Comparison of Effectiveness of Four Open Boundary Conditions for Incompressible Unbounded Flows*

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ABSTRACT

Four open boundary conditions for incompressible unbounded flows are evaluated in the framework of the Leith type third-order upwind scheme (QUICKEST scheme), and the effectiveness of each is compared by two means: the difference of flows among open boundary conditions, and between short and long open boundaries. Three test problems used for the open boundary condition evaluation are the backward-facing step flow, the blunt based body flow and the rectangular cylinder obstacle flow in a channel. The investigated open boundary conditions are (1) the one first used by Thoman and Szewczyk (1966), (2) the one proposed by Mehta and Lavan (1975), (3) the Sommerfeld radiation condition first used by Orlanski (1976), and (4) the Sommerfeld radiation condition used by Bottaro (1990) and Kobayashi et al. (1993). In (3), Orlanski proposed a method which numerically evaluates the phase speed at the closest interior points every time. In (4), Bottaro and Kobayashi et al. proposed taking the mean channel velocity as the phase speed. The author proposes taking the uniform inlet velocity as the phase speed. As the conclusion, we show that (4) is the optimum open boundary condition of the four conditions. The effects of several values of constant phase speed are shown.

Keywords: Leith type 3o upwind scheme, open boundary condition, Sommerfeld radiation condition, backward-facing step flow, blunt based body flow, rectangular cylinder obstacle flow

概　　要

本欄では、非圧縮・非有界流れのための４つの open boundary conditions が Leith type third-order upwind scheme を用いて評価され、その各々の有効性が open boundary conditions の間の流れの差異と長短の２つの open boundary の間の流れの差異の２つの手段によって比較検討される。open boundary condition の評価のために使用された３つの実験問題は流路管内の backward-facing step flow, blunt based body flow 及び rectangular cylinder obstacle flow である。検討された open boundary condition は(1)Thoman と Szeweczyk(1966)によって最初に用いられたものと、(2)Mehta と Lavan(1975)によって提案されたもの(3) Orlanski(1976)によって最初に用いられた Sommerfeld radiation condition 及び(4)Bottaro (1990)と Kobayashi ら(1993)によって用いられた Sommerfeld radiation condition とである。(3)に於いて、Orlanski は各シュミレーション時間毎に、open boundary の１つ左の最内点において位相速度を数値的に評価する方法を提案した。一方(4)において、Bottaro と Kobayashi らは位相速度として平均流路内速度をとることを提案した。これに対して、著者は本稿において、位相速度として一様インレット速度をとることを提案している。各種の数値実験の全結果は著者の提案した(4)が最も優れた open boundary condition であることを示したことを付言しておく。

* **Received 29 July 1998**

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1. INTRODUCTION

In many computational problems, we are faced with infinite domains, which for computational reasons must be made finite. One possibility is to introduce an artificial boundary in order to reduce the infinite computational domain to a finite one. Then, the introduction of the artificial boundary makes it necessary to formulate appropriate artificial boundary conditions, the open boundary conditions (OBCs). However, mathematics does not tell us how to select the OBCs. While we are not able to state the best OBCs, we can list some qualities that they would display: They should permit the flow to exit the domain gracefully and passively, and not have any effect on the behaviour of the solution in the domain near the open boundary, especially far from it. They should be transparent, and lead to the same solution inside the common domain no matter where truncation occurred [1].

Historically, Thoman and Szewczyk (1966) firstly developed less restrictive OBCs [2]. But use of their zero-gradient OBCs causes premature smoothing of the wake. For the sake of this phenomenon, by Lugt and Haussling (1974), and Metha and Lavan (1975), the computation of local velocities at the exit was selected as the best method of allowing vortices to leave the domain with minimal interference [3, 4]. Afterwards, studies for the OBCs have been done by many researcheres [1, 5-7]. The recent trend of this study seems to separate two branches. One is the course of studying ‘no boundary condition at outflow’ [8-11]. This has been developed for the finite element method, but cannot apply to the finite difference method yet. The other is the course of studying the Sommerfeld radiation condition [12-16]. This seems to be the most promising as the OBCs at the present time.

In this paper, we compute the numerical solutions of the backward-facing step flows, the blunt based body flows and the rectangular cylinder obstacle flows in the framework of the Leith type third order upwind scheme. Four OBCs for incompressible unbounded flows are applyed to each numerical solution, its effectiveness is compared with each other, and whether its OBC can bear practically or not is confirmed. As the conclusion, we show that the Sommerfeld radiation condition is the most excellent OBC within four OBCs, when we take the uniform inlet velocity as the phase speed.

2. NUMERICAL METHOD

Basic equations

The two-dimensional viscous incompressible flow is governed by the following equations : The vorticity (ζ) transport equation in conservation form in case that *Re* is the Reynolds number is given by

$$\zeta_t + (u\zeta)_x + (v\zeta)_y = \frac{1}{Re}(\zeta_{xx} + \zeta_{yy}), \tag{1}$$

and the Poisson equation for the stream-function (ψ) by

$$\psi_{xx} + \psi_{yy} = -\zeta, \tag{2}$$

where t is the time, x and y the axtial and normal coordinates, respectively. The subscripts t, x and y refer to partial derivatives with respect to t, x and y, respectively. The x and y components of the velocity (u, v) are given by

$$\psi_y = u,\ \psi_x = -v. \tag{3}$$

Finite difference schemes

The Leith type third order upwind finite difference schemes for equation (1) are given as follows [17]:

$$\begin{aligned}
\zeta^{n+1}_{i,j} &= \zeta^{n}_{i,j} + WR + WL + WU + WD \\
&+ \frac{1}{2}c^2_{i+1/2}(\zeta_{i+1,j} - \zeta_{i,j}) - \frac{1}{2}c^2_{i-1/2}(\zeta_{i,j} - \zeta_{i-1,j}) \\
&+ \frac{1}{2}c^2_{j+1/2}(\zeta_{i,j+1} - \zeta_{i,j}) - \frac{1}{2}c^2_{j-1/2}(\zeta_{i,j} - \zeta_{i,j-1}) \\
&+ \gamma_x(\zeta_{i+1j} - 2\zeta_{i,j} + \zeta_{i-1,j}) \\
&+ \gamma_y(\zeta_{i,j+1} - 2\zeta_{i,j} + \zeta_{i,j-1}),
\end{aligned} \tag{4}$$

where

$$c_{i+1/2} = \frac{\Delta t u_{i+1/2,j}}{\Delta x},\ c_{i-1/2} = \frac{\Delta t u_{i-1/2,j}}{\Delta x},$$

$$c_{j+1/2} = \frac{\Delta t v_{i,j+1/2}}{\Delta y}, \tag{5}$$

$$c_{j-1/2} = \frac{\Delta t v_{i,j-1/2}}{\Delta y},\ \gamma_x = \frac{\Gamma \Delta t}{\Delta x^2},\ \gamma_y = \frac{\Gamma \Delta t}{\Delta x^2},$$

$$\Gamma = \frac{1}{Re}, \qquad (6)$$

$$u_{i+1/2,j} = \frac{1}{2}(u_{i+1,j} + u_{i,j}),$$

$$u_{i-1/2,j} = \frac{1}{2}(u_{i,j} + u_{i-1,j}), \qquad (7)$$

$$v_{i,j+1/2} = \frac{1}{2}(v_{i,j+1} + v_{i,j}),$$

$$v_{i,j-1/2} = \frac{1}{2}(v_{i,j} + v_{i,j-1}), \qquad (8)$$

if $u_{i+1/2,j} \geq 0$,

$$WR = -\frac{1}{2}c_{i+1/2}(\zeta_{i+1,j} + \zeta_{i,j}) + c_{i+1/2}\left(\frac{1}{6} - \gamma_x - \frac{1}{6}c^2_{i+1/2}\right) * (\zeta_{i+1,j} - 2\zeta_{i,j} + \zeta_{i-1,j}), \qquad (9)$$

if $u_{i+1/2,j} \leq 0$,

$$WR = -\frac{1}{2}c_{i+1/2}(\zeta_{i+1,j} + \zeta_{i,j}) + c_{i+1/2}\left(\frac{1}{6} - \gamma_x - \frac{1}{6}c^2_{i+1/2}\right) * (\zeta_{i+2,j} - 2\zeta_{i+1,j} + \zeta_{i,j}), \qquad (10)$$

if $u_{i-1/2,j} \geq 0$,

$$WL = \frac{1}{2}c_{i-1/2}(\zeta_{i,j} + \zeta_{i-1,j}) - c_{i-1/2}\left(\frac{1}{6} - \gamma_x - \frac{1}{6}c^2_{i-1/2}\right) * (\zeta_{i,j} - 2\zeta_{i-1,j} + \zeta_{i-2,j}), \qquad (11)$$

if $u_{i-1/2,j} \leq 0$,

$$WL = \frac{1}{2}c_{i-1/2}(\zeta_{i,j} + \zeta_{i-1,j}) - c_{i-1/2}\left(\frac{1}{6} - \gamma_x - \frac{1}{6}c^2_{i-1/2}\right) * (\zeta_{i+1,j} - 2\zeta_{i,j} + \zeta_{i-1,j}), \qquad (12)$$

if $v_{i,j+1/2} \geq 0$,

$$WU = -\frac{1}{2}c_{j+1/2}(\zeta_{i,j+1} + \zeta_{i,j}) + c_{j+1/2}\left(\frac{1}{6} - \gamma_x - \frac{1}{6}c^2_{j+1/2}\right) * (\zeta_{i,j+1} - 2\zeta_{i,j} + \zeta_{i,j-1}), \qquad (13)$$

if $v_{i,j+1/2} \leq 0$,

$$WU = -\frac{1}{2}c_{j+1/2}(\zeta_{i,j+1} + \zeta_{i,j}) + c_{j+1/2}\left(\frac{1}{6} - \gamma_x - \frac{1}{6}c^2_{j+1/2}\right) * (\zeta_{i,j+2} - 2\zeta_{i,j+1} + \zeta_{i,j}), \qquad (14)$$

if $v_{i,j-1/2} \geq 0$,

$$WD = \frac{1}{2}c_{j-1/2}(\zeta_{i,j} + \zeta_{i,j-1}) - c_{j-1/2}\left(\frac{1}{6} - \gamma_x - \frac{1}{6}c^2_{j-1/2}\right) * (\zeta_{i,j} - 2\zeta_{i,j-1} + \zeta_{i,j-2}), \qquad (15)$$

if $v_{i,j-1/2} \leq 0$,

$$WD = \frac{1}{2}c_{j-1/2}(\zeta_{i,j} + \zeta_{i,j-1}) - c_{j-1/2}\left(\frac{1}{6} - \gamma_x - \frac{1}{6}c^2_{j-1/2}\right) * (\zeta_{i,j+1} - 2\zeta_{i,j} + \zeta_{i,j-1}), \qquad (16)$$

Boundary conditions

Figure 1 shows geometry definition of three test problems. In (A), B1, B2, B3 and B5 are the no-slipe solid walls, B4 the inlet and B6 an open boundary. Coordinates of points 1 and 2 are (2JH, JH) and (IN, 2JH), respectly. We take IN = 14JH as the short open boundary and IN = 20JH as the long open boundary. In (B), B1, B2, B3, B4 and B6 are the no-slipe solid walls, B5 the inlet and B7 an open boundary. Coordinates of points 1, 2 and 3 are (0, JH), (2JH, 2JH) and (IN, 3JH), respectly. We take IN = 14JH as the short open boundary and IN = 20JH as the long open boundary. In (C), B1, B2, B3, B4, B6 and B7 are the no-slipe solid walls, B5 the inlet and B8 an open boundary. Coordinates of points 1, 2 and 3 are (8JH, 2JH), (9JH, 3JH) and (IN, 5JH), respectly. We take IN = 30JH as the short open boundary and IN = 35JH as the long open boundary. In all the numerical computations, grid size is decided based on JH = 40. At the inlet, a uniform inlet u-velocity profile

$$u\,(y) = 1 \tag{17}$$

is chosen. We note that truncation occurs at x = IN.

