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Studies on Satellite-Based Navigation and Communication Utilizing Precise Clock Synchronization Between Radio Stations

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Abstract

This paper summarizes studies on the utilization of precise clock synchronization in satellite-based navigation and communication. The user's clock synchronization with one of the precise clocks installed in a navigation satellite or ground radio station, which maintains the time standard for the generation of range measurement signals, enables the adoption of new satellite navigation concepts, and the simplicity will open up avenues for efficient methods of mobile communications. Precise clock synchronization between two radio stations, which really means the detection of the time offset between two clocks, is performed, in principle, by using bi-directional communication between these stations and by detecting the difference in radio propagation times between one direction and its opposite. Satellite positioning based on clock synchronization will reduce the number of deployed satellites and mitigate geometrical requirements for satellite placement. Application of synchronized timing to spread-spectrum communications will produce a technology combining code division multiple access (CDMA) with time division multiple access (TDMA), with which optional communications between radio stations and multiple access circuit control for mobile message communications can be achieved efficiently.

Another subject of this paper is the formulation and integration of several kinds of satellite-based navigation algorithms. Such formation and integration can be skillfully achieved by applying the weighted least squares (WLS) method. Nearly optimal estimation of positioning is done by regulating the weights of the WLS algorithm, referring to the numerical values of dilution of precision (DOP), which are calculated from the covariance matrix $\sigma_0^2(\underline{H}^T \underline{H})^{-1}$ discussed later. Characteristics of the WLS navigation algorithm are discussed in relation to the geometry of satellite placement.

1. Introduction

Satellite positioning done by measuring one-way radio propagation time, such as GPS/NAVSTAR, which uses several satellites as points of reference for range measurement, requires knowledge of the standard time of the satellite by which radio transmitting timing is generated; therefore, for GPS, the time difference between the clocks installed in the navigation satellite and the user's equipment is estimated together with the user's position vector. In this case, the range measured by the user's equipment is called the pseudo-range, which is shifted from the real range by the above clock offset.

A clock-asynchronous positioning method like GPS, as described above, has special requirements for satellite deployment; it requires that at least four satellites be widely distributed in the sky and arranged visibly far from the shape of a line or a circle. Therefore, clock-asynchronous positioning requires that more satellites be deployed and that there be a wide field of view in the sky to maintain positioning accuracy.

In contrast, the new positioning method, based on clock-synchronous positioning, requires that the user's clock be synchronized with the satellite clock by means of other measurements. Clock synchronization is done via two-way communication between the satellite and the user's equipment, as shown in the following section. Clock synchronization frequency can be reduced by using a highly stable clock, such as an atomic clock. This positioning method reduces the number of satellites to be deployed and mitigates the geometrical requirements for satellite placement.

Clock synchronization of all radio stations, including mobile vehicles with navigation means, produces a new mobile communication network. Generally, in mobile communications that use satellites as the only means of radio relay, the radio signal transmitted by a mobile radio station is received by all ground radio stations linked to the satellites and reprocessed for connection with the public telephone network. In other words, in satellite-based mobile communications, mobile stations cannot select the ground stations with which they want to connect, because the radio signals relayed by the satellite are transmitted widely without focus. However, the use of clock synchronism for communications makes it possible for ground stations to process only the radio signals arriving at a pre-assigned time slot. As a result, the mobile station is able to select the ground station by the control of radio transmission timing. This timing can be generated by using the clock offset and the range between radio stations, and spread-spectrum communications will permit radio signal superpositioning if there are slight time lags in radio signal epochs, even if they have been coded with the same pseudo-random number.

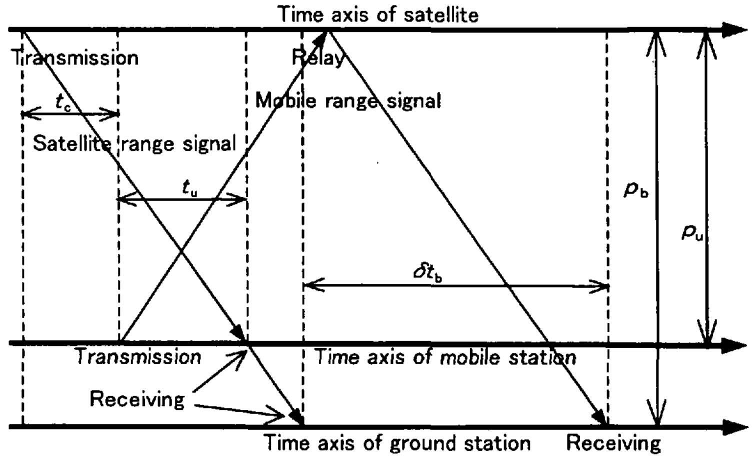

Fig. 1 Timing Chart of Mobile-Satellite Synchronization

2. User's Clock Synchronization

The time difference between two radio stations, or "clock offset", as it is commonly called, is estimated by detecting the time lag in two-way propagation of range measurement signals.

In case of the measurement of user's clock offset to the satellite clock, the aid of the ground radio station is needed. As illustrated in Figure 1, the user's equipment, which will often be called "mobile station" hereafter, is able to measure the difference , t_u, between the arrival time of the range measurement signal transmitted by the satellite and the transmission time of the range measurement signal generated by the mobile station itself. The ground station linked with the mobile station is able to measure the time difference, δt_b , between the arrival of the range measurement signals transmitted from the mobile station relayed by the satellite and from the satellite itself. By having them inform each other of the time differences mentioned above, the clock offset at this moment in case of the use of geostationary satellite, t_c, can be easily calculated by

$$t_c = (\delta t_b - t_u)/2 - \omega_e \cdot (r \times L)/c^2 \quad (1)$$

where the last term of Eq.1 is the Sagnac correction term and ω_e is the earth rotation rate, r the position vector of mobile station, L the position vector of satellite and c is the speed of light. It should be noted that no knowledge of precise distances between radio stations is needed in this synchronization processing, and nearly all errors related to radio propagation are cancelled, allowing t_c to be accurately derived.

