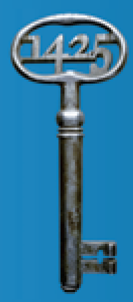
KU LEUVEN



Efficient time- and frequency-domain simulation methods for vibro-acoustics and flow acoustics

Wim Desmet (*)
Noise and Vibration Research Group
KU Leuven – Department of Mechanical Engineering
Leuven, Belgium

(*), Antonio Ammirati, Anna Caiazzo, Hervé Denayer, Wim De Roeck, Maria Muriel Gracia, Stijn Jonckheere, Vyacheslav Korchagin, Bart Vanelderen, Thomas Toulorge,..... and many more

Overview

- Who we are
- Introduction:
  - Flow-induced acoustics and vibrations
  - Importance of virtual (acoustic) tools
- Mathematical formulations:
  - Flow-induced vibrations
  - Flow-induced acoustics
- Numerical solution strategies (@ KU Leuven):
  - for time-domain LEE
  - for frequency-domain vibro-acoustics
- Conclusions

KU LEUVEN 2

Overview

- Who we are
- Introduction:
  - Flow-induced acoustics and vibrations
  - Importance of virtual (acoustic) tools
- Mathematical formulations:
  - Flow-induced vibrations
  - Flow-induced acoustics
- Numerical solution strategies (@ KU Leuven):
  - for time-domain LEE
  - for frequency-domain vibro-acoustics
- Conclusions

KU LEUVEN 3

Who we are

KU LEUVEN 4

Our team

- KU Leuven (1425)
  - Department of Mechanical Engineering
    - Division of Production engineering, Machine design and Automation (PMA)
      - *Noise and Vibration Research Group (MOD)*
- Research staff
  - 5 academic and 1 associated
  - 1 industrial research manager
  - 11 postdoctoral researchers
  - 61 PhD incl. 10 industrial PhD res.

- Areas of research
  - vibro-acoustics
  - aero-acoustics
  - multi-body dynamics
  - smart system dynamics
  - structural reliability & uncertainty

Application domains
- energy and environment
- transport and mobility
- health
- advanced manufacturing

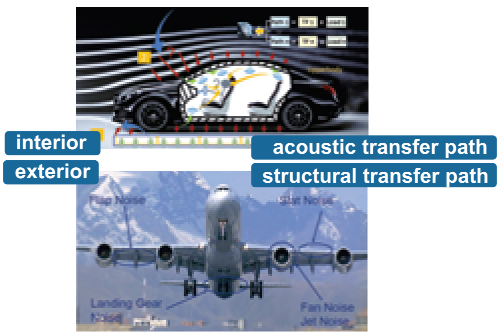
KU LEUVEN 5

Overview

- Who we are
- Introduction:
  - Flow-induced acoustics and vibrations
  - Importance of virtual (acoustic) tools
- Mathematical formulations:
  - Flow-induced vibrations
  - Flow-induced acoustics
- Numerical solution strategies (@ KU Leuven):
  - for time-domain LEE
  - for frequency-domain vibro-acoustics
- Conclusions

KU LEUVEN 6

Flow-induced acoustics and vibrations



KU LEUVEN 7

Importance of virtual (acoustic) tools

Modern product design:

- Ever expanding and often conflicting design criteria
- Shortening of the design cycle
- Acoustic performance: growing importance
  - Customer demands
  - Legal regulations

Numerical prediction techniques (virtual prototyping)
(+) Faster and cheaper than real prototype testing
(+) Sensitivity analyses
(+) Acoustic evaluation at all stages of design

KU LEUVEN 8

Overview

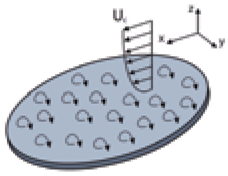
- Who we are
- Introduction:
  - Flow-induced acoustics and vibrations
  - Importance of virtual (acoustic) tools
- Mathematical formulations:
  - Flow-induced vibrations
  - Flow-induced acoustics
- Numerical solution strategies (@ KU Leuven):
  - for time-domain LEE
  - for frequency-domain vibro-acoustics
- Conclusions

KU LEUVEN

Mathematical formulation

Flow-induced vibrations

- Aerodynamic excitation:
  - Unsteady flow
    - Stationary
    - Homogeneous
    - Fully developed
    - Zero mean pressure gradient
- Structural vibrations:
  - Out-of plane bending



→ Weak (one-way) coupling between aerodynamic field and elastic vibrations in the structure

KU LEUVEN

Mathematical formulation

Flow-induced vibrations

- Procedure (weak coupling)

Identification of Turbulent Boundary Layer (TBL) wall pressure from CFD

↓

Structural dynamic calculation using TBL loading function

→ Response PSD from TBL wavenumber-frequency PSD and wave admittance

$$S_{vv}(\mathbf{r},\omega) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{PP}(k_x,k_y,\omega)\,|H_v(\mathbf{r},k_x,k_y,\omega)|^2\,dk_x dk_y,$$

$$\approx \frac{1}{4\pi^2}\sum_{k_1}\sum_{k_2} S_{PP}(k_{\Gamma,x},k_{\Gamma,y},\omega)\,|H_v(\mathbf{r},k_{\Gamma,x},k_{\Gamma,y},\omega)|^2\,\Delta_{Nyq,x}\Delta_{Nyq,y}.$$

S_{PP}: Corcos, Chase, Smol'yakov-Tkachenko, ...

KU LEUVEN

Overview

- Who we are
- Introduction:
  - Flow-induced acoustics and vibrations
  - Importance of virtual (acoustic) tools
- Mathematical formulations:
  - Flow-induced vibrations
  - Flow-induced acoustics
- Numerical solution strategies (@ KU Leuven):
  - for time-domain LEE
  - for frequency-domain vibro-acoustics
- Conclusions

KU LEUVEN

Mathematical formulation

Flow-induced acoustics

- Aerodynamic noise generation: unsteady flow features
- Acoustic waves: time-harmonic compressibility waves

governed by the same type of equations: Navier-Stokes equations

KU LEUVEN

Mathematical formulation

Flow-induced acoustics

Compressible Navier-Stokes equations:

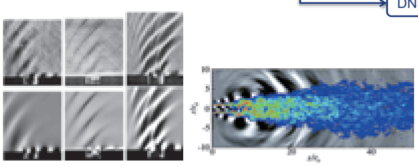
- Description of:
  - fluid motion
  - heat transport
  - compressibility effects: a.o. acoustic wave propagation
  - viscous effects
- All flow-acoustic interaction phenomena
  - aerodynamic noise generation
  - convective noise propagation
- Non-linear acoustic propagation effects
- Visco-thermal effects

KU LEUVEN

Direct Noise Computation (DNC)

- Use scale-resolved CFD techniques to solve the Navier-Stokes equations directly

