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Application of the Inverse Wiedemann Effect to Torque Measurements and to Torque Variation Recordings.

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I. Introductory

If a ferro-magnetic wire conducting an electric current is twisted, it is longitudinally magnetised. This phenomenon—the inverse Wiedemann effect—has been investigated by many authors, the magnetisation being given in relation to the longitudinal electric current and to the angle of twist. However, as these measurements were made with rather thin wires, the magnetism was very weak, and consequently the accuracy of the results of the measurements are not sufficient for use in the quantitative discussion of this phenomenon. For this reason, the author measured the effect with iron rods of 2—2.5 cm. in diameter and published the results in a previous paper,(1) showing these with respect to their dependency upon the torsional stress (instead of upon the angle of twist), and further added that this phenomenon could be

(1) On the inverse Wiedemann effect and its allied phenomena. Japanese Journal of Physics, Vol. V, No. 1, 1928.

applied to the measurements of torque and to recording the variations of torque on a rotating shaft transmitting power.

For convenience' sake, the results of the same measurements as explained on p. 2 of the above mentioned paper, made with a mild steel rod of 2 cm. in diameter, are shown below. The rod was inserted in a massive iron yoke in order to prevent the rod from inducing free poles at its ends and producing a self-demagnetising field in its centre part. The rod was a little longer than the length of the yoke, and arms were attached to both the ends. The torque was applied by these arms, the yoke serving as bearings. Thick wires were soldered on to the end surfaces, and a longitudinal electric current was sent through them. The rod was insulated from the yoke with pieces of thin mica film, in order to prevent the longitudinal current from shunting through the yoke. A coil was wound over the middle part of the rod and connected to a ballistic galvanometer. The measurement was carried out in the following way. After the required torque had been applied to the rod, a longitudinal current of a certain amount was passed and reversed many times to establish a cyclic state. Then the ballistic galvanometer circuit was closed and the longitudinal current was reversed. The longitudinal magnetisation was calculated from half the ballistic effect observed at this time. In this way we get the following results. (The values for mild steel given in the previous paper were obtained with rods which had been heated and annealed before the measurements. In the present experiment the rods were not heated after they were lathed down to the required diameter. Rods not heated become much less strongly magnetised than annealed ones at any stage of torsion and circular field.)

These results are shown graphically in Figs. 1 and 2.

Now suppose a longitudinal electric current to be sent to part of a shaft transmitting power through brushes which are in contact with the shaft at two points some distance apart, and a coil wound over the centre part of the shaft between the brushes to be connected to an

Longitudinal magnetisation (total amount of flux) due to the inverse Wiedemann effect of a low carbon mild steel rod of 2 cm. in diameter.

Long. curr. in amp.		10	20	30	40	50	60
Torque in m. kg.	2	31	202	501	740	880	925
	4	64	410	1020	1480	1765	1865
	6	95	585	1450	2185	2575	2775
	8	117	695	1780	2735	3235	3605
	10	135	765	2005	3110	3790	4230
	12	147	805	2165	3370	4185	4715
	15	155	840	2270	3620	4525	5155

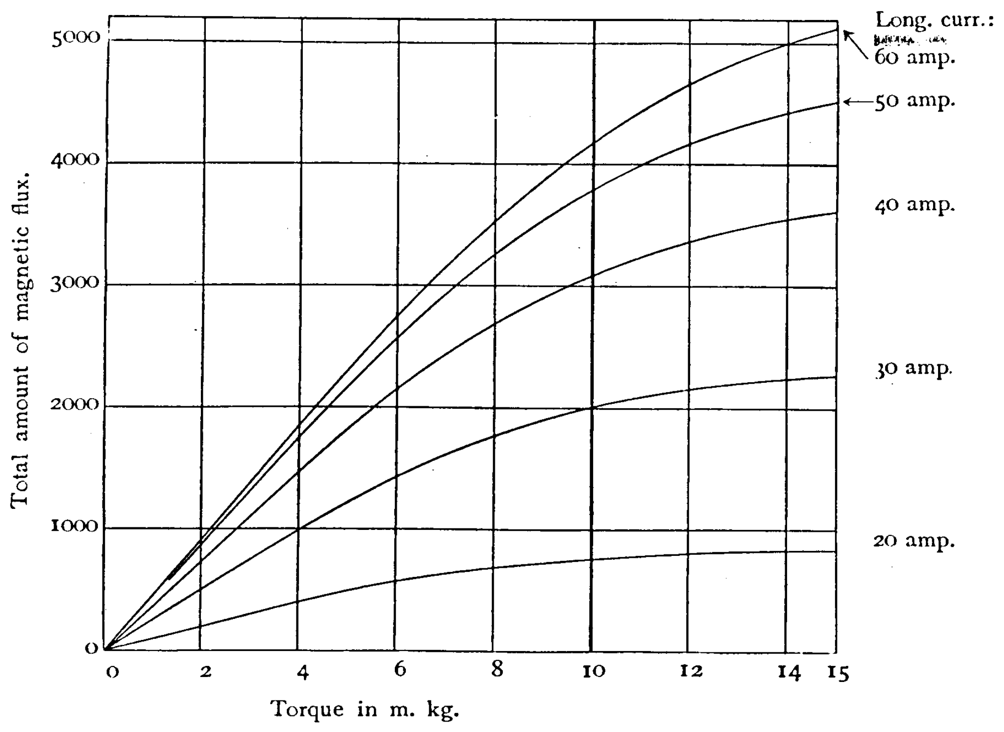


Fig. 1.

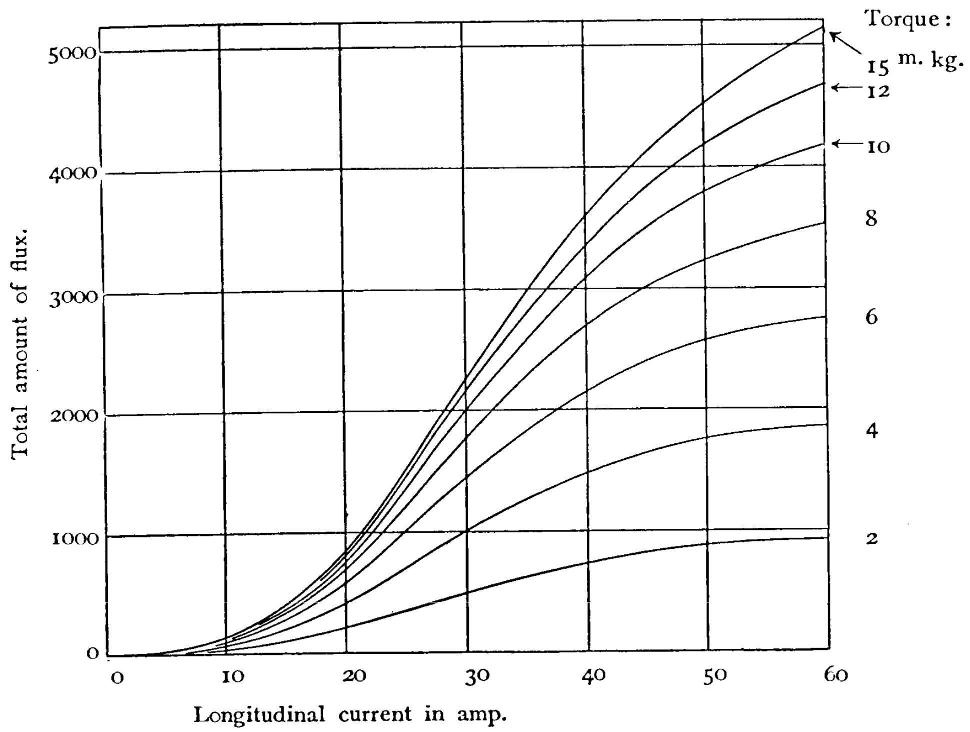


Fig. 2.

oscillograph. (The coil may be held so as not to rotate with the shaft.) If the torque on the shaft remains unchanged, the longitudinal mageti-sation of the shaft is constant. But, if the torque varies, the longitu-dinal magnetisation changes at the same moment and E. M. F. is induced in the coil. Therefore, in this way we can record the variation of the torque acting on the shaft.