The adoption of the highly stable clock reduces the frequency of the above processing. In this case, the drift rate of user's clock offset must be estimated accurately. This is done by observing the trend in the position vector error derived from the user's equipment installed in the mobile station while the latter is at a standstill.

We now address the problem of deriving a relationship between the positioning error and the clock drift rate. Assuming that $\delta \boldsymbol{r}$ is the error of the user's position vector $\boldsymbol{r}$, $\delta \boldsymbol{L}_i$ the error of the i-th satellite position vector $\boldsymbol{L}_i$, δp_i the error of the range $p_i\,(=|\boldsymbol{L}_i-\boldsymbol{r}|)$ and $\boldsymbol{m}_i$ the unit vector defined by $(\boldsymbol{L}_i-\boldsymbol{r})/\,p_i$, the following equation can be derived:

$$\boldsymbol{m}_i\cdot\delta\boldsymbol{r}=\boldsymbol{m}_i\cdot\delta\boldsymbol{L}_i-\delta p_i \tag{2}$$

Therefore, in three-dimensional positioning, the change, $\delta\boldsymbol{r}_e$, of the position error, $\delta\boldsymbol{r}$, at the time interval from t_0 to t_1 is written as

$$\delta\boldsymbol{r}_e=\frac{1}{\boldsymbol{m}_3\cdot(\boldsymbol{m}_1\times\boldsymbol{m}_2)}\begin{bmatrix}(\boldsymbol{m}_2\times\boldsymbol{m}_3)^T\\(\boldsymbol{m}_3\times\boldsymbol{m}_1)^T\\(\boldsymbol{m}_1\times\boldsymbol{m}_2)^T\end{bmatrix}^T\begin{bmatrix}\delta p_1(t_0)-\delta p_1(t_1)\\\delta p_2(t_0)-\delta p_2(t_1)\\\delta p_3(t_0)-\delta p_3(t_1)\end{bmatrix} \tag{3}$$

where it is assumed that $\delta p_i(t_0)-\delta p_i(t_1)$ changes due to the drift rate of user's clock offset $\delta\varepsilon_c$, written as follows:

$$\delta p_i(t_1)-\delta p_i(t_0)=c\int_{t_0}^{t_1}\delta\varepsilon_c(t)dt \tag{4}$$

For simplicity, it has been assumed here that $\boldsymbol{m}_i$ is fixed during the measurement of $\delta\boldsymbol{r}_e$. If $\delta\varepsilon_c$ is constant, the following equation is easily derived:

$$\delta\varepsilon_c=\frac{-\boldsymbol{m}_i\cdot\delta\boldsymbol{r}_e}{c(t_1-t_0)}\qquad (i=1,2,3) \tag{5}$$

3. Clock-Synchronous Positioning

The navigation algorithm of three-dimensional clock-asynchronous positioning that applies the range measurements of four satellites is written as

$$\begin{bmatrix}\delta\boldsymbol{r}\\\delta q\end{bmatrix}=\frac{1}{W_4}\begin{bmatrix}(\boldsymbol{n}_{24}\times\boldsymbol{n}_{34})^T & -V_{423}\\(\boldsymbol{n}_{34}\times\boldsymbol{n}_{14})^T & -V_{431}\\(\boldsymbol{n}_{14}\times\boldsymbol{n}_{24})^T & -V_{412}\\(\boldsymbol{n}_{12}\times\boldsymbol{n}_{31})^T & V_{123}\end{bmatrix}^T\begin{bmatrix}\delta s_1\\\delta s_2\\\delta s_3\\\delta s_4\end{bmatrix} \tag{6}$$

where δq is the range offset error due to clock offset error δt_c and δs_i is the difference between the measured pseudo-range and the calculated one derived from the latest estimate of position $\boldsymbol{r}$. Other variables are defined as follows.

$$W_4=\boldsymbol{n}_{14}\cdot(\boldsymbol{n}_{24}\times\boldsymbol{n}_{34})$$
$$=V_{123}-V_{423}-V_{431}-V_{412} \tag{7}$$
$$\boldsymbol{n}_{ij}=\boldsymbol{m}_i-\boldsymbol{m}_j\qquad (i,\ j=1,\ 2,\ 3,\ 4) \tag{8}$$
$$V_{ijk}=\boldsymbol{m}_i\cdot(\boldsymbol{m}_j\times\boldsymbol{m}_k)\qquad (i,\ j,\ k=1,\ 2,\ 3,\ 4) \tag{9}$$

For synchronous positioning applying three satellites, the following equation is obtained from Eq.6 with $p_4=0$ and $L_4=\boldsymbol{r}_0$:

$$\delta\boldsymbol{r}=\frac{1}{V_{123}}\begin{bmatrix}(\boldsymbol{m}_2\times\boldsymbol{m}_3)^T\\(\boldsymbol{m}_3\times\boldsymbol{m}_1)^T\\(\boldsymbol{m}_1\times\boldsymbol{m}_2)^T\end{bmatrix}^T\begin{bmatrix}\delta s_1\\\delta s_2\\\delta s_3\end{bmatrix}+\frac{1}{V_{123}}(\boldsymbol{n}_{12}\times\boldsymbol{n}_{31})\delta q \tag{10}$$

Here, δs_i is the same pseudo-range error as used in Eq.6, which contains the range offset δq to be determined from the measured clock offset.