3. TESTED OPEN BOUNDARY CONDITIONS

OBC: no.1

The following open boundary condition was firstly used by Thoman and Szewczyk (1966):

$$\frac{\partial u}{\partial x}\Big|_{OB} = -\frac{\partial^2 \psi}{\partial x^2}\Big|_{OB} = 0,\ \frac{\partial \zeta}{\partial x}\Big|_{OB} = 0. \tag{18}$$

OBC: no. 2

The following open boundary condition was proposed by Mehta and Lavan (1975):

$$\frac{\partial \zeta}{\partial t} = -\frac{\partial (u\zeta)}{\partial x} - \frac{\partial (v\zeta)}{\partial y},\ \frac{\partial \psi}{\partial x} = -v,\ \frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - \upsilon\frac{\partial v}{\partial y} = -\left(v\zeta + \frac{1}{2}\frac{\partial\,(u^2+v^2)}{\partial y}\right) \quad \text{at } OB, \tag{19}$$

in case that at the open boundary the inertia terms are dominant.

OBC: no. 3

The following open boundary condition is the Sommerfeld radiation condition firstly used by Orlanski (1976):

$$\frac{\partial \phi}{\partial t} + c\frac{\partial \phi}{\partial t} = 0 \text{ at } OB, \tag{20}$$

where ϕ is any variable, and c is the phase velocity of the waves. Orlanski proposed the following method which numerically evaluates the phase speed at the closet interior points every time: Using a leapfrog finite-difference representation, we have

$$\frac{\phi^{n+1}_{OB} - \phi^{n-1}_{OB}}{2\Delta t} = -\frac{c}{2\Delta x}\left(\phi^{n+1}_{OB} + \phi^{n-1}_{OB} - 2\phi^{n}_{OB-1}\right). \tag{21}$$

Hence the phase speed is numerically evaluated at the closet interior points from the above equation as follows:

$$c = -\frac{\Delta x}{\Delta t}\,\frac{\phi^{n}_{OB-1} - \phi^{n-2}_{OB-1}}{\phi^{n}_{OB-1} + \phi^{n-2}_{OB-1} - 2\phi^{n-1}_{OB-2}}. \tag{22}$$

From the above two equations, we can also obtain the boundary conditions $\{\phi^{n+1}_{OB}\}$ as follows:

$$\phi^{n+1}_{OB} = \frac{1 - c\Delta t/\Delta x}{1 + c\Delta t/\Delta x}\,\phi^{n-1}_{OB} + \frac{2c\Delta t/\Delta x}{1 + c\Delta t/\Delta x}\,\phi^{n}_{OB-1}. \tag{23}$$

OBC: no. 4

The following open boundary condition is the Sommerfeld radiation condition used by Bottaro (1990) and Kobayashi, Pereira and Sousa (1993):

$$\frac{\partial \phi}{\partial t} + c\frac{\partial \phi}{\partial x} = 0 \text{ at } OB, \tag{24}$$

where ϕ is any variable, and c is the phase velocity of the waves. Bottaro took the average streamwise speed in the channel as c, and Kobayashi et al. the mean channel velocity as c. The author proposes to take the uniform inlet velocity as c. Therefore, c = 1.

4. RESULTS ON THE BACKWARD-FACING STEP FLOWS

In this problem, we show results for Re = 1,000. Firstly we compare difference of flows among four OBCs in case of IN = 20JH = 800. Figure 2 shows three aspects of flows according to advance of the simulation time t. In (A), we can see that variation of flow does not yet arrive at the open boundary at t = 30. Meanwhile in (B), we can find that variation of flow already arrives at the open boundary at t = 35, and hence it is the same as well at t = 40, as showed in (C). As seen from (B) and (C), there is severe difference among four OBCs in flows in the domain near the open boundary. Hence we cannot at all conclude which of four OBCs gives the most excellent solution, only from Figure 2. While it is surely true in the domain near the open boundary, we can also show complete coincidence in flows among four OBCs in the domain within x = 14 JH = 560. That is, Figure 3 shows that there is no difference of flows among four OBCs, which is drawed in piles the stream function profile and the vorticity profile of flow by each OBC on several vertical internal points within x = 560 in case of IN = 800. This fact promotes us next step. (We here note that computation of flow by OBC: no. 2 was breaked off due to occure overflow in computation of ζ at $t > 35.1$. Hence [3] of Figure 3 does not contain data by OBC: no. 2.)

Secondly we compare difference of flows between short and long open boundaries by each OBC. Figure 4 (A) and (B) show its aspects of flows by OBC: no. 1, and (C) its difference of flows between IN = 560 and IN = 800, which is drawed similarly to Figure 3. Here we strongly note that the profile for IN = 560 is data at the open boundary, and on the other hand the profile for IN = 800 is data on the vertical internal points. Clearly from (C), there is severe difference of flows by OBC: no. 1 in the domain near the open boundary of IN = 560. Hence OBC: no. 1 cannot at all say as a good OBC. (In (A) and (B) of Figure 4, we use symbols of DTHP and DTHN. DTHP expresses the pitch drawed on streanlines for $\psi \geq 0$, and DTHN expresses the pitch drawed on streanlines for $\psi \leq 0$.) Next we examine the case of OBC: no. 2. Figure 5 shows its difference of flows between short and long open boundaries. As seen from (C), OBC: no. 2 shows comparatively good coincedence of flows in the domain near the open boundary of IN = 560. Regretably this OBC cannot bear practically due to occure overflow at $t > 35.1$. Next we look into the case of OBC: no. 3. Figure 6 shows its difference of flows between short and long open boundaries. Clearly from (C), there is severe difference of flows by OBC: no. 3, especially at the open boundary of IN = 560. Hence OBC: no. 3 is better than OBC: no. 1, but we cannot yet say that it is a good OBC.

Finally we examine the case of OBC: no. 4. Figure 7 shows its difference of flows between short and long open boundaries. As seen from (C), OBC: no. 4 shows tolerable good coincedence of flows even at the open boundary of IN = 560. Hence we can conclude that OBC: no. 4 is the best OBC among four OBCs, and is the excellent OBC.

5. RESULTS ON THE BLUNT BASED BODY FLOWS

This problem is more complicated than the previous one, and hence OBC: no. 2 could not bear practically for this problem due to this complexity. We discuss about OBCs for this problem similarly to the previous problem. We show here results for Re = 1,000. Firstly we compare difference of flows among three OBCs in case of IN = 20JH = 800. Figure 8 shows three aspects of flows according to advance of the simulation time t. In (A), we can see that variation of flow does not yet arrive at the open boundary at t = 30. Meanwhile in (B), we can find that variation of flow already arrives at the open boundary at t = 35, and hence it is the same as well at t = 40, as showed in (C). As seen from (B) and (C), there is severe difference among three OBCs in flows in the domain near the open boundary. Hence we cannot at all conclude which of three OBCs gives the most excellent solution, only from Figure 8. While it is surely true in the domain near the open boundary, we can also show complete coincidence in flows among three OBCs in the domain within x = 14JH = 560. That is, Figure 9 shows that there is no difference of flows among three OBCs, which is drawed in piles the stream function-profile and the vorticity-profile of flow by each OBC on several vertical internal points within x = 560 in case of IN = 800. This fact promotes us next step.

Secondly we compare difference of flows between short and long open boundaries by each OBC. Figure 10 (A) and (B) show its aspects of flows by OBC: no. 1, and (C) its difference of flows between IN = 560 and IN = 800, which is drawed similarly to Figure 9. Here we strongly note that the profile for IN = 560 is data at the open boundary, and on the other hand the profile for IN = 800 is data on the vertical internal points. Clearly from (C), there is severe difference of flows by OBC: no. 1 in the domain near the open boundary of IN = 560. Hence OBC: no. 1 cannot at all say as a good OBC. Next we look into the case of OBC: no. 3. Figure 11 shows its difference of flows between short and long open boundaries. Clearly from (C), there is severe difference of flows by OBC: no. 3, especially at the open boundary of IN = 560. Hence OBC: no. 3 is better than OBC: no. 1, but we cannot yet say that it is a good OBC.

Finally we examine the case of OBC: no. 4. Figure 12 shows its difference of flows between short and long open boundaries. As seen from (C), OBC: no. 4 shows tolerable coincedence of flows even at the open boundary of IN = 560. Hence we can conclude that OBC: no. 4 is the best OBC among three OBCs, and is the comparatively good OBC for this problem.

6. RESULTS ON THE RECTANGULAR CYLINDER OBSTACLE FLOWS

This problem is the most complicated among three problem, and hence even OBC: no. 1 could not bear practically for this problem due to this complexity. We discuss about OBCs for this problem similarly to the previous problem. We show here results for Re = 1,000. Firstly we compare difference of flows between two OBCs in case of IN = 35 JH = 1400. Figure 13 shows three aspects of flows according to advance of the simulation time t. In (A), we can see that variation of flow does not yet arrive at the open boundary at t = 45. Meanwhile in (B), we can find that variation of flow already arrives at the open boundary at t = 55, and hence it is the same as well at t = 65, as showed in (C). As seen from (B) and (C), there is severe difference between two OBCs in flows in the domain near the open boundary. Hence we cannot at all conclude which of two OBCs gives the most excellent solution, only from Figure 13. While it is surely true in the domain near the open boundary, we can also show comparatively good coincidence in flows between two OBCs in the domain within x = 30 JH = 1200. That is, Figure 14 shows that there is a little small difference of flows between two OBCs, which is drawed in piles the streamfunction profile and the vorticity profile of flow by each OBC on several vertical internal points within x = 1200 in case of IN =1400. This fact promotes us next step.

Secondly we compare difference of flows between short and long open boundaries by each OBC. Figure 15 (A) and (B) show its aspects of flows by OBC: no. 3, and (C) its difference of flows between IN = 1200 and IN = 1400, which is drawed similarly to Figure 14. Here we strongly note that the profile for IN = 1200 is data at the open boundary, and on the other hand the profile for IN = 1400 is data on the vertical internal points. Clearly from (C), there is severe difference of flows by OBC: no. 3, especially at the open boundary of IN = 1200. Hence we cannot yet say that OBC: no.3 is a good OBC.

Finally we examine the case of OBC: no. 4. Figure 16 shows its difference of flows between short and long open boundaries. As seen from (C), OBC: no. 4 shows considerably smaller difference of flows than OBC: no. 3 at the open boundary of IN = 1200. Hence we can conclude that OBC: no. 4 is better than OBC: no. 3, and bears more well practically for this problem.

7. DISCUSSION

On the backward-facing step flows

(1) Figure 17 shows difference of flows between short and long open boundaries computed under the condition of Re = 800 and OBC: no. 1. We computed this flows in the framework of the first order upwind scheme. As seen from (C), OBC: no. 1 can bear well practically for such the problem as $Re \leq 800$.

(2) We examined the case of IN = 400 where truncation occurs. This value of IN is fairly smaller than the case of Figure 7. We show its result in Figure 18. As seen from this, there is a little difference of flows, especially at the open boundary of IN = 400. Hence we had better not shorten location of truncation to IN = 400.

On the blunt based body flows

(1) As we compare Figure 7 with Figure 12, we clearly see that the complete coincidence of flows between short and long open boundaries as Figure 7 cannot obtain when the problem becomes more complicated.

(2) This fact suggests that OBC: no. 4 no longer is the complete OBC for this problem, although it is the excellent OBC for the backward-facing step problem. Hence we must be studying to search for a better OBC.

On the rectangular cylinder obstacle flows

(1) Firstly we note that numerical solution of flows for $IN \leq 1000$ cannot give the right solutions even by OBC: no. 4. Because reflection occurs at the open boundary, its effect is changed the behaviour of the solution in the domain far from the open boundary, and at last its accumulation leads the wrong solution.

(2) In this paper, the author proposes to take c = 1 as the phase speed of the Sommerfeld radiation condition. Its result is given in Figure 16. Figure 19 and 20 show difference of flows between short and long open boundaries by OBC: no. 4 in case of c = 0.7 and c = 1.3, respectively. When we

compare each (C) of Figure 19, 16 and 20, c = 1.3 seems to be the best among three phase speeds.

