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RNG 乱流モデルにおける遷移の計算について

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TRANSITION PREDICTION USING THE RNG BASED ALGEBRAIC TURBULENCE MODEL

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ABSTRACT

A new length scale for flow with pressure gradient is proposed in the RNG algebraic turbulence model. The dissipation rate is modeled, so that the equation for eddy viscosity takes the cubic form. The model shows the continuity of eddy viscosity near the wall. This model was applied to predict transition location for flows at pressure gradient. The characteristics of the predicted transition well agree with the experimental and empirical data.

1. INTRODUCTION

The computation of practical boundary layer is often greatly dependent on the modeling of the laminar - turbulent flow zone. It is well known that transition is influenced by freestream turbulence, pressure gradient, Mach number, Reynolds number, accoustic noise, surface conditions and so on [1].

Several attempts to empirically determine the location of transition have been made. Cebeci [2] proposed an empirical relation between x Reynolds number, Re_x and the momentum thickness Reynolds number, Re_θ.

The application of this model to predict transition location has been evaluated by Kawai [3] for subsonic flow around airfoils. The Baldwin-Lomax turbulence model [4] is employed to calculate the turbulence effect starting from the location predicted by this model. It was shown that the drag coefficient is in better agreement with experimental data when this model is employed than the model proposed in the Baldwin-Lomax turbulence model [4].

Van Driest and Blummer [5] developed a transition formulation by taking account of the influence of pressure gradient and freestream turbulence on transition. This model is derived from the assumption that transition occurs when the local vorticity within the boundary layer exceeds a threshold. This is subsequently modified by Abu-Gnaham and Shaw [6] so as to incorporate the minimum Reynolds number calculated from the instability analysis.

These models are not directly related to turbulence model employed in the fully turbulent flow and usually only used to identify the location where the turbulence model begins to be effective. In actual case, to simulate natural transition from laminar to turbulence, a streamwise intermittency is needed.

An example of transition models which are directly combined with turbulence model has been developed by McDonald and Fish [7]. The solution of this model needs to calculate the streamwise development of a turbulent mixing length, the magnitude of which is determined by the turbulent kinetic energy. That is, at least one additional equation to the governing equations must be computed.

We are concerned with zero equation turbulence model which is faster than one or two equation models since there are no additional differential equations to the governing ones. Baldwin and Lomax [4] assume that the transition takes place when the ratio of the turbulent viscosity to the freestream molecular viscosity exceeds a certain value. Although the model provides a reasonably good prediction of transition location for some cases, the transition process from laminar to turbulent flow, however, occurs suddenly, which is quite unnatural.

Recently, applications of the RNG based algebraic turbulence model, which was originally derived by Yakhot and Orszag [8], to the study of transition have started [9] [10]. The key factor of this model lies in the employment of the *heaviside* function in which the parameter as an argument of the function determines the onset of turbulence. This model requires the definition of the length scale and the formulation of the dissipation rate in the calculation of turbulent viscosity.

In the present paper, a new length scale taking account of freestream with pressure gradient is proposed for the RNG. The dissipation rate is modeled such that the turbulent viscosity can be directly calculated from

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the model equation avoiding the iterative method. The capability of the present model to capture the onset of turbulence is evaluated and its transition characteristics are compared with the experimental and empirical data.

In the next section, the algebraic RNG turbulence model with a proposed parameter will be discussed. The results and evaluations of this model for transition prediction are described in Section 3. Finally, in section 4 the concluding remarks will be made.

2. RNG TURBULENCE MODEL

2.1 The Modeling

The algebraic kinematic eddy viscosity derived from the RNG theory takes the following form [8] [11] :

$$\nu_e = \nu\{1 + H(\beta - C_R)\}^{\frac{1}{3}} \tag{1}$$

with

$$\beta = \hat{a}\frac{\epsilon\Lambda^{-4}}{\nu^3} \tag{2}$$

where H is the *heaviside* function and $\hat{a} = 0.12$. C_R experimentally lies between 75 and 200, and ϵ is the dissipation rate. The relation $\tilde{L} = \pi\Lambda^{-1}$ is assumed to relate the integral length scale $\tilde{L}$ with the wave number Λ, whose value represents the allowable turbulence scale in the inertial range. Relating $\tilde{L}$ with the mixing length L at large Reynolds numbers, we have the relation that $L = (\hat{a}^{1/4}/\pi)\tilde{L}$, where L is given. The effective turbulent viscosity $\nu_e = \nu_t + \nu$ is defined as the sum of the turbulent viscosity ν_t and the laminar one ν.

