第百二十七號

抄　錄

熱對流の問題，特にシュミットの實驗に對する理論的説明に就て

所員　佐々木達治郎

冷い流体が温い流体の下を流れる時は冷い流体は層をなして温い流体を押し除けて進み，層の前部は張れ上り頭の如き形をなすことはシュミットの實驗に於て知られて居ることである．本論文は熱對流の問題として取扱ひ此實驗に對する理論的説明を與へたものである．

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On a Problem of Heat Convection with Special Reference to the Theoretical Explanation of Schmidt's Experiment.

By

Tatudirô SASAKI, *Rigakuhakusi*,

Member of the Institute.

I. Introduction.

It is a well known fact that when a current of cold fluid encounters a warm fluid, it sinks down and rushes forward buoying up the adjacent warm fluid. The mode of motion of the fluid in such a case was investigated a long time ago by W. Schmidt[1], in order to explain the mechanism of line squall. Schmidt's experiments were carried out in a long rectangular vessel by making a colder and heavier fluid flow under a warmer and lighter fluid. Beneath the warmer fluid the colder fluid rushes forward, forming a sheet, the farthest end of which forms not only a wedge like shape, but is also a little swollen up like a head. It is evident by the observation of the motions of fluid particles that a vortex is set up in this part of the fluid and mixing of cold and warm fluids takes place.

An attempt is made here to examine how far the interesting results obtained by Schmidt can be explained theoretically.

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(1) W. Schmidt, Met. Zeitschr. 1911, S. 357.

II. Theoretical Investigation.

Instead of the cold fluid flows under the warm fluid, we consider the cold fluid is at rest and the warm fluid flows over the cold one with a mean velocity U opposite that of the cold fluid. In order to avoid the complication of the problem, we limit it to two dimensions and moreover neglect the viscosity of the fluid.

The equations of fluid motion are in the usual notation : —

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \ , \quad \frac{Dv}{Dt} = -g-\frac{1}{\rho}\frac{\partial p}{\partial x} \ ,$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t}+(U+u)\frac{\partial}{\partial x}+v\frac{\partial}{\partial y} \ .$$

In these equations the density ρ is variable in consequence of variable temperature and pressure. But, as was shown by Boussinesq and justified by Lord Rayleigh[2], in the case under consideration the influence of pressure is not important and the variation with temperature is small, so we neglect the variation of density, except in as far as they modify the action of gravity.

Hence we can wright

$$\rho = \rho_0 - \rho_0 \alpha \theta \ ,$$

where θ is the temperature reckoned from the initial temperature of the warmer fluid, ρ_0 is the density at that temperature and α is the expansion coefficient. If we consider u, v and θ are small, and neglect the squares of small quantities, the equations of motion reduce to

$$\frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x} \ , \quad \frac{\partial v}{\partial t}+U\frac{\partial v}{\partial x} = -\frac{1}{\rho_0}\frac{\partial P}{\partial y}+g\alpha\theta \ ,$$

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(2) Scientific Papers Vol. VI, p. 435.

where

$$P = p + g\rho_0 y .$$

Since we treat the fluid as incompressible the equation of continuity becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 .$$

The equation for the conduction of heat becomes when θ is small

$$\frac{\partial \theta}{\partial t} + U \frac{\partial \theta}{\partial x} = \kappa \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) ,$$

in which κ is the diffusibility for temperature.

For the sake of simplicity we now treat the problem in the following way. A fluid of density ρ_0 and the temperature Θ_0 flows from $-\infty$ in the direction of positive x-axis with a mean velocity U in a rectangular vessel of depth h, the upper wall and left half of the bottom of which are maintained at the temperature Θ_0 and the right half of the bottom is maintained at the temperature $\Theta_0 - \Theta$. When the motion is steady the equations of motion and the heat conduction reduce to

$$\left. \begin{array}{l} U \dfrac{\partial u}{\partial x} = -\dfrac{1}{\rho_0} \dfrac{\partial P}{\partial x} , \quad U \dfrac{\partial v}{\partial x} = -\dfrac{1}{\rho_0} \dfrac{\partial P}{\partial y} + \gamma\theta , \\[4mm] U \dfrac{\partial \theta}{\partial x} = \kappa \left(\dfrac{\partial^2 \theta}{\partial x^2} + \dfrac{\partial^2 \theta}{\partial y^2} \right) \end{array} \right\} \quad \dots\dots\dots (1)$$

with the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 ,$$

where γ stands for ga.

