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Curvature Effects in Three - Dimensional Boundary Layers

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ABSTRACT

The effect of including wall and streamline curvature terms in swept-wing boundary-layer stability calculations is studied. The linear disturbance equations are cast on a fixed, body-intrinsic, curvilinear coordinate system. Those nonparallel terms which contribute mainly to the streamline-curvature effect are retained in this formulation and approximated by their local finite-difference values. Convex wall curvature has a stabilizing effect, while streamline curvature is destabilizing.

Keywords: crossflow instability, swept-wing flows, laminar-turbulent transition

1. INTRODUCTION

Three-dimensional boundary-layer stability and transition on a swept wing has become an increasingly important research subject for the effective application of Laminar Flow Control. A recent review is given by Reed & Saric [1]. Although there are a variety of instability mechanisms which may cause transition in a swept-wing boundary layer, the present work focuses on the study of the stationary crossflow instability.

For non-rotating three-dimensional boundary-layer flow, Malik & Poll [2] adopted Stuart's theory [3] and studied the incompressible flow over the windward face of a yawed infinite cylinder and reported that the location of transition had an N factor of 11, as compared with 17 when the effects of curvature were neglected. However, this work contained the surface curvature and was inconclusive as far as the streamline-curvature effects were concerned. To apply Stuart's stability theory, one must guess the wave direction, which is part of the solution and is generally difficult to approximate. Malik & Poll suggested iteratively rotating the coordinate system until one of the coordinate axes aligns with the wavenumber vector. Then the local growth rates are obtained. Two metric terms m_{12} and m_{21} [3] associated with the final coordinate system are claimed to provide the effects of streamline curvature. This seems to be difficult to justify for three reasons.

First, there is no unique coordinate system satisfying tangency to the wavenumber vector, and hence the values of m_{12} and m_{21} can depend on the choice of the trial coordinate system. Second, to overcome the pressure force in the crossflow direction, near the wall in a swept-wing boundary layer, the flow must follow streamlines that are more curved than the local inviscid streamlines. Thus, streamline curvature must vary with the normal direction and typically m_{12} and m_{21} are locally taken as constants. Third, streamline curvature is actually induced by the spatial variations of the basic state, which have long been known as "nonparallel effects". In previous work, these have been routinely neglected.

2. DISCUSSION

In this work the effects of wall curvature and streamline curvature on the stationary crossflow instability of a swept-wing boundary layer are addressed. The disturbance equations are written on a fixed body-intrinsic coordinate system; thus the wall curvature is consistently formulated. The length scale chosen is

$$L = (R\nu / U_\infty)^{1/2}$$

where R is the local radius of wall curvature. The streamwise derivatives of basic-state quantities are retained and approximated by their local finite-difference values. These terms represent the streamline curvature and cannot be neglected; in all previous works, they have been routinely neglected. The complete, new equations are found in Lin & Reed [4]; they are too lengthy to be included here.

Results on a highly swept wing [4,5] indicate that curvature has only a minor effect on wave angle. By comparing with solutions of the Orr-Sommerfeld equation, wall curvature is found to be stabilizing and streamline curvature is found to be destabilizing for the stationary crossflow instability. This is evident in Figure 1 which shows comparisons of amplification factors for different curvature formulations:

O-S (no curvature)

bdy01 (body curvature only)

bdy11 (body and streamline curvature)

Results were verified by more computer-resource-intensive linear Navier-Stokes calculations [4,5]. Also, recently, Schrauf et al. [6] have reported a destabilizing effect of streamline curvature in a swept-wing boundary layer. However, their formulation does not include the nonparallel terms included here.

One curious observation is that streamline curvature is stabilizing on a rotating disk, but destabilizing on a swept wing. Pfenninger [7] points out that "the mean boundary layer crossflow of a rotating disk is directed from the concave towards the convex edge of the disturbance vortices, in contrast to swept wings, where it is directed from the convex towards the concave vortex edge." For the swept wing, the resulting centrifugally unstable stratification in the crossflow direction of the streamtubes destabilizes the boundary-layer flow. The opposite occurs for the rotating disk, where the stratification is centrifugally stable.

The strength of linear theories is in their use for design by comparing N factors from one configuration to another. A configuration with a smaller N factor (using the same theory) is likely to remain laminar longer. It has been demonstrated here that the new theory at least qualitatively contains the appropriate relationships between body and surface curvature for 3-D boundary layers and in this sense will aid in the evaluation of new airfoil shapes for swept wings.

Acknowledgements

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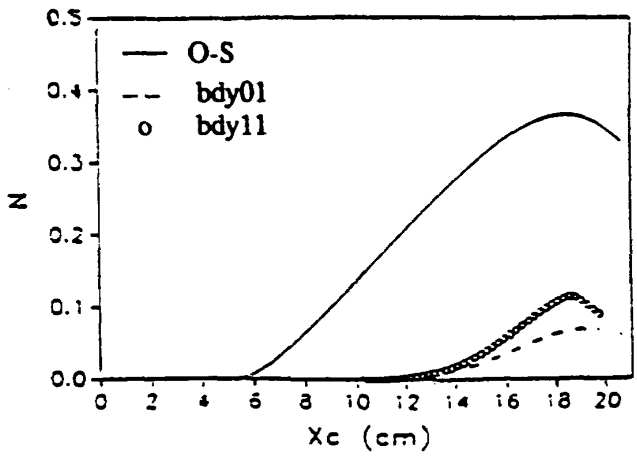

Figure: N-factor calculations for λ_z=3cm with various linear theories. O-S(——): without curvature effect; bdy01(– –) with wall-curvature effect; bdy11(o): with both streamline- and wall-curvature effects.