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抄錄

空氣密度がピトー係數に及ぼす影響

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從來ピトー係數は空氣密度に關係する場合はレーノルヅ數の變化によつてのみ影響があると考へて居たのであるが，色々の空氣密度で實驗して居る際にレーノルヅ數以外に空氣密度によつてピトー係數が變ると思はれることが多々あつた．それでこの實驗を行つたのであるが，何分空氣速度の絶對的測定は困難であつて精密な測定が行はれて居ないので，標準となるべきピトー管がなく次の三つの方法による測定を比較して見た．

先づトンネルでレールの上にピトー管を裝置した台車を走らせて實驗した．この實驗では台車の速度が小さくて壓力測定が困難なので風車を用ひて實驗し台車の影響を除去して風車の絶對的測定の基準とし，次に風洞內で風車の實驗を以つて同一速度に於ける風車の回轉數をトンネル實驗値と風洞實驗値と比較して見た．この實驗はよく一致するので風車を基準としてピトー管の實驗を行つた．無論トンネル內でもピトー管の實驗を行つて點檢して見たがよく一致することが判つた．

以上の實驗で先づ一氣壓の場合に於けるピトー係數の測定が出來たので，空氣密度の小さい場合の測定を行ふことにした．然るに空氣密度の小さい場合に於ける風速の絶對測定の標準計器がないので又風車を基準とすることにしたのである．風車を廻轉腕上に乘せて圓周速度と風車の廻轉數との關係を調べて見た處，Swirl の補正を行へば前記トンネル及び風洞實驗の結果とよく一致することが判つた．

次に低壓室內にて風車を廻轉腕に乘せて前述の如き實驗を行つたのであるが，Swirl の影響は一氣壓の場合と同じであることが判り，空氣密度が小さい場合に於ける風車の廻轉と速度との關係の最も信ずべき結果を得た．從つてこれを標準として低壓低溫風洞內でピトー係數を測定したのであつてその結果は第28圖に示す通りピトー係數はレーノルヅ數によるよりは空氣密度による方が遙かに大なることを知つたのである．

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On the Effect of the Density of the Air upon the Pitot-static Tube Coefficient.

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Introduction.

Some years ago we have constructed a low pressure and low temperature wind tunnel in our institute. The velocity of the air stream in the tunnel was measured by means of a standard pitot-static tube, assuming the pressure difference generated by the differential pressure head is proportional to the air density. But as the measurements were carried out further, we found the fact that the pitot-static tube coefficient itself seemed to be affected by the air density. Since then we were occupied ourselves by the absolute determination of the pitot-static tube coefficient, by causing it to move in a straight path through still air. The experiments were made in a tunnel of a railway in Tiba district, which is now out of use owing to the crack of its part by the earthquake. As the speed, however, of the car on the rail is very low, the pressure differences set up by the pitot-static tube are very small. This difficulty in the measurements of speed would be overcome by the development of a specially sensitive manometer, if the experiments were made in the laboratory. But the use of a sensitive manometer on the car moving on the rail is difficult owing to the vibration of the car. Hence we determined to calibrate a windmill causing it to move on the rail and then to use it as a standard for the calibration of the pitot-static tube in the wind tunnel. The determination of the pitot-static tube coefficient on the rail was also carried out in order to check the results obtained in the wind tunnel. The experiments in low air density were made in a low pressure chamber by the use of a whirling arm and the results obtained in the low pressure and low temperature wind tunnel were checked.

The arrangement of Experiment in a Tunnel at Iwai.

The tunnel employed is situated between Iwai station and Tomiura station, the length of which is 730 m. and the rail is almost in a straight line. During the experiments both entrances are shut by doors and no current of air is made to occur in the tunnel. The rail is welded and smoothed and rubber rings are wound on the wheels to prevent the vibration of the car when it is moved on the rail. The car on which the measuring apparatus is put is pulled by a wire rope which is tugged by a 20 HP D.C. motor. The maximum speed attained by the car is 15 m./s. Along a side of the rail electric lines are prepared and with 5m. (± 5 mm.) intervals electric contact metals are provided. On these contact metals slides a brush which is fixed to the car as the latter moves on the rail. Whenever the contact is made an electric relay is operated, which pulls a chronogragh pen and puts a mark on the chronogram. The chronograph used has three pens: the first of which is used for time mark, the second for bench mark and the third for windmill revolution. These marks are put on a paper which is drawn out at the rate of 120 mm./s. by a 6 volt 30 watt 2000 r.p.m. D. C. shunt motor. The length of the paper let out in 0·01 sec. is 1·2 mm., which is easily measured by the use of a rule. At every 50 revolutions of the windmill an electric relay is opperated putting a mark on the chronogram. Fig. 1 shows a part of the chronogram, in which (A) shows a mark at every 50 revolutions of the windmill, (B) the time mark at each 1 sec. and (C) the mark at each contact of the brush on the contact metal along the rail. The velocity of the car and the revolutions per second of the windmill are calculated from the marks on the chronogram. The apparatus is put on a box which is suspended on the car by rubber strips, and is protected against the vibration of the car caused by its motion on the rail.

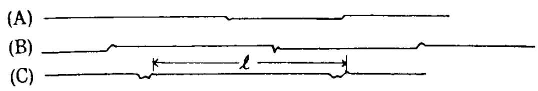


Fig. 1.

A small windmill, which is carefully constructed so that the friction of the bearing is very small, is fixed on a framework fastend to the car as shown in Fig. 2. This framework is carefully constructed so as

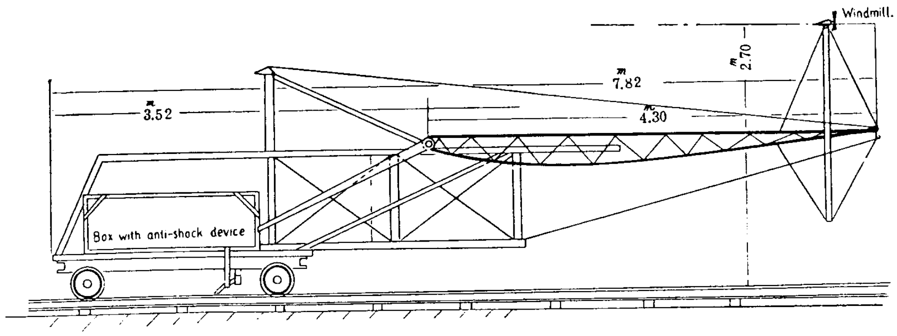
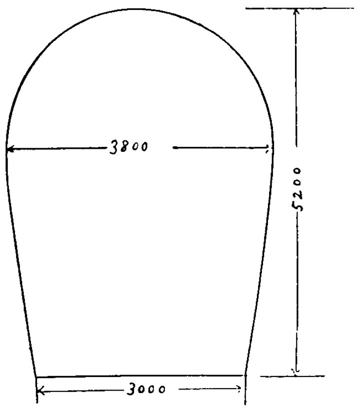


Fig. 2.

to avoid vibrations caused by the motion of the car. The windmill is fixed on a pole, which can be moved on the framework so that the distance of the windmill from the body of the car can be altered. The height of the windmill from the rail is so adjusted that it moves along nearly the central line of the tunnel. The cross section of the tunnel is shown in Fig. 3.

In determining the coefficient of the pitot-static tube, the same apparatus is used, altering the arrangement a little to make it convenient for that purpose. The windmill is removed and a pitot-static tube is installed at the top of the post, inside of which rubber tubes are housed connecting the pitot-static tube to the liquid manometer. The liquid column of the manometer is photographed by a small cinematograph camera driven by a 6 volt 50 watt D.C. shunt motor. The speed of the film is so adjusted that from 3 to 9 pictures are taken per unit time according to the speed of the car. To the cinematograph camera is attached a small shutter, which is opperated by an electric relay whenever a time signal at each second is send out from the chronometer and a time mark is put on the margin of the film. The film developed is shown in Fig. 4, and the apparatus is shown in Fig. 5.



Cross section of the tunnel.
Fig. 3.

Fig. 4.

