ISSN 0389-4010
UDC 519.23
519.245
620.178.3
620.184
620.187

TECHNICAL REPORT OF NATIONAL AEROSPACE LABORATORY

TR-1270T

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May 1995

NATIONAL AEROSPACE LABORATORY

CHŌFU, TOKYO, JAPAN

Analysis of Fatigue Fractographic Data of a Rod End Housing Using Monte Carlo Simulation*

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ABSTRACT

This paper presents a new method using Monte Carlo simulation to estimate a life distribution of fatigue crack propagation on the basis of crack length versus striation spacing data. Moreover, to compare with the results obtained by this method, simple stochastic crack growth models using Monte Carlo simulation and other probabilistic methods without a simulation procedure provide the life distributions. The proposed method uses the distributions of two parameter estimates of a regression line with a reasonable correlation between the two parameter estimates. One cycle of the Monte Carlo scheme generates a set of the two parameter estimates and they give a life of crack propagation. In this study, these methods are applied for the striation spacing·data measured on the fatigue fracture surface of a rod end housing of a hydraulic actuator used for a main landing gear of a transport aircraft. The life distributions predicted are discussed and compared to clarify the features of each method. The proposed method approximates the true fatigue life as the *B* allowable life when the initial crack length is assumed to be 0 mm.

Keywords: Fractography, SEM observation, Fatigue crack propagation, Regression analysis, Monte Carlo simulation, Stochastic process models, Life distribution, Reliability analysis, *B*-allowable life.

概　　　要

本研究は、き裂長さとストライエーション幅のデータをもとに、き裂進展寿命分布を推定するためのモンテカルロ・シミュレーションを使用する新しい方法を提案する。また、この方法により得られる結果と比較するために、モンテカルロ・シミュレーションを使う単純な確率過程モデルとシミュレーションを使用しない他の確率的方法により寿命分布を導く。提案する方法は、適切な相関を持つ回帰直線の2個のパラメータ推定値における分布を使用する。1回のモンテカルロ・シミュレーションにより、1組の二つのパラメータ推定値を発生し、これらは一つのき裂進展寿命を与える。この研究では上記した種々の方法を、輸送機主脚用油圧アクチュエータのロッド・エンド・ハウジングに現われた疲労破面上で計測したストライエーション・データに応用する。それぞれの方法の特徴を明らかにするために、予測された寿命分布を検討し比較する。さらに提案する方法は、ロッドエンドハウジングの疲労寿命が初期き裂長さを0 mmとした場合の*B* 許容値として近似されることを示す。

Symbols Used

a = crack length
a_0 = initial crack length
a_f = final crack length
b = striation spacing
b_e = location parameter of b in Type-I extreme-value distribution
b_c = scale parameter of b in two-parameter Weibull distribution
C = a parameter of a and b relationship

* Received 8 March 1995
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E = a function of R
$f(\cdot)$ = probability density function
$F_E(\cdot)$ = cumulative distribution function for Type-I extreme-value distribution
$F_W(\cdot)$ = cumulative distribution function for two-parameter Weibull distribution
k = sample size
m = a parameter of a and b relationship
n = number of load cycles
N = life of crack propagation
N^* = deterministic life of crack propagation
N_B = B allowable life or 90% confidence limit of N
N_M = median life from averaged log-life
N_{MR} = median life by ranking
$q(a)$ = scale parameter of b at a in Type-I extreme-value distribution
R = reliability
u = standard normal variable
α = shape parameter of b in two-parameter Weibull distribution
η = coefficient of variation of b
η_E = equivalent coefficient of variation of b in Type-I extreme-value distribution
$\mu(a)$ = mean or median of b at a
$\mu_L(a)$= mean of log b at a
$\sigma(a)$ = standard deviation of b at a
$\sigma(\cdot)$ = standard deviation
σ_E = standard error
σ_L = log standard deviation
$\sigma_L(a)$= standard deviation of log b at a
$\sigma^2(\cdot)$ = variance
σ_E^2 = error variance

1. Introduction

Each striation spacing observed by a scanning electron microscope (SEM) on a fatigue fracture surface is widely recognized to represent the increment of crack length by one load cycle. However, this spacing is only the result of one-point observation in a wide area of the fatigue fracture surface. Moreover, striation spacing b versus crack length a data are plotted on one graph, the data points are not distributed on a single line but in a wide band, even if they are obtained from one fracture surface. This means that there is, at least locally, no deterministic relationship between crack length and striation spacing.

On the other hand, the purpose of measuring striation spacing on the fracture surface formed in an actual structure is to get information on the relationship between crack length and the number of load cycles, life of fatigue crack propagation, stress intensity factor at arbitrary crack length, etc. For these purposes, the data points distributed in a wide band are generally assumed to have a deterministic relationship and can be approximated by a single linear regression line on double logarithmic graph paper. Statistical and probabilistic analyses can also be applicable for this kind of data and provide probabilistic prediction with respect to crack propagation. The following concepts are acceptable for practical probabilistic and statistical analyses.

(1) b occurs as a random variable at arbitrary crack length a. (2) If the a and b data are approximated by a regression line, the estimates of the two parameters have statistical distributions respectively. In the latter case, two ideas are effective, namely, (2a) the slope parameter of a regression line is regarded as deterministic because of the small scatter in its estimates, and only the estimates of the intersection parameter have a statistical distribution, and (2b) each of the two parameter estimates has a statistical distribution.

Concept (1) described above was used in the stochastic crack growth models proposed by Virkler et al. [1, 2], Yang et al. [3, 4], and Artley [5], the features of these models were clarified to some extent. On the other hand, on the basis of concept (2a), the relationship between stress intensity factor and crack growth rate was discussed by Besuner and Tetelman [6], Harris and Lim [7], and Shimokawa and Hamaguchi [8]. The studies by Yang et al. [3, 4] included this concept also. Since the authors are not aware of any research reports which analyze fatigue crack propagation using concept (2b), the analysis using this concept is considered to be a new proposal. Since the life of crack propagation can not be analytically solved on the basis of concepts (1) or (2b) without an approximation, Monte Carlo simulation is indispensable.

The data analyzed in this study were measured by SEM observation of a fracture surface which emerged in a rod end housing of a hydraulic actuator used for a main landing gear of a transport aircraft.

This part is made of a 7075 forged aluminum alloy. The stochastic and other probabilistic analysis procedures mentioned above including the authors' proposal are described and formulated. The data is analyzed by linear regression. A deterministic crack growth approach predicts a deterministic life of crack propagation. Using the results of the regression analysis, the life distributions of crack propagation are predicted by the described methods and the obtained results are discussed and compared. Furthermore, the probabilistic methods on the basis of concepts (2a) and (2b) compute B allowable lives on the assumption of an initial crack length of 0 mm. These B allowable lives are compared with the true life of the rod end housing.

