No. 46.

(English Abstract from the Japanese Original.)

On the Effect of the Wall of a Wind Tunnel upon the Lift Coefficient of a Model.

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Introduction.

Assuming the lift coefficient of a model is not changed in the channel, Prandtl has calculated the variation of the drag coefficient introducing the horse shoe vortices and their images. In reality lift is changed as well as the drag, but so far as I am aware no correction factor has been found. Treating the matter as a two-dimensional problem, the correction factor for the case of N.P.L. type wind tunnel is calculated in part 1 of this paper and that for the cases of Göttingen and Eiffel's type wind tunnels is calculated in part 2.

Part 1.

(a) *On the lift coefficient of an inclined plate placed between two parallel planes.*

Let $z=x+iy$, where (x, y) are rectangular coordinates in the plane of motion. The z-plane is shown in Fig. 1. Let $f=\phi+i\psi$, where ϕ is the velocity potential, and ψ is the stream function. The f-plane is shown in Fig. 2.

Transforming the f-plane into a rectangle in an s-plane (Fig. 4) we get the relation

$$f=\frac{\psi_1+\psi_2}{\pi}\left\{[\zeta(\mu+\nu)-\zeta(\mu-\nu)](s-\mu)-\log\frac{\sigma(s+\nu)\,\sigma(\mu-\nu)}{\sigma(s-\nu)\,\sigma(\mu+\nu)}\right\}.$$

Transforming the s-plane into a ring region in a Z-plane (Fig. 5) by the relation

$$s=\omega_1+\omega_3-\frac{\omega_1}{i\pi}\log Z,$$

and putting

$$\frac{df}{dz}=e^{-i\Omega},$$

the velocity of the general flow being unity, Ω can be calculated by Villat's formula

Teisei. (Corrigenda.)

Hôkoku 46 Gô ni oite,

159 Peizi 2 Gyô. Zentai no Nagare no Sokudo wa 1 de naku U_∞ de aru. Sitagatte 169 P. 1 Gyô de $2\psi_1 = dU_\infty$ to naru. Soreyueni kono Peizi de d wa mina dU_∞ ni naru. Koko de $U_\infty = e^{-i\Omega(\nu)}$ de aru. $\psi_1 = \psi_2$ no tokubetu na Baai dewa $U_\infty = e^{\eta_1 \omega_1 \frac{\delta_1}{\pi}} \Big/ \sigma_2\left(\omega_1 \frac{\delta_1}{\pi}\right)\sigma_3\left(\omega_1 \frac{\delta_1}{\pi}\right)$ de aru. Mata $\frac{P}{\rho b}$ wa $\frac{P}{\rho b U_\infty^2}$ ni naru. Sitagatte Hyô I wa kawaru. (*Rosenhead* no Ronbun wo sansyô !)

Tugini 164 Peigi de z wa Ita no mawari wo hitomawari-sitemo onazi Atai wo toru koto kara eta Dyôken $\theta_4 = \theta_3 - \pi$ wa $\psi_1 = \psi_2$ no Baai de aru. Ippan no Baai wa $M_1 + M_2 = 0$ de aru. Noti no Keisan niwa betu ni tukatte inai kara ato no Siki niwa eikyôsinai.

Insatu no Ayamari.

190 P. no sanban-meno Siki no naka no $x_2 - \nu$ wa $x_2 + \nu$ no Ayamari.

191 P. no sanban-me to goban-me no Siki de $\frac{\sigma(2\nu)}{\sigma(s_1+\nu)\sigma(s_1-\nu)}$ wa $\frac{\sigma(2\nu)}{\sigma(s_1+\nu)\sigma(s_1-\nu)} \cdot \frac{1}{e_3 - e_2}$ no Ayamari.

$$\Omega(Z)=\frac{i\omega_1}{\pi^2}\int_0^{2\pi}\Phi(\theta)\zeta\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta\right)d\theta$$

$$-\frac{i\omega_1}{\pi^2}\int_0^{2\pi}\Psi(\theta)\zeta_3\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta\right)d\theta$$

with the condition

$$\int_0^{2\pi}\Phi(\theta)\,d\theta=\int_0^{2\pi}\Psi(\theta)\,d\theta\,,$$

where $\phi(\theta)$ is the angle which the direction of the flow along the face of the inclined plate makes with the positive direction of x-axis, expressed as a function of central angle θ in the Z-plane; and $\Psi(\theta)$, that of two parallel planes, is zero, as the flow along the planes is in the positive direction of x-axis.

From the expression for Ω thus calculated and that for f, we get the expression for dz by

$$dz=e^{i\Omega}df\,.$$

Integrating, we get the expression for z, which is expressed in (7).

Superimposing a complex potential function $f_1=i\,\Gamma\log Z$ and defining the circulation Γ so that the stream rejoins at C, the lower end, we get, adding f and f_1, the flow as shown in Fig. 7.

The force acting on the plate can be calculated by

$$P=-i\frac{\rho}{2}\int\left(\frac{df}{dz}+\frac{df_1}{dz}\right)^2dz\,,$$

the path of integration being taken round the plate.

The value for $\int\left(\frac{df}{dz}+\frac{df_1}{dz}\right)dz$ is expressed in (8), and as it is real, P is an imaginary quantity, i.e. there is no drag.

The special cases, in which $\psi_1=\psi_2$ and $\psi_1=\infty$ have also been treated.

(b) *On the lift coefficient of a Joukowsky's aerofoil in N.P.L. type wind tunnel.*(1)

By the relation (7) the ring region composed of circles of radii 1 and q in the Z-plane is transformed conformally into a doubly connected region in the z-plane, the boundaries of which are two parallel straight lines and an inclined slit between them.

The next ring region of radii 1 and $\frac{1}{q}$ in the Z-plane is transformed into the same doubly

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(1) A special case has been treated in my previous paper. Proc. of the Phys. Math. Soc. of Japan, 3rd Ser. Vol. 9, No. 11.

connected region as the former, but situated in the lower sheet of Riemann surface. So if we transform the eccentric ring region, with the inner circle of radius q and the outer one crossing the circle of radius 1 at the point $Z=e^{i\theta_4}$, into the z-plane, we get a doubly connected region, the boundaries of which consist of two parallel straight lines and a curve similar to Joukowsky's profile between them.

The lift is calculated as in the former case, and we have got the result that the lift depends not only on the value of the strength of circulation, but also on the ratio of the chord to the distance between the walls, the curvature of the aerofoil, and the angle of incidence.

Part 2.

Göttingen and Eiffel's type wind tunnel.

In these cases the model is hang in a jet of air, so that we must have the boundary condition that the pressure is constant on the surface of the jet. Villat's formula is not applicable in its original form in these cases, so that we have to derive a formula for $\Omega(Z)$ which fulfills the boundary conditions, that on the outer circle of radius 1 in the Z-plane the real part of $\Omega(Z)$ takes the value of the angle which the direction of the flow along the face of the model makes with the positive direction of x-axis, expressed as a function of central angle θ in the Z-plane, and that on the inner circle of radius q imaginary part of $\Omega(Z)$ vanishes.

For that sake we have to change Villat's formula as follows:

$$\Omega(Z)=\frac{i\omega_1}{\pi^2}\int_0^{2\pi}\Phi(\theta)\,\zeta\left[\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta\right)\,\Big|\,\omega_1,\quad 2\omega_3\right]d\theta$$

$$-\frac{i\omega_1}{\pi^2}\int_0^{2\pi}\Psi(\theta)\zeta_3\left[\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta\right)\,\Big|\,\omega_1,\quad 2\omega_3\right]d\theta\,,$$

and

$$\int_0^{2\pi}\Phi(\theta)\,d\theta=\int_0^{2\pi}\Psi(\theta)\,d\theta\,, \qquad (a)$$

and make $\Phi(\theta)=\Psi(\theta)$, then at the point Z' where Z is inversed geometrically with modulus q, $\Omega(Z')$ takes a value conjugate to $\Omega(Z)$. This means that the imaginary part of $\Omega(Z)$ vanishes on the circle of radius q. It can be proved that the other boundary condition is at the same time fulfilled. We also see that the condition (a) is fulfilled in making $\Phi(\theta)=\Psi(\theta)$.

Hence the formula which we sought is as follows:

$$\Omega(Z)=\frac{i\omega_1}{\pi^2}\int_0^{2\pi}\Phi(\theta)\left\{\zeta\left[\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta\right)\,\middle|\,\omega_1\,,\,2\omega_3\right]-\zeta_3\left[\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta\right)\,\middle|\,\omega_1\,,\,2\omega_3\right]\right\}d\theta$$

where $\Phi(\theta)$ has the same meaning as that in Villat's formula.

On the lift coefficient of an inclined plate.

The flow in this case is shown in Fig. *12*. The complex potential function is expressed in the s-plane as follows:

$$w=\frac{\psi_1+\psi_2}{\pi}\left\{[\zeta(\mu+\nu)-\zeta(\mu-\nu)]\,(s-\mu)-\log\frac{\sigma(s+\nu)\,\sigma(\mu-\nu)}{\sigma(s-\nu)\,\sigma(\mu+\nu)}\right\}+\Gamma\frac{\pi}{\omega_1}(s-\omega_1-\omega_3)\,,$$

where Γ is the circulation and μ, ν have the same meanings as those in the former case. From the condition that $\dfrac{dw}{ds}=0$ at s_1, (the point in the s-plane which corresponds to the point B) we can determine the circulation Γ. Calculating the function Ω as in the former case, we finally get

$$e^{i\Omega(s)}=e^{i\left(-\frac{\pi}{2}+\delta\right)}\frac{1}{e_3-e_2}\cdot\frac{[\sigma_2(s-s_2)-\sigma_3(s-s_2)]\,[\sigma_2(s+s_1)-\sigma_3(s+s_1)]}{\sigma(s-s_2)\,\sigma(s+s_1)}\,,$$

where s_2 is the point in the s-plane which corresponds to the point C. Hence z can be calculated from the relation $z=\int e^{i\Omega}dw$, the result of which is given in (b) p. 189. The constants C_0, C_1 etc. are given in p. 188.

z takes the same value as we go completely round the plate, so that it must have the period $2\omega_1$. Therefore we get the condition (11) in p. 189, where x_1 and x_2 are the real parts of s_1 and s_2 respectively. From the condition (11) and one more condition that $\Omega(\nu)=0$, we can determine x_1 and x_2.

The breadth b of the inclined plate is calculated in (c) p. 190, where α is the angle of down wash, which is calculated from the relation $\Omega(-\nu)=-\alpha$.

The force acting upon the inclined plate is calculated from the relation

$$P=-\frac{i\rho}{2}\int\left(\frac{dw}{dz}\right)^2 dz\,,$$

where the path of integration is to be taken round the plate.

Carrying out the integration we get

$$\frac{P}{\rho b}=\frac{d}{b}(1-\cos\alpha+i\sin\alpha)\,,$$

where d is the diameter of the undisturbed part of the jet. We see from this result that there is drag as well as the lift. It is certainly unfavourable that the drag other than that caused by viscosity is induced, which is not the case in N.P.L. type wind tunnel.

Carrying out the numerical calculation for the case in which $\psi_1=\psi_2$ and $\frac{\pi}{2}-\delta=10^\circ$, we get the result shown in table II (p. 192), where the symbols with suffixes o correspond to those in the case where $d=\infty$. The values of L/L_0 are plotted against b/d in Fig. 14. In order to compare these values of L/L_0 with those calculated in the case of N.P.L. type wind tunnel, the latter are also plotted in the same figure.

Dai 46 Gô.

(Showa 3n. (1928) 12 gwatu hakko.)

Hûtô no Kabe ga Mokei no Yôryoku-keisû ni oyobosu Eikyô ni tuite.

Syoin Rigakusi, SASAKI-Tatudirô.

Hasigaki

Hûtô-zikken de iroirona Hûto de nasareta Zikken no Kekkwa ga, taihen tigatte iru koto wa wareware no ôkina Nayami de aru. Motiron Kûki no Nebasa to Nagare ga itiyô de nai koto wa Tigai no ôkina Gen'in dewa aru ga, Hûto no Kabe no Eikyô mo naozarini dekinai. Nagare ga itiyô de nai koto wa, Hûtô wo kaizensureba yoku naru ga, Kabe no Eikyô wa sonoyôniwa yukanai kara, Syûsei-keisû wo motomeru koto ga hituyô de aru. *Prandtl*[1] wa Yôryoku-keisû ga Hûto no naka de tigawanai to kateisite, Uma no Kutu no Katati wo sita Udu to sore no Zô to wo tukatte Kôryoku-keisû wo keisansita. Zissai niwa Yôryoku-keisû mo tigau no de aru ga, ima made kore no Syûsei-keisû wa nai yô de aru.

Yôryoku-keisû no Tigai wo kûkantekini keisansuru koto wa, taihen mudukasii kara, koko dewa heimentekino Mondai tosite keisansita.

Konoyôni site keisansita Yôryoku-keisû no Tigai wa, ikubunka wareware no Nayami wo yawaragete kureru de arô. Mata, Hikôki no Heikô wo siraberu Baai niwa, Syuyoku no simono Nagare wa Biyoku ni hataraku Tikara ni ôkina Eikyô ga aru kara, Syuyoku no sugu simono Nagare wo siraberu koto ga hituyô de aru. Kono Baai niwa kono Keisan wa, taihen yakudatu to omou.

(1) Prandtl: Gottinger Nachrichten, 1919.

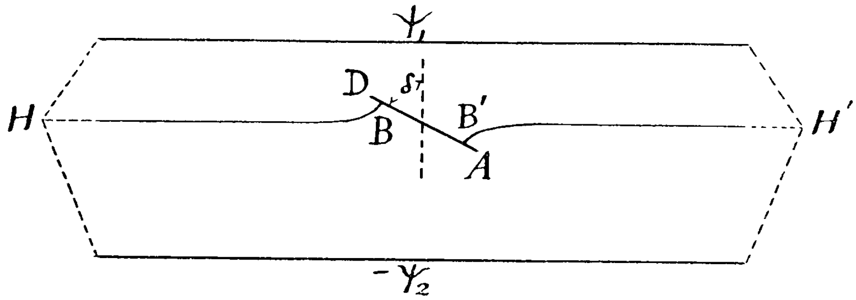
N.P.L.—gata no Baai ni tuite wa *Glauert* ya Terazawa Kyôzyu no Kenkyû ga aru. Terazawa Kyôzyu no Keisan no hô ga seikaku de sono Kekkwa ga kwantan de aru. Proc. of the Imperial Academy, IV (1918), No. 4.

I.

Eikoku-sikino Hûtô no Baai wa Keisan ga kantan de aru kara, madu kono Baai no Keisan wo suru.

(*a*) *Eikoku-sikino Hûtô no nakani Ita ga nanameni okareta Baai.*

Kono Baai wo heimentekini kangaeruto, Du 1 ni simesu yôni nimaino heikôna Ita no tyûkan ni Ita wo nanameni oita tokino Nagare ni naru.

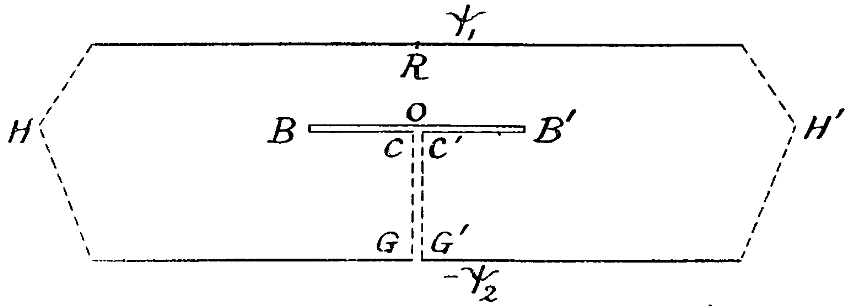


Du 1. z-Men.

Kono Baai no hurenzokuna Nagare wa, sudeni sirarete iru ga, wareware no Baai niwa renzokuna Nagare ga hituyô de atte, *Schwartz-Christoffel* no Henkwan dewa taihen mudukasiku natte tokenai. Sokode wareware wa Villat no Hôhô[1] wo tukatte keisansuru.

Ima, $z=x+iy$ to kaku, koko de (x, y) wa Undo no Heimen ni okeru Tyokkaku-zahyô de aru. ϕ wo Sokudo-*potential* to si, ψ wo Nagare no Kansû to suru. $f=\phi+i\psi$ to sureba, f wa z no Kaisekikansû de aru.

Du 1 ni sôtôsuru f no Men wa Du 2 no tôri de aru.



Du 2. f-Men.

(1) Henri Villat: Scientia No. 38.

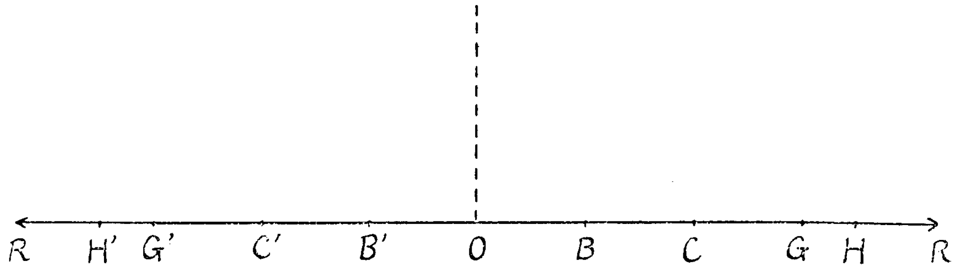
$CC'GG'$ ni Kirime wo irete f-Men wo t-Men no ueno hanbun ni tugino Kwankei ni yotte henkwansuru:

$$\frac{df}{dt}=M\frac{t^2-b^2}{(t^2-h^2)\sqrt{(t^2+c^2)(t^2-g^2)}},$$

koko de b wa B ni, $-b$ wa B' ni sôtôsuru. Hokano mono mo dôyô de aru. Sosite Zyôsû M wo, t ga h wo tôrikosu toki, kyûni tatediku-hôkôno Atai ga $\psi_1+\psi_2$ dake kawaru yôni kimeruto,

$$M=\frac{\psi_1+\psi_2}{\pi}\cdot\frac{2h\sqrt{(h^2-c^2)(h^2-g^2)}}{h^2-b^2}.$$

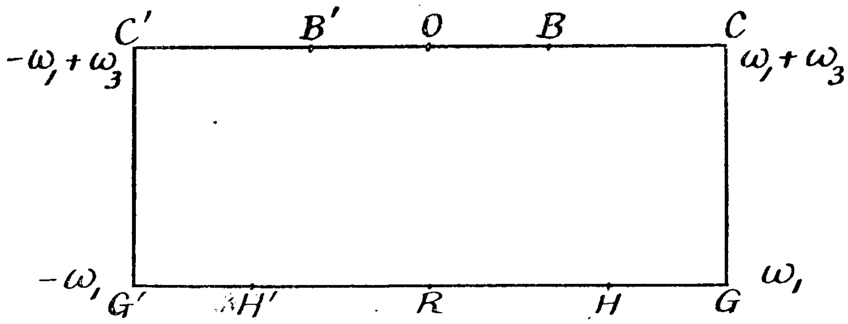
to naru.



Du 3. t-Men.

Kono t-Men no Ryôiki wa, s-Men ni okeru Hen ga $2\omega_1$ to $\frac{\omega_3}{i}$ to no Kukei ni tugino Kwankei ni yotte, seikaku-tekini (conformally) henkwansareru

$$t^2=\wp s-e_3.$$



Du 4. s-Men.

B ni taisite $s=\mu$, H ni taisite $s=\nu$ to sureba,

$$\frac{df}{ds}=\frac{\psi_1+\psi_2}{\pi}\cdot\frac{\wp'\nu}{\wp\nu-\wp\mu}\cdot\frac{\wp s-\wp\mu}{\wp s-\wp\nu},$$

$$=\frac{\psi_1+\psi_2}{\pi}\Big[\zeta(\mu+\nu)-\zeta(\mu-\nu)-\zeta(s+\nu)+\zeta(s-\nu)\Big]$$

to naru.

Kono Kansû wo sekibunsite, $s=\mu$ ni oite $f=o$ ni naru yôni suruto:

$$f=\frac{\psi_1+\psi_2}{\pi}\left\{\Big[\zeta(\mu+\nu)-\zeta(\mu-\nu)\Big](s-\mu)-\log\frac{\sigma(s+\nu)\sigma(\mu-\nu)}{\sigma(s-\nu)\sigma(\mu+\nu)}\right\}.$$

Zyunkwan (circulation) wa nai mono to kateisita kara, f wa $2\omega_1$ no Syûki wo motu, sokode tugino Dyôken wo eru:

$$\zeta(\mu+\nu)-\zeta(\mu-\nu)=\frac{2\eta_1\nu}{\omega}. \qquad (1)$$

f wa, $s=w$, no Tokoro to $s=\omega_1+\omega_3$ no Tokoro to dewa $i\psi_2$ dake tigau kara,

$$\frac{i\psi_2\pi}{\psi_1+\psi_2}=\Big[\zeta(\mu+\nu)-\zeta(\mu-\nu)\Big]\omega_3-2\eta_3\nu \qquad (2)$$

to naru.

(1) to (2) to kara

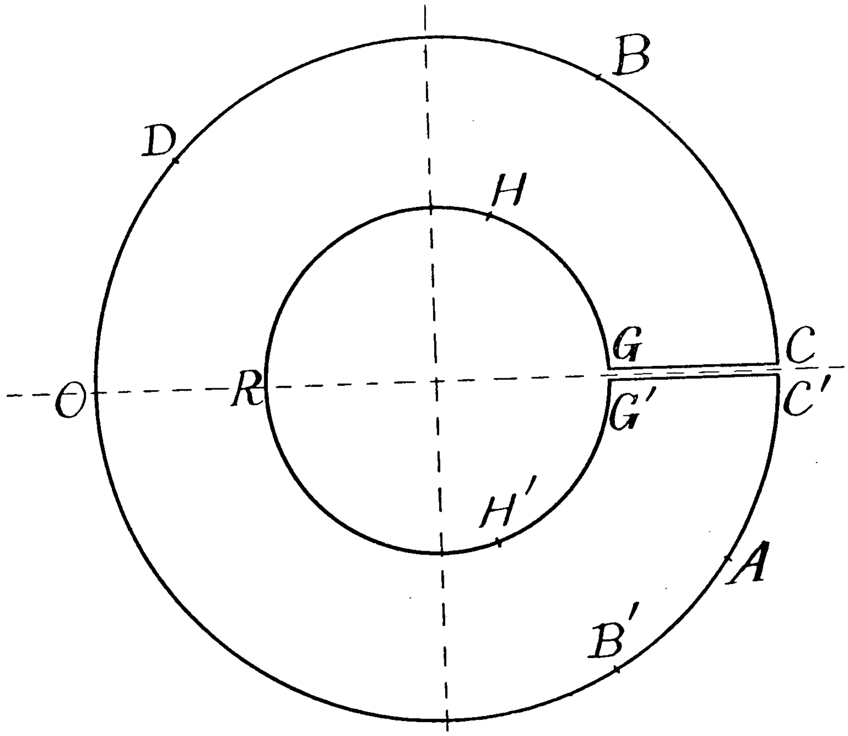
$$\nu=\frac{\psi_2}{\psi_1+\psi_2}\omega_1$$

naru Kwankei ga erareru.

s-Men ni okeru Kukei no Utigawa wa, Z-Men ni okeru Hankei ga sorezore 1 to q $(=e^{\frac{i\pi\omega_3}{\omega^1}}<1)$ to no Dôsin-en ni yotte kagirareta Wagata-ryôiki ni, tugino Kwankei ni yotte, seikaku-tekini henkwansareru.

$$s=\omega_1+\omega_3-\frac{\omega_1}{i\pi}\log Z.$$

Nanamena Ita no Men wa sotogawano En ni, heikôna nimaino Ita wa utigawano En ni sôtôsuru.



Du 5. Z-Men.

$A:\quad e^{i\theta_4}$

$B:\quad e^{i\theta_1}$

$B':\quad e^{i(2\pi-\theta_1)}$

$D:\quad e^{i\theta_3}$

$H:\quad qe^{i\theta_2}$

$H':\quad qe^{i(2\pi-\theta_2)}$

Sokode Z-Men ni okeru Kaku wo motte arawasuto,

$$\mu=\omega_3+\omega_1\left(1-\frac{\theta_1}{\pi}\right),$$

$$\nu=\omega_1\left(1-\frac{\theta_2}{\pi}\right)$$

to naru.

Tugini

$$\frac{df}{dz}=e^{-i\Omega} \tag{3}$$

to suru. Zentaino Nagare no Sokudo wa *1* de aru.

Genkai-men ni okeru Sokudo ga, *Z*-Men ni okeru Tyûsin-kaku no Kansû tosite arawasu koto ga dekiruto, *Villat* no Kôsiki[1] ni yotte Ω wo keisansuru koto ga dekiru.

Sunawati,

$$\Omega(Z)=\frac{i\omega_1}{\pi^2}\int_0^{2\pi}\Phi(\theta)\zeta\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta\right)d\theta$$

$$-\frac{i\omega_1}{\pi^2}\int_0^{2\pi}\Psi(\theta)\zeta_3\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta\right)d\theta\,, \tag{4}$$

koko de $\Phi(\theta)$ to $\Psi(\theta)$ to no aidaniwa tugino Kwankei ga aru:—

$$\int_0^{2\pi}\Phi(\theta)\,d\theta=\int_0^{2\pi}\Psi(\theta)\,d\theta. \tag{5}$$

Tadasi, $\Phi(\theta)$ wa nanamena Ita no Men ni okeru, $\Psi(\theta)$ wa heikôna Ita no Men ni okeru Nagare no Hôkô ga sorezore x-diku no seino Hôkô to nasu Kaku de, kono Baai $\Psi(\theta)=0$ de aru.

Ueno Kôsiki (5) kara

$$\int_{\theta_1}^{\theta_4}\left(\frac{\pi}{2}+\delta\right)d\theta+\int_{\theta_4}^{\theta_1}\left(-\frac{\pi}{2}+\delta\right)d\theta+\int_{\theta_1}^{\theta_3}\left(\frac{\pi}{2}+\delta\right)d\theta+\int_{\theta_3}^{2\pi-\theta_1}\left(-\frac{\pi}{2}+\delta\right)d\theta=0;$$

$$\therefore\quad \theta_3+\theta_4-\pi+2\delta=0.$$

(1) Mae to onazi, p. 16. § II.

Mata, Kôsiki (4) kara

$$\Omega(Z)=\frac{i\omega_1}{\pi^2}\int_0^{2\pi}\Phi(\theta)\zeta\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta\right)d\theta$$

$$=-\frac{i}{\pi}\left[\delta\log\frac{\sigma\left(\frac{\omega_1}{i\pi}\log Z+\frac{\omega_1}{\pi}\theta_1-2\omega_1\right)}{\sigma\left(\frac{\omega_1}{i\pi}\log Z+\frac{\omega_1}{\pi}\theta_1\right)}\right.$$

$$\left.+\frac{\pi}{2}\log\frac{\sigma^2\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta_4\right)\sigma^2\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta_3\right)}{\sigma^2\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta_1\right)\sigma\left(\frac{\omega_1}{i\pi}\log Z+\frac{\omega_1}{\pi}\theta_1\right)\sigma\left(\frac{\omega_1}{i\pi}\log Z+\frac{\omega_1}{\pi}\theta_1-2\omega_1\right)}\right].$$

Kantanni suru tameni kore wo s no Kansû tosite arawaseba,

$$\Omega(s)=-i\log\left\{\left[\frac{\sigma(s+\mu-2\omega_3)}{\sigma(s+\mu+2\omega_2)}\right]^{\frac{\delta}{\pi}+\frac{1}{2}}\cdot\frac{\sigma(s-s_4)\sigma(s-s_3)}{\sigma(s-\mu)\sigma(s+\mu-2\omega_3)}\right\},$$

koko de

$$s_3=\omega_1+\omega_3-\frac{\omega_1}{\pi}\theta_1,$$

$$s_4=\omega_1+\omega_3-\frac{\omega_1}{\pi}\theta_4.$$

∴ (3) kara

$$dz=e^{i\Omega}dJ$$

$$=\frac{\psi_1+\psi_2}{\pi}\cdot\frac{\wp'\nu}{\wp\nu-\wp\mu}\cdot\frac{\sigma^2\nu}{\sigma^2\mu}(-1)^{\frac{\delta}{\pi}-\frac{1}{2}}\cdot\frac{\sigma(s-s_4)\sigma(s-s_3)}{\sigma(s+\nu)\sigma(s-\nu)}$$

$$\times e^{2\eta_1(\mu+2\omega_2+\omega_1)\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+2\eta_3(\mu-\omega_3)+2\eta_1 s\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+2\eta_3 s}\cdot ds. \tag{6}$$

Wareware wa kono Kansû wo sekibunsinakereba naranai. Sore ga tameniwa kore wo Kyûsû ni tenkaisite notini sekibunsureba yoi.

$$F(s)=e^{2\eta_1 s\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+2\eta_3 s}\cdot\frac{\sigma(s-s_4)\sigma(s-s_3)}{\sigma(s+\nu)\sigma(s-\nu)}$$

to okeba

$$F(s+2\omega_1)=F(s),$$

$$F(s+2\omega_3)=e^{2i\left(\delta+\frac{\pi}{2}\right)}F(s)$$

to naru kara, kono Kansû wa dai-nisyuno Daen-kansû de aru.

Sokode $F(s)$ wo kantanna Kansû ni wakeru. Sore niwa

$$A(s)=-\frac{\sigma\left[s-2\omega_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]}{\sigma s\,\sigma\left[2\omega_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]}e^{2\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)s}$$

naru Kansû wo tukau.

$F(s)$ wa $s=\nu$ oyobi $s=-\nu$ ni oite kantanna Kyoku wo motu, sosite

$$\underset{s=\nu}{\text{Res.}}\ F(s)=e^{2\left[\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+\eta_3\right]\nu}\cdot\frac{\sigma(\nu-s_4)\sigma(\nu-s_3)}{\sigma(2\nu)}=C_\nu,$$

$$\underset{s=-\nu}{\text{Res.}}\ F(s)=e^{-2\left[\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+\eta_3\right]\nu}\cdot\frac{\sigma(\nu+s_4)\sigma(\nu+s_3)}{\sigma(2\nu)}=C_{-\nu}$$

de aru kara,

$$F(s)=C_\nu A(s-\nu)+C_{-\nu}A(s+\nu)$$

to naru.

$A(s-\nu)$ to $A(s+\nu)$ to wa tugino yôni site Kyûsû ni tenkai dekiru:

$u=\frac{s}{2\omega_1}$, $\tau=\frac{\omega_3}{\omega_1}$ to sureba, s ga O kara ω_3 made kawaruto, $u-\frac{\tau}{2}$ wa $-\frac{\tau}{2}$ kara O made kawaru kara,

$$-\Re\left(\frac{\tau}{i}\right)<2\Re\left(\frac{u-\frac{\tau}{2}}{i}\right)<\Re\left(\frac{\tau}{i}\right)$$

naru Dyôken wo mitasu yueni,

$$A(s-\nu)=\frac{1}{2w_1}\cdot\frac{\vartheta_1'(0)\vartheta_3\left[u-\frac{\nu}{2\omega_1}-\frac{1+\tau}{2}-\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]}{\vartheta_3\left[u-\frac{\nu}{2\omega_1}-\frac{1+\tau}{2}\right]\vartheta_1\left[-\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]}e^{i\left(\delta+\frac{\pi}{2}\right)}$$

$$=\frac{2\pi}{\omega_1}\cdot e^{i\left(\delta+\frac{\pi}{2}\right)}\left[-\frac{1}{4\sin\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi}\right.$$

$$+\sum_{n=1}^{\infty}(-1)^n q^n$$

$$\left.\cdot\frac{\sin\pi\left[2n\left(\frac{s-\nu}{2\omega_1}-\frac{1+\tau}{2}\right)-\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]-q^{2n}\sin\pi\left[2n\left(\frac{s-\nu}{2\omega_1}-\frac{1+\tau}{2}\right)+\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]}{1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}}\right],$$

$$A(s+\nu)=\frac{2\pi}{\omega_1}e^{i\left(\delta+\frac{\pi}{2}\right)}\left[-\frac{1}{4\sin\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi}\right.$$

$$+\sum_{n=1}^{\infty}(-1)^n q^n$$

$$\left.\cdot\frac{\sin\pi\left[2n\left(\frac{s+\nu}{2\omega_1}-\frac{1+\tau}{2}\right)-\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]-q^{2n}\sin\pi\left[2n\left(\frac{s+\nu}{2\omega_1}-\frac{1+\tau}{2}\right)+\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]}{1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}}\right].$$

Korerano Kansû wo sekibunsureba

$$\int A(s-\nu)\,ds=2e^{i\left(\delta+\frac{\pi}{2}\right)}\left[-\frac{s}{4\cos\delta}\right.$$

$$+\sum_{n=1}^{\infty}(-1)^{n+1}q^n$$

$$\left.\cdot\frac{\cos\pi\left[2n\left(\frac{s-\nu}{2\omega_1}-\frac{1-\tau}{2}\right)-\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]-q^{2n}\cos\pi\left[2n\left(\frac{s-\nu}{2\omega_1}-\frac{1+\tau}{2}\right)+\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]}{n\left[1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}\right]}\right],$$

$$\int A(s+\nu)\,ds = 2e^{i\left(\delta+\frac{\pi}{2}\right)}\left[-\frac{s}{4\cos\delta}\right.$$

$$+\sum_{n=1}^{\infty}(-1)^{n+1} q^n$$

$$\left.\cdot\frac{\cos\pi\left[2n\left(\frac{s+\nu}{2\omega_1}-\frac{1+\tau}{2}\right)-\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]-q^{2n}\cos\pi\left[2n\left(\frac{s+\nu}{2\omega_1}-\frac{1+\tau}{2}\right)+\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]}{n\left[1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}\right]}\right],$$

de aru kara, $s=\omega_1+\omega_3-\frac{\omega_1}{i\pi}\log Z$ naru Kwankei wo tukatte, Z de arawaseba,

$$\int A(s+\nu)\,ds = ie^{i\left(\delta+\frac{\pi}{2}\right)}\left[-\frac{1}{4\cos\delta}\left(\omega_1+\omega_3-\frac{\omega_1}{i\pi}\log Z\right)\right.$$

$$\left.+\sum_{n=1}^{\infty}(-1)^{n+1} q^n\frac{Z^n e^{-i\frac{n\pi\nu}{\omega_1}}(e^{i\delta}+q^{2n}e^{-i\delta})-Z^{-n}e^{\frac{in\pi\nu}{\omega_1}}(e^{-i\delta}+q^{2n}e^{i\delta})}{n\left[1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}\right]}\right],$$

$$\int A(s-\nu)\,ds = ie^{i\left(\delta+\frac{\pi}{2}\right)}\left[-\frac{1}{4\cos\delta}\left(\omega_1+\omega_3-\frac{\omega_1}{i\pi}\log Z\right)\right.$$

$$\left.+\sum_{n=1}^{\infty}(-1)^{n+1} q^n\frac{Z^n e^{i\frac{n\pi\nu}{\omega_1}}(e^{i\delta}+q^{2n}e^{-i\delta})-Z^{-n}e^{-\frac{in\pi\nu}{\omega_1}}(e^{-i\delta}+q^{2n}e^{i\delta})}{n\left[1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}\right]}\right],$$

to naru.

$$\therefore\quad z = M_1\int A(s-\nu)ds + M_2\int A(s+\nu)ds + Zy\hat{o}s\hat{u}.$$

Koko de,

$$M_1=\frac{\psi_1+\psi_2}{\pi}e^{-2\eta_1\omega_1\left(\frac{\theta_1+\theta_2}{\pi}-1\right)\left(\frac{\delta_1}{\pi}-1\right)}\times\frac{\sigma_3\left[\frac{\omega_1}{\pi}\left(\theta_4-\theta_2\right)\right]\sigma_3\left[\frac{\omega_1}{\pi}\left(\theta_3-\theta_2\right)\right]}{\sigma_3\left[\frac{\omega_1}{\pi}\left(\theta_1+\theta_2\right)\right]\sigma_3\left[\frac{\omega_1}{\pi}\left(\theta_1-\theta_2\right)\right]},$$

$$M_2=-\frac{\psi_1+\psi_2}{\pi}e^{-2\eta_1\omega_1\left(\frac{\theta_1-\theta_2}{\pi}-1\right)\left(\frac{\delta_1}{\pi}-1\right)}\times\frac{\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2+\theta_4)\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2+\theta_3)\right]}{\sigma_3\left[\frac{\omega_1}{\pi}(\theta_1+\theta_2)\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_1-\theta_2)\right]};$$

mata $$\delta_1=\frac{\pi}{2}-\delta.$$

z wa nanamena Ita no mawari wo hitomawari-sitemo onazi Atai wo toru kara, Z-Men de wagatano Ryôiki wo hitomawari sekibunsita mono wo O to okeba, $\theta_4=\theta_3-\pi$ to naru. Mata, maeni eta Kwankei $\theta_3+\theta_4=\pi-2\delta$ wo tukauto, $\theta_3=\pi-\delta$, $\theta_4=-\delta$ to naru.

$$\therefore\quad z=\frac{\psi_1+\psi_2}{\pi}e^{i\left(\delta+\frac{\pi}{2}\right)}\frac{2\pi}{\omega_1}$$

$$\cdot\left[e^{-2\eta_1\omega_1\left(\frac{\theta_1+\theta_2}{\pi}-1\right)\left(\frac{\delta}{\pi}-\frac{1}{2}\right)}\cdot\frac{\sigma_3\left[\frac{\omega_1}{\pi}(\theta_4-\theta_2)\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_3-\theta_2)\right]}{\sigma_3\left[\frac{\omega_1}{\pi}(\theta_1+\theta_2)\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_1-\theta_2)\right]}\right.$$

$$\times\left\{\frac{1}{4\cos\delta}\left(\omega_1+\omega_3-\frac{\omega_1}{i\pi}\log Z\right)\right.$$

$$\left.-\sum_{n=1}^{\infty}(-1)^{n+1}q^n\frac{\omega_1 i}{\pi}\cdot\frac{Z^n e^{-\frac{in\pi\nu}{\omega_1}}(e^{i\delta}+q^{2n}e^{-i\delta})-Z^{-n}e^{i\frac{n\pi\nu}{\omega_1}}(e^{-i\delta}+q^{2n}e^{i\delta})}{2n\left[1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}\right]}\right\}$$

$$-e^{-2\eta_1\omega_1\left(\frac{\theta_1-\theta_2}{\pi}-1\right)\left(\frac{\delta}{\pi}-\frac{1}{2}\right)}\cdot\frac{\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2+\theta_4)\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2+\theta_3)\right]}{\sigma_3\left[\frac{\omega_1}{\pi}(\theta_1+\theta_2)\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_1-\theta_2)\right]}$$

$$\times\left\{\frac{1}{4\cos\delta}\left(\omega_1+\omega_3-\frac{\omega_1}{i\pi}\log Z\right)\right.$$

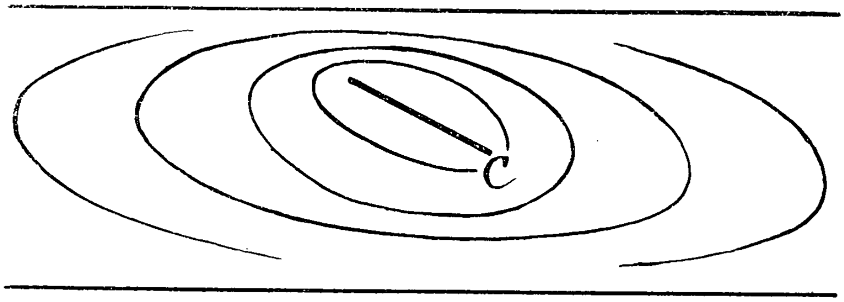
$$\left.\left.-\sum_{n=1}^{\infty}(-1)^{n+1}q^n\frac{\omega_1 i}{\pi}\cdot\frac{Z^n e^{i\frac{n\pi\nu}{\omega_1}}(e^{i\delta}+q^{2n}e^{-i\delta})-Z^{-n}e^{-i\frac{n\pi\nu}{\omega_1}}(e^{-i\delta}+q^{2n}e^{i\delta})}{2n\left[1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}\right]}\right\}\right]$$

$$+\ Zy\hat{o}s\hat{u}. \qquad (7)$$

Kono Zyôsû wa katteni totte yoi kara, O ni sitemo yoi.

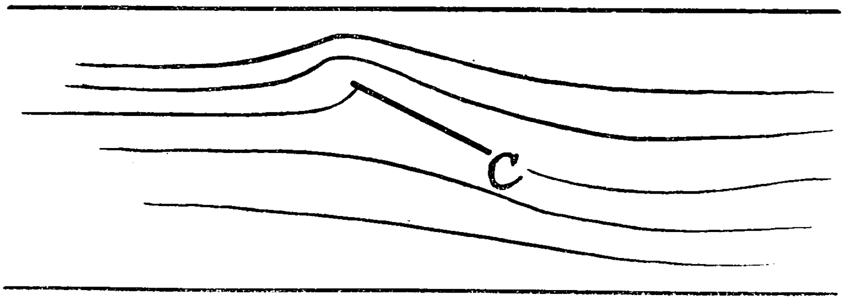
Tugini nanamena Ita ni hataraku Tikara wo keisansuru.

Kore ga tameniwa Ita no mawarini zyunkwansuru Nagare wo tukekuwaeru Hituyô ga aru. Kono Zyunkwan no Z-Men ni okeru Hukusosû-*potential*-kansû wa $f_1 = i\Gamma \log Z$ de arawasareru. Kono de Γ wa Zyunkwan no Tuyosa de aru. z-Men ni okeru kono Zyunkwan wo Du ni arawaseba Du 6 no tôri de aru.



Du 6.

Zyunkwan Γ no Tuyosa wa Nagare ga hutatabi C (Ita no sitagawano Hazi) de issyoni naru yôni kimeru. Sokode maeno Nagare to kumiawaseruto, Du 7 ni simesu yôni naru.



Du 7.

$w = f + f_1$

to okeba Ita ni hataraku Tikara wa tugino Siki de keisan dekiru:—

$$P = -i\frac{\rho}{2}\int\left(\frac{dw}{dz}\right)^2 dz;$$

tadasi ρ wa Kûki no Mitudo, mata Sekibun no Miti wa Ita no mawarini toru.

C ni oite Nagare ga hutatabi issyoni naru tameniwa, sono Ten de

$$\frac{dw}{dz}=0,$$

sunawati Z-Men dewa $Z=e^{i\theta_4}$ de

$$\frac{df}{dZ}+\frac{df_1}{dZ}=0$$

to naru Hituyô ga aru.

$$\therefore \quad \Gamma=(e_3-e_1)(e_3-e_2)\frac{\sigma\left(2\frac{\omega_1}{\pi}\theta_2\right)\sigma\left[\frac{\omega_1}{\pi}(\theta_1-\theta_4)\right]\sigma\left[\frac{\omega_1}{\pi}(\theta_1+\theta_4)\right]}{\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2+\theta_4)\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2-\theta_4)\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_1+\theta_2)\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_1-\theta_2)\right]}.$$

Tugini,

$$\int\left(\frac{dw}{dz}\right)^2 dz=\int\left(\frac{df}{dz}\right)^2 dz+\int\left(\frac{df_1}{dz}\right)^2 dz+2\int\left(\frac{df}{dz}\right)\left(\frac{df_1}{dz}\right)dz$$

de atte, hazimeno hutatuno Kô wa O ni natte saigono mono nomi ga nokoru. Saigono Kô wa Z-Men de arawaseba

$$2i\Gamma\int e^{-i\Omega}\frac{dZ}{Z}$$

to naru. Koko de, Sekibun no Miti wa hutatuno Dôsin-en no aidano Dôsinen de aru.

Sate,

$$e^{-i\Omega}=(-1)^{-\left(\frac{\delta}{\pi}+\frac{1}{2}\right)}e^{-2\eta_1(s+\mu+2\omega_2+\omega_1)\left(\frac{\delta}{\pi}+\frac{1}{2}\right)-2\eta_3(s+\mu-\omega_3)}\cdot\frac{\dot{\sigma}(s-\mu)\,\sigma(s+\mu)}{\sigma(s-s_4)\,\sigma(s-s_3)}$$

de atte, kono Kansû wa dai-nisyu no Daen-kansû de aru kara, mae to onaziyôni site kantanna Kansû ni wake, sosite Kyûsû ni tenkai dekiru.

Sunawati

$$e^{-i\Omega}=(-1)^{-\left(\frac{\delta}{\pi}+\frac{1}{2}\right)}e^{-2\eta_1(\mu+2\omega_2+\omega_1)\left(\frac{\delta}{\pi}+\frac{1}{2}\right)-2\eta_3(\mu-\omega_3)}$$

$$\times e^{i\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\frac{2\pi}{\omega_1}}\left\{e^{-2\left[\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+\eta_3\right]s_4}\cdot\frac{\sigma(s_4+\mu)\sigma(s_4-\mu)}{\sigma(s_4-s_3)}\right.$$

$$\times\left[\frac{1}{4\sin\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi}+\sum_{n=1}^{\infty}(-1)^n q^n\right.$$

$$\left.\cdot\frac{\sin\pi\left[2n\left(\frac{s}{2\omega_1}+\frac{\theta_4}{2\pi}\right)+\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]-q^{2n}\sin\pi\left[2n\left(\frac{s}{2\omega_1}+\frac{\theta_4}{2\pi}\right)-\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]}{1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}}\right]$$

$$+e^{-2\left[\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+\eta_3\right]s_3}\cdot\frac{\sigma(s_3+\mu)\sigma(s_3-\mu)}{\sigma(s_3-s_4)}$$

$$\times\left[\frac{1}{4\sin\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi}+\sum_{n=1}^{\infty}(-1)^n q^n\right.$$

$$\left.\left.\cdot\frac{\sin\pi\left[2n\left(\frac{s}{2\omega_1}+\frac{\theta_3}{2\pi}\right)+\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]-q^{2n}\sin\pi\left[2n\left(\frac{s}{2\omega_1}+\frac{\theta_3}{2\pi}\right)-\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]}{1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}}\right]\right\};$$

mata
$$s=\omega_1+\omega_3-\frac{\omega_1}{i\pi}\log Z$$

ni yotte kono Kansû wa Z no Kansû ni naosareru.

Sokode

$$\int e^{-i\Omega}\frac{dZ}{Z}$$

wo Z-Men de hutatuno Dôsin-en no aidano Dôsin-en ni sotte sekibun-sureba Z^{-1} no Kô nomi ga nokotte, atono Kô wa mina O ni naru. Sosite,

$$\int\left(\frac{dw}{dz}\right)^2 dz=\frac{2\pi^2\Gamma}{\omega_1\cos\delta}e^{2\eta_1\omega_1\left(\frac{\theta_1}{\pi}-1\right)\left(\frac{\delta}{\pi}-\frac{1}{2}\right)}$$

$$\times\left[e^{2\eta_1\omega_1\frac{\theta_4}{\pi}\left(\frac{\delta}{\pi}-\frac{1}{2}\right)}\cdot\frac{\sigma\left[\frac{\omega_1}{\pi}(\theta_1+\theta_4)\right]\sigma\left[\frac{\omega_1}{\pi}(\theta_1-\theta_4)\right]}{\sigma\left[\frac{\omega_1}{\pi}(\theta_3-\theta_4)\right]}\right.$$

$$\left.-e^{2\eta_1\omega_1\frac{\theta_3}{\pi}\left(\frac{\delta}{\pi}-\frac{1}{2}\right)}\cdot\frac{\sigma\left[\frac{\omega_1}{\pi}(\theta_1+\theta_3)\right]\sigma\left[\frac{\omega_1}{\pi}(\theta_1-\theta_3)\right]}{\sigma\left[\frac{\omega_1}{\pi}(\theta_3-\theta_4)\right]}\right]. \quad (8)$$

to naru. Kono Atai wa zituno Ryô de aru kara,

$$P=-\frac{i\rho}{2}\int\left(\frac{dw}{dz}\right)^2 dz$$

wa kyono Ryô de aru. Sunawati, Ita ni hataraku Tikara wa Huyô-ryoku nomi de aru koto ga wakaru.

Tokubetuna Baai (1.)

$\psi_1=\psi_2$ no tokiwa, nanamena Ita no Tyûsin ga hutatuno heikôna Ita kara hitosii Kyori ni aru Baai ni naru.

$$\therefore\quad \nu=\frac{1}{2}\omega_1,\qquad \mu=\frac{1}{2}\omega_1+\omega_3,$$

$$\theta_1=\theta_2=\frac{\pi}{2},$$

$$\theta_4=-\delta,\qquad \theta_3=\pi-\delta$$

to naru kara,

$$z=\frac{2\psi_1}{\pi}e^{-i\delta_1-\eta_1\omega_1\frac{\delta_1}{\pi}}\cdot\sigma_2\left(\frac{\omega_1}{\pi}\delta_1\right)\sigma_3\left(\frac{\omega_1}{\pi}\delta_1\right)\cdot\sum\frac{Z^n e^{\frac{in\pi}{2}}}{in\sin\left(\delta_1-\frac{n\pi\omega_3}{\omega_1}\right)},$$

koko de n wa $-\infty$ kara $+\infty$ made no Kisû no Seisû no subeteno Atai wo toru.

Ita no Haba wo b to sureba, $2\psi_1=d$ to site,

$$b=\frac{4d}{\pi}\sigma_2\left(\frac{\omega_1}{\pi}\delta_1\right)\sigma_3\left(\frac{\omega_1}{\pi}\delta_1\right)e^{-\eta_1\omega_1\frac{\delta_1}{\pi}}\cdot\sum\frac{q^n\cos(n-1)\delta_1-q^{3n}\cos(n+1)\delta_1}{n(1-2q^{2n}\cos 2\delta_1+q^{4n})};$$

tadasi $n=1,\ 3,\ 5,\ \ldots\ldots\ \infty$, mata d wa hutatuno heikôna Ita no aidano Kyori de aru.

Mata,
$$\Gamma=-\frac{d}{\pi^2}\omega_1(e_2-e_3)\frac{\sigma\left(\frac{\omega_1}{\pi}\delta_1\right)\sigma_1\left(\frac{\omega_1}{\pi}\delta_1\right)}{\sigma_2\left(\frac{\omega_1}{\pi}\delta_1\right)\sigma_3\left(\frac{\omega_1}{\pi}\delta_1\right)},$$

sitagatte,

$$P=2i\rho d\,(e_2-e_3)\,e^{\eta_1\omega_1\frac{\delta_1}{\pi}-2\eta_1\omega_1\left(\frac{\delta_1}{\pi}\right)^2}\cdot\frac{\sigma^2\left(\frac{\omega_1}{\pi}\delta_1\right)\sigma_1^2\left(\frac{\omega_1}{\pi}\delta_1\right)}{\sin\delta_1\,\sigma_2\left(\frac{\omega_1}{\pi}\delta_1\right)\sigma_3\left(\frac{\omega_1}{\pi}\delta_1\right)}.$$

b to P to kara d wo kesite

$$\frac{P}{\rho b}=i\frac{(e_2-e_3)\pi}{2\sin\delta_1\cdot Q}e^{2\eta_1\omega_1\left(1-\frac{\delta_1}{\pi}\right)\frac{\delta_1}{\pi}}\cdot\frac{\sigma^2\left(\frac{\omega_1}{\pi}\delta_1\right)\sigma_1^2\left(\frac{\omega_1}{\pi}\delta_1\right)}{\sigma_2^2\left(\frac{\omega_1}{\pi}\delta_1\right)\sigma_3^2\left(\frac{\omega_1}{\pi}\delta_1\right)};$$

kokode
$$Q=\sum\frac{q^n\cos(n-1)\delta_1-q^{3n}\cos(n+1)\delta_1}{n(1-2q^{2n}\cos 2\delta_1+q^{4n})},$$

tadasi $n=1,\ 3,\ 5,\ \ldots\ldots\ \infty$.

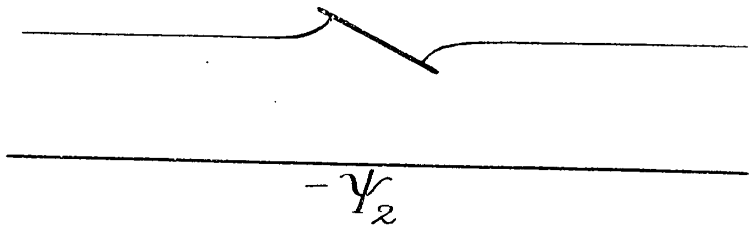
P_0 ga $d=\infty$ no Baai ni Ita ni hataraku Tikara to si, $\frac{b}{d}$ no iroirona Atai ni tuite $\frac{P}{P_0}$ wo keisansuruto, $\delta_1=10^\circ$ no toki wa tugino yôni naru.

Hyô I.

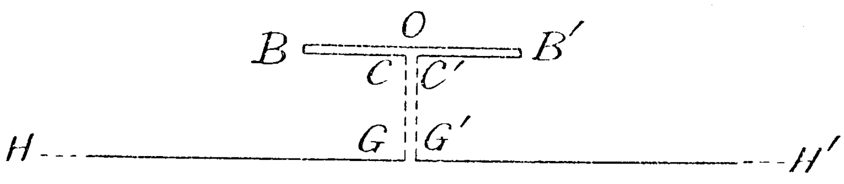
q	b/d	P/P_0
0	0	1
0·1	0·1271	0·9789
0·15	0·2055	0·9485
0·2	0·2996	0·9068

Tokubetuna Baai (2.)

$\psi_1 = \infty$ no Baai. Konotokiwa ippanno Baai kara suguni mitibikidasu koto wa dekinai. z-Men oyobi f-Men wa Du 8, Du 9 ni simesu yôni naru.



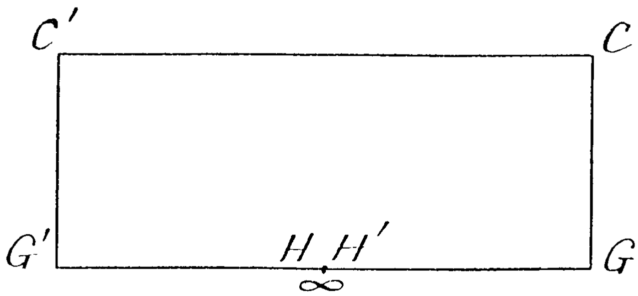
Du 8. z-Men.



Du 9. f-Men.

f wa s-Men no Kukei ni tugino Kwankei ni yotte henkwansareru:—

$$\frac{df}{ds} = -M(\wp s - \wp \mu).$$



Du 10. s-Men.

$$\therefore \quad f = M(\zeta s + \wp\mu \cdot s) + Zy\hat{o}s\hat{u}.$$

f wa $2\omega_1$ no Syûki wo motu kara,

$$\zeta(s+2\omega_1) + \wp\mu \cdot 2\omega_1 = \zeta s,$$

$$\therefore \quad \wp\mu + \frac{\eta_1}{\omega_1} = 0.$$

Mata $\quad s = \omega_1 \quad$ dewa $\quad f = \varphi_1 - i\psi_2,$

$\quad s = \omega_1 + \omega_3 \quad$ dewa $\quad f = \varphi_1.$

$$\therefore \quad i\psi_2 = M(\eta_3 + \wp\mu \cdot \omega_3) = \frac{M}{\omega_1}(\eta_3\omega_1 - \eta_1\omega_3) = -\frac{i\pi}{2} M,$$

$$\therefore \quad M = -\frac{2\psi_2}{\pi},$$

$$\therefore \quad \frac{df}{ds} = \frac{2\psi_2}{\pi}(\wp s - \wp\mu)$$

to naru.

Mata

$$dz = -(-1)^{\frac{\delta}{\pi} - \frac{1}{2}} \cdot e^{2\eta_1(s+\mu+2\omega_2+\omega_1)\left(\frac{\delta}{\pi}+\frac{1}{2}\right) + 2\eta_3(s+\mu-\omega_3)}$$

$$\times \frac{2\psi_2}{\pi} \cdot \frac{\sigma(s-s_3)\sigma(s-s_4)}{\sigma^2 s\, \sigma^2 \mu}.$$

Ima $\quad F_1(s) = e^{2\left[\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right) + \eta_3\right]s} \cdot \dfrac{\sigma(s-s_3)\sigma(s-s_4)}{\sigma^2 s}$

to okeba,

$$F_1(s+2\omega_1) = F_1(s),$$

$$F_1(s+2\omega_3) = e^{2i\left(\frac{\delta}{\pi}+\frac{1}{2}\right)} F_1(s)$$

to naru kara, $F_1(s)$ wa dai-nisyuno Daen-kansû de aru. Sosite $s=0$ de hitotuno nidyûna Kyoku ga aru kara, tugino yôni site kantanna Kansû ni wakeru koto ga dekiru. h wo taihen tiisai Ryô to sureba,

$$F_1(h)=e^{2\left[\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+\eta_3\right]h}\frac{\sigma(h-s_3)\sigma(h-s_4)}{\sigma^2 h}$$

$$=\frac{A_1}{h}+\frac{A_2}{h^2}+\textit{ renzokuna Kansû}.$$

$$e^{2\left[\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+\eta_3\right]h}\cdot\sigma(h-s_3)\,\sigma(h-s_4)$$
$$=\sigma(s_3)\sigma(s_4)+\left[-\sigma'(s_3)\,\sigma(s_4)-\sigma(s_3)\,\sigma'(s_4)+2\left\{\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+\eta_3\right\}\sigma(s_3)\sigma(s_4)\right]h$$
$$+\ldots\ldots,$$

mata

$$\sigma^2 h=\sigma^2(0)+2\sigma'(0)\,h+2\left[\sigma'^2(0)+\sigma'(0)\sigma''(0)\right]\frac{h^2}{2!}+\ldots\ldots$$
$$=h^2+\ldots\ldots$$

de aru kara,

$$A_2=\sigma s_3\sigma s_4,$$
$$A_1=A_2\left\{-\zeta s_3-\zeta s_4+2\left[\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+\eta_3\right]\right\},$$

$$\therefore\quad F_1(s)=\sigma s_3\sigma s_4\left[\left\{-\zeta s_3-\zeta s_4+2\left[\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+\eta_3\right]\right\}A(s)-A'(s)\right];$$

kokode

$$A(s)=\frac{2\pi}{\omega_1}e^{i\left(\delta+\frac{\pi}{2}\right)}\left[-\frac{1}{4\sin\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi}+\sum_{n=1}^{\infty}(-1)^n q^n\right.$$
$$\left.\cdot\frac{\sin\pi\left[2n\left(\frac{s}{2\omega_1}-\frac{1+\tau}{2}\right)-\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]-q^{2n}\sin\pi\left[2n\left(\frac{s}{2\omega_1}-\frac{1+\tau}{2}\right)+\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]}{1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}}\right].$$

$$A'(s)=\frac{2\pi^2}{\omega_1^2}e^{i\left(\delta+\frac{\pi}{2}\right)}\left[\sum_{n=1}^{\infty}(-1)^n q^n n\right.$$
$$\left.\cdot\frac{\cos\pi\left[2n\left(\frac{s}{2\omega_1}-\frac{1+\tau}{2}\right)-\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]-q^{2n}\cos\pi\left[2n\left(\frac{s}{2\omega_1}-\frac{1+\tau}{2}\right)+\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\right]}{1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}}\right].$$

Yueni

$$z=(-1)^{\frac{\delta}{\pi}-\frac{1}{2}}\, e^{2\eta_1(\mu+2\omega_2+\omega_1)\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+2\eta_3(\mu-\omega_3)}\cdot\frac{2\psi_2}{\pi}\cdot\frac{\sigma s_3\sigma s_4}{\sigma^2\mu}$$

$$\times\left[\left\{\zeta s_3+\zeta s_4-2\left[\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+\eta_3\right]\right\}\int A(s)ds+A(s)\right]$$

to naru. Ima $\int A(s)ds$ oyobi $A(s)$ wo Z no Kansû to site arawaseba,

$$\int A(s)ds=ie^{i\left(\delta+\frac{\pi}{2}\right)}\left[-\frac{1}{4\cos\delta}\left(\omega_1+\omega_3-\frac{\omega_1}{i\pi}\log Z\right)\right.$$

$$\left.+\sum_{n=1}(-1)^{n+1}q^n\frac{Z^n(e^{i\delta}+q^{2n}e^{-i\delta})-Z^{-n}(e^{-i\delta}+q^{2n}e^{i\delta})}{n(1+2q^{2n}\cos 2\delta+q^{4n})}\right],$$

$$A(s)=\frac{\pi}{\omega_1}e^{i\left(\delta+\frac{\pi}{2}\right)}\left[-\frac{1}{4\cos\delta}\right.$$

$$\left.+\sum_{n=1}^{\infty}(-1)^{n+1}q^n\frac{Z^n(e^{i\delta}+q^{2n}e^{-i\delta})+Z^{-n}(e^{-i\delta}+q^{2n}e^{i\delta})}{1+2q^{2n}\cos 2\delta+q^{4n}}\right].$$

Yôryoku wa ippanno Baai to onaziyôni nari, tada Γ ga tigau nomi de aru. Kono Baai niwa

$$\Gamma=-\frac{2\psi_2}{\pi}(\wp s_4-\wp\mu)\frac{\omega_1}{\pi}$$

de aru.

(b) *Eikokusikino Hûtô no nakani* Joukowsky *no Hane ga okareta Baai.*.

Maeni dasita Kwankei (6) ni yotte Z-Men ni okeru Hankei ga sorezore *1* to q to no Dôsin-en no aidano Wagata-ryôiki wa, z-Men ni okeru hutatuno heikôna Sen to sorera no aidani aru nanamena Kirime kara naru nidyûni tunagatta Ryôiki ni seikaku-tekini henkwansareru.

Z-Men ni okeru Hankei ga sorezore 1 to $\frac{1}{q}$ to kara naru Dôsinen no aidano Wagata-ryôiki wa, z-Men dewa *Riemann*-Men no Sitagawa no Men (*leaf*) ni oite mae to onazi Ryôiki ni henkwansareru. Sore de aru kara, Z-Men de Utigawa no En ga Hankei ga q de, Sotogawa no En ga $Z=e^{i\theta_4}$ ni oite Hankei 1 no En to maziwaru yôna dosin de nai Wagata-ryôiki wo (6) no Kwankei de z-Men ni henkwansuru to, hutatuno heikôna Tyokusen to sorera no aidani *Joukowsky* no Hane ni nita Genkai no aru Ryôiki ni naru.

Joukowsky no Hane no Baai ni oitemo, f-Men wa maeno Baai to onazi de Du 2 no tôri de aru ga, tada ψ_1, ψ_2 no kawarini ψ_3, ψ_4 to sureba yoi. Mata s-Men wa ω_1, ω_3 no kawarini Ω_1, Ω_3 naru betuno Syûki wo moti, μ, ν no kawarini μ_1, ν_1 to sureba yoi. Sore de aru kara, f no Siki matawa μ_1, ν_1, ψ_3, ψ_4, Ω_1, Ω_3 nado no Kwankeisiki wa mae to onaziyôni naru. Sunawati:

$$f=\frac{\psi_3+\psi_4}{\pi}\left\{\left[\zeta(\mu_1+\nu_1)-\zeta(\mu_1-\nu_1)\right](s-\mu_1)-\log\frac{\sigma(s+\nu_1)\sigma(\mu_1-\nu_1)}{\sigma(s-\nu_1)\sigma(\mu_1+\nu_1)}\right\},$$

$$\zeta(\mu_1+\nu_1)-\zeta(\mu_1-\nu_1)=\frac{2H_1\nu_1}{\Omega_1},$$

$$\frac{i\psi_4\pi}{\psi_3+\psi_4}=\left[\zeta(\mu_1+\nu_1)-\zeta(\mu_1-\nu_1)\right]\Omega_3-2H_3\nu_1,$$

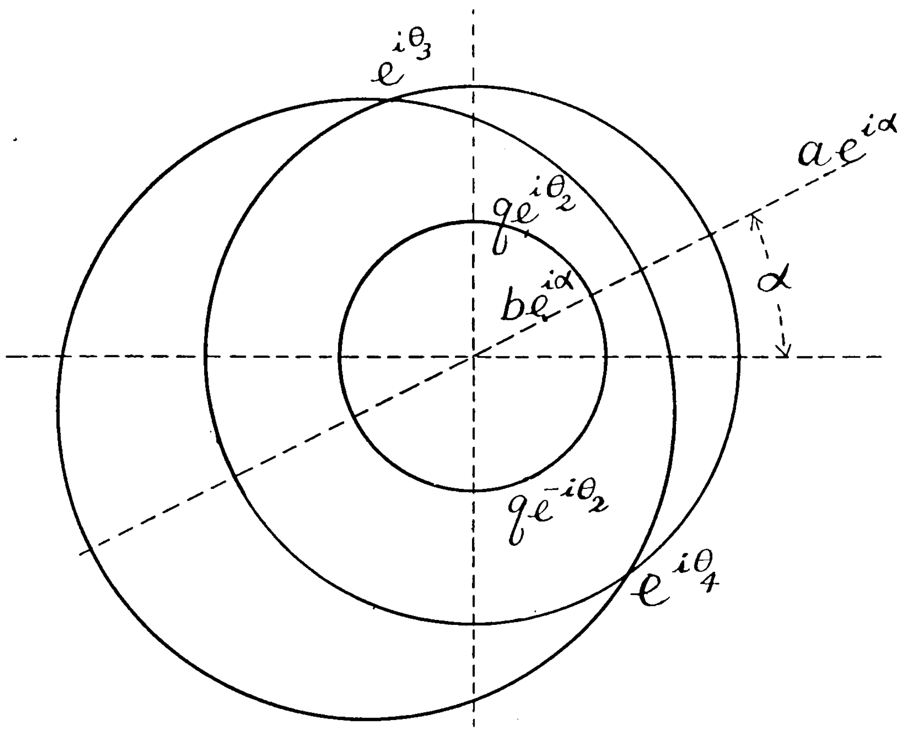
$$\nu_1=\frac{\psi_4}{\psi_3+\psi_4}\Omega_1.$$

Sokode s-Men ni okeru kono Kukei wo ueni nobeta dôsin de nai Wagata-ryôiki ni henkwansuru tameniwa, tugino yôni sureba yoi:

$$s=ig+h-\frac{\Omega_1}{i\pi}\log\frac{Z-be^{i\alpha}}{Z-ae^{i\alpha}}=\xi+i\eta,$$

mata

$$Z=X+iY$$



Du 11. *Z*-Men.

to okeba

$$\xi=h-\frac{\omega_1}{\pi}\tan^{-1}\frac{(b-a)(X\sin\alpha+Y\cos\alpha)}{(X-b\cos\alpha)(X-a\cos\alpha)+(Y+b\sin\alpha)(Y+a\sin\alpha)},$$

$$\eta=g+\frac{\omega_1}{2\pi}\log\frac{(X-b\cos\alpha)^2+(Y+b\sin\alpha)^2}{(X-a\cos\alpha)^2+(Y+a\sin\alpha)^2},$$

to naru.

$\eta=0$ ga $X^2+Y^2=q^2$ no En ni $\eta=\frac{\Omega_3}{i}$ ga $Z=e^{i\theta_4}$ ni oite Hankei *1* no En ni maziwaru En ni naru yôni Zyôsû a, b wo kimeru to

$$e^{-\frac{2\pi g}{\Omega_1}}=\frac{b}{a},\qquad q^2=ab,$$

$$\frac{a}{b}\cdot\frac{b^2+1-2b\cos(\theta_4-\alpha)}{a^2+1-2a\cos(\theta_4-\alpha)}=e^{\frac{2\pi\Omega_3}{i\Omega_1}}=\frac{1}{Q^2}$$

to naru.

$\xi=\nu_1$ ga $Z=qe^{i\theta_2}$ ni,

$\xi=-\nu_1$ ga $Z=qe^{i(2\pi-\theta_2)}$ ni sôtôsuru yôni sureba,

$$\nu_1-h=-\frac{\Omega_1}{\pi}$$

$$\cdot\tan^{-1}\frac{(b-a)(-q\cos\theta_2\sin\alpha+q\cos\alpha\sin\theta_2)}{(q\cos\theta_2-b\cos\alpha)(q\cos\theta_2-a\cos\alpha)+(q\sin\theta_2-b\sin\alpha)(q\sin\theta_2-a\sin\alpha)}$$

$$=-\frac{\Omega_1}{\pi}\tan^{-1}\frac{(b-a)q\sin(\theta_2-\alpha)}{q^2-q(a+b)\cos(\theta_2-\alpha)+ab},$$

$$-\nu_1-h=-\frac{\Omega_1}{\pi}\tan^{-1}\frac{-(b-a)q\sin(\theta_2+\alpha)}{q^2-q(a+b)\cos(\theta_2+\alpha)+ab}.$$

$$\therefore\quad \nu_1=-\frac{\Omega_1}{2\pi}\left[\tan^{-1}\frac{(b-a)\,q\sin(\theta_2-\alpha)}{q^2-q(a+b)\cos(\theta_2-\alpha)+ab}\right.$$

$$\left.-\tan^{-1}\frac{-(b-a)\,q\sin(\theta_2+\alpha)}{q^2-q(a+b)\cos(\theta_2+\alpha)+ab}\right],$$

$$h=\frac{\Omega_1}{2\pi}\left[\tan^{-1}\frac{(b-a)\,q\sin(\theta_2-\alpha)}{q^2-q(a+b)\cos(\theta_2-\alpha)+ab}+\tan^{-1}\frac{-(b-a)\,q\sin(\theta_2+\alpha)}{q^2-q(a+b)\cos(\theta_2+\alpha)+ab}\right].$$

Mata Zyunkwan wa Z-Men dewa

$$f_4=i\Gamma\log\frac{Z-be^{i\alpha}}{Z-ae^{i\alpha}}$$

de arawasareru. Γ no Tuyosa wo kimeru niwa maeno Baai to dôyôni $Z=e^{i\theta_4}$ de

$$\frac{df}{dZ}+\frac{df_4}{dZ}=0,$$

ni naru yôni sureba yoi.

$Z=e^{i\theta_4}$ ni sôtôsuru s no Atai wo s_0 to sureba,

$$s_0=h-\frac{\Omega_1}{\pi}\tan^{-1}\frac{(b-a)\sin(\theta_4-\alpha)}{ab-(a+b)\cos(\theta_4-\alpha)+1}+\Omega_3$$

de aru kara,

$$\Gamma = -\frac{\Omega_1}{\pi}\left(\frac{df}{ds}\right)_{s=s_0}$$

de Γ wa kimaru.

Joukowsky no Hane ni hataraku Tikara wa, $f+f_4=w$ to sureba

$$P = -\frac{i\rho}{2}\int\left(\frac{dw}{dz}\right)^2 dz$$

de keisan dekiru. Koko de Sekibun no Miti wa *Joukowsky* no Hane no mawarini toru.

$$\int\left(\frac{dw}{dz}\right)^2 dz = \int\left(\frac{df}{dz}\right)^2 dz + \int\left(\frac{df_4}{dz}\right)^2 dz + 2\int\frac{df}{dz}\frac{df_4}{dz}dz.$$

Hazimeno hutatuno Kô wa O de aru koto wa sugu wakaru. Saigono Kô no Sekibun wo okonau tameniwa, z-Men ni okeru yorimo, Z-Men ni okeru hô ga benri de aru kara, Sekibun-hugô no naka wo Z no Kansû ni naosu.

$$\frac{df}{dz}\frac{df_4}{dz}dz = \frac{df}{dZ}\cdot\frac{df_4}{dZ}\cdot\frac{dZ}{dz}dZ,$$

$$\frac{df}{dZ}\cdot\frac{df_4}{dZ} = \frac{\Gamma\Omega_1}{\pi}\left(\frac{1}{Z-be^{i\alpha}} - \frac{1}{Z-ae^{i\alpha}}\right)^2\left(\frac{df}{ds}\right)_{s=h+ig-\frac{\Omega_1}{i\pi}\log\frac{Z-be^{i\alpha}}{Z-ae^{i\alpha}}}.$$

mata (6) kara

$$\frac{dZ}{dz}dZ = -\frac{i\pi}{\omega_1}\cdot\frac{ZdZ}{M_1}\left[e^{-2\eta_1 s\left(\frac{\delta}{\pi}+\frac{1}{2}\right)-2\eta_3 s}\cdot\frac{\sigma(s+\nu)\,\sigma(s-\nu)}{\sigma(s-s_3)\sigma(s-s_4)}\right]_{s=\omega_1+\omega_3-\frac{\omega_1}{i\pi}\log Z}$$

de aru; koko de

$$M_1 = \frac{\psi_1+\psi_2}{\pi}\cdot\frac{\wp'\nu}{\wp\nu-\wp\mu}\cdot\frac{\sigma^2\nu}{\sigma^2\mu}e^{i\left(\delta-\frac{\pi}{2}\right)}$$

$$\times e^{-2\eta_1\left[\omega_3+\frac{\omega_1}{\pi}\theta_1\right]\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+2\eta_3\left(1-\frac{\theta_1}{\pi}\right)\omega_1}.$$

Kono Kansû wa Hankei q no En no ue dewa zituno Sû de aru kara, dôsin de nai Wagata-ryôiki wo q no Ritu (*modulus*) de sakasimani sita Ryôiki no nakani kaiseki-tekini entyôsuru koto ga dekiru. Sosite, kikagaku-tekini sakasimani sita Ten dewa, motono Ten no kyôyakuna Atai wo motu.

Ima

$$I=\int_{c>|q|}\frac{df}{dZ}\cdot\frac{df_4}{dZ}\cdot\frac{dZ}{de}dZ-\int_{c<|q|}\frac{df}{dZ}\cdot\frac{df_4}{dZ}\cdot\frac{dZ}{dz}\cdot dZ,$$

koko de hazimeno Kô no Sekibun-miti wa motono Ryôiki ni, atono Kô no Sekibun-miti wa sakasimani sita Ryôiki ni okeru todita Kyokusen de aru.

X_1, Y_1 wo zituno Sû to sureba

$$I=(X_1+iY_1)-(X_1-iY_1)=2iY_1$$

to naru. Sikaruni, I wa $q\,e^{i\theta_2}$ oyobi $q\,e^{-i\theta_2}$ ni okeru *Residua* no Wa de aru kara kore wo keisansi

$$\omega_1+\omega_3-\frac{\omega_1}{i\pi}\log\frac{e^{i\alpha}\left(b-ae^{\frac{i\pi}{\Omega_1}(h+ig+\nu_1)}\right)}{1-e^{\frac{i\pi}{\Omega_1}(h+ig+\nu_1)}}=\nu,$$

oyobi
$$\omega_1+\omega_3-\frac{\omega_1}{i\pi}\log\frac{e^{i\alpha}\left(b-ae^{\frac{i\pi}{\Omega_1}(h+ig-\nu_1)}\right)}{1-e^{\frac{i\pi}{\Omega_1}(h+ig-\nu_1)}}=-\nu.$$

naru Dyôken wo ireruto

$$I=-\frac{i}{\omega_1}M_1\Gamma\Omega_1\left\{e^{-i\alpha}\frac{(b-ae^{\frac{i\pi}{\Omega_1}(h+ig+\nu_1)})(1-e^{\frac{i\pi}{\Omega_1}(h+ig+\nu_1)})}{(b-a)^2e^{\frac{i\pi}{\Omega_1}(h+ig+\nu_1)}}\right.$$

$$\times e^{-2\left\{\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+\eta_3\right\}\nu}\cdot\frac{\sigma(2\nu)\,\sigma(\nu-\nu)}{\sigma(\nu-s_3)\sigma(\nu-s_4)}\cdot\frac{\sigma(\nu_1+\mu_1)\sigma(-\nu_1+\mu_1)}{(\sigma 2\nu_1)}$$

$$+e^{-i\alpha}\frac{(b-ae^{\frac{i\pi}{\Omega_1}(h+ig-\nu_1)})(1-e^{\frac{i\pi}{\Omega_1}(h+ig-\nu_1)})}{(b-a)^2e^{\frac{i\pi}{\Omega_1}(h+ig-\nu_1)}}$$

$$\left.\times e^{+2\left\{\eta\left(\frac{\eta}{\pi}+\frac{1}{2}\right)+\eta_3\right\}\nu}\cdot\frac{\sigma(-\nu+\nu)\sigma(-2\nu)}{\sigma(-\nu-s_3)\sigma(-\nu-s_4)}\cdot\frac{\sigma(\nu_1-\mu_1)\sigma(\mu_1+\nu_1)}{\sigma(2\nu_1)}\right\}=0$$

de aru. Sunawati, Kôryoku wa O de aru koto ga wakaru.

Sore de aru kara, Tikara wa Yôryoku nomi ni naru. Kore wo keisansuru tameniwa, $\frac{df}{dZ}\cdot\frac{df_4}{dZ}\cdot\frac{dZ}{dz}\cdot dZ$ wo Z no Kyûsû ni tenkaisuru no ga itiban Benri de aru.

Sate,

$$\zeta(s+\nu_1)-\zeta(s-\nu_1)=\zeta_3(s+\nu_1-\Omega_3)-\zeta_3(s-\nu_1-\Omega_3)$$

de aru; mata $s+\nu_1-\Omega_3$ to $s-\nu_1-\Omega_3$ to no kyono Bubun wa, s ga O kara Ω_3 madeno aidano Atai wo toru tokiwa, O to $-\Omega_3$ to no aidano Atai wo toru kara, tugino yôna Tenkai-siki ga erareru:[1]

$$\zeta(s+\nu_1)-\zeta(s-\nu_1)$$

$$=\frac{2H_1\nu_1}{\Omega_1}+\frac{2\pi}{\Omega_1}\sum_{n=1}^{\infty}\frac{Q^n}{1-Q^{2n}}\left\{\sin\frac{n\pi}{\Omega_1}(s+\nu_1-\Omega_3)-\sin\frac{n\pi}{\Omega_1}(s-\nu_1-\Omega_3)\right\};$$

mata

$$\zeta(\mu_1+\nu_1)-\zeta(\mu_1-\nu_1)-\frac{2H_1\nu_1}{\Omega_1}=0$$

(1) J. Tannery et J. Molk: Éléments de la theory des Fonctions elliptiques. Tome IV p. 101 (2).

de aru kara,

$$\frac{df}{ds}=-4\frac{\psi_3+\psi_4}{\pi}\sum_{n=1}^{\infty}\frac{Q^n}{1-Q^2{}_n}\sin\frac{n\pi\nu_1}{\Omega_1}\left\{\cos\frac{n\pi\Omega_3}{\Omega_1}\cos\frac{n\pi s}{\Omega_1}+\sin\frac{n\pi\Omega_3}{\Omega_1}\sin\frac{n\pi s}{\Omega_1}\right\}$$

$$=-2\frac{\psi_3+\psi_4}{\pi}\sum_{n=1}^{\infty}\sin\frac{n\pi\nu_1}{\Omega_1}\left\{\frac{1+Q^{2n}}{1-Q^{2n}}\cos\frac{n\pi s}{\Omega_1}+i\sin\frac{n\pi s}{\Omega_1}\right\}.$$

$$\therefore\quad\left(\frac{df}{ds}\right)_{s=ig+h-\frac{\Omega_1}{i\pi}\log\frac{Z-be^{i\alpha}}{Z-ae^{i\alpha}}}$$

$$=-\frac{\psi_3+\psi_4}{\pi}\sum_{n=1}^{\infty}\sin\frac{n\pi\nu_1}{\Omega_1}\left[\frac{1-Q^{2n}}{1-Q^{2n}}\left\{e^{-\frac{n\pi g}{\Omega_1}+\frac{in\pi h}{\Omega_1}}\left(\frac{Z-ae^{i\alpha}}{Z-be^{i\alpha}}\right)^n\right.\right.$$

$$\left.+e^{\frac{n\pi g}{\Omega_1}-\frac{in\pi h}{\Omega_1}}\left(\frac{Z-be^{i\alpha}}{Z-ae^{i\alpha}}\right)^n\right\}$$

$$\left.+\left\{e^{-\frac{n\pi g}{\Omega_1}+\frac{in\pi h}{\Omega_1}}\left(\frac{Z-ae^{i\alpha}}{Z-be^{i\alpha}}\right)^n-e^{\frac{n\pi g}{\Omega_1}-\frac{in\pi h}{\Omega_1}}\left(\frac{Z-be^{i\alpha}}{Z-ae^{i\alpha}}\right)^n\right\}\right].$$

Tugini

$$\chi(s)=e^{-2\left[\eta_1\left(\frac{\delta}{\pi}+\frac{1}{2}\right)+\eta_3\right]s}\cdot\frac{\sigma(s+\nu)\sigma(s-\nu)}{\sigma(s-s_3)\sigma(s-s_4)}.$$

to sureba

$$\chi(s+2\omega_1)=\chi(s),$$

$$\chi(s+2\omega_3)=e^{-2i\left(\delta+\frac{\pi}{2}\right)}\chi(s)$$

to nari, $\chi(s)$ wa dai-nisyuno Daen-kansû de aru kara, hutatuno kantanna Kansû ni wakeru koto ga dekiru. Keisan wa mae to onazi de aru kara, ryakusite

$$\frac{dZ}{dz}dZ=\frac{i\pi_3}{\omega_1^2(\psi_1+\psi_2)}\cdot\frac{Z}{(e_1-e_3)(e_2-e_3)}$$

$$\cdot\frac{\sigma_3\left[\frac{\omega_1}{\pi}(\theta_1+\theta_2)\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_1-\theta_2)\right]}{\sigma\omega_1\,\sigma\left[2\frac{\omega_1}{\pi}\theta_2\right]}e^{2\eta_1\omega_1\left(\frac{1}{2}-\frac{\delta}{\pi}\right)\left(1-\frac{\theta_1-\delta}{\pi}\right)}$$

$$\times\left\{\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2-\delta)+\omega_1\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2+\delta)-\omega_1\right]e^{-2\eta_1\omega_1\left(\frac{1}{2}-\frac{\delta}{\pi}\right)}\right.$$

$$\times\left[\frac{1}{2\cos\delta}+\sum_{n=1}^{\infty}(-1)^n\frac{Z^n(e^{-i\delta}+q^{2n}e^{i\delta})e^{in\delta}+q^{2n}Z^{-n}(e^{i\delta}+q^{2n}e^{-i\delta})e^{-in\delta}}{1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}}\right]$$

$$-\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2-\delta)\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2+\delta)\right]$$

$$\left.\times\left[\frac{1}{2\cos\delta}+\sum_{n=1}^{\infty}\frac{Z^ne^{in\delta}(e^{-i\delta}+q^{2n}e^{i\delta})+q^{2n}Z^{-n}(e^{i\delta}+q^{2n}e^{-i\delta})e^{-in\delta}}{1-2q^{2n}\cos 2\left(\frac{\delta}{\pi}+\frac{1}{2}\right)\pi+q^{4n}}\right]\right\}.$$

Konoyôni site $\frac{df}{dZ}\cdot\frac{df_4}{dZ}\cdot\frac{dZ}{dz}dZ$ no kaku Bubun no Tenkai-siki wo eta kara, ikkô dutu sekibunsureba yoi. Sekibun no Miti wa Hankei q no En to kore to dôsin de nai En tono aida no todida Kyokusen de aru. Sono Kekkwa wa taihen hukuzatu de aru.

Tokubetuna Baai $\psi_1=\psi_2$ no tokiwa,

$$\theta_1=\theta_2=\frac{\pi}{2},$$

$$\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2-\delta)+\omega_1\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2+\delta)-\omega_1\right]e^{-2\eta_1\omega_1\frac{\delta_1}{\pi}}=e^{-\eta_1 1\frac{\delta_1}{\pi}}\sigma_3\omega_1\sigma_2\left(\frac{\omega_1}{\pi}\delta_1\right),$$

$$\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2+\delta)\right]\sigma_3\left[\frac{\omega_1}{\pi}(\theta_2-\delta)\right]=e^{-\eta_1\omega_1\frac{\delta_1}{\pi}}\sigma_3\omega_1\sigma_2\left(\frac{\omega_1}{\pi}\delta_1\right),$$

$$e^{2\eta_1\omega_1\left(\frac{1}{2}-\frac{\delta_1}{\pi}\right)\left(1-\frac{\theta_1-\delta}{\pi}\right)}=e^{2\eta_1\omega_1\frac{\delta_1}{\pi}\left(1-\frac{\delta_1}{\pi}\right)},$$

mata $$\frac{\sigma_3^2\omega_1}{\sigma^2\omega_1}=e_3-e_1$$

de aru kara,

$$P=-\frac{i\rho}{2}\int\left(\frac{dw}{dz}\right)^2 dz$$

$$=i\rho\frac{\pi^3 32}{d(e_2-e_3)\,\omega_1^2}\cdot e^{\eta_1\omega_1\left(1-2\frac{\delta_1}{\pi}\right)}\cdot\sigma_2\!\left(\frac{\delta_1}{\pi}\omega_1\right)\sigma_3\!\left(\frac{\delta_1}{\pi}\omega_1\right)\Gamma.\,(a-b)^2$$

$$\times\left\{\frac{1+Q^2}{1-Q^2}\sin\frac{\pi\nu_1}{\Omega_1}\cdot\frac{1}{(a-b)^3}\left[aq\frac{\sin\left(\frac{\pi h}{\Omega_1}-a\right)+q^2\sin\left(2\delta_1-\frac{\pi h}{\Omega_1}-a\right)}{1-2q^2\cos 2\delta_1+q^4}\right.\right.$$

$$-bq\frac{\sin\left(\frac{\pi h}{\Omega_1}+a\right)-q^2\sin\left(2\delta_1+\frac{\pi h}{\Omega_1}+a\right)}{1-2q^2\cos 2\delta_1+q^4}-q\frac{b^2}{a}\sin\left(\frac{\pi h}{\Omega_1}+3a+2\delta_1\right)$$

$$\left.+b^3q(3b^2-8ab+6a^2)\sin\left(\frac{\pi h}{\Omega_1}-3a-2\delta_1\right)+\ldots\ldots\right]$$

$$+\frac{1+Q^4}{1-Q^4}\sin\frac{2\pi\nu_1}{\Omega_1}\cdot\frac{b}{a}\left[4b\sin\left(\frac{2\pi h}{\Omega_1}-3a-2\delta_1\right)+20b^3\sin\left(\frac{2\pi h}{\Omega_1}-5a-4\delta_1\right)\right.$$

$$\left.+\ldots\ldots\ldots\right]$$

$$+\frac{1+Q^6}{1-Q^6}\sin\frac{3\pi\nu_1}{\Omega_1}\cdot\frac{bq}{a^2}\left[(5b-a)\sin\left(\frac{3\pi h}{\Omega_1}-3a-2\delta_1\right)\right.$$

$$\left.+5b^2(7b-3a)\sin\left(\frac{3\pi h}{\Omega_1}-5a-4\delta_1\right)+\ldots\ldots\right]$$

$$+\frac{1+Q^8}{1-Q_8}\sin\frac{4\pi\nu_1}{\Omega_1}\cdot\frac{b^2}{a^2}\left[(6b-2a)\sin\left(\frac{4\pi h}{\Omega_1}-3a-2\delta_1\right)\right.$$

$$\left.+2b(3a^2-21ab+28b^2)\sin\left(\frac{4\pi h}{\Omega_1}-5a-4\delta_1\right)+\ldots\ldots\right]$$

$$\left.+\ldots\ldots\right\},$$

to naru.

Kono Kekkwa kara miruto, Yôryoku wa tanni Γ no Atai nomi de naku, q, a, δ_1 oyobi α ni kwankeisuru. Sunawati, Hane no Gen to hutatuno heikôna Kabe no aidano Kyori to no Hi, Hane no Kyokuritu oyobi Syôkaku ni kwankeisuru koto ga wakaru.

II.

Göttingen *oyobi* Eiffel *no Kata no Hûtô no Baai.*

Kono Baai niwa hukidasu Nagare no nakani Mokei ga atte, Hûtô no Kabe ga naku, sonokawarini Nagare no Sotogawa dewa Kiatu ga ittei de aru kara, maeno Baai no yôni *Villat* no Kôsiki wo sonomama tukau wake niwa yukanai.

Villat no Kôsiki wo

$$\Omega(Z)=\frac{i\omega_1}{\pi^2}\int_0^{2\pi}\Phi(\theta)\zeta\left[\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{i\pi}\theta\right)\middle|\,\omega_1,\ 2\omega_3\right]d\theta$$

$$-\frac{i\omega_1}{\pi_2}\int_0^{2\pi}\Psi(\theta)\zeta_3\left[\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta\right)\middle|\,\omega_1, 2\omega_3\right]d\theta$$

oyobi
$$\int_0^{2\pi}\Phi(\theta)d\theta=\int_0^{2\pi}\Psi(\theta)d\theta \qquad (a)$$

to kakikae, $\Phi(\theta)=\Psi(\theta)$ to okeba Z wo q no *Modulus* de kikagaku-tekini gyakunisita Ten $\frac{q^2}{x-iy}$ ni oitewa $\Omega\left(\frac{q^2}{x-iy}\right)$ wa Ω (Z) to kyôyakuna Atai wo motu. Soreyueni $Z=q\,e^{i\theta}$ naru En no ue dewa $\Omega\,(Z)$ no kyono Bubun wa O de aru, sunawati hukidasu Nagare no Sotogawa no Men dewa Hayasa ga ittei de aru. Mata $\Phi(\theta)=\Psi(\theta)$ to oku koto ni yotte Dyôken (a) wa mitasareru.

Sunawati,

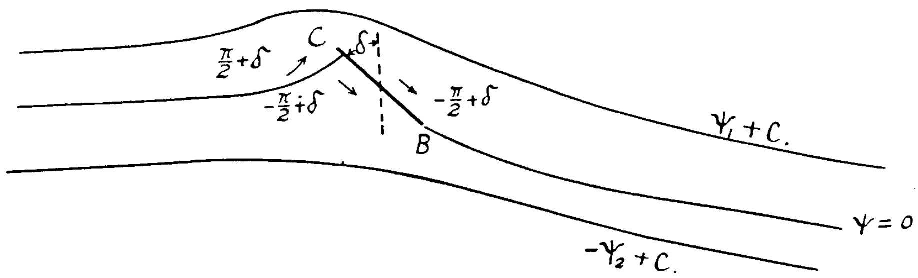
$$\Omega(Z)=\frac{i\omega_1}{\pi^2}\int_0^{2\pi}\Phi(\theta)\left\{\zeta\left[\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta\right)\Big|\,\omega_1,\ 2\omega_3\right]\right.$$

$$\left.-\zeta_3\left[\left(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta\right)\Big|\,\omega_1,\ 2\omega_3\right]\right\}d\theta$$

to oite, Mokei no Hyômen no Nagare ga x-Diku no seino Hôkô to nasu Kaku wo Z-Men de Hankei 1 no Ensyû no ue de Tyûsinkaku no Kwansû $\Phi(\theta)$ tosite arawaseba, Hankei q no Ensyû wa Nagare no Sotogawa no Men ni sôtôsi, sikamo Hayasa ga itteini naru.

Ita wo Nagare no Hôkô ni nanameni oita Baai.

Kono Baai no Nagare wa Du 12 ni simesu tôri de aru.



Du 12. z-Men.

Ita no Sitagawa no Hazi B de Nagare ga issyoni naru yôni Zyunkwan wo sadameru. Sore de aru kara, Hukuso-*Potential*-Kansû wa mae no Mondai no Zyunkwan no nai Baai no f-Kansû to Zyunkwan ni yoru Kansû $f_1=i\Gamma\log Z$ to wo kuwaeta mono de aru.

Sunawati s-Men dewa

$$f=\frac{\psi_1+\psi_2}{\pi}\left\{\left[\zeta(\mu+\nu)-\zeta(\mu-\nu)\right](s-\mu)-\log\frac{\sigma(s+\nu)\,\sigma(\mu-\nu)}{\sigma(s-\nu)\,\sigma(\mu+\nu)}\right\},$$

$$f_1=\Gamma\frac{\pi}{\omega_1}(s-\omega_1-\omega_3)$$

to naru, kodo de μ, ν, Γ wa maeno Baai to onazi Imi wo motu.

$\therefore$ $w=f+f_1$ wa motomeru Hukuso-*Potential*-Kansû de aru.

Yueni

$$\frac{dw}{ds}=\frac{\psi_1+\psi_2}{\pi}\cdot\frac{\wp'\nu}{\wp\nu-\wp\mu}\cdot\frac{\wp s-\wp\mu}{\wp s-\wp\nu}+\frac{\pi\Gamma}{\omega_1}.$$

s_1 wo B ni sôtôsuru Ten to sureba, s_1 de $\frac{dw}{ds}=O$ de aru kara,

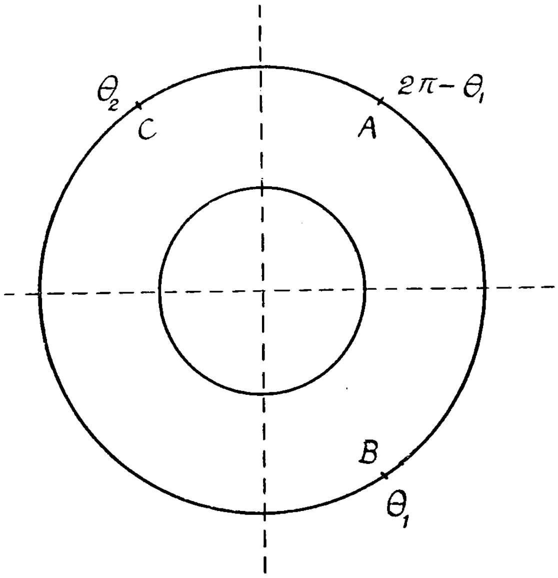
$$\frac{\pi\Gamma}{\omega_1}=-\frac{\psi_1+\psi_2}{\pi}\cdot\frac{\wp'\nu}{\wp\nu-\wp\mu}\cdot\frac{\wp s_1-\wp\mu}{\wp s_1-\wp\nu}$$

de Γ ga sadamaru. Kono Γ no Atai wo tukauto $\frac{dw}{ds}$ wa tugino yôni kakikaerareru.

$$\frac{dw}{ds}=-\frac{\psi_1+\psi_2}{\pi}\cdot\frac{\sigma(2\nu)}{\sigma(s_1+\nu)\sigma(s_1-\nu)}\cdot\frac{\sigma(s+s_1)\sigma(s-s_1)}{\sigma(s+\nu)\sigma(s-\nu)}$$

Yueni $\frac{dw}{ds}$ wa $s=-s_1+2\omega_3$ demo O ni naru. Kono Ten wa A ni sôtô suru.

Tugini Ω (Z) wo keisansuru.



Z-Men ni citewa

A wa $e^{i(2\pi-\theta_1)}$ ni

B wa $e^{i\theta_1}$ ni

C wa $e^{i\theta_2}$ ni sôtôsuru.

Du 13. Z-Men.

$$\Omega(Z) = -\frac{i}{\pi}\Bigg[\Big(-\frac{\pi}{2}+\delta\Big)\log(-1)$$

$$+\pi\log\frac{\sigma\Big[\Big(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta_2\Big)\Big|\omega_1,\ 2\omega_3\Big]\sigma_3\Big[\Big(\frac{\omega_1}{i\pi}\log Z+\frac{\omega_1}{\pi}\theta_1-2\omega_1\Big)\Big|\omega_1,\ 2\omega_3\Big]}{\sigma_3\Big[\Big(\frac{\omega_1}{i\pi}\log Z-\frac{\omega_1}{\pi}\theta_2\Big)\Big|\omega_1,\ 2\omega_3\Big]\sigma\Big[\Big(\frac{\omega_1}{i\pi}\log Z+\frac{\omega_1}{\pi}\theta_1-2\omega_1\Big)\Big|\omega_1,\ 2\omega_3\Big]}\Bigg].$$

Mata

$$\frac{\omega_1}{i\pi}\log Z=\omega_1+\omega_3-s$$

$$\frac{\omega_1}{\pi}\theta_2=\omega_1+\omega_3-s_2$$

de aru kara, s-Men dewa

$$\Omega(s) = -\frac{i}{\pi}\Bigg[\Big(-\frac{\pi}{2}+\delta\Big)\log(-1)$$

$$+\pi\log\frac{\sigma[(s-s_2)\,|\,\omega_1,\ 2\omega_3]\,\sigma_3[(s+s_1-2\omega_3)\,|\,\omega_1,\ 2\omega_3]}{\sigma_3[(s-s_2)\,|\,\omega_1,\ 2\omega_3]\,\sigma\,(s+s_1-2\omega_3)\,|\,\omega_1,\ 2\omega_3]}\Bigg].$$

$$\therefore \quad e^{i\Omega(s)}=e^{i\left(-\frac{\pi}{2}+\delta\right)}\times\frac{\sigma[(s-s_2)\mid\omega_1,2\omega_3]\,\sigma_3[(s+s_1-2\omega_3)\mid\omega_1,2\omega_3]}{\sigma_3[(s-s_2)\mid\omega_1,2\omega_3]\,\sigma[(s+s_1-2\omega_3)\mid\omega_1,2\omega_3]}$$

$$=e^{i\left(-\frac{\pi}{2}+\delta\right)}\times\frac{1}{e_3-e_2}\cdot\frac{[\sigma_2(s-s_2)-\sigma_3(s-s_2)][\sigma_2(s+s_1)-\sigma_3(s+s_1)]}{\sigma(s-s_2)\,\sigma(s+s_1)}$$

Mata z-Men de Nagare no Kamino mugenni tôi Tokoro dewa x-Diku no seino Hôkô ni ittisi, simono mugenni tôi Tokoro dewa x-Diku no seino Hôkô to $-\alpha$ no Kaku wo nasu mono to sureba,

$\frac{dw}{dz}=e^{-i\Omega}$ de aru kara,

$s=\nu$ ni oitewa $\Omega(s)=0$,

$s=-\nu$ ni oitewa $\Omega(s)=-\alpha$.

$\therefore \quad s_1=x_1+\omega_3, \qquad s_2=x_2+\omega_3$ to sureba,

$$\log\left[\frac{\sigma_1(x_2-\nu)\sigma_1(x_1+\nu)-(e_1-e_3)\sigma(x_2-\nu)\sigma(x_1+\nu)}{\sigma_3(x_2-\nu)\sigma_3(x_1+\nu)}\right.$$
$$\left.-i\sqrt{e_1-e_3}\,\frac{\sigma_1(x_2-\nu)\sigma(x_1+\nu)+\sigma(x_2-\nu)\sigma_1(x_1+\nu)}{\sigma_3(x_2-\nu)\sigma_3(x_1+\nu)}\right]+i\left(-\frac{\pi}{2}+\delta\right)=0 \qquad (9)$$

oyobi

$$\log\left[\frac{\sigma_1(x_2+\nu)\sigma_1(x_1-\nu)-(e_1-e_3)\sigma(x_2+\nu)\sigma(x_1-\nu)}{\sigma_3(x_2+\nu)\sigma_3(x_1-\nu)}\right.$$
$$\left.-i\sqrt{e_1-e_3}\,\frac{\sigma_1(x_2+\nu)\sigma(x_1-\nu)+\sigma(x_2+\nu)\sigma_1(x_1-\nu)}{\sigma_3(x_2+\nu)\sigma_3(x_1-\nu)}\right]+i\left(-\frac{\pi}{2}+\delta\right)=-i\alpha. \qquad (10)$$

$$\therefore \quad \frac{\sigma_1(x_2-\nu)\sigma_1(x_1+\nu)-(e_1-e_3)\sigma(x_2-\nu)\sigma(x_1+\nu)}{\sigma_3(x_2-\nu)\sigma_3(x_1+\nu)}=\cos\left(\frac{\pi}{2}-\delta\right)$$

$$\sqrt{e_1-e_3}\,\frac{\sigma_1(x_2-\nu)\sigma(x_1+\nu)+\sigma(x_2-\nu)\sigma_1(x_1+\nu)}{\sigma_3(x_2-\nu)\sigma_3(x_1+\nu)}=-\sin\left(\frac{\pi}{2}-\delta\right)$$

$$\frac{\sigma_1(x_2+\nu)\sigma_1(x_1-\nu)-(e_1-e_3)\sigma(x_2+\nu)\sigma(x_1-\nu)}{\sigma_3(x_2+\nu)\sigma_3(x_1-\nu)}=\cos\left(\frac{\pi}{2}-\delta-\alpha\right)$$

$$\sqrt{e_1-e_3}\,\frac{\sigma_1(x_2+\nu)\sigma(x_1-\nu)+\sigma(x_2+\nu)\sigma_1(x_1-\nu)}{\sigma_3(x_2+\nu)\sigma_3(x_1-\nu)}=-\sin\left(\frac{\pi}{2}-\delta-\alpha\right)$$

to naru.

Tugini z no Keisan wo suru.

$$dz=e^{i\Omega}dw$$

$$=\frac{\psi_1+\psi_2}{\pi}e^{i\left(-\frac{\pi}{2}+\delta\right)}\frac{\sigma(2\nu)}{\sigma(s_1+\nu)\sigma(s_1-\nu)}\cdot\frac{1}{e_3-e_2}$$

$$\times\frac{\sigma(s-s_1)}{\sigma(s+\nu)\sigma(s-\nu)}\cdot\frac{\left[\sigma_2(s-s_2)-\sigma_3(s-s_2)\right]\left[\sigma_2(s+s_1)-\sigma_3(s+s_1)\right]}{\sigma(s-s_2)}ds.$$

Ima

$$F_1(s)=\frac{\sigma(s-s_1)}{\sigma(s+\nu)\sigma(s-\nu)}\cdot\frac{\sigma_2(s-s_2)\sigma_2(s+s_1)+\sigma_3(s-s_2)\sigma_3(s+s_1)}{\sigma(s-s_2)},$$

$$F_2(s)=\frac{\sigma(s-s_1)}{\sigma(s+\nu)\sigma(s-\nu)}\cdot\frac{\sigma_2(s-s_2)\sigma_3(s+s_1)+\sigma_3(s-s_2)\sigma_2(s+s_1)}{\sigma(s-s_2)}$$

to sureba, $F_1(s)$ wa $2\omega_1$, $2\omega_3$ no Syûki wo motu Daen-kansû de aru. Mata

$$F_2(s+2\omega_1)=F_2(s),$$

$$F_2(s+2\omega_3)=-F_2(s)$$

de aru kara, $F_2(s)$ wa dai-ni-syuno Daen-kansû de aru. Sorede aru kara, $F_1(s)$, $F_2(s)$ wa tugino yôni kantanna Kansû ni wakeru koto ga dekiru.

Sunawati,

$$F_1(s)=C_0+C_1\zeta(s+\nu)+C_2\zeta(s-\nu)+C_3\zeta(s-s_2)$$

$$F_2(s)=B_1\xi_{10}(s+\nu)+B_2\xi_{10}(s-\nu)+B_3\xi_{10}(s-s_2),$$

koko de $\xi_{10}(s)=\frac{\sigma_1(s)}{\sigma(s)}$ de aru. Mata

$$C_1=\operatorname*{Res}_{s=-\nu}F_1(s)=-\frac{\sigma(s_1+\nu)}{\sigma(2\nu)}\cdot\frac{\sigma_2(s_2+\nu)s_2(s_1-\nu)+\sigma_3(s_2+\nu)\sigma_3(s_1-\nu)}{\sigma(s_2+\nu)},$$

$$C_2=\operatorname*{Res}_{s=\nu}F_1(s)=\frac{\sigma(s_1-\nu)}{\sigma(2\nu)}\cdot\frac{\sigma_2(s_2-\nu)\sigma_2(s_1+\nu)+\sigma_3(s_2-\nu)\sigma_3(s_1+\nu)}{\sigma(s_2-\nu)},$$

$$C_3=\operatorname*{Res}_{s=s_2} F_1(s)=-\frac{\sigma(s_1-s_2)}{\sigma(s_2+\nu)\sigma(s_2-\nu)}\left\{\sigma_2(s_1+s_2)+\sigma_3(s_1+s_2)\right\},$$

$$B_1=\operatorname*{Res}_{s=-\nu} F_2(s)=-\frac{\sigma(s_1+\nu)}{\sigma(2\nu)}\cdot\frac{\sigma_2(s_2+\nu)\sigma_3(s_1-\nu)+\sigma_3(s_2+\nu)\sigma_2(s_1-\nu)}{\sigma(s_2+\nu)},$$

$$B_2=\operatorname*{Res}_{s=\nu} F_2(s)=\frac{\sigma(s_1-\nu)}{\sigma(2\nu)}\cdot\frac{\sigma_2(s_2-\nu)\sigma_3(s_1+\nu)+\sigma_3(s_2-\nu)\sigma_2(s_1+\nu)}{\sigma(s_2-\nu)},$$

$$B_3=\operatorname*{Res}_{s=s_2} F_2(s)=C_3,$$

$$C_0=-C_1\zeta(s_1+\nu)-C_2\zeta(s_1-\nu)-C_3\zeta(s_1-s_2).$$

$$\therefore\quad z=e^{i\left(-\frac{\pi}{2}+\delta\right)}\cdot\frac{\psi_1+\psi_2}{\pi}\cdot\frac{\sigma(2\nu)}{\sigma(s_1+\nu)\sigma(s_1-\nu)}\cdot\frac{1}{e_3-e_2}$$

$$\times\Big\{C_0s+C_1\log\sigma(s+\nu)+C_2\log\sigma(s-\nu)+C_3\log\sigma(s-s_2)$$

$$-B_1\log\left[\xi_{30}(s+\nu)-\xi_{20}(s+\nu)\right]-B_2\log\left[\xi_{30}(s-\nu)-\xi_{20}(s-\nu)\right]$$

$$-B_3\log\left[\xi_{30}(s-s_2)-\xi_{20}(s-s_2)\right]\Big\},\qquad (10)$$

koko de $\xi_{\alpha 0}(s)=\frac{\sigma_\alpha(s)}{\sigma(s)}$ de aru.

z wa Ita wo hitomawarisitemo onazi Atai wo motu koto ga hituyô de aru kara, $2\omega_1$ no Syûki wo motu Hituyô ga aru. Sunawati

$$C_0 2\omega_1+2\eta_1[\nu C_1-\nu C_2-s_2C_3]+i\pi(B_1+B_2+B_3)=0$$

Kore no kyono Bubun wa

$$-2\eta_1\omega_3(C_1+C_2)-2\eta_1\omega_3C_3+i\pi B_3=0$$

$$\therefore\quad 2\omega_1\eta_3-2\eta_1\omega_3+i\pi=0$$

Kore wa tuneni hitosii. Mata zituno Bubun wa

$$C_1\left[\zeta_3(x_1+\nu)-\frac{\eta_1(x_1+\nu)}{\omega_1}\right]+C_2\left[\zeta_3(x_1-\nu)-\frac{\eta_1(x_1-\nu)}{\omega_1}\right]$$

$$+C_3\left[\zeta(x_1-x_2)-\frac{\eta_1(x_1-x_2)}{\omega_1}\right]=\frac{i\pi}{2\omega_1}(B_1+B_2).\qquad (11)$$

Sunawati

$$-\cos\left(\frac{\pi}{2}-\delta-\alpha\right)\left[\zeta_3(x_1+\nu)-\frac{\eta_1(x_1+\nu)}{\omega_1}\right]+\cos\left(\frac{\pi}{2}-\delta\right)\left[\zeta_3(x_1-\nu)-\frac{\eta_1(x_1-\nu)}{\omega_1}\right]$$

$$+\left[\cos\left(\frac{\pi}{2}-\delta-\alpha\right)-\cos\left(\frac{\pi}{2}-\delta\right)\right]\left[\zeta(x_1-x_2)-\frac{\eta_1(x_1-x_2)}{\omega_1}\right]$$

$$=\frac{\pi}{2\omega_1}\left[\sin\left(\frac{\pi}{2}-\delta\right)-\sin\left(\frac{\pi}{2}-\delta-\alpha\right)\right] \qquad (12)$$

naru Dyôken ga hituyô de aru.

Soreyueni (9) oyobi (11) kara x_1 oyobi x_2 wo kimeru koto ga dekiru. Zissaino Keisan niwa (9), (10) wo tugino yôni kakinaosi

$$\frac{\pi}{2}-\delta+\tan^{-1}\frac{\sqrt{e_1-e_3}\,\sigma(x_2-\nu)}{\sigma_1(x_2-\nu)}+\tan^{-1}\frac{\sqrt{e_1-e_3}\,\sigma(x_1+\nu)}{\sigma_1(x_1+\nu)}=0$$

oyobi

$$\frac{\pi}{2}-\delta-\alpha+\tan^{-1}\frac{\sqrt{e_1-e_3}\,\sigma(x_2-\nu)}{\sigma_1(x_2-\nu)}+\tan^{-1}\frac{\sqrt{e_1-e_3}\,\sigma(x_1-\nu)}{\sigma_1(x_1-\nu)}=0,$$

korerano Siki to (12) to kara x_1, x_2 oyobi α wo motomeru hô ga kantan de aru.

Ita no Haba wo b to sureba, b wa $\int_B^C dz$ no Zettaiti de aru kara,

$$b=\frac{\psi_1+\psi_2}{\pi}\Bigg\{(x_1-x_2)\left[\cos\left(\frac{\pi}{2}-\delta-\alpha\right)\zeta_3(x_1+\nu)-\cos\left(\frac{\pi}{2}-\delta\right)\zeta_3(x_1-\nu)\right.$$

$$\left.+\left(\cos\left(\frac{\pi}{2}-\delta\right)-\cos\left(\frac{\pi}{2}-\delta-\alpha\right)\right)\zeta(x_1-x_2)\right]$$

$$-\cos\left(\frac{\pi}{2}-\delta-\alpha\right)\log\frac{\sigma_3(x_1+\nu)}{\sigma_3(x_2+\nu)}+\cos\left(\frac{\pi}{2}-\delta\right)\log\frac{\sigma_3(x_1-\nu)}{\sigma_3(x_2-\nu)}$$

$$+2\left[\cos\left(\frac{\pi}{2}-\delta-\alpha\right)-\cos\left(\frac{\pi}{2}-\delta\right)\right]\log\sigma_2\left(\frac{x_1-x_2}{2}\right)\sigma_3\left(\frac{x_1-x_2}{2}\right)$$

$$-\sin\left(\frac{\pi}{2}-\delta-\alpha\right)\left[\tan^{-1}\frac{\sqrt{e_1-e_3}\,\sigma(x_1+\nu)}{\sigma_1(x_1+\nu)}-\tan^{-1}\frac{\sqrt{e_1-e_3}\,\sigma(x_2+\nu)}{\sigma_1(x_2+\nu)}\right]$$

$$+\sin\left(\frac{\pi}{2}-\delta\right)\left[\tan^{-1}\frac{\sqrt{e_1-e_3}\,\sigma(x_1-\nu)}{\sigma_1(x_1-\nu)}-\tan^{-1}\frac{\sqrt{e_1-e_3}\,\sigma(x_2-\nu)}{\sigma_1(x_2-\nu)}\right]\Bigg\}. \qquad (c)$$

Tugini Ita ni hataraku Tikara wo keisansuru. P de kono Tikara wo arawaseba

$$P=-\frac{i\rho}{2}\int\left(\frac{dw}{dz}\right)^2 dz,$$

Sekibun no Miti wa Ita wo hitomawarisuru. Sate

$$\int\left(\frac{dw}{dz}\right)^2 dz=\int e^{-i\Omega}dw.$$

Kono Sekibnn wo s-Men de okonau tokiniwa, kattena Ten s kara $s+2\omega_1$ made $y=0$ oyobi $y=\frac{\omega_3}{i}$ no hutatuno Sen wo yokogiranai yôna Miti wo tôte sekibunsureba yoi.

Sunawati

$$\int e^{-i\Omega}dw=\frac{\psi_1+\psi_2}{\pi}\cdot\frac{\sigma(2\nu)}{\sigma(s_1+\nu)\sigma(s_1-\nu)}e^{i\left(\frac{\pi}{2}-\delta\right)}$$

$$\times\left\{C_0 2\omega_1+2\eta_1(\nu C_1-\nu C_2-s_2C_3)-i\pi\,(B_1+B_2+B_3)\right\}$$

koko de maeni eta

$$C_0 2\omega_1+2\eta_1(\nu C_1-\nu C_2-s_2C_3)+i\pi(B_1+B_2+B_3)=0$$

naru Dyôken wo tukauto,

$$\int e^{-i\Omega}dw=-2i\pi\cdot\frac{\psi_1+\psi_2}{\pi}\cdot\frac{\sigma(2\nu)}{\sigma(s_1+\nu)\,\sigma\,(s_1-\nu)}e^{i\left(\frac{\pi}{2}-\delta\right)}$$

$$\times(B_1+B_2+B_3)$$

$$=2i\pi\frac{\psi_1+\psi_2}{\pi}(1-\cos\alpha+i\sin\alpha).$$

$$\therefore\quad P=\rho(\psi_1+\psi_2)(1-\cos\alpha+i\sin\alpha).$$

Nagare no kamino Hôko de mugenni tôi Tokoro no Nagare no Haba wo d to sureba, $\psi_1+\psi_2=d$ (Hayasa wa *1* de aru) de aru kara

$$\frac{P}{\rho b}=\frac{d}{b}\times(1-\cos\alpha+i\sin\alpha).$$

Sunawati, kono Baai niwa Ita ni hataraku Tikara no Hôkô wa Nagare to $\tan^{-1}\frac{\sin\alpha}{1-\cos\alpha}$ no Kaku wo nasi, Yôryoku to Kôryoku to ni naru. Nebasa ni yoru Teikô no hokani konoyôna Teikô ga hairu koto wa tasikani hurieki de aru.

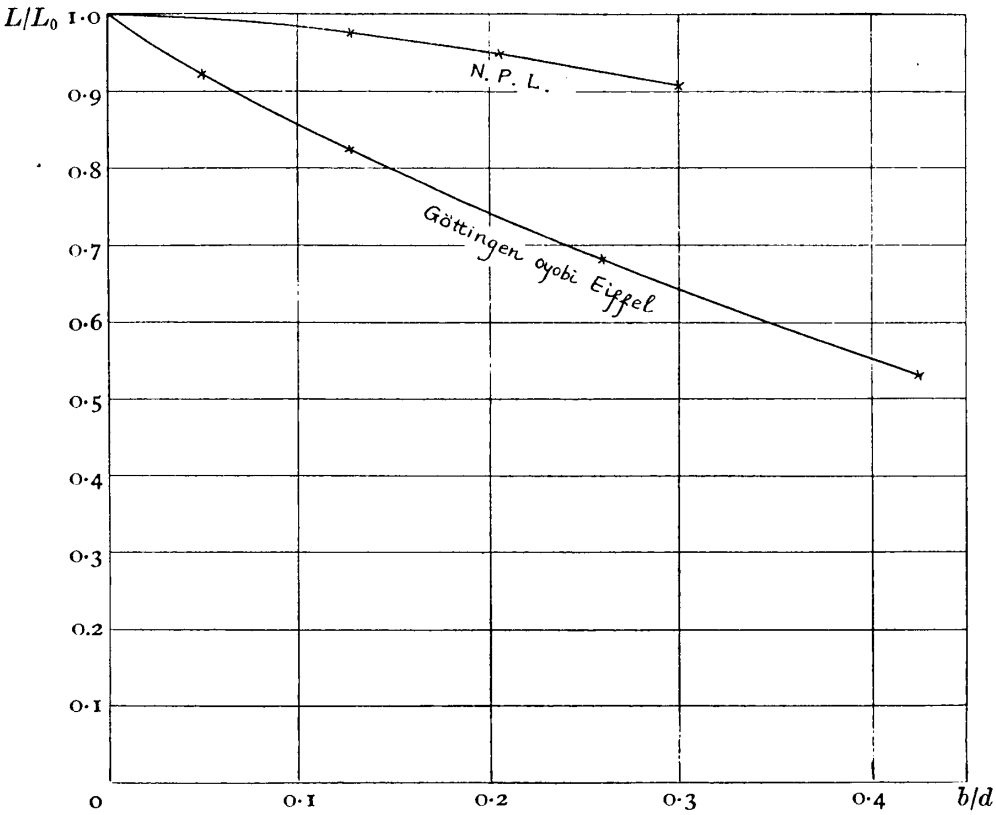
Sûryôtekini Keisan wo okonatte maeni keisansita N.P.L. gatano Hûtô no Baai to hikakusureba tugino yô de aru.

$\psi_1=\psi_2$ naru Baai niwa, $\nu=\frac{\omega_1}{2}$, $\mu=\frac{\omega_1}{2}+\omega_3$ de aru. $\frac{\pi}{2}-\delta=10^\circ$ to sureba tugino Kekkwa wo eru. Koko de L_0 wa mugenni hiroi Nagare no Baai ni okeru Yôryoku L wo arawasu.

Hyô II.

q	$\frac{\pi x_1}{2\omega_1}$	$\frac{\pi x_2}{2\omega_1}$	α	b/d	L/L_0	D/L
0	-50°	40°	0°	0	1	0
0·02	-49°37′	40°22′ 3″	1°28′19″	0·0510	0·9241	0·0128
0·05	-49° 5′	40°48′49″	3°17′45″	0·1277	0·8255	0·0287
0·1	-48°20′	41°22′57″	5°32′ 2″	0·2601	0·6796	0·0483
0·15	-47°42′	41°46′59″	7° 5′16″	0·4253	0·5318	0·0619

Maeni keisansita N.P.L.—gata no Baai no L/L_0 to kono Baai no mono to wo Du ni arawasite hikakusite miruto, Du 14 no tôri de aru. Kore ni yotte miruto, N.P.L.—gatano Hûtô ni yotte eta Zikken no Kekkwa no hô ga Zissai ni tikai Atai wo ataeru koto ga wakaru.



Du 14.

Owarini Tamaru Kyôzyu oyobi Terazawa Kyôzyu ni iroiro yûekina Gotyûkoku wo uketa koto wo hukaku kansyasuru.

(Hûtô no Kabe no Eikyô ni tuite Nisii-Udi ga zikkentyû de aru.)