No. 77.

(English Abstract from the Japanese Original.)

On the Effect of the Walls of a Wind Tunnel upon the Lift Coefficient of a Model. (*Continued*)

Tatudirô SASAKI, *Rigakuhakusi*,

Member of the Institute.

Introduction.

In the previous paper(1) I have investigated the variation of lift and drag on a model wing in a wind tunnel. The first part of the paper dealt with the case of a flat plate between parallel walls. Recently Rosenhead(2) re-investigated this case and obtained a different result. In the second part of my paper I have dealt with the case of a flat plate in an unbounded jet of air. For this case von Kàrmàn(3) has obtained an approximate formula which is in close agreement with mine.

It was considered that the case in which the model is put in an unbouned jet of air does not represent the case of a wind tunnel of Göttingen and Eiffel type, and the mouthes of the tunnel must have some effect. For the effect of the exit wall L. Poggi(4) has derived a formula, which shows this effect is very small. Ono(5) also has calculated this effect

(1) Report No. 46, Dec. 1928.

(2) Rosenhead: The lift on a flat plate between parallel walls. Proc. of the Roy. Soc. July, 1931 p. 127.

Glauert has pointed out that my result does not agree with that derived from an approximate theory, but at that time I couldn't find any fault. It was made clear by the paper of Rosenhead. (See corrigenda at the end of this paper.) I express my thanks to Mr. Glauert and Dr. Rosenhead for this point.

(3) von Kàrmàn: Beitrag zur Theorie des Auftriebes (Vorträge aus dem Gebiete der Aerodynamik und Verwandter Gebiete. 1929 s. 98.)

(4) L. Poggi: Sulla variazione da apportarsi ai resultati delle esperienze eseguite al tunnel aerodinamico su di un modello alare. L'Aerotecnica Aprile 1931.

(5) Ono: Journal of Aeronautical Research Institute. No. 86 Oct. 1931.

independently. In the present paper I have treated the problem in a different way, and in the third part of the paper I have investigated the effect of both the exit and the collector walls of the wind tunnel.

I. The effect of an exit wall.

The form of the flow in this case is shown in Fig. 1. The air flows from infinity with the velocity $-U$ between parallel walls till the exit, out of which the air flows as a free jet and impinging upon the model the direction of it is deflected. It is theoretically possible to treat this kind of flow as a two dimensional problem in hydrodynamics, but as it is mathematically very complicated we treat the case shown in Fig. 2, which represents infinite number of parallel walls and models. It is enough to consider the region ABPP′B′A′, if we obtain some functions with the period id. If we consider the region between two stream lines along AB and A′B′ the form of flow resembles very much to that of Fig. 1, and so the lift and drag on the model calculated in both cases will be nearly equal. This was proved formerly in my paper(1) in the case where the walls are absent.

The analysis is made and the result is shown in the expression (6).

II. The case in which the model is replaced by a point vortex.

The case resembles to the case treated by Poggi, and the result is the same as that obtained by him. The result is shown in the expression (7).

III. The effect of both the exit and the collector walls.

The form of flow is shown in Fig. 12, and the result is shown in the expression (8).

It is shown from the results of investigation made in this paper, the effects of both the exit and the collector walls are very small, and so we can consider as though both the mouthes are absent and the model is put in a free jet in treating the lift and drag on a wing model in a wind tunnel of Göttingen and Eiffel type.

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(1) Sasaki: Journal of the Aeronautical Research Institute. No. 83 July 1931.

Dai 77 Gô.

(Shôwa 6n. (1931) 12 gwatu hakkô.)

Hûtô no Kabe ga Mokei no Yôryoku-keisû ni oyobosu Eikyô ni tuite. (*Tuduki*)

Syoin Rigakuhakusi, SASAKI-Tatudirô.

Hasigaki.

Kono maeno Ronbun dewa Mokei ga nimaino heikôna Ita no aidani okareta Baai to ziyûna Hukidasi no nakani okareta Baai to wo toriatukatta. Zensha wa N.P.L no Hûtô ni sôtosuru no de atte, saikinni *Rosenhead*(1) ga Kenkyû wo sinaosita. Kôsya wa *Göttingen* oyobi *Eiffel*-Gata no Hûtô ni sôtôsuru no de atte, sono noti *Kàrmàn*(2) ni yotte Ryakkin-siki ga motomerareta.

Göttingen oyobi *Eiffel*-gata no Hûtô no Baai ni ziyuna Hukidasi to sita no dewa dyûbun dewa naku, Hukidasi-guti oyobi Suikomi-guti no Eikyô ga ikubun ka aru ni tigai nai to omowarete ita. Hukidasi-guti no Eikyô wa saikinni *L. Poggi*(3) ni yotte, mata betuni *Ono*(4) ni yotte ryakkin-tekini kenkyû-sareta. Sorerano Kekkwa ni yoreba Hukidasi-guti no Eikyô wa taihen sukunai koto ga wakaru.

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(1) Rosenhead: The lift on a flat plate between parallel walls. Proc. of the Roy. Soc. July 1931 p. 127.

Glauert ga keisan-sita noto tigau to itte yokosareta ga sono toki wa yoku wakaranakatta. *Rosenhead* in yotte kono Ten ga akiraka ni sareta. (Saigo ni tuketa Teisei wo miyo!) *Glauert* to *Rosenhead* to ni Orei wo môsiageru.

(2) Kàrmàn: Reitrag zur Theorie des Auftriebes (Vorträge aus dem Gebiete der Aerodynamik und verwandter Gebiete. 1929 s. 98.)

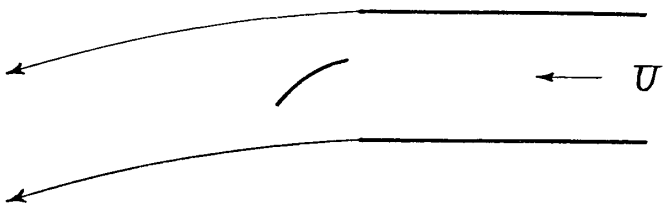
(3) L. Poggi: Sulla variazione da apportarsi ai resultati delle esperienze eseguite al tunnel aerodinamico su di un modello alare. L'Aerotecnica Aprile 1931.

(4) Ono: Kôkûkenkyûsyo Ihô 86 Gô. 10 gt. 1931.

Kono Ronbun dewa nao tigatta Hôhô ni yotte keisan-sita Hukidasi-guti no Eikyô to Suikomi-guti no Eikyô to wo noberu.