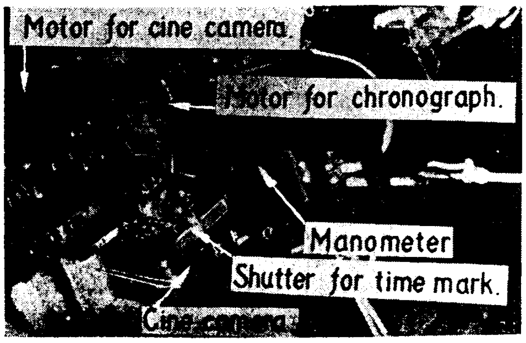
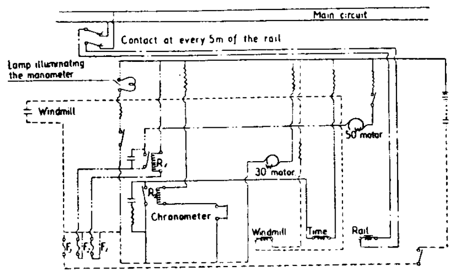


Fig. 5.



Pig. 6.

For the sake of economic consumption of the film, the motor is rotated only in the interval when the velocity of the car is constant.

For that purpose an electric circuit shown in Fig. 6 is used. Two circuit brakers of uneaqual height are provided at a side of the rail: the lower one at a place where the car is just reached to a constant velocity, and the higher one at a place where the car begins to deccelerate. Two strips of fusible metal F_1 and F_2 are streched out of the car, one to meat the lower braker and the other to meat the higher one. When F_1 is cut off the relay R_1 is released and switches the circuit to the motor to drive the cinematograph camera, and when F_2 is cut off the motor is stopped.

There is another circuit braker at a side of the rail and a third fusible metal F_3 projecting from the car. This fusible metal F_3 is cut off just before the car is stopped, and the lamp illuminating the manometer and the motor for chronograph are switched off. The time mark on the film and the corresponding time mark on the chronogram is made evident, since as the impulse at 0 sec. is not send out from the chronometer the time mark at 1 sec. (next to pause) and that at 59 sec. (before pause) are evident on the film and on the chronogram.

Results of Experiment.

As a first step we carried out the calibration of the rotating speed of a windmill, as shown in Fig. 7, when it was moved on the rail, setting it very close to the car, i.e. to the root of the framework. The results of 20 observations are shown in Fig. 8, in which n/v is plotted against v. Where n is the rotating speed of the windmill and v is its translational velocity.

It is widely known that n and v are in the following relation

$$n = a + bv$$

Fig. 7.

where a and b are constants. Hence n/v approaches an assymptotic value when v is large, which is clearly shown in the figure. By the method of least square we have got an empirical formula:

$$n = (-0{\cdot}0695 \pm 0{\cdot}0015) + (1{\cdot}290 \pm 0{\cdot}011)v\ .$$

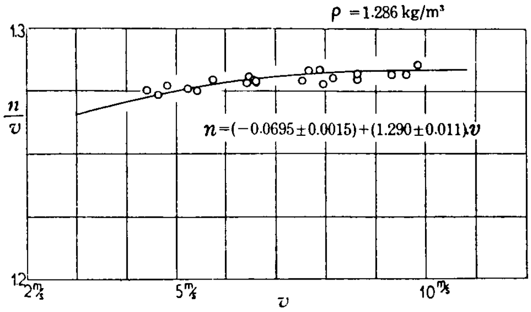


Fig. 8.

From this formula we can calculate the ratio n/v at large v. Above the speed 10 m./s. the effect of friction on the value of n/v is less than 0·5 % .

The same windmill was tested in our low pressure and temperature wind tunnel (diameter of the exit is 75 cm., max. wind velocity at 1 atmosphric pressure is 65 m./s., at 1/2 atmosphric pressure is 100 m./s., lowest pressure is 10 cm. Hg., lowest temperature is −43°C) at one atmospheric pressure, using a Prandtl pitot-static tube (coefficient 0·998) as a standard for the measurement of the wind velocity.

The velocity distributions in various cross sections of the jet are shown in Fig. 9, where x is the distance of the cross section from the exit. We set the windmill at a position symmetric with that of the pitotstatic tube. Taking into account of the velocity distribution in the cross section concerned, we got the following result, in the speed range from 5·5 m./s. to 26·2 m./s.

$$n = (-0{\cdot}0680 \pm 0{\cdot}0010) + (1{\cdot}219 \pm 0{\cdot}012)v\ .$$

In Fig. 10, n/v is plotted against v.

The results of experiment in the tunnel and those in the wind tunnel are different, the difference in the value of n/v being about 5%. The causes of this difference were considered to be

(1) a particular characteristic of the windmill used.

(2) imperfect setting of the windmill, i.e. the shaft of the windmill would be inclined to the direction of the wind,

(3) the effect of the car and the framework on the velocity of the relative wind in the experiment on the rail.

To investigate the cause (1) several other types of windmill were tested in the same way, but we have got the results same as those described before. The effect of the wind direction on the rotation of the

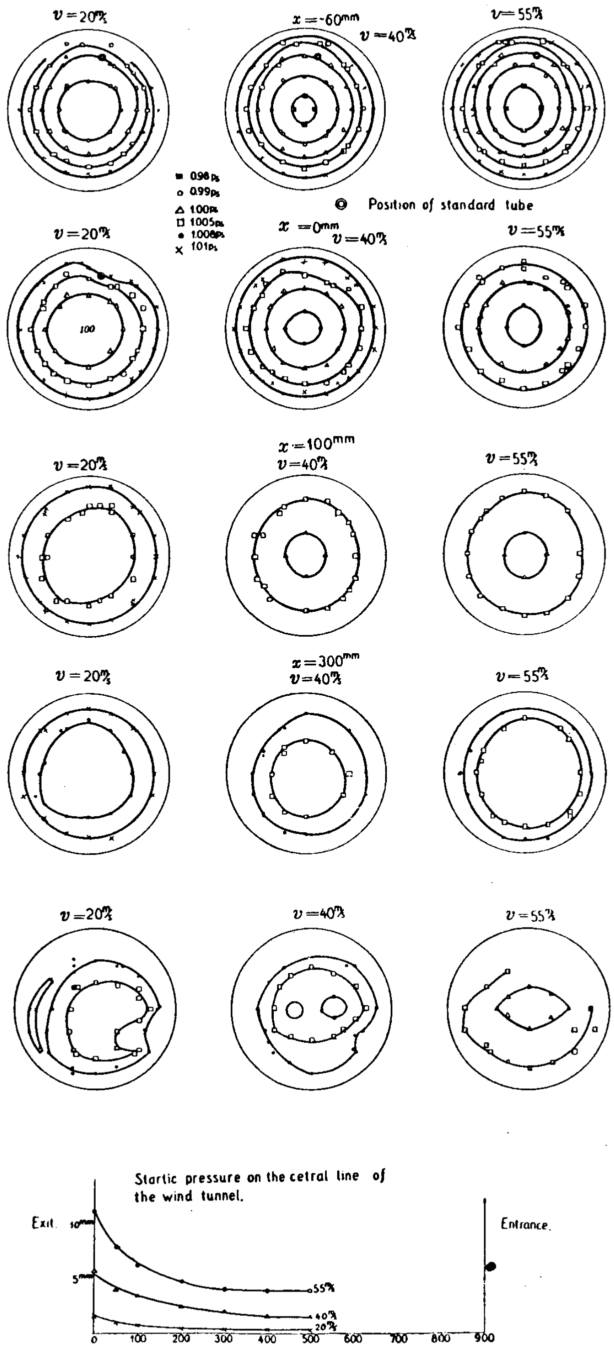


Fig. 9.

windmills were tested using No. 2 and No. 3 windmills, which are of the same type but the twist of the blades is opposite. The results of

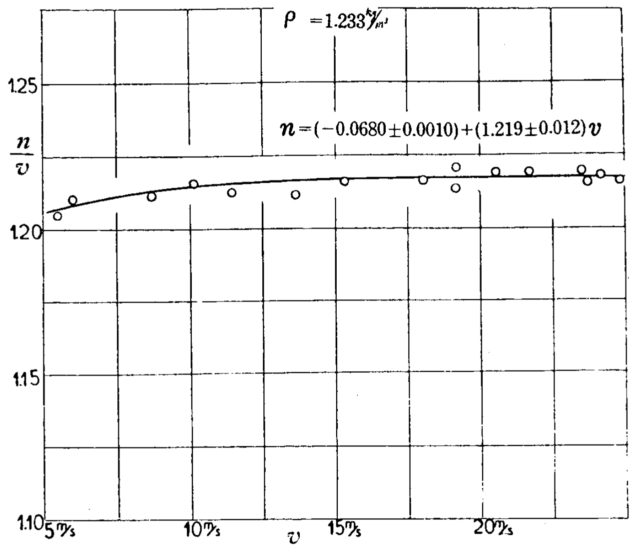


Fig. 10.

experiment in the wind tunnel are shown in Figs. 11 and 12, where the angle θ between the wind direction and the windmill shaft is positive when it was turned anticlockwise.

