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NUMERICAL SOLUTIONS OF INVISCID & VISCOUS FLOWS ABOUT AIRFOILS BY TVD METHOD

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ABSTRACT

The need to properly compute steady and unsteady viscous flows surrounding airfoils at transonic speeds remains an outstanding problem in fluid dynamics. In transonic flow where viscous effects such as shock-boundary interactions and separation are dominant, methods based on the *Navier-Stokes* equations are needed. Calculations of unsteady transonic flow about oscillating airfoils where flutter, dynamic stall, buffet and moving shock waves on these surfaces change the entire flow fields and aerodynamic characteristics, are still stiff problems that stimulate more work and studies to be done. To simulate these problems correctly a robust computer program is needed. This report shows the works have been done up to now i.e., developing a computer program and verifying it by applying to steady viscous and inviscid flow calculations and inviscid flow about an oscillating airfoil.

INTRODUCTION

With the recent development in super computers either in speed and memory storage a remarkable progress has been made in the field of computational fluid dynamics. Recent improvement in the theoretical and computational fluid dynamics has been largely based on finite difference methods. Several Reynolds-average Navier-Stokes codes for numerical prediction of airfoil flows have been developed and applied to many areas of fluid dynamics, successfully. These schemes are also extended to unsteady Navier-Stokes codes by many researchers. Among them, schemes based on Total Variation Diminishing (TVD) properties are more actively followed and extended to several different problems of fluid dynamics nowadays. The TVD property guarantees that these schemes avoid spurious oscillations but sharp approximation to shocks and discontinuities. Generally, TVD schemes have made a possible very robust algorithm for *Euler* equations and other hyperbolic systems.

The implicit TVD schemes originally developed by *Yee & Harten* [2]and extended for general curvilinear coordinates by Yee et al, for hyperbolic conservation laws have been used as numerical method.

In the present paper this scheme was applied to steady inviscid and viscous flows about NACA0012 airfoil under several cases. Unsteady inviscid calculations using moving grid system around the same airfoil undergoing pitching oscillation about 1/4 chord were also conducted.

For viscous steady-state application, a simple algorithm utilizing the TVD scheme for the Navier-Stokes equations is to difference the hyperbolic terms the same way as for Euler equations, and then central differencing the viscous term. The algebraic *Baldwin-Lomax* [1] turbulence model is coupled to the governing equations. The scheme was also applied to viscous flows for several cases and obtained results and comparison of them with the other results are given in the end.

1. Conservative Form of Governing Equations in Curvilinear Coordinates

Governing equations of viscous fluid flows are Navier-Stokes equations. The strong conservative form of these equations in two dimensions can be written in vector form as follows

$$\frac{\partial \hat{Q}}{\partial t}+\frac{\partial \hat{F}}{\partial \xi}+\frac{\partial \hat{G}}{\partial \eta}=R_a^{-1}\left(\frac{\partial \hat{S}_1}{\partial \xi}+\frac{\partial \hat{S}_2}{\partial \eta}\right), \qquad (1.1)$$

where

$$\hat{Q}=\frac{1}{J}\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ e\end{pmatrix}, \qquad \hat{F}=\frac{1}{J}\begin{pmatrix}\rho U\\ \rho u U+\xi_x p\\ \rho v U+\xi_y p\\ U(e+p)-\xi_t p\end{pmatrix},$$

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$$\hat{G}=\frac{1}{J}\begin{pmatrix}\rho V\\ \rho uV+\eta_x p\\ \rho vV+\eta_y p\\ V(e+p)-\eta_t p\end{pmatrix},$$

$$\hat{S}_1=\frac{1}{J}\begin{pmatrix}0\\ \xi_x\tau_{xx}+\xi_y\tau_{xy}\\ \xi_x\tau_{xy}+\xi_y\tau_{yy}\\ \xi_x R_1+\xi_y R_2\end{pmatrix},\quad \hat{S}_2=\frac{1}{J}\begin{pmatrix}0\\ \eta_x\tau_{xx}+\eta_y\tau_{xy}\\ \eta_x\tau_{xy}+\eta_y\tau_{yy}\\ \eta_x R_1+\eta_y R_2\end{pmatrix},$$