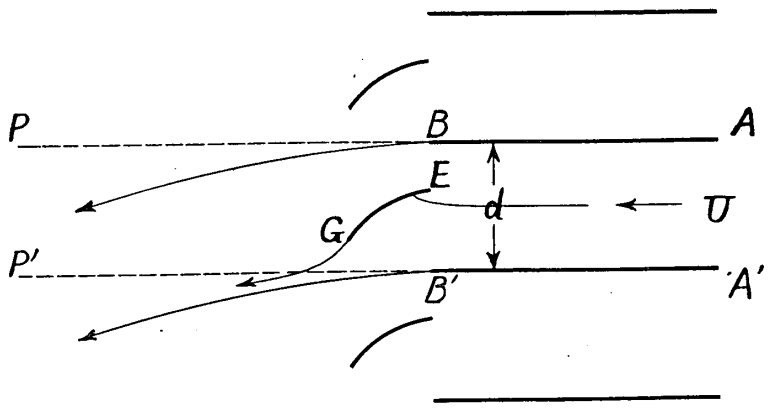
1. *Hukidasi-guti no Eikyô.*

Kono mae to dôyô ni nizi-genno Mondai to site toriatukau. Sunawati, z-Men ni okeru Dukei wa Du 1 no yôni naru.



Du 1. z-Men.

Mokei wo Ten no Zyunkwan to sita Baai wa *Poggi* no toriatukatta Mondai de atte, ryakkin-tekini keisan-suru koto ga dekiru. Sikasi, Mokei ga aru Katati wo motu Baai niwa riron-tekini wa Mondai wo toku koto ga dekiruga, zissaino Keisan wa mudukasikute dekinai. Sokode Mondai wo Du 2 no yôni musûno heikôna Ita ga aru Baai ni suru. Kono Baai wa Du 1 no Baai to hotondo hitosii Kekkwa wo ataeru koto wa kono mae ni nobeta tôri de aru.(1)



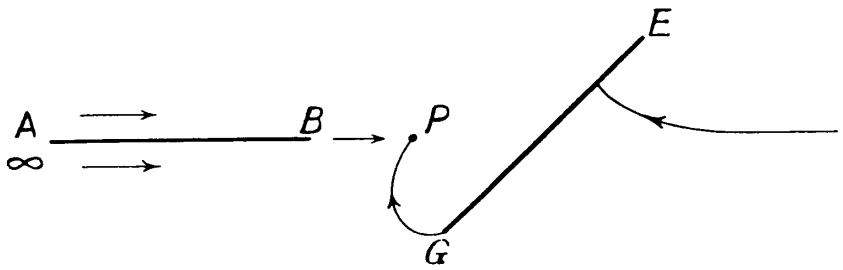
Du 2. z-Men.

(1) Sasaki: Kôkûkenkyûsyo Ihô 83 Gô. 7 gt. 1931.

z-Men ni okeru ABP to $A'B'P'$ to no aida no Ryôiki wa

$$z = \frac{d}{2\pi}\log(t-t_p)+B$$

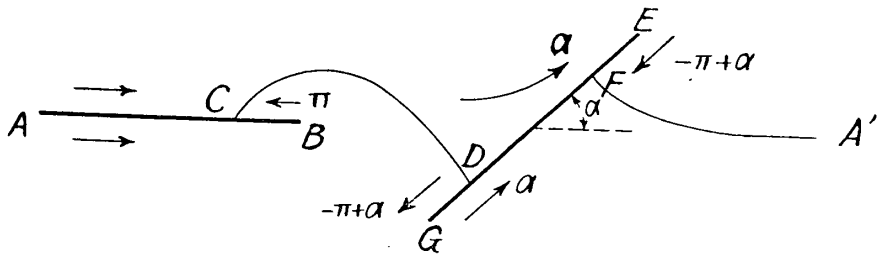
nite, z-Men kara t-Men ni henkwan-sareru. Koko de B wa Zyôsû de aru.



Du 3. t-Men.

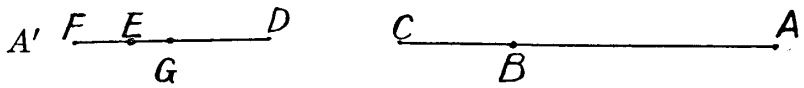
t-Men de GE wo Tyokusen to sureba z-Men dewa GE wa *exponential* Kyokusen ni naru.

Tugini, t-Men wa tugino yôni site s-Men ni okeru $CC'D'D$ naru Kukei ni henkwan-sareru. (Du 6)



Du 4. t-Men.

Ima Du 4 ni oite $-\infty$ no Tokoro ni aru A kara nagare-dasite $+\infty$ no A' ni yuku Nagare wo kangaereba, AB oyobi GE ni sôte nagareru Ryûsen wa Du ni simesu tôri de aru. Kore ni sôtô-suru w-Men wa Du 5 no yôni naru.

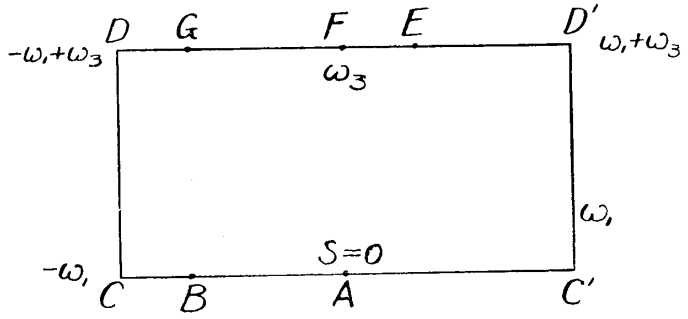


Du 5. w-Men.

w-Men wa

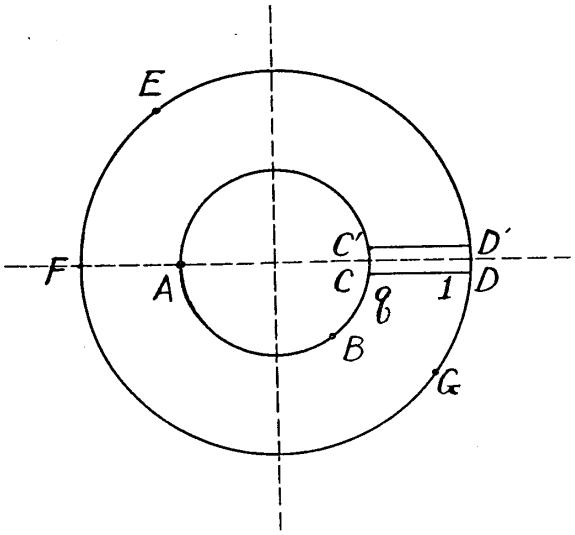
$$w = C_0 \wp s$$

naru Kwankei ni yotte s-Men ni okeru Kukei ni henkwan-sareru. C_0 wa kattena zituno Sû de aru.



Du 6. s-Men.