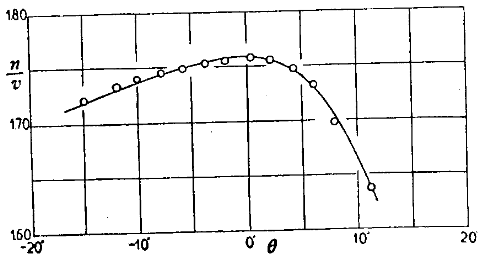


Fig. 11.

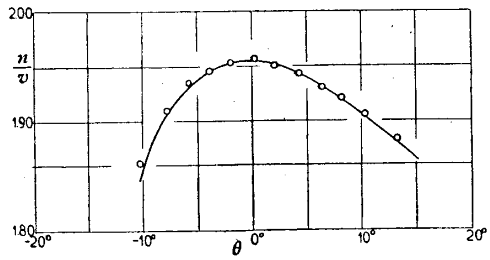


Fig. 12.

The error on θ less than 1°, which would occur in setting the windmill, has not a recognizable effect on the value of n/v. Hence (2) was not considered as the cause of the difference. We were now in a position to investigate the cause (3).

We have constructed the No. 5 windmill (diameter 15·5 cm.) as shown in Fig. 13, and set it on the framework at various distances from the car. The distance x was measured from the front edge of the car. The experiment was carried out in the speed range of the car from 4 m./s. to 13 m./s., and 19 observations were made at each x.

Fig. 13.

Writing as before

$$n = a + bv$$

and calculating a and b from the results of experiment using the method of least square, we have got the following figures. We have also calculated from the above formula the value of n/v at the speed of the car 10 m./s.,

No. 5 windmill.

x	a	b	$(n/v)_{10\,\text{m./s.}}$
m 0·00	−0·502±0·0013	1·336±0·011	1·286
0·66	−0·534±0·0013	1·324±0·011	1·270
1·37	−0·582±0·0012	1·319±0·011	1·261
2·00	−0·487±0·001	1·307±0·010	1·259
2·80	−0·575±0·0016	1·305±0·014	1·247
3·34	−0·501±0·0020	1·294±0·017	1·244
4·09	−0·523±0·0010	1·286±0·009	1·233
4·63	−0·504±0·0014	1·290±0·012	1·240

as it was convenient to compare the results of experiment in the tunnel with those in the wind tunnel, since the velocity ranges are different in both cases.

These results are plotted in Fig. 14. In Fig. 15 are plotted the values of a and b against x.

Plotting the values of $\left(\frac{n}{v}\right)_{10\,\text{m.}\,\text{s.}}$ on a semi-logarithmic paper, we found the points lie on a straight line, except two points at $x = 2{\cdot}00$ m. and at $x = 4{\cdot}09$ m., as shown in Fig. 16. Hence we can wright

$$\left(\frac{n}{v}\right)_{10\,\text{m./s.}} = C + Ae^{-Bx}\,.$$

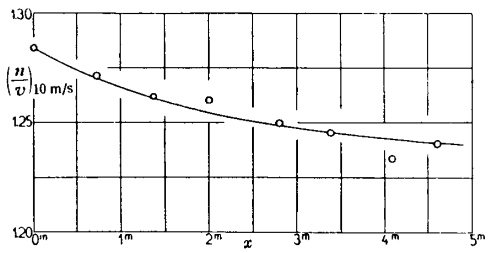


Fig. 14.

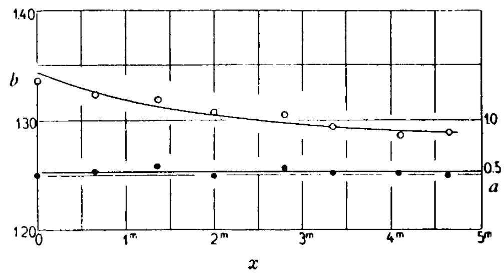


Fig. 15.

The values of the constants A, B and C were calculated from the above results and we have got

$$\left(\frac{n}{v}\right)_{10\,\text{m./s.}} = 1{\cdot}232 + 0{\cdot}065e^{-0{\cdot}62x}\,.$$

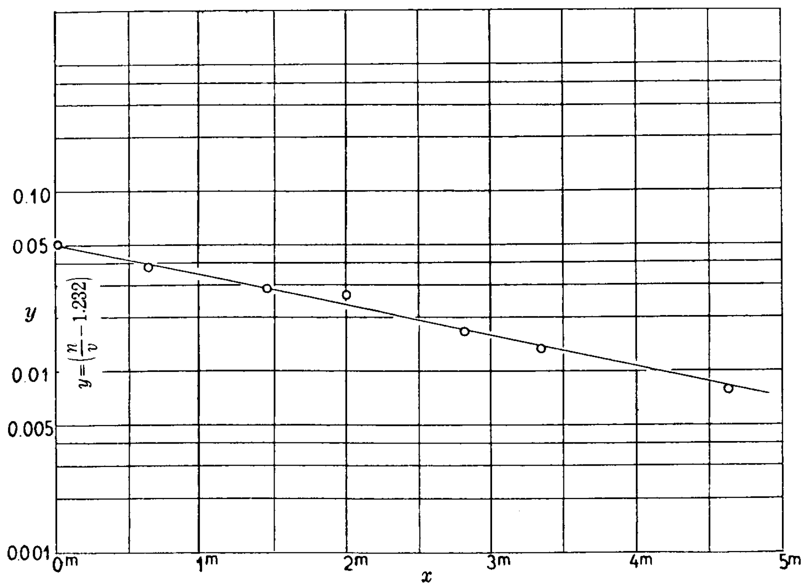


Fig. 16.

The value of $\left(\frac{n}{v}\right)_{10\,m./s.}$ which is not influenced by the presence of the car can be taken as the value of it at $x=\infty$. The rotating speed of the same windmill was measured in the wind tunnel, using the pitotstatic tube as a standard for the measurement of the speed of air. Fifteen observations were made at the speed range from 23 m./s. to 58 m./s., and we obtained the following empirical formula by the use of the method of least square

$$n=(-0{\cdot}535\pm0{\cdot}001)+(1{\cdot}290\pm0{\cdot}044)v$$

at $\rho=1{\cdot}285$ kg./m^3.

The value of $\left(\frac{n}{v}\right)_{10\,m./s.}$ calculated from the above formula is 1·236. This value agree very closely with that obtained in tunnel experiment, reduced to $x=\infty$, which is 1·232, the difference being about 0·3%. Hence we can conclude that the difference in values of n/v in tunnel experiment from those in wind tunnel experiment is due to the effect of the car upon the motion of air in the vicinity of the windmill. This is also proved from the other stand point. In Fig. 15 the value of a, which concerns with the friction, is constant; whereas the value of b diminishes with x, which proves that v is increased as x is diminished, so that we may consider the fact as an effect of the car upon the motion of air in the vicinity of the windmill.

The Experiments on Pitot-static Tubes.