The boundary conditions to be satisfied are

$$
\left.
\begin{aligned}
& v = 0 \quad \text{at} \quad y = 0 \quad \text{and} \quad y = h, \\
& \theta = 0 \quad \text{at} \quad y = h, \\
& \theta = F(x) \quad \text{at} \quad y = 0, \\
& \text{where} \quad F(x) = 0 \quad \text{when} \quad x < 0, \\
& \qquad\qquad\quad = -\theta \quad \text{when} \quad x > 0.
\end{aligned}
\right\} \quad \cdots\cdots\cdots (2)
$$

The appropriate solution of the equation of heat conduction is

$$
\theta = \Re\frac{1}{\pi}\int_0^\infty\int_{-\infty}^\infty F(\lambda)e^{ia(x-\lambda)}\frac{\sinh\beta(h-y)}{\sinh\beta h}\,da\,d\lambda \cdots\cdots\cdots (3)
$$

where $\quad \beta = \sqrt{a^2 + i\dfrac{U}{\kappa}a}$.

If we introduce the stream function ψ, we have from equations (1)

$$
U\frac{\partial}{\partial x}\nabla^2\psi = -\gamma\frac{\partial\theta}{\partial x}.
$$

From this relation we have, since $v = -\dfrac{\partial\psi}{\partial x}$,

$$
\nabla^2 v = \Re\frac{i\gamma}{\pi U}\int_0^\infty\int_{-\infty}^\infty F(\lambda)e^{ia(x-\lambda)}a\frac{\sinh\beta(h-y)}{\sinh\beta h}\,da\,d\lambda.
$$

The solution of this equation which satisfies the conditions (2) is

$$
v = -\Re\frac{\kappa\gamma}{\pi U^2}\int_0^\infty\int_{-\infty}^\infty F(\lambda)e^{ia(x-\lambda)}\left[\frac{\sinh a(h-y)}{\sinh ah} - \frac{\sinh\beta(h-y)}{\sinh\beta h}\right]da\,d\lambda.
$$

$$
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (4)
$$

The expression for ψ can be obtained from this equation, integrating it with respect to x and changing the sign, as follows :—

$$\psi = \Re \frac{\kappa \gamma}{\pi U^2} \int_0^\infty \int_{-\infty}^\infty F(\lambda) \frac{e^{ia(x-\lambda)}}{ia} \left[\frac{\sinh a(h-y)}{\sinh ah} - \frac{\sinh \beta(h-y)}{\sinh \beta h} \right] da\, d\lambda .$$

$$\dotfill (5)$$

III. Integration of the expression for θ.

In the first place we integrate the expression for θ. For that sake we first perform the integration with respect to a i.e.

$$I_1 = \int_0^\infty e^{ia(x-\lambda)} \frac{\sinh \sqrt{a^2 + i\frac{U}{\kappa}a}\,(h-y)}{\sinh \sqrt{a^2 + i\frac{U}{\kappa}a} \cdot h}\, da .$$

Let

$$I_2 = \int_0^\infty e^{-ia(x-\lambda)} \frac{\sinh \sqrt{a^2 - i\frac{U}{\kappa}a}(h-y)}{\sinh \sqrt{a^2 - i\frac{U}{\kappa}a} \cdot h}\, da .$$