On the open boundary conditions

(1) Such the OBCs as OBC: no. 1 and 2 force to prescribe any condition at the open boundary. Hence they seem to oppose some qualities that an ideal OBC would display. As its poofs, flows by these OBCs are necessarily influenced heavily whenever variation of flow arrives at the open boundary, as seen in Figure 4, 5 and 10.
(2) Such the OBCs as OBC: no. 3 and 4 (that is, the Sommerfeld radiation condition) do not force to prescribe any condition at the open boundary, but seem to aid to permit the flow to exit the domain gracefully and passively. Such phenomenon is one of qualities that an ideal OBC would display
(3) The Sommerfled radiation condition is used by some researchers, but the method of deciding its phase speed is different by each researcher. However there is not a firm ground why the phase speed would be decided by their methods. Then the author proposes to take a constant as the phase speed, despite of being not able to state a firm ground.
(4) Clearly from comparison of Figure 6 and 7, Figure 11 and 12, and Figure 15 and 16, OBC: no. 4 is more excellent than OBC: no. 3 for all the problems. Moreover clearly from comparison of each (C) of Figure 19, 16 and 20, the case of c = 1.3 seems to show the best result.
(5) Hence we will say that to take a constant as the phase speed is also better than to take a mean channel velocity every time as the phase speed. Because a mean value necessarily becomes to $c < 1$. From its reason, we can say that a larger value of c is profitable to premit the flow to exit the domain gracefully and passively.

8. CONCLUSION

In this paper we studied about the open boundary conditions for incompressible unbounded flows, reported numerical solutions of flows by four OBCs for the backward-facing step problem, the blunt based body problem and the rectangular cylinder obstacle problem, and evaluated these results by means of difference of flows among four OBCs and between short and long open boundaries.

As the conclusion in all the cases, we showed that the OBC proposed by the author is the most excellent among the investigated OBCs. It is a very simple method which uses the Sommerfeld radiation condition as the OBC, and take a constant as its phase speed. This OBC showed to be the excellent OBC for the backward-facing step problem. However for the blunt based body problem and the rectangular cylinder obstacle problem, that is, as the problem becomes more complicated, this OBC no longer is the complete OBC. Hence we must be studying to search for a better OBC.

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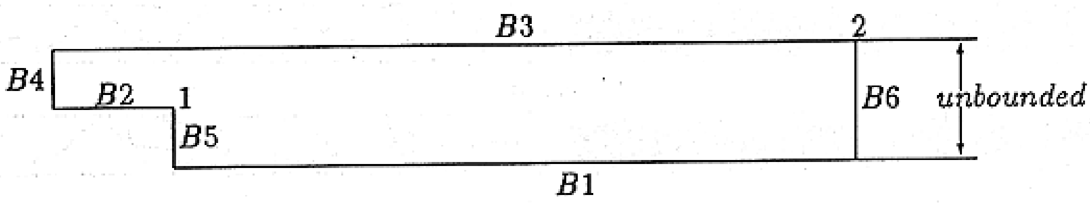
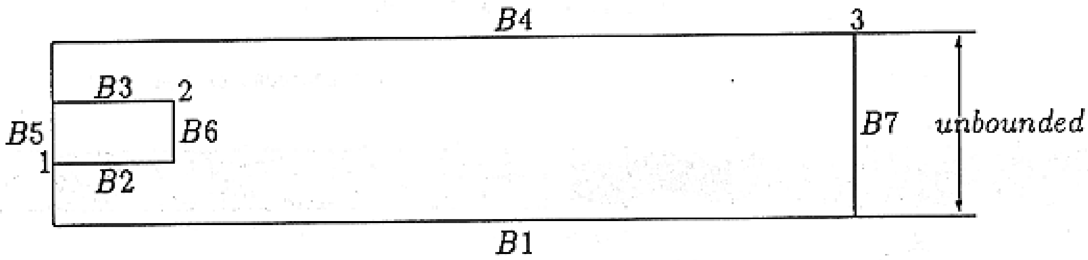
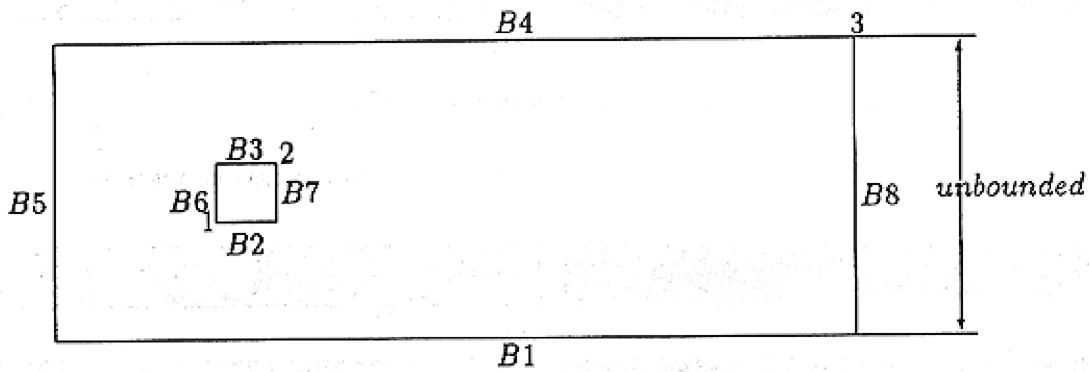


(A) Backward-facing step problem

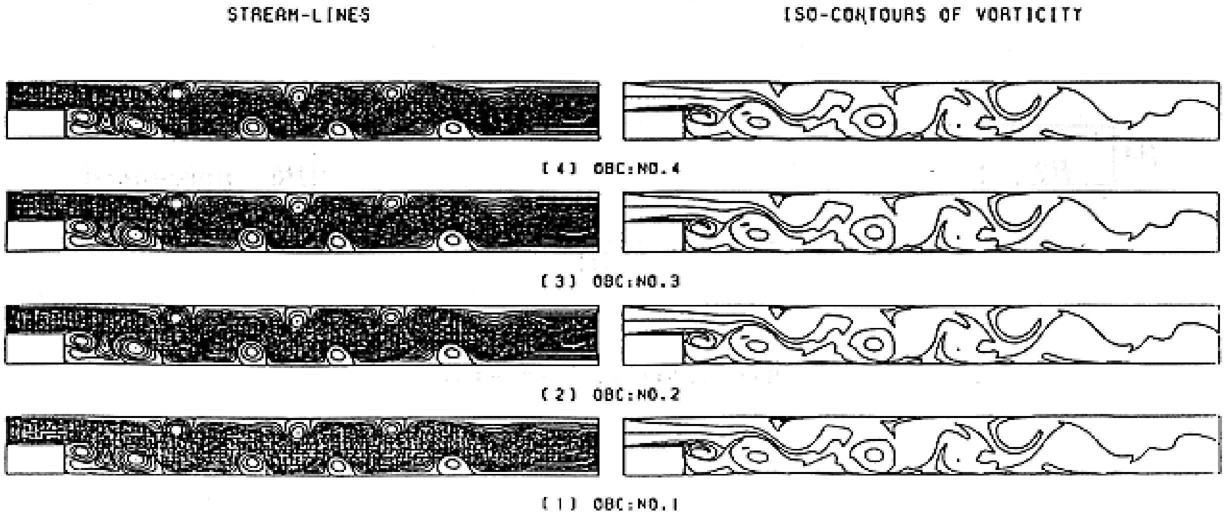


(B) Blunt based body problem

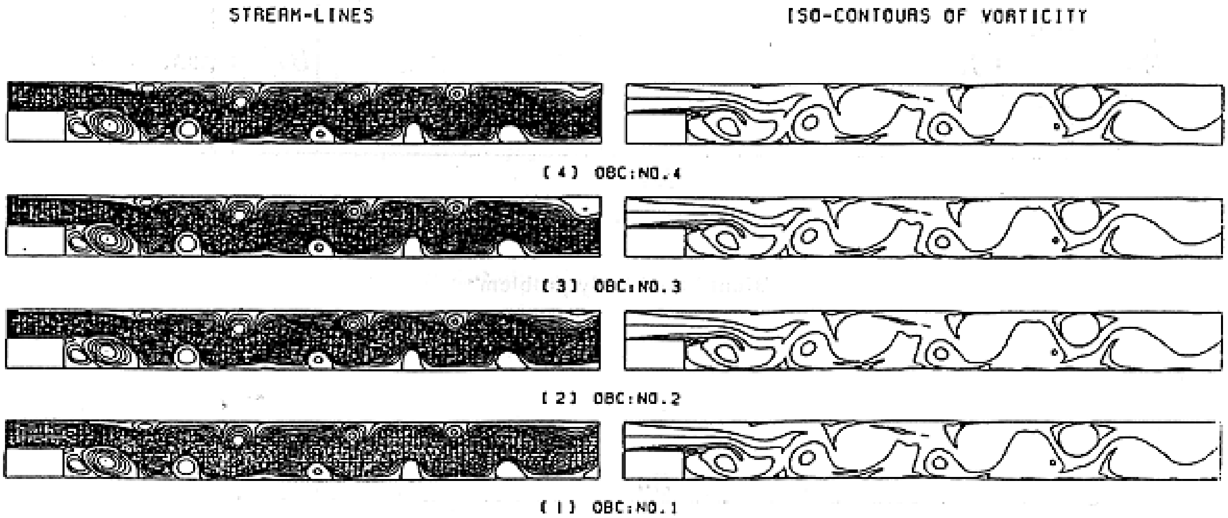


(C) Rectangular cylinder obstacle problem

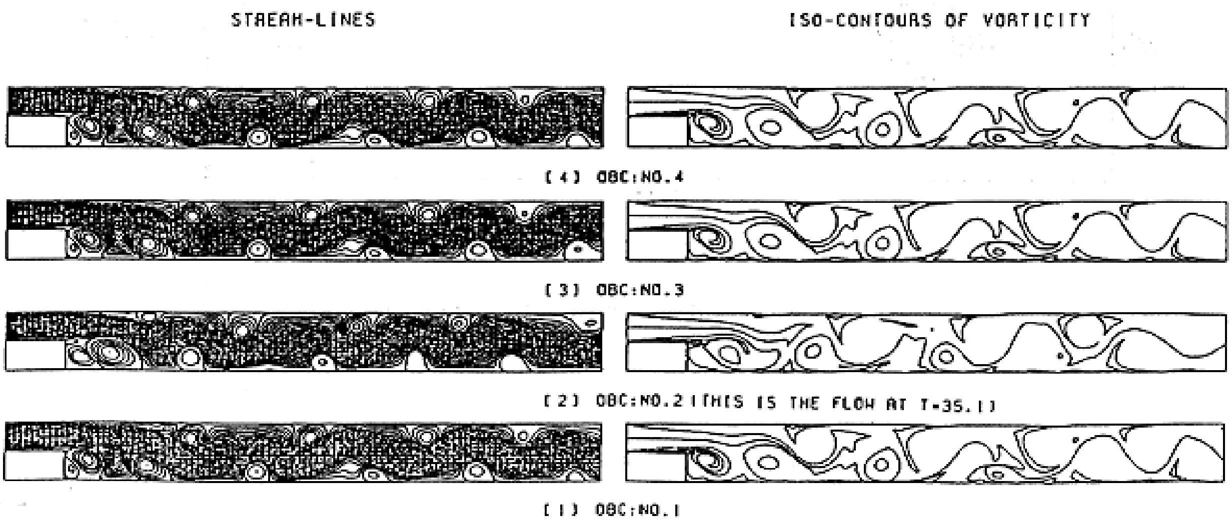
Fig. 1 Geometry definiton of three test problems



(A) The aspect of flow at t = 30



(B) The aspect of flow at t = 35



(C) The aspect of flow at t = 40

Fig. 2 Difference of flows among four OBCs (Re = 1,000, IN = 800)