We have constructed two pitot-static tubes of the same form as shown in Fig. 17, which will be named S. No. 1 and S. No. 2. These tubes and four other pitot-static tubes as shown in Fig. 18 were calibrated in our wind tunnel using N.P.L. type pitot-static

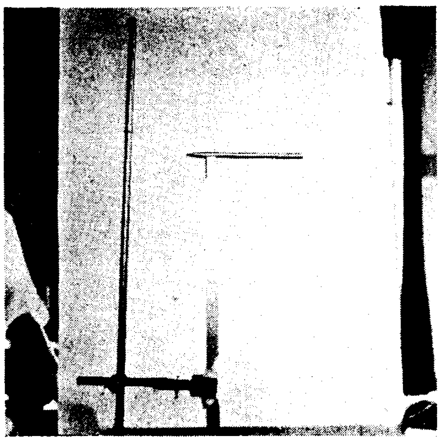
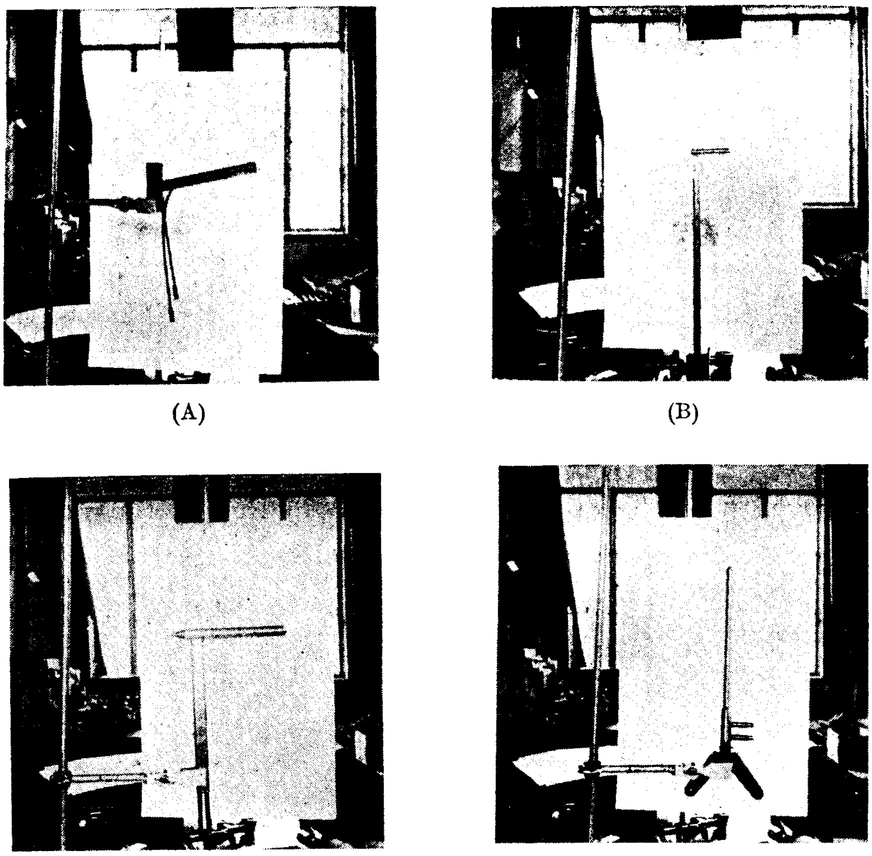


Fig. 17.

tube (coefficient 1·006) as a standard. The effect of angle of setting is also tested. The results of experiment are shown in Fig. 19.

Judging from the results of the experiment, S. No. 1 and S. No. 2 have fairly good characteristics, and of the two S. No. 2 is better. These pitot-static tubes are particularly constructed suitable for tunnel experiments. Hence we decided to use S. No. 2 for further researches.



(A) (B) (C) (D)

Fig. 18.

Using the apparatus shown in Fig. 1, and setting the pitot-static tube S. No. 2 instead of the windmill, we carried out several experiments in the tunnel. The velocity of the car was measured in the same way as that in the experiments for windmill. The pressure difference generated by the pitot-static tube was measured by the liquid manometer. On a 35 mm. film was photographed liquid column, the height of which was measured by a comparator after the film was developed, and the pitot-static tube coefficient was calculated by the formula

$$K = \frac{2p}{\rho v^2}$$

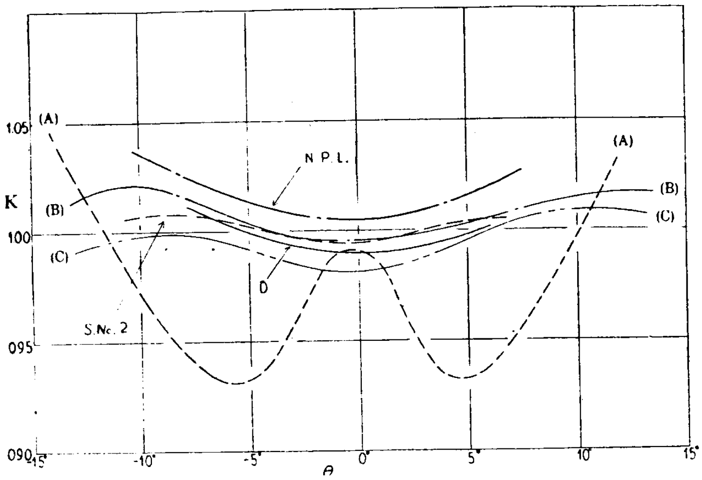


Fig. 19.

where p is the pressure difference measured by the manometer and v is the velocity of the car. The measurements were made at various x and the results are shown in the following table.

x	K	v
m		m./s.
0·00	1·089	12·70
0·00	1·096	12·78
0·00	1·096	12·03
1·46	1·046	12·74
1·46	1·048	"
1·46	1·053	11·50
2·04	1·032	11·66
2·04	1·036	13·82
2·04	1·037	10·11
2·04	1·040	11·57
2·81	1·020	13·75
2·81	1·023	11·62
2·81	1·025	13·72
2·81	1·028	12·71
4·74	1·009	12·70
4·74	1·010	12·73
4·74	1·012	11·85

The values of K are plotted against x in Fig. 20 and 21.

In Fig. 21 the values of K are plotted on a semi-logarithmic paper, in which the points lie nearly on a straight line. Hence we can write

$$K = C + Ae^{-Bx}.$$

The constants A, B and C were calculated from the above results and we obtained

$$K = 0{\cdot}997 + 0{\cdot}094e^{-0{\cdot}42x}.$$

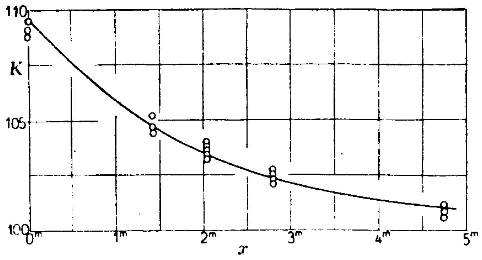


Fig. 20.

The value of K when x is infinity is 0·997, which agree very closely with that obtained in wind tunnel experiments, i. e. 0·996. Hence we can conclude that the absolutely determined value of K of the S. No. 2 pitot-static tube would be 0·997 at one atmospheric pressure.

Experiment with a Whirling Arm.

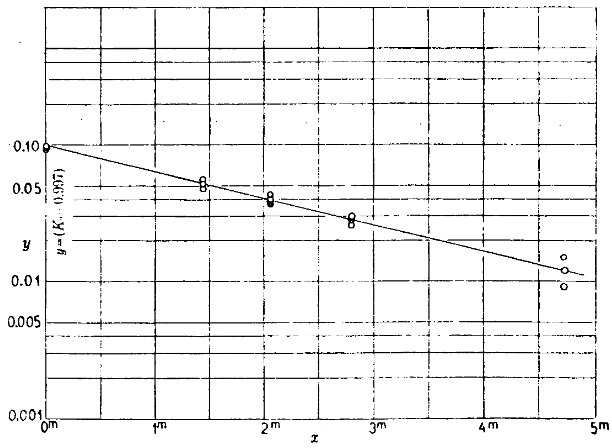


Fig. 21.