Since I_1 and I_2 are conjugate, if $I_1 = X_1 + iY_1$, then $I_2 = X_1 - iY_1$. Therefore

$$I_1 + I_2 = 2X_1 = 2\,\Re I_1 \dotfill (6)$$

But since

$$I_2 = -\int_0^{-\infty} e^{ia(x-\lambda)} \frac{\sinh \sqrt{a^2 + i\frac{U}{\kappa}a}(h-y)}{\sinh \sqrt{a^2 + i\frac{U}{\kappa}a} \cdot h}\, da$$

$$= \int_{-\infty}^0 e^{ia(x-\lambda)} \frac{\sinh \sqrt{a^2 + i\frac{U}{\kappa}a}(h-y)}{\sinh \sqrt{a^2 + i\frac{U}{\kappa}a} \cdot h}\, da ,$$

we have

$$I_1 + I_2 = \int_{-\infty}^{\infty} e^{ia(x-\lambda)} \frac{\sinh\sqrt{a^2 + i\dfrac{U}{\varkappa}a}(h-y)}{\sinh\sqrt{a^2 + i\dfrac{U}{\varkappa}a \cdot h}}\, da\ .$$

This integral can be performed as follows. Let a be a complex quantity and carry out the contour integration in the complex a-plane along the pathes shown in Fig. 1.
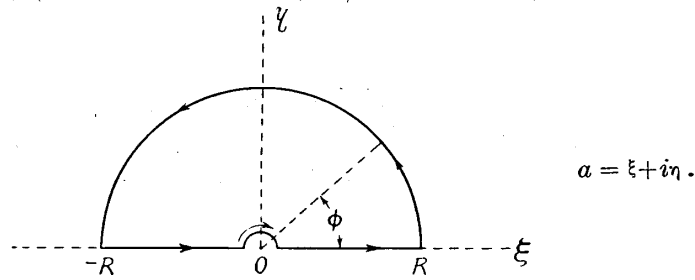

$$a = \xi + i\eta\ .$$

Fig. 1.

On the large semi-circle, the radius R of which tends to infinity in the limit,

$$\lim_{R\to\infty} \frac{\sinh\sqrt{a^2 + i\dfrac{U}{\varkappa}a}(h-y)}{\sinh\sqrt{a^2 + i\dfrac{U}{\varkappa}a \cdot h}} = \lim_{n\to\infty}\left(\frac{h-y}{h}\right)^{2n+1} = \begin{cases} 1, & y = 0\ , \\ 0, & 0 < y < h\ , \end{cases}$$

and

$$\int e^{ia(x-\lambda)}\, da = \int_0^\pi e^{iR(x-\lambda)(\cos\phi + i\sin\phi)} Rie^{i\phi} d\phi$$

$$= \int_0^\pi e^{-R(x-\lambda)\sin\phi - iR(x-\lambda)\cos\phi} Rie^{i\phi} d\phi$$

$$< \int_0^\pi e^{-R(x-\lambda)\sin\phi} Rd\phi = 2\int_0^{\frac{\pi}{2}} Re^{-R(x-\lambda)\sin\phi}\, d\phi$$

$$< 2\int_0^{\frac{\pi}{2}} Re^{-R(x-\lambda)\phi\frac{2}{\pi}}\, d\phi = -2\frac{R\pi}{2R(x-\lambda)}\left| e^{-R(x-\lambda)\phi\frac{2}{\pi}} \right|_0^{\frac{\pi}{2}}$$

$$= -\frac{\pi}{x-\lambda}\left[e^{-R(x-\lambda)} - 1 \right]\ .$$

Hence when $x-\lambda>0$, $0<y\leq h$ the integral along the large semi-circle tends to zero as R tends to infinity . $a=0$ is a branch point of the function to be integrated but it is not a pole. Therefore the integral along the real axis is equal to $2\pi i$ times the sum of the residues. So we have

$$\int_{-\infty}^{\infty} e^{ia(x-\lambda)} \frac{\sinh \beta(h-y)}{\sinh \beta h} da$$

$$= 2\pi i \sum_{n=1}^{\infty} (-1)^n i \frac{e^{\left(\frac{U}{2\kappa}-\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)(x-\lambda)} \sin n\pi \frac{(h-y)}{h}}{\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}} \cdot \frac{n\pi}{h^2}$$

$$= \frac{2\pi^2}{h^2} \sum_{n=1}^{\infty} \frac{ne^{\left(\frac{U}{2\kappa}-\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)(x-\lambda)}}{\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}} \sin n\pi \frac{y}{h} .$$