$$U=\xi_x u+\xi_y v+\xi_t$$
$$V=\eta_x u+\eta_y v+\eta_t \qquad (1.2)$$

where J is the *JACOBIAN* of transformation and

$$\tau_{xx}=(\lambda+2\mu)\frac{\partial u}{\partial x}+\lambda\frac{\partial v}{\partial y},$$
$$\tau_{yy}=\lambda\frac{\partial u}{\partial x}+(\lambda+2\mu)\frac{\partial v}{\partial y},$$
$$\tau_{xy}=\tau_{yx}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right), \qquad (1.3)$$
$$R_1=u\tau_{xx}+v\tau_{yx}+\frac{\mu}{P_r(\gamma-1)}\frac{\partial a^2}{\partial x},$$
$$R_2=u\tau_{xy}+v\tau_{yy}+\frac{\mu}{P_r(\gamma-1)}\frac{\partial a^2}{\partial y},$$
$$\lambda=-\frac{2}{3}\mu .$$

Pressure is related to the conservative flow variables, Q, by the equation of state

$$p=(\gamma-1)\left[\frac{e}{\rho}-\frac{1}{2}\left(u^2+v^2\right)\right]. \qquad (1.4)$$

2. NUMERICAL METHOD

2.1. Description of Algorithm

Here the algorithm for inviscid equation (Euler Eqs) is expressed and application to viscous flow is given in the following. In generalized coordinates conservation form of the governing equations for inviscid flow has the form

$$\frac{\partial\hat{Q}}{\partial t}+\frac{\partial\hat{F}}{\partial\xi}+\frac{\partial\hat{G}}{\partial\eta}=0. \qquad (2.1)$$

Let $\hat{A}$ and $\hat{B}$ be Jacobians of $\hat{F}$ and $\hat{G}$ as $\hat{A}=\frac{\partial\hat{F}}{\partial\hat{Q}}$, $\hat{B}=\frac{\partial\hat{G}}{\partial\hat{Q}}$ and let the eigenvalues of $\hat{A}$ be $(\Lambda^1{}_\xi,\Lambda^2{}_\xi,\ldots,\Lambda^4{}_\xi)$ and the eigenvalues of $\hat{B}$ be $(\Lambda^1{}_\eta,\Lambda^2{}_\eta,\ldots,\Lambda^4{}_\eta)$

Denote $\hat{R}_\xi$ and $\hat{R}_\eta$ as the matrices whose columns are eigenvectors of $\hat{A}$ and $\hat{B}$, and denote $\hat{R}_\xi^{-1}$ and $\hat{R}_\eta^{-1}$ as the inverse of $\hat{R}_\xi$ and $\hat{R}_\eta$.

Let the grid spacing be denoted by $\Delta\xi$ and $\Delta\eta$ such that $\xi=j\Delta\xi$ and $\eta=k\Delta\eta$. Define $\hat{Q}_{j+1/2,k}$ as some symmetric average of $\hat{Q}_{j,k}$ and $\hat{Q}_{j+1,k}$ (for example *Roe's* average for gas dynamic[8]). Let $\alpha^l_{j+1/2,k}$, $\hat{R}_{j+1/2,k}$, $\hat{R}^{-1}_{j+1/2,k}$ denote the quantities α^l_ξ , $\hat{R}_\xi$, $\hat{R}_\xi^{-1}$ related to $\hat{A}$ evaluated at $\hat{Q}_{j+1/2,k}$. Similarly the indices ($k+1/2$) show values evaluated at k and $k+1$.

Define $\alpha_{j+1/2,k}$ as the difference of the characteristic variables in the local ξ direction and $\alpha_{j,k+1/2}$ in η direction as $\alpha_{j+1/2,k}=\hat{R}^{-1}_{j+1/2,k}\dfrac{\hat{Q}_{j+1,k}-\hat{Q}_{j,k}}{0.5*(J_{j+1,k}+J_{j,k})}$.