Soreyueni kono Kukei wo $s = \omega_1 + \omega_3 - \frac{\omega_1}{i\pi} \log Z$ naru Kwakei ni yotte Z-Men ni okeru Wagata-ryôiki ni henkwan-sureba *Villat*[1] no Kôsiki wo motiite keisan-suru koto ga dekiru.



Du 7. Z-Men.

(1) Villat: Scientia No. 38.

Sunawati

$$\int_0^{\theta_E} a d\theta + \int_{\theta_E}^{\theta_F} (-\pi + a)\, d\theta + \int_{\theta_F}^{\theta_G} a d\theta + \int_{\theta_G}^{2\pi} (-\pi + a) d\theta = \int_{\theta_B}^{2\pi} \pi d\theta .$$

$$\therefore \quad \theta_E + \theta_G + \theta_B = 5\pi - 2a \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Mata

$$\Omega(Z) = -\frac{i}{\pi}\left[a \log(-1)\, e^{-2\eta_1\left(\frac{\omega_1}{i\pi}\log Z - \omega_1\right)} - \pi \log \frac{\sigma\left(\frac{\omega_1}{i\pi}\log Z - \omega_1\right)\sigma\left(\frac{\omega_1}{i\pi}\log Z - 2\omega_1\right)}{\sigma\left(\frac{\omega_1}{i\pi}\log Z - \frac{\omega_1}{\pi}\theta_E\right)\sigma\left(\frac{\omega_1}{i\pi}\log Z - \frac{\omega_1}{\pi}\theta_G\right)} - \pi \log \frac{\sigma_3\left(\frac{\omega_1}{i\pi}\log Z - 2\omega_1\right)}{\sigma_3\left(\frac{\omega_1}{i\pi}\log Z - \frac{\omega_1}{\pi}\theta_B\right)} \right]$$

to naru.

$$\frac{\omega_1}{\pi}\theta_E = \omega_1 + \omega_3 - s_E , \qquad \frac{\omega_1}{\pi}\theta_G = \omega_1 + \omega_3 - s_G ,$$

$$\frac{\omega_1}{\pi}\theta_B = \omega_1 - s_B$$

to site Ω wo s-Men de arawaseba

$$\Omega(s) = -i\left[\frac{a}{\pi}\log(-1)\, e^{-2\eta_1(\omega_3 - s)} - \log \frac{\sigma(\omega_3 - s)\sigma(\omega_3 - \omega_1 - s)}{\sigma(s_E - s)\sigma(s_G - s)} - \log \frac{\sigma_3(\omega_3 - s - \omega_1)}{\sigma_3(s_B - s + \omega_3)} \right]$$

$$= -i\log\left[\frac{1}{\sigma^2\omega_3\sigma\omega_1\sigma_3\omega_1}e^{i(-\pi+\alpha)}e^{-2\eta_1\omega_3\frac{\alpha}{\pi}+2\eta_3\omega_1+\eta_3 s_B}\right.$$

$$\left.\times e^{2\eta_1\left(\frac{\alpha}{\pi}-1\right)s+2\eta_3 s}\times\frac{\sigma(s-s_E)\,\sigma(s-s_G)\,\sigma(s-s_B)}{\sigma_1 s\,\sigma_2 s\,\sigma_3 s}\right]$$

to naru.

Soreyueni

$$dt = e^{i\Omega(s)}dw = \frac{2C_0 e^{i(-\pi+\alpha)}e^{-2\eta_1\omega_3\frac{\alpha}{\pi}+2\eta_3\omega_1+\eta_3 s_B}}{\sigma^2\omega_3\,\sigma\omega_1\,\sigma_3\omega_1}$$

$$\times e^{2\eta_1\left(\frac{\alpha}{\pi}-1\right)s+2\eta_3 s}\times\frac{\sigma(s-s_E)\,\sigma(s-s_G)\,\sigma(s-s_B)}{\sigma^3 s}\,ds$$

to naru. Ima

$$F(s) = e^{2\eta_1\left(\frac{\alpha}{\pi}-1\right)s+2\eta_3 s}\times\frac{\sigma(s-s_E)\,\sigma(s-s_G)\,\sigma(s-s_B)}{\sigma^3 s}$$

to sureba

$$F(s+2\omega_1) = F(s)\ ,$$

$$F(s+2\omega_3) = e^{2i\alpha}F(s)$$

de aru kara $F(s)$ wa daini-syuno Daen-kwansû de aru.

Soreyueni $F(s)$ wa

$$A(s) = -\frac{\sigma\left(s-2\omega_1\frac{\alpha}{\pi}\right)}{\sigma s\,\sigma\left(2\omega_1\frac{\alpha}{\pi}\right)}e^{2\eta_1\frac{\alpha}{\pi}s}$$

naru Kwansû wo motiite tugino yôni kaku koto ga dekiru.

$$F(s) = C_1A(s)+C_2A'(s)+C_3A''(s)\ .$$

Kokode

$$C_3 = -\frac{1}{2}\sigma s_E \sigma s_G \sigma s_B ,$$

$$C_2 = 2C_3\left\{\zeta s_E + \zeta s_G + \zeta s_B - 2\left[\eta_1\left(\frac{\alpha}{\pi}-1\right)+\eta_3\right]\right\} ,$$

$$C_1 = -2C_3(\wp s_E + \wp s_G + \wp s_B) - \frac{C_2^2}{2C_3}$$

de aru.

Mata

$$\frac{2C_0 e^{i(-\pi+\alpha)-2\eta_1\omega_3\frac{\alpha}{\pi}+2\eta_3\omega_1+\eta_3 s_B}}{\sigma^2\omega_3\sigma\omega_1\sigma_3\omega_1}$$

$$= \frac{2C_0}{\sigma^2\omega_3\sigma\omega_1\sigma_3\omega_1} e^{-i\pi+\eta_3\omega_1\left(3-2\frac{\alpha}{\pi}-\frac{\theta_E}{\pi}\right)} (= M)$$

de atte, zituno Sû de aru. Ima kore wo M de arawasu.

C_1, C_2, C_3 mo zituno Sû de aru kara $MC_1 = K_1$, $MC_2 = K_2$, $MC_3 = K_3$ to sureba K_1, K_2, K_3 wa zituno Sû de aru. Soreyueni

$$dt = \left[K_1 A(s) + K_2 A'(s) + K_3 A''(s)\right] ds \ldots\ldots\ldots\ldots \quad (2)$$

to naru. t wa GE wo hitomawari-sitemo onazi Atai wo motu kara

$$K_1 = 0$$

to naru. Soreyueni

$$\wp s_E + \wp s_G + \wp s_B + \left[\zeta s_E + \zeta s_G + \zeta s_B - 2\left\{\eta_1\left(\frac{\alpha}{\pi}-1\right)+\eta_3\right\}\right]^2 = 0 \quad (3)$$

naru Dyôken ga erareru.