In Fig. 1 we see that the magnetic variation is practically linear up to the torque of about 4 m. kg., when the longitudinal current is 20 amp.; and up to 6 m. kg., when the current is 40 amp.; and up to 7 m. kg., when the current is 60 amp. It is much more convenient to use only this part of the curve, though, of course, we can make use of the part of the curve beyond these limits, if its inclination is steep. As was shown on p. 12 of the previous paper, if two cylindrical rods of the same material and of different diameters carry longitudinal electric currents proportional to the diameters and undergo torques propotional to the cubes of the diameters, the longitudinal magnetisations (total amounts of flux) are proportional to the squares of the diameters. From this fact we can find the limit of torque to be applied to a mild steel shaft of any diameter in order to keep the variation of longitudinal magnetisation linear under a given longitudinal current. (The author carried out the same measurements with rods of the same size made of many different materials, and found that the materials generally called "mild steel" gave the same results to within ± 20 per cent.) For instance, for a shaft of 3 cm. in diameter the limit is 13.5 m. kg., when the longitudinal current is 30 amp.; 20 m. kg., when the current is 60 amp.; and 23.5 m. kg., when the current is 90 amp. But again, the "safety load" for the 2 cm. shaft is about 10 m. kg. Therefore, if a 3 cm. shaft is to be loaded up to its "safety load", the curve for 60 amp. in Fig. 1 becomes the curve for 90 amp., the torque scale giving the proportion to the safety load which comes on the line of 10 m. kg. in Fig. 1, and the magnetisation scale showing 9/4 times that indicated in Fig. 1.

The voltage of the source of the longitudinal current must be high enough (say several volts) not to make the current fluctuate by the change of contact resistances of the brushes. However, as it can be seen by Fig. 2 (still better by Fig. 7 in the previous paper), a small alteration in longitudinal current gives but very small effect to the longitudinal magnetisation.

The values of longitudinal magnetisation shown above are for the case when the influence of the ends does not exist, that is to say the rod is infinitely long. If the rod is short, the longitudinal magnetisation is smaller than the value in the endless case owing to the action of the demagnetising field of the free poles developed at the ends. Its ratio to the value in the endless case (which however varies considerably with the degree of magnetisation) is found in the following table.

Ratio of longitudinal magnetisation at the centre of the rod to the value in the endless case (rough average values).

Length/diameter	40	30	20	10
Ratio	.42	.27	.15	.06

It is a well known fact that after any change has taken place in a magnetic field acting on a piece of iron, some time elapses before the corresponding change of magnetic state is complete, but the change of magnetism due to the change of stress in the specimen takes place and is completed practically in an instant. Figs. 3 and 4 show this in the case of the inverse Wiedemann effect. A mild steel rod of 2 cm. in diameter and 70 cm. in length was laid on brass bearings, and arms were attached to both ends of it. One of the arms was fixed and the other was pulled up until the torque on the rod reached 10 m. kg. When this arm was let go, it rested on the top surface of a pillar erected on the floor. The height of the pillar was so adjusted that the torque on the rod became 3 m. kg. at that time. A longitudinal current of 40 amp. was passed through the rod and a coil wound over the centre part of the rod was connected to an oscillograph. Fig. 3 is the record at the time when the arm was let go and struck the top of the pillar and so stopped. The curve marked 1, which gives the position of the arm, was taken by making a metal piece attached to the arm slide on a resistance wire conducting a current. The curve marked 2 shows the E.M.F. in the coil. Fig. 4 is the record when a heavy weight was dropped upon the arm from a

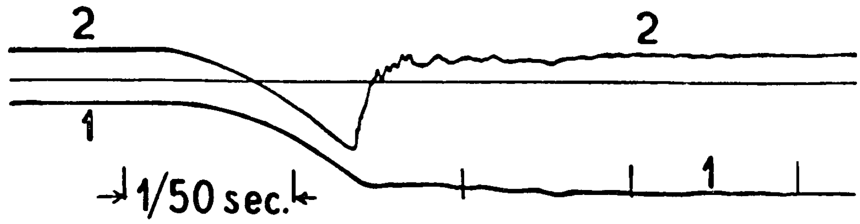


Fig. 3.

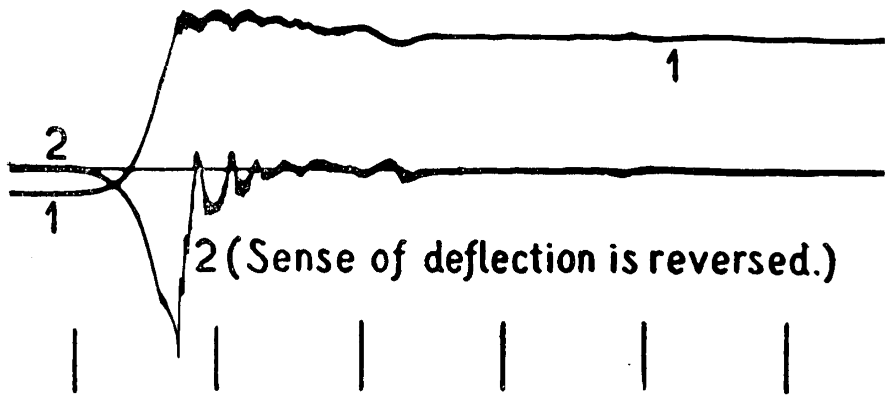


Fig. 4.

height of about 1 m., (The initial torque was about 1 m. kg. caused by the weight of the arm itself.) and the arm, after descending a certain distance, was held by a pillar erected on the floor. (The torque at this time was 10 m. kg.) Neither of the records is very accurate, owing to the vibrations of various part as well as the floor. However, it can be clearly seen that the changes of the torque and of the longitudinal magnetisation take place at the same moment. Therefore, our method is suitable for recording very quick changes of torque.

The fact that the hysteresis is small in the magnetic change due to the change of stress is also an advantage of our method.

II. Recording torque variations of an automobile engine

The author has taken many oscillograms of the fluctuations of torque from a six-cylinder automobile engine, connecting a Froude's water dynamometer to it. Fig. 5 shows diagramatically the arrangement for our measurements.

The part of the shaft used for the measurement was about 30 cm. long and 3 cm. in diameter. This part was a separate piece from the whole shaft, and was connected to the parts of the shaft on both sides rigidly by brass couplings, keeping small gaps of about 2 cm. in the shaft, for the purpose of intercepting the magnetic circuit. This is especially advantageous for the alternating current method explained in

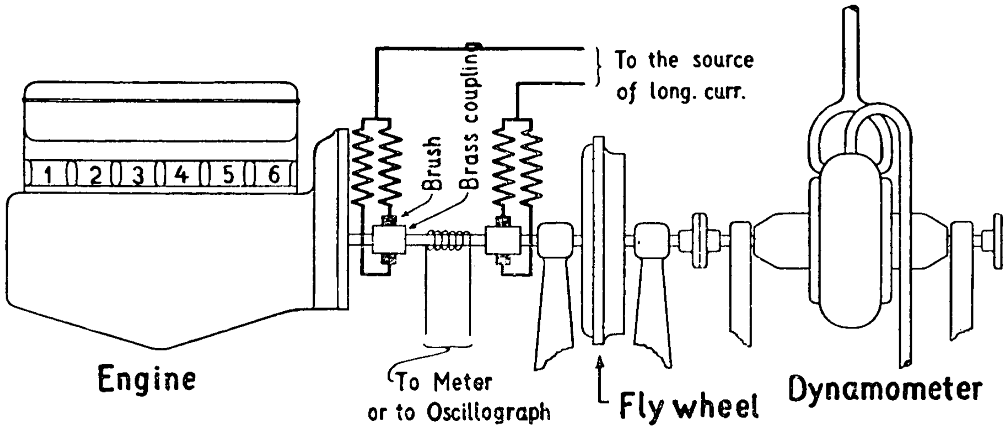


Fig. 5.