Navier-Stokes equations → DNS / LES → Sound generation and propagation



KU LEUVEN

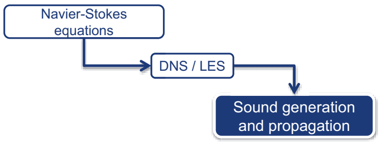
Scale discrepancies in CAA

	Aerodynamic field	Acoustic field
Smallest length scale	Turbulent structures: $l_\eta \approx D\,\mathrm{Re}_D^{-3/4}$ (~1µm)	Acoustic wavelength: $\lambda = c/f$ (~1 cm)
Propagation speed	Convected with the mean flow velocity U_0 ($M = U_0/c_0$~0,3)	Propagating at the speed of sound: $U_0 + c_0$ ($c_0 \approx 340$ m/s)
Propagation distance	Short (~cm)	Long (~km)
Energy level	p': 100-1000 Pa v': 1.0 m/s	p': 0.2 Pa (80dB) v': 5e^{-4} m/s (80 dB)

!! low Mach, high(er) Reynolds

KU LEUVEN

Direct Noise Computation (DNC)



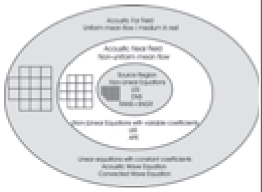
- Most restrictive requirements of aerodynamic and acoustic variables:
  - Fine grid (turbulent length scales)
  - Large grid (acoustic propagation distance)
  - Numerical scheme with low dissipation and dispersion error
  - …

extremely demanding, not applicable for industrial applications

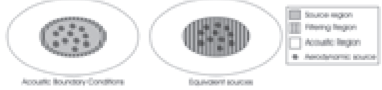
KU LEUVEN

Hybrid methodologies

- Domain decomposition:
  - Source region: noise generation
  - Acoustic domain: noise propagation



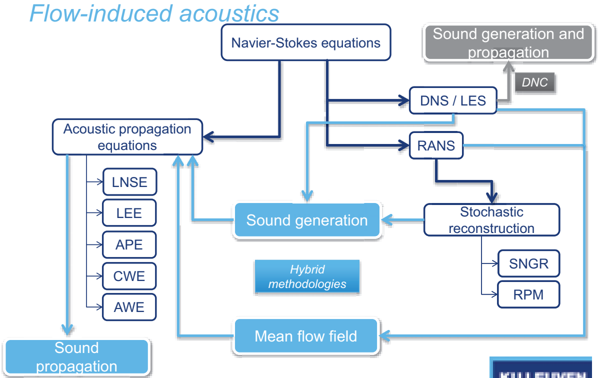
  - Coupling technique:
    - Equivalent sources (Acoustic analogies)
    - Boundary conditions (Kirchhoff, Ffowcs Williams-Hawkings)



KU LEUVEN

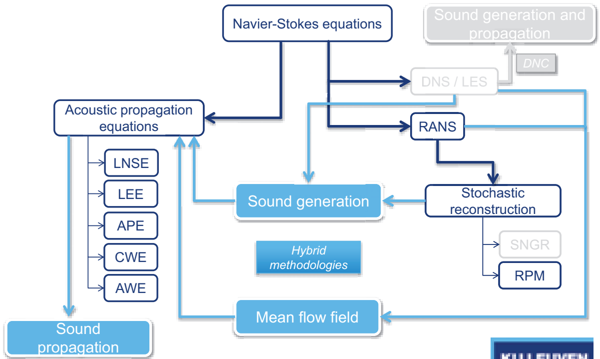
Mathematical formulation

Flow-induced acoustics



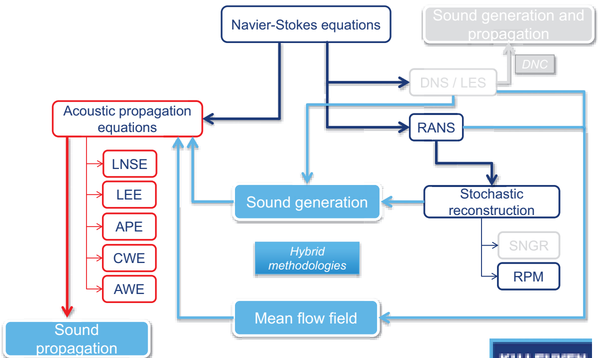
KU LEUVEN

KU Leuven research activities



KU LEUVEN

KU Leuven research activities

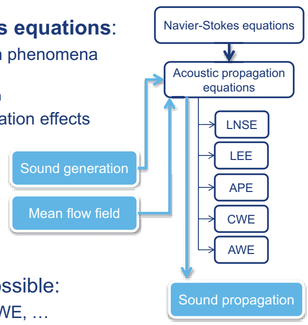


KU LEUVEN

Acoustic propagation equations

Compressible Navier-Stokes equations:

- All flow-acoustic interaction phenomena
  - aerodynamic noise generation
  - **convective noise propagation**
- non-linear acoustic propagation effects
- visco-thermal effects
- …



- Often simplifications are possible:
  - LNSE, LEE, APE, CWE, AWE, …

KU LEUVEN

Acoustic propagation equations

Linearized Navier-Stokes Equations (LNSE)

Assumptions:

- variable decomposition is possible and mean flow field is known (from e.g. RANS-simulation)

$$\rho(\vec{x},t) = \rho'(\vec{x},t) + \rho_0(\vec{x},t)$$
$$\vec{u}(\vec{x},t) = \vec{u}'(\vec{x},t) + \vec{U}_0(\vec{x},t)$$
$$p(\vec{x},t) = p'(\vec{x},t) + P_0(\vec{x},t)$$

- non-linear effects can be neglected: linearization of the equations
- the acoustic waves do not influence the mean flow field: one-way interaction
- viscous effects are considered small enough to assume isentropic flow

KU LEUVEN

Acoustic propagation equations

(isentropic) Linearized Navier-Stokes Equations (LNSE):

- Continuity:

$$\frac{\partial \rho'}{\partial t} + \frac{\partial(\rho_0 u'_r + \rho' u_{0r})}{\partial x_r} = 0$$

- Momentum:

$$\frac{\partial \rho_0 u'_s}{\partial t} + \frac{\partial \rho_0 u'_s u_{0r}}{\partial x_r} + \frac{\partial p'}{\partial x_s} + (\rho_0 u'_r + \rho' u_{0r})\frac{\partial u_{0s}}{\partial x_r} - \frac{\partial \tau'_{sr}}{\partial x_r} = 0$$

$$\tau'_{sr} = \mu\left(\frac{\partial u'_s}{\partial x_r} + \frac{\partial u'_r}{\partial x_s} - \frac{2}{3}\frac{\partial u'_k}{\partial x_k}\delta_{rs}\right)$$

- Energy: isentropic flow

$$\frac{\partial p'}{\partial \rho'} = c_0^2 = \frac{\gamma p_0}{\rho_0}$$

KU LEUVEN

Acoustic propagation equations

Linearized Euler Equations (LEE):

additional assumption: viscous effects negligible

- Einstein notation:

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho_0 u_i' + \rho' U_{i0}\right) = \Phi_{cont}$$

$$\frac{\partial}{\partial t}\left(\rho_0 u_i'\right) + \frac{\partial}{\partial x_j}\left(\rho_0 u_i' U_{j0}\right) + \frac{\partial p'}{\partial x_i} + \left(\rho_0 u_j' + \rho' U_{j0}\right)\frac{\partial U_{i0}}{\partial x_j} = \Phi_{mom,i}$$

$$\frac{\partial p'}{\partial t} + \frac{\partial}{\partial x_i}\left(\gamma P_0 u_i' + p' U_{i0}\right) + (\gamma - 1) p' \frac{\partial U_{i0}}{\partial x_i} - (\gamma - 1) u_i' \frac{\partial P_0}{\partial x_i} = \Phi_{ener}$$

- Vector notation: $\frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{E}}{\partial x} + \frac{\partial \vec{F}}{\partial y} + \frac{\partial \vec{G}}{\partial z} + \vec{H} = \vec{S}$
- Matrix notation: $\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{A}_i \boldsymbol{U}}{\partial x_i} + \boldsymbol{C}\boldsymbol{U} = \boldsymbol{S}$

KU LEUVEN

Acoustic propagation equations

avoiding unstable solutions of the LEE (no viscous damping – vorticity and entropy modes):

- Neglecting terms containing mean flow gradients
  = mean flow gradient suppression
- Alternative formulations based on the assumption of an irrotational and isentropic acoustic field = APE

$$\frac{\partial p'}{\partial t} + c_0^2 \nabla \cdot \left(\rho_0 \vec{u}' + \vec{U}_0 \frac{p'}{c_0^2}\right) = c_0^2 \Phi_{cont}$$

$$\frac{\partial \vec{u}'}{\partial t} + \nabla\left(\vec{U}_0 \cdot \vec{u}'\right) + \nabla\left(\frac{p'}{\rho_0}\right) = \frac{\vec{\Phi}_{mom}}{\rho_0}$$

KU LEUVEN

Acoustic propagation equations

Convective Wave Equation (CWE):

additional assumption: uniform mean flow

$$\frac{D^2}{Dt^2}p' - c_0^2 \nabla^2 p' = c_0^2 \frac{D}{Dt}\Phi_{cont} - c_0^2 \nabla \cdot \vec{\Phi}_{mom}$$

$$\frac{\partial^2}{\partial t^2}p' + \underbrace{\frac{\partial}{\partial t}\left(2(\vec{U}_0 \cdot \nabla)p'\right)}_{I} - \underbrace{c_0^2\left(1 - M_0^2\right)}_{II}\nabla^2 p' = \Phi_{ac}$$

where:

I = convective amplification/dissipation

II = shift in wavenumber with factor $(1\text{-}M_0)$ or $(1+M_0)$

KU LEUVEN

Acoustic propagation equations

Linear acoustic wave equation:

additional assumption: quiescent medium

Acoustic Wave Equation (AWE):

$$\frac{1}{c_0^2}\frac{\partial^2 p'(\vec{x},t)}{\partial t^2} - \nabla^2 p'(\vec{x},t) = \Phi_{ac}(\vec{x},t)$$

KU LEUVEN

Acoustic propagation equations

Comparison of propagation equations

	NS	Linearized NS	Euler	LEE	CWE	AWE
non-linear effects	(green)	(red)	(green)	(red)	(red)	(red)
viscous effects	(green)	(green)	(red)	(red)	(red)	(red)
type of mean flow	any	any	any	any	uniform	rest
convective: refraction	(green)	(green)	(green)	(green)	(red)	(red)
convective: amplification	(green)	(green)	(green)	(green)	(green)	(red)
convective: wavenumber shift	(green)	(green)	(green)	(green)	(green)	(red)
convective: directivity change	(green)	(green)	(green)	(green)	(green)	(red)

KU LEUVEN

Acoustic propagation equations

Simulation of acoustic propagation:

- *equations defined in time-domain:*
  - natural description of source mechanisms
  - convenient interfacing with source information from unsteady CFD
  - efficient formulation for broadband and transient phenomena
- *assuming time-harmonic behaviour: frequency domain*
  - faster
  - only steady-state behaviour

KU LEUVEN

Acoustic propagation equations
time domain vs. frequency domain

- small perturbations
- no viscosity
- homogeneous
  (no flow, no (T,v) gradients)

- speed of sound c
- fluid density ρ_0
- source distribution q

$$\nabla^2 p(r,t) - \frac{1}{c^2}\frac{\partial^2 p(r,t)}{\partial t^2} = -\rho_0 \frac{\partial q(r,t)}{\partial t}$$

Acoustic Wave Equation — **time domain**

$$\nabla^2 p(r) + k^2 p(r) = -j\rho_0 \omega q(r)$$

frequency domain — Helmholtz equation

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$c = \lambda . f$$

assuming steady-state response for time-harmonic excitation:

$$p(x,y,z,t) = \mathrm{Re}\{p(x,y,z)e^{j\omega t}\}$$

KU LEUVEN

Overview

- Who we are
- Introduction:
  - Flow-induced acoustics and vibrations
  - Importance of virtual (acoustic) tools
- Mathematical formulations:
  - Flow-induced vibrations
  - Flow-induced acoustics
- Numerical solution strategies (@ KU Leuven):
  - for time-domain LEE
  - for frequency-domain vibro-acoustics
- Conclusions

KU LEUVEN

Modeling tools for the LEE

Overview spatial discretization:

Structured Grids	Unstructured Grids
Finite Difference Methods	Finite Volume Methods
Spectral Methods	Continuous Finite Element Methods
Pseudo-Spectral Methods	Discontinuous Finite Element Methods
	Residual Distribution Methods

KU LEUVEN

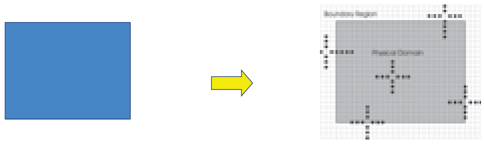
33

Modeling tools for the LEE

Finite differences

Basic principle:

domain discretization



$$\frac{\partial U}{\partial t}+\frac{\partial A_i U}{\partial x_i}+CU=S$$

+ boundary conditions

$$\frac{\partial f(x)}{\partial x}\approx\frac{1}{\Delta x}\sum_{j=-N}^{N}a_j f(x+j\Delta x)$$

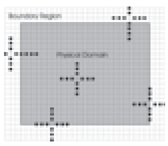
Add artificial selective damping to avoid 'unphysical' and 'unstable' high-frequency oscillations.