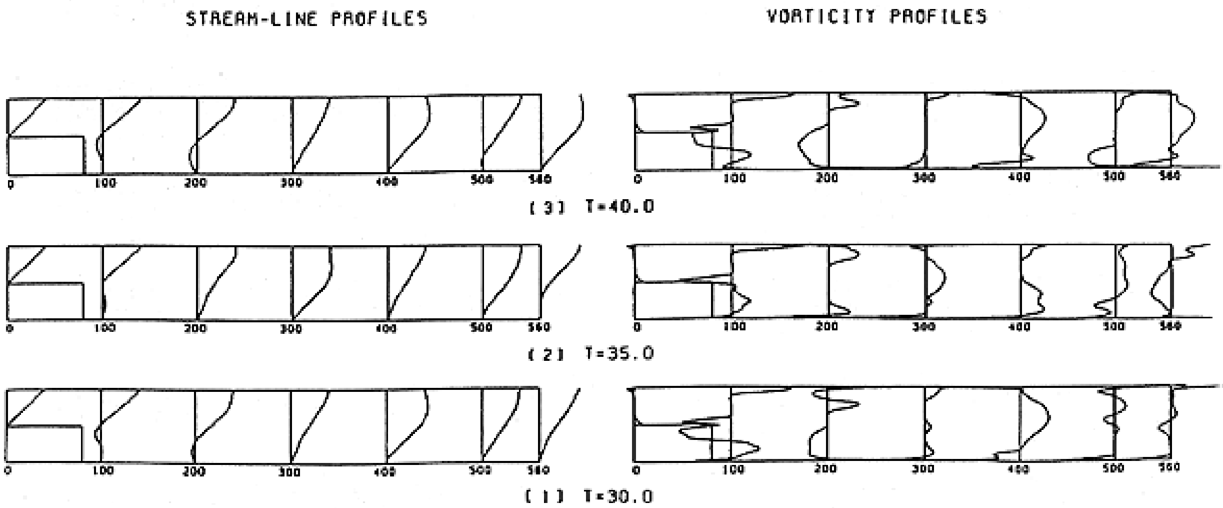
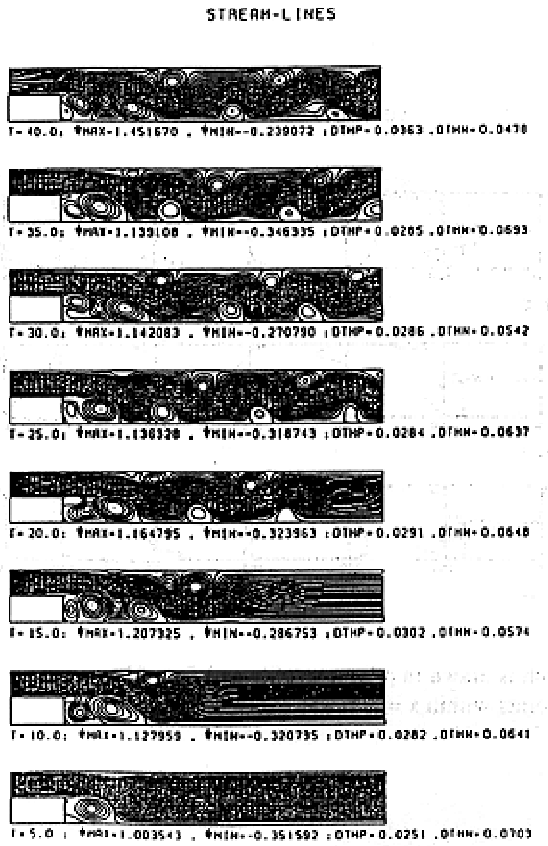
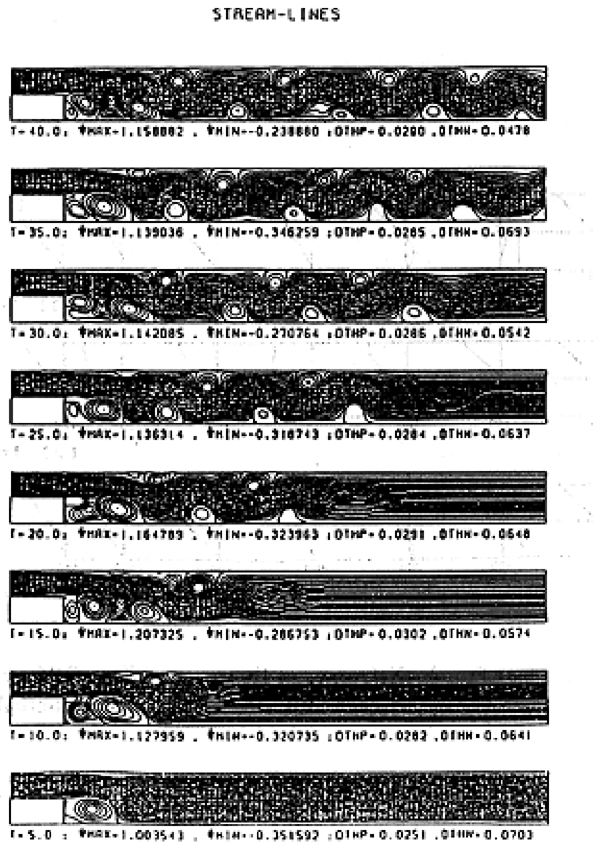


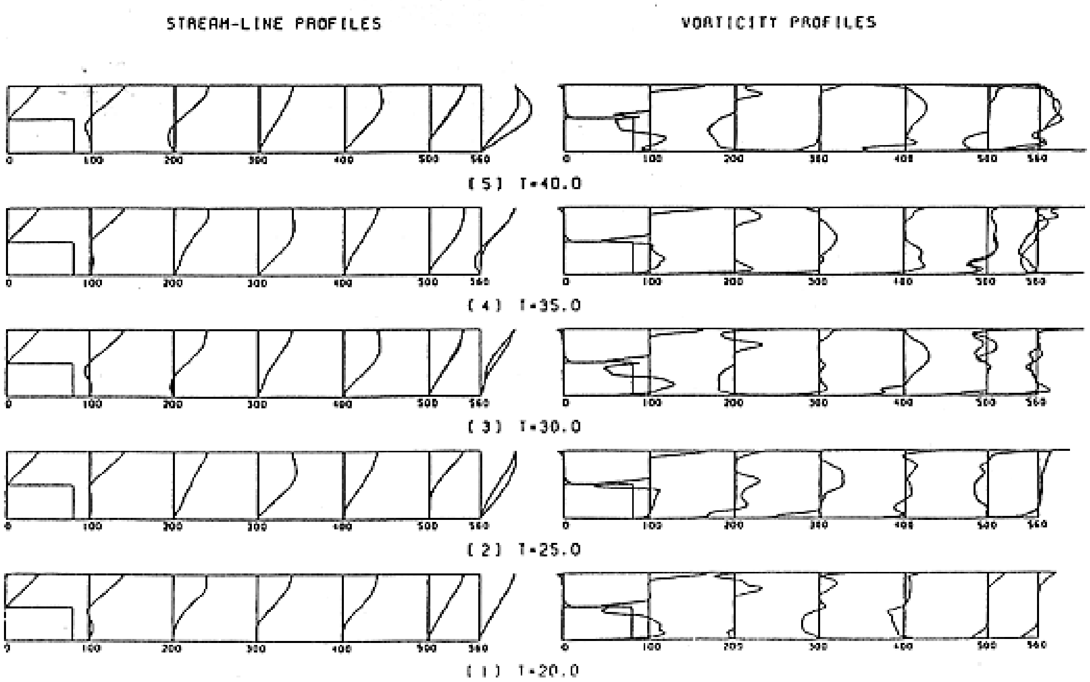
Fig. 3 Difference of flows among four OBCs which is drawn in piles ψ profile and ζ profile by each OBC on several vertical internal points within x = 560 (Re = 1,000, IN = 800)



(A) Aspects of flow every t = 5 (IN = 560)

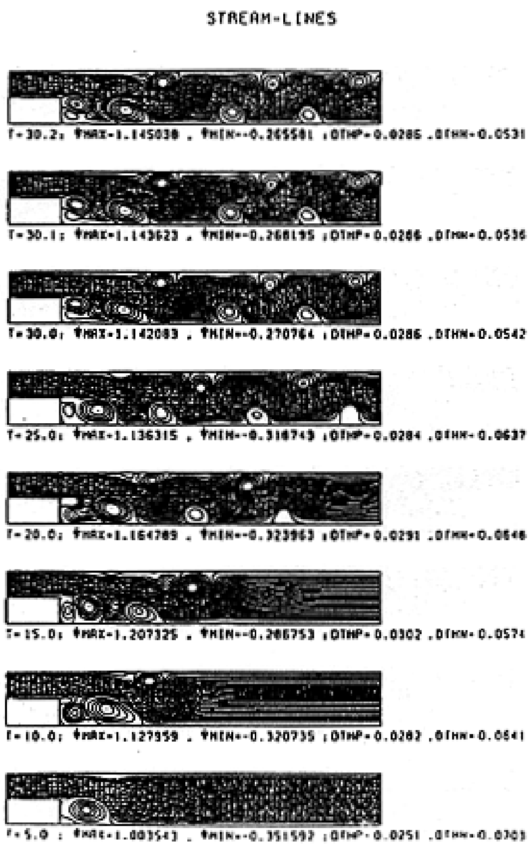


(B) Aspects of flow every t = 5 (IN = 800)

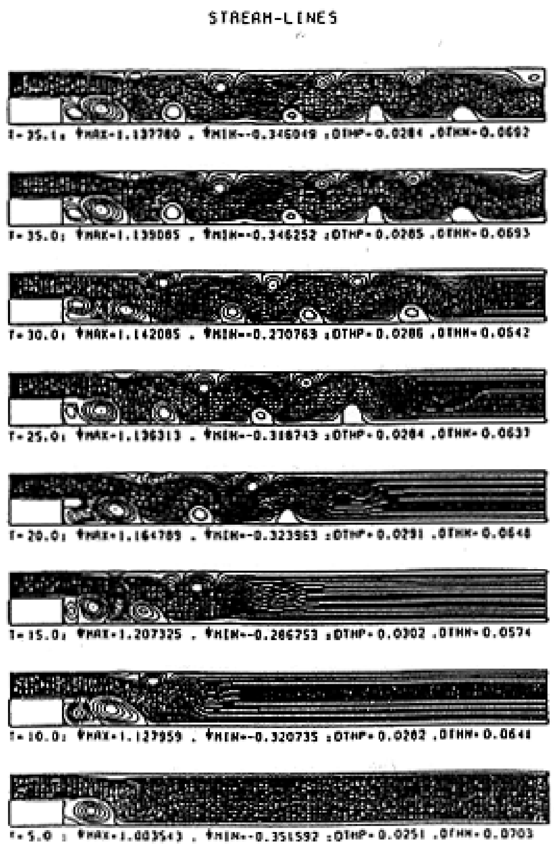


(C) Difference of flows between IN = 560 and IN = 800 which is drawn in piles ψ profile and ζ profile on several vertical internal points each t

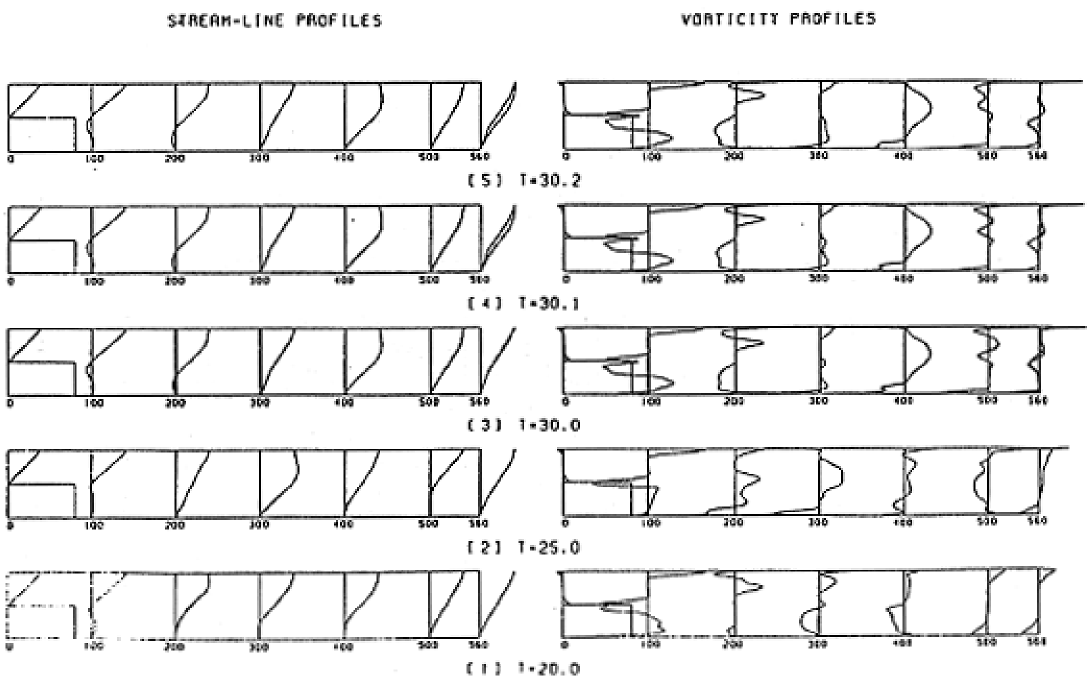
Fig. 4 Difference of flows between short and long open boundaries (Re = 1,000, OBC: no. 1)