Considering the equilibrium condition, where the production of kinetic energy is equal to the dissipation rate ϵ, the following formulation for ϵ is employed :

$$\epsilon = \frac{u_\tau^3}{\kappa y} \tag{3}$$

Because the calculated drag strongly depends on the first grid point next to the body surface, a fine grid is normally employed, that is, typically $y^+ \leq 15$. In this region, the damping action due to the molecular viscosity manifest themselves near the wall, that is, in the low Reynolds number region [13].

Thus, we devised a damping function for the dissipation rate ϵ in Eq.(3), taking into account the experimental fact that its dimensionless value, which is defined by $\epsilon^+ \equiv \epsilon\nu/u_\tau^4 = 1/\kappa y^+$, approaches to 0.1 [14] at the wall as follows:

$$\epsilon = u_\tau^3 D_R/\kappa y \tag{4}$$

where $D_R = 1 - exp(-0.1\kappa u_\tau y/\nu)$ and $u_\tau = \sqrt{\tau_w/\rho}$. As can be seen from Eq.(4), the dissipation rate ϵ is free from the effective turbulent viscosity ν_e.

Substituting Eq. (4) into Eq.(2), we get

$$(\nu_e/\nu)^3 = 1 + H(\frac{u_\tau^3 L^4}{\kappa y \nu^3} D_R \gamma_R - C_R) \tag{5}$$

where the intermittency function γ_R is added. This is defined as

$$\gamma_R = \{1 + 5.5(\eta)^6\}^{-1} \tag{6}$$

where $\eta = y/\delta$ and δ is the boundary layer thickness. The reason for using the intermittency function is to ensure reduction in turbulent viscosity near the edge of the boundary layer.

2.2 Length Scale

Wiegahrt and Tillmann [15] showed that under adverse pressure gradient the mixing length increases from $\kappa\ y/\theta$ to $2\ y/\theta$, where $\kappa(=0.4)$ is the von Karman constant and θ is the momentum thickness. That is, the von Karman constant κ varies in the flow with pressure gradient. Nakayama *et al.* [16] also derived the same conclusion from the kinetic energy equation.

In the logarithmic region, the momentum equation can be written as [17]

$$\frac{d\tau}{dy} = \frac{dp}{dx} + \rho u_\tau \frac{du_\tau}{dx} f^2(y^+) \tag{7}$$

where τ is the total shear stress composed of both the turbulent shear stress τ_t and the laminar shear stress τ_l, and $u/u_\tau = f(y^+)$. We can integrate Eq.(7) from the wall to y_s, where y_s is the location where the turbulent viscosity becomes maximum. As a result, we have

$$\tau_t = \tau_w + \frac{dp}{dx}y_s + \rho\nu\frac{du_\tau}{dx}\int_0^{y_s^+} f^2 dy^+ \tag{8}$$

Writing the turbulent shear stress τ_t in terms of the mixing length, $\tau_t = \rho L^2(du/dy)^2$. Making Eq.(8) nondimensional, we get

$$\begin{aligned} L^+ \quad = \quad & \kappa y^+\{1 + \frac{\delta}{\rho u_\tau^2}\frac{dp}{dx}\eta_s \\ & +(\frac{u_e^2}{u_\tau^2}\frac{d(u_\tau/u_e)}{dx} + \frac{1}{u_\tau}\frac{du_e}{dx}) \\ & \times\frac{u_e\delta}{u_\tau}\int_0^{\eta_s}(\frac{u}{u_e})^2 d\eta\}^{1/2} \end{aligned} \tag{9}$$

where $L^+ = Lu_\tau/\nu$, $u_\tau = \sqrt{\tau_w/\rho}$, and u_e is the freestream velocity.

In the flow with zero or very mild pressure gradient, the contribution of the third term of Eq. (9) is assumed to be small. The second and fourth terms including dP/dx and du_e/dx, respectively, show the influences of pressure gradient on the shearing stress for a very mild pressure gradient flow.