Mata A ni okeru Sokudo wa

$$e^{-i\Omega(o)} = -\frac{1}{M}\times\frac{1}{\sigma s_E \sigma s_G \sigma s_B} = \frac{1}{2MC_3} = \frac{1}{2K_3}$$

de aru. Ima kore wo $\frac{1}{2}$ ni naru yôni C_0 wo kimeru to $K_3 = 1$ to naru.

GE to AB no Entyô to ga maziwaru Ten wo $t = 0$ to si (2) Siki wo sekibun-si, sonoue maeni nobeta Dyôken wo ireruto

$$t = K_2 A(s) + A'(s) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

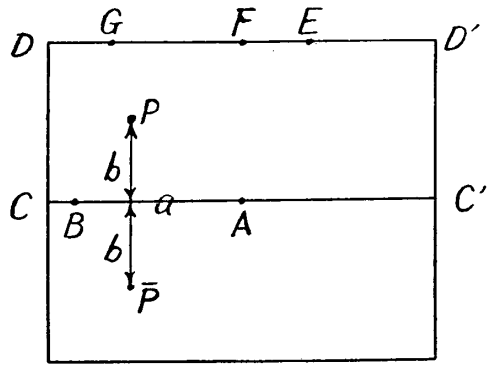
to naru. Kokode

$$K_2 = MC_2 = 2MC_3\left\{\zeta s_E + \zeta s_G + \zeta s_B - 2\left[\eta_1\left(\frac{\alpha}{\pi} - 1\right) + \eta_3\right]\right\}$$

$$= 2\left\{\zeta s_E + \zeta s_G + \zeta s_B - 2\left[\eta_1\left(\frac{\alpha}{\pi} - 1\right) + \eta_3\right]\right\}$$

de aru.

Tugi ni Hukuso-*potential* Kwansû wa s-Men dewa tuginoyôni naru.



$P(a,\ b)$,
$\bar{P}(a,\ -b)$

Du 8. s-Men.

$$\chi = Cs + \frac{m}{2\pi}\log\frac{\vartheta_1^2\left(\frac{s}{2\omega_1}\right)}{\vartheta_1\left(\frac{s-s_P}{2\omega_1}\right)\vartheta_1\left(\frac{s-\bar{s}_P}{2\omega_1}\right)}.$$

Tadasi s_P wa t_P ga zitu de t_B to o to no aida ni aru yôni erabu.

G ni oite nameraka ni nagareru tame niwa $s = s_G$ nite $\frac{d\chi}{ds} = 0$ to naru hituyô ga aru. Soreyueni

$$C = -\frac{m}{4\pi\omega_1}\left[2\frac{\vartheta_1'\left(\frac{s_G}{2\omega_1}\right)}{\vartheta_1\left(\frac{s_G}{2\omega_1}\right)} - \frac{\vartheta_1'\left(\frac{s_G - s_P}{2\omega_1}\right)}{\vartheta_1\left(\frac{s_G - s_P}{2\omega_1}\right)} - \frac{\vartheta_1'\left(\frac{s_G - \bar{s}_P}{2\omega_1}\right)}{\vartheta_1\left(\frac{s_G - \bar{s}_P}{2\omega_1}\right)}\right]$$

to naru.

Mokei no mawari no *circulation* wo Γ to sureba

$$\Gamma = 2\omega_1 C$$

de aru.

(1) Siki oyobi (3) Siki no hutatuno Dyôken dewa s_E, s_G, s_B wo kimeru koto ga dekinai. Soreyueni B ni oitewa Nagare ga nameraka de aru koto kara $s = s_B$ de

$$\frac{d\chi}{ds} = 0$$

ni naru yôni s_B wo kimeru.

Mata z-Men ni oite A ni okeru Sokudo wo $-U$ to sureba

$$\left(\frac{d\chi}{dz}\right)_{s=0} = -\frac{m}{d} = -U$$

de aru kara

$$m = Ud$$

to nari m ga kimaru.

Tugini Mokei ni hataraku Tikara wo keisansuru. *Circulation* nomi matawa *circulation* ga nai Nagare no naka dewa, Mokei ni Tikara ga hatarakanai kara, $\left(\frac{d\chi}{ds}\right)^2$ no uti de korerano Kô wa habuite

$$2C\frac{m}{4\pi\omega_1}\left[2\frac{\vartheta_1'\left(\frac{s}{2\omega_1}\right)}{\vartheta_1\left(\frac{s}{2\omega_1}\right)}-\frac{\vartheta_1'\left(\frac{s-\bar{s}_P}{2\omega_1}\right)}{\vartheta_1\left(\frac{s-s_P}{2\omega_1}\right)}-\frac{\vartheta_1'\left(\frac{s-\bar{s}_P}{2\omega_1}\right)}{\vartheta_1\left(\frac{s-\bar{s}_P}{2\omega_1}\right)}\right]$$

$$=C\frac{Ud}{\pi}\left[2\zeta s-\zeta(s-s_P)-\zeta(s-\bar{s}_P)-\frac{\eta_1}{\omega_1}(s_P+\bar{s}_P)\right]$$

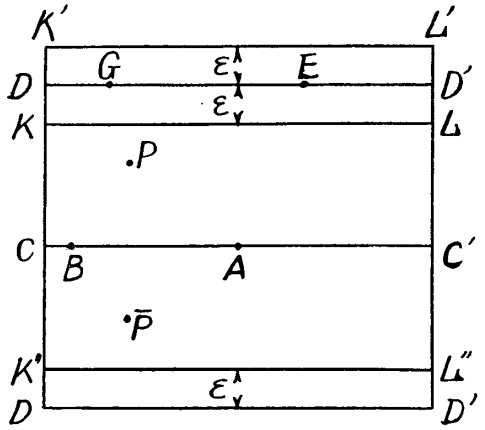
ga nokoru. Soreyueni

$$\int\left(\frac{d\chi}{dz}\right)^2dz=\int\left(\frac{d\chi}{ds}\right)^2\frac{ds}{dt}\cdot\frac{dt}{dz}\cdot ds$$

$$=-\int C\frac{Ud}{\pi}\left[2\zeta s-\zeta(s-s_P)-\zeta(s-\bar{s}_P)-\frac{\eta_1}{\omega_1}(s_P+\bar{s}_P)\right]\cdot\frac{\sigma s_E\,\sigma s_G\,\sigma s_B}{2}$$

$$\times e^{-2\eta_1\left(\frac{\alpha}{\pi}-1\right)s-2\eta_3 s}\times\frac{\sigma^3 s}{\sigma(s-s_E)\,\sigma(s-s_G)\,\sigma(s-s_B)}$$

$$\times\frac{2\pi}{d}\left[K_2A(s)+A'(s)-K_2A(s_P)-A'(s_P)\right]ds \quad \cdots\cdots\cdots\cdots \quad (5)$$



