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HELICOPTERS.

III. Climbing of Helicopter.

By

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By the analogous way as was shown in my previous paper on airscrew at fixed point we can obtain the fundamental equations of most general airscrew.

$$dT=\rho N\Gamma(\omega r-W_t)dr \qquad (1)$$

$$dQ=\rho N\Gamma r(v+W_a)dr \qquad (2)$$

$$dT=2\pi r\rho(v+W_a)W'_a dr \qquad (3)$$

$$dQ=2\pi rr'\rho(v+W_a)W_t'dr \qquad (4)$$

$$r(v+W_a)dr=r'(v+W'_a)dr' \qquad (5)$$

$$2W_t r=W'_t r' \qquad (6)$$

$$\omega dQ=\pi r\rho(v+W_a)(W_a'^2+W_t'^2+2vW'_a)dr \qquad (7)$$

As the velocity of ascent of helicopter can not be very great without very great surplus of power, we will solve these equations under the supposition that

I) v is small compared with W_a, and higher than the second order of v/W_a can be neglected.

2) v and W_a are of same order of magnitude and putting $W_a=v+w_a$ higher than second order of w_a/W_a can be neglected. In the first case from (2) and (4) we have

$$W'_t=\frac{N\Gamma}{2\pi r} \qquad (8)$$

and from (6)

$$W_t=\frac{N\Gamma}{4\pi r} \qquad (5)$$

From (4) and (7)

The kôkûkenkûzyo Zaturoku No. 13 and also No. 10 p 140.

$$2\omega r' W'_t = W'^2_a + W'^2_t + 2v W'_a$$

$$W_a' = \frac{W'_t(2\omega r' - W'_t)}{W_a' + 2v} = \frac{2\omega r' W'_t}{W'_a}\left[1 - \frac{W_t'}{2\omega r'} - \frac{2v}{W_a'}\right]$$

$$\therefore \quad W_a' = \sqrt{2\omega r' W'_t}\left[1 - \frac{W_t'}{4\omega r'} - \frac{v}{W'_a}\right]$$

$$\doteqdot \frac{\sqrt{N\omega\Gamma}}{\sqrt{\pi}}\left[1 - \frac{W'_t}{4\,\omega r'} - \frac{v\sqrt{\pi}}{\sqrt{N\omega\Gamma}}\right]$$

As $W'_t/4\omega r'$ is very small in the most effective portion of the blade, neglecting this we have

$$W_a' = \frac{\sqrt{N\omega\Gamma}}{\sqrt{\pi}} - v \quad \text{......(10)}$$

From (1) and (3)

$$N\Gamma(\omega r - W_t) = 2\pi r(v + W_a) W'_a$$

$$v + W_a = \frac{N\Gamma(\omega r - W_t)}{2\,\pi r W_a'}$$

$$= \frac{\sqrt{N\omega\Gamma}}{2\sqrt{\pi}}\left(1 + \frac{v\sqrt{\pi}}{\sqrt{N\omega\Gamma}}\right)$$

$$\therefore \quad W_a = \frac{1}{2}\left[\frac{\sqrt{N\omega\Gamma}}{\sqrt{\pi}} - v\right] \quad \text{......(11)}$$

In the second case, putting $W'_a = 2(v + w_a)$

We have from (4) and (7)

$$2W'_a(W'_a - w_a) = W_t'(2\omega r' - W_t')$$

$$W'^2_a = \frac{N\omega\Gamma}{2\pi}\left(1 + \frac{w_a}{W_a'}\right)$$

$$\therefore \quad W_a' = \frac{\sqrt{N\omega\Gamma}}{\sqrt{2\pi}}\left(1 + \frac{1}{2}\frac{w_a}{W_a'}\right)$$

$$= \frac{\sqrt{N\omega\Gamma}}{\sqrt{2\pi}}\left[1 + \frac{1}{2}\left(\frac{1}{2} - \frac{v}{W'_a}\right)\right]$$