(A) Aspects of flow every t = 5 (IN = 560)

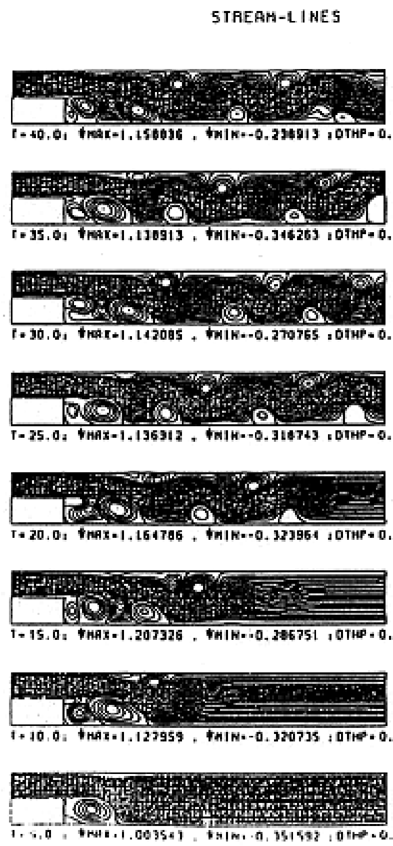


(B) Aspects of flow every t = 5 (IN = 800)

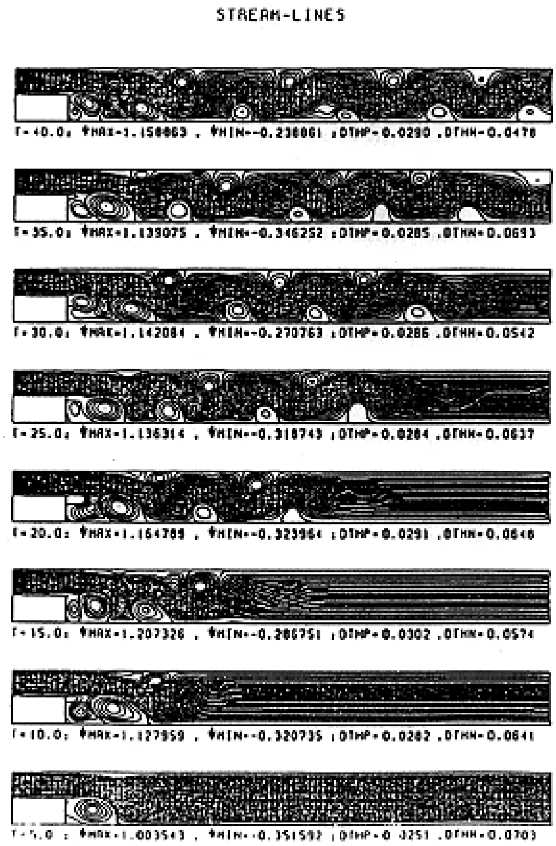


(C) Difference of flows between IN = 560 and IN = 800 which is drawn in piles ψ profile and ζ profile on several vertical internal points each t

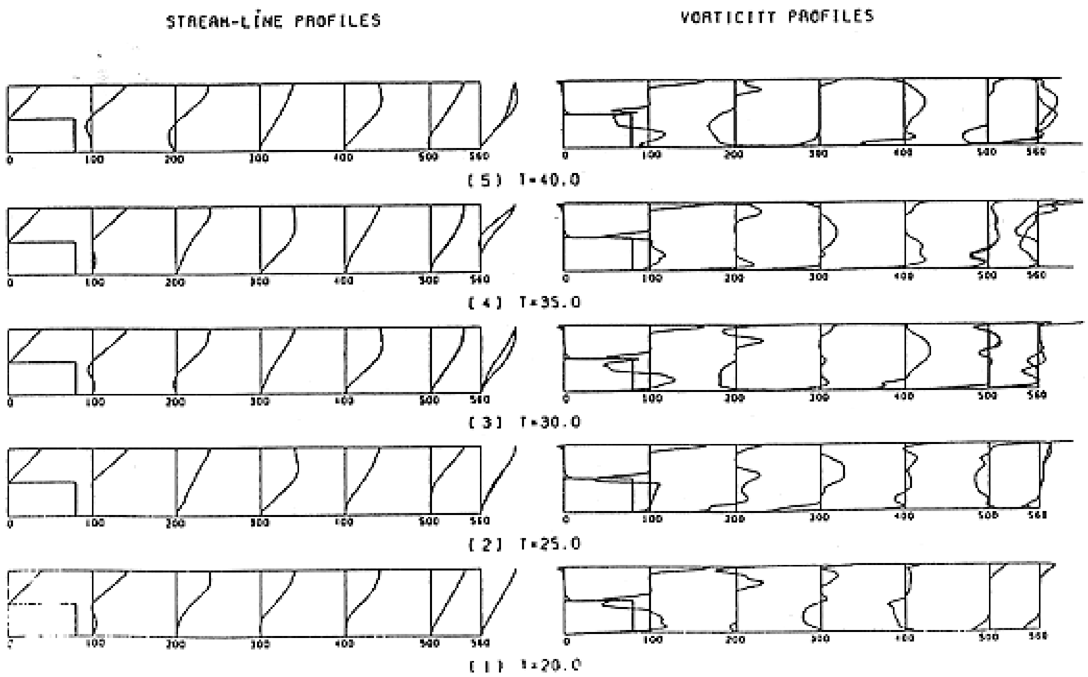
Fig. 5 Difference of flows between short and long open boundaries (Re = 1,000, OBC: no. 2)



(A) Aspects of flow every t = 5 (IN = 560)

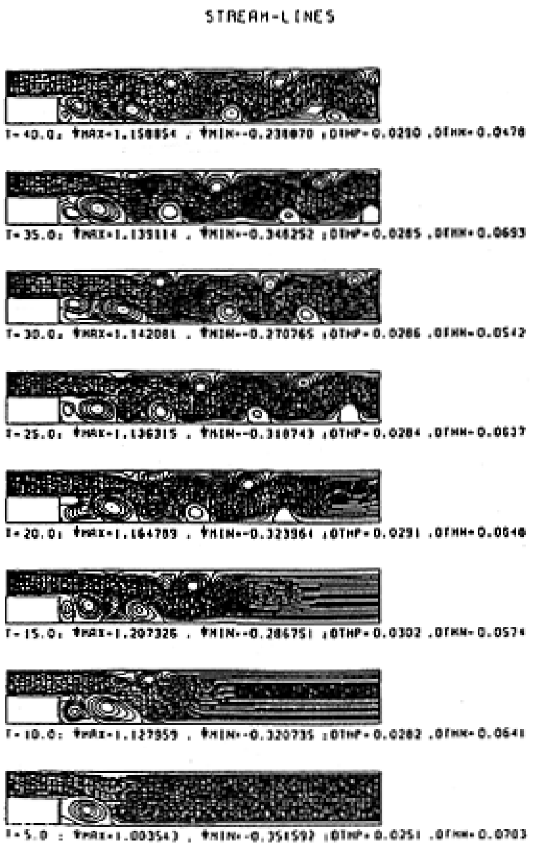


(B) Aspects of flow every t = 5 (IN = 800)

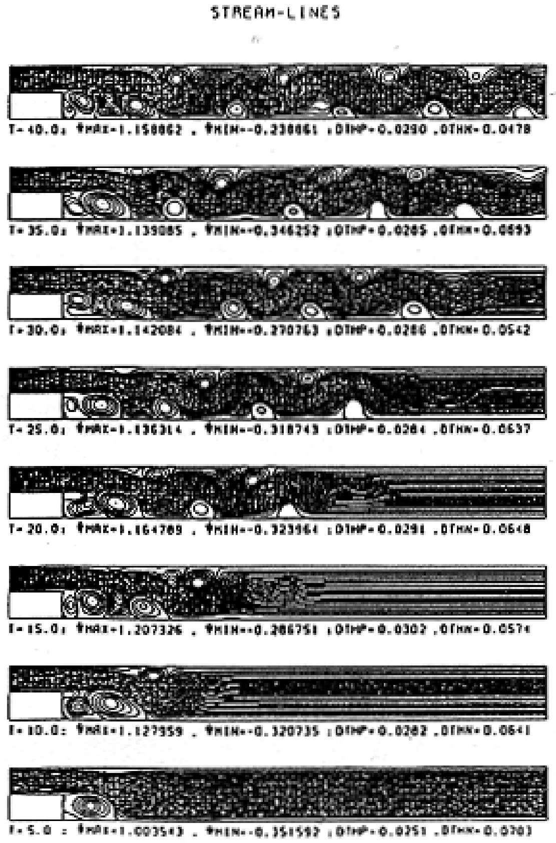


(C) Difference of flows between IN = 560 and IN = 800 which is drawn in piles ψ profile and ζ profile on several vertical internal points each t

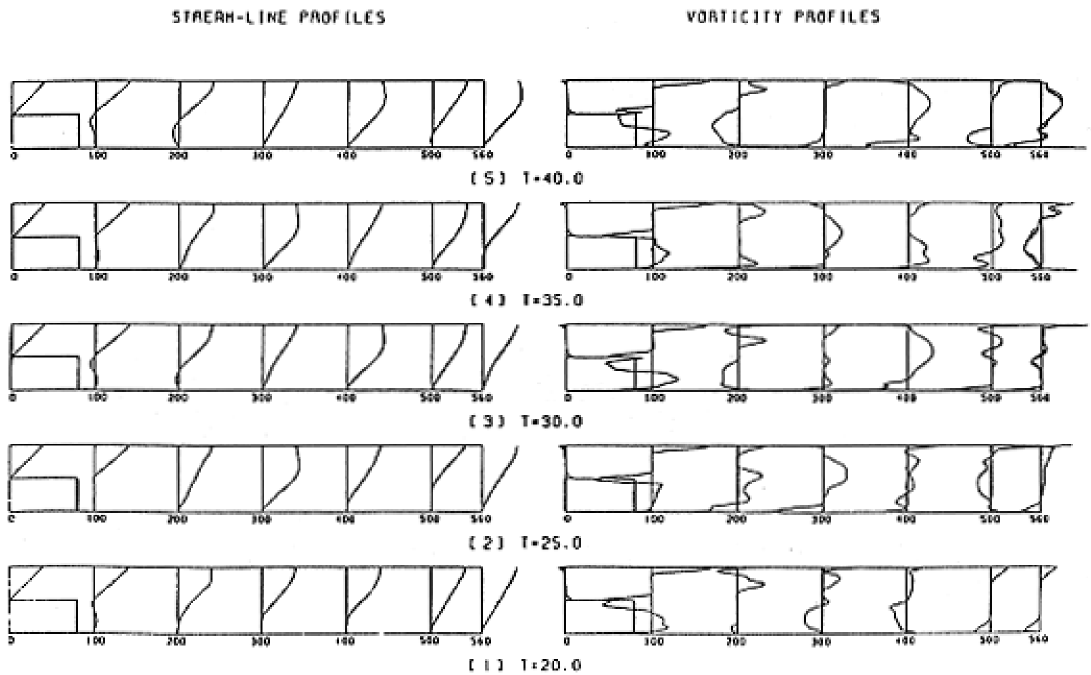
Fig. 6 Difference of flows between short and long open boundaries (Re = 1,000, OBC: no. 3)



(A) Aspects of flow every t = 5 (IN = 560)