Du 9. s-Men.

s-Men ni okeru Sesibun no Miti wa KL de aru. Ima

$$F_1(s) = \frac{\sigma_3 s}{\sigma(s-s_E)\,\sigma(s-s_G)\,\sigma(s-s_B)} e^{-2\eta_1\left(\frac{\alpha}{\pi}-1\right)s-2\eta_3 s}$$
$$\times\left[2\zeta s-\zeta(s-s_P)-\zeta(s-\bar{s}_P)-\frac{\eta_1}{\omega_1}(s_P+\bar{s}_P)\right]$$
$$\times\left[K_2A(s)+A'(s)\right]$$

to sureba $F_1(s)$ wa Daen-kwansû de aru. Soreyueni

$$F_1(s) = \bar{C}_0+\bar{C}_1\zeta(s-s_E)+\bar{C}_2\zeta(s-s_G)+\bar{C}_3\zeta(s-s_B)$$
$$+\bar{C}_4\zeta(s-s_P)+\bar{C}_5\zeta(s-\bar{s}_P)$$

to naru. Kokode

$$\bar{C}_1 = e^{-2\eta_1\left(\frac{\alpha}{\pi}-1\right)s_E-2\eta_3 s_E}\times\frac{\sigma^3 s_E}{\sigma(s_E-s_G)\,\sigma(s_E-s_B)}$$
$$\times\left[2\zeta s_E-\zeta(s_E-s_P)-\zeta(s_E-\bar{s}_P)-\frac{\eta_1}{\omega_1}(s_P+\bar{s}_P)\right]\left[K_2A(s_E)+A'(s_E)\right],$$

$$\bar{C}_2 = e^{-2\eta_1\left(\frac{\alpha}{\pi}-1\right)s_G-2\eta_3 s_G}\times\frac{\sigma^3 s_G}{\sigma(s_G-s_E)\,\sigma(s_G-s_B)}$$
$$\times\left[2\zeta s_G-\zeta(s_G-s_P)-\zeta(s_G-\bar{s}_P)-\frac{\eta_1}{\omega_1}(s_P+\bar{s}_P)\right]\left[K_2A(s_G)+A'(s_G)\right],$$

$$\bar{C}_3 = e^{-2\eta_1\left(\frac{\alpha}{\pi}-1\right)s_B-2\eta_3 s_B}\times\frac{\sigma^3 s_E}{\sigma(s_B-s_E)\,\sigma(s_B-s_G)}$$
$$\times\left[2\zeta s_B-\zeta(s_B-s_P)-\zeta(s_B-\bar{s}_P)-\frac{\eta_1}{\omega_1}(s_P+\bar{s}_P)\right]\left[K_2A(s_B)+A'(s_B)\right],$$

$$\bar{C}_4 = -e^{-2\eta_1\left(\frac{\alpha}{\pi}-1\right)s_P-2\eta_3 s_P}\times\frac{\sigma^3 s_P}{\sigma(s_P-s_E)\,\sigma(s_P-s_G)\,\sigma(s_P-s_B)}$$
$$\times\left[K_2A(s_P)+A'(s_P)\right],$$

$$\bar{C}_5 = -e^{-2\eta_1\left(\frac{\alpha}{\pi}-1\right)\bar{s}_F-2\eta_3\bar{s}_P} \times \frac{\sigma^3\bar{s}_P}{\sigma(\bar{s}_P-s_E)\,\sigma(\bar{s}_P-s_G)\,\sigma(\bar{s}_P-s_B)}$$

$$\times\left[K_2A(\bar{s}_P)+A'(\bar{s}_P)\right]$$

de atte

$$\bar{C}_1+\bar{C}_2+\bar{C}_3+\bar{C}_4+\bar{C}_5=0$$

de aru.

Mata

$$F_2(s) = e^{i\alpha}\,e^{-2\eta_1\left(\frac{\alpha}{\pi}-1\right)s-2\eta_3 s} \times \frac{\sigma^3 s}{\sigma(s-s_E)\,\sigma(s-s_G)\,\sigma(s-s_B)}$$

$$\times\left[2\zeta s-\zeta(s-s_P)-\zeta(s-\bar{s}_P)-\frac{\eta_1}{\omega_1}(s_P+\bar{s}_P)\right]$$

to sureba, $F_2(s)$ wa daini-syuno Daen-kwansû de atte,

$$\bar{A}(s) = \frac{\sigma\left(s+2\omega_1\frac{\alpha}{\pi}\right)}{\sigma s\,\sigma\left(2\omega_1\frac{\alpha}{\pi}\right)}e^{-2\eta_1\frac{\alpha}{\pi}s}$$

naru Kwansû wo motiiruto

$$F_2(s) = e^{i\alpha}\left[B_1\bar{A}(s-s_E)+B_2\bar{A}(s-s_G)+B_3\bar{A}(s-s_B)+B_4\bar{A}(s-s_P)\right.$$

$$\left.+B_5\bar{A}(s-\bar{s}_P)\right]$$

to naru. Kokode

$$B_1 = \frac{\bar{C}_1}{K_2A(s_E)+A'(s_E)},$$

$$B_2 = \frac{\bar{C}_2}{K_2A(s_G)+A'(s_G)},$$

$$B_3 = \frac{\bar{C}_3}{K_2 A(s_E) + A'(s_E)},$$

$$B_4 = \frac{\bar{C}_4}{K_2 A(s_P) + A'(s_P)},$$

$$B_5 = \frac{\bar{C}_5}{K_2 A(\bar{s}_P) + A'(\bar{s}_P)},$$

de aru.