From the relation (5) we have

$$\Re \int_{0}^{\infty} e^{ia(x-\lambda)} \frac{\sinh \beta(h-y)}{\sinh \beta h} da$$

$$= \frac{\pi^2}{h^2} \sum_{n=1}^{\infty} \frac{ne^{\left(\frac{U}{2\kappa}-\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)(x-\lambda)}}{\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}} \sin n\pi \frac{y}{h} \dots\dots\dots (7)$$

when $x-\lambda>0$, $0<y\leq h$.

We now treat the case, in which $x-\lambda<0$. In this case the pathes of integration should be taken as shown in Fig. 2.

In the case $x-\lambda<0$ the integral along the large semi-circle tends to zero, as R tends to infinity, just as proved above in the case $x-\lambda>0$. A is a branch point of the function to be integrated, but it is not a

pole. Hence the integral twice round A is zero. The integral along OA and AO cancel each other, since we integrate the function twice round the branch point A and return on the same leaf of the Riemann's surface. O is a branch point, but it is not a pole. Hence we have when $x - \lambda < 0$,
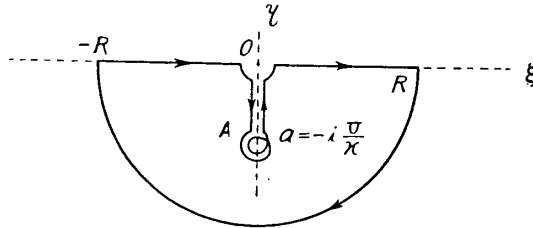

Fig. 2.

$$\Re \int_0^\infty e^{ia(x-\lambda)} \frac{\sinh \beta(h-y)}{\sinh \beta h} da$$

$$= \frac{\pi^2}{h^2} \sum_{n=1}^\infty \frac{ne^{\left(\frac{U}{2\kappa}+\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)(x-\lambda)}}{\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}} \sin n\pi\frac{y}{h} \quad\dots\dots\ (8)$$

We now perform the integration with respect to λ . Since $F(x) = 0$ when $x < 0$ and $F(x) = -\Theta$ when $x > 0$, we have

$$\theta = -\Re \frac{\Theta}{\pi} \int_0^\infty \int_0^\infty e^{ia(x-\lambda)} \frac{\sinh \beta(h-y)}{\sinh \beta h} da d\lambda .$$

When $x < 0$, since $\lambda > 0$, we have always $x - \lambda > 0$. But in the case $x > 0$, $x - \lambda$ is positive when $0 < \lambda < x$ and $x - \lambda$ is negative when $\lambda > x$. Therefore when $x < 0$, we take (8) and integrate it with respect to λ from 0 to ∞ . In this case the expression for θ becomes

$$\theta = -\frac{\Theta\pi}{h^2} \sum_{n=1}^\infty \frac{ne^{\left(\frac{U}{2\kappa}+\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)x}}{\left(\frac{U}{2\kappa}+\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}} \sin n\pi\frac{y}{h} \quad\dots\dots\ (9)$$

When $x > 0$, we divide the region into two parts. i) When $0 < \lambda < x$, since $x - \lambda > 0$, we take (7) and integrate it with respect to λ from 0 to x. Hence we have

$$\Re \int_0^\infty \int_0^x e^{ia(x-\lambda)} \frac{\sinh \beta(h-y)}{\sinh \beta h} da d\lambda$$

$$= \frac{\pi^2}{h^2} \sum_{n=1}^\infty \frac{n \left[e^{\left(\frac{U}{2\kappa} - \sqrt{\frac{n^2\pi^2}{h^2} + \frac{U^2}{4\kappa^2}} \right) x} - 1 \right]}{\left(\frac{U}{2\kappa} - \sqrt{\frac{n^2\pi^2}{4\kappa^2} + \frac{U^2}{4\kappa^2}} \right) \sqrt{\frac{n^2\pi^2}{h^2} + \frac{U^2}{4\kappa^2}}} \sin n\pi \frac{y}{h}.$$