2. Relationship Between Crack Length and Striation Spacing

The relationship between crack length a and striation spacing b can empirically be expressed by

$$b = Ca^m, \tag{1}$$

where m and C are treated as constants when argued in a deterministic crack growth approach. By transforming Equation (1) into logarithm, then

$$\log b = \log C + m \log a. \tag{2}$$

In other words, the relationship between a and b is a straight line on double logarithmic graph paper. If b is considered to be the crack propagation rate, Equation (1) corresponds to Paris-Erdogan's law [9] for a panel infinitely wide with a central crack. This is shown in the appendix.

3. Observation of a Fracture Surface of a Rod End Housing

Figure 1 depicts the external appearance of the hydraulic actuator for a main landing gear of a transport aircraft. Figure 2 shows the location of fatigue fracture surfaces. Figure 3 presents an outline of the variation of internal pressure acting on the rod end housing in one flight cycle. Other hydraulic pulsating pressure may occur other than this pressure change. Figure 4 is an example of photographs taken by a field-emission type SEM. Since the load history of Figure 3 suggests that one pair of two coarse striations are formed in each flight cycle, the striation spacing in one flight cycle is defined as shown in Figure 4, i.e., the interval at every pair of coarse striations. Figure 5 indicates the observed a and b data and the regression line determined later. Equation (2) reasonably approximates these data points.

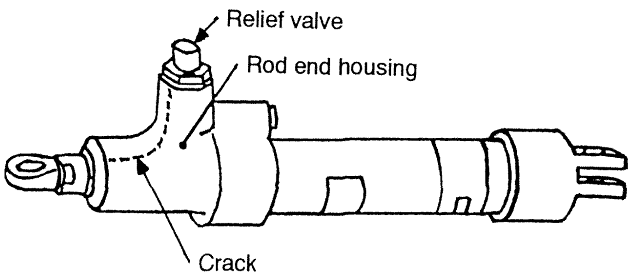

Figure 1 A hydraulic actuator for a main landing gear.

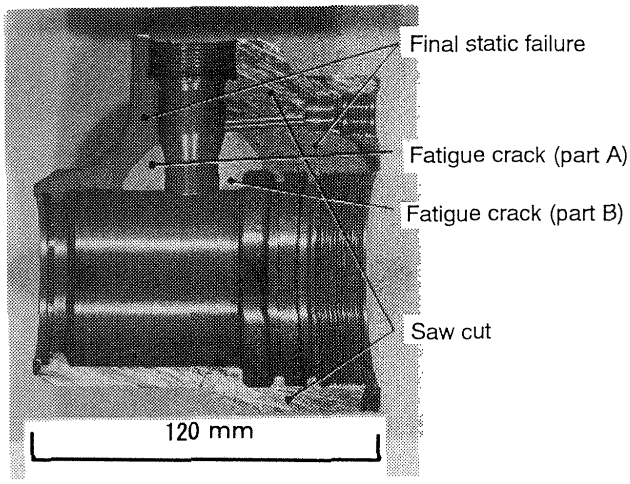

Figure 2 Fracture surface of a rod end housing.

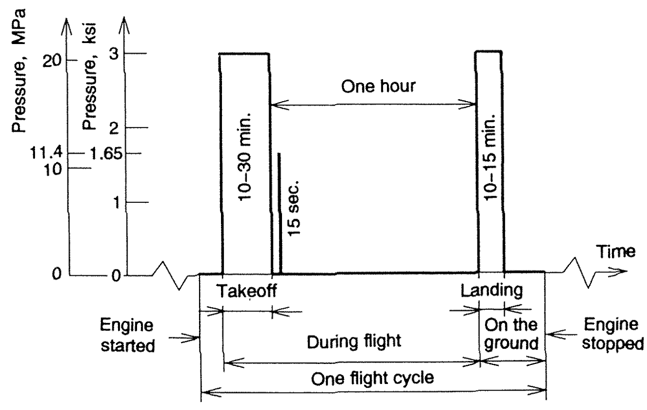

Figure 3 Pressure variation of the hydraulic actuator in one flight cycle.

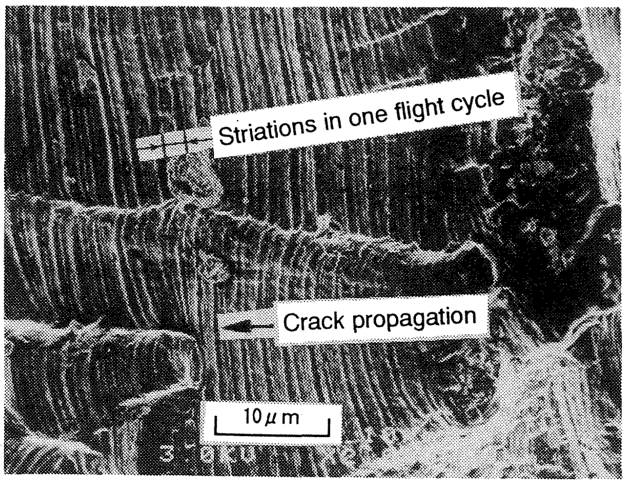

Figure 4 Striations observed by SEM.

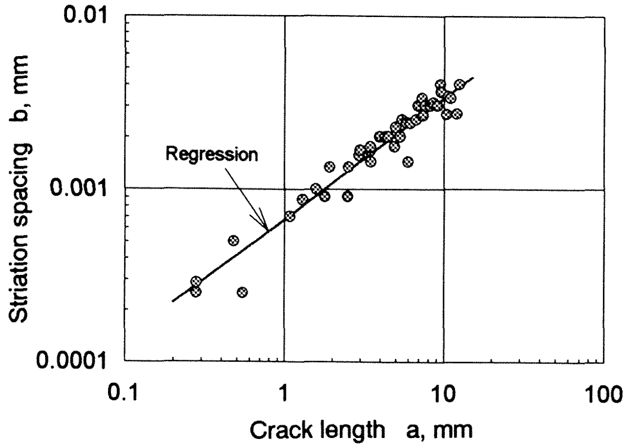

Figure 5 Observed striation spacing versus crack length.