KU LEUVEN

34

Modeling tools for the LEE

Finite differences



$$\frac{\partial f(x)}{\partial x}\approx\frac{1}{\Delta x}\sum_{j=-N}^{N}a_j f(x+j\Delta x)$$

some key properties

+ straightforward implementation
- difficult BCs on large stencils
- difficult to change integration scheme
- low to moderate geometrical complexities
- stability restrictions on time step

KU LEUVEN

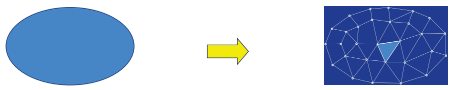
35

Modeling tools for the LEE

Discontinuous Galerkin

Basic principle:

domain discretization



$$\frac{\partial U}{\partial t}+\frac{\partial A_i U}{\partial x_i}+CU=S$$

+ boundary conditions

$$U_i^e(\vec{r},t)=\sum_{k=1}^{N(p,d)}b_k(\vec{r})U_{i,k}^e(t)$$

$$\frac{\partial U_i^e}{\partial t}=(M^{-1}A)\cdot F-\sum_f (M^{-1}B_f)F_n^{Rn}$$

KU LEUVEN

36

Modeling tools for the LEE

Discontinuous Galerkin

Galerkin approach:

- Multiplying LEE with base function b_k
- Integrating over each element Ω
- Weighted residual formulation + partial integration

$$\int_\Omega b_k\frac{\partial U^e_{r,i}b_i}{\partial t}d\Omega-\int_\Omega\frac{\partial b_k}{\partial x_i}(A_iU^e)_r d\Omega+\int_{\partial\Omega}b_kF^{Rn}_{n,r}(U^e,U^{nb})d\partial\Omega=0,\quad k=1,\ldots,N$$

$$\underbrace{\left(\int_\Omega bb^Td\Omega\right)}_{M}\frac{\partial(U^e)^T}{\partial t}-\underbrace{\left(\int_\Omega\frac{\partial b}{\partial x_i}b^Td\Omega\right)}_{K_r}(A_iU^e)^T+\sum_{edges}\underbrace{\left(\int_{\partial\Omega}bb^Td\partial\Omega\right)}_{B_{edge}}(F_n^{Rn})^T=0$$

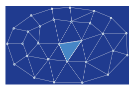
Only communication between elements through inter-element flux

KU LEUVEN

37

Modeling tools for the LEE

Discontinuous Galerkin



$$U_i^e(\vec{r},t)=\sum_{k=1}^{N(p,d)}b_k(\vec{r})U_{i,k}^e(t)$$

some key properties

+ easy parallellization
+ straightforward implementation of B.C.
+ arbitrary order of accuracy
+ grid flexibility
+ efficiency
- storage cost for 5 DOF's
- quadrature demands CPU-time

KU LEUVEN

38

Modeling tools for the LEE

time-integration

Time advancing: p-stage explicit Runge-Kutta scheme

$$U^0=U(t_n),\quad U^i=U(t_n)+\alpha_i\Delta tF(U^{i-1}),\quad U(t_n+\Delta t)=U^p\qquad for\ i=1,\ldots,p$$

Spatial DG discretization: optimized Runge-Kutta integration:

- High-order accuracy
- Low dispersion and dissipation error

Time step imposed by stability considerations

39 Toulorge, T., Desmet, W. (2012). Optimal Runge-Kutta Schemes for Discontinuous Galerkin Space Discretizations Applied to Wave Propagation Schemes. *Journal of Computational Physics, 231*, 2067-2091

KU LEUVEN

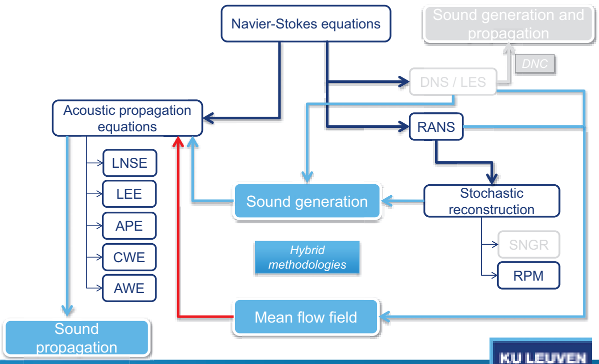
KU Leuven innovations

1. optimized Runge Kutta schemes
2. time domain impedance boundary condition based on recursive convolution
3. **source mean flow mapping** and filtering
4. **Random Particle Mesh (RPM) method**
5. modelling tools for the Linearized Navier-Stokes Equations (LNSE)

KU LEUVEN

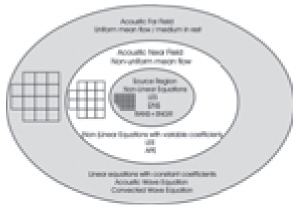
40

KU Leuven research activities

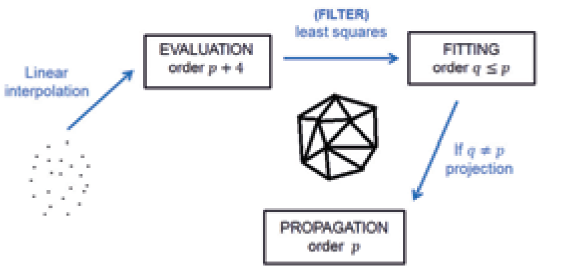


Hybrid methods: Coupling techniques

- Hybrid methods: domain decomposition
  - Fine CFD grid (source domain)
  - Coarser acoustic grid (propagation domain)
- Coupling: mapping of data from the CFD grid to the acoustic grid
  - **Mean flow field**
  - Source data

