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Gust Response Analysis for SST with Navier-Stokes Equations

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ABSTRACT

Based on Navier-Stokes equations and structural and flight dynamic equations of motion, dynamic responses in vertical gust flow perturbation are investigated for a supersonic transport. A tightly coupled method was developed by the subiteration between aerodynamic equations and dynamic equations of motion. First, the results of direct-coupling method are compared with the results of two model methods. Then gust responses for the one-minus-cosine gust profile are analyzed for the rigid and flexible airplane configurations.

1. Introduction

Gust load is one of the important dynamic loads considered in aircraft structure design. Due to its multidisciplinary nature with aerodynamics, flight dynamics, aeroelasticity and atmospheric turbulence, up to now, only the doublet-lattice, unsteady linear aerodynamic code (DLM) coupled with the equation of motion of flexible vehicle was used for the gust response analysis [1-4].

Gusts in nature tend to random. The early design methods for gust loads were based on a single discrete gust having one-minus-cosine velocity profile. Recently the statistical discrete gust (SDG) method and the power spectral density (PSD) method [5] in the frequency domain are used to define the gust loads, however, which are still hard to combine with the modern Navier-Stokes numerical method.

In the paper, the fully implicit multiblock Navier-Stokes aeroelastic solver implemented by the authors [6], coupled with the flight and structural dynamic equations of motion, has been developed to simulate gust dynamic responses for the supersonic transport (SST) designed by National Aerospace Laboratory of Japan (NAL) [7]. To study the effects of dynamic response due to flow perturbation and airplane motion, a comparative study was first done for the rigid airplane in the harmonic flow perturbation with the direct-coupling method and other model methods. Then the gust responses in a one-minus-cosine gust velocity profile are analyzed for the rigid and flexible airplane models.

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2. Aerodynamic Equations and Numerical Method

Aerodynamic governing equations are the unsteady, three-dimensional thin-layer Navier-Stokes equations in strong conservation law form, which can be written in curvilinear coordinates as

$$\partial_t \hat{Q} + \partial_\xi F + \partial_\eta G + \partial_\zeta H = \partial_\zeta H_v + S_{GCL} \tag{1}$$

The source term S_{GCL} is obtained from the geometric conservation for a moving mesh. In the formulation, all variables are normalized by the appropriate combination of freestream density, freestream velocity and mean aerodynamic chord length. The viscosity coefficient μ in H_v is computed as the sum of laminar and turbulent viscosity coefficients, which are evaluated by the Sutherland's law and Baldwin-Lomax model.

LU-SGS method, employing a Newton-like subiteration, is used for solving Equation 1. Second order temporal accuracy is obtained by utilizing three-point backward difference in the subiteration procedure. The numerical algorithm can be deduced as

$$\begin{aligned} &LD^{-1}U\Delta Q \\ &= -\phi^i\{(1+\phi)Q^p - (1+2\phi)Q^n + \phi Q^{n-1} \\ &-J\Delta t Q^p S_{GCL}^p + J\Delta t(\delta_\xi F^p + \delta_\eta G^p + \delta_\zeta(H^p - H_v^p))\} \end{aligned} \tag{2}$$

where

$$L = \overline{\rho} I + \phi^i J\Delta t(A_{i-1,j,k}^+ + B_{i,j-1,k}^+ + C_{i,j,k-1}^+)$$

$$D = \overline{\rho} I$$

$$U = \bar{\rho}I - \phi^i J\Delta t(A^-_{i+1,j,k} + B^-_{i,j+1,k} + C^-_{i,j,k+1})$$

and

$$\bar{\rho} = 1 + \phi^i J\Delta t(\bar{\rho}(A) + \bar{\rho}(B) + \bar{\rho}(C))$$

$$\phi^i = 1/(1+\phi)$$

$$\Delta Q = Q^{p+1} - Q^p$$

Here $\phi = 0.5$ and Q^p is the subiteration approximation to Q^{n+1}. As $p \to \infty$, $Q^p \to Q^{n+1}$. The deduced subiteration scheme reverts to the standard first-order LU-SGS scheme for $\phi = 0$ and $p = 1$. In the following calculations, the number of subiteration is set to 3.

The inviscid terms in Equation 1 are approximated by the modified third-order upwind HLLEW scheme of Obayashi et al [8]. For the isentropic flow, the scheme results in the standard upwind-biased flux-difference splitting scheme of Roe, and as the jump in entropy becomes large in the flow, the scheme turns into the standard HLLEW scheme. Thin-layer viscous term in Equation 1 is discretized by second-order central difference.