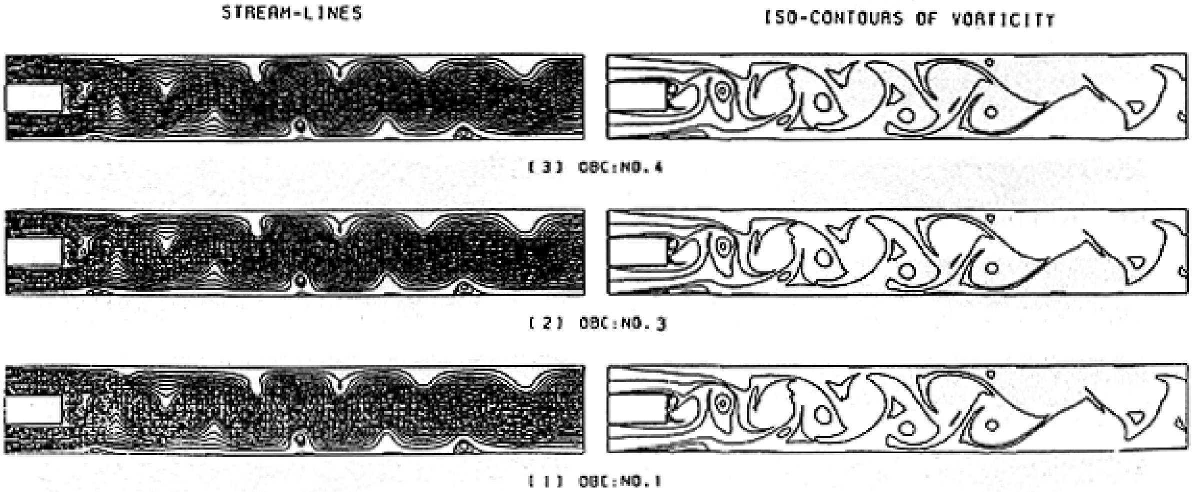


(B) Aspects of flow every t = 5 (IN = 800)

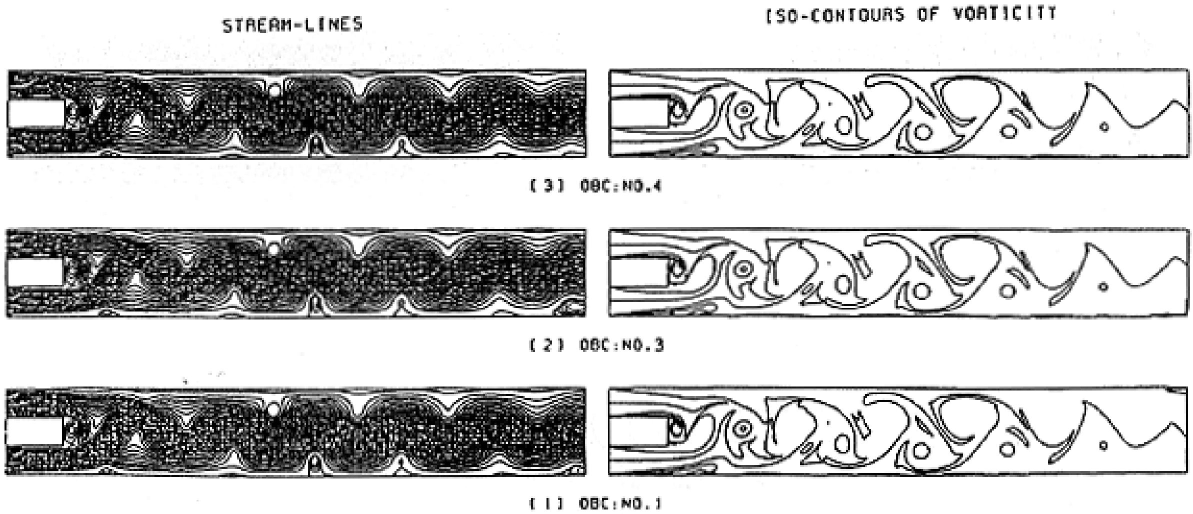


(C) Difference of flows between IN = 560 and IN = 800 which is drawn in piles ψ profile and ζ profile on several vertical internal points each t

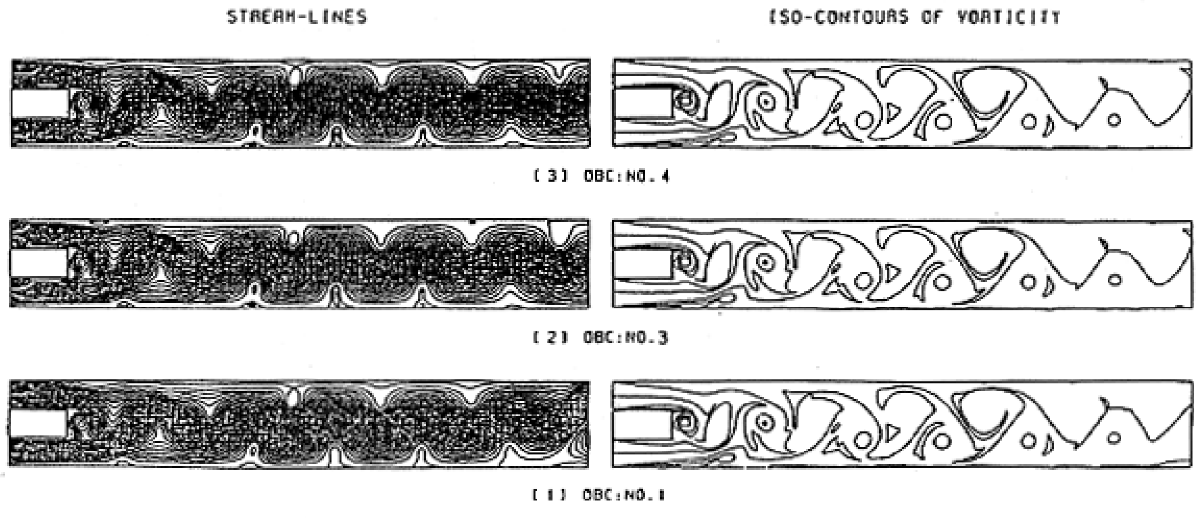
Fig. 7 Difference of flows between short and long open boundaries (Re = 1,000, OBC: no. 4)



(A) Aspects of flow at t = 30



(B) Aspects of flow at t = 35



(C) Aspects of flow at t = 40

Fig. 8 Difference of flows among three OBCs (Re = 1,000, IN = 800)

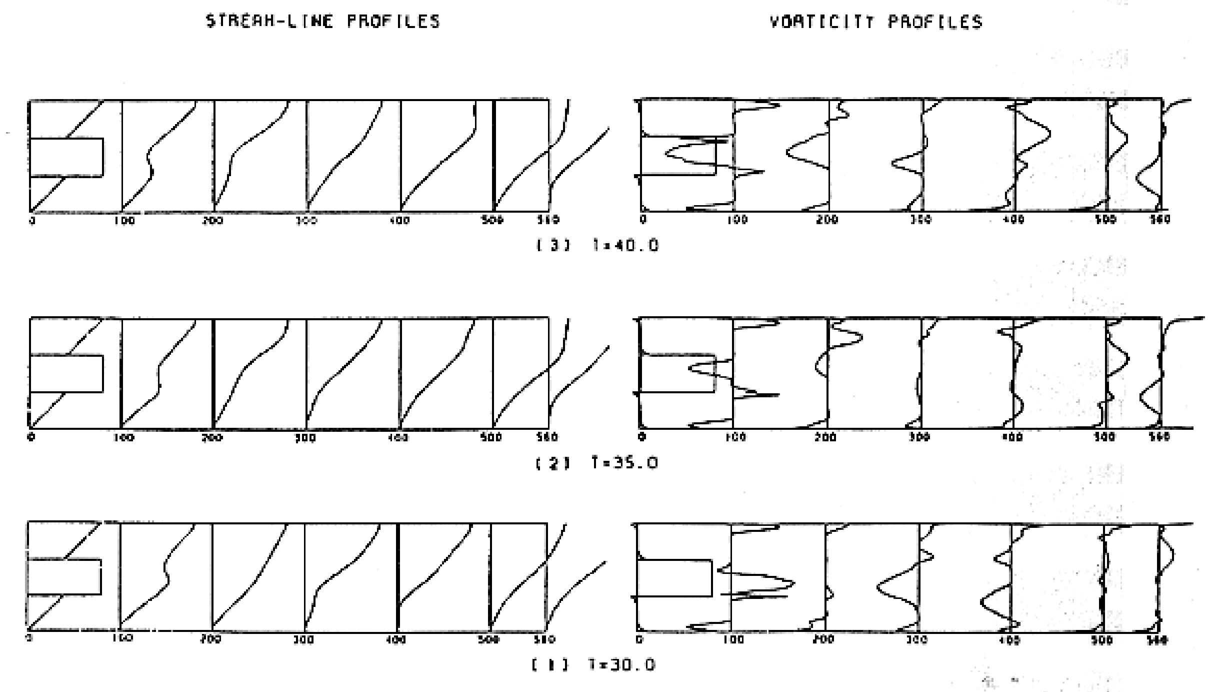
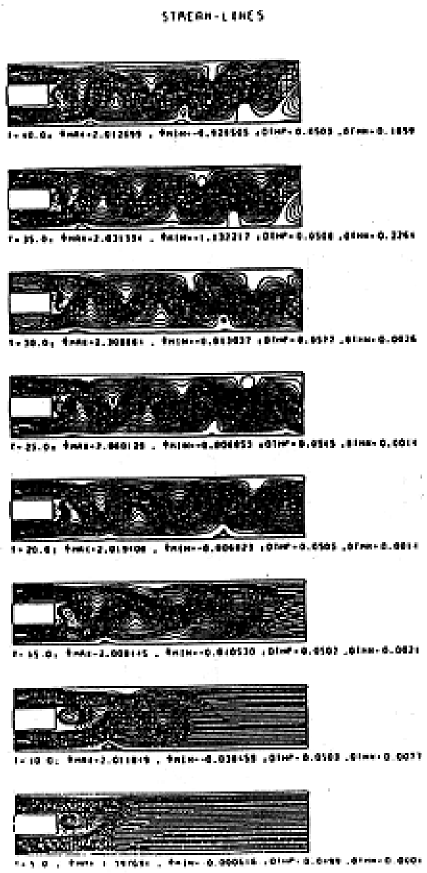
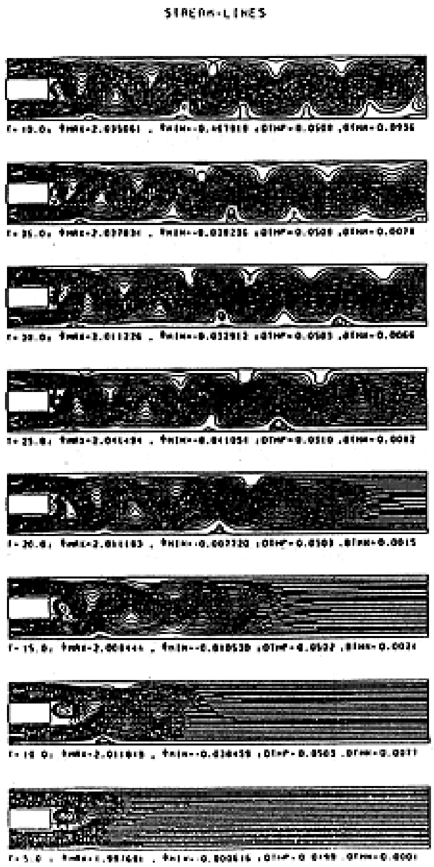