4. Estimation of Crack-Propagation Life and Its Distribution from the Crack Length Versus Striation Spacing Data

The life of crack propagation, N, and its distribution can be derived from the data shown in Figure 5. Principally, this is effective only in the interpolated region of crack length in which striations were observed. Let the initial crack length be a_0 and the final crack length to failure be a_f. The symbols of parameters, estimators, and estimates are not distinguished for simplicity of the expression later. A summation symbol $\sum_{i=1}^{k}$ is simply written as $\sum$.

4.1 Deterministic Crack Growth Approach

A deterministic crack growth approach yields only the deterministic life N^*. At first, a regression analysis determines the relationship between a and b using their measured data. In this case, a is treated as an independent variable. The estimates of m and C in Equation (2) are calculated by

$$m = \frac{\sum \log a_i \cdot \log b_i - k \cdot \overline{\log a} \cdot \overline{\log b}}{\sum (\log a_i)^2 - k\left(\overline{\log a}\right)^2}, \quad (3)$$

$$C = \exp_{10}(\overline{\log b} - m \cdot \overline{\log a}), \quad (4)$$

where k is the number of observations, $\exp_{10}(x) = 10^x$,

$$\overline{\log a} = (\sum \log a_i)/k, \quad (5)$$

and

$$\overline{\log b} = (\sum \log b_i)/k, \quad (6)$$

Then let n be the number of load cycles and

$$b = da/dn. \quad (7)$$

By using Equations (1) and (4), N^* can be led to

$$\left.\begin{aligned} N^* &= \frac{a_f^{1-m} - a_0^{1-m}}{C(1-m)}, \quad (m \neq 1) \\ &= \frac{\ln(a_f/a_0)}{C}, \quad (m = 1) \end{aligned}\right\} \quad (8)$$

In the following argument, the case $m = 1$ is not discussed for simplicity of discussion.

4.2 Stochastic Crack Growth Models

In considering crack growth by stochastic process models, this study employs the following simple concepts. Namely, (a) one striation spacing b formed at the crack tip of arbitrary crack length a by one load cycle appears randomly and independently of the previous load history. One load cycle means one flight cycle in the later discussion. (b) The distribution of b follows one of the four distribution models, i.e., normal, log-normal, Type-I extreme-value, and two-parameter Weibull distributions. (c) The median of b at a is defined as $\mu(a)$, which is represented by Equation (2) and the estimates of m and C obtained by the regression analysis. (d) Concerning the scatter in b, the coefficient of variation η is assumed to be constant regardless of a. This means that the log-standard deviation σ_L in the log-normal distribution, the equivalent coefficient of variation η_E in the Type-I extreme-value distribution, and the shape parameter α in the two-parameter Weibull distribution are constant independently of a. Figure 6 shows the concept of crack growth by a stochastic process model. As additional information of correspondence

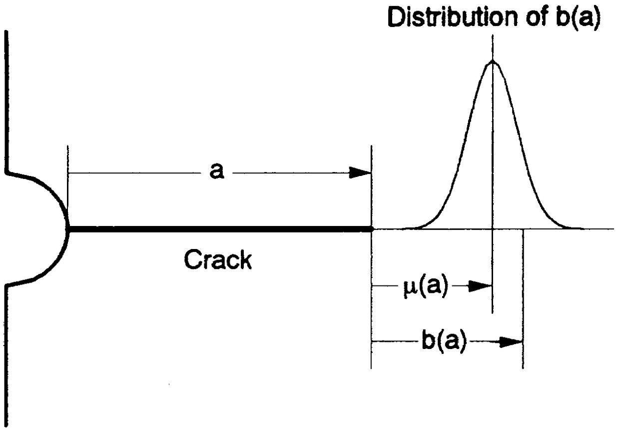

Figure 6 Concept of crack growth by a stochastic process model.

in the distributions of the normal group and the Weibull group, the normal distribution corresponds to the Type-I extreme-value distribution and the log-normal distribution to the two-parameter Weibull distribution [10].

4.2.1 Normal crack growth rate model

In the case of a normal random process model, the probability density function of b at arbitrary crack length a, $f(b)$, is written as

$$f(b) = \frac{1}{\sqrt{2\pi}\ \sigma(a)} \exp\left[- \frac{\{b - \mu(a)\}^2}{2\{\sigma(a)\}^2} \right], \qquad (9)$$

where $\mu(a)$ and $\sigma(a)$ are the mean and standard deviation of b at a respectively. Since the median agrees with the mean in the case of the normal distribution, $\mu(a)$ is given by Equation (1) and the estimates of m and C determined by the regression analysis. The coefficient of variation η is

$$\eta = \sigma(a)/\mu(a). \qquad (10)$$

From the assumption described above η is constant regardless of a.

Denote crack length after the number of load cycles j as a_j and crack growth by one load cycle after j cycles as $b(a_j)$. In the case of the normal random process model, $b(a_j)$ is given by

$$\left.\begin{aligned} b(a_j) &= \mu(a_j)\cdot(1+\eta\cdot u_{j+1}), & (1+\eta\cdot u_{j+1}>0) \\ &= 0 & (1+\eta\cdot u_{j+1}\leq 0) \end{aligned}\right\} (11)$$

where $\mu(a_j) = Ca_j^m$. When $1+\eta\cdot u_{j+1}\leq 0$, the crack does not grow; however, one cycle is added to the total load cycles. If the value of u_{j+1} is given as a normal random number by Monte Carlo simulation, crack growth follows a normal random process. Hence, the relationship between a_{j+1} and a_j is written as

$$a_j = a_{j-1} + Ca_{j-1}^m\ (1+\eta\cdot u_j). \qquad (12)$$

The relation between crack length and number of load cycles is calculated by adding the crack growth from a_0 to a_j at the j-th cycle. The life of crack propagation, N, is defined by j when a_j exceeds a_f for the first time. Accordingly, the distribution of N can be led by the repetition of this procedure.

4.2.2 Log-normal crack growth rate model

In the case of the log-normal random process model, the probability density function of log$b$ at a, $f(\log b)$, is given by replacing b by log$b$, $\mu(a)$ by $\mu_L(a)$, the mean of log$b$, and $\sigma(a)$ by $\sigma_L(a)$, the standard deviation of log$b$, in Equation (9). The antilogarithm of $\mu_L(a)$ becomes the median $\mu(a)$. $\sigma_L(a)$ is constant independently of a according to the assumption described above. In this case, $b(a_j)$ is represented by

$$b(a_j) = \mu(a_j)\cdot\exp_{10}(\sigma_L\cdot u_{j+1}). \qquad (13)$$

Therefore,

$$a_j = a_{j-1} + Ca_{j-1}^m\cdot\exp_{10}(\sigma_L\cdot u_j). \qquad (14)$$

Using these equations, the relation between crack length and number of load cycles and the life of crack propagation can be calculated in a manner similar to that of the normal random process model. Equation (14) agrees with that proposed by Yang et al. [3, 4], though the form of the equation is different.