For multiblock-grid application, the Navier Stokes equations are solved in each block separately. To calculate the convective and viscous fluxes in the block boundary, data communication is performed through two-level halo cells. The detail about the multiblock Navier-Stokes solver can be found in references [6] [9].

3. Equations of Motion and Numerical Method

In the present study of dynamic response, the airplane is permitted freedom in vertical translation and pitch, and the following assumptions are made,

1. The disturbed motion is symmetrical with respect to the airplane's longitudinal plane of symmetry.
2. The airplane is initially in horizontal flight at cruise velocity.
3. The vertical flow perturbation is normal to the flight path, and is uniform in the spanwise direction.
4. The deformation of the wing is approximated to the elastic plate model.

3.1 Direct-coupling method

With the above assumptions, the equilibriums of total force along the z-axis and total pitching moment about the y-axis are:

$$\iint_S \ddot{w}(x,y,t)\rho dxdy = \iint_S \Delta p(x,y,t)dxdy \quad (3a)$$

$$\iint_S \ddot{w}(x,y,t)\rho x dxdy = \iint_S \Delta p(x,y,t)xdxdy \quad (3b)$$

For the equilibrium of an element, we obtain:

$$w(x,y,t) - w(0,0,t) - x\frac{\partial w(0,0,t)}{\partial x} = \iint_S C(x,y;\xi,\eta)[\Delta p(\xi,\eta,t) - \rho(\xi,\eta)\ddot{w}(\xi,\eta,t)]d\xi d\eta \quad (3c)$$

In the system of equations, the unknown quantity is $w(x,y,t)$, which represents the disturbed displacement of elastic airplane from its original equilibrium configuration. The pressure change of $\Delta p(x,y,t)$ based on cruise condition is calculated by the aerodynamic equations, which depends on the instantaneous values of the displacement, velocity, acceleration of airplane, as well as the past history of the motion.

Introducing natural modes, we have,

$$w(x,y,t) = \sum_{i=1}^{n} \phi_i(x,y)q_i(t) \quad (4)$$

where $\phi_i(x,y)$ is normalized natural mode shapes of the airplane including rigid modes and $q_i(t)$ normal coordinate. Then Equations (3a-3c) can be deduced to

$$\ddot{q}_i + 2\zeta_i\omega_i\dot{q}_i + \omega_i^2 q_i = F_i / M_i \quad (5)$$

$$(i = 1,2,\cdots,n; \omega_1 = \omega_2 = 0)$$

with the initial conditions $q_i(0) = \dot{q}_i(0) = 0$ and where

$$M_i = \iint_S \phi_i^2(x,y)\rho(x,y)dxdy$$

$$F_i = \iint_S \Delta p(x,y,t)\phi_i(x,y)dxdy$$

It can be seen that the generalized mass M_1 represents the mass of airplane, and q_1 the plunging displacement. Similarly, M_2, q_2 represent the pitching moment of inertia and angular displacement in pitch, respectively.

The same subiteration method for the aerodynamic equation can be used for Equation 5. The resulting numerical scheme is

$$
\begin{bmatrix} 1 & -\phi^i \Delta t \\ \phi^i \Delta t \omega_i^2 & 1+2\phi^i \omega_i \zeta_i \Delta t \end{bmatrix} \Delta S
$$
$$
= -\phi^i \{(1+\phi)S^p - (1+2\phi)S^n + \phi S^{n-1}
$$
$$
+ \Delta t \begin{bmatrix} 0 & -1 \\ \omega_i^2 & 2\omega_i \zeta_i \end{bmatrix} S^p - \Delta t \begin{bmatrix} 0 \\ F_i^p / M_i \end{bmatrix} \} \tag{6}
$$

where $S = [q, \dot{q}]$, $\Delta S = S^{p+1} - S^p$.

As $p \to \infty$, a full implicit second-order temporal accuracy scheme for the numerical simulation of dynamic response is formed by the coupling solutions of Eq. 2 and Eq. 6. In the following calculation, the number of subiteration is set to 3.

If the airplane is assumed as the rigid configuration, then only the first two equations of Eq. 5 coupled with the aerodynamic equations need to be solved. If the pitching motion can be further neglected, the dynamic response is only considered in the motion of vertical translation. For the simpler case, two model methods can be introduced as follows.