With the above notation, a one parameter family of TVD scheme can be written as

$$\hat{Q}^{n+1}_{j,k}+\lambda^\xi\theta\left[\tilde{F}^{n+1}_{j+1/2,k}-\tilde{F}^{n+1}_{j-1/2,k}\right]+\lambda^\eta\theta\left[\tilde{G}^{n+1}_{j,k+1/2}-\tilde{G}^{n+1}_{j,k-1/2}\right]$$
$$=\hat{Q}^n_{j,k}-\lambda^\xi(1-\theta)\left[\tilde{F}^n_{j+1/2,k}-\tilde{F}^n_{j-1/2,k}\right]$$
$$-\lambda^\eta(1-\theta)\left[\tilde{G}^n_{j+1/2,k}-\tilde{G}^n_{j-1/2,k}\right] \qquad (2.2)$$

Where θ is a parameter, $\lambda^\xi=\Delta t/\Delta\xi$, and $\lambda^\eta=\Delta t/\Delta\eta$. A space second-order form of the numerical flux function $\tilde{F}_{j+1/2}$ can be expressed as

$$\tilde{F}_{j+1/2}=\frac{1}{2}\left[\hat{F}_{j,k}+\hat{F}_{j+1,k}+\hat{R}_{j+1/2,k}\Phi_{j+1/2,k}\right], \qquad (2.3)$$

where the element of the $\Phi_{j+1/2,k}$ denoted by $\phi^l_{j+1/2,k}$, ($l=1,\ldots,m$) are

$$\phi^l_{j+1/2,k}=\frac{1}{2}\sigma\left(\Lambda^l{}_{j+1/2,k}\right)\left(g^l_{j,k}+g^l_{j+1,k}\right)-\psi\left(\Lambda^l{}_{j+1/2,k}+\gamma^l_{j+1/2,k}\right)\alpha^l_{j+1/2,k}, \qquad (2.4)$$

with

$$g^l_{j,k}=S.\max\left[0,\min\left(\left|\alpha^l_{j+1/2,k}\right|,S.\alpha^l_{j-1/2,k}\right)\right],$$
$$S=sign\left(\alpha^l_{j+1/2,k}\right). \qquad (2.5)$$

The function $\psi(z)$ is sometimes referred to as the coefficient of numerical viscosity.
and can be defined as

$$\psi(z) = \begin{cases} |z| & |z| \geq \varepsilon \\ \left(z^2 + \varepsilon^2\right)/2\varepsilon & |z| < \varepsilon \end{cases}, \tag{2.6}$$

where ε is a small positive number and

$$\gamma^l_{j+1/2,k} = \frac{1}{2}\sigma\left(\Lambda^l_{j+1/2,k}\right)\begin{cases}\left(g^l_{j+1,k} - g^l_j\right)/\alpha^l_{j+1/2,k} & \alpha^l_{j+1/2,k} \neq 0 \\ 0 & \alpha^l_{j+1/2,k} = 0\end{cases}, \tag{2.7}$$

where $\alpha^l_{j+1/2,k}$ are elements of Eq.(2.4). The scheme is first-order accurate in time for steady state calculation with the selection of $\sigma(z)$ as $\sigma(z) = \frac{1}{2}\psi(z)$ and second-order accurate in time suitable for time accurate calculation with $\sigma(z) = \frac{1}{2}\psi(z) + \lambda\left(\theta - \frac{1}{2}\right)z^2$. Similarly we can define the numerical flux $\tilde{G}_{j,k+1/2}$.

The scheme (2.2) is a mixed implicit-explicit scheme. When $\theta = 0$, Eq.(2.5) is an explicit method; when $\theta = 1$ is an implicit scheme and if $\theta = 1/2$, the time differencing is the trapezoidal formula, and scheme is second-order in time.