$F_1(s)$ oyobi $F_2(s)$ wa tomoni s-Men de KL no ue to $K'L'$ no ue to de kyôyakuna Atai wo motu. (5) no Sekibun wa KL no ue de okonau node aru ga $K'L'$ no ue de okonaeba $\int_K^L$ to $\int_{K''}^{L''}$ to wa kyôyakuna Atai wo motu.

$$\int_{K'}^{L'} F_1(s)ds = \bar{C}_0 2\omega_1 - 2\eta_1(\bar{C}_1 s_E + \bar{C}_2 s_G + \bar{C}_3 s_E + \bar{C}_4 s_P + \bar{C}_5 \bar{s}_P)$$

de aru ga

$$\bar{C}_0 = \frac{2}{\sigma s_E \sigma s_G \sigma s_E} + \bar{C}_1 \zeta s_E + \bar{C}_2 \zeta s_G + \bar{C}_3 \zeta s_E + \bar{C}_4 \zeta s_P + \bar{C}_5 \zeta \bar{s}_P$$

de aru kara

$$\begin{aligned}\int_{K'}^{L'} F_1(s)\,ds = 2\omega_1 \Bigg[& \frac{2}{\sigma s_E \sigma s_G \sigma s_E} + \bar{C}_1\left(\zeta s_E - \frac{\eta_1}{\omega_1} s_E\right) + \bar{C}_2\left(\zeta s_G - \frac{\eta_1}{\omega_1} s_G\right) \\ & + \bar{C}_3\left(\zeta s_E - \frac{\eta_1}{\omega_1} s_E\right) + \bar{C}_4\left(\zeta s_P - \frac{\eta_1}{\omega_1} s_P\right) \\ & + \bar{C}_5\left(\zeta \bar{s}_P - \frac{\eta_1}{\omega_1} \bar{s}_P\right)\Bigg]\end{aligned}$$

to naru.

Mata

$$\int_{K''}^{L''} F_2(s)\,ds = e^{2i\alpha} \int_{K'}^{L'} F_2(s)ds$$

de aru kara

$$\int_{K'}^{L'} F_2(s)ds - \int_{K''}^{L''} F_2(s)\,ds = (1 - e^{2i\alpha}) \int_{K'}^{L'} F_2(s)ds$$

$$= -2\pi i(B_1+B_2+B_3+B_4+B_5)e^{i\alpha}.$$

Soreyueni

$$\int_{K'}^{L'} F_2(s)\,ds = \frac{\pi}{\sin\alpha}(B_1+B_2+B_3+B_4+B_5)$$

to naru.

Kôryoku $+i$ Yoryoku

$$= -\frac{i\rho}{2}\int_{K'}^{L'} \sigma s_E \sigma s_G \sigma s_B\, CU\Big[F_1(s) - e^{-i\alpha}F_2(s)\Big\{K_2A(s_P) + A'(s_P)\Big\}\Big]ds$$

$$= -\frac{i\rho}{2} CU\sigma s_E \sigma s_G \sigma s_B\Big[\int_{K'}^{L'} F_1(s)ds - e^{-i\alpha}\Big\{K_2A(s_P) + A'(s_P)\Big\}\int_{K'}^{L'} F_2(s)\,ds\Big]$$

$$= -i\rho\omega_1 CU\sigma s_E \sigma s_G \sigma s_B\Big[\frac{2}{\sigma s_E \sigma s_G \sigma s_B} + \bar{C}_1\Big(\zeta s_B - \frac{\eta_1}{\omega_1}s_E\Big)$$

$$+ \bar{C}_2\Big(\zeta s_G - \frac{\eta_1}{\omega_1}s_G\Big) + \bar{C}_3\Big(\zeta s_B - \frac{\eta_1}{\omega_1}s_B\Big) + \bar{C}_4\Big(\zeta s_P - \frac{\eta_1}{\omega_1}s_P\Big)$$

$$+ \bar{C}_5\Big(\zeta \bar{s}_P - \frac{\eta_1}{\omega_1}\bar{s}_P\Big)$$

$$- e^{-i\alpha}\Big\{K_2A(s_P) + A'(s_P)\Big\}\frac{2}{\sin\alpha}(B_1+B_2+B_3+B_4+B_5)\Big]$$

$$= i\rho\frac{U^2d}{2\pi}\left[2\frac{\vartheta_1'\left(\frac{s_G}{2\omega_1}\right)}{\vartheta_1\left(\frac{s_G}{2\omega_1}\right)}-\frac{\vartheta_1'\left(\frac{s_G-s_P}{2\omega_1}\right)}{\vartheta_1\left(\frac{s_G-s_P}{2\omega_1}\right)}-\frac{\vartheta_1'\left(\frac{s_G-\bar{s}_P}{2\omega_1}\right)}{\vartheta_1\left(\frac{s_G-\bar{s}_P}{2\omega_1}\right)}\right]$$

$$\times\left[1+\frac{\sigma s_E\sigma s_G\sigma s_B}{2}\left\{\bar{C}_1\left(\zeta s_E-\frac{\eta_1}{\omega_1}s_E\right)+\bar{C}_2\left(\zeta s_G-\frac{\eta_1}{\omega_1}s_G\right)\right.\right.$$

$$+\bar{C}_3\left(\zeta s_B-\frac{\eta_1}{\omega_1}s_B\right)+\bar{C}_4\left(\zeta s_P-\frac{\eta_1}{\omega_1}s_P\right)$$

$$+\bar{C}_5\left(\zeta\bar{s}_P-\frac{\eta_1}{\omega_1}\bar{s}_P\right)$$

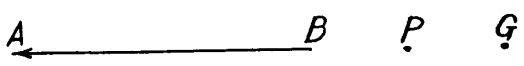
$$\left.\left.-e^{-i\alpha}\left(K_2A(s_P)+A'(s_P)\right)\frac{\pi}{\sin\alpha}(B_1+B_2+B_3+B_4+B_5)\right\}\right] \quad (6)$$

to naru.

II. *Mokei ga Ten no Udu de okikaerareta Baai.*

Ima made nobeta yôni sureba Hukidasi-guti no Eikyô wa tonikaku keisan dekiru, sikasi taihen hukuzatu de atte α ga taihen tiisai Baai ni oitemo amari kantan ni naranai. Soreyueni mottomo kantanna Baai to site Mokei ga Ten no Udu ni yotte oki-kaerareta Baai wo kangaete miyô.

Kono Baai niwa t-Men wa Du 10 no yôni naru.

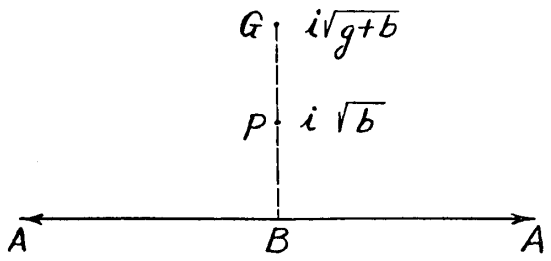


Du 10. t-Men.

Ima

$$s = i\sqrt{t+b}$$

naru Kwankei ni yotte t-Men wo s-Men ni henkwan-sureba Du 11 no yôni naru.



Du 11. s-Men.

