van der Waals 気体中の衝撃波の不安定解析

石井隆次　由比政年

Instability of Shock Waves in a van der Waals Gas

by

Ryuji ISHII
Department of Engineering Science, Kyoto University,

and

Masatoshi YUHI
Department of Civil Engineering, Kanazawa University,

ABSTRACT

Instability or stability of shock waves in van der Waals gas is invesitigated numerically. First possibility of presense of several types of shock transition is proved. Next an evolution condition is applied to these shocks numerically. It is shown that only the supersonic-subsonic shock can exist stably. However, all the supersonic-subsonic shock can not always exist as an stable entity and then the supersonic-subsonic condition is only a necessary condition for the shock to be stable. Shock splitting process or the time evolutions of unstable shock transition is also simulated.

1. Introduction

Real gas near the phase boundary has some peculiar effects that are different from those caused by viscosity, heat conduction, relaxation of internal modes and cemical reactions. The most important effects are "degradicity" and "retrogradicy". The former is responsible for the expansion shock and the latter the evapolation (or condensation) in adiabatic expansion (or adiabatic compression).

A fundamental problem concerning the nature of shock waves in a van der Waals gas has long been studied. One of the most interesting facts proved so far is that the existence of expansion shocks cannot be ruled out from the point of view of stability. It is well known that the compression shocks occur in gases having a fundamental derivative $\Gamma = (C^4/2V^3)(\partial^2V/\partial p^2)_s > 0$, where V, p, C and s are the specidic volume, the pressure, the speed of sound and the specific entropy, respectively, and expansion shocks occur for those having $\Gamma < 0$.[1-4]

Shock waves in a large-heat-capacity gas emerging from a tube were investigated numerically and experimentally by Thompson[3] et al. Weak shock waves in which the local value of the fundamental derivative changes were studied in detail by Cramer and Kluwick[4]. In the previous studies, analyses were made mainly for relatively weak shock waves. In the full nonlinear problem, the shocks exhibit much more complicated behavior than that of the weak shocks. In this paper, one-dimensional shock waves are considered. First,the Rankine-Hugoniot relations are obtained by using the Rayleigh line and the shock adiabatic in the pV-plane. characteristics of the shocks are investigated in detail. Next, to justify and confirm the analytical results, numerical experiments are made on a supercomputer with a TVD-scheme. All the numerical simulations were performed on the supercomputer Fujitsu VP-2600 at the Date Processing Center of Kyoto University.

2. Possible Shock Transitions

Detailed investigation of shock transitions satisfying the Rankine-Hugoniot relations and the entropy condition yields the result; there can be seven types of shock transition in the van der Waals gas[5]. These are

Case 1; $M_1 > 1$, $M_2 > 1$
Case 2; $M_1 > 1$, $M_2 < 1$ } for compression shocks,
Case 3; $M_1 < 1$, $M_2 < 1$

Case 4; $M_1 > 1$, $M_2 > 1$
Case 5; $M_1 > 1$, $M_2 < 1$ } for expansion shocks.
Case 6; $M_1 < 1$, $M_2 > 1$
Case 7; $M_1 < 1$, $M_2 < 1$

The sample flow conditions found here are listed in Table 1.

3. Tine Evolution of Shock Discontinuity

The evolution condition is that the solution is unique and the transition must be stable to

Table 1. Possible shock transitions

Case	upstream conditions (V_1, p_1, M_1)	downstream conditions (V_2, p_2, M_2)
1	(1.80, 0.92862, 1.01188)	(1.30, 1.06221, 1.00810)
2-1	(1.90, 0.90515, 1.06542)	(0.90, 1.19528, 0.70261)
2-2	(1.80, 0.92862, 1.01262)	(1.450, 1.02227, 0.99729)
2-3	(1.80, 0.92862, 1.01123)	(1.060, 1.12608, 0.95806)
3-1	(1.60, 0.97549, 0.99563)	(1.0, 1.13272, 0.90064)
3-2	(1.40, 1.01939, 0.98961)	(1.010, 1.11558 0.92042)
4-1	(1.40 1.01939, 1.00952)	(1.760, 0.92699, 1.00401)
4-2	(1.20, 1.05513, 1.04721)	(2.0, 0.86080, 1.01495)
5-1	(1.40, 1.01939, 1.00963)	(1.650, 0.95521, 0.99759)
5-2	(1.20, 1.05513, 1.04080)	(1.610, 0.95675, 0.97703)
6	(1.0, 1.080, 0.96212)	(2.40, 0.76586, 1.05653)
7	(0.950, 1.08671, 0.85949)	(1.650 0.93281, 0.95591)