2.2. ADI Form of Linearized Scheme

An ADI form of this equation will be adopted and can be expressed in conservation form as

$$\begin{aligned}&\left[I + \lambda^\xi \theta H^x_{j+1/2,k} - \lambda^\xi \theta H^x_{j-1/2,k}\right]D^* = \\ &-\lambda^\xi\left[\tilde{F}^n_{j+1/2,k} - \tilde{F}^n_{j-1/2,k}\right] - \lambda^\eta\left[\tilde{G}^n_{j,k+1/2} - \tilde{G}^n_{j,k+1/2}\right] \\ &\left[I + \lambda^\eta \theta H^y_{j,k+1/2} - \lambda^\eta \theta H^y_{j,k-1/2}\right]D = D^*, \quad \hat{Q}^{n+1} = \hat{Q}^n + D.\end{aligned} \tag{2.8}$$

where

$$\begin{aligned}H^x_{j+1/2,k} &= 1/2\left[\hat{A}_{j+1,k} + \Omega^x_{j+1/2,k}\right]^n, \\ H^y_{j,k+1/2} &= 1/2\left[\hat{B}_{j,k+1} + \Omega^y_{j,k+1/2}\right]^n,\end{aligned} \tag{2.9}$$

and

$$\begin{aligned}\Omega^x_{j+1/2} &= \left(R_x diag\left[\beta^l - \psi\left(\Lambda^l + \gamma^l\right)\right]R_x^{-1}\right)_{j+1/2}\Delta_{j+1/2}, \\ \Omega^y_{k+1/2} &= \left(R_y diag\left[\beta^l - \psi\left(\Lambda^l + \gamma^l\right)\right]R_y^{-1}\right)_{k+1/2}\Delta_{k+1/2}, \quad l = 1,\dots,m\end{aligned} \tag{2.11.a}$$

where

$$\beta^l_{j+1/2} = \frac{\left(g^l_j + g^l_{j+1}\right)}{\alpha^l_{j+1/2}}, \qquad \beta^l_{k+1/2} = \frac{\left(g^l_k + g^l_{k+1}\right)}{\alpha^l_{k+1/2}}. \tag{2.11.b}$$

2.3. Application of TVD Scheme to Viscous Flow

TVD schemes (2.8) can be applied to Navier-Stokes equations by simply adding a second-order central finite difference discretization of viscous terms to the right hand side of this equation for full Navier-Stokes Eqs. For Thin-Layer approximation first we linearize viscous term in η direction as

$$R_e^{-1}\partial_\eta \hat{S}^{n+1} = R_e^{-1}\partial_\eta\left(\hat{S}^n + J^{-1}\hat{M}^n\hat{Q}^{n+1}\right). \tag{2.12}$$

where $J^{-1}\hat{M}^n = \dfrac{\partial\hat{S}^n}{\partial\hat{Q}^n}$,

then we add central finite difference of (2.12) to (2.8)

3. BOUNDARY CONDITIONS

Boundary Conditions for Euler and Navier-Stokes Solutions

A particular set of boundary conditions employed in airfoil computations are described below. The geometry is mapped onto the computational rectangle such that all the boundary surfaces are edge of the rectangle. This application is for a "C" mesh topology.

Initial condition is set equal to nondimensional free stream values of $\hat{Q}$ plus boundary conditions. Stretched grids are usually used to place far field boundaries far away from the body surface. When bow shocks and attached shocks are generated at a body surface care is taken to ensure that the shocks are sufficiently weak when they reach far field boundaries so that they are not reflected or at least they reflect outside the flow domain. At outer boundaries stream values are set equal to free stream values or can be calculated from *Riemann* invariants.

At a rigid body surface, tangency must be satisfied for inviscid flow and the no slip condition for viscous flow. In two dimensions, body surfaces are usually mapped to η = constant coordinates. The normal component of velocity in terms of the curvilinear metrics is given by

$V_n = \dfrac{\eta_x u + \eta_y v}{\sqrt{\left(\eta_x^2 + \eta_y^2\right)}}$ and the tangential component by

$V_t = \dfrac{\eta_y u - \eta_x v}{\sqrt{\left(\eta_x^2 + \eta_y^2\right)}}$. Therefore, tangency is satisfied by $V_n = 0$ (no flow through the body). The tangential velocity V_t is obtained at the body surface through linear extrapolation for inviscid cases and is set equal to zero for viscous cases. The Cartesian velocities are then formed from the inverse relation between them which leads to

$$\begin{pmatrix} u \\ v \end{pmatrix} = \frac{1}{\sqrt{\left(\eta_x^2 + \eta_y^2\right)}}\begin{bmatrix} \eta_y & \eta_x \\ -\eta_x & \eta_y \end{bmatrix}\begin{pmatrix} V_t \\ V_n \end{pmatrix}. \tag{3.1}$$

The extrapolation of V_t procedures less error if the mesh lines are clustered to the body surface. The normal and tangential velocities in Eq(3.1) are scaled such that the metric variations are removed which decrease the errors in the extrapolations for nonorthogonal meshes.