$$= \frac{\sqrt{N\omega\Gamma}}{\sqrt{2\pi}}\left[\frac{5}{4} - \frac{4}{5}\frac{\sqrt{2\pi}}{\sqrt{N\omega\Gamma}}v\right]$$

$$= \frac{5\sqrt{N\omega\Gamma}}{4\sqrt{2\pi}} - \frac{4}{5}v \quad \text{......(12)}$$

and we have

$$v + W_a = \frac{N\Gamma(\omega r - W_t)}{2\,\pi r' W_a'}$$

$$=\frac{4}{5}\frac{\sqrt{N\omega\Gamma}}{\sqrt{2\pi}}+\frac{64}{125}v$$

$$\therefore\quad W_a=\frac{4}{5}\frac{\sqrt{N\omega\Gamma}}{\sqrt{2\pi}}-\frac{61}{125}v$$

$$=1.13\times\frac{\sqrt{N\omega\Gamma}}{2\sqrt{\pi}}-0.977\times\frac{v}{2}\qquad\qquad(13)$$

Therefore the expressions of thrust and torque are from (1) and (2)

$$dT=\rho N\Gamma\omega r dr\ ^{(1)}\qquad\qquad(14)$$

$$dQ=\frac{\rho N\Gamma r}{2}\left[v+\frac{\sqrt{N\omega\Gamma}}{\sqrt{\pi}}\right]dr\qquad\text{(for 1st case)}\qquad(15)$$

$$dQ=\frac{\rho N\Gamma r}{2}\left[1.023\ v+1.13\frac{\sqrt{N\omega\Gamma}}{\sqrt{\pi}}\right]dr\qquad\text{(for 2nd case.)}\qquad(16)$$

Next we must take into consideration the profil resistance of the blade. The profil resistance has very small influence on thrust and let us neglect this.

The torque necessary to overcome the profil resistance is

$$c_f\frac{1}{2}\rho Nt\omega r\,dr\quad\text{approximately.}\qquad\qquad(17)$$

Therefore we have for 1st case:

$$dT=\rho N\Gamma\omega r dr\qquad\qquad(18)$$

$$dQ=\left[\frac{\rho N\Gamma r}{2}\left(v+\frac{\sqrt{N\omega\Gamma}}{\sqrt{\pi}}\right)d+c_f\frac{1}{2}\rho Nt\omega^2r^3\right]dr\qquad\qquad(19)$$

Let us integrate these expressions supposing Γ=constant.

Then

$$T=\rho N\Gamma\omega\frac{R^2}{2}\qquad\qquad(20)$$

$$Q=\frac{\rho N\Gamma}{2}\frac{R^2}{2}\left(v+\frac{\sqrt{N\omega\Gamma}}{\sqrt{\pi}}\right)+c_f\frac{1}{2}\rho Nt\omega^2\frac{R^4}{4}\qquad\qquad(21)$$

(21) can be written

$$Q=\frac{\rho N\Gamma}{2}\frac{R^2}{2}v+Q_0\qquad\qquad(22)$$

where Q_0=torque when $v=0$ and other conditions are identical. Eliminating Γ between (20) and (22) we have

(1) $\frac{N\Gamma}{4\pi r}$ is neglected compared with ωr.

$$v=\frac{2(P-P_0)}{T} \quad \cdots\cdots (23)$$

where $P=\omega Q$, $P_0=\omega Q_0$.

Similarly in second case we have

$$v=\frac{1.92(P-P_0)}{T} \quad \cdots\cdots (24)$$

Therefore we have the following theorem

velocity of vertical ascent of helicopter is proportional to the excess of power and inversely proportional to its weight.

Thrust T or the weight of helicopter at fixed point and power are connected by the relation

$$T=K(P_0R)^{\frac{2}{3}} \quad \cdots\cdots (25)$$

where $K=(2\pi\rho)^{\frac{1}{3}}$ for optimum airscrew at fixed point when the frictional resistance is neglected.