Again, with the real range δp_i defined

$$\delta p_i=\delta s_i+\delta q \tag{11}$$

Eq.10 can be simplified as follows:

$$\delta\boldsymbol{r}=\frac{1}{V_{123}}\begin{bmatrix}(\boldsymbol{m}_2\times\boldsymbol{m}_3)^T\\(\boldsymbol{m}_3\times\boldsymbol{m}_1)^T\\(\boldsymbol{m}_1\times\boldsymbol{m}_2)^T\end{bmatrix}^T\begin{bmatrix}\delta p_1\\\delta p_2\\\delta p_3\end{bmatrix} \tag{12}$$

Positioning accuracy is generally expressed by several kinds of DOP, which is the ratio of the position error to the range measurement error (RME) in the statistical sense, and is derived from the diagonal elements of the covariance matrix $\boldsymbol{P}$ of the position vector error $\delta\boldsymbol{x}$ ($=[\ \delta\boldsymbol{r}^T\ \delta q]^T$), which is written as follows for the asynchronous positioning:

$$\boldsymbol{P}=E[\,\delta\boldsymbol{x}\,\delta\boldsymbol{x}^T]$$
$$=\sigma_0^2(\underline{H}^T\underline{H})^{-1} \tag{13}$$

where σ_0 is the standard deviation of the range measurement error and $\underline{H}$ is the measurement matrix, shown in Section 4, for instance.

DOP is governed by the geometrical relations between the positions of applied satellites and the measurement point. For convenience, several kinds of DOP have been defined: *VDOP* is vertical

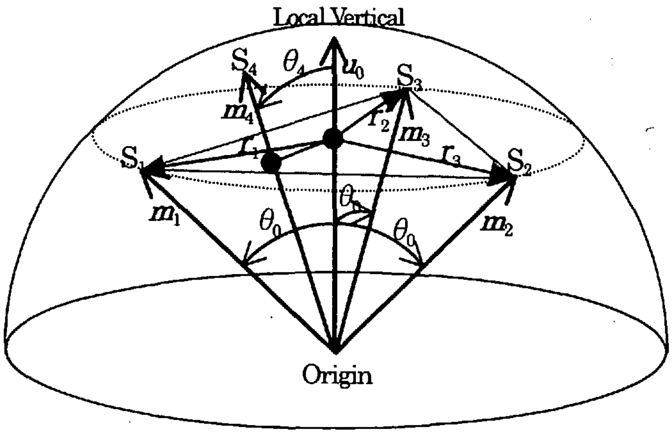

Fig. 2 Satellite Arrangement Geometry

DOP, *HDOP* is horizontal DOP, *TDOP* is temporal DOP, which relates to the range offset due to the clock offset, and *PDOP* is positional DOP, which is the root-sum squares of *VDOP* and *HDOP*. Under normal conditions, it is difficult to obtain convenient functional expressions of the above DOPs. However, when all error sources have been replaced with the equivalent RME, each equivalent RME has the same standard deviation, σ_0, and four satellites applied have been placed as shown in Figure 2, then the approximate DOPs of the clock-asynchronous positioning will be derived as follows:

$$VDOP=(4/3)^{1/2}/|\cos\theta_4-\cos\theta_0| \qquad (14)$$

$$HDOP=(4/3)^{1/2}/\sin\theta_0 \qquad (15)$$

$$TDOP=(1/3+\cos^2\theta_0)^{1/2}/|\cos\theta_4-\cos\theta_0| \qquad (16)$$

For three-dimensional clock-synchronous positioning, it is assumed that no 4th satellite is used; DOPs are written as follows:

$$VDOP=(1/3+\mu^2)^{1/2}/\cos\theta_0 \qquad (17)$$

$$HDOP=(4/3)^{1/2}/\sin\theta_0 \qquad (18)$$

where μ is the ratio of the standard deviation σ_c of range offset error to the standard deviation σ_0 of equivalent RME.

For two-dimensional clock-asynchronous positioning, the 4th satellite is not applied in the same way for three-dimensional clock-synchronous positioning. Approximate DOPs are derived as follows:

$$HDOP=(4/3)^{1/2}/\sin\theta_0 \qquad (19)$$

$$TDOP=(1/3+\lambda^2\cos^2\theta_0)^{1/2} \qquad (20)$$

where λ is the ratio of the standard deviation σ_x of the altitude error to the standard deviation σ_0.

Two-dimensional clock-synchronous positioning uses only two satellites. Approximate *HDOP* is written as follows:

$$HDOP=[2/(\sin\theta_V\sin\phi)^2 + \lambda^2/\{\tan\theta_V\cos(\phi/2)\}^2 + \mu^2/\{\sin\theta_V\cos(\phi/2)\}^2]^{1/2} \qquad (21)$$

where θ_V ($\theta_V=\theta_0$ in this case) is the zenith angle of the applied satellites and ϕ is the difference in the azimuth of the satellites.