The pressure on the body surface is obtained from the normal momentum equation. In viscous flow either steady and unsteady, the contravariant velocities U , V on the body surface are zero and the equation can be written as

$$\rho\left[-\ddot{x}\eta_x-\ddot{y}\eta_y\right]+R_e^{-1}\left(2\eta_x\eta_y\tau_{xy_\eta}+\eta_x^2\tau_{xx_\eta}+\eta_y^2\tau_{yy_\eta}\right)=p_\xi\left(\eta_x\xi_x+\eta_y\xi_y\right)+p_\eta\left(\eta_x^2+\eta_y^2\right) \quad (3.2)$$

where $\ddot{x},\ddot{y}$ are second derivatives with respected to time.

In inviscid flow the viscous terms and only V are equal to zero. In this case the normal momentum equation for inviscid flow can be simplified to

$$\rho\left[-\ddot{x}\eta_x-\ddot{y}\eta_y\right]-\rho U\left(\eta_x u_\xi+\eta_y v_\xi\right)=p_\xi\left(\eta_x\xi_x+\eta_y\xi_y\right)+p_\eta\left(\eta_x^2+\eta_y^2\right). \quad (3.3)$$

For subsonic outflow boundary, three eigenvalues of A are positive and one is negative thus one of the stream variables can be determined from down stream and three other one from extrapolation. We set the pressure equal to free stream value

4. UNSTEADY SOLUTION OF EULER EQUATIONS

In this method[3], first a steady state grid is generated around the airfoil at mean angle of attack and since the motion of airfoil undergoing a force oscillation is described in advance as a function of time, the instantaneous gird is generated at each time step. The displacement of grid points on the surface is determined from the equation of motion of airfoil and then whole grid system is updated in the following manner

$$\begin{aligned} x_{j,k}^{n+1} &= x_{j,k}^n+\Delta x_{j,1}^n S(s), \\ y_{j,k}^{n+1} &= y_{j,k}^n+\Delta y_{j,1}^n S(s), \end{aligned} \quad (4.1)$$

where $x_{j,k}^n, y_{j,k}^n$ are the coordinates of grid points at $t=n\Delta t$ and $x_{j,k}^{n+1}, y_{j,k}^{n+1}$ are those at time $t=(n+1)\Delta t$. The displacements of grid points on surface $\Delta x_{j,1}^n$, $\Delta y_{j,1}^n$ are calculated by

$$\begin{aligned} \Delta x_{j,1}^n &= x_{j,1}^{n+1}-x_{j,1}^n, \\ \Delta y_{j,1}^n &= y_{j,1}^{n+1}-y_{j,1}^n. \end{aligned} \quad (4.2)$$

In Eq.(4.1) S is a function of s which is the distance of a grid point from body surface along a grid line. Function S(s) is chosen as

$$\begin{aligned} S(s) &= 1 && \text{for } s<s_{ref} \\ S(s) &= \sqrt{1-\Xi^2} && \text{for } s>s_{ref}, \end{aligned} \quad (4.3)$$

where

$$\Xi=\left(s-s_{ref}\right)/\left(s_{max}-s_{ref}\right), \quad (4.4)$$

where s_{ref} is some adequate value between zero and one. Here $2/3s_{max}$ is selected.