3.2 Unsteady model method

Assume the airplane is rigid and is permitted freedom only in vertical translation, the flight dynamic equation of motion can be written as

$$
M\ddot{z} = L - Mg \tag{7}
$$

M is the total mass of the vehicle and L the total lift. z is the vertical displacement (positive upward). The equation after normalized similar to the flow governing equations becomes as:

$$
\ddot{z} = C_1 C_L - C_2 \tag{8}
$$

$$
C_1 = \frac{\rho_\infty Sc}{2M}, \quad C_2 = \frac{gc}{V_\infty^2}
$$

If the lift coefficient in the equation is obtained from the pre-calculation for the fixed airplane in the same vertical flow perturbation, then the equation of motion can be solved independently. Through the comparison of this method with the direct-coupling method, the effect of dynamic responses neglecting airplane motion in buildup of lift can be studied.

3.3 Quasi-steady model method

If the time lag in buildup of lift is neglected and the incremental lift is considered only due to the change of angle of attack, then the model equation of motion 7 can be further written as.

$$
M\ddot{z} = \frac{1}{2}\rho_\infty V_\infty^2 S C_{L\alpha} \left(\frac{w(t)}{V_\infty} - \frac{\dot{z}}{V_\infty} \right) \tag{9}
$$

Here $w(t)$ represents the vertical perturbation velocity profile. The normalized equation can be written as:

$$
\ddot{z} + C_1 C_{L\alpha} \dot{z} = C_1 C_{L\alpha} w(t) \tag{10}
$$

$C_{L\alpha}$ is the derivative of lift coefficient which is determined by steady flow calculations.

4. Results and Discussions

Dynamic responses in vertical flow perturbation are studied for the SST wing/fuselage model [7]. For the experimental aircraft, the fuselage length is 11.5m, the mean aerodynamic chord 2.754m, the reference area $S = 10.12m^2$. The design cruise point is at $M_\infty = 2.0$, $\alpha = 2°$ and $\mathrm{Re} = 27.5 \times 10^6$. In the calculation, the flight altitude of the airplane is assumed at the 8,000m from sea level. The H-H type multiblock grid with 30 blocks was generated for the SST configuration. The aircraft is initially assumed at cruise flight, and then encounters a gust turbulence atmosphere. So the calculation of gust dynamic response needs to start from the cruise steady flowfield.

The cruise lift coefficient at cruise condition $M_\infty = 2.0$, $\alpha = 2°$ is $C_{L0} = 0.108$, which is in correspondence with the experimental value of 0.110 (the experimental model contains horizontal and vertical wings). To determine the derivative of lift coefficient for the quasi-steady model equation of motion, the steady flow at $M_\infty = 2.0$, $\alpha = 1°$ is also calculated. The calculated lift coefficient is 0.073, which also agrees with the experimental value of 0.0745. The derivative of lift coefficient can be approximately calculated as $C_{L\alpha} = 2.0$.

In addition, at the cruise flight, due to the equilibrium of various forces, the cruise lift should be equal to the total weight of the airplane. The total mass of the aircraft can be calculated with

$$M = \frac{1}{2g} \rho_\infty V_\infty^2 S C_{L0} \quad (11)$$

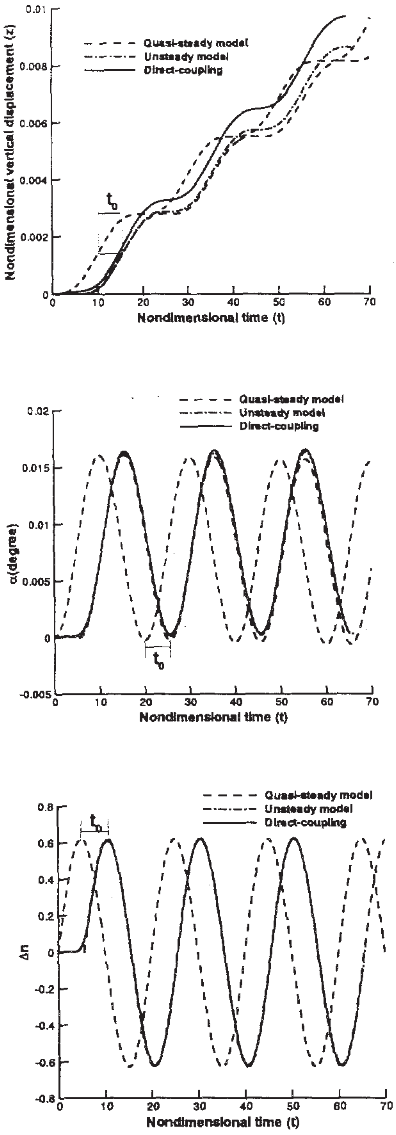

Fig. 1 Time histories of vertical displacement, angle of attack and incremental load factor for SST at $M_\infty = 2.0$, $\alpha = 2°$

4.1 Supersonic dynamic response at $M_\infty = 2.0$

As the aircraft is cruising at $M_\infty = 2.0$, $\alpha = 2°$, a vertical harmonic flow perturbation is added to the aircraft with

$$w(t) = w_0 \sin(\omega t) \quad (12)$$

the amplitude of the flow perturbation is taken as $w_0 = V_\infty / 30$ and the reduced frequency $k = \omega c / V_\infty = 0.314$.