Figure 3 shows the relations between the DOPs of three-dimensional synchronous / asynchronous positioning and the zenith angle θ_0. In asynchronous positioning, *VDOP* increases as θ_0 decreases. In contrast, *VDOP* in synchronous positioning is nearly equal to the ratio μ at the range of $\theta_0 < 30$ degrees and increases as θ_0 increases. *HDOP* in synchronous positioning shows the same tendency as in asynchronous position-

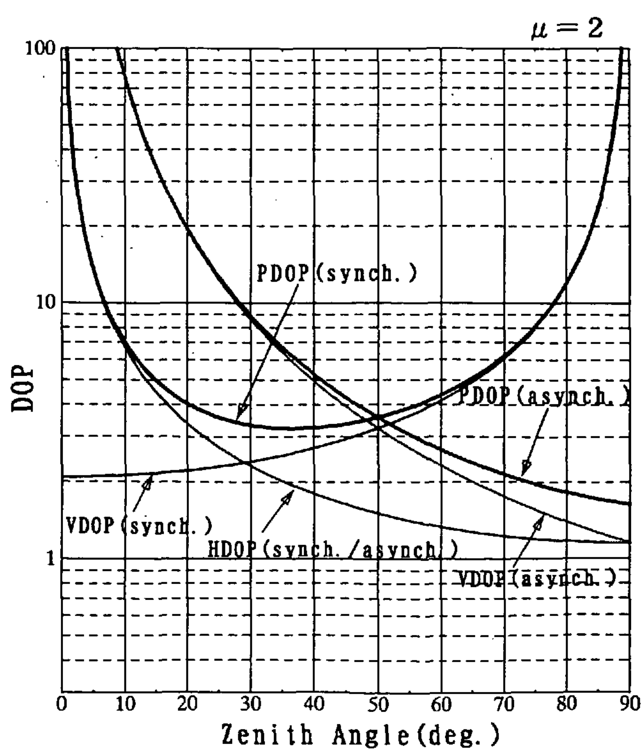

Fig. 3 DOP Comparisons for Three--Dimensional Synchronous/Asynchronous Positioning

ing, and decreasing as θ_0 increases. Therefore, in asynchronous positioning the zenith angle must be enlarged to reduce *PDOP*, which means that a wide field of view is required to obtain accurate positioning. However, synchronous positioning does not require a wide field of view and has a minimum value of *PDOP* at around 35 degrees of the zenith angle.

As evident from the above, synchronous positioning is advantageous not only for reducing DOP but also in terms of the number of satellites to be deployed in orbit. Only two geostationary satellites are required to perform two-dimensional clock-synchronous positioning, which has the positioning accuracy indicated by Eq.21. For example, *HDOP* reaches nearly 5 around Japan when both λ and μ are 2.

We can conclude in this section that clock synchronous positioning has the favorable features of providing integrated satellite services with regard to regional positioning and mobile communications on small-scale constellations through the application of several geosynchronous satellites.

4. Integration of Navigation Algorithms

Any type of navigation algorithms is generally expressed by a common weighted least squares (WLS) algorithm that enables the adoption of all measurements and does not require the selection of 4 satellites in a good geometric arrangement.

The WLS positioning algorithm is also convenient for deriving approximate equations for DOPs. Assuming that σ_0 is the reference value for the standard deviation (SD) of measurement errors, σ_i (i=1,2,$\cdots$N) is the SD of the range measurement error of the i-th satellite, σ_x, σ_y and σ_z are the SDs of the local vertical, local horizontal east and local horizontal north position measurement errors, and σ_c is the SD of the range offset measurement error, then measurement error ratios can be defined as follows:

$$\kappa_i = \sigma_i / \sigma_0 \quad (i=1,2,\cdots,N) \tag{22}$$

$$\kappa_x = \sigma_x / \sigma_0 \tag{23}$$

$$\kappa_y = \sigma_y / \sigma_0 \tag{24}$$

$$\kappa_z = \sigma_z / \sigma_0 \tag{25}$$

$$\kappa_c = \sigma_c / \sigma_0 \tag{26}$$

The WLS positioning algorithm is

$$\delta \mathbf{x} = (\underline{H}^T \underline{H})^{-1} \underline{H}^T \delta\rho \tag{27}$$

where $\delta\rho$ is the vector of the normalized measurement errors, all the SDs of which have been normalized to σ_0, and defined as follows:

$$\begin{aligned}\delta\rho &= (\delta\rho_1\, \delta\rho_2 \cdots \delta\rho_N\, \delta\rho_x\, \delta\rho_y\, \delta\rho_z\, \delta\rho_c)^T \\ &= (\delta Z_1/\kappa_1 \cdots \delta Z_N/\kappa_N \\ &\qquad \delta r_{xm}/\kappa_x\, \delta r_{ym}/\kappa_y\, \delta r_{zm}/\kappa_z\, \delta q_m/\kappa_c)^T\end{aligned} \tag{28}$$

Here δZ_i (i=1,2,$\cdots$,N) is the range measurement error, δr_{xm}, δr_{ym} and δr_{zm} are position measurement errors, δq_c is the range offset measurement error, and $\underline{H}$ is the measurement matrix to relate $\delta \mathbf{x}$ with $\delta\rho$, and is defined as follows:

$$\underline{H} = \begin{bmatrix} m_{11}/\kappa_1 & \cdots & m_{N1}/\kappa_N & 1/\kappa_x & 0 & 0 & 0 \\ m_{12}/\kappa_1 & \cdots & m_{N2}/\kappa_N & 0 & 1/\kappa_y & 0 & 0 \\ m_{13}/\kappa_1 & \cdots & m_{N3}/\kappa_N & 0 & 0 & 1/\kappa_z & 0 \\ 1/\kappa_1 & \cdots & 1/\kappa_N & 0 & 0 & 0 & 1/\kappa_c \end{bmatrix}^T \tag{29}$$

Eq. 27 is the integrated navigation algorithm from which all navigation algorithms are derived by proper setting of the measurement error ratios in Eq.28 and Eq.29. For example, the three-dimensional clock-asynchronous positioning algorithm that is Eq.6 was derived with κ_x, κ_y, κ_z and κ_c set to infinity and N=4. The three-dimensional clock synchronous positioning algorithm that is Eq.10 was derived with κ_x, κ_y and κ_z set infinity and N=3.