In this method outer boundaries are hold constant when the inner boundary changes with airfoil motion. Grid points near to the surface move in the same motion as that of airfoil surface and the displacements of grid points far from surface decreases as we move away from the surface and vanished at outer boundary. It is needed in this method to calculate Jacobian and metrics at each time step. The boundary conditions for inviscid calculation on the body surface are implemented as follow

$$V_n=0,$$

$$\begin{pmatrix} u+x_t \\ v+y_t \end{pmatrix}=\frac{1}{\sqrt{\left(\eta_x^2+\eta_y^2\right)}}\begin{bmatrix} \eta_y & \eta_x \\ -\eta_x & \eta_y \end{bmatrix}\begin{pmatrix} V_t \\ V_n \end{pmatrix}. \quad (4.5)$$

where x_t, y_t surface velocities are calculated numerically at time step "n" and" n+1". As steady state calculation pressure is computed from normal momentum Eq.(3.3).

A first-order backward difference approximation of accelerations can be written as

$$\begin{aligned} \ddot{x}^{n+1} &= \frac{x^{n+1}-2x^n+x^{n-1}}{\Delta t^2}, \\ \ddot{y}^{n+1} &= \frac{y^{n+1}-2y^n+y^{n-1}}{\Delta t^2}. \end{aligned} \quad (4.6)$$

To satisfy Kutta condition physical velocities at trailing edge are set equal to airfoil surface velocities

5. NUMERICAL RESULTS

A C-mesh topology with 251x41 grids for inviscid steady flows, 299x71 for unsteady inviscid flow and with 259x61 grid points about NACA0012 airfoil have been used in all of calculations.

Since the actual grids have widely varying cell size a space varying Δt is used as a vehicle to improve convergence rate. For inviscid steady flows it is considered as $\Delta t = \Delta t_{ref}/1.+\sqrt{J}$, and for steady viscous flows a $\Delta t = \Delta t_{ref}/\left(|U|+|V|+\sqrt{\xi_x^2+\xi_y^2+\eta_x^2+\eta_y^2}\right)$.

The ε is set equal to .125 for all cases. The cases considered here for inviscid flow are a) M=.8, α=1.25 b) M=.85 α=1.00 and obtained results are compared with results obtained by Yee & Harten. The numerical results are given in Fig.1,2 which show good agreement with those of Harten The required CPU time is 0.6 sec/iter on single processor of NWT.

The second case considered here was unsteady inviscid flow calculation about airfoil undergoing pitching oscillation about 1/4 chord with the reduced frequency of 0.1628, Mach number of .755, amplitude 2.51 deg and zero incidence. The mesh is regeneaed at each time step by the method mentioned earlier. Fig.3 shows how the mesh is renewed by this method at two different time step. Time history of aerodynamic coefficients for six cycles, pressure and Mach contours for some instances are given in Fig.4 and Fig.5. This calculation requires 1510 iterations for one cycle and 15 minutes CPU time.

Finally steady viscous flows about the same airfoil were considered. Some of the obtained results are given here. Fig.6 show pressure and friction coefficient at four different cases and Fig.7 pressure coefficient and mach contour of two cases. Comparison of current results with the other numerical results and also experimental results are given in table.1. The obtained results are close to the others. For viscous flows second order accurate in space and time show better stability. The minimum distance of the first grid away from surface was considered about order of $.1/R_e$ to resolve boundary layer. Required CPU time in this case is 1.6 sec/iter.

CONCLUSION

A computer program based on TVD scheme was developed and applied to inviscid and viscous flows. Current results show good agreement with the other results. With more improvement in convergence rate this code is applicable to unsteady viscous flow calculations such as flutter or dynamic stall prediction which were our objectives for doing this work.

Finally I appreciate Mr. Kawai (of NAL) for his permiting to use his block thridiagonal solver.