It is well known that in the log-law region, $L^+ = \alpha y^+$; that is, it varies linearly with the distance from the wall up to η_s in the overlapping region. In the zero, or very mild pressure gradient, this α can be the von Karman constant κ. Furthermore, from Eq.(9), we obtain

$$\alpha = \kappa P^+ \tag{10}$$

where

$$P^+ = \{1 - (\eta_s - \int_0^{\eta_s}(\frac{u}{u_e})^2 d\eta)\frac{u_e^2}{u_\tau^2 Re_\delta}\Lambda\}^{1/2} \tag{11}$$

and $\Lambda = \delta^2/\nu\ du_e/dx$ is the Pohlhausen parameter. η_s can be approximated from the zero pressure condition as 0.225.

Thus, in the present calculation we employed a modification indicated by Eq.(10) for the length scale L, assuming that the effect of freestream pressure gradient on the length scale across the boundary layer is the same, that is P^+. Consequently, the length scale is defined as [18]

$$\frac{L}{\delta} = \frac{0.2\eta(1 - 0.46\eta)}{0.45 + \eta} P^+ \qquad (12)$$

2.3 Transition Model

The transition captured with the present model is associated with the *heaviside* function, which is defined as

$$H(x) = \begin{cases} x \text{ for } & x > 0 \quad \text{turbulent} \\ 0 \text{ for } & x \leq 0 \quad \text{laminar} \end{cases} \qquad (13)$$

Specifically in the RNG algebraic model the turbulence starts when satisfying the condition

$$\frac{\hat{a}\epsilon}{\nu^3}\Lambda^{-4} > C_R \qquad (14)$$

Quite recently, Yakhot *et al.* [10] also showed the possibility of the RNG model to capture the transition, where a new length scale and dissipation rate were proposed. They acquired a quartic equation for ν_e and simulated a flat plate flow with $C_R = 160$. On the other hand, Kirtley [12] employed $C_R = 200$ to apply the RNG model to a turbomachinery problem.

Substituting Eqs.(4) and (12) into Eq.(14) for zero pressure gradient case, using $\eta_s = 0.225$ where the effective viscosity is known to become maximum, and setting C_R to 75, we have the criteria for transition location, that is when

$$\frac{u_\tau \delta}{\nu} > 77.75 \qquad (15)$$

As the theoretical formulas for laminar flow [20], we have

$$c_f = 2\left(\frac{u_\tau}{u_e}\right)^2 = \frac{0.664}{\sqrt{Re_x}} \qquad (16)$$

$$\delta = \frac{5\,x}{\sqrt{Re_x}} \qquad (17)$$

Substituting Eqs. (16) and (17) into Eq. (15), we find that the critical Reynolds number Re_x^{cr} is about 5.3×10^5, which is close to the experimental data [20].

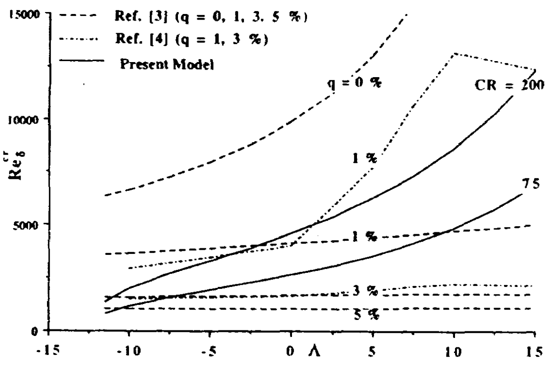

Fig. 1 Characteristics of the captured transition for various values of pressure gradient.

3. RESULTS AND EVALUATIONS

To observe the characteristics of the predicted transition location with the present model, a family of laminar velocity profiles were calculated. The boundary layer thickness, the skin friction and the velocity profile in the y-direction were computed in terms of the Pohlhausen parameter Λ. The complete model of Eq.(1) was computed up to the transition location is captured, that is when Eq. (15) is satisfied.

The characteristics of the captured transition locations under pressure gradient with this model are presented in Fig. 1. The solid curves illustrate the two cases for $C_R = 75$ and 200. Results calculated by using the van Driest-Blumer [5] and the Gnaham-Shaw [6] empirical formulas for various turbulence intensities q are also presented for comparison. In addition to the fact that the change in C_R can represent the effect of turbulence intensity in the freestream, the qualitative tendency is reasonable. That is, in the favourable pressure gradient region, the transition is delayed, while at the adverse pressure gradient the transition is accelerated.