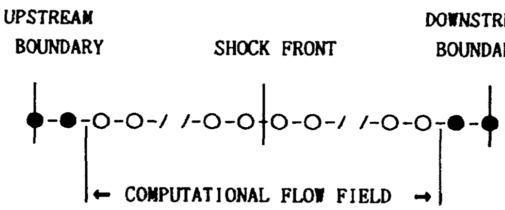

Fig.1 Initial and boundary conditions.

small disturbances.Since all the shock transitions listed in Table 1 are determined uniquely in the p-V plane, here the evolution condition requires that these shock transitions are stable to small disturbances[6].

Numerically we investigate the shock stability or instability in the coordinate system moving with the shock front. As shown in Fig. 1, first flow conditions satisfying the Rankine-Hugoniot relations and the entropy condition are specified as the initial conditions across the discontinuous plane. For the boundary conditions, the initial conditions are always fixed at two points both upstream and downstream of the boundaries. A time-dependent approach is adopted with a finite difference TVD-scheme[7]. As is well known, numerically any shock wave has some numerical (artificial) structure extending into several meshes. Then after the initiation of numerical simulation, the specified discontinuous shock structure begins to shift into a numerically stable structure. During this transition process, some numerical (artificial) fluctuations are produced. If the upstream flow is subsonic, these propagate upstream and on arriving at the boundaries, some part of the fluctuations may be reflected back from the boundary. On the other hand, those always propagate downstream and some part will be reflected back, if the downstream flow is subsonic. These disturbances will fluctuate again the shock.

In this paper, we define a shock is stable if the shock wave stay substantially at the original position, its profile extending into several meshes tends to have a converged steady profile, and the flow conditions both upstream and downstream of the numerical shock (having a structure with several meshes) converge exactly to those specified initially. Here it must be emphasized that any shock in the ideal gas has been confirmed to be stable in the similar numerical experiment with the present TVD-scheme.

Samples of time evolutions of the shock transition are shown in Fig.2, where the CFL and mesh numbers are taken to be 0.4 and 100, respectively. The first shows an unstable supersonic-supersonic shock. In such an unstable case, the original discontinuous shock is splitted into a few types of waves which spread out with increasing time step. Once the wave splitting occurs, any time converged shock profile can never be constructed. Figs. 2-b) and c) show stable shocks and then their numerical structures converge to steady ones. Fig.2-d) shows an unstable shock.It is interesting that this shock is supersonic-subsonic shock. Figs.2-e) and f) show expansion shocks, where the former is an unstable supersonic-supersonic shock and the latter is a stable supersonic-subsonic shock.

It has to be stressed that for the stable shocks, the results were checked for various time steps up to 500,000 and it was also confirmed that the instabilities do not depend on the mesh number (50~5000) and the CFL number