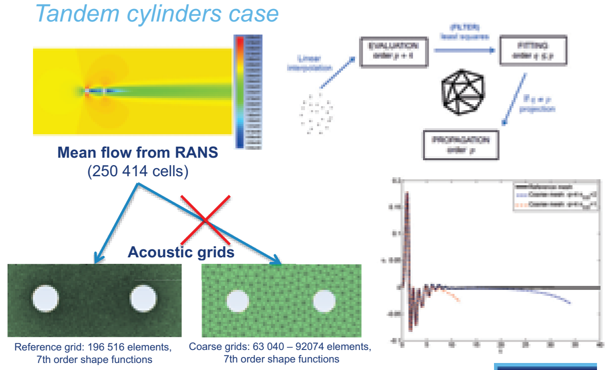


Interfacing schemes: Mean flow mapping



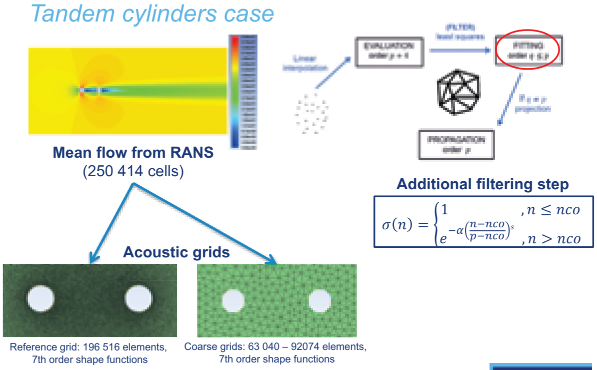
Interfacing schemes: Mean flow mapping

Tandem cylinders case



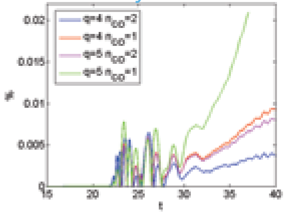
Interfacing schemes: Mean flow mapping

Tandem cylinders case



Interfacing schemes: Mean flow mapping

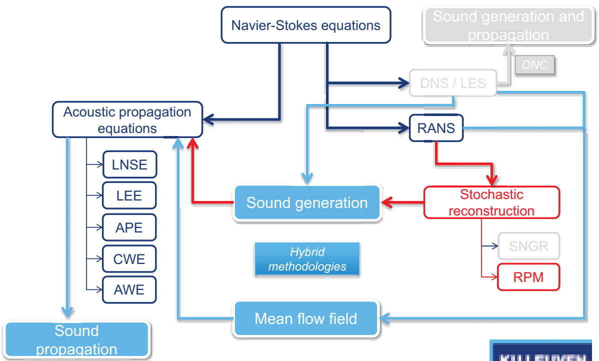
Tandem cylinders case



- Error values associated with the intermediate mesh
- Different orders used for the fitting mesh and different cut-off degrees used for the filter

- $q - n_{co} \geq 4$ ➡ not working
- different mean flow approximations of the same order ➡ similar results
- the lower the order of the mean flow approximation, the more stable the simulation
- error less than 0.1%

KU Leuven research activities



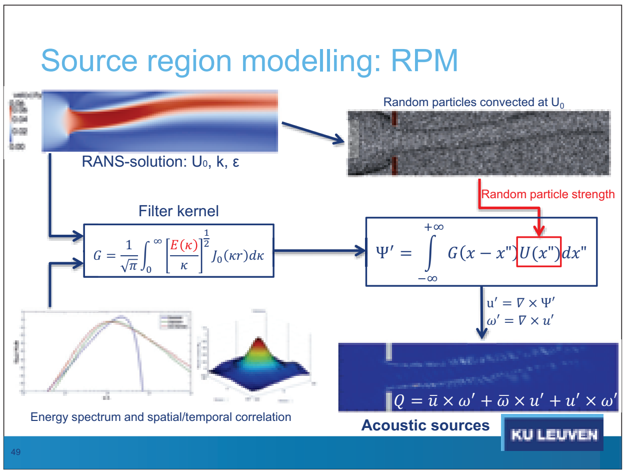
Source region modelling: Numerical

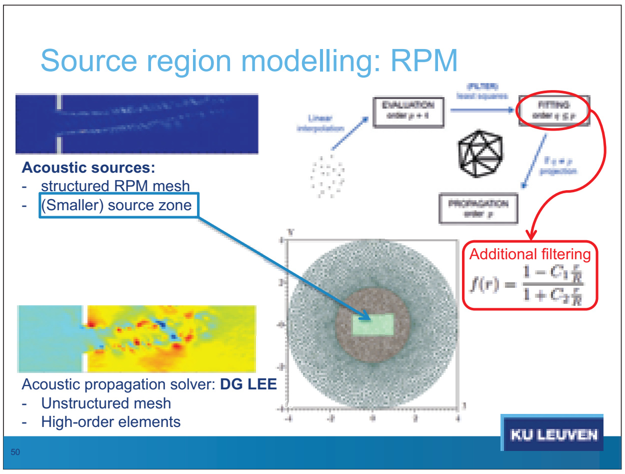
Noise generation: unsteady flow field

- scale-resolving simulations (DNS, LES)
  - computationally demanding
  - conservative interfacing scheme (mapping) needed
- steady flow simulations (RANS) + stochastic reconstruction
  - reconstruct unsteady variables from steady RANS

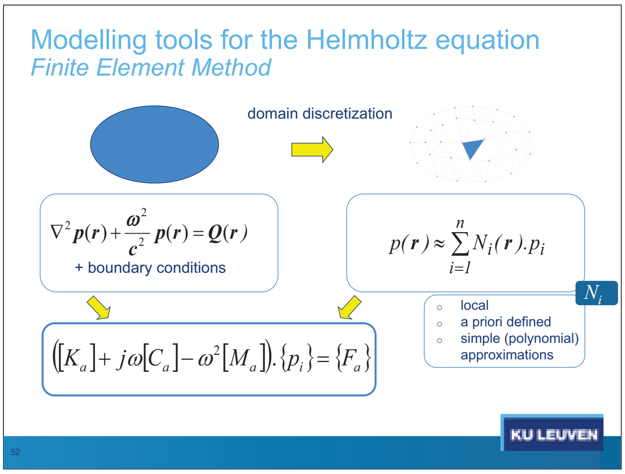
Stochastic Methods

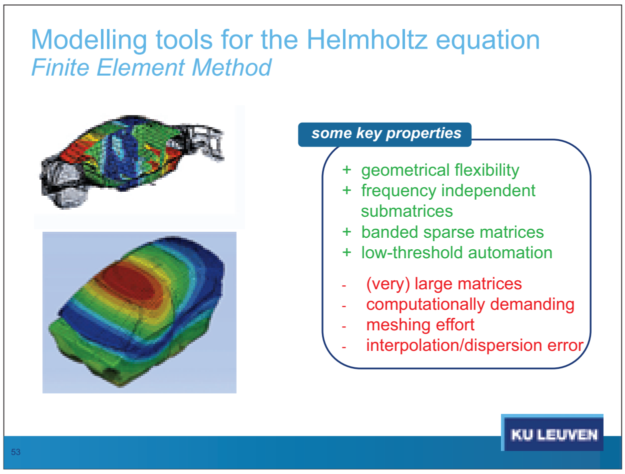
- SNGR (frequency-domain)
- RPM (time-domain)

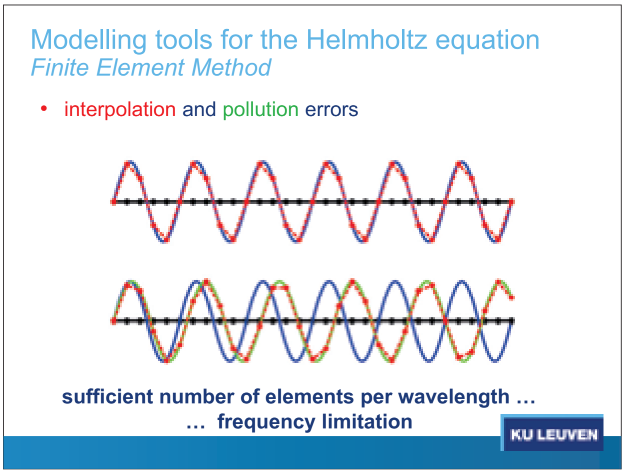

Source region modelling: RPM
RANS-solution: U₀, k, ε
Random particles convected at U_0
Random particle strength
Filter kernel
$G = \frac{1}{\sqrt{\pi}} \int_0^\infty \left[\frac{E(\kappa)}{\kappa}\right]^{\frac{1}{2}} J_0(\kappa r) d\kappa$
$\Psi' = \int_{-\infty}^{+\infty} G(x - x'') U(x'') dx''$
$u' = \nabla \times \Psi'$
$\omega' = \nabla \times u'$
$Q = \bar{u} \times \omega' + \bar{\omega} \times u' + u' \times \omega'$
Energy spectrum and spatial/temporal correlation
Acoustic sources
KU LEUVEN
49