As a standard for calibrating a pitot-static tube at low air density, we wanted to use the windmill which was calibrated in the tunnel. It is generally supposed that the windmill is less influenced by the air density than a pitot-static tube, but we cannot use the windmill as a standard for calibration without testing it at low air density. For that sake we constructed a whirling arm as shown in Fig. 22. At the end of this whirling arm, the length of which can be altered, the windmill was set with its axis coinciding with the tangent of the circular path, which the end of the arm would describe.

The height, from the bed to the arm, of the whirling arm is 140 cm., and the length of the arm can be altered from 98·2 cm. to 128·2 cm. The tangential speed at the end of the whirling arm can be varied from 1 m./s. to 23 m./s.

The whirling arm was put in the low pressure and low temperature wind tunnel to investigate the effect of the air density upon the rotating speed of the windmill. As it was thought that the wall of the wind tunnel would have an effect upon the rotating speed of the windmill, we tested it, putting the whirling arm inside and outside of the wind tunnel, but we could not find any appreciable difference.

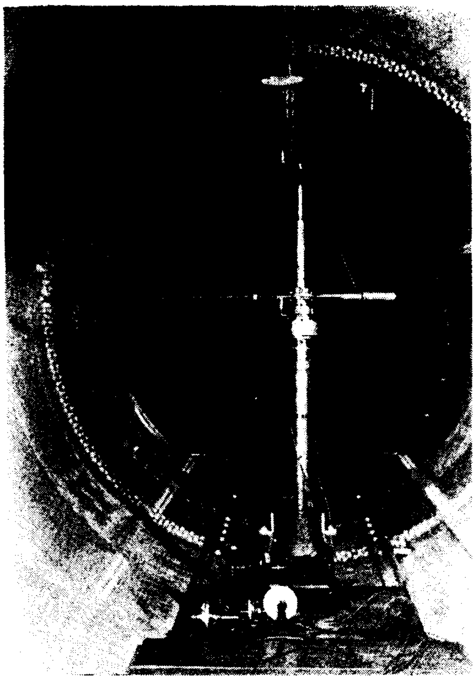


Fig. 22.

Next, the effect of the whirling arm upon the revolving speed of a windmill was tested, intending to detect the swirl of the air set up by the motion of the windmill in the air. Experiments were made varying the length of the arm from 108·2 cm. to 128·2 cm. with 5 cm. step, but we could not find any noticiable effect: the difference in each case falls within the error of the experiment.

If no swirl of the air exist, the rotating speed n of the windmill, measured on the whirling arm, will be the same as that measured in the tunnel or in the wind tunnel. The value of n measured on the whirling arm, at $\rho = 1{\cdot}286\,\mathrm{kg/m^3}$, and expressed in an empirical formula is

$$n = (-0{\cdot}395 \pm 0{\cdot}001) + (1{\cdot}262 \pm 0{\cdot}053)v\,.$$

The value of $\left(\frac{n}{v}\right)_{10\,\mathrm{m./s.}}$ is 1·222, which is less than that obtained from the results of the tunnel experiments and the wind tunnel experiments; the difference being about 0·7% . Probably this difference is due to the swirl of the air.

The Measurements of the Swirl.

We decided to use the hot-wire anemometer for the measurement of the swirl, as it was thought the most convenient method for that purpose.(1) The hot-wire was first calibrated on the whirling arm at the tangential speeds from 0·07 m./s. to 0·53 m./s. . The wire used was platinum, 0·03 mm. in diameter. We measured the increase of the current through the wire and has obtained the results shown in Fig. 23.

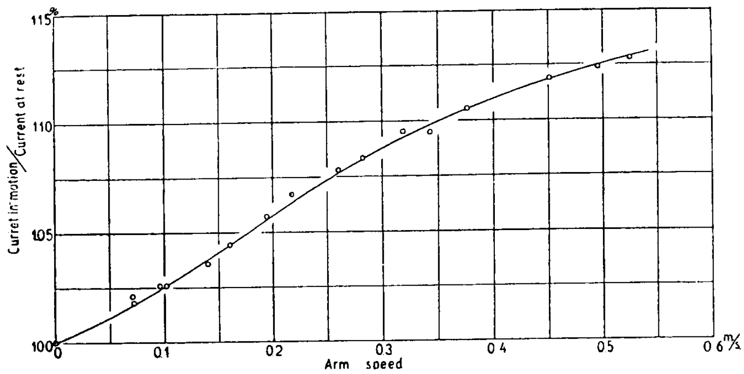
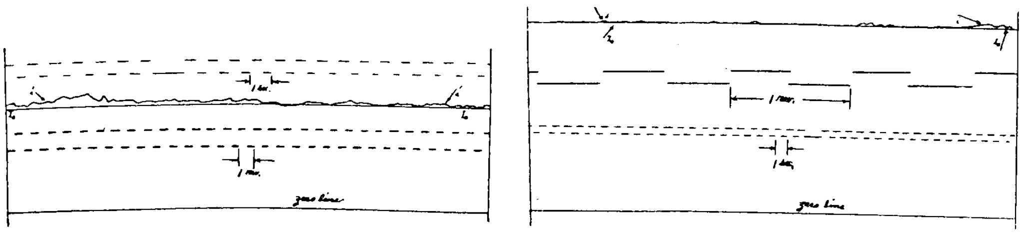


Fig. 23.

After the wire had been calibrated it was removed from the arm and fixed in the tunnel close to the path of the windmill. The clearance between the wire and the windmill as it passed was 10 mm. The swirl set up by the No. 5 windmill were measured at the tangential speeds from 5·5 m./s. to 14·2 m./s. The changes of current in the wire were recorded on the oscillograms, which are shown in Fig. 24. The 0—0 lines on the oscillograms are zero lines of the current, I_0—I_0 lines show

(1) E. Ower and F. C. Johansen, On the determination of the Pitot-Static Tube Factor at Low Reynolds Numbers, with Special Reference to the Measurement of Low air Speeds. Proc. Roy. Soc. Vol. 136, 1932, p. 153.

the current through the platinum wire when the windmill is at rest, i.e. when there were no swirl, and the $i—i$ lines show the current through the wire when the windmill was rotated. Hence the distances between the $i—i$ lines and the $I_0—I_0$ lines show the increase of current through the platinum wire due to the swirl of air generated by the motion of the windmill.



(a) (b)

Fig. 24.

The values of swirl of air which influence the tangential speed of the windmill in still air are not the mean values, but the swirl which remained when the windmill came once more to the place where the swirl was measured. The values are smaller than the mean values and is zero when the speed of the windmill is small as shown in Fig. 24b. The swirl seems to decay in about 3·3 sec. As it is very difficult to measure the electric current through the wire which changes so irregularly, we decided to measure the swirl by the use of a small windmill setting it close to the path of the No. 5 windmill. The results at one atmospheric pressure are shown in Fig. 25.

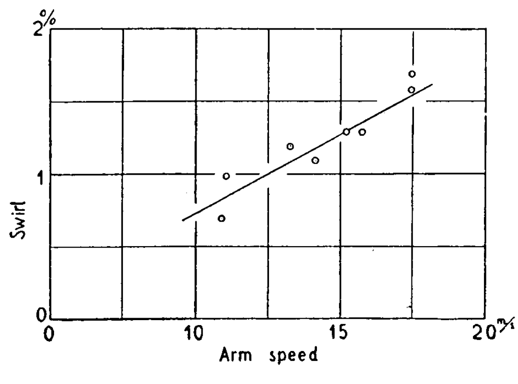


Fig. 25.

The corrections are made on the translational speeds of the windmill and the relations between n and v_1 are tabulated in Table I, and are shown in Fig. 26. Where v_1 is the arm speed corrected for the swirl. The value of n/v_1 at the speed 10 m./s. was interpolated from the Table I and Fig. 26, and is estimated to be 1·233, which agrees very closely to the values obtained from the experiments in the tunnel and in the wind tunnel.