4.2.3 Type-I extreme-value crack growth rate model

In the case of the Type-I extreme value random process model, the cumulative distribution function of b at a, $F_E(b)$, is represented by

$$F_E(b) = 1 - \exp\left[- \exp\left\{ \frac{b - b_e(a)}{q(a)} \right\} \right], \qquad (15)$$

where $b_e(a)$ and $q(a)$ are the location and scale parameters of b at a respectively. Since the relationship between a and b obtained by the regression analysis is regarded as the median relationship, $b_e(a)$ becomes

$$b_e(a) = \mu(a) - q(a) \cdot \ln(\ln 2). \quad (16)$$

In the Type-I extreme-value distribution, the equivalent coefficient of variation, η_E, is defined as

$$\eta_E = q(a)/\mu(a), \quad (17)$$

where η_E is constant regardless of a. Let R be reliability and

$$E_{j+1} = \ln\{\ln(1/R_{j+1})\} - \ln(\ln 2). \quad (18)$$

Then

$$\left.\begin{aligned} b(a_j) &= \mu(a_j) \cdot (1+\eta_E \cdot E_{j+1}), && (1+\eta_E \cdot E_{j+1} > 0) \\ &= 0, && (1+\eta_E \cdot E_{j+1} \le 0) \end{aligned}\right\} \quad (19)$$

When $1+\eta_E \cdot E_{j+1} \le 0$, a crack does not grow as in the case of the normal random process model. This equation leads to

$$a_j = a_{j-1} + Ca_{j-1}^m (1+\eta_E \cdot E_j). \quad (20)$$

Monte Carlo simulation generates a pseudo random number and gives it to R_j. Equation (18) presents the value of E_j and Equation (20) provides the relationship between crack length and the number of load cycles and the life of crack propagation.

In comparing Equation (19) with Equation (11) or Equation (20) with Equation (12), it is clarified that η_E corresponds to η, and E_j to u_j.

4.2.4 Weibull crack growth rate model

In the case of the two-parameter Weibull random process model, the cumulative distribution function of b at a, $F_w(b)$, is written as

$$F_W(b) = 1 - \exp\left[-\left\{\frac{b}{b_C(a)}\right\}^{\alpha(a)}\right], \quad (21)$$

where $b_c(a)$ and $\alpha(a)$ are the scale and shape parameters at a, respectively. α is constant regardless of a according to the assumption mentioned before. Since the relationship between a and b obtained by the regression analysis is regarded as the median relationship, $b_c(a)$ becomes

$$b_C(a) = \mu(a) \cdot \exp\left\{\frac{\ln(\ln 2)}{\alpha}\right\}. \quad (22)$$

Therefore, $b(a_j)$ is given by

$$b(a_j) = \mu(a_j) \cdot \exp(E_{j+1}/\alpha). \quad (23)$$

This equation leads to

$$a_j = a_{j-1} + Ca_{j-1}^m (E_j/\alpha). \quad (24)$$

Equation (24) provides the relationship between crack length and the number of load cycles and the life of crack propagation.

In comparing Equation (23) with Equation (13) or Equation (24) with Equation (14), it is clarified that $1/\alpha$ corresponds to σ_L, and E_j to u_j.

4.3 Application of Distributions of Parameter Estimates of a Regression Line

A proposed method is presented and a conventional probabilistic method assuming m to be deterministic is described as a special case of the proposed method. The life and parameters discussed here refer to the estimates.

4.3.1 In case of distributions of both *m* and *C* estimates considered

When the data plotted in Figure 5 are approximated by a regression line on the basis of Equation (2), the estimates of m and $\log C$ can be statistically analyzed. These results can be applied for probabilistic inference. If the distribution of the combinations of both parameter estimates can be obtained, this distribution leads to the life distribution of crack propagation. However, Monte Carlo simulation is necessary to give the distribution of the sets of both parameter estimates.

Since the estimates of m and $\log C$ are correlated, it is difficult for any simulation to directly give a proper set of both parameter estimates. Hence, let a Monte Carlo scheme generate the combination of the estimates of m and $\overline{\log b}$, because $\overline{\log b}$ is independent of m. Then $\log C$ is calculated using Equation (4). $\overline{\log a}$ in Equation (4) is a deterministic value determined by the method of measurement. Thus a set of the properly correlated estimates of m and $\log C$ can be obtained. Equation (4) indicates the correlation between the two estimates of m and $\log C$.

A regression theory provides an unbiased error variance σ_E^2 as

$$\sigma_E^2 = \frac{\sum(\log b_i - \log C - m \cdot \log a_i)^2}{k-2}. \quad (25)$$

Unbiased variances of m and $\overline{\log b}$, $\sigma^2[m]$ and $\sigma^2[\overline{\log b}]$, are given as

$$\sigma^2[m] = \sigma_E^2/\{\Sigma(\log a_i)^2 - k\,(\overline{\log a})^2\}, \quad (26)$$

$$\sigma^2[\overline{\log b}] = \sigma_E^2/k. \quad (27)$$

The estimate of m or $\overline{\log b}$ calculated from the data is known to follow a t-distribution. However, its population should be a normal distribution of infinite sample size. Therefore, in this Monte Carlo simulation the populations of m and $\overline{\log b}$ are considered to be normal. With respect to m, the mean is defined by Equation (3) and the variance by Equation (26). As to $\overline{\log b}$, the mean is represented by Equation (6) and the variance by Equation (27). One cycle of the Monte Carlo scheme generates a set of m and $\overline{\log b}$ values, then Equation (4) gives the value of C using the $\overline{\log a}$ observed. Thus, these values of m and C present a life N of crack propagation with given values of a_0 and a_f using Equation (8). Repetition of this procedure provides a distribution of N.

The distribution of N derived in this manner is strictly based on the data of k observations. As the sample size k becomes larger and the observed region of crack length expands, m and $\overline{\log b}$ converge to their means in their populations regardless of the value of σ_E^2 due to Equations (26) and (27). Accordingly, N converges to the value estimated by a deterministic crack growth rate approach. In other words, the scatter of N predicted here is dependent on the amount of scatter in each of the two parameter estimates and does not mean the variability of the life inherent to the material. However, in reality it is impossible to obtain ideal data regarding the sample size and the measured region of the crack length. In this context, this proposal is an analytical method which uses Monte Carlo simulation to compensate the insufficient measurements.