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FIGURES

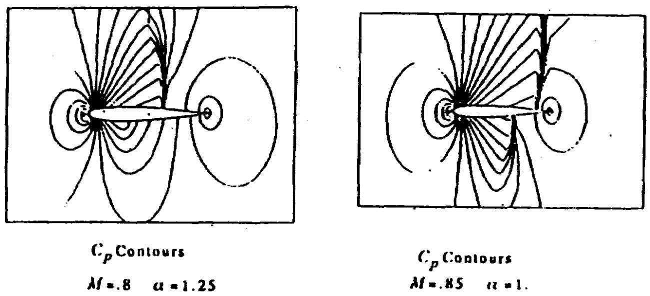


Fig. 1. Inviscid flow solutions about NACA0012

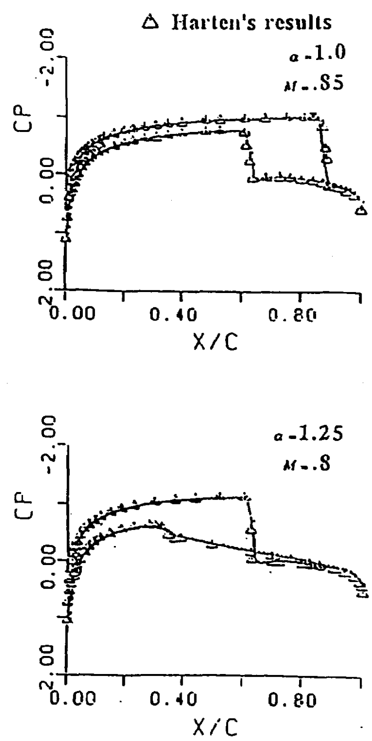


Fig. 2. Comparision of current results with those of Harten

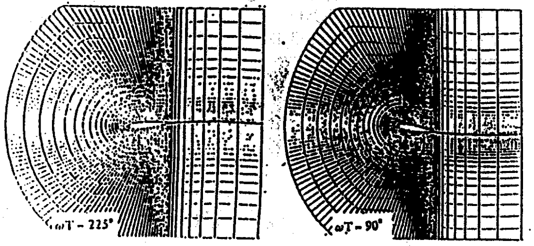


Fig. 3. Grid Distributions Around NACA0012 in Unsteady Calculations at Two Instances Using Moving Grid System

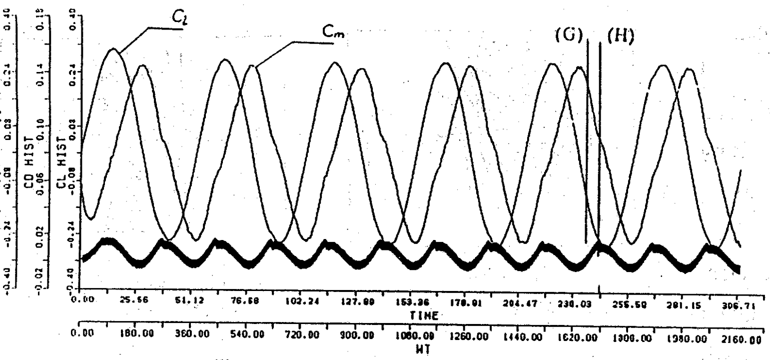
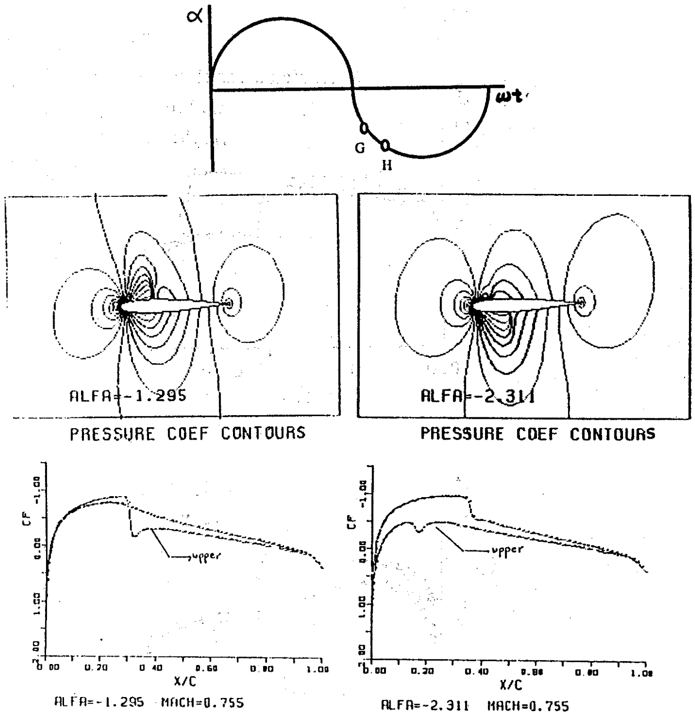