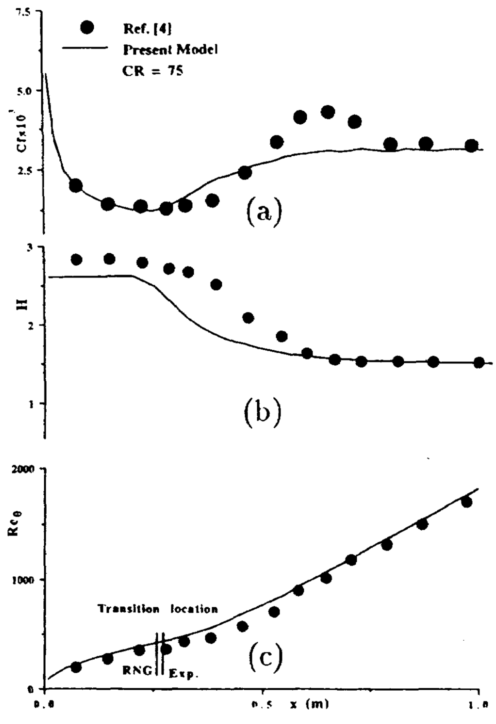

Fig. 2 Boundarylayer characteristics at zero pressure gradient (a) c_f ,(b) H ,(c) Re_θ.

As shown in Fig. 1, the characteristics of the captured transition qualitatively agree with the empirical formulation. However, the quantitative result is slightly different from the experimental data. This is because the transition model described by the RNG has a fixed value for C_R. On the other hand, in the experiment, the transition does not only depend on the Reynolds number, but also other factors such as turbulence intensity, accoustic noise and so on.

The constant C_R determines transition location not only in the streamwise direction but also in the normal one to the wall, that is, the laminar sub-layer thickness.

Estimation of the laminar sub-layer thickness expressed as y^+_{lam} can be made directly from Eq. (5), by defining $L = \kappa y$. Using $C_R = 75$ will lead to $y^+_{lam} \approx 13.5$ which is close to Fediaevski's [21] prediction ($y^+_{lam} \approx 12$). On the other hand, when C_R is increased to 200, the y^+ is approximately 18. Because y^+_{lam} increases monotonously between $C_R = 75$ and 200, $C_R = 75$ is considered the best value in the present model. All results presented in this paper are obtained using $C_R = 75$.

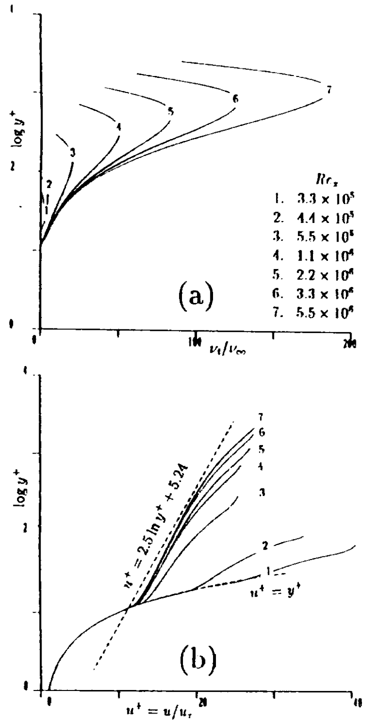

Fig. 3 The growth of turbulent viscosity ν_t/ν and velocity $u^+ = u/u_\tau$ during transition period (a) ν_t/ν, (b) $u^+ = u/u_\tau$.

For $C_R = 75$, the calculated result is expected to be in reasonable agreement with the experimental data which are collected under the conditions of more than 1% turbulence intensity and small pressure gradient.

It is suspected that the present model can give a good prediction for adverse pressure gradient flow problem with less than 2% turbulence intensity even if the pressure gradient is small when $C_R = 75$. Moreover, in the separated flow region the velocity gradient is negative, the turbulent viscosity will be zero in the separated region, since the argument inside the *heaviside* function becomes negative.

All computations were carried out using the popular box-method [2]. The laminar viscosity ν was held constant ($= 1.5 \times 10^{-5} m^2/s$). The original governing equations were transformed into the well known Falkner-Skan equation.

The zero pressure gradient case was simulated with a freestream velocity $U_e = 22 m/s$. The experiment of Gnaham-Shaw [6] shows that transition occurs at $28cm$ from the leading edge. Using the present model, turbulence occurs at $25cm$, which is close to the experimental result.

The local skin friction c_f, the Reynolds number based on the momentum thickness Re_θ and the shape factor H are presented in Fig. 2. Except for the slight difference in skin friction c_f, Re_θ and H are in close agreement with the experiment.