4.3.2 In case of deterministic *m* assumed

If in Equation (26) the region of a is fully wide and the number of observations k is sufficiently large, $\sigma[m]$ becomes small and the scatter in m is not considerable. In other words, only C has scatter, though the value of m is defined as that obtained by the regression analysis. From this assumption, the scatter in lives of crack propagation can be obtained. Equation (8) leads to the standard deviation of $\log N$, $\sigma[\log N]$, as

$$\sigma[\log N] = \sigma[\log C]. \quad (28)$$

In other words, the standard deviation of $\log N$ agrees with that of $\log C$. Furthermore, if the distribution form of $\log C$ is normal, $\log N$ also follows a normal distribution.

On the other hand, the following two cases with respect to C can be considerable. Those are (a) C obtained by a regression analysis with deterministic m, and (b) C in Equation (1) with deterministic m.

(a) *C* given by a regression analysis with deterministic *m*

The variance of $\log C$ is given by Equations (4) and (28) as

$$\sigma^2[\log C] = \sigma^2[\overline{\log b}] = \sigma^2[\log N] = \sigma_E^2/k. \quad (29)$$

Therefore, the variances of $\log C$, $\overline{\log b}$, and $\log N$ are equal to each other. Since σ_E^2 of Equation (25) is an unbiased estimator independent of sample size k, each variance in Equation (29) is macroscopically in inverse proportion to k. Consequently, N is estimated as a deterministic value for very large k. The unbiased variance of $\log N$ is given by Equation (29) and the mean of N is calculated by Equation (8) using the m and C values estimated by the regression analysis. The content described above is the discussion about N given by the regression analysis.

(b) In case of *C* with deterministic *m* in Equation (1)

In a deterministic crack growth approach, m and C are considered to be material constants. Here, let only m be deterministic and C have scatter inherent to a material. In this case, the measured data is assumed to be a sample from the population. Then Equation (2) leads to the scatter of $\log C$ equivalent to that of $\log b$. Consequently, together with Equation (28), the following equation

$$\sigma^2[\log N] = \sigma^2[\log C] = \sigma^2[\log b] = \sigma_E^2 \quad (30)$$

is formulated. Because the expected value of σ_E^2 does not depend on k, that of $\sigma[\log N]$ is also estimated free from k. Then this is regarded as a material constant. This idea is also a special case of a stochastic crack growth rate model described in the introduction of this paper, i.e., crack growth by one load cycle is completely correlated with that by previous load cycles [3, 4]. The mean of $\log N$ is regarded as the life calculated by m and $\log C$ which are estimated by the regression analysis.

Every study assuming deterministic m described in the introduction is based on idea (b) written above. The investigation on the basis of idea (a) indicated above is not discussed yet.

5. Calculated Results and Considerations

The results calculated by the procedures described above are presented below. The true life of the rod end housing was 9,176 flights.

5.1 Parameter Estimates from the Observed Data and Life Estimated by Deterministic Crack Growth Rate Approach

A regression analysis for the observed data shown in Figure 5 gave the parameter estimates and their standard deviations as indicated in Table 1. The estimate of the standard error σ_E is about 0.08.

Table 2 presents the lives of crack propagation calculated by a deterministic crack growth rate approach using m and C listed in Table 1. In this calculation, the initial crack length a_0 was taken as three kinds, i.e., 0, 0.01566, and 0.28mm. When

Table 1 Parameters and standard deviations estimated by a regression analysis.

k*	m	C	σ_E	$\sigma[m]$	$\sigma_E/\sqrt{k}$
43	0.6937	0.0006731	0.07829	0.02723	0.01194

*Number of observations.

Table 2 Crack-propagation life calculated by parameter estimates obtained by the regression analysis.

a_0	a_f	N*
0	12.58	10533
0.01566	12.58	9176
0.28	12.58	7250

a_0, a_f = initial, final crack length (mm).

a_0=0.01566 mm, the estimated deterministic life agrees with the true life. a_0=0.28 mm was the minimum crack length at which striations were observed. The final crack length, a_f=12.58 mm, was the maximum length at which striations were observed. Around the crack initiation point of the rod end housing no material defect or damage was observed. a_0=0.01566 mm is fairly small but not zero. When a_0=0 mm, the estimated life of 10,533 flights is approximately 15% longer than the true life.

5.2 Distribution of Striation Spacing

A distribution of striation spacing b at arbitrary crack length a is estimated as follows. The distribution form of b is assumed to be independent of a and the coefficient of variation η of b to be invariant to a. Invariant η means that the standard deviation of $\log b$, σ_L, the equivalent coefficient of variation in a Type-I extreme-value distribution, η_E, and the shape parameter of a two-parameter Weibull distribution, α, are also constant independently of a. On the basis of this assumption, a residual of $\log b$ is defined by

$$\Delta(\log b)_i = \log b_i - \log(C a_i^m) \tag{31}$$

and it has the same variance regardless of a. Then, the values of $\Delta(\log b)_i$ are collected and analyzed. Figure 7 presents the obtained results of $\Delta(\log b)$ plotted on log-normal graph paper with the median ranks. The four distribution lines are depicted by the parameter estimates, which were calculated by the least-squares method for the plotted data with the median ranks on each probability paper. This figure indicated the Type-I extreme-value distribution to be the best fit, followed by the two-parameter Weibull,

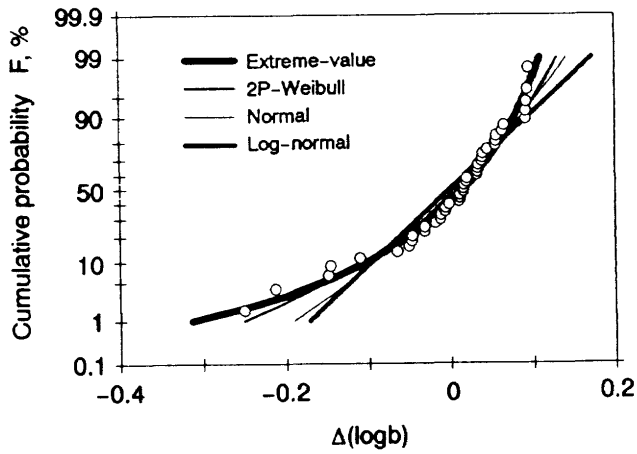

Figure 7 Distribution of residuals of striation spacing.

normal, and log-normal distributions. According to the assumption described above, the distribution of residuals in this figure agrees with the distribution of b at any a. Among the four stochastic crack growth rate models the Type-I extreme-value crack growth rate model is the most realistic. This result is different from that reported by Yang et al. [4] who stated that crack growth rates measured from striation spacing with an SEM follow a log-normal distribution well.