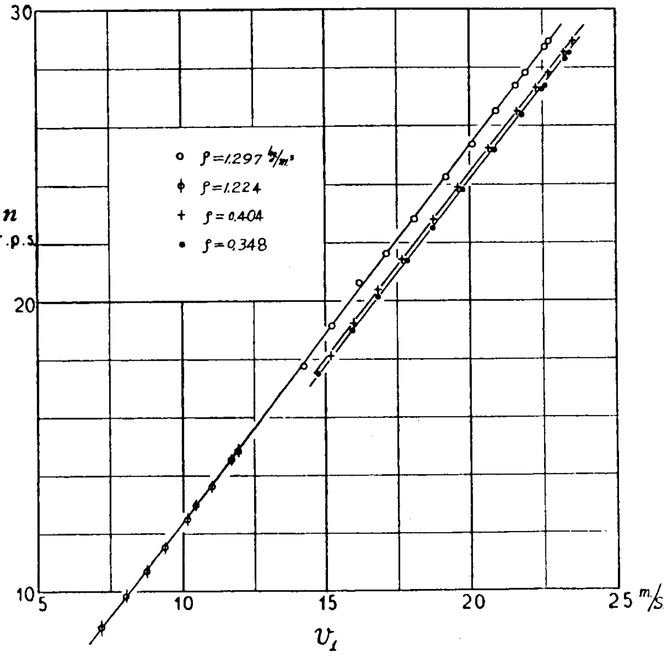


Fig. 26

The calibration of the windmill at low air density.

The relation between the rotating speed n of the No. 5 windmill and its translational speed v was investigated at various air densities using the whirling arm described before. The results are shown in Table I.

The values of the swirl set up by the motion of the windmill on a circular path were measured at various air densities, but we could not find any influence of the air density upon the swirl. Considering this negative effect of the air density upon the swirl was due to the weakness of the swirl, since very minute difference was not measured accurately, we decided to measure the strong swirl set up by the motion of a disc having the diameter 10 cm. at various air densities and to observe the effect of the air density upon the swirl. This disc was set on the whirling arm and the swirl generated by its motion was measured by the use of a small windmill setting it close to the path of the disc. The results are shown in the following table and in Table II.

ρ kg./m.^{3}	Range of arm speed v m./s.	Swirl %
1·153	6·9~17·0	12·6
0·904	8·0~17·0	12·8
0·800	9·0~17·3	12·6
0·602	10·8~17·2	12·6
0·406	11·6~17·4	12·3
0·356	12·4~17·2	12·3

It is evident from these results that the swirl is not influenced by the density of the air in the range of the density from 1·225 kg./m.^{3} to 0·356 kg./m.^{3}

The translational speed v_1 of the No. 5 windmill corrected for the swirl at various air densities are shown in Table I and in Fig. 26.

The relation between n and v_1 were calculated from these results at various air densities and the empirical formulae are shown in the following table:

ρ kg./m.³	$n=$	$(n/v_1)_{10}\times\rho$	ρ kg./m.³	$n=$	$(n/v_1)_{10}\times\rho$
1·286	$-0{\cdot}80+1{\cdot}312\,v$	1·585	0·612	$-0{\cdot}73+1{\cdot}280\,v$	0·739
1·243	$-0{\cdot}67+1{\cdot}298\,v$	1·530	0·578	$-0{\cdot}74+1{\cdot}275\,v$	0·695
1·224	$-0{\cdot}65+1{\cdot}300\,v$	1·513	0·536	$-0{\cdot}78+1{\cdot}275\,v$	0·642
1·152	$-0{\cdot}78+1{\cdot}301\,v$	1·409	0·465	$-0{\cdot}85+1{\cdot}270\,v$	0·551
1·105	$-0{\cdot}70+1{\cdot}293\,v$	1·351	0·404	$-0{\cdot}85+1{\cdot}265\,v$	0·475
1·020	$-0{\cdot}74+1{\cdot}295\,v$	1·250	0.348	$-1{\cdot}05+1{\cdot}260\,v$	0·402
0·965	$-0{\cdot}72+1{\cdot}296\,v$	1·180			
0·902	$-0{\cdot}76+1{\cdot}294\,v$	1·100			
0·864	$-0{\cdot}74+1{\cdot}288\,v$	1·048			
0·796	$-0{\cdot}74+1{\cdot}289\,v$	0·967			
0·734	$-0{\cdot}70+1{\cdot}284\,v$	0·891			
0.696	$-0{\cdot}74+1{\cdot}281\,v$	0·840			

It is extraordinary that $\frac{n}{v_1}\times\rho$ and ρ are in a linear relation, for instance the relation at $v_1 = 10$ m./s. is shown in Fig. 27. Hence we can write

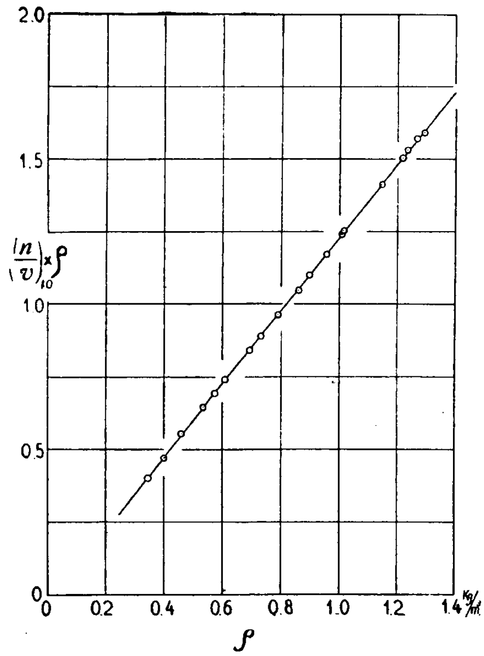


Fig. 27.

$$\frac{n}{v_1}\rho = p\rho + q\,, \qquad (1)$$

where p and q are constants.

Putting $n = a + bv_1$ in the above equation we get

$$\frac{a}{v_1} + b = p + \frac{q}{\rho}\,.$$

The values of p and q can be calculated from the above table, leaving v_1 as a parameter, in the form

$$p = \alpha + \frac{\beta}{v_1}\,, \quad q = \alpha' + \frac{\beta'}{v'}\,.$$

Hence putting these values in equation (1), we get

$$n=\left(\alpha+\frac{\alpha'}{\rho}\right)v_1+\left(\beta+\frac{\beta'}{\rho}\right).$$

Calculating the constants in this equation we get

$$n = 1{\cdot}321\left(1-0{\cdot}015\frac{\rho_0}{\rho}\right)v_1-0{\cdot}664\left(1+0{\cdot}09\frac{\rho_0}{\rho}\right). \qquad (2)$$

where n = revolutions per second, at the air density ρ,

v_1 = air speed in m./s. corrected,

ρ = density of air, kg./m.^{3},

ρ_0 = density of the air at the ground level, 1·225 kg./m.^{3}.

By the use of the formula (2) n at any ρ and v_1 can be calculated within the error 0·3%.

The calibration of the pitot-static tube at low air density.

Using the No. 5 windmill as a standard for the measurements of the wind velocity, we calibrated the pitot-static tube S. No. 2 at various air densities from ρ = 1·285 kg./m.^{3} to ρ = 0·392 kg./m.^{3} in the speed range between 27 m./s. and 71 m./s. in the low pressure and low temperature wind tunnel. From 13 observations we have got the results as shown in Fig. 28 and in the following table. It is evident from these results that the pitot-static tube coefficient decreases about 4% when the air density is decreased as low as 0·4 kg./m.^{3}. This decrease of the pitot-static tube coefficient is far greater than the errors involved in the measurements and in the calculations. Hence we can conclude that the pitot-static tube coefficient is influenced by the density of the air.

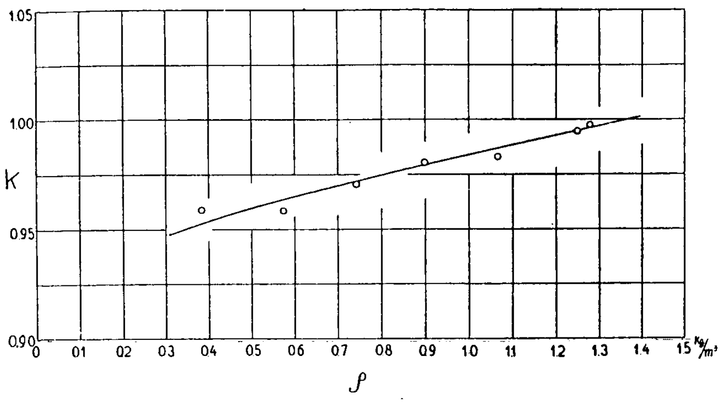


Fig. 28.