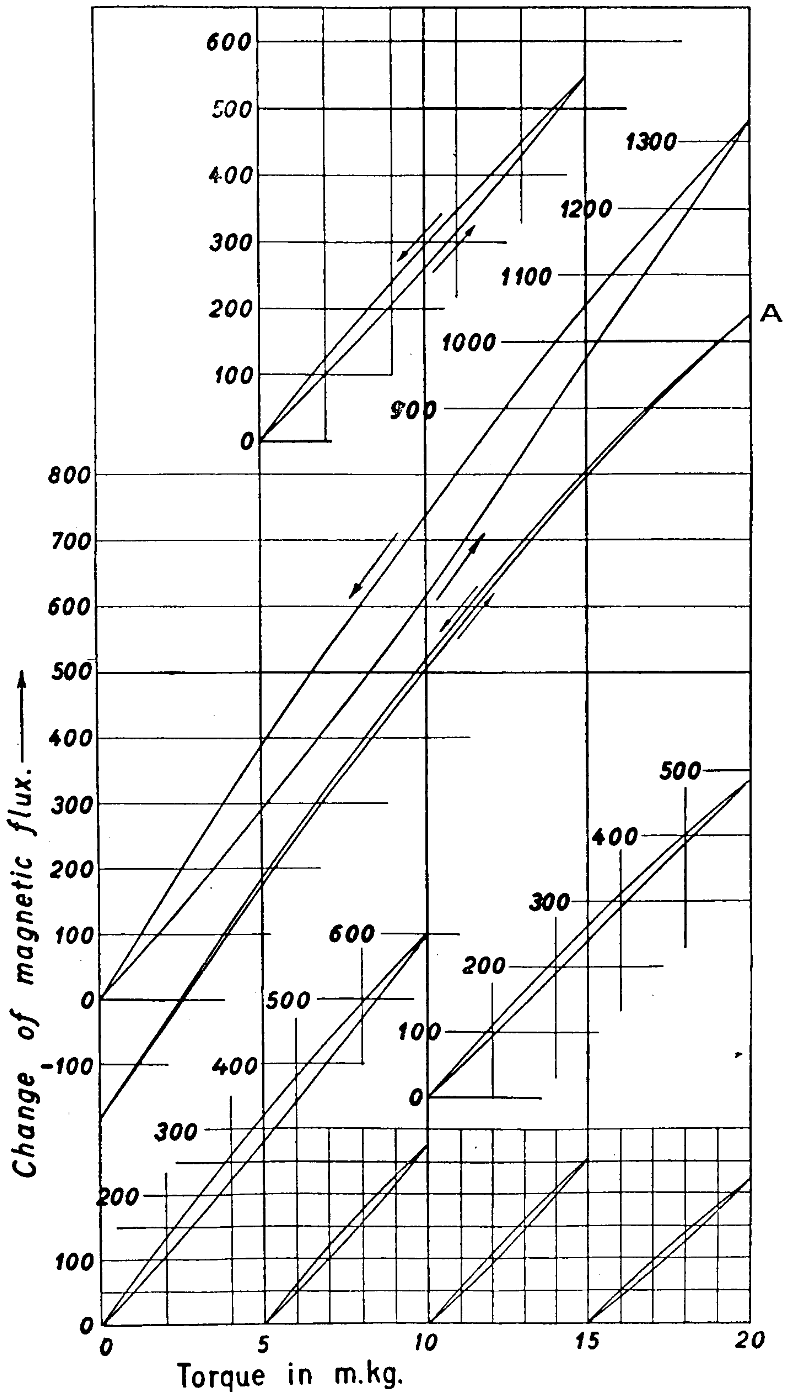
the next chapter. But this is not necessarry if it is so arranged that a still larger part of the shaft is traversed by the longitudinal current. The fly wheel was detached from the engine and fixed on the dynamometer side of the measuring part.

A longitudinal current of 50 amp. was sent from a 10 volt storage battery. This longitudinal current for the estimated maximum torque 20 m. kg. corresponds to 33 amp. for 6 m. kg. on the 2 cm. shaft. Fig. 1 shows us that the magnetisation curve is not straight in our range and, if it is wanted straight, the longitudinal current must be raised above 60 amp. Two coils were prepared, by winding fine insulated wire 10,000 turns and thicker wire 500 turns (both with many taps). The former was for raising the E. M. F. up to several volts in order to connect it to electronic vacuum tubes, and the latter for the direct measurements of the current.

In this way, if we take a record of torque variation, when the engine is running, connecting a suitable number of turns of the coil to an oscillograph, we get such a curve as that marked **O** in Fig. 7. (The curve marked **O** in Fig. 7. was taken by amplifying the voltage by triod vacuum valves, as this was necessary in order to record at the same time its integrated curve marked **X**. The direct connection of 20—500 turns of coil gives sufficient amplitude in our case for an oscillograph of ordinary sensibility.)

For the purpose of calibrating the relation between the torque and longitudinal magnetisation in our arrangement, arms were attached to both the ends of the whole system of shafts, and the torque was applied by fixing one of the arms and loading on the other. The curve marked A in Fig. 6 shows the longitudinal magnetisation of the shaft part during loading and unloading cyclically between 0—20 m. kg. when measured by the method of reversing longitudinal current. (The fact that the longitudinal magnetisation is not zero at torque zero is due either to the influence of the couplings or to the existence of internal stress or residual magnetism in the shaft piece). The other long magnetisation curve in Fig. 6 was obtained, when the shaft was loaded and unloaded cyclically between 0 and 20 m. kg., the longitudinal current being kept constant, in such a way that a certain small amount of torque (say 2 m. kg. each time) was applied impulsively, the change of the longitudinal magnetisation being observed by the throw of the ballistic galvanometer, and this was repeated until the torque 20 m. kg. was reached, and then it was unloaded in the same manner, and so on. It is only natural that the magnetic change in this way is much greater than the value found by the former method. Further observations were carried out in many other ranges of torque, 0—10, 5—15, 10—20, 5—10, 10—15 and 15—20 m. kg., and the results are shown in Fig. 6. It can be seen in that diagram that, as the longitudinal current is not strong enough, the inclination of the curve decreases as the torque increases, and that when the range of torque variation is small, the inclination of the magnetisation curve is also small.

Assume that the spot of light on the focusing plane of the oscillograph deflect f mm. when the E.M.F. in the coil is 1 volt, (if vacuum valves are used, their action inclusive) and that the number of flux lines in the shaft piece changes by N when the change of 1 m. kg. takes place in the torque τ. Then, when the torque varies, the change of



Change of magnetic flux.
Torque in m.kg.
0
5
10
15
20
100
200
300
400
500
600
700
800
-100
1000
1100
1200
1300
900
A

Fig. 6.

flux is $N\frac{d\tau}{dt}$. Accordingly, if the coil is of T turns, the E. M. F. is $\frac{TN}{10^8}\frac{d\tau}{dt}$ volts. Therefore, the deflection of 1 mm. shows a rate of change of torque of $\frac{10^8}{fTN}$ m. kg. per sec.

In the case of Fig. 7, N is 48, the mean torque being 14.7 m. kg., T 4500 and f 6.35, hence the deflection of 1 mm. corresponds to 73 m. kg. per sec.

The regular curve below the centre line (marked △) shows the primary current of the ignition induction-coil. The upward jumps of that curve indicate the times of sparking. The figure above each jump shows the number of the cylinder which was ignited. (See Fig. 5.) The longer downward projection marked D6 shows the top dead centre of the cylinder No. 6 before the explosion stroke and the shorter one marked D'6 gives the same before the suction stroke. The short parallel straight lines at the bottom indicate timings of 1/50 sec.