5.3 Life Distributions Predicted by Stochastic Crack Growth Models

Using the parameter estimates from the data, the four kinds of stochastic crack growth rate models simulated crack propagation and led to the life distributions. The initial crack length a_0 is set to be 0.001 mm instead of 0 mm, because the calculation is impossible for a_0=0 mm. Table 3 presents the results for the normal group models and Table 4 for

Table 3 Central tendency and scatter of crack-propagation lives simulated by two kinds of stochastic crack growth rate models in the normal group and Monte Carlo simulation.

Simulation conditions						
Initial crack length = 0.001 mm, Final crack length = 12.58 mm; C = 0.0006731, m = 0.6937, Sample size = 50, Deterministic life = 9949						
Normal random process				Log-normal random process		
$\eta[b]$ %	Mean life	$\eta[N]$ %	Mean invalid cycles	$\sigma[\log b]$	Median life	$\sigma[\log N]$
5	9954	0.0551	0	0.01	9951	0.000109
10	9955	0.111	0	0.05	9890	0.000582
*18.2	9957	0.205	0	*0.0783	9797	0.000883
20	9958	0.226	0	0.1	9698	0.00114
30	9960	0.343	4	0.3	7855	0.00345
50	9924	0.564	227	0.5	5154	0.00985
80	9586	0.718	1014	0.7	2793	0.0268

$\eta[\cdot]$ = coefficient of variation, $\sigma[\cdot]$ = standard deviation,
*indicates the values calculated from the data.

Table 4 Central tendency and scatter of crack-propagation lives simulated by two kinds of stochastic crack growth rate models in the Weibull group and Monte Carlo simulation.

Simulation conditions						
Initial crack length = 0.001 mm, Final crack length = 12.58 mm; C = 0.0006731, m = 0.6937, Sample size = 50, Deterministic life = 9949						
Type-I extreme-value random process				2-P Weibull random process		
$\eta_E[b]$ %	Median life	$\eta_E[N]$ %	Mean invalid cycles	$\alpha[b]$	Median life	$\alpha[N]$
2.5	10006	0.0249	0	1	6915	98
5	10059	0.0531	0	2	9356	222
10	10168	0.117	0	4	10022	464
*13.1	10236	0.145	3	6	10095	633
20	10383	0.208	47	*6.98	10099	730
40	10615	0.409	585	10	10086	1028
60	10491	0.512	1283	20	10038	1968

$\eta_E[\cdot]$ = equivalent coefficient of variation in Type-I extreme-value distribution,
$\alpha[\cdot]$ = shape parameter, *indicates the values calculated from the data.

the Weibull group models. The scatter of striation spacing calculated from the data indicates the value with a symbol*. To investigate the effect of this scatter on life distribution, the parameter to indicate the scatter of b are changed in wide regions. Since the scatter in simulated lives is very small, the simulation provided 50 lives for each case. Tables 3 and 4 indicate that the scatter of N is one or two orders of magnitude smaller than that of b. In other words, the scatter of life derived by the stochastic process models in this study is very small. This tendency is the same for the necessary number of flight cycles to arbitrary crack length.

The predicted properties of the mean or median life are as follows. The mean of N in the case of the normal random process model agrees well with the estimate by a deterministic crack growth approach in general, though as the coefficient of variation of b takes larger values, invalid cycles at which crack does not grow increases. For the coefficient of variation of b to be 80% the mean of N slightly dropped. These results come from the symmetric distribution of b. On the other hand, in the case of the log-normal random process model, the median life (i.e., the antilogarithm of $\overline{\log N}$) becomes shorter as the scatter of b becomes larger. This can be explained by the unsymmetric distribution of b. The facts described above indicate that the scatter of b influences not the scatter of N but the mean or median life. In the case of the Type-I extreme-value random process model, the median life is close to the estimate by the deterministic approach, though as the equivalent coefficient of variation of b becomes larger, the median of N becomes slightly larger. When the invalid cycles increase, the median life shows a little decrease. In the case of the two-parameter Weibull random process model, as the shape parameter α is larger, the amount of scatter is smaller. When $\alpha[b]$ is equal to one, the median life becomes very small. For $\alpha[b]$ larger than or equal to 2, the median life becomes slightly larger then decreases, but remains very close to the deterministic life estimate. This tendency is due to the shape of the distribution of b. The above results show that the stochastic crack growth rate models present the effect of the distribution form and scatter of b on the mean or median life. In this context, these models are not effective in practical terms.

Though the log-normal random process model of crack growth proposed by Virkler et al. [1,2] is different to some extent from that in this study, their results, i.e., very small scatter in lives and the predicted lives very close to the life estimate by the deterministic approach, are similar to those described above.

5.4 Application of Distributions of Parameter Estimates of a Regression Line

5.4.1 In case of deterministic *m* assumed

According to ideas (a) and (b) described in **4.3.2**, the standard deviation of $\log N$ is given by Equations (29) and (30), respectively, and their estimates are given in Table 1.

5.4.2 Distributions of both *m* and *C* considered

The proposed Monte Carlo simulation generated 200 sets of m and C. These 200 sets were used for every case of the different three initial crack lengths. Table 5 presents the median lives and standard deviation of $\log N$ calculated from the 200 sets of m and C. The median lives agree well with those listed in Table 2 obtained by the deterministic approach. When a_0= 0.28 mm, N was calculated in the interpolated region of crack length. $\sigma[\log N]$ is slightly larger than the value of $\sigma_E/\sqrt{k}$ in Table 1. This comes from the additional scatter of m. When the extrapolated region is included especially extended to small crack length, such as a_0= 0 and 0.01566 mm, $\sigma[\log N]$ becomes larger.

Figure 8 presents the 200 sets of m and C derived by the simulation. A coarse line described by Equation (4) represents the relationship between m and C well on average. A fine line is derived by the least-squares method with m as an independent variable. Equation (4) almost agrees with this relationship.

Table 5 Medians and scatter of crack-propagation lives predicted by the proposed Monte Carlo simulation.

h*	a_0**	a_f**	N_M***	N_{MR}****	$\sigma[\log N]$
200	0	12.58	10541	10525	0.02932
200	0.01566	12.58	9160	9144	0.01798
200	0.28	12.58	7237	7232	0.01281

*sample size, **a_0,a_f= initial, final crack length (mm), ***median life from averaged log-life, ****median life by ranking.