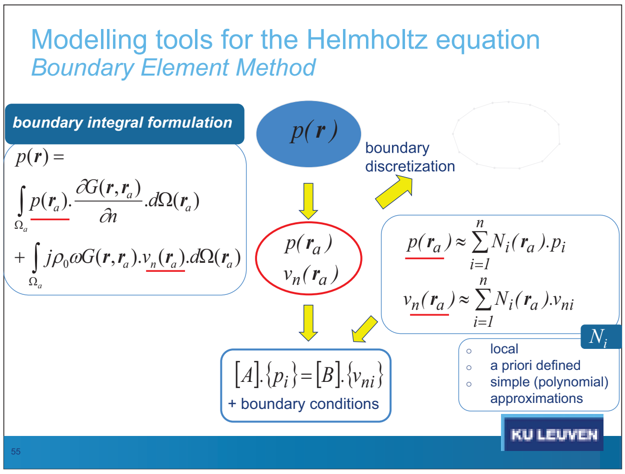

Source region modelling: RPM
Acoustic sources:
- structured RPM mesh
- (Smaller) source zone
Acoustic propagation solver: DG LEE
- Unstructured mesh
- High-order elements
Additional filtering
$f(r) = \frac{1 - C_1 \frac{r}{R}}{1 + C_2 \frac{r}{R}}$
KU LEUVEN
50

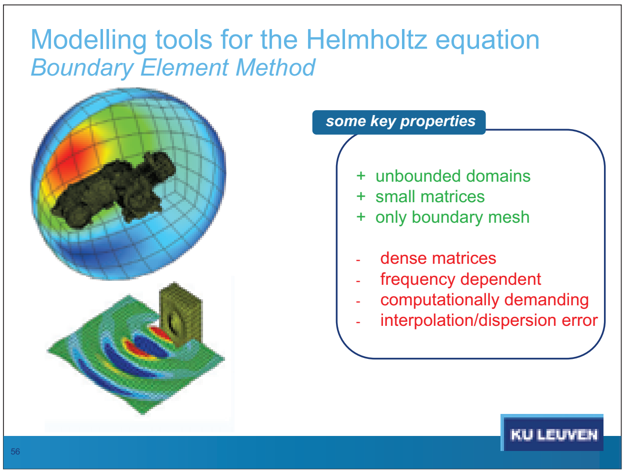

Overview
• Who we are
• Introduction:
○ Flow-induced acoustics and vibrations
○ Importance of virtual (acoustic) tools
• Mathematical formulations:
○ Flow-induced vibrations
○ Flow-induced acoustics
• Numerical solution strategies (@ KU Leuven):
○ for time-domain LEE
○ for frequency-domain vibro-acoustics
• Conclusions
KU LEUVEN
51

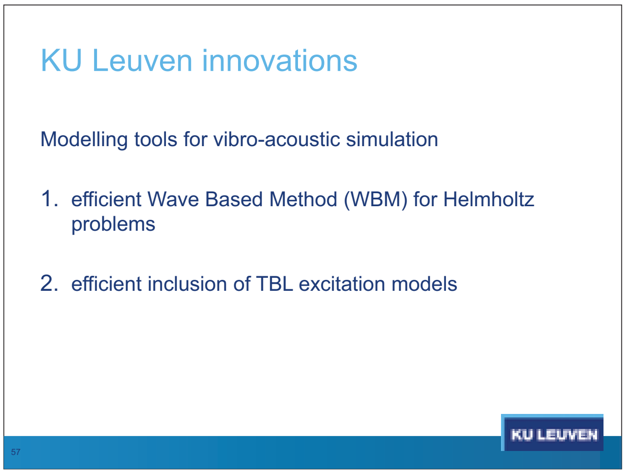

Modelling tools for the Helmholtz equation
Finite Element Method
domain discretization
$\nabla^2 p(\mathbf{r}) + \frac{\omega^2}{c^2} p(\mathbf{r}) = Q(\mathbf{r})$
+ boundary conditions
$p(\mathbf{r}) \approx \sum_{i=1}^{n} N_i(\mathbf{r}).p_i$
N_i
○ local
○ a priori defined
○ simple (polynomial) approximations
$([K_a] + j\omega[C_a] - \omega^2[M_a]).\{p_i\} = \{F_a\}$
KU LEUVEN
52

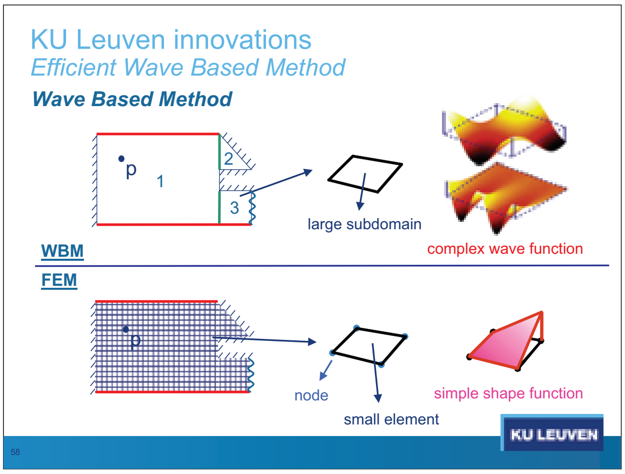

Modelling tools for the Helmholtz equation
Finite Element Method
some key properties
+ geometrical flexibility
+ frequency independent submatrices
+ banded sparse matrices
+ low-threshold automation
- (very) large matrices
- computationally demanding
- meshing effort
- interpolation/dispersion error
KU LEUVEN
53

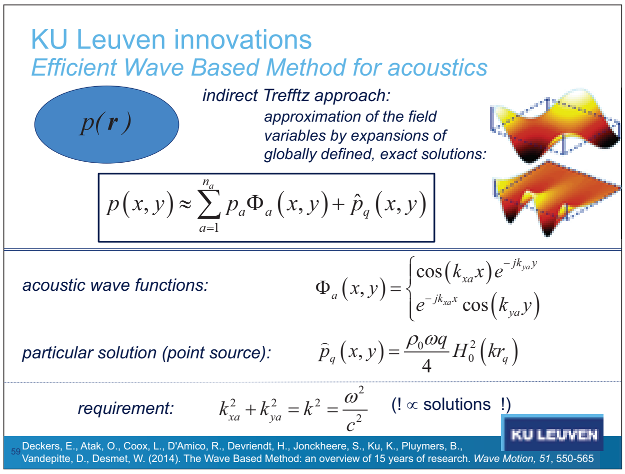

Modelling tools for the Helmholtz equation
Finite Element Method
• interpolation and pollution errors
sufficient number of elements per wavelength …
… frequency limitation
KU LEUVEN