Supposing the uniform deployment of satellites in the earth surface, the approximate DOPs for WLS positioning can be derived as follows:

$$VDOP = \frac{(1+1/N\kappa_c^2)^{1/2}}{\{N\beta^2 + \gamma^2/\kappa_c^2 + (1+1/N\kappa_c^2)/\kappa_v^2\}^{1/2}} \tag{30}$$

$$HDOP = \frac{2}{\{N(1-\gamma^2) + 2/\kappa_H^2\}^{1/2}} \tag{31}$$

$$TDOP = \frac{(\gamma^2 + 1/N\kappa_v^2)^{1/2}}{(1+1/N\kappa_c^2)^{1/2}} VDOP \tag{32}$$

where κ_x is replaced with κ_v, κ_y and κ_z are replaced with κ_H. Moreover, β and γ are defined as

$$\beta = \{(1/N)\sum_{i=1}^{N} \cos^2\theta_i - \alpha^2\}^{1/2} \qquad (33)$$

$$\gamma = (\alpha^2 + \beta^2)^{1/2} \qquad (34)$$

$$\alpha = (1/N)\sum_{i=1}^{N} \cos\theta_i \qquad (35)$$

where θ_i is the zenith angle of the i-th satellite.

Eqs.14, 15 and 16 were derived from above equations by assuming that κ_v, κ_H and κ_c are infinite, N=4 and $\theta_i = \theta_0$ for i=1,2,3.

Figure 4, 5 and 6, which were prepared using the above equations, show the relations between the DOPs and the limited zenith angle with six satellites uniformly disposed in the sky. It is recognized from these figures that measurement of either vertical position or range offset is effective to improve the accuracy of estimates of both vertical position and clock offset. However, only horizontal position measurements are effective for improving horizontal position accuracy. Therefore, backup using velocity integral positioning, etc., will ensure stable performance.

The integrated positioning algorithm that uses the WLS method effectively reflects the conditions of a very high stable clock of 10^{-11}sec/sec, for instance, installed in the user's equipment as the clock for the range measurement. This stability is equivalent to roughly 10m/hr of the range drift that causes 30m/hr of three-dimensional positioning error at *PDOP*=3. Even if satellites visibility changes frequently and the number of visible satellites is reduced to three, three-dimensional positioning will continue without a fatal loss of accuracy.

The weights of the WLS positioning algorithm can be regulated using the numerical values of the DOPs calculated from the covariance matrix $P\,(=\sigma_0^2(\underline{H}^T\underline{H})^{-1})$. This regulation results in nearly optimal estimation of positioning, because the WLS algorithm is the same as in the special case of Kalman filtering. For example, DOPs decrease as the number of updating times n increases with the mobile station at a standstill. Approximate equations are written as follows:

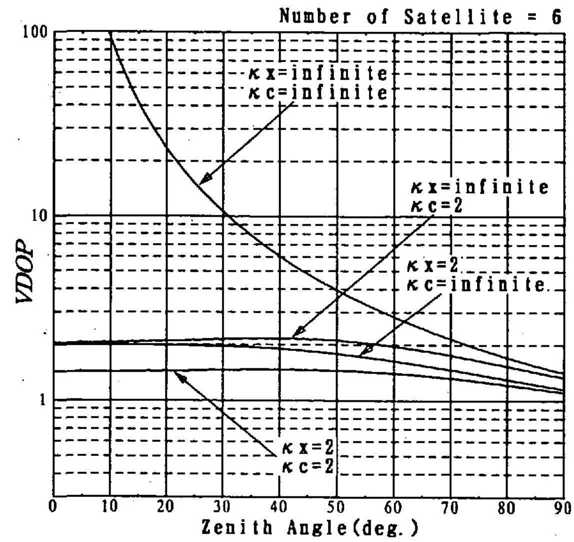

Fig. 4 *VDOP* of WLS Positioning for Zenith Angle

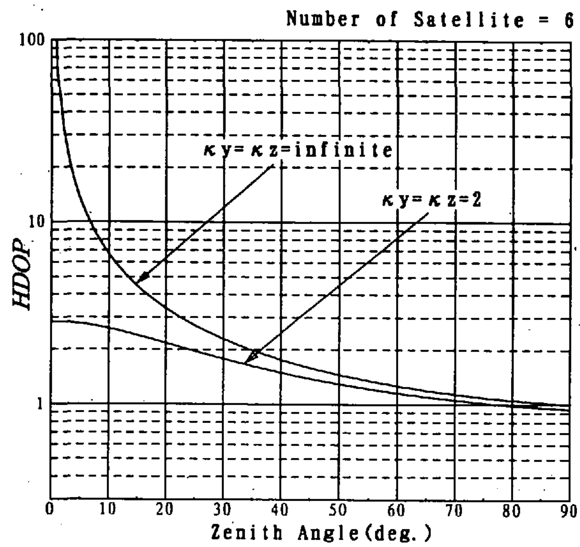

Fig. 5 *HDOP* of WLS Positioning for Zenith Angle

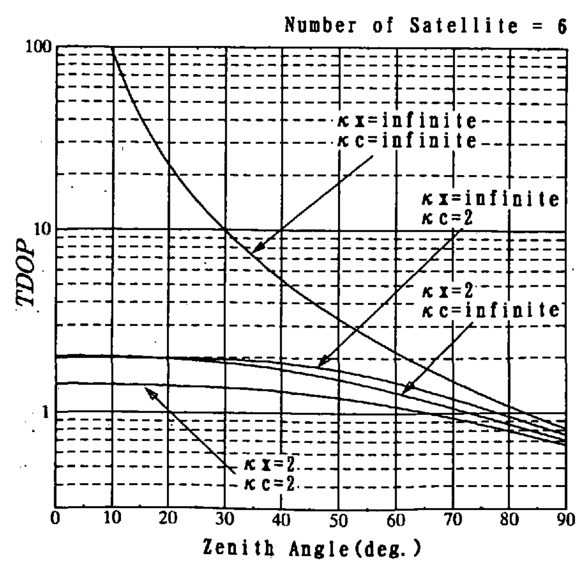

Fig. 6 *TDOP* of WLS Positioning for Zenith Angle

$$VDOP(n\Delta t) \cong (1/n)^{1/2} VDOP(0) \quad (36)$$
$$HDOP(n\Delta t) \cong (1/n)^{1/2} HDOP(0) \quad (37)$$
$$TDOP(n\Delta t) \cong (1/n)^{1/2} TDOP(0) \quad (38)$$

where Δt is the time interval of positioning.