ii) When $\lambda > x$, since $x - \lambda < 0$, we take (8) and integrate it with respect to λ from x to ∞. In this case we have

$$\Re \int_0^\infty \int_x^\infty e^{ia(x-\lambda)} \frac{\sinh \beta(h-y)}{\sinh \beta h} da d\lambda$$

$$= \frac{\pi^2}{h^2} \sum_{n=1}^\infty \frac{n}{\left(\frac{U}{2\kappa} + \sqrt{\frac{n^2\pi^2}{h^2} + \frac{U^2}{4\kappa^2}} \right) \sqrt{\frac{n^2\pi^2}{h^2} + \frac{U^2}{4\kappa^2}}} \sin n\pi \frac{y}{h}.$$

Hence when $x > 0$ the expression for θ becomes

$$\theta = -\frac{\Theta\pi}{h^2} \sum_{n=1}^\infty \left[\frac{2h^2}{n\pi^2} + \frac{ne^{\left(\frac{U}{2\kappa} - \sqrt{\frac{n^2\pi^2}{h^2} + \frac{U^2}{4\kappa^2}} \right) x}}{\left(\frac{U}{2\kappa} - \sqrt{\frac{n^2\pi^2}{h^2} + \frac{U^2}{4\kappa^2}} \right) \sqrt{\frac{n^2\pi^2}{h^2} + \frac{U^2}{4\kappa^2}}} \right] \sin \frac{n\pi y}{h}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (10)$$

At $y = 0$ we have

$$\theta = -\frac{\Theta}{\pi} \int_0^\infty \int_0^\infty \cos a(x-\lambda) da d\lambda = \begin{cases} 0, & x < 0, \\ -\Theta, & x > 0. \end{cases} \cdots\cdots\cdots (11)$$

IV. Integration of the expressions for v and ψ .

The expression for v is as shown in (4)

$$v = -\Re \frac{\kappa\gamma}{\pi U^2}\int_0^\infty \int_{-\infty}^\infty F(\lambda)e^{ia(x-\lambda)}\left[\frac{\sinh a(h-y)}{\sinh ah} - \frac{\sinh\beta(h-y)}{\sinh\beta h}\right]dad\lambda .$$

The second integral is the same as that which appeared in the expression for θ , and the first one can be obtained by only replacing β by a in the second integral. Hence the expression for v becomes

$$v = \frac{\Theta\kappa\gamma}{\pi U^2}\sum_{n=1}^\infty\left[\frac{e^{\frac{n\pi x}{h}}}{n} - \frac{\pi^2}{h^2}\frac{ne^{\left(\frac{U}{2\kappa}+\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)x}}{\left(\frac{U}{2\kappa}+\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}}\right]\sin\frac{n\pi y}{h} ,$$

$$\dotfill (12)$$

when $x < 0$, and

$$v = -\frac{\Theta\kappa\gamma}{\pi U^2}\sum_{n=1}^\infty\left[\frac{e^{-\frac{n\pi x}{h}}}{n} + \frac{\pi^2}{h^2}\frac{ne^{\left(\frac{U}{2\kappa}-\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)x}}{\left(\frac{U}{2\kappa}-\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}}\right]\sin\frac{n\pi y}{h} ,$$

$$\dotfill (13)$$

when $x > 0$.

v is continuous at $x = 0$. Hence it is continuous everywhere in the field of flow.

