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抄　錄

風洞試驗に於ける周壁補正に就いて

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圓形風洞による翼の試驗に於ける周壁補正の式は翼を風洞の中央に置ける場合に對して既に Prandtl の論ぜるものなり. こゝに述べんとする問題は翼を風洞の中央に置かずして實驗せる場合に對する補正の式に關することにして,最も簡單なる一例として翼の特別の形即ち揚力分布の一樣なるものに就いて Glauert の導入せる補正係數 $\bar{\eta}$ の値として

$$\bar{\eta} = \frac{1}{32\lambda^2}\log\frac{\{(1-\zeta_1)^2+4\lambda\mu(1-\zeta_1)+4\lambda^2\}\{(1-\zeta_2)^2-4\lambda\mu(1-\zeta_2)+4\lambda^2\}}{(1-\zeta_1)^2(1-\zeta_2)^2}$$

を得たり. 但し a は風洞の半徑, $2b$ は翼の幅, y_0 は風洞の中心より翼に下せる垂線の長さ, x_0 はその垂線の足と翼の中心との距離を表はし, ζ_1, ζ_2, λ, μ 等は

$$\zeta_1 = \frac{(x_0+b)^2+y_0^2}{a^2}, \qquad \zeta_2 = \frac{(x_0-b)^2+y_0^2}{a^2},$$

$$\lambda = \frac{b}{a}, \qquad \mu = \frac{x_0}{a}$$

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On the Wall Interference of a Circular Wind Tunnel.

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§ 1. In 1933[1] H. Glauert[2] summalized the existing results of discussions on the wall interference of wind tunnels on wings, etc. Most practically important cases are already treated and only a few problems on this subject are left untouched. The correction formula concerning the wing placed at the centre of a wind tunnel of circular cross-section is obtained as is well known by Prandtl and is proved to be very usefull in practice.

The problem which I propose to discuss in this note is that to obtain the correction formula for the wing of which the position is not at the centre of the tunnel. The result may not be so important as Prandtl's and only be of seldom use in a special problem of wind tunnel measurements.

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(1) Aeronautical Research Committee, London, R. & M. No. 1566.

(2) In last summer I was informed that a fatal accident caused the death of Mr. Hermann Glauert. This short note is written partly in the meaning to extend my deepest sympathy of condolence to the bereaved and to express my greatest regret of the tragically suddden departure of one of my scientific friends. Various symbols used here are almost the same as those in his last paper quoted above.

§ 2. In the annexed figure the section of the wind tunnel, open or jet type, in which the wing is placed is shown. BA is a wing of which the aerodynamical characteristics are to be measured, C being its centre.

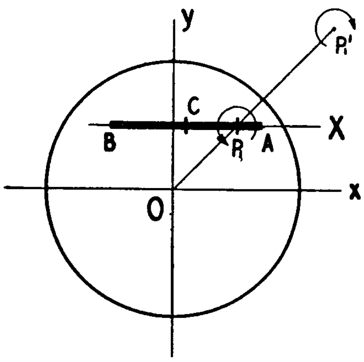

Fig. 1.

Take the coordinate axes (x, y) passing throught the centre of the circular section, the x-axis being parallel to the wing to starboard. Along the wing take another axis X, the origin being at C. Let the coordinates of the point C be (x_0, y_0), and the radius of the section be a. Suppose that at a point P_1 on the span of the wing to starboard of which $CP_1 = X_1$ there is a trailing vortex of strength K. The image vortex of strength $-K$ occurs at the inverse point P_1' of the point P_1 with respect to the circle. The coordinates of the points P_1 and P_1' are (x_1, y_1) and (x_1', y_1') respectively, where

$$x_1 = x_0 + X_1 , \qquad y_1 = y_0 ,$$

$$x_1' = \frac{a^2(x_0+X_1)}{(x_0+X_1)^2+y_0^2} , \qquad y_1' = \frac{a^2 y_0}{(x_0+X_1)^2+y_0^2} .$$

At any point P whose coordinates are (x_0+X, y_0) on the span of the wing there is an upward induced velocity, due to the image, of magnitude

$$v = -\frac{K}{4\pi} \frac{x_0+X-x_1'}{(x_0+X-x_1')^2+(y_0-y_1')^2} ,$$

in which the factor 4π is used instead of 2π as usual.