ρ kg./m.³	*K.*
1·285	0·998
1·260	0·994
1·081	0·983
0·909	0·980
0·750	0·970
0·582	0·959
0·392	0·959

In concluding, we express our thanks to Messrs. K. Hukuda, A. Iwasaki and Y. Inaba for their assistances in the course of the experiments.

TABLE I. The relations between n and v, swirl speed, corrected arm speed.

$\rho = 1{\cdot}297$ kg./m.³

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
28·90	23·22	0·49	22·73
27·79	22·38	0·46	21·92
26·49	21·32	0·41	20·91
25·38	20·48	0·38	20·10
24·23	19·55	0·34	19·21
22·80	18·40	0·32	18·08
21·68	17·46	0·29	17·17
20·62	16·65	0·24	16·21
19·15	15·46	0·21	15·25
17·77	14·42	0·18	14·24
27·40	22·00	0·44	21·56
28·73	23·05	0·48	22·57

$\sigma = 1{\cdot}286$ kg./m.³

n r.p.s.	v m./s	Swirl speed m./s.	Corrected arm speed m./s.
30·22	24·29	0·53	23·76
29·10	23·36	0·50	22·86
27·60	22·19	0·45	21·74
26·65	21·39	0·40	20·99
24·97	20·10	0·36	19·74
24·00	19·37	0·33	19·04
22·61	18·23	0·30	17·93
21·17	17·08	0·26	16·82
20·13	16·28	0·23	16·05
18·84	15·25	0·20	15·05
28·39	22·82	0·47	22·35
29·67	23·87	0·52	23·35

$\rho = 1{\cdot}243$ kg./m.³

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
17·68	14·31	0·18	14·13
19·31	15·60	0·21	15·39
20·38	16·44	0·24	16·20
21·60	17·44	0·27	17·17
23·00	18·56	0·31	18·25
24·40	19·71	0·35	19·36
24·78	19·95	0·36	19·59
25·82	20·81	0·39	20·42
26·50	21·35	0·42	20·93

$\rho = 1{\cdot}224$ kg./m.³

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
14·88	12·06	0·11	11·95
14·54	11·82	0·10	11·72
13·64	11·08	0·09	10·99
13·00	10·55	0·08	10·47
12·48	10·19	0·07	10·12
11·51	9·42	0·06	9·36
10·72	8·78	0·06	8·72
9·78	8·05	0·04	8·01
8·76	7·24	0·02	7·22

$\rho = 1{\cdot}152$ kg./m.^{3}

n r.p.s	*v* m./s.	Swirl speed m./s.	Corrected arm speed m./s.
28·80	23·20	0·49	22·71
27·54	22·27	0·45	21·82
18·22	14·80	0·19	14·61
19·58	15·85	0·22	15·63
20·70	16·76	0·25	16·51
21·78	17·61	0·28	17·33
23·12	18·75	0·32	18·43
24·31	19·74	0·35	19·39
25·48	20·64	0·39	20·25
26·52	21·38	0·41	20·97
27·60	22·35	0·45	21·90
28·92	23·30	0·49	22·81

$\rho = 1{\cdot}105$ kg./m.^{3}

n r.p.s.	*v* m./s.	Swirl speed m./s.	Corrected arm speed m./s.
25·30	20·49	0·38	20·11
24·65	19·90	0·36	19·54
24·08	19·50	0·34	19·16
23·33	18·96	0·32	18·64
22·91	18·56	0·31	18·25
22·25	18·12	0·29	17·83
21·61	17·63	0·28	17·35
20·54	16·70	0·25	16·45
19·38	15·68	0·22	15·46
18·14	14·73	0·19	14·54
16·53	13·50	0·16	13·34

$\rho = 1{\cdot}020$ kg./m.^{3}

n r.p.s.	*v* m./s.	Swirl speed m./s.	Corrected arm speed m./s.
30·20	24·46	0·55	23·91
28·82	23·29	0·49	22·80
27·70	22·39	0·45	21.94
26·50	21·38	0·42	20·96
24·95	20·17	0·37	19·80
23·85	19·32	0·34	18·98
22·50	18·23	0·30	17·93
19·63	15·90	0·22	15·68
20·15	16·36	0·24	16·12
18·67	15·16	0·20	14·96
28·39	22·98	0·48	22·50
29·60	23·90	0·52	23·38

$\rho = 1{\cdot}019$ kg./m.^{3}

n r.p.s.	*v* m./s.	Swirl speed m./s.	Corrected arm speed m./s.
29·18	63·68	0·51	23·17
27·91	22·60	0·46	22·14
26·88	21·80	0·43	21·37
25·78	20·90	0·40	20·50
24·57	19·95	0·36	19·59
23·38	18·95	0·32	18·63
21·90	17·82	0·29	17·53
20·71	16·90	0·25	16·65
19·56	15·89	0·22	15·67
18·69	15·21	0·20	15·01
27·76	22·53	0·46	22·07
29·00	23·47	0·50	22·97

$\rho = 0{\cdot}965$ kg./m.^{3}

n r.p.s.	v. m./s.	Swirl speed m./s.	Corrected arm speed m./s.
16·51	13·42	0·15	13·27
17·87	14·50	0·18	14·32
19·25	15·62	0·22	15·40
20·41	16·60	0·25	16·35
21·78	17·66	0 28	17·38
23·20	18·75	0·32	18·43
23·60	19·14	0·33	19·81
24·43	19·78	0·35	19·43
25·13	20·35	0·38	19·97
25·95	20·94	0·40	20·54

$\rho = 0{\cdot}902$ kg./m.^{3}

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
29·21	23·69	0·51	23·18
29·03	23·60	0·51	23·09
28·22	22·88	0·48	22·40
27·88	22·60	0·46	22·14
26·90	21·85	0·43	21·42
25·64	20·79	0·39	20·40
24·45	19·88	0·36	19·52
23·15	18·87	0·32	18·55
21·78	17·77	0·28	19·49
20·87	16·97	0·26	16·71
19·64	15·96	0·23	15·73
19·04	15·52	0·21	15·31

$\rho = 0{\cdot}864$ kg./m.^{3}

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
17·64	14·38	0·18	14·20
19·00	15·55	0·21	15·34
20·18	16·51	0·24	16·27
21·33	17·39	0·27	17·12
22·13	18·02	0·29	17·73
22·80	18·60	0·31	18 29
23·65	19·25	0·33	18·92
24·35	19·83	0·36	19·47
24·80	20·17	0·37	19·80
25·50	20·75	0·39	20·36

$\rho = 0{\cdot}796$ kg./m.^{3}

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
29·10	23·65	0·51	23·14
28·98	23·52	0·50	23·02
28·19	22·92	0·48	22·44
27·73	22·60	0·46	22·14
26·99	21·96	0·44	21 52
25·60	20·82	0·39	20·43
24·50	19·95	0·36	19·59
23·32	19·03	0·32	18·71
22·02	17·98	0·29	17·69
20·67	16·82	0·25	16·57
19·86	16·18	0·23	15·95
19·18	15·68	0·22	15·46

$\rho = 0{\cdot}794\,\text{kg./m.}^3$

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
29·71	24·18	0·54	23·64
29·57	23·99	0·52	23·47
28·73	23·39	0·50	22·89
28·40	23·07	0·48	22·59
27·57	22·40	0·45	21·95
26·37	21·48	0·42	21·06
24·30	19·80	0·35	19·45
23·72	19·32	0·34	18·98
22·58	18·41	0·30	18·11
21·28	17·37	0·27	17·10
19·98	16·34	0·23	16·11
18·75	15·33	0·20	15·13

$\rho = 0{\cdot}734\,\text{kg./m.}^3$

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
25·82	21·02	0·40	20·62
25·10	20·44	0·38	20·06
24·40	19·85	0·36	19·49
23·73	19·32	0·34	18·98
23·01	18·75	0·32	18·43
21·60	17·59	0·28	17·31
20·40	16·63	0·25	16·38
19·30	15·77	0·22	15·55