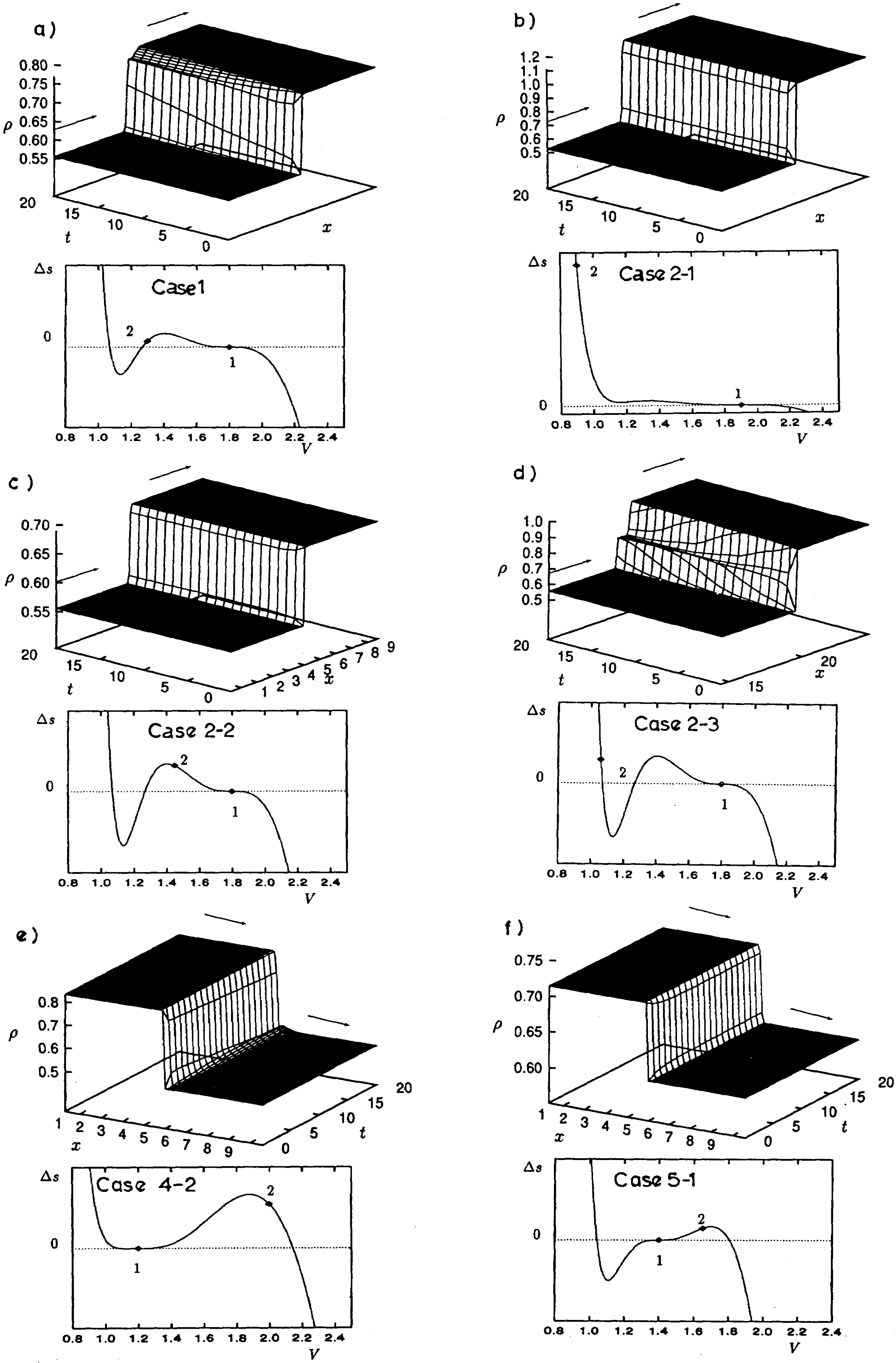

Fig.2 Stable and unstable shock transitions.

Table 2. Stable and unstable shocks.

Case	Present Numerical	Theoretical		
		C & K	F & H	K
1 (CS,Sup.-Sup.)	●	●	★	★
2-1 (CS,Sup.-Sub.)	○	○	○	○
2-2 (CS,Sup.-Sub.)	○	○	○	○
2-3 (CS,Sup.-Sub.)	●	●	○	○
3-1 (CS,Sub.-Sub.)	●	●	○	○
3-2 (CS,Sub.-Sub.)	●	●	○	○
4-1 (ES,Sup.-Sup.)	●	●	★	★
4-2 (ES,Sup.-Sup.)	●	●	★	★
5-1 (ES,Sup.-Sub.)	○	○	○	○
5-2 (ES,Sup.-Sub.)	○	○	○	○
6 (ES,Sub.-Sup.)	●	●	★	★
7 (ES,Sub.-Sub.)	●	●	○	○

○; stable, ●; unstable, ★; not applicable,
C & K; Cramer and Kluwick[4], F & H; Fowles and Houwing[8], K;Kontorovich[9].

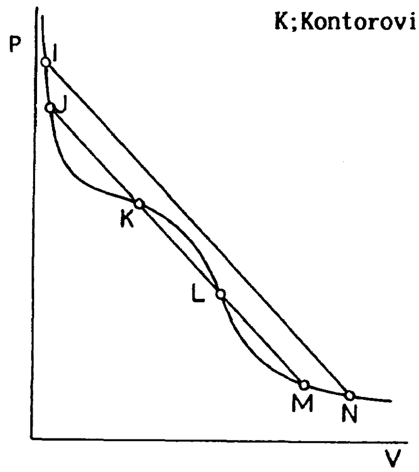

Fig.3 Rankine-Hugoniot relations in the p-V plane.