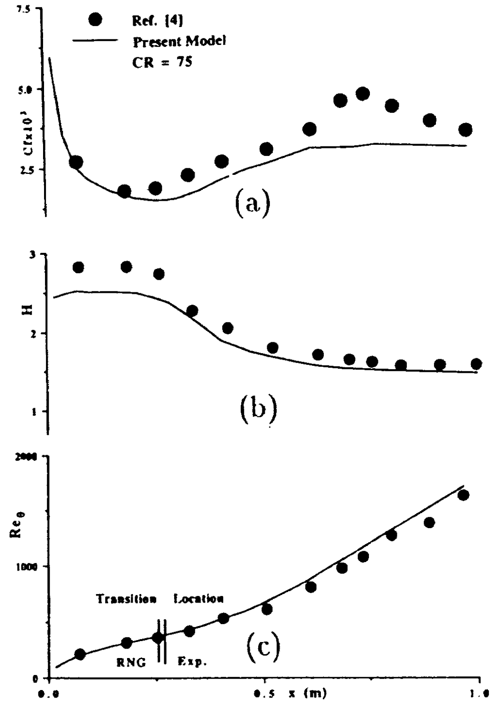

Fig. 4 Boundary layer characteristics at favourable pressure gradient (a) c_f ,(b) H ,(c) Re_θ.

Furthermore, the RNG model can simulate a gradual transitional process from laminar to turbulent flow. The value of c_f is likely to increase gradually until the Reynolds number becomes high enough at which the result merges with the experimental one. In the range where the laminar flow gradually approaches the turbulent flow, the streamwise intermittency appears. Although, in fact, this transitional process is quite different from the intermittency in the normal direction to the wall.

This gradual transitional process is due to the effective turbulent viscosity ν_e. The ν_e is calculated based on the parameter β included in the *heaviside* function. This ν_e is filtered locally with regard to the assigned threshold value, that is, the value of C_R. When β is larger than C_R, the ν_e/ν is directly calculated as a cubic root of $\{1 + \beta - C_R\}$. Otherwise, it becomes unity; that is, $\nu_t = 0$. At a certain streamwise location, where the Reynolds number becomes large enough, the effective turbulent viscosity ν_t begins to have a value.

Figure 3 shows the turbulent viscosity (ν_t/ν) and the related non-dimensionalized velocity distribution ($u^+ = u/u_\tau$) in terms of $y^+ = u_\tau y/\nu$ at several streamwise locations. The dashed lines are the theoretical expressions of velocity for comparison.

When ν_t starts to have a value (see curve 1), the velocity deviates slightly from the fully laminar one. The effect of the growing ν_t can be clearly seen in the outer part of the log-law distribution. As the Reynolds number becomes higher, the magnitude of turbulent viscosity increases, and the velocity comes close to the empirical formulation for the velocity in the turbulent region. It is this spatial growth that prevents the skin friction c_f from discontinuously reaching the value shown by the experimental data. It is conceived that the difference in skin friction in Fig 2. is due to this effect.

Furthermore, the turbulent viscosity predicted by this model is shown to be continuous in the normal direction to the wall and to increase smoothly with the Reynolds number. In addition, the laminar sub-layer is reasonably predicted. Moreover, in the transition region, no oscillations are observed.

One example of favourable pressure gradient cases is presented in Fig. 4. The calculation was performed using the same streamwise pressure distribution as in ref. [6], where the reference velocity is $24.4 m/s$. The transition occurs at the location of $x = 25cm$. This reasonably agrees with the experimental data of Gnaham-Shaw [6] in which the transition occurs at $x = 28cm$. Despite the discrepancy in skin friction, the boundary layer growth is well predicted, which is revealed in the distributions of Reynolds number Re_θ and the shape factor H.

4. Concluding Remarks

- A dissipation rate for the RNG based algebraic turbulence model has been devised. The present model maintained the same cubic equation as the originally proposed one. The variation of length scale inside the boundary layer is defined by considering pressure gradient.
- The eddy viscosity computed with this model is smooth in the region near the wall. The effective viscosity ν_e was shown to depend on the Reynolds number. As the Reynolds number increases, so does ν_e. The gradual increase in ν_e which can be associated with the onset of turbulence seems to represent the intermittency effect.
- The present model has been used to evaluate the transition phenomena for flow with pressure gradient. The characteristics of the captured transition location qualitatively agree with the empirical formulations.
- To see the capability of the present model concerning transition location, the simulated result has been compared with the experimental one. They showed good agreement for cases with pressure gradient.

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