A steady state of estimation errors depends on the prediction errors of state variables or system noises generated for a period of time in position updating and the proper regulation of WLS weights. Assuming that $\phi_V^{1/2}$ and $\phi_H^{1/2}$ are ratios of the standard deviation of position prediction error to the standard deviation σ_0 and that $\phi_T^{1/2}$ is the ratio of the standard deviation of the range offset prediction error to the standard deviation σ_0, then the steady-state values of the above DOPs would be approximately as follows:

$$VDOP_\infty^2 = \gamma^{-1}(\phi_V \phi_T)^{1/2} \quad (39)$$
$$HDOP_\infty^2 = \{-\phi_H + (\phi_H^2 + 4\phi_H HDOP_0)^{1/2}\}/2 \quad (40)$$
$$TDOP_\infty^2 = \gamma(\phi_V \phi_T)^{1/2} \quad (41)$$

where γ^2 is derived from Eq.34. The above equations do not hold when $\phi_V^{1/2}$, $\phi_H^{1/2}$ or $\phi_T^{1/2}$ are larger than 3, roughly speaking.

Figure 7 shows the tendency of DOP convergence due to iterative updating when the limited zenith angle θ_L is 30degrees, $\phi_V = \phi_H = \phi_T = 0.5$ and N=6. It can be clearly seen that vertical and temporal DOP quickly decrease to around 1 after several position updates, even if the DOP's initial value is over 5. It is concluded that regulation of the WLS weights is highly useful in the navigation of motor cars or ships in terms of attaining good positioning, same as the application of Kalman filtering, because unexpected movements are comparatively small and the values for ϕ_V, ϕ_H and ϕ_T remain low.

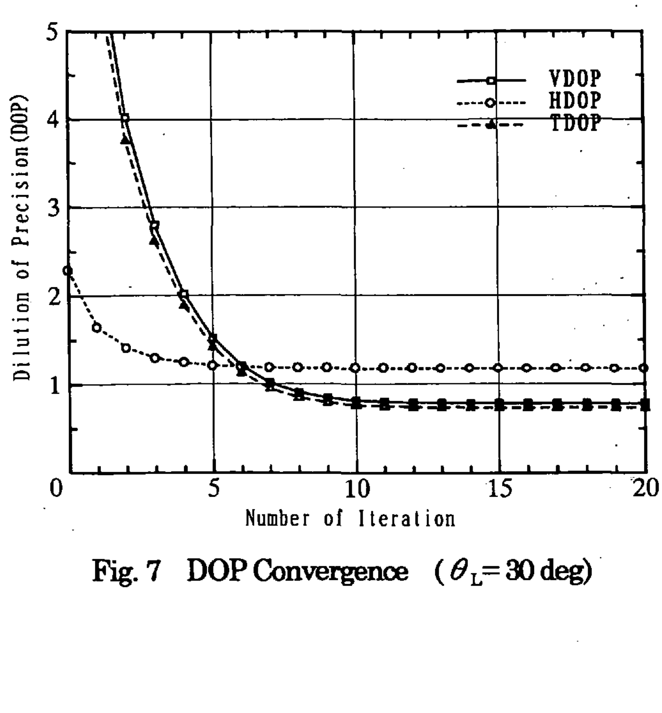

Fig. 7 DOP Convergence (θ_L= 30 deg)

5. Clock-Synchronous Communication

A potential application for clock synchronization is mobile communications.

In general, the great advantage of satellite communications exists in the possibility of wide-area broadcasting from space and in the fact that many mobile users are able to receive radio waves everywhere, selecting the information which they need. However, the mobile communication from a mobile user to a ground radio station is not always efficient, owing to the redundant communication links unintentionally formed with multiple ground radio stations. To put it concretely, the radio waves transmitted from a mobile user are received by all satellites visible from the mobile user and transmitted to all visible ground stations. Therefore, each ground station receives and processes the radio waves to provide the other users with information from the mobile user, consequently reducing the efficiency of mobile communications.

The problem related to the above disadvantage of satellite-based mobile communications can be solved by applying precise clock synchronization to communication circuit control. When mobile users have the positioning means to calculate the distance of radio propagation to the other radio station, the mobile station will be able to transmit the radio waves so that they arrive at the time that has been pre-assigned for each ground station. Each ground station will then be able to receive and process only the radio waves that have been transmitted at the pre-assigned timing. As a result of the above procedures, mobile users will be able to select the radio stations and the radio waves transmitted by mobile users linked with public telephone networks, without redundancy. Such optional communication will be possible when all radio stations have been clock-synchronized precisely and mobile stations know their own locations. Also, the application of radio waves with spectra spread by the appropriate pseudo-random number (PRN) codes is essential for reducing the interference in multiple access communications.

In CDMA communication, which uses "spread spectrum" radio waves modulated by ideal PRN codes, radio waves have no correlation to those whose heads (epoch) of the PRN code has some time lag, even if waves are modulated by the same PRN codes. Therefore, the radio station is able to select and process only those which arrive within a very narrow time slot, based on pre-assigned timing.

