀態となる可きものにして復傾斜の結果固定式は(17)と(18)式廻轉式は(45)と(46)式で與へらるる廻轉能率を惹起す．後者は前者より能率曲線に及ぼす影響大にしてその大さ前者の三倍に相當す．然れども後者に於ては廻轉する氣筩及び曲柱室がはずみ車の役目を演じその大なる影響を減殺するものと考へ得る。

5. 發動機の「フレーム」に與ふる偶力も復傾斜の結果固定式は(23)と(24)

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式廻轉式は (54) と (55) 式で示さるゝ不平衡偶力を生す. 而して此れ等の不平衡偶力の「フレーム」の能率曲線に與ふる影響は略等くして廻轉式の場合の廻轉能率の影響と等しき程度のものなり。

本論文は栖原教授の懇切なる指導の賜なり. 玆に厚く謝意を表する次第なり。

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