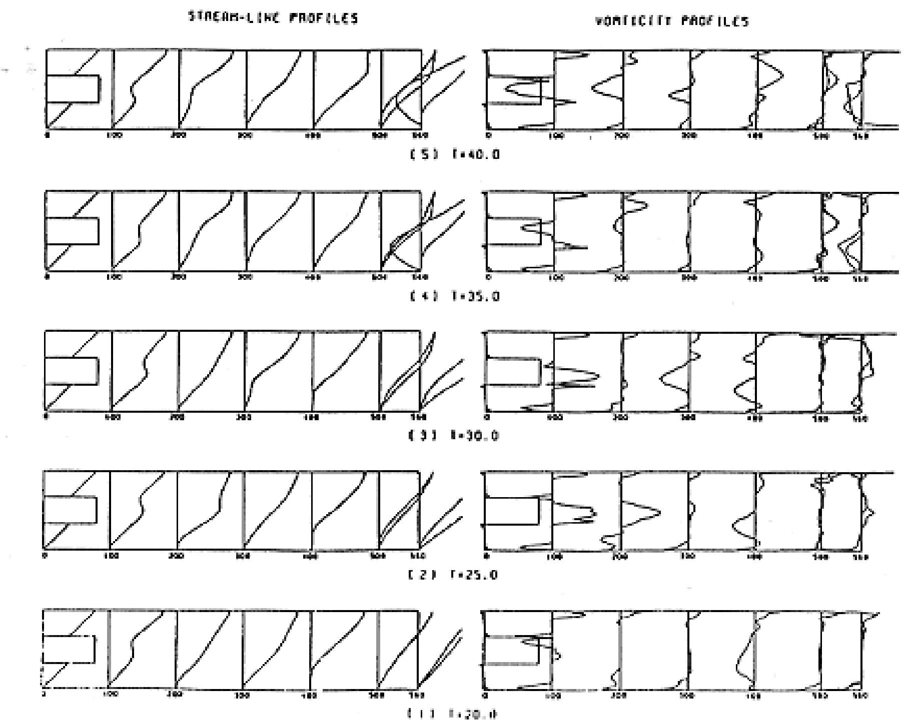
Fig. 9 Difference of flows among three OBCs which is drawn in piles ψ profile and ζ profile by each OBC on several vertical internal points within x = 560 (Re = 1,000, IN = 800)



(A) Aspects of flow every t = 5 (IN = 560)

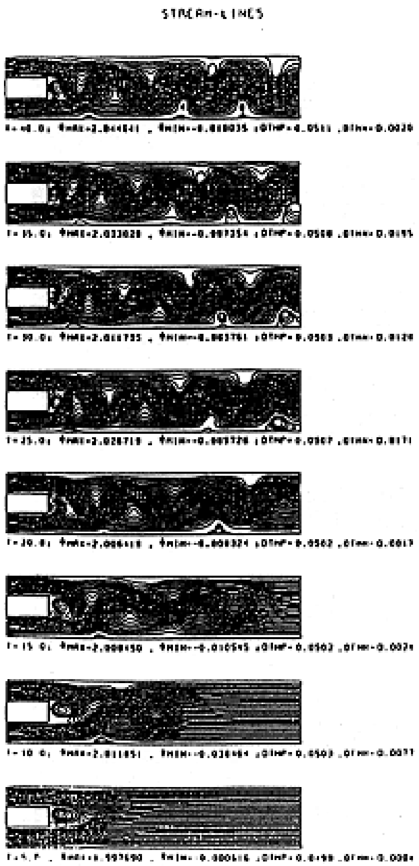


(B) Aspects of flow every t = 5 (IN = 800)

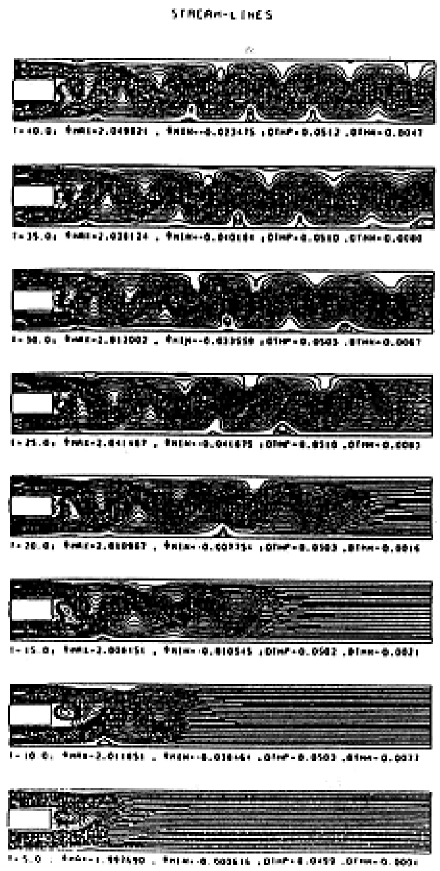


(C) Difference of flows between IN = 560 and IN = 800 which is drawn in piles ψ profile and ζ profile on several vertical internal points each t

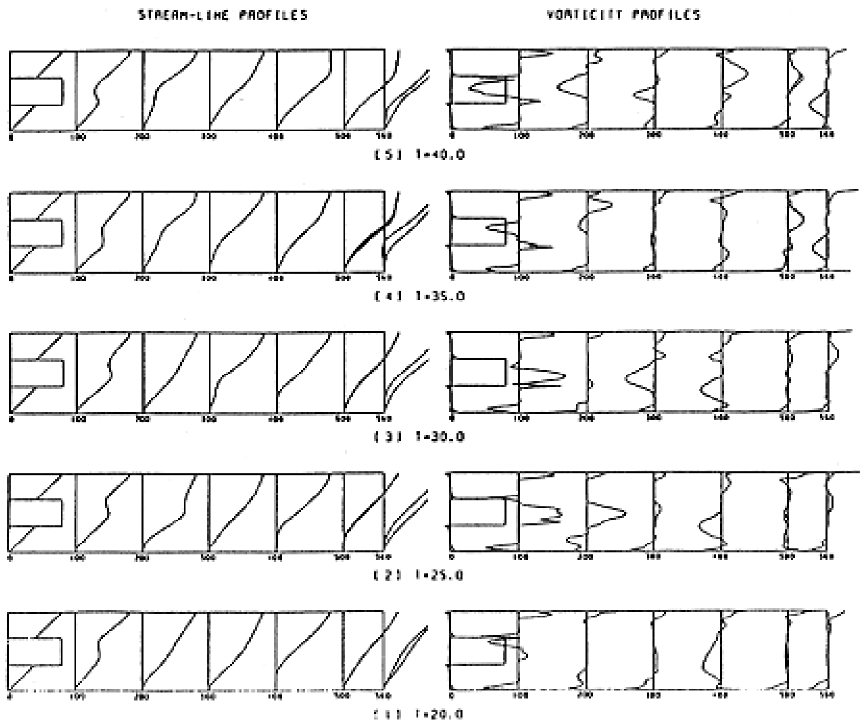
Fig. 10 Difference of flows between short and long open boundaries (Re = 1,000, OBC: no. 1)



(A) Aspects of flow every t = 5 (IN = 560)

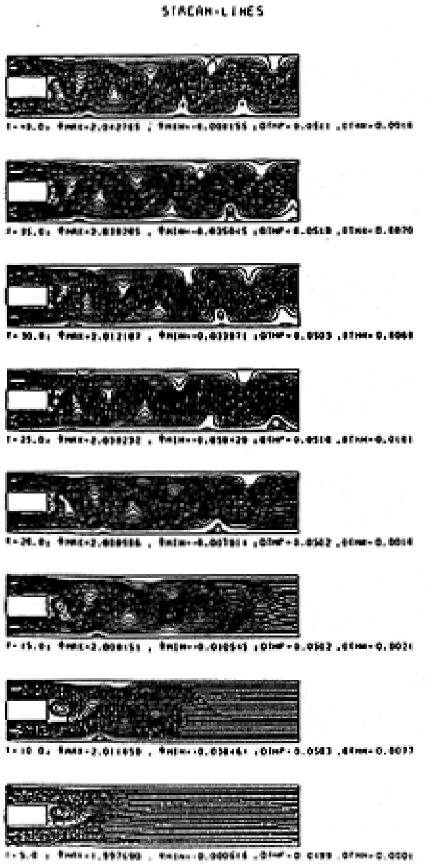


(B) Aspects of flow every t = 5 (IN = 800)

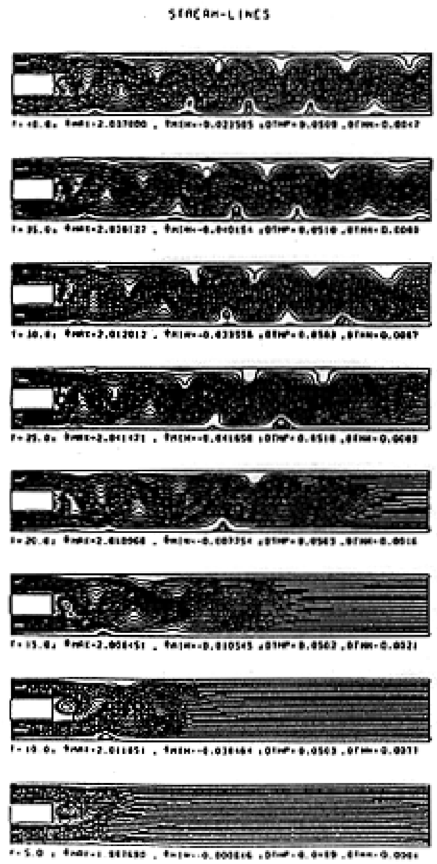


(C) Difference of flows between IN = 560 and IN = 800 which is drawn in piles ψ profile and ζ profile on several vertical internal points each t

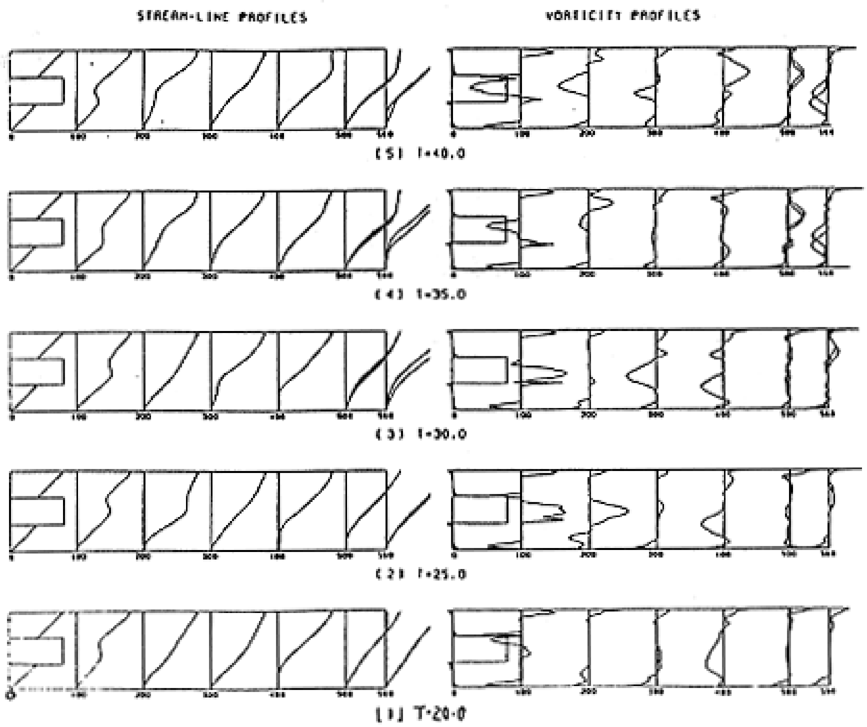
Fig. 11 Difference of flows between short and long open boundaries (Re = 1,000, OBC: no. 3)



(A) Aspects of flow every t = 5 (IN = 560)

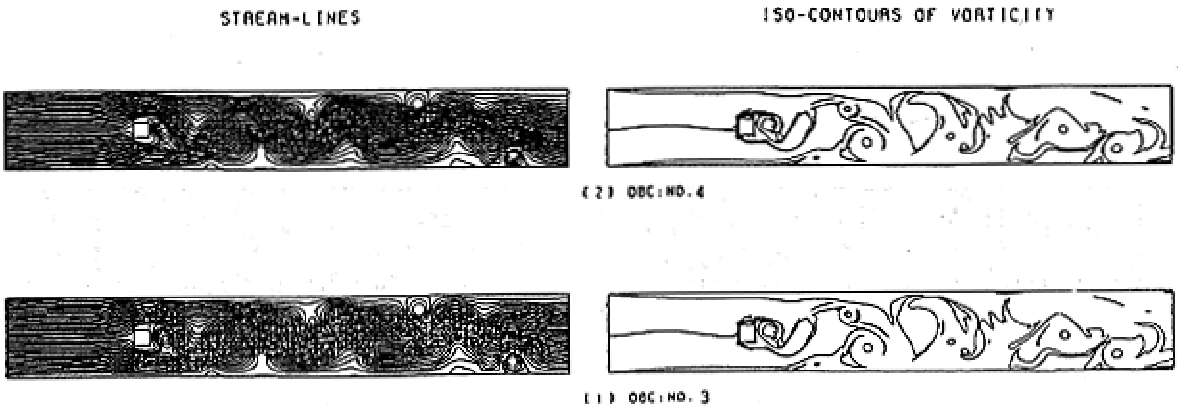


(B) Aspects of flow every t = 5 (IN = 800)