Kokode g, b wa t-Men ni okeru G, B no Atai de aru.

z-Men ni oite nozomu yôna Hukuso-*potential* Kwansû wa s-Men dewa

$$w = -\frac{iI}{2\pi}\log\frac{s-i\sqrt{g+b}}{s+i\sqrt{g+b}} - \frac{m}{2\pi}\log(s-i\sqrt{b})(s+i\sqrt{b})$$

$$+\frac{iI'}{2\pi}\log\frac{s-i\sqrt{b}}{s+i\sqrt{b}}$$

de aru. Sore yueni w wa z-Men dewa

$$w = -\frac{i\Gamma}{2\pi}\log\frac{\sqrt{ge^{\frac{2\pi z}{d}}+b}-i\sqrt{g+b}}{\sqrt{ge^{\frac{2\pi z}{d}}+b}+i\sqrt{g+b}} - \frac{mz}{d}$$

$$+\frac{i\Gamma'}{2\pi}\log\frac{\sqrt{ge^{\frac{2\pi z}{d}}+b}-i\sqrt{b}}{\sqrt{ge^{\frac{2\pi z}{d}}+b}+i\sqrt{b}}.$$

$$\therefore \quad \frac{dw}{dz} = -\frac{i\Gamma}{d}\sqrt{g+b}\,\frac{e^{\frac{2\pi z}{d}}}{\sqrt{ge^{\frac{2\pi z}{d}}+b\left(e^{\frac{2\pi z}{d}}-1\right)}} - \frac{m}{d}$$

$$+\frac{i\Gamma'}{d}\sqrt{b}\,\frac{1}{\sqrt{ge^{\frac{2\pi z}{d}}+b}}\,.$$

$z=+\infty$ ni oitewa $u=-U$ de aru kara

$$-\frac{m}{d} = -U\,.$$

$$\therefore \quad m = Ud$$

de m ga kimaru.

B ni oite namerakani nagareru tame niwa, B ni oitewa $ge^{\frac{2\pi z_B}{d}}+b=0$ de aru kara

$$-\frac{i\Gamma}{d}\sqrt{g+b}\,\frac{e^{\frac{2\pi z_B}{d}}}{e^{\frac{2\pi z_B}{d}}-1}+\frac{i\Gamma'}{d}\sqrt{b}=0$$

de aru. Sore yueni

$$\Gamma' = \Gamma\sqrt{\frac{b}{g+b}}$$

to naru.

$z=-\infty$ ni oite wa

$$\left(\frac{dw}{dz}\right)_{z=-\infty} = -U+\frac{i\Gamma}{d}\sqrt{\frac{b}{g+b}}\,.$$

Soreyueni Nagare wa x-Diku ni taisite sitamukini $\frac{\Gamma}{Ud}\sqrt{\frac{b}{b+g}}$ no Kaku wo nasu.

Tugini $z=0$ ni oitewa $z=0$ ni okeru *circulation* no Eikyô wo nozokeba

$$\left(\frac{dw}{dz}+\frac{i\Gamma}{2\pi z}\right)_{z=0}=-\frac{i\Gamma}{2d}\cdot\frac{b}{g+b}-U+\frac{i\Gamma}{d}\cdot\frac{b}{g+b}$$

$$=-U+\frac{i\Gamma}{2d}\cdot\frac{1}{1+\frac{g}{b}}$$

de aru. Sikaruni Hukidasi-guti kara Mokei ni itaru x-Diku Hôkô no Hedatari wo L to sureba

$$\frac{b}{g}=e^{\frac{2\pi L}{d}}$$

de aru kara, Mokei no Tokoro ni oitewa Nagare wa sitamukini

$$\frac{\Gamma}{2dU}\cdot\frac{1}{1+e^{-\frac{2\pi L}{d}}}=\Delta\alpha$$

no Kaku dake magaru.

Yoku no Gen wo l to sureba

$$C_z=\frac{\rho\Gamma U}{\frac{1}{2}\rho l U^2}$$

de aru kara

$$\Gamma=\frac{1}{2}C_z l U$$

to naru. Mata

$$\frac{\partial C_z}{\partial\alpha}=a$$

to sureba

$$\Delta C_z=a\Delta\alpha$$

de aru kara

$$\Delta C_z = a\frac{C_z l}{4d}\cdot\frac{1}{1+e^{-\frac{2\pi L}{d}}}\,.$$

$d=\infty$ no Baai no C_z wo $C_{z\infty}$ to sureba

$$C_z = C_{z\infty} - \Delta C_z\,.$$

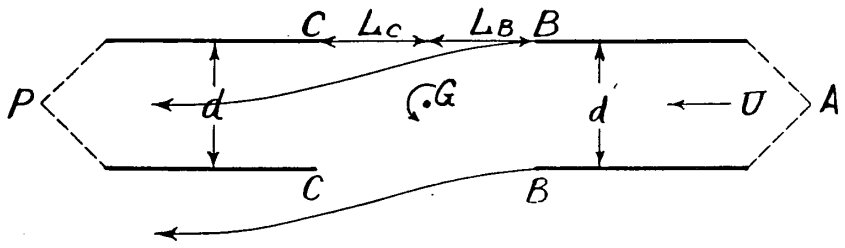
$$\therefore\quad \frac{C_z}{C_{z\infty}} = \frac{1}{1+a\frac{l}{4d}\cdot\frac{1}{1+e^{-\frac{2\pi L}{d}}}} \qquad \cdots\cdots\cdots\cdots\cdots\cdots (7)$$

Kore wa *Poggi* ga dasita Siki to onazi de aru.

Ima toriatukatta Baai wa Mokei ga Hûtô no mannakano Diku no ueni aru Baai de aru ga, yoko ni soreta Baai mo G no Iti wo zurasu nomi de kantan ni dekiru.

III. *Hukidasi-guti oyobi Suikomi-guti no Eikyô.*

Kono Baai niwa z-Men wa Du 12 ni simesu yôni naru.



Du 12. z-Men.

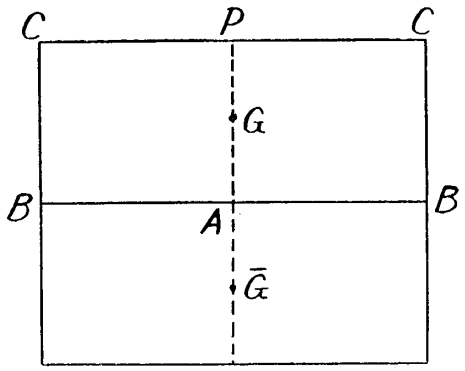
$t = ge^{\frac{2\pi z}{d}}$ naru Kwankei ni yotte t-Men ni henkwan-sureba Du 13 no yôni naru. Kokode g wa t-Men ni okeru PG no Hedatari de aru.

Du 13. *t*-Men.

t-Men wa

$$t = -\wp s + e_3$$

naru Kwankei ni yotte *s*-Men ni henkwan dekiru.