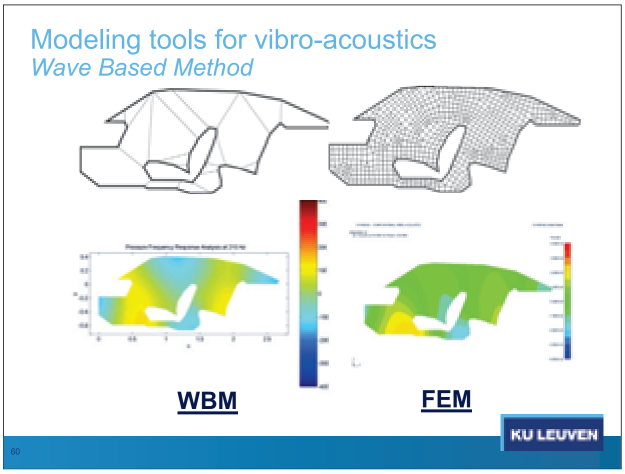

Modelling tools for the Helmholtz equation
Boundary Element Method
boundary integral formulation
$p(\mathbf{r}) =$
$\int_{\Omega_a} p(\mathbf{r}_a).\frac{\partial G(\mathbf{r},\mathbf{r}_a)}{\partial n}.d\Omega(\mathbf{r}_a)$
$+ \int_{\Omega_a} j\rho_0\omega G(\mathbf{r},\mathbf{r}_a).v_n(\mathbf{r}_a).d\Omega(\mathbf{r}_a)$
$p(\mathbf{r})$
boundary discretization
$p(\mathbf{r}_a)$
$v_n(\mathbf{r}_a)$
$p(\mathbf{r}_a) \approx \sum_{i=1}^{n} N_i(\mathbf{r}_a).p_i$
$v_n(\mathbf{r}_a) \approx \sum_{i=1}^{n} N_i(\mathbf{r}_a).v_{ni}$
N_i
○ local
○ a priori defined
○ simple (polynomial) approximations
$[A].\{p_i\} = [B].\{v_{ni}\}$
+ boundary conditions
KU LEUVEN
55

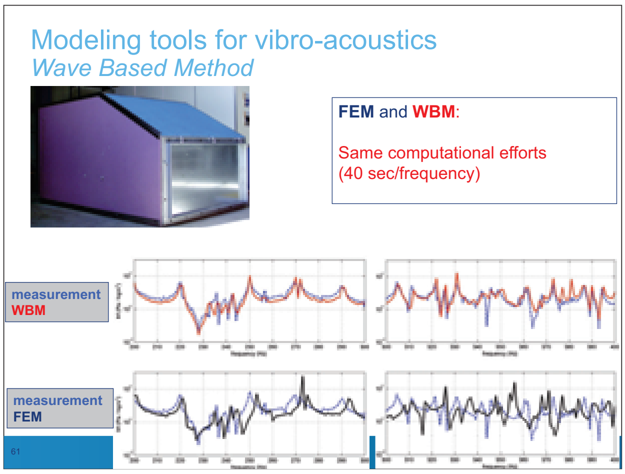

Modelling tools for the Helmholtz equation
Boundary Element Method
some key properties
+ unbounded domains
+ small matrices
+ only boundary mesh
- dense matrices
- frequency dependent
- computationally demanding
- interpolation/dispersion error
KU LEUVEN
56

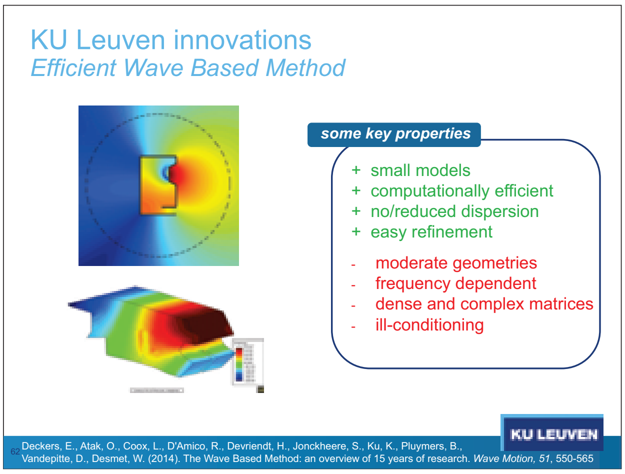

KU Leuven innovations
Modelling tools for vibro-acoustic simulation
1. efficient Wave Based Method (WBM) for Helmholtz problems
2. efficient inclusion of TBL excitation models
KU LEUVEN
57

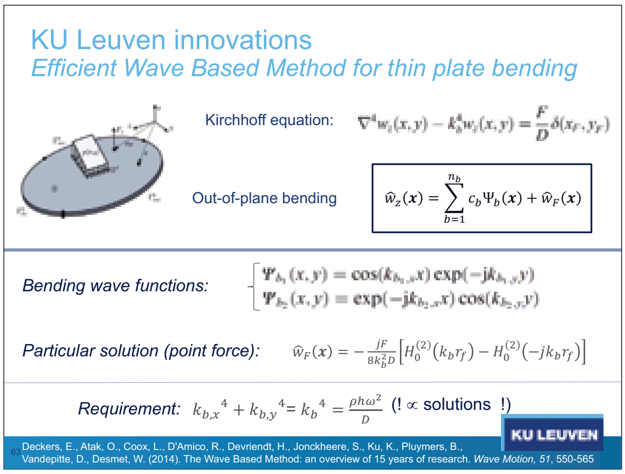

KU Leuven innovations
Efficient Wave Based Method
Wave Based Method
p
1
2
3
large subdomain
complex wave function
WBM
FEM
p
node
small element
simple shape function
KU LEUVEN
58