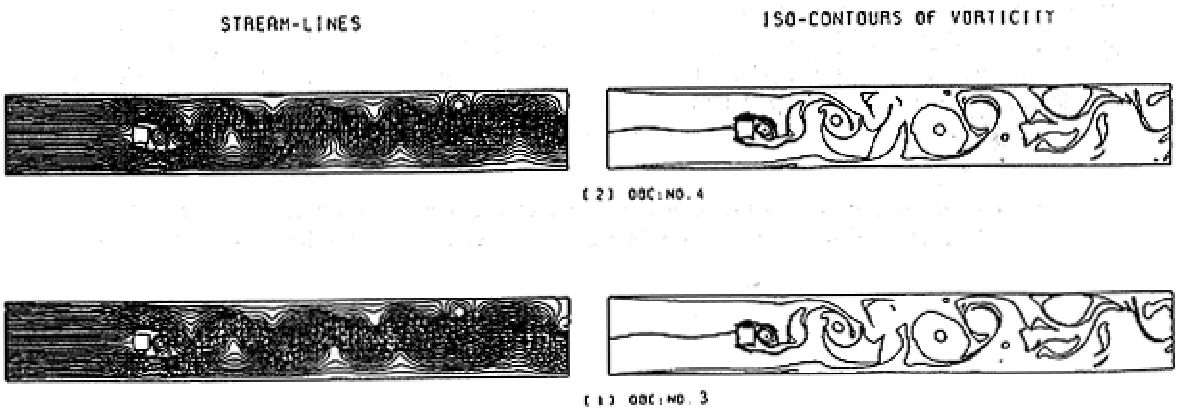


(C) Difference of flows between IN = 560 and IN = 800 which is drawn in piles ψ profile and ζ profile on several vertical internal points each t

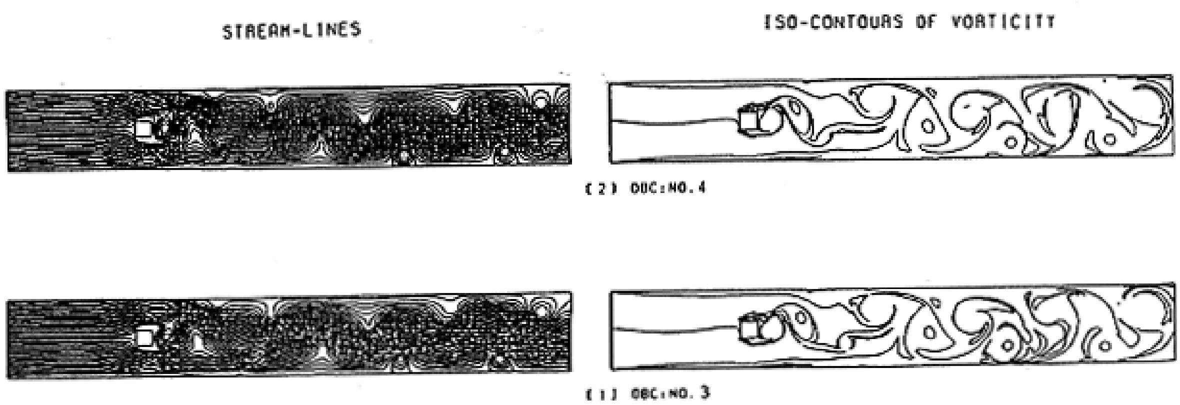
Fig. 12 Difference of flows between short and long open boundaries (Re = 1,000, OBC: no. 4)



(A) Aspects of flow at t = 45



(B) Aspects of flow at t = 55



(C) Aspects of flow at t = 65

Fig. 13 Difference of flows between two OBCs (Re = 1,000, IN = 1400)

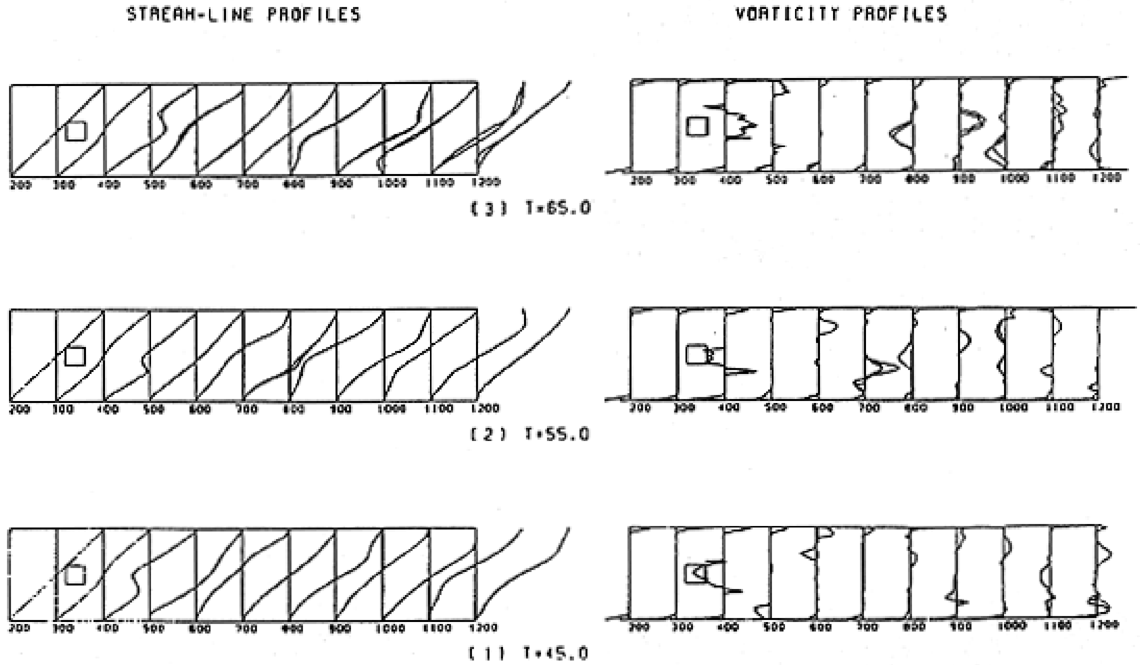
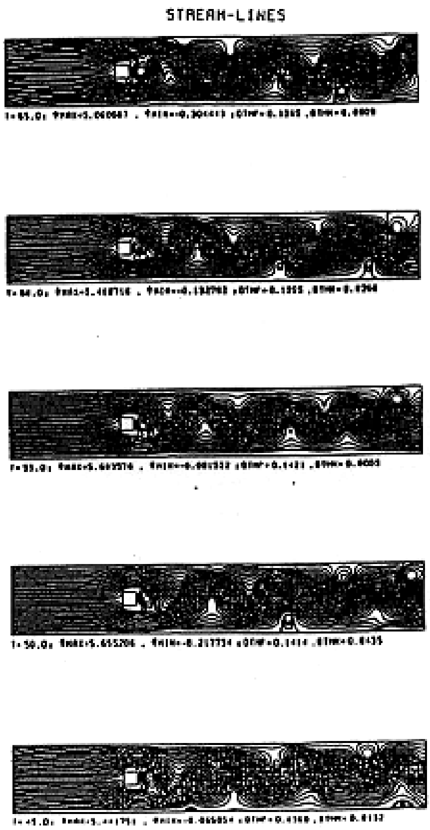
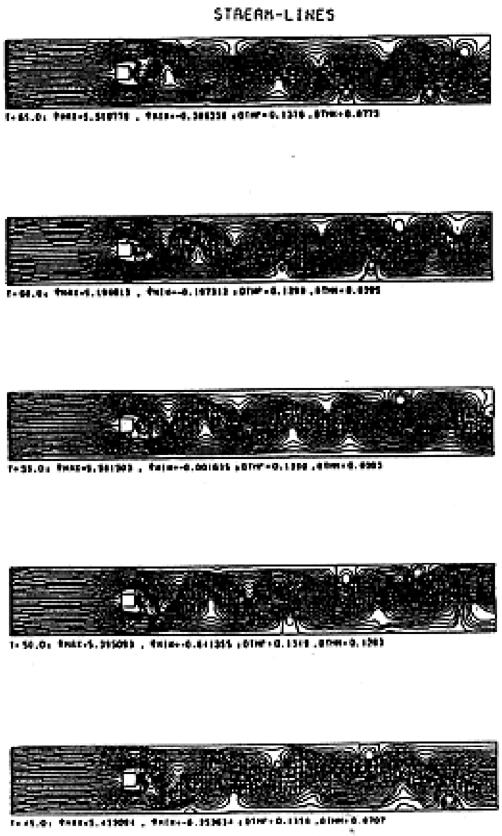


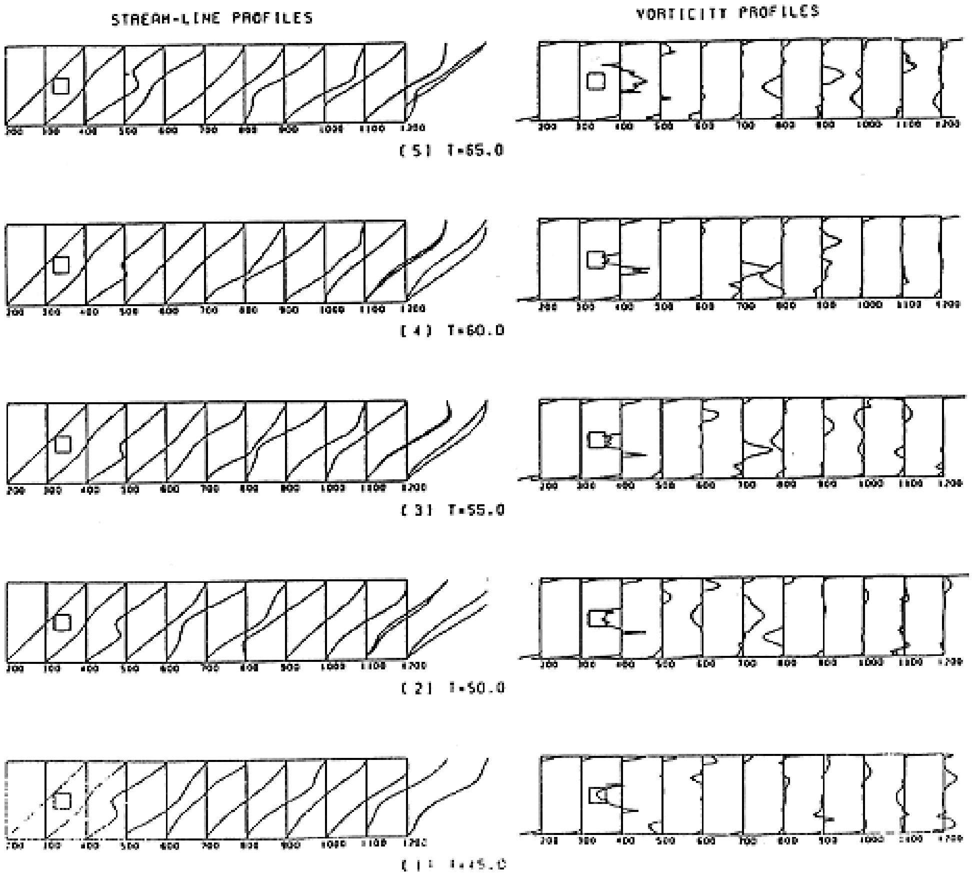
Fig. 14 Difference of flows between two OBCs which is drawn in piles ψ profile and ζ profile by each OBC on several vertical internal points within x = 1200 (Re = 1,000, IN = 1400)



(A) Aspects of flow every t = 5 (IN = 1200)