The aforementioned property of radio waves modulated by a PRN code is useful for the multiple access of mobile communication circuits described previously. Especially in message communications, the probability of interference can be reduced even if simple ALOHA is applied for the method of circuit access.

The probability of interference P is expressed as follows:

$$P = 1 - (r/k)\exp(-r/k) \qquad (42)$$

where r is the average rate of circuit application (connection) in the mobile communications and k is the number of multiple access channels, defined as follows:

$$k = \frac{\text{Time Interval of PRN Code Sequence}}{\text{Available Time Slot in Inversed SS}} \qquad (43)$$

Figure 8 which is derived from Eq.42 shows that the preparation of multiple access channels improves the efficiency of spread-spectrum communication circuits. Specifically, the probability of interference P without multiple access, that is $k=1$, is 0.95 at around $r=5$, but P is reduced to almost 1/10 when k is 10. Therefore, multiple access in mobile message communications utilized by many mobile users is indispensable for

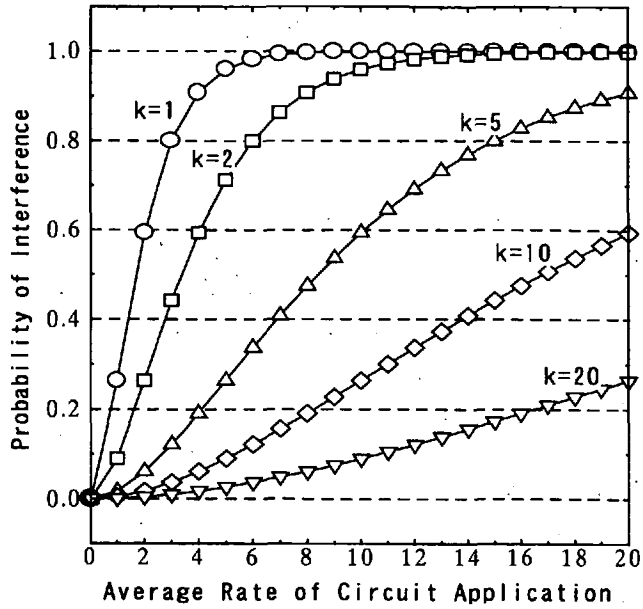

Fig. 8 Probability of Interference vs. Multiple Access

avoiding frequent interference, and multiple access channels are formed by preparing several time slots that have time lags for radio receiving based on the pre-assigned timing, which is maintained by the precise synchronism.

6. Satellite Constellation for Positioning

Satellite positioning based on clock synchronization, or "clock-synchronous positioning", as it is called in the previous sections, has highly favorable features. Especially in the application of geosynchronous satellites to navigation satellites, clock-synchronous positioning will realize higher availability than GPS by a small-scale satellite constellation.

A minimum satellite constellation for continuous positioning can be constructed by two geostationary satellites. Figure 9 shows $HDOP$ of two-dimensional clock-synchronous positioning. It can be seen from this figure that $HDOP$ less than 10 can be attained when the latitude of measurement position is higher than 20 degrees and the difference in geostationary longitude of the two satellites is within 10 to 90 degrees. Such a regional navigation system, using geostationary communication satellites, will suit GPS augmentation.

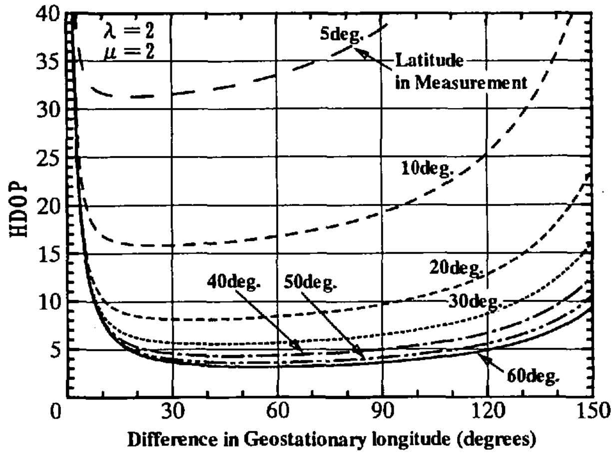

Fig. 9 HDOP of Two-Dim. Synchronous Positioning

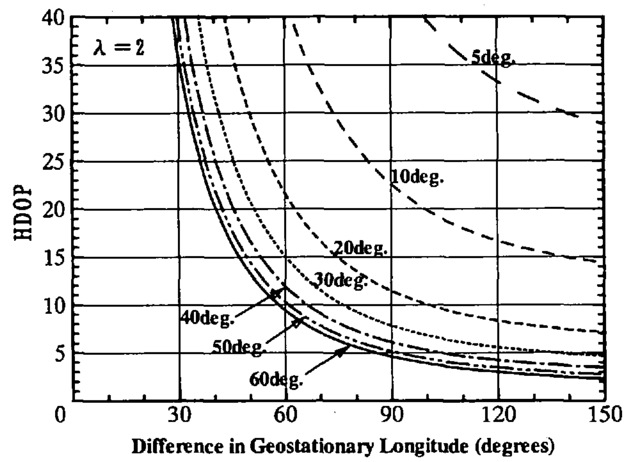

Fig. 10 HDOP of Two-Dim. Asynchronous Positioning

In principle, a constellation composing of three satellites enables two-dimensional clock-asynchronous and three-dimensional clock-synchronous positioning, and a constellation of more than three satellites enables three-dimensional clock-asynchronous positioning. However, when geosynchronous satellites are applied to navigation satellites, two-dimensional clock-asynchronous positioning is the only available one in terms of the geometrical dilution of precision. Figure 10 shows *HDOP* of two-dimensional clock-asynchronous positioning. It can be seen from this figure that *HDOP* less than 10 can be attained when the latitude of measurement position is higher than 20 degrees and the maximum difference in geostationary longitude of the three satellites is larger than 100 degrees. Therefore, the navigation serviceable area is narrower than that in two-dimensional clock-synchronous positioning. It means that more satellites are needed in two-dimensional clock-asynchronous positioning.