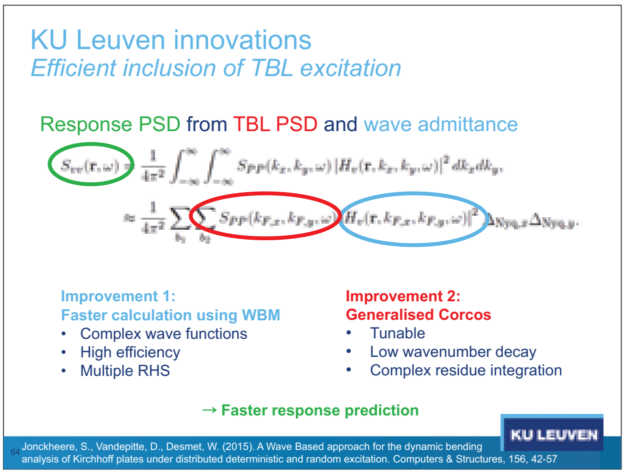

KU Leuven innovations
Efficient Wave Based Method for acoustics
$p(\mathbf{r})$
indirect Trefftz approach:
approximation of the field variables by expansions of globally defined, exact solutions:
$p(x,y) \approx \sum_{a=1}^{n_a} p_a \Phi_a(x,y) + \hat{p}_q(x,y)$
acoustic wave functions:
$\Phi_a(x,y) = \begin{cases} \cos(k_{xa}x)e^{-jk_{ya}y} \\ e^{-jk_{xa}x}\cos(k_{ya}y) \end{cases}$
particular solution (point source):
$\hat{p}_q(x,y) = \frac{\rho_0 \omega q}{4} H_0^2(kr_q)$
requirement:
$k_{xa}^2 + k_{ya}^2 = k^2 = \frac{\omega^2}{c^2}$
(! ∝ solutions !)
KU LEUVEN
Deckers, E., Atak, O., Coox, L., D'Amico, R., Devriendt, H., Jonckheere, S., Ku, K., Plumyers, B., Vandepitte, D., Desmet, W. (2014). The Wave Based Method: an overview of 15 years of research. Wave Motion, 51, 550-565


Modeling tools for vibro-acoustics
Wave Based Method
WBM
FEM
KU LEUVEN
60


Modeling tools for vibro-acoustics
Wave Based Method
FEM and WBM:
Same computational efforts
(40 sec/frequency)
measurement
WBM
measurement
FEM
61


KU Leuven innovations
Efficient Wave Based Method
some key properties
+ small models
+ computationally efficient
+ no/reduced dispersion
+ easy refinement
- moderate geometries
- frequency dependent
- dense and complex matrices
- ill-conditioning
KU LEUVEN
Deckers, E., Atak, O., Coox, L., D'Amico, R., Devriendt, H., Jonckheere, S., Ku, K., Plumyers, B., Vandepitte, D., Desmet, W. (2014). The Wave Based Method: an overview of 15 years of research. Wave Motion, 51, 550-565


KU Leuven innovations
Efficient Wave Based Method for thin plate bending
Kirchhoff equation:
$\nabla^4 w_z(x,y) - k_b^4 w_z(x,y) = \frac{F}{D}\delta(x_F, y_F)$
Out-of-plane bending
$\hat{w}_z(x) = \sum_{b=1}^{n_b} c_b \Psi_b(x) + \hat{w}_F(x)$
Bending wave functions:
$\Psi_{b_1}(x,y) = \cos(k_{b_1,x}x)\exp(-jk_{b_1,y}y)$
$\Psi_{b_2}(x,y) = \exp(-jk_{b_2,x}x)\cos(k_{b_2,y}y)$
Particular solution (point force):
$\hat{w}_F(x) = -\frac{jF}{8k_b^2 D}\left[H_0^{(2)}(k_b r_f) - H_0^{(2)}(-jk_b r_f)\right]$
Requirement: $k_{b,x}{}^4 + k_{b,y}{}^4 = k_b{}^4 = \frac{\rho h \omega^2}{D}$ (! ∝ solutions !)
KU LEUVEN
Deckers, E., Atak, O., Coox, L., D'Amico, R., Devriendt, H., Jonckheere, S., Ku, K., Plumyers, B., Vandepitte, D., Desmet, W. (2014). The Wave Based Method: an overview of 15 years of research. Wave Motion, 51, 550-565


KU Leuven innovations
Efficient inclusion of TBL excitation
Response PSD from TBL PSD and wave admittance
$S_{vv}(\mathbf{r},\omega) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{PP}(k_x,k_y,\omega)\,|H_v(\mathbf{r},k_x,k_y,\omega)|^2\,dk_x dk_y$
$\approx \frac{1}{4\pi^2}\sum_{b_1}\sum_{b_2} S_{PP}(k_{F,x},k_{F,y},\omega)\,|H_v(\mathbf{r},k_{F,x},k_{F,y},\omega)|^2\,\Delta_{Nyq,x}\Delta_{Nyq,y}$
Improvement 1:
Faster calculation using WBM
• Complex wave functions
• High efficiency
• Multiple RHS
Improvement 2:
Generalised Corcos
• Tunable
• Low wavenumber decay
• Complex residue integration
→ Faster response prediction
KU LEUVEN
Jonckheere, S., Vandepitte, D., Desmet, W. (2015). A Wave Based approach for the dynamic bending analysis of Kirchhoff plates under distributed deterministic and random excitation. Computers & Structures, 156, 42-57

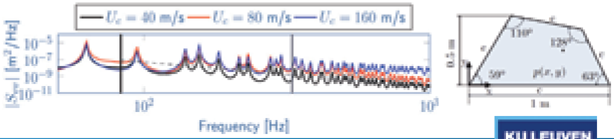
KU Leuven innovations
Efficient inclusion of TBL excitation

Response PSD from TBL PSD and wave admittance

$$S_{vv}(\mathbf{r},\omega) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{PP}(k_x,k_y,\omega)\,|H_v(\mathbf{r},k_x,k_y,\omega)|^2\,dk_x dk_y$$

$$\approx \frac{1}{4\pi^2}\sum_{b_1}\sum_{b_2} S_{PP}(k_{P,x},k_{P,y},\omega)\,|H_v(\mathbf{r},k_{P,x},k_{P,y},\omega)|^2\,\Delta_{Nyq,x}\Delta_{Nyq,y}.$$

2mm clamped aluminium plate under Corcos TBL



Jonckheere, S., Vandepitte, D., Desmet, W. (2015). A Wave Based approach for the dynamic bending analysis of Kirchhoff plates under distributed deterministic and random excitation. Computers & Structures, 156, 42-57

KU LEUVEN

65

Overview

- Who we are
- Introduction:
  - Flow-induced acoustics and vibrations
  - Importance of virtual (acoustic) tools
- Mathematical formulations:
  - Flow-induced vibrations
  - Flow-induced acoustics
- Numerical solution strategies (@ KU Leuven):
  - for time-domain LEE
  - for frequency-domain vibro-acoustics
- Conclusions

KU LEUVEN

66

Conclusions

- Aerodynamic versus vibro-acoustics phenomena:
  Large disparity in energy/length/time scales
- Numerical schemes:
  Balance between accuracy – stability - efficiency
- Hybrid approaches:
  noise/vibration generation – noise/vibration propagation
- KU Leuven approaches:
  - DG methods for LEE
  - mapping techniques (for mean flow and sources)
  - RPM for stochastic source reconstruction
  - Wave Based Method for (TBL excited) vibro-acoustics

KU LEUVEN

67

Thank you

Wim Desmet

Celestijnenlaan 300B – box 2420
3001 Leuven, Belgium
+32 16 32 25 27
wim.desmet@kuleuven.be

www.mech.kuleuven.be/mod

NOISE & VIBRATION Research Group

KU LEUVEN

68