$\rho = 0{\cdot}696\,\text{kg./m.}^3$

n r.p.s.	v m./s.	Swirl speed m./s.	Ccrrected arm speed m./s.
29·17	23·80	0·52	23·28
28·98	23·62	0·50	23·12
28·31	23·09	0·48	22·61
27·61	22·51	0·46	22·05
27·08	22·03	0·44	21·59
25·10	20·54	0·38	20·16
24·40	19·90	0·36	19·54
23·24	18·98	0·32	18·66
22·01	18·00	0·29	17·71
20·53	16·78	0·25	16·53
19·68	16·10	0·23	15·87
18·31	15·01	0·20	14·81

$\rho = 0{\cdot}612\,\text{kg./m.}^3$

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
29·23	24·00	0·53	23·47
29·10	23·94	0·52	23·42
28 22	23·13	0·49	22·64
27·90	22·75	0·47	22·28
27·03	22·09	0·44	21·65
25·94	21·23	0·41	20·83
24·28	19·91	0·36	19·55
23·38	19·14	0·33	18·81
22·08	18·16	0·30	18·86
20·76	17·08	0·26	16·82
19·65	16·15	0·23	15·92
18·27	15·05	0·20	14·85

$\rho = 0{\cdot}612$ kg./m.^{3}

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
29·38	24·06	0·53	23·53
29·34	24·00	0·53	23·47
28·50	23·37	0·50	22·87
28·06	23·00	0·48	22·52
27·60	22·60	0·46	22·14
26·09	21·43	0·41	21·02
25·21	20·71	0·39	20·32
23·59	19·40	0·34	19·06
22·36	18·40	0·30	18·10
21·09	17·33	0·27	17·06
19·86	16·35	0·24	16·14
18·66	15·34	0·21	15·23

$\rho = 0{\cdot}578$ kg./m.^{3}

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
25·42	20·88	0·40	20·48
24·93	20·41	0·38	20·03
24·31	19·93	0·36	19·59
23·61	19·37	0·34	19·03
22·75	18·63	0·31	18·32
21·80	17·90	0·29	17·61
20·35	16·73	0·25	16·48
19·25	15·83	0·22	15·61
17·83	14·65	0·19	14·46
16·50	13·60	0·16	13·44

$\rho = 0{\cdot}536$ kg./m.^{3}

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
29·29	24·06	0·53	23·53
29·20	23·92	0·52	23·40
28·11	23·10	0·48	22·62
27·62	22·84	0·47	22·37
27·02	22·33	0·45	21·88
25·75	21·22	0·41	20·81
24·30	20·03	0·36	19·67
23·20	19·12	0·33	18·79
21·92	18·10	0·29	17·81
20·63	17·03	0·26	17·77
19·56	16·20	0·23	15·97
18·00	14·93	0·20	14·73

$\rho = 0{\cdot}465$ kg./m.^{3}

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
29·13	24·18	0·53	23·65
28·70	23·96	0·52	23·44
28·19	23·33	0·49	22·84
27·71	22·91	0·48	22·43
26·90	22·28	0·45	21·83
25·85	21·40	0·41	20·99
24·05	19·94	0·36	19·58
23·19	19·23	0·33	18·90
21·75	18·12	0·30	17·82
20·80	17·32	0·27	17·05
19·45	16·21	0·23	15·98
18·25	15·22	0·20	15·02

$\rho = 0{\cdot}465$ kg./m.³

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
29·11	24·12	0·55	23·59
29·04	23·98	0·53	23·45
28·19	23·29	0·50	22·79
27·82	23·00	0·48	22·52
26·78	22·20	0,45	21·75
25·63	21·28	0·41	20·87
24·39	20·20	0·37	29·83
23·05	19·22	0·33	18·89
22·09	18·35	0·30	18·05
20·74	17·23	0·26	16·97
19·54	16·25	0·23	16·02
18·16	15·15	0·20	14·95

$\rho = 0{\cdot}404$ kg./m.³

n r.p.s.	v m./s.	Swirl speed m./s.	Corrected arm speed m./s.
28·97	24·12	0·53	23·59
28·55	23·78	0·52	23·26
27·87	23·19	0·49	22·70
27·33	22·78	0·48	22·30
26·50	22·12	0·44	21·68
25·28	21·08	0·40	20·68
23 94	19·96	0·36	19·60
22·84	19·10	0·33	18·77
21·45	17·94	0·29	17·65
20·44	17·11	0·26	16·85
19·29	16·22	0·23	15·99
18·18	15·28	0·20	15·20

$\rho = 0{\cdot}348$ kg./m.³

n r.p.s.	v. m./s.	Swirl speed m./s.	Corrected arm speed m./s.
28·50	23·95	0·53	23·42
28·31	23·78	0·52	23·26
27·40	23·10	0·49	22·61
27·31	22·98	0·47	22·51
26·40	22·23	0·44	21·79
25·20	21·25	0·40	20·85
23·88	20·11	0·36	19·75
22·53	19·04	0·33	18·71
21·41	18·11	0·29	17·82
20·11	17·08	0·26	16·82
18·99	16·16	0·23	15·93
17·53	14·93	0·20	14·79

TABLE II. The influence of air density upon the swirl generated by a disc having the diameter 10 cm.

$\rho = 1{\cdot}153$ kg /m.^{3}

Arm speed	Swirl speed	%
m./s.	m./s.	
6·87	0·64	9·3
8·06	0·92	11·4
9·02	1·11	12·3
10·04	1·23	12·3
11·38	1·50	13·2
12·02	1·56	13·0
13·05	1·71	13·1
13·85	1·81	13·1
14·95	2·00	13·4
15·91	2·19	13·8
16·77	2·26	13·5
16.98	2·23	13·2

12·6%

$\rho = 0{\cdot}904$ kg./m.^{3}

Arm speed	Swirl speed	%
m./s.	m./s.	
8·04	0·88	11·0
8·82	1·03	11·7
9·40	1·15	12·2
10·19	1·19	11·7
10·98	1·36	12·4
11·53	1·53	13·3
12·11	1·65	13·6
12·85	1·70	13·2
13·35	1·82	13·6
14·18	1·99	14 0
14·68	2·00	13·6
15·25	2·02	13·3
15·92	2·07	13·0
16·95	2·13	12·6

12·8%

$\rho = 0{\cdot}800$ kg./m.^{3}

Arm speed	Swirl speed	%
m./s	m./s.	
8·95	0·92	10·3
10·13	1·13	11·2
10·73	1·22	11·4
11·68	1·64	14·0
12·40	1·73	13·9
13·12	1·85	14·1
13·85	2·02	14·6
14·80	1·80	12·2
14·71	1·89	12·8
15·73	1·73	11·0
16·54	2·22	13·4
17·30	2·14	12·3

12·6%

$\rho = 0{\cdot}602$ kg./m.^{3}

Arm speed	Swirl speed	%
m./s.	m./s.	
10·77	1·19	11·0
11·76	1·43	12·2
12·45	1·66	13·3
14·75	1·71	11·6
14·07	1·80	12·8
14·70	1·91	13·0
15·25	2·13	14·0
15·56	2·09	13·4
16·07	2·06	12·8
17·17	2·03	11·8

12·6%

$\rho = 0{\cdot}460$ kg./m.^{3}

Arms speed	Swirl speed	%
m./s.	m./s.	
11·59	1·30	11·2
12·32	1·43	11·6
12·80	1·43	11·2
13·33	1·58	11·9
13·76	1·72	12·5
14·55	1·86	12·8
15·10	1·87	12·4
15·88	2·05	12·9
16·55	2·14	12·9
17·37	2·33	13·4

12·3%

$\rho = 0{\cdot}356$ kg./m.^{3}

Arm speed	Swirl speed	%
m./s.	m./s.	
12·40	1·33	10·7
12·90	1·45	11·3
13·22	1·61	12·2
13·72	1·60	11·7
14·25	1·74	12·2
14·65	1·84	12·6
15·15	1·86	12·3
15·75	2·06	13·1
16·29	2·21	13·6
17·20	2·24	13·0

12·3%