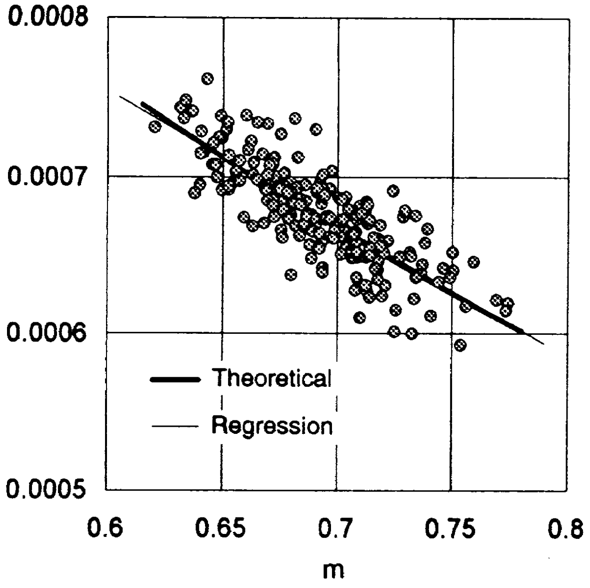

Figure 8 200 sets of m and C values given by Monte Carlo simulation and their mean theoretical relationship and regression line.

Moreover, this figure indicates the correlation between m and C.

Figure 9 indicates the life distributions of crack propagation calculated by the 200 sets of m and C for three kinds of a_0 plotted on log-normal probability paper. For a_0=0.28 mm, i.e., the interpolated region of crack length, the distribution form fits well with a log-normal distribution. When an extrapolated region is included, namely for a_0=0 and 0.01566mm, the distribution form deviates from a log-normal distribution and the tail of the longer life region distributes in a much longer life region than those predicted by the log-normal distribution. This tendency comes from the accumulation of load cycles in the small crack length region.

Figure 10 shows the measured data of a and b plotted in Figure 5 again and the regression line as a coarse line and two fine lines drawn by the sets of m and C which predicted the maximum and minimum lives for a_0=0 mm among 200 sets. The value of b on the line which gives the maximum life is small in the short region of a. This relation is reversed on the line which gives the minimum life. This suggests that the distribution form is convex upward when the extrapolated region is extended in the smaller crack length. If a_0 is changed, the different set of m and C gives the maximum or minimum life.

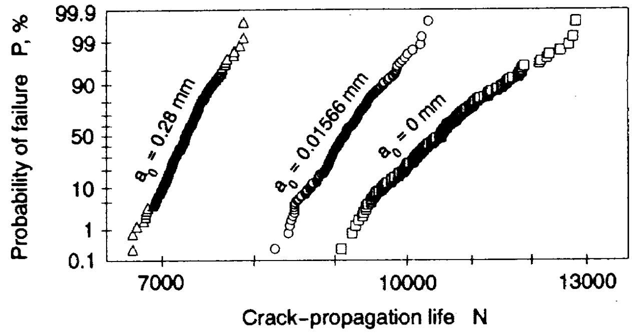

Figure 9 Distributions of crack-propagation lives.

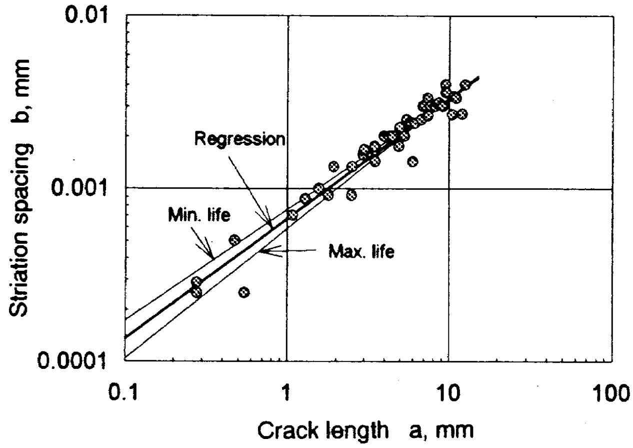

Figure 10 The regression line and the relationships of a and b to present the minimum and maximum lives among 200 relationships, where initial crack length assumed to be 0 mm.

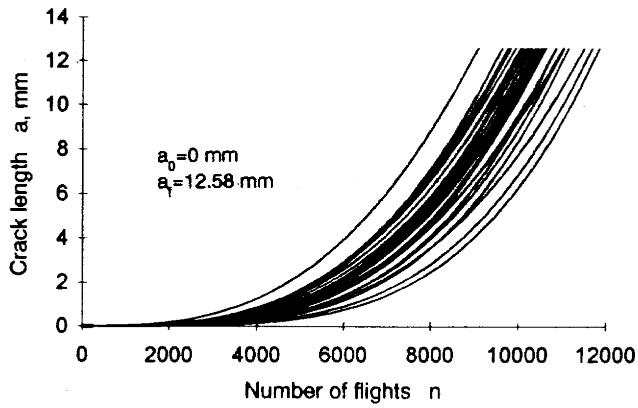

Figure 11 Crack propagation curves of first 30 trials, where initial crack length assumed to be 0 mm.

Figure 11 presents 30 curves of crack propagation for a_0= 0 mm depicted by the first 30 sets of m and C generated by this simulation in order to show the intersections of these curves. The reason to limit 30 curves is to make a clear distinction between these curves. A lot of intersections can be recognized. If m is deterministic as described before, these curves do not intersect each other and broaden toward the end.

5.4.3 *B* allowable life

If the true life can be predicted on the basis of a_0=0 mm, this is of practical value. Thus, the investigation was conducted to determine if it can predict the true life as *B* allowable life [11] (one-sided tolerance limit for probability of survival = 0.9 and confidence level = 0.95).

Table 6 gives the calculated results. Table 6(a) shows the results for the case of deterministic *m* and the scatter of log*C* as the estimate by Equation (29). The term "*B* allowable" seems not to be appropriate in this case; thus, the confidence limit of the mean is calculated. Accordingly, this value should be called as the confidence limit of *N*. The deviation of this value from the true life is fairly large to the unconservative side. Table 6(b) presents the results for the case of deterministic *m* and the scatter of log*C* as the estimate by Equation (30). The *B* allowable in this case is conservative but too small in comparison with the true life. Table 6(c) lists the results of *B* allowable lives by the Monte Carlo simulation on the basis of the distributions of both the *m* and *C* estimates. There are two kinds of estimates. One is calculated assuming a log-normal distribution of *N* by the use of the one-sided tolerance limit factor for the sample size k=200 [12]. The other is estimated by a non-parametric method for an unknown distribution. The thirteenth ascending order of observations corresponds to the *B* allowable for k=200 [11]. These *B* allowable lives are very close to the true life of 9,176 flights. The procedure proposed in this paper predicted closely the true life as the *B* allowable for a_0= 0 mm.