A satellite constellation composing of two geostationary satellites and three geosynchronous satellites with 45 degrees of orbital inclination, for instance, it is called five stars network (FSN) for convenience, will enable three-dimensional clock-synchronous positioning roughly in the area of one third of the earth's surface. Figure 11 shows the navigation serviceable areas that achieve positioning accuracy of PDOP<5 at 10 degrees of the limited elevation angle. It is clearly seen from this figure that clock-synchronous positioning is superior to clock-asynchronous in the availability as a means of navigation.

Figure 12 shows the global positioning accuracy of a satellite constellation which composes of three FSNs deployed globally. Numbers in the world map means the percentage of $PDOP > 5$ and minus sign (-) means no dilution in all time. It is seen from this figure that it is possible to provide navigation service of $PDOP < 5$ almost 100 % in the world, using FSN constellations.

7. Conclusion

The results clearly demonstrate that clock-synchronous positioning is superior to clock-asynchronous positioning in terms of the number of satellites deployed and the geometrical requirements for satellite placement, as well as the fact that any positioning algorithm can be effectively integrated by applying the weighted least squares estimation algorithm shown in Section 4.

The integrated positioning algorithm behaves like asynchronous positioning without clock synchronization and offers more accurate positioning data, depending on the frequency of synchronization and the user's clock stability. Optimal processing can be done by properly regulating the measurement error ratios.

Precise clock synchronization of around 10^{-9} sec will be achieved using bi-directional communication, and this synchronism will be applied not only to clock-synchronous positioning but also to clock-synchronous communications, realizing the efficient use of spread-spectrum radio waves.

Satellite-based navigation and mobile communications will be satisfactorily unified by clock synchronism, as is expected in the above discussions, and will establish complementary relations. The integrated satellite system that results will decrease total-system life-cycle cost and will supply positioning and message communication services simultaneously at a low charge for numerous mobile users.

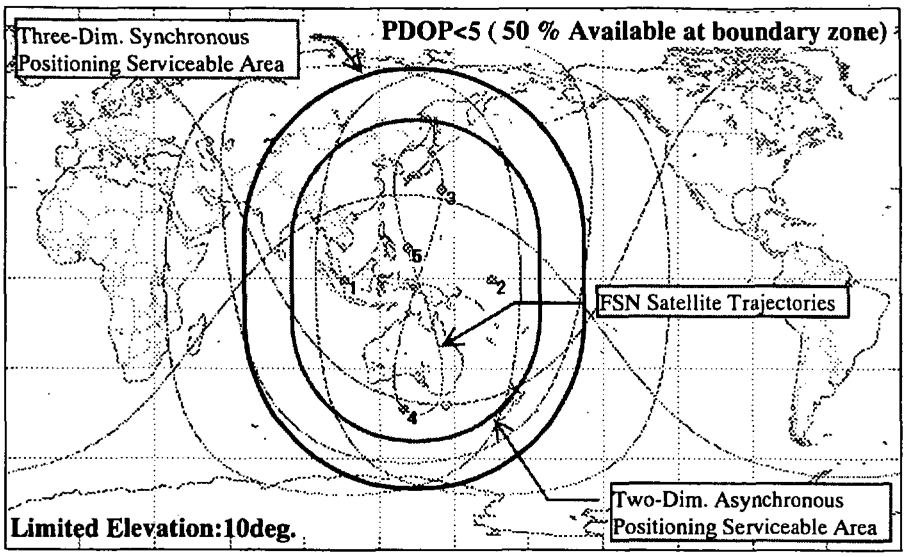

Fig. 11 Comparison of FSN Navigation Serviceable Area

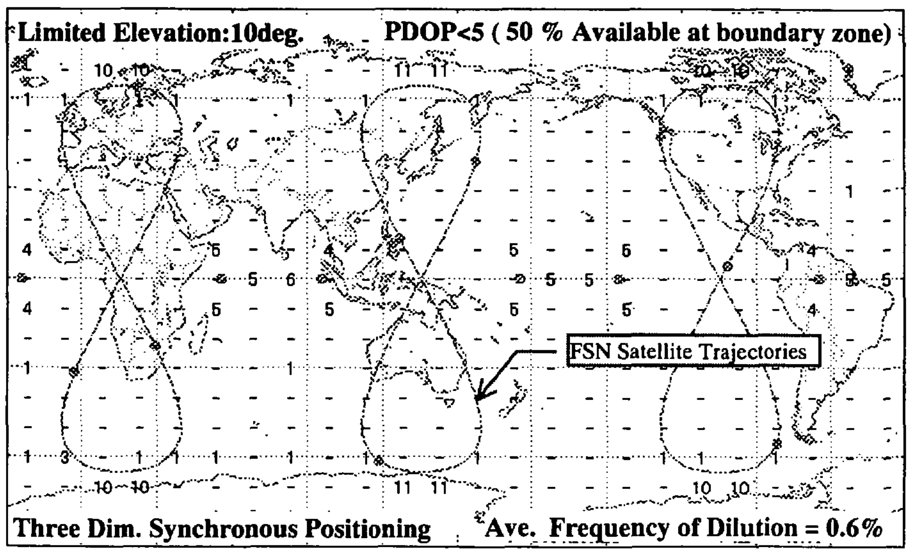

Fig. 12 Global Positioning Performance of Three FSNs Constellation

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