The airplane is assumed as rigid body and only the vertical translation is considered. Dynamic responses are calculated with the above three methods. For the solution of the equations of motion, the initial conditions are assumed as $z_{t=0} = 0, \dot{z}_{t=0} = 0$. Fig. 1 shows the time histories of vertical displacement $z(t)$, angle of attack due to motion $\alpha(t) \approx \dot{z}(t)/V_\infty$ and incremental load factor $\Delta n(t) = \ddot{z}(t)/g$. For the comparison, the dynamic responses of 'quasi-steady model' method after the translation of lag time $t_0 = 5.543$ are also depicted in the figure. If the time lag of the 'quasi-steady model' method can be ignored, the time histories of angle of attack and incremental load factor show nearly no difference although the vertical displacement of 'direct-coupling' method increases with the time a little faster than other two model methods. In fact, the incremental load factor is equivalent to the lift coefficient, comparing the incremental load factor of direct-coupling solution with the unsteady model method. It indicates, in this case, the contribution for the buildup of lift due to airplane motion is small and can be neglected.

Through the above comparison, although all the three methods can be used for dynamic response analyses, the computational expenses are completely different. Comparing the unsteady model method and direct-coupling method, the time cost of the quasi-steady method can be ignored, but the lag time is not known before the other method is implemented.

4.2 Transonic dynamic response at $M_\infty = 0.9$

The SST experimental model is designed for cruise flight at $M_\infty = 2.0$, $\alpha = 2°$ and the flight altitude 15,000m. Because there is nearly no gust at that high altitude, in the above study, the cruise altitude 8,000m is assumed and the total mass of airplane is calculated with the lift coefficient at the corresponding condition. Due to the strong nonlinearity of transonic flows, the transonic dynamic response may be interested. To investigate the dynamic response at a transonic Mach number, here, we still assume the airplane can cruise at $M_\infty = 0.9$, $\alpha = 2°$ and flight altitude 8,000m, which means the total mass of the airplane is alleviated artificially. The parameters of vertical flow perturbation and the flight altitude are taken the same values of the above supersonic dynamic response calculation.

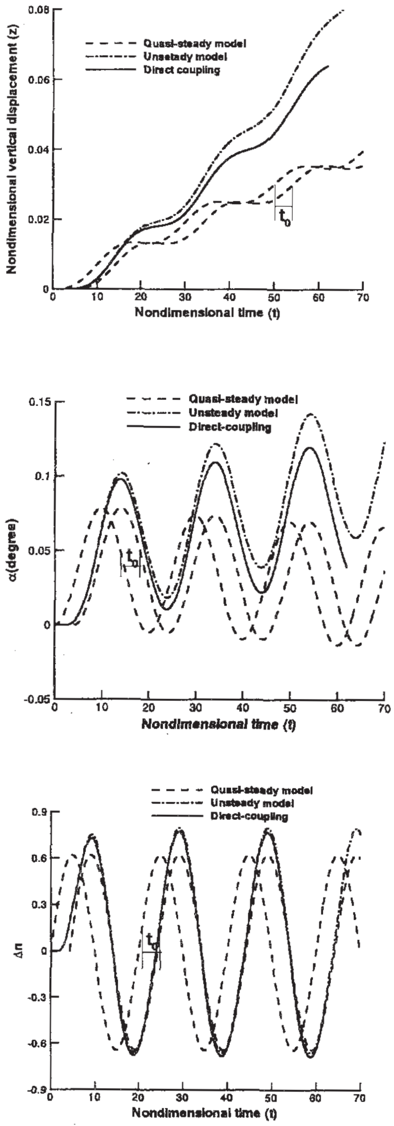

Fig. 2 Time histories of vertical displacement, angle of attack and incremental load factor for SST at $M_\infty = 0.9$, $\alpha = 2°$

The time histories of dynamic responses are shown in Fig. 2, in which the dynamic responses of the 'quasi-steady model' method after the translation of lag time $t_0 = 4.22$ are also depicted in the figure. Even no consideration of time lag, comparing unsteady model and direct-coupling methods, the quasi-steady method predicts the slower growth of displacement with time, the reverse tendency of change of angle of attack and the smaller maximum load incremental factor. It indicates the quasi-steady method is unsuitable for the analyses of transonic dynamic response. For the unsteady model method, the vertical displacement and angle of attack also increase faster with time than the direct coupling method.