The expression for ψ can be obtained by directly integrating (5), but it is easier to integrate (12) and (13) with respect to x and change the sign. In this way we get

$$\psi = -\frac{\Theta\kappa\gamma}{U^2}\sum_{n=1}^\infty\left[\frac{h}{n^2\pi^2}e^{\frac{n\pi x}{h}} - \frac{\pi}{h^2}\cdot\frac{ne^{\left(\frac{U}{2\kappa}+\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)x}}{\left(\frac{U}{2\kappa}+\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)^2\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}}\right]$$

$$\times \sin\frac{n\pi y}{h} + f_1(y) ,$$

when $x < 0$ and

$$\psi = -\frac{\Theta\kappa\gamma}{U^2}\sum_{n=1}^{\infty}\left[\frac{h}{n^2\pi^2}e^{-\frac{n\pi x}{h}} - \frac{\pi}{h^2}\frac{ne^{\left(\frac{U}{2\kappa}-\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)x}}{\left(\frac{U}{2\kappa}-\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)^2\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}}\right]$$

$$\times \sin\frac{n\pi y}{h} + f_2(y)$$

when $x > 0$.

The functions $f_1(y)$ and $f_2(y)$ should be so chosen that $\psi = 0$ at $x = -\infty$ and ψ is continuous at $x = 0$.

Hence we get finally

$$\psi = -\frac{\Theta\kappa\gamma}{U^2}\sum_{n=1}^{\infty}\left[\frac{h}{n^2\pi^2}e^{\frac{n\pi x}{h}} - \frac{\pi}{h^2}\frac{ne^{\left(\frac{U}{2\kappa}+\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)x}}{\left(\frac{U}{2\kappa}+\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)^2\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}}\right]$$

$$\times \sin\frac{n\pi y}{h} \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad(14)$$

when $x < 0$, and

$$\psi = -\frac{\Theta\kappa\gamma}{U^2}\sum_{n=1}^{\infty}\left[\frac{h}{n^2\pi^2}e^{-\frac{n\pi x}{h}} - \frac{\pi}{h^2}\cdot\frac{ne^{\left(\frac{U}{2\kappa}-\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)x}}{\left(\frac{U}{2\kappa}-\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}\right)^2\sqrt{\frac{n^2\pi^2}{h^2}+\frac{U^2}{4\kappa^2}}}\right.$$

$$\left.+\frac{2Uh^2}{\kappa n^3\pi^3}\right]\sin\frac{n\pi y}{h} \quad\dots\dots\dots\dots\dots\dots\dots\dots \quad(15)$$

when $x > 0$.

V. Numerical calculations.

In carrying out the numerical calculations it is necessary to know the relative magnitudes of the unknown quantities κ, U, h and Θ. h and Θ can be determined immediately from the conditions of experiment, but the value of κ largely depends upon the condition of mixing of fluids of different temperature and not upon the ordinary heat conduction when the fluid is at rest. The relation between U and Θ seems to us a little obscure in Schmidt's experiment, and there is no datum to determine the value of κ. Hence in the following numerical calculations I derermined the value of h, κ and U arbitrarily and from the map of ψ (the relative values of ψ at all points in the field can be calculated from the formulae of ψ, but the absolute values are not determined because the value of Θ is still unknown) calculated, we determine the value of Θ in such a way that when the stream function, which represents the uniform flow of velocity U, is superimposed on the above map of ψ, the head of the sheet of cold fluid nearly coincides the centre of the vortex, which is the case in experiment.

We consider the fluid is water, and determine $\kappa = 60$ cm.^{2}/sec. so that the mode of fluid motion resembles as nearly as that of the experiment. For the value of U we took $1\cdot8$ cm./sec. calculating from the velocity of fluid particles shown in the figure in Schmidt's paper. We also determined $h = 31$ cm. and $\gamma = 0\cdot17$ cm./sec. .

The values of ψ/Θ calculated from (13) and (14) using above values of κ, U, γ and h are shown in Table I.