When there is another vortex of strength $-K$ at the point P_2 which is situated on the wing and at an equidistance as P_1 from C but to the port side. The induced velocity, due to both images, at any point P on the span of the wing becomes

$$v = \frac{K}{4\pi}\left\{\frac{\zeta_1(x_0+X_1)(1-\xi_1)}{(x_0+X_1)^2(1-\xi_1)^2+y_0^2(1-\zeta_1)^2} - \frac{\zeta_2(x_0-X_1)(1-\xi_2)}{(x_0-X_1)^2(1-\xi_2)^2+y_0^2(1-\zeta_2)^2}\right\},$$

where

$$\zeta_1 = \frac{(x_0+X_1)^2+y_0^2}{a^2}, \quad \xi_1 = \frac{x_0+X}{x_0+X_1}\zeta_1,$$

and

$$\zeta_2 = \frac{(x_0-X_1)^2+y_0^2}{a^2}, \quad \xi_2 = \frac{x_0+X}{x_0-X_1}\zeta_2.$$

The coefficient η introduced by Glauert is defined by

$$\eta = \frac{v}{V}\cdot\frac{C}{S}\cdot\frac{1}{k_L},$$

where V denotes the tunnel velocity, C the area of the wing, S the area of the cross-section of the tunnel and k_L the lift coefficient. The mean value of η, weighted according to the distribution of lift L across the span, is

$$\bar{\eta} = \frac{1}{L}\int \eta\, dL.$$

This is equivalent to the correction coefficient calculated by Prandtl in special cases. Thus for any distribution of the lift L, using the above expression of v, we can find the value of $\bar{\eta}$ in the present case by a single integration.

§ 3. Let us try the calculation of $\bar{\eta}$ in the simplest case in which the distribution of lift is uniform along the span of the wing, *i.e.* a system of horseshoe vortices. If we denote the span $2b$, we may put b in place of X_1 in the expressions of v, ζ and ξ. On remembering that

$$VSk_L = 2bK. \quad \frac{dL}{L} = \frac{dX}{2b}, \quad S = \pi a^2,$$

we have

$$\bar{\eta} = \frac{a^2}{16b^2}\int_{-b}^{b}\left\{\frac{\zeta_1(x_0+b)(1-\xi_1)}{(x_0+b)^2(1-\xi_1)^2+y_0^2(1-\zeta_1)^2} - \frac{\zeta_2(x_0-b)(1-\xi_2)}{(x_0-b)^2(1-\xi_2)^2+y_0^2(1-\zeta_2)^2}\right\}dX,$$

where

$$\zeta_1 = \frac{(x_0+b)^2+y_0^2}{a^2} = \frac{\mathrm{OA}^2}{a^2}, \quad \xi_1 = \frac{x_0+X}{x_0+b}\zeta_1,$$

and

$$\zeta_2 = \frac{(x_0-b)^2+y_0^2}{a^2} = \frac{\mathrm{OB}^2}{a^2}, \quad \xi_2 = \frac{x_0+X}{x_0-b}\zeta_2.$$

Or changing the integration variable,

$$\bar{\eta} = \frac{a^2}{16b^2}\left\{\int_{\xi_1(-b)}^{\zeta_1}\frac{(x_0+b)^2(1-\xi_1)}{(x_0+b)^2(1-\xi_1)^2+y_0^2(1-\zeta_1)^2}d\xi_1\right.$$

$$\left.-\int_{\zeta_2}^{\xi_2(b)}\frac{(x-b)^2(1-\xi_2)}{(x_0-b)^2(1-\xi_2)^2+y_0^2(1-\zeta_2)^2}d\xi_2\right\},$$

where

$$\xi_1(-b) = \frac{x_0-b}{x_0+b}\zeta_1, \quad \xi_2(b) = \frac{x_0+b}{x_0-b}\zeta_2.$$

Performing the integration and introducing non-dimensional quantities

$$\lambda = \frac{b}{a}, \quad \mu = \frac{x_0}{a},$$

we have

$$\bar{\eta} = \frac{1}{32\lambda^2}\log\frac{\{(1-\zeta_1)^2+4\lambda\mu(1-\zeta_1)+4\lambda^2\}\{(1-\zeta_2)^2-4\lambda\mu(1-\zeta_2)+4\lambda^2\}}{(1-\zeta_1)^2(1-\zeta_2)^2}.$$

If we put $x_0 = 0$, $y_0 = 0$ in the above, then

$$\zeta_1 = \zeta_2 = \frac{b_2}{a^2} = \lambda^2 ,$$

and, writing ζ instead of λ^2, we obtain

$$\bar{\eta} = \frac{1}{8} \log \frac{1+\zeta}{1-\zeta} ,$$ (1)

which is the result obtained by Glauert in the case where the wing is placed at the central part of the tunnel.

(1) R. & M. *loc. cit.* p. 14.