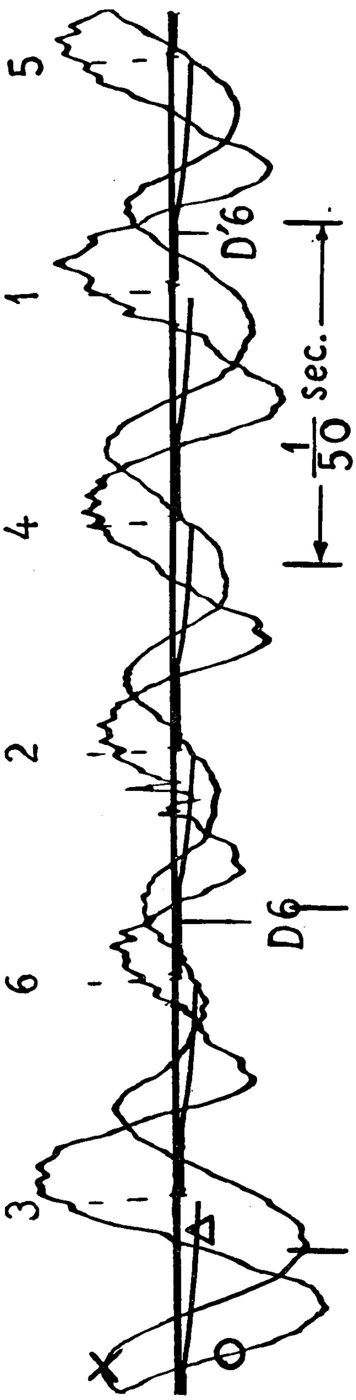


Fig. 7. Mean torque : 14.7 m. kg. O 1 mm.=73 m. kg./sec. × 1 mm.=.214 m. kg.

As the curve marked **O** gives the rate of change of torque, we get the curve showing the torque itself by integrating it graphically (after correcting by Fig. 6 if necessary). If this curve **O**, which is the record during two revolutions, is divided into six pieces of 120° rotation each, and the mean of them is taken, we get the curve down in thin line in Fig. 8. If this curve is graphically integrated (assuming 1 mm. = 73 m. kg. per sec.), we get the curve drawn in thick line in Fig. 8. This represents the "average" fluctuation of torque from the mean value.

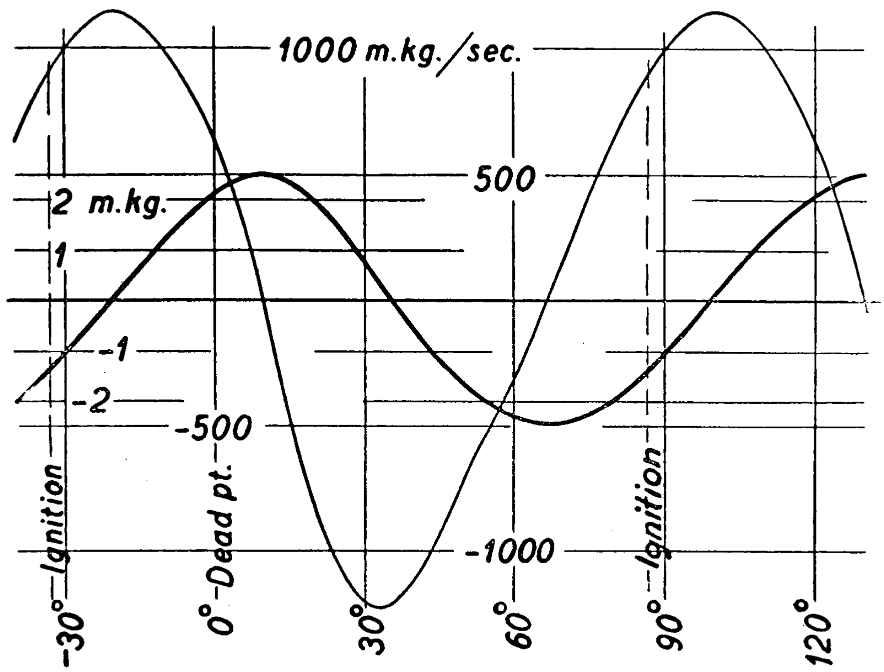


Fig. 8.

The torque curve, which is found by integrating its variation curve O, can be obtained otherwise by integrating the E. M. F. of the coil wound over the shaft by the aid of resistance and capacity and record-

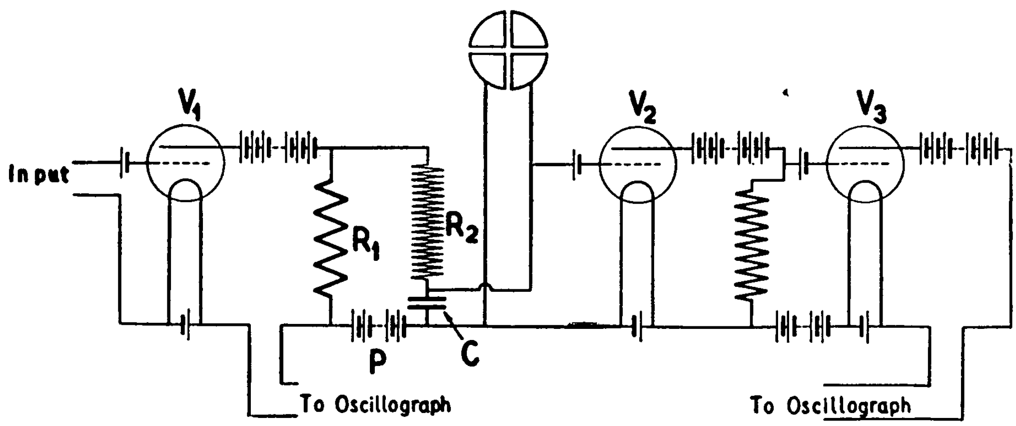


Fig. 9.

ing it directly by an oscillograph. The diagram of connections used by the author is shown in Fig. 9. The E. M. F. of the coil was amplified by the triod vacuum valve V_1 and recorded by one of the vibrators of the oscillograph (curve O). A resistance of 1000—3000 ohms (current capacity 300 ma.) R_1 was inserted in this circuit in series. A mica condenser of 1—3 microfarads C was connected so as to be charged through a high resistance of 100,000—250,000 ohms R_2 by the fluctuating potential difference at both the terminals of R_1, the amplitude of which was 30—100 volts, the steady E. M. F. being cancelled by the battery P. The exactness of the cancellation was watched by a quadrant electrometer connected to both the terminals of C and adjusted by changing P or R_1. The variation of the potential difference of the poles of C was amplified by V_2 and V_3, and was recorded by another vibrator of the oscillograph. The record taken in this way is the torque curve marked X in Fig. 7.

Let V denote the instantaneous voltage of the periodic component of the E.M.F. at the terminals of R_1, when the steady component (the mean voltage) is cancelled away. V changes in proportion to the change of E. M. F. in the coil. Let the potential difference of the poles of the condenser C be v. If K and R denote the capacity of C and the resistance of R_2 respectively, we can write

$$\frac{dv}{dt}=\frac{V-v}{KR} .$$

The solution is

$$ve^{\frac{t}{KR}}=\frac{1}{KR}\int e^{\frac{t}{KR}}V\,dt+\text{const} .$$

According to Fourier's theory, any periodic curve can be considered to be the sum of a series of curves, each of which is a sine curve. Therefore, taking one of the components, let us put

$$V=\overline{V}\cos(2\pi nt) ,$$

where n is the frequency and $\overline{V}$ a certain constant value of voltage. Then the solution becomes

$$v=\frac{\overline{V}}{1+4\pi^2n^2K^2R^2}\left\{2\pi nKR\sin(2\pi nt)+\cos(2\pi nt)\right\}+Ce^{\frac{-t}{KR}},$$

or

$$v=\frac{\overline{V}}{\sqrt{1+4\pi^2n^2K^2R^2}}\sin(2\pi nt+\varphi)+Ce^{\frac{-t}{KR}},$$