Du 14. *s*-Men.

z-Men ni oite nozomu yôna Hukuso-*potential* Kwansû wa *s*-Men ni oite wa

$$w = -\frac{i\Gamma}{2\pi}\log\frac{\sigma(s-s_G)}{\sigma(s-\bar{s}_G)}+\frac{m}{2\pi}\log\frac{\sigma^2 s}{\sigma(s-\omega_3)\sigma(s+\omega_3)}+C_s$$

de aru. $\bar{s}_G$ wa s_G no kyôyaku na Atai de atte, kono Baai $\bar{s}_G = -s_G$ de aru.

$$\frac{dw}{dz} = -\frac{i\Gamma}{2\pi}\Big[\zeta(s-s_G)-\zeta(s-\bar{s}_G)\Big]+\frac{m}{2\pi}\Big[2\zeta s-\zeta(s-\omega_3)-\zeta(s+\omega_3)\Big]+C$$

$$= -\frac{i\Gamma}{2\pi}\Big[-2\zeta s_G+\frac{\wp' s_G}{\wp s-\wp s_G}\Big]+\frac{m}{\pi}(\zeta s-\zeta_3 s)+C\,.$$

Tugini

$$\frac{ds}{dt} = -\frac{1}{\wp' s}, \quad \frac{dt}{dz} = \frac{2\pi}{d}(-\wp s + e_3)$$

de aru kara

$$\frac{dw}{dz} = \frac{dw}{ds} \cdot \frac{ds}{dt} \cdot \frac{dt}{dz}$$

$$= \frac{2\pi}{d} \cdot \frac{dw}{ds} \cdot \frac{\wp s - e_3}{\wp' s}$$

to naru.

B ni oitewa nameraka ni nagareru yueni $\left(\frac{dw}{ds}\right)_{s=\omega_1} = 0$.

$$\therefore \quad C = -\frac{i\Gamma}{2\pi}\left[2\zeta s_E + \frac{\wp' s_G}{\wp s_G - e_1}\right]$$

to naru.

Mata A ni oite wa $u = -U$ de aru kara

$$\left(\frac{dw}{dz}\right)_{s=0} = -\frac{m}{d} = -U .$$

$$\therefore \quad m = Ud$$

de aru

Tugini

$$\frac{dw}{dz} + \frac{i\Gamma}{2\pi z} = \frac{2\pi}{d} \cdot \frac{\wp s - e_3}{\wp' s}\left\{-\frac{i\Gamma}{2\pi}\left(\frac{\wp' s_G}{\wp s_G - e_1} + \frac{\wp' s_G}{\wp s - \wp s_G}\right)\right.$$

$$\left. + \frac{m}{\pi}(\zeta s - \zeta_3 s)\right\} + \frac{i\Gamma}{d} \cdot \frac{1}{\log(\wp s - e_3) - \log(\wp s_G - e_3)}$$

de aru. s ga s_G ni taihen tikazuku Baai niwa

$$\lim_{\varepsilon=0} \frac{1}{\log[\wp(s_G+\epsilon)-e_3]-\log(\wp s_G-e_3)}$$

$$= \lim_{\varepsilon=0} \frac{1}{\frac{\epsilon\wp' s_G}{\wp s_G-e_3}+\frac{\wp'' s_G(\wp s_G-e_3)-(\wp' s_G)^2}{(\wp s_G-e_3)^2}\cdot\frac{\epsilon^2}{2}+\cdots.}$$

$$= \lim_{\varepsilon=0} \frac{\wp s_G-e_3}{\wp' s_G}\left[\frac{1}{\epsilon}-\frac{1}{2}\left(\frac{\wp'' s_G}{\wp' s_G}-\frac{\wp' s_G}{\wp s_G-e_3}\right)+\cdots.\right].$$

Mata

$$\lim_{\varepsilon=0} \frac{\wp(s_G+\epsilon)-e_3}{\wp'(s_G+\epsilon)[\wp(s_G+\epsilon)-\wp s_G]}$$

$$= \lim_{\varepsilon=0} \frac{\wp s_G-e_3+\epsilon\wp' s_G+\cdots.}{[\wp' s_G+\epsilon\wp'' s_G+\ldots]\left[\epsilon\wp' s_G+\frac{\epsilon^2}{2}\wp'' s_G+\cdots.\right]}$$

$$= \lim_{\varepsilon=0} \frac{\wp s_G-e_3}{(\wp' s_G)^2}\left[\frac{1}{\epsilon}+\frac{\wp' s_G}{\wp s_G-e_3}-\frac{3}{2}\frac{\wp'' s_G}{\wp' s_G}+\cdots.\right]$$

de aru. Soreyueni

$$\lim_{s=s_G}\left(\frac{dw}{dz}+\frac{i\Gamma}{2\pi z}\right)$$

$$= -U-\frac{i\Gamma}{d}\left[-\frac{\wp'' s_G}{\wp' s_G}+\frac{\wp' s_G}{\wp s_G-e_1}+\frac{1}{2}\frac{\wp' s_G}{\wp s_G-e_3}\right]\frac{\wp s_G-e_3}{\wp' s_G}$$

de aru ga

$$\frac{\wp'' s_G}{\wp' s_G}=\frac{1}{2}\left[\frac{\wp' s_G}{\wp s_G-e_1}+\frac{\wp' s_G}{\wp s_G-e_2}+\frac{\wp' s_G}{\wp s_G-e_2}\right]$$

de aru kara, G ni oite wa

$$u=-U$$

$$v = -\frac{\Gamma}{2d}\left[\frac{\wp s_G - e_3}{\wp s_G - e_3} - \frac{\wp s_G - e_3}{\wp s_G - e_1}\right].$$

$$= -\frac{\Gamma}{2d}\left[\frac{1}{1+e^{-\frac{2\pi L_C}{d}}} - \frac{1}{1+e^{+\frac{2\pi L_B}{d}}}\right]$$

de aru. Kokode L_B, L_C wa Du 12 ni simesu tôri de aru.

Soreyueni mae no Baai to dôyôni site

$$\frac{C_z}{C_{z\infty}} = \frac{1}{1+a\frac{l}{4d}\left[\frac{1}{1+e^{-\frac{2\pi L_C}{d}}} - \frac{1}{1+e^{+\frac{2\pi L_B}{d}}}\right]} \quad \cdots\cdots\cdots (8)$$

to naru. $L_C = \infty$ no Baai niwa

$$\frac{C_z}{C_{z\infty}} = \frac{1}{1+a\frac{l}{4d}\cdot\frac{1}{1+e^{-\frac{2\pi L_B}{d}}}}$$

to nari mae no Baai to onazi ni naru.

Ima made keisan-sita Kekkwa de miru to Hukidasi-guti oyobi Suikomi-guti no Eikyô wa sukunaku, ziyûna Hukidasi to sita Baai no Heri no 10% gurai Herikata ga sukunaku naru dake de aru.