Fig. 4. Unsteady Inviscid Solution About NACA0012 Airfoil in Pitching Oscillation(f=.1628,Xa=.25).
$\alpha = 0.0 + 2.51\sin(\omega t)$, $M = .03384$, $M = .755$
C_l, C_m, C_d History



(G) (H)

Fig. 5. Unsteady Inviscid Flow Results at Two Time Levels at Sixth Cycle

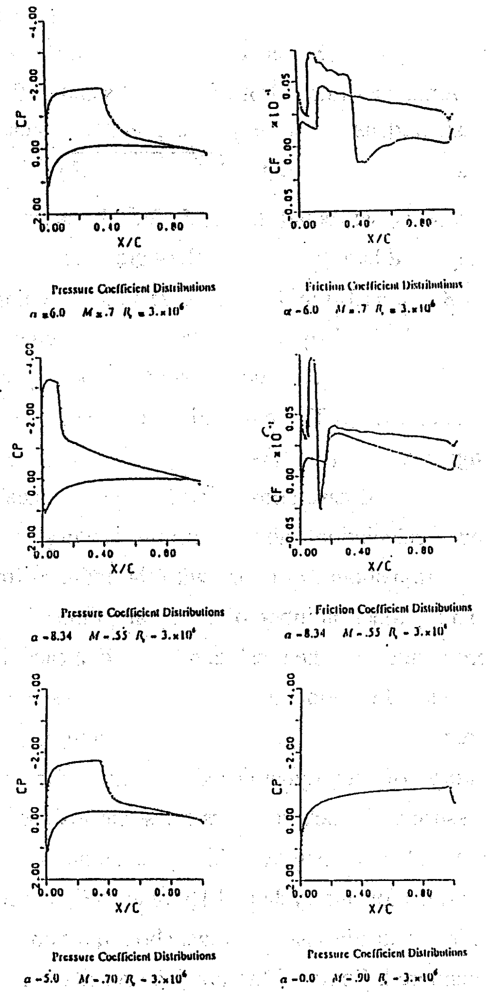
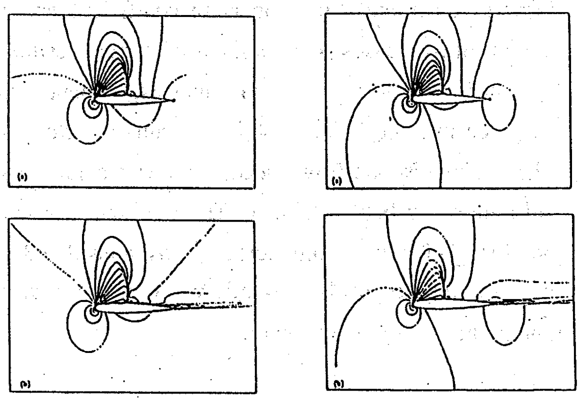


Fig.6 Numerical Results of Navier-Stokes Eqs.



(a) C_p Contours (b) Mach Contours
$M = .7$ $\alpha = 5.$ $R_e = 3.\times10^6$

(a) C_p Contours (b) Mach Contour
$M = .7$ $\alpha = 6.$ $R_e = 3.\times10^6$

Fig.7 Numerical Solutions of N-S Eqs. pressur coeff and Mach number Contours

Table. 1. Comparison of current viscous results with numerical results of coekley AIAA 87-0412 and experimental results of Harris NASA TM-81927

comparison of current results								
current results				Coekley		Harris		
M	α	C_l	C_d	C_l	C_d	α_{exp}	C_l	C_d
0.2	0.00		0.0118					
0.55	8.34	0.9815	0.0452	0.994	0.0358	9.86	.983	.0253
0.70	0.00		0.0088		0.0081			
0.70	5.00	0.7267	0.0334	0.766	0.0428			
0.70	6.00	0.8338	0.0645	0.850	0.0621			
0.755	0.00		0.0198					
0.799	2.26	0.4135	0.0428	0.476	0.0446	2.86	.39	.0331
0.8	0.00		0.028		0.0153			
0.9	0.00		0.1139		0.1165			