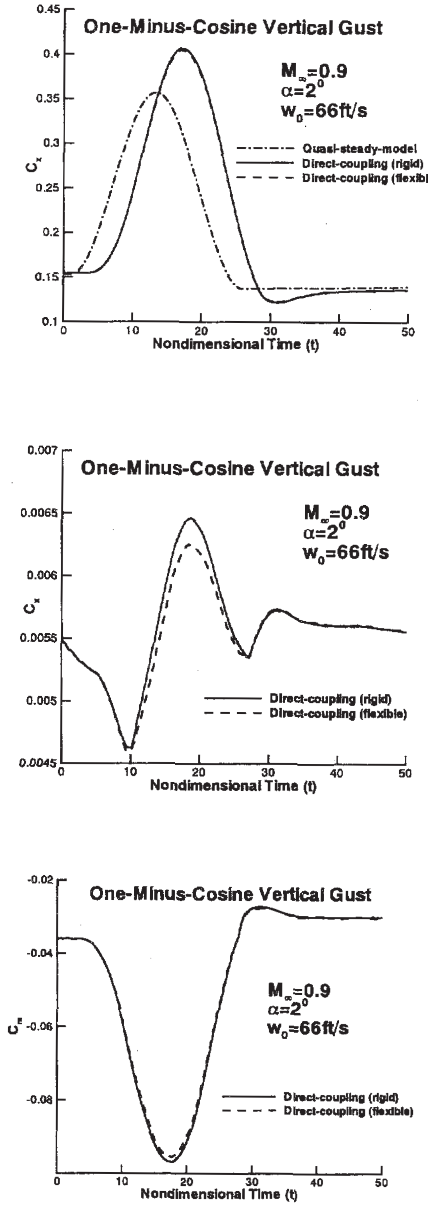

Fig. 3 Time histories of the coefficients of normal, axial forces and pitching moments for rigid and flexible configurations at $M_\infty = 0.9$, $\alpha = 2°$

4.3 Dynamic response for one-minus-cosine gust

The early design methods for gust loads were based on the discrete gust having one-minus-cosine pulse, namely,

$$W = \frac{1}{2} W_0 \left(1 - \cos \frac{2\pi x}{2H} \right) \tag{13}$$

W_0 is the gust velocity, which is specified as functions of altitudes. In the present calculation of cruise altitude of 8,000 m, W_0 is assumed as 66ft/s. Based on the experimental evidence [5], the gust gradient distance H is taken as the 12.5 times mean geometric chordlengths.

Considering structural deformation of wing, the aeroelastic natural modes are taken from the natural modes of the flutter experimental model of rigid structure. Due to the difference of structure and mass distributions of the real airplane and the experimental model, the structural data of experimental model cannot be used for the gust response analyses of the real airplane. But the calculated results should be nearly identical for the rigid and flexible analyses due to the rigidity of structure of the experimental model. And in the future, when the normalized natural modes shapes of the real unrestrained airplane can be provided, the solver can be used directly for the dynamic analyses of gust response with the consideration of the elastic deformation.

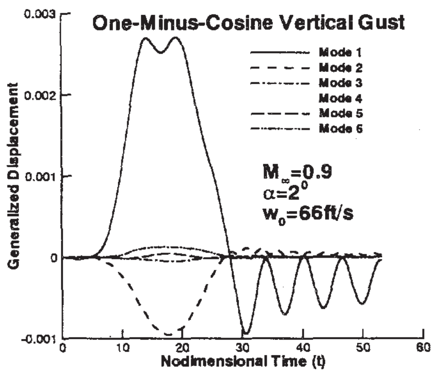

Fig. 4 Time histories of the structural deformation of the first six modes at $M_\infty = 0.9$, $\alpha = 2°$

The time histories the coefficients of normal force, axial force and pitching moments are shown in figure 3. When the airplane flight through the gust pulse, the force and moments also experience a pulse, then tend to recover the equilibrium state. The maximum of lift can reach 166.7% larger than the value of cruise flight and a larger pitching-down moment is also produced. As stated above, the results of flexible analyses are similar as those of rigid analyses except smaller difference on the maximum area. Figure 4 gives the time histories of structural deformation of the generalized displacements. For the airplane of rigidity, although the deformation is smaller, the airplane experiences a larger structural deformation in the gust process, which should be considered in the structure design, especially for the design of large civil aircraft.

Acknowledgement

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