TABLE I.

y \ x	3 cm.	6 cm.	9 cm.	12 cm.	15 cm.
0·1 h	—2·23	—2·01	—1·98	—2·02	—2·10
0·2 h	—3·82	—3·33	—3·35	—3·40	—3·51
0·3 h	—4·48	—3·92	—4·04	—4·07	—4·20
0·4 h	—4·40	—3·74	—3·76	—4·14	—4·35
0·5 h	—4·20	—3·68	—3·90	—4·16	—4·09
0·6 h	—3·68	—3·10	—3·35	—3·59	—3·57

TABLE I.—(*Continued*)

y \ x	o cm.	—3 cm.	—6 cm.	—9 cm.
0·1 h	—2·79	—1·74	—1·24	—0·89
0·2 h	—4·11	—2·81	—1·82	—1·52
0·3 h	—4·69	—3·38	—2·69	—1·96
0·4 h	—4·39	—3·63	—2·87	—2·19
0·5 h	—4·15	—3·40	—2·75	—2·16
0·6 h	—3·45	—2·95	—2·37	—1·94

The stream lines corresponding to this case are shown in Fig. 3, and those in the case where the velocity U is superimposed are shown in Fig. 4. In Fig. 3 we see clearly a vortex is formed at the head of the sheet of cold water.

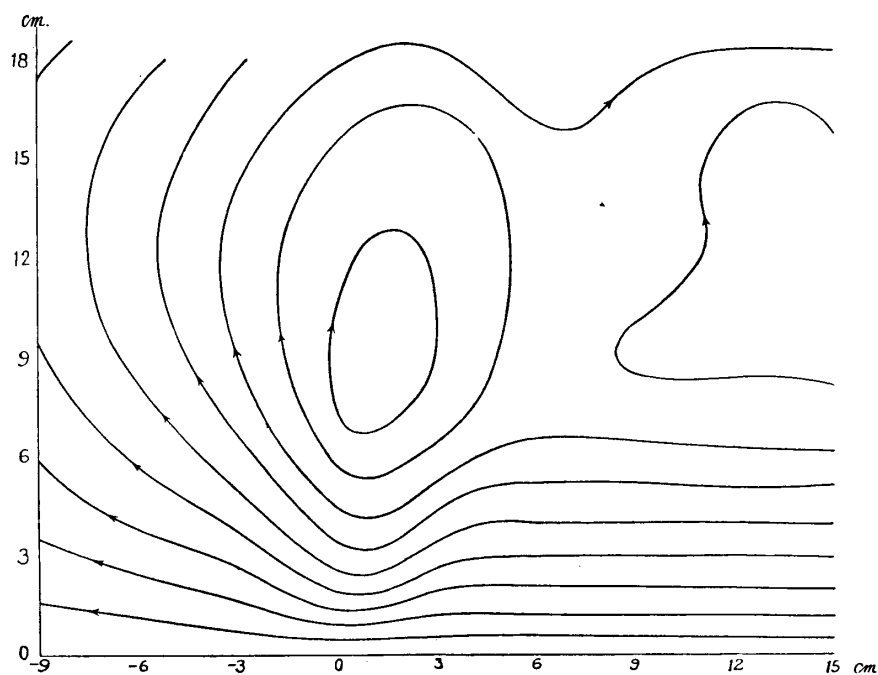The value of θ calculated in such a way as written above is 2·7°C.
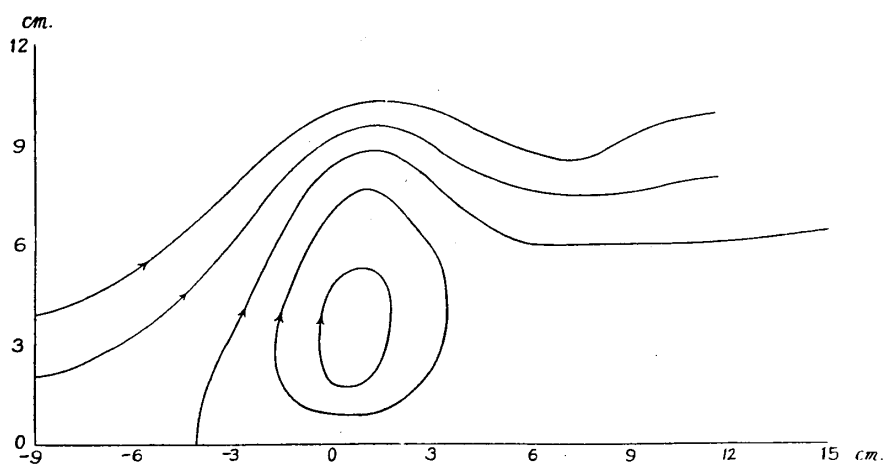

Fig. 3.