where C is an arbitrary constant, and

$$\varphi=\tan^{-1}\frac{1}{2\pi nKR}\ .$$

Since the frequency of the principal wave in Fig. 7 is about 75, the value of $2\pi nKR$ is, when K is 3 microfarads and R 250,000 ohms, about 350, and when 1 microfarad and 100,000 ohms (the most unfavourable case), about 47. If 1 is neglected in comparison with the square of this value, we can write

$$v=\frac{\overline{V}}{2\pi nKR}\sin(2\pi nt+\varphi)\ .$$

The values of φ are about 10′ and 1° 15′ (3′ 15″ and 25′ of rotation of the shaft) in the above mentioned cases, and still smaller for the components of smaller wave-lengths. (If a still longer wave corresponding to two revolutions of the shaft is considered, φ becomes 58′ and 7° 20′.) If we neglect such small changes of the phase, we can consider the curve **X** as the integrated curve of the curve **O**, and put

$$\frac{dv}{dt}=\frac{V}{KR}\ .$$

The fact shown above is true when the dielectric of the condenser is practically perfect and the leakage between the terminals of it negligible. To test the reliability of the connections, the author took an oscillogram as shown in Fig. 10, applying the voltage of a cell to the "in put" alternating in direction 100 times per second by means of a

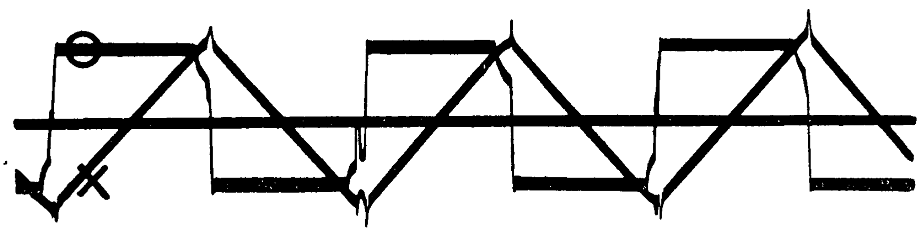


Fig. 10.

quickly rotating commutator. The fact that each part of the broken line X in the record is straight and its inclination coincides with the value calculated show that our instruments works satisfactorily.

For the calibration of the relation between the torque and the deflection of the integrated curve X, it is convenient to connect an A. C. source of several volts to the "in put" and measure the amplitude of the spot of light of the oscillograph.

Let the voltage of A. C. connected to the "in put" be $\tilde{V}$. The instantaneous value of 50 cycle A. C. of $\tilde{V}$ volts is expressed by $\tilde{V}\sqrt{2}\cos(100\pi t)$. The E. M. F. at the terminals of R_1 varies as this value. Therefore we can put

$$\frac{dv}{dt}=A\tilde{V}\sqrt{2}\cos(100\pi t) ,$$

where v is the potential difference between the poles of the condenser C, and A a certain constant. Hence we get

$$v=\frac{A\tilde{V}\sqrt{2}}{100\pi}\sin(100\pi t) .$$

Since the deflection h of the light spot of the oscillograph changes in proportion to v, we can write

$$h=Bv=\frac{AB\tilde{V}\sqrt{2}}{100\pi}\sin(100\pi t) ,$$

where B is another constant.

Let the amplitude of the spot of light be $\tilde{V}w$ mm. when A. C. of $\tilde{V}$ volts is connected to the "in put". Then we get

$$h_{\text{max.}}=\frac{\tilde{V}w}{2}=\frac{AB\tilde{V}\sqrt{2}}{100\pi} .$$

Hence we can fined the value of AB.

$$AB=25\sqrt{2}\,\pi w .$$

The E. M. F. in the coil wound over the shaft is, as explained above, $\frac{TN}{10^8}\cdot\frac{d\tau}{dt}$. Therefore

$$\frac{dv}{dt}=\frac{ATN}{10^8}\,\frac{d\tau}{dt} .$$

Integrating we get

$$v=\frac{ATN}{10^8}(\tau-\tau_0) .$$

Finally we obtain the relation

$$h=Bv=\frac{ABTN}{10^8}(\tau-\tau_0)=\frac{25\sqrt{2}\,\pi wTN}{10^8}(\tau-\tau_0) .$$

In the case of Fig. 7, w, amplitude per 1 volt A. C., is 19.5 mm. Hence 1 mm. of deflection of the curve X corresponds to torque of .214 m. kg.

If the curve X of Fig. 7 is divided, as was done with the curve O, into six pieces of 120° rotation each, and the mean curve of these is drawn, we get very nearly the same curve as the thick line curve obtained by graphical integration in Fig. 8.

Fig. 11 was taken by rotating the photographic film slowly, all other conditions being nearly the same as in Fig. 7, to show the regular trains of waves. Fig. 12 shows the record when the cylinders

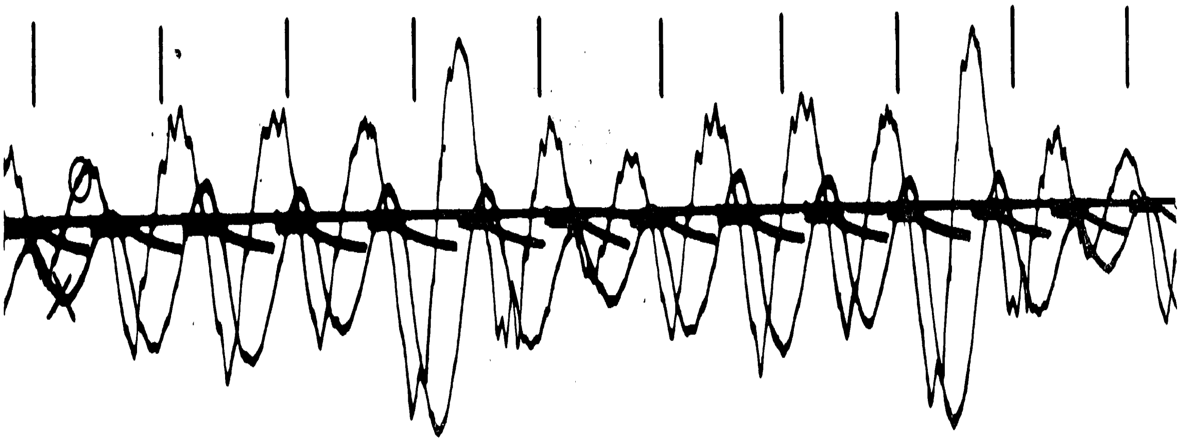


Fig. 11. R. P. M.: 1400 Mean torque: 14.5 m. kg.

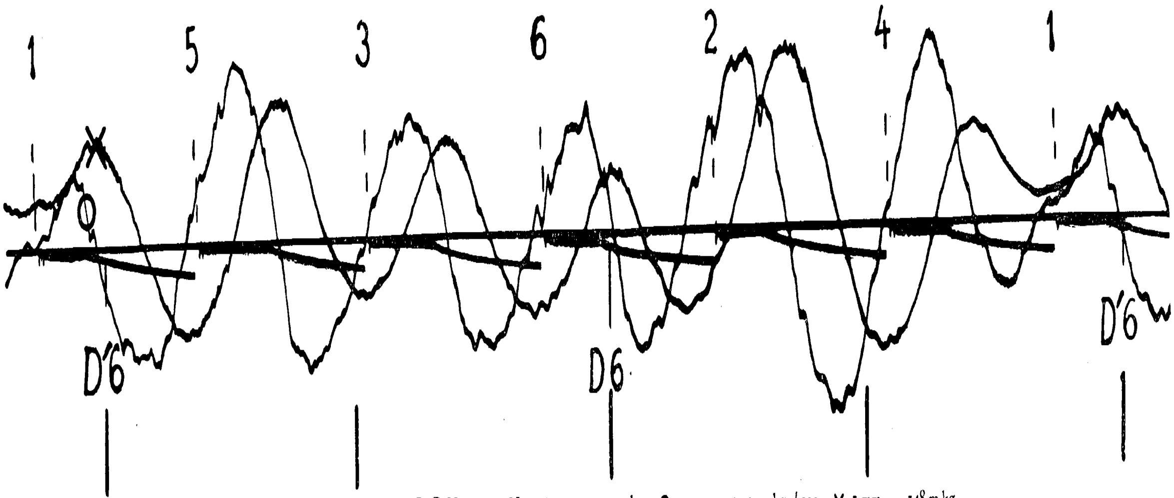


Fig. 12. Ignition advanced. R. P. M.: 1495 Mean torque: 14.9 m. kg. O 1 mm. = 49.4 m. kg. / sec. X 1 mm. = .148 m.kg.