(0.4~0.8).

For the shock instability, we can summarize the results as in Table 1, where a few theoretical predictions are compared. Obviously the present results agree completely with & Kluwic's predictions[4] but do not always with the other predictions. Since the stability theories of Kontorovich[8] and Fowling & Fouwing[9] are discussed in the previous paper, here Cramer & Kluwic's theory is described briefly. Their analysis is based on the Navier-Stokes equations which can be applied only to weak shock waves. And they assumed their result is applicable to shocks with arbitrary strength. They concluded that the shock adiabatic and the Rayleigh line intersect only at two points in the p-V plane, describing the upstream and downstream flow conditions across the shock discontinuity, for the shock to be stable.

Fig. 3 shows schematically the stable and unstable shock transitions in the p-V plane. All transitions between two points (I,N), (J,K), (J,L), (L,M), (K,L), (K,M) and (L,M) are possible. Although their transition directions are determined by the entropy condition, only those between the points (I,N), (J,K), (K,L) and (L,M) are stable.

From Cramer and Kluwic's criterion and the present numerical results, it can be said that the supersonic-subsonic condition is a necessary condition but not a sufficient condition. It is rather remarkable that the present results about the shock instability are completely well explained by their theory. Obviously the present conclusion is partly contradictory to the results in the previous paper, where it was concluded that the supersonic-supersonic shock is stable at least numerically. This wrong result comes from the fact that for a shock transition where the fundamental derivative Γ takes very small absolute values, the wave profile changes very slowly. Then the accurate investigation of shock instability requires a sufficiently large number of time steps. Then although the previous Cramer numerical results are correct but the time steps are not always satisfactory to observe the shock instability.

4. Conclusions

Normal shock waves in the van der Waals gas are investigated theoretically and numerically. Some interesting features of shock transitions, which are different from those in the ideal gas, were found: 1) In the p-V plane, the shock adiabatic and the Rayleigh line can intersect at four points. 2) The supersonic-subsonic condition is only a necessary condition but not always a sufficient condition. 3) The stable shocks satisfy the criterion proposed by Cramer & Kluwick but do not always those by Fowles & Houwing and Kontrovich. 4) One-dimensional shock splitting occurs for unstable shock transitions.

As described in the introduction, there are two kinds of shock instability; one is the shock splitting and the other the corrugation instability. The latter could not be treated in the present one-dimensional analysis and then will be studied in the near future.

References

1) Thompson, P. A.:A Fundamental derivatives in gasdynamics, Phys. Fluids 14 (1971), pp. 1843-1849.
2) Thompson, P. A. & Lambrakis, K.G.:Negative shock waves, J. Fluid Mech. 60 (1973), pp. 187-208.
3) Thompson, P. A., Carofano, G. C. & Kim,Y-G, :Shock waves and phase changes in a large-heat-capacity fluid emerging from a tube, J. Fluid Mech. 166 (1986), pp.57-92.
4) Cramer, M. S. and Kluwick A.:On the propagation of waves exhibiting both positive and negative nonlinearity, J. Fluid Mech. 142 (1984), pp.9-37.
5) Ishii, R; Shock Waves in a van der Waals Gas (part 1), Special Cases, Trans. Japan Soc. Aero. Space Sci. Vol.38, No.121 (1995), pp.282-292.
6) Lax, P. D.:"Shock waves and entropy," in Contributions to Nonlinear Functional Analysis edited by Zarantonello, Academic Press, New York, 1971.
7) Chakravarthy S.R. and Osher,S.:A new class of high accuracy TVD schemes for hyperbolic conservation laws, AIAA Paper 85-0363 (1985).
8) Kontorovich, V. M.:Concerning the stability of shock waves. Sov. Phys: Tech. Phys. 6 (1957), pp.1179-1181.
9) Fowles,G. R. & Houwing, A. F.P.:Instability of shock and detonation waves. Phys. Fluids, 27 (1984), pp.1982-1990.