T. Sasaki.

Fig. 4.

The values of θ (C°) calculated from (8) and (9) are shown in Table II.

TABLE II.

y \ x	3 cm.	6 cm.	9 cm.	12 cm.	15 cm.
0·1 h	—1·67	—1·96	—2·08	—2·14	—2·22
0·2 h	—1·42	—1·66	—1·84	—1·95	—2·07
0·3 h	—1·17	—1·35	—1·53	—1·63	—1·79
0·4 h	—0·82	—0·94	—1·09	—1·19	—1·36
0·5 h	—0·68	—0·73	—0·86	—0·97	—1·13
0·6 h	—0·51	—0·55	—0·59	—0·71	—0·86

y \ x	0 cm.	—3 cm.	—6 cm.	—9 cm.
0·1 h	—1·07	—0·49	—0·23	—0·13
0·2 h	—0·99	—0·58	—0·34	—0·22
0·3 h	—0·86	—0·58	—0·38	—0·26
0·4 h	—0·72	—0 54	—0·37	—0·26
0·5 h	—0·57	—0·46	—0·34	—0·25
0·6 h	—0·44	—0·37	—0·28	—0·21

Lines of equal values of θ are shown in Fig. 5, which show the temperature distribution in the water.
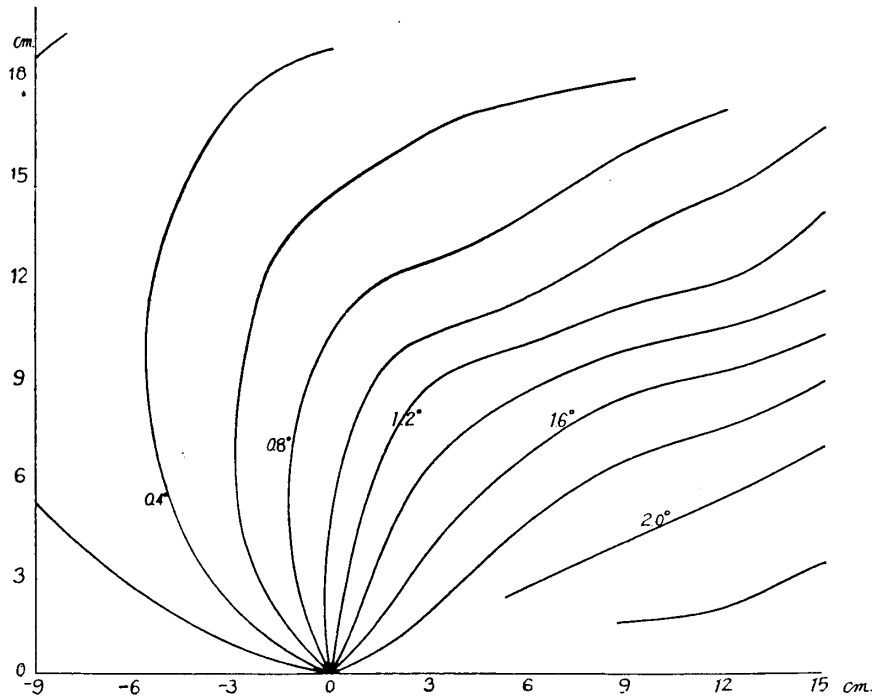

Fig. 5.

VI. Conclusion.

Although rough assumptions are made in deriving differential equations of fluid motion and the equation of continuity, they are sufficient in making rough calculation of the problem of heat convection as in the present case. If the experiment is made more precisely and the relations between unknown quantities are determined, we are able to see whether or not the present theoretical investigation agree with the experiment and afford a theoretical explanation of the experiment.