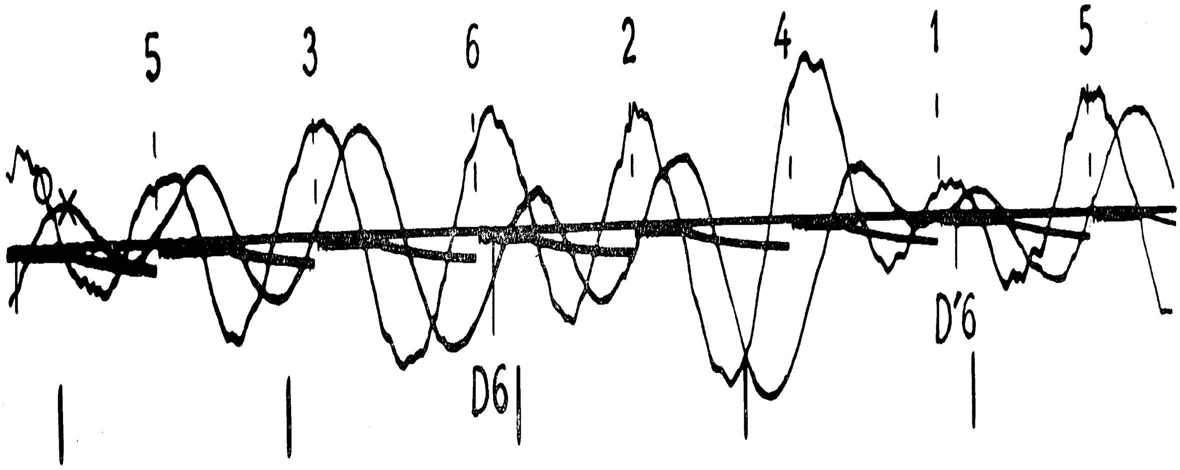


Fig. 13. Ignition retarded. R. P. M.: 1470 Mean torque: 11.8 m. kg. O 1 mm. = 100.5 m. kg. / sec. X 1 mm. = .297 m. kg.

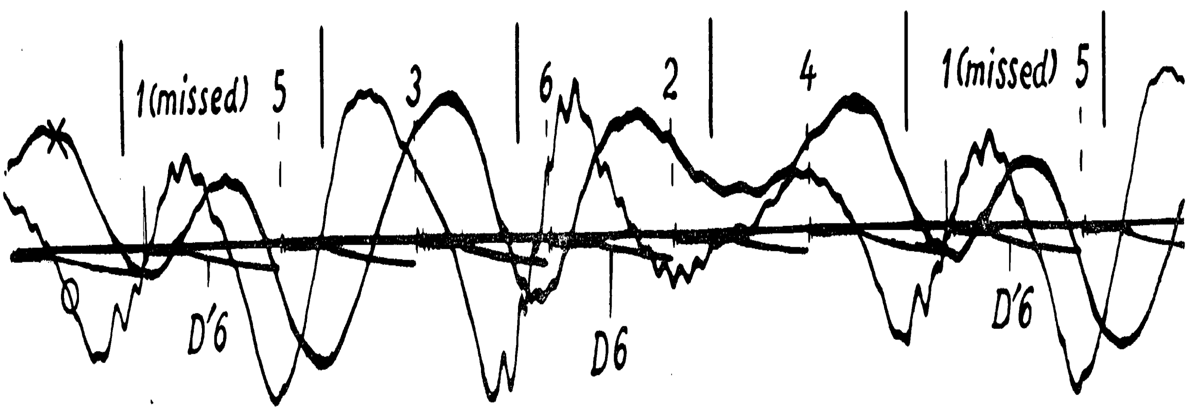


Fig. 14. Cylinder No. 1 missing. R. P. M.: 1470 Mean torque: 9.6 m. kg. O 1 mm. = 236 m. kg. / sec. X 1 mm. = .695 m. kg.

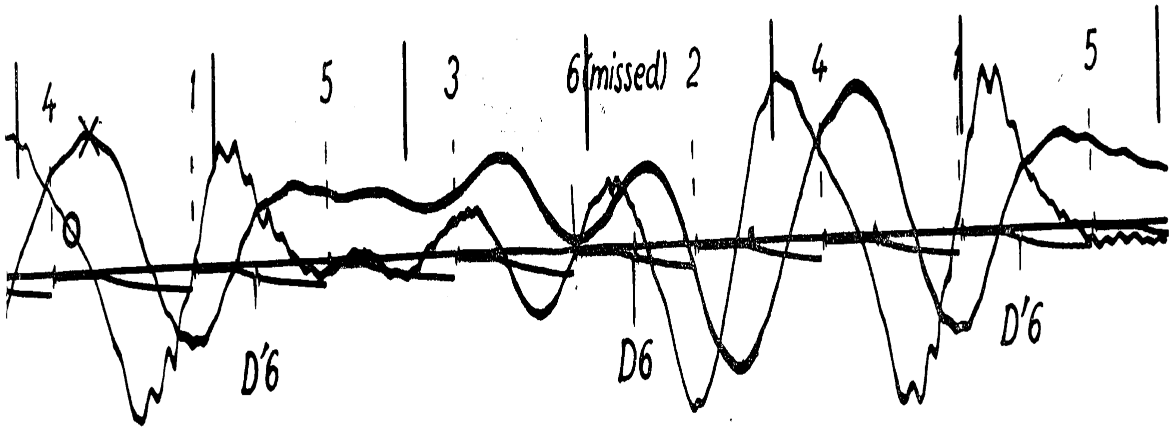


Fig. 15. Cylinder No. 6 missing. R. P. M.: 1470 Mean torque: 9.0 m. kg. O 1 mm. = 236 m. kg./sec. X 1 mm. = .695 m. kg.

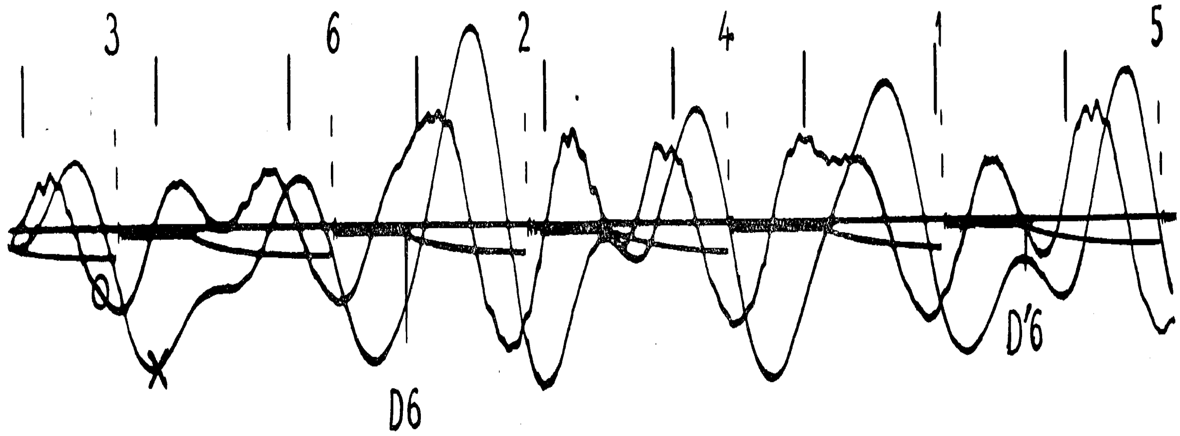


Fig. 16. Ignition advanced. R. P. M.: 638 Mean torque: 9.3 m. kg. O 1 mm. = 91.1 m. kg./sec. X 1 mm. = .268 m. kg.