Table 6 *B* values and 90% confidence value of crack-propagation life.

(a) m = constant, $\sigma[\log C] = \sigma_E/\sqrt{k}$
N_B = 90% confidence value

DOF	a_0	a_f	N_B
41	0	12.58	10163

(b) m = constant, $\sigma[\log C] = \sigma_E$
N_B = *B* value

DOF	a_0	a_f	N_B
41	0	12.58	7774

(c) Monte Carlo simulation

DOF	a_0	a_f	N_B*	N_B**
199	0	12.58	9554	9529

DOF = degree of freedom, a_0, a_f= initial, final crack length (mm),
*B value for the log-normal distribution,
**B value from 13/200 (rank/observations) for unknown distribution.

As mentioned before, according to the methods used for Table 6(a) and 6(c), if the number of observations is increased in all regions of crack length, the lives estimated will distribute in a narrow range and will reach the deterministic life. Moreover, it ought to approach the true life of the article discussed. However, in order to get the estimate very close to the true life, the striation spacing data are necessary in the very small crack length region. This is difficult because of the limitations inherent in SEM performance. In other words, the accuracy of life estimation by the measured data of striation spacing is dependent on the number of cycles to crack initiation, identification of the initial defect size, or crack growth rate in very small crack length region; all of these factors are very difficult to discuss clearly.

At least the behavior of crack growth predicted by the proposed method is considered to be reasonable after a crack grows to some extent and striation spacing is observed. This kind of analysis can be effective to determine the inspection schedule in maintenance activities of aircraft.

6. Conclusions

On the basis of the crack length and striation spacing data, a new method using Monte Carlo simulation was proposed to predict the life distribution of crack propagation. Stochastic and other probabilistic methods including the proposed one were applied to the data obtained by SEM observation of the fracture surface of a rod end housing. The life distributions were predicted by these methods and were discussed. Major conclusions obtained are as follows:

(1) The crack length *a* versus striation spacing *b* data analyzed were approximated with a straight line on double logarithmic graph paper.

(2) The standard error σ_E of log*b* was estimated to be about 0.08.

(3) The life of crack propagation N estimated by the deterministic crack growth approach, assuming an initial crack length of $a_0 = 0$ mm, was approximately 15% longer than the true life.

(4) The distribution of the residuals with respect to $\log b$ fitted best with a Type-I extreme-value distribution among the four kinds of distribution models. This suggests that the distribution of b at arbitray a fits well with this distribution form.

(5) The median lives of crack propagation predicted by the four kinds of stochastic crack growth rate models were very close to the life estimated by the deterministic crack growth approach except for the cases of the large scatter of b in the log-normal and two-parameter Weibull random process models. The deviation of the median life from the deterministic life is dependent on the unsymmetrical distribution of b in the two models. The scatter of N obtained by each of the four models was one or two orders of magnitude smaller than that of b.

(6) If m is determinstic, the distribution form and variance of $\log N$ are equal to those of $\log C$. When the unbiased variance of $\log C$ is calculated by a regression analysis, it is equal to that of $\overline{\log b}$, i.e., σ_E^2/k. Therefore, as k becomes larger, N converges to a certain value regardless of the value of σ_E^2. On the other hand, when the scatter of $\log C$ is taken as a material constant, the variance of $\log C$ is equal to that of $\log b$, i.e., σ_E^2.

(7) If the measured region of a is extended and k is increased, the variances of m and $\log C$ used in the proposed Monte Carlo scheme will converge to zero. Thus, the lives of crack propagation given by this scheme will converge to a certain value at the same time.

(8) The median life (the antilogarithm of $\overline{\log N}$ or the median by ranking) obtained by the 200 sets of m and C generated by the simulation approximately agreed with the life estimated by the deterministic approach. The standard deviation of $\log N$ was a slightly greater than the value of $\sigma_E/\sqrt{k}$ when load cycles were integrated in the interpolated region of crack length measured. Though, the standard deviation of $\log N$ become larger as the extrapolated region of crack growth was extended to shorter crack length. The distribution form of N fitted a log-normal distribution well in the interpolated region of a. When this region was extended to shorter crack length, the distribution form deviated from the log-normal and presented the convex curve upward on log-normal probability paper.

(9) The averaged relationship between m and C given by the simulation was approximated well by Equation (4) determined by the regression analysis for the measured a vs. b data. This equation agreed well with the regression line calculated from the data of m and C, also.

(10) The relation between a and b to give the minimum life in the simulation is that b is large in the region of small a. Conversely, the relation to give the maximum life is that b is small in the region of small a.

(11) There appeared intersections among the crack propagation curves derived by the simulation.

(12) On the assumption of $a_0 = 0$ mm, the proposed Monte Carlo simulation approximately predicted the true life of the rod end housing as the B allowable life.

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Appendix

Correspondence of Equation (1) to Paris-Erdogan's Law

For a sheet infinitely wide with a central crack Paris-Erdogan's law [9] of crack growth rate is represented by

$$\begin{aligned} da/dn &= C_0 \cdot \Delta K^M \\ &= C_0 \, \Delta S^M \pi^{M/2} a^{M/2}, \end{aligned} \tag{a1}$$

where da/dn is the crack growth rate, ΔS is the stress range, ΔK is the stress intensity factor range, and C_0 and M are constants. Then, let

$$b = da/dn \tag{a2}$$

$$C = C_0 \, \Delta S^M \pi^{M/2} \tag{a3}$$

$$m = M/2. \tag{a4}$$

Equation (a1) is transformed as

$$b = C \cdot a^m. \tag{a5}$$

This equation agrees with Equation (1). Accordingly, Equation (1) is shown to correspond to Paris-Erdogan's law.

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TECHNICAL REPORT OF NATIONAL AEROSPACE LABORATORY
TR-1270T

航空宇宙技術研究所報告1270T号（欧文）

平成７年５月発行

発行所　航空宇宙技術研究所
東京都調布市深大寺東町７丁目44番地１
電話（0422）47-5911（大代表）〒182
印刷所　株式会社　三　興　印　刷
東京都新宿区西早稲田2－1－18

Published by
NATIONAL AEROSPACE LABORATORY
7-44-1 Jindaiji-Higashi, Chōfu, Tokyo 182
JAPAN

Printed in Japan