The constant K is proportional to $\rho^{\frac{1}{3}}$ and the power necessary to sustain the helicopter for different values of ρ is

$$P_0\times\frac{\rho_0}{\rho}$$

as P is proportional to ρ.

Therefore the expression of v at any altitude is

$$v=\frac{2\left(P\times\frac{\rho}{\rho_0}-P_0\times\frac{\rho_0}{\rho}\right)}{T} \quad \cdots\cdots (26)$$

and for the second case

$$v=\frac{1.92\left(P\times\frac{\rho}{\rho_0}-P_0\times\frac{\rho_0}{\rho}\right)}{T} \quad \cdots\cdots (27)$$

Whilst between $\frac{\rho}{\rho_0}$ and altitude we have the following relation[1]

$$\frac{\rho}{\rho_0}=e^{-\frac{h}{9380}} \quad \cdots\cdots (28)$$

Hence v can be written

$$v=\frac{a}{T}\left\{P\,e^{-\frac{h}{9380}}-P_0\,e^{\frac{h}{9380}}\right\} \quad \cdots\cdots (29)$$

where a is 2 under the first assumption and 1.92 under the second assumption.

(1) See Rateau: Théorie des Hélice. p. 109.

The time required to climb to the hieght H is

$$t=\int_0^H \frac{dh}{v}$$

$$=\frac{T}{a}\int_0^H \frac{dh}{Pe^{-\frac{h}{9380}}-P_0e^{\frac{h}{9380}}}$$

Let us expand the expression under $\int$ and take the first term only. Then we have

$$e^{-\frac{h}{9380}}=1-\frac{h}{9380}$$

$$e^{\frac{h}{9380}}=1+\frac{h}{9380}$$

Therefore

$$t=\frac{T}{a}\int_0^H \frac{dh}{P\left(1-\frac{h}{9380}\right)-P_0\left(1+\frac{h}{9380}\right)}$$

$$=\frac{9380\,T}{a\,(P+P_0)}\log\frac{1}{1-\frac{P+P_0}{P-P_0}\cdot\frac{H}{9380}} \quad\cdots\cdots\cdots\cdots(30)$$

Table 1.

Time of climb.

Helicopter	to 500 meters	to 1000 meters
Cornu	29.1 sec.	62.5 sec.
P-K-Z	57.0	148
Oemichen No. 1	142	347
Bothezat	254	—

The height of ceiling can be easily obtained.

At ceiling v is zero i.e.,

$$P\times\frac{\rho}{\rho_0}=P_0\times\frac{\rho_0}{\rho}$$

Substituting $e^{-\frac{h}{9380}}$ for $\frac{\rho}{\rho_0}$ we have

$$e^{-\frac{2h}{9380}}=\frac{P_0}{P}$$

$$h_c=4690\log\frac{P}{P_0} \qquad \text{(31)}$$

where h_c=ceiling in meters,

P_0=power necessary to sustain the helicopter at fixed point at sea level,

P=power of engine at sea level.

In Table 2 the values of ceilling and climbing velocity at sea level of the helicopters in Table 1 are calculated.

To calculate the value P_0 the formula

$$T=K(P_0R)$$

was used taking $K=0.80$.

Table 2.

Ceiling and climbing velocity.

Helicopter	P_0	P	P/P_0	h_c
Cornu	9.1 H.P.	25	2.75	4700
P-K-Z	127	180	1.42	1640
Oemichen	8.35	12.5	1.50	1900
Bothezat	41.5	50	1.20	855

We must remember that these results were obtained supposing that the blade angle was adequately adjusted during manoeuvres. The conclution from the present analysis is most encouraging. A helicopter having the dimension and power of the existing ones will have a tolerable ceiling and climbing velocity when the airscrews are properly designed.

A remarkable feature of helicopter is that the time required to reach a certain height increases abruptly near the ceiling. This property enables a helicopter to reach a certain height sooner than an aeroplane of the same ceiling.