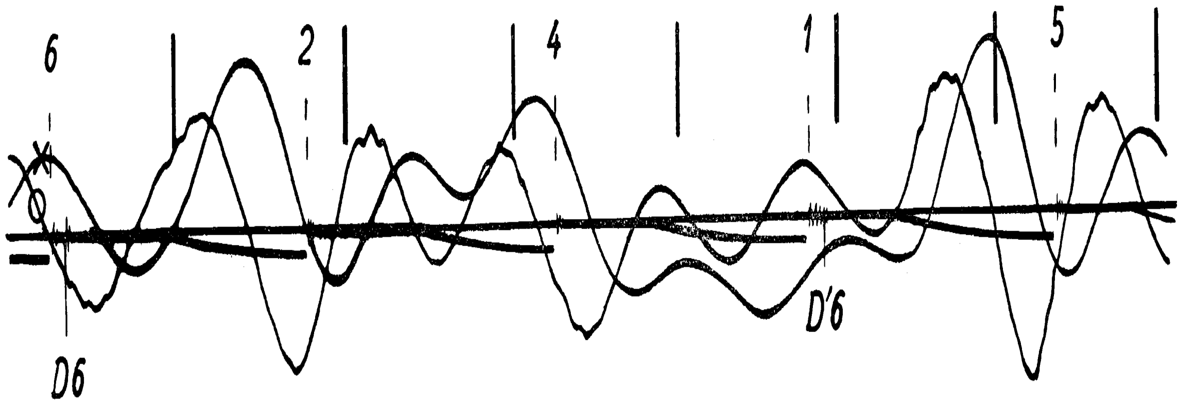


Fig. 17. Ignition retarded. R. P. M.: 655 Mean torque: 13.0 m. kg. O 1.mm. = 178 kg./sec. X 1 mm. = .522 m. kg.

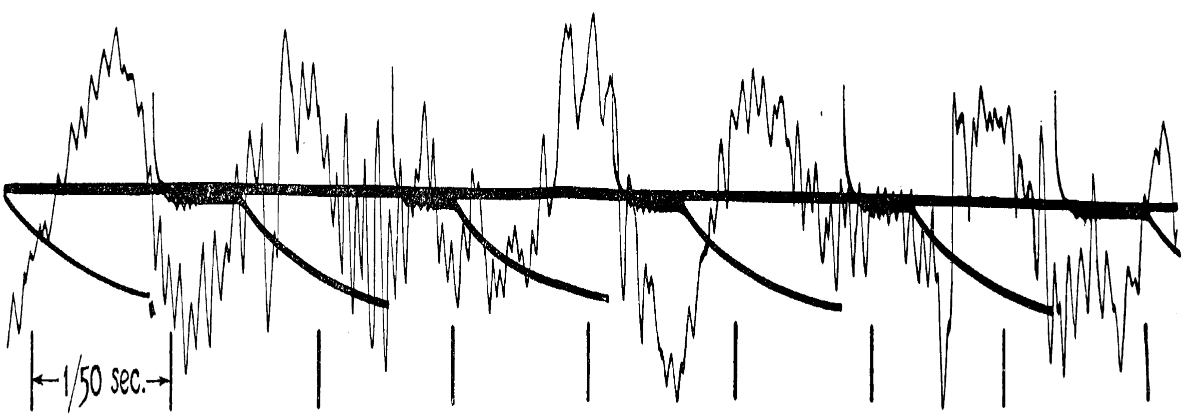


Fig. 18. Flywheel detached.

were ignited with the sparks advanced, and Fig. 13 when they were ignited with the sparks retarded. In the either case of Figs. 14 and 15, one cylinder was missing of set purpose. In Figs. 16 and 17, the number of revolutions of the engine was in each case about half of that in the other cases, the resistance of the dynamometer being increased. Fig. 18 is the record when the flywheel was removed from the shaft. (The fact that the quick vibrations in this case are conspicuous was perhaps due to the action of the water dynamometer.)

III. Measurement of mean torque

It was shown in the previous paper that, if an A. C. is passed through a twisted iron rod, change of magnetisation as in the curve marked × in Fig. 19 takes place in the rod, and, consequently, E.M.F. as shown in the curve marked O is induced in the coil wound over the rod; and further, if an A. C. is passed through a longitudinally magnetised rod, a record as shown in Fig. 20 is obtained. (The sine curves express the phases of the longitudinal currents.)

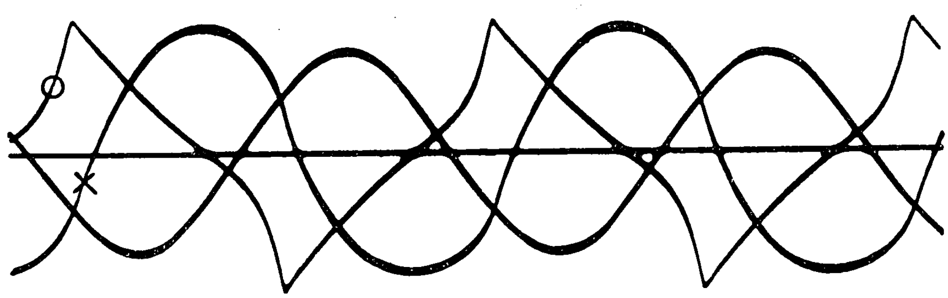


Fig. 19.

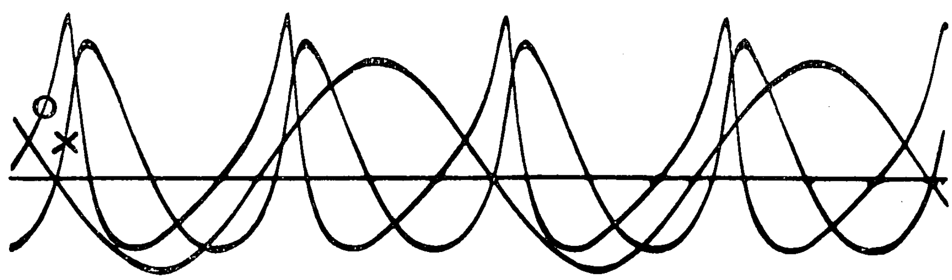


Fig. 20.

As the zigzag curve marked **O** of Fig. 19 varies its amplitude with the torque applied, we can find the torque by measuring the E. M. F. induced in the coil, if the relation between them is calibrated previously. If the longitudinal current is sufficiently strong and the torque is not too large for the thickness of the rod, the E. M. F. changes linearly with the torque, as does the magnetisation in the case of direct longitudinal current.

We now go on to explain experiments carried out with the engine shaft as described in the preceding chapter, but sending an A. C. through it. A longitudinal current of 42.5 amp. was sent from a small transformer of 1 KW., by lowering the voltage down to 10 volts. In calibration, torque was applied by the arms attached to both the ends of the shaft as before. The current was measured by connecting the coil to a "vacuum-thermo-element" (The number of turns of the coil was 100, and the resistance of the cercuit, the heating wire of the "thermo-element" and the coil together, about 45 ohms). The upper curve in Fig. 21 shows the results converted by calculation to E. M. F. per turn of the coil. The E. M. F. of the coil can be measured otherwise by rectifying it by means of a commutator attached to a synchronous motor turned by A. C. from the same source as the longitudinal current. The position of the brushes should be adjusted so that the millivoltmeter needle points to the maximum. The lower curve in Fig. 21 gives the results obtained by this method. These two methods do not give the same results, because the former gives the square root of the mean square value of the wave, while the latter gives the mean of the absolute value. (For sine waves the ratio of these two values is 1.11, but larger in our case.) When the shaft is thick and the torque and the longitudinal current are strong, and consequently it is possible to take greater power from the coil, we can measure the current by an A. C. meter directly.

If the shaft is permanently magnetised in longitudinal direction the wave shown in Fig. 20 is superimposed on the wave due to torque. Especially in such a case as that where the part of the shaft traversed

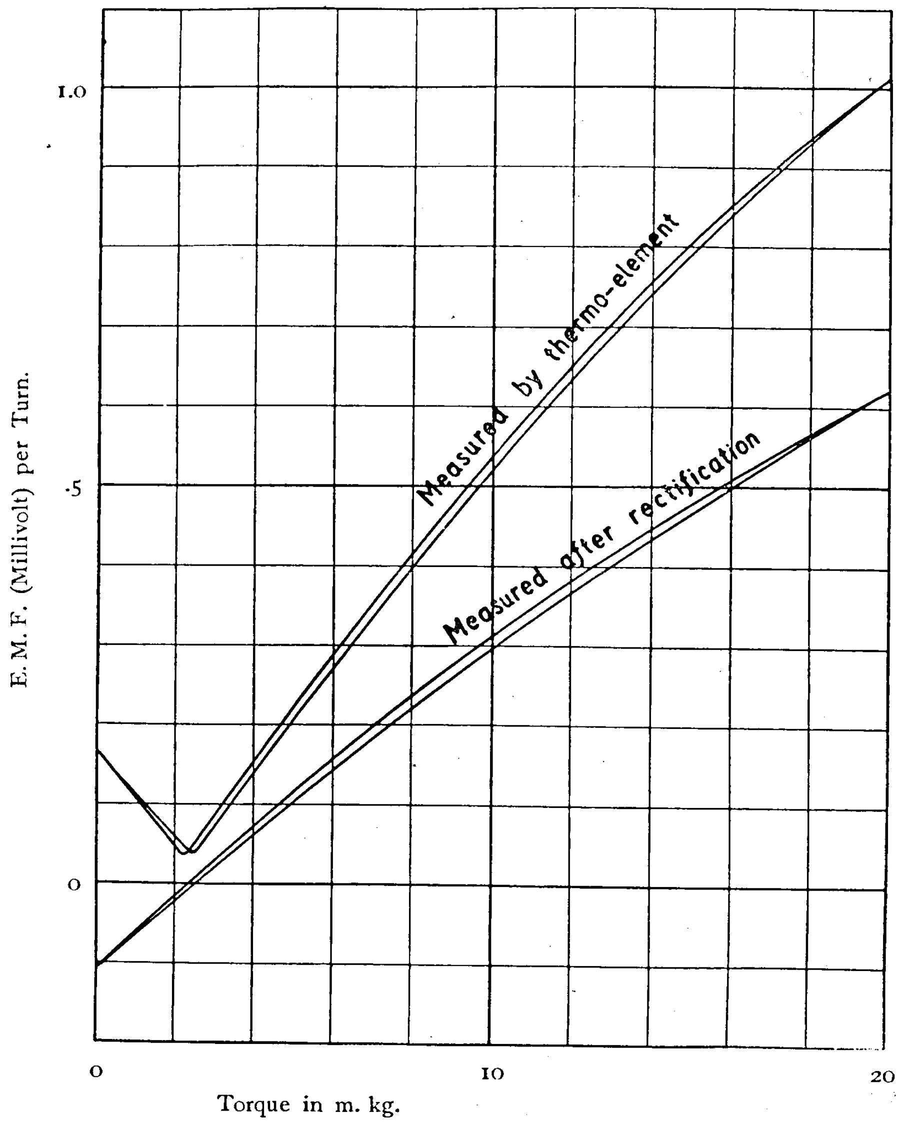


Fig. 21.

by the longitudinal current is very short compared to the whole length of the shaft (without gaps), it is difficult to get rid of this disturbance. Since the frequency of this wave of Fig. 20 is double that of the A. C. sent through the shaft, the influence of this current wave is nearly eliminated, if the mixed current due to both torque and magnetism is measured after being rectified by a synchronous motor and a commutator.

The method explained in this chapter is applicable with great accuracy, when the amount of variation of torque is small compared to the mean value of torque, or when the period of change of torque is much longer than that of the A. C. If the period is long enough and the mater used is sufficiently quick in action, we can observe the feature of torque fluctuation as well as instantaneous values of torque.

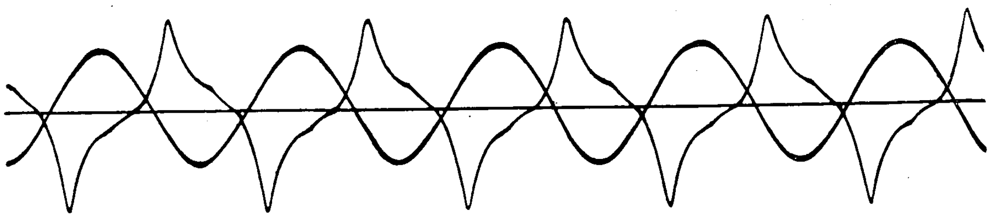


Fig. 22.

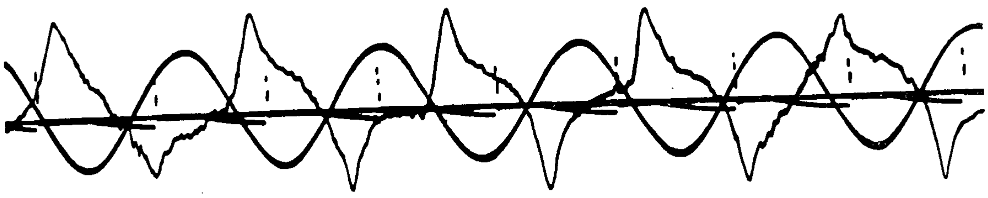


Fig. 23.

Figs. 22 and 23 are records of the "zigzag current" produced by the coil wound over the engine shaft under torque of 10 m. kg.; in the former case the torque was applied by the arms attached to the shaft when the engine was at rest, in the latter the torque was exercised by the dynamometer when the engine was running at the speed of 1700 R. P. M. We see that the mean values of the currents in both cases are nearly the same, though the torque fluctuation in the latter case was considerable.

In the case where the variation of torque is slow in comparison with the cycle of the A. C., if the "zigzag current" is recorded by rotating the photographic film slowly, the row of tops of the curve gives a torque curve. Fig. 23 was taken by sending a 50 cycle A. C.

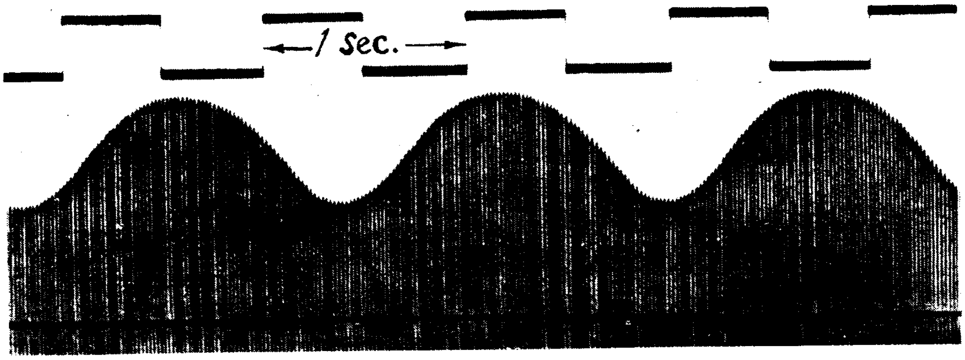


Fig. 24.

through a shaft, which was undergoing a torque oscillating approximately in simple harmonic between 5 and 10 m. kg. with a period of about 1.6 sec. Some records of this nature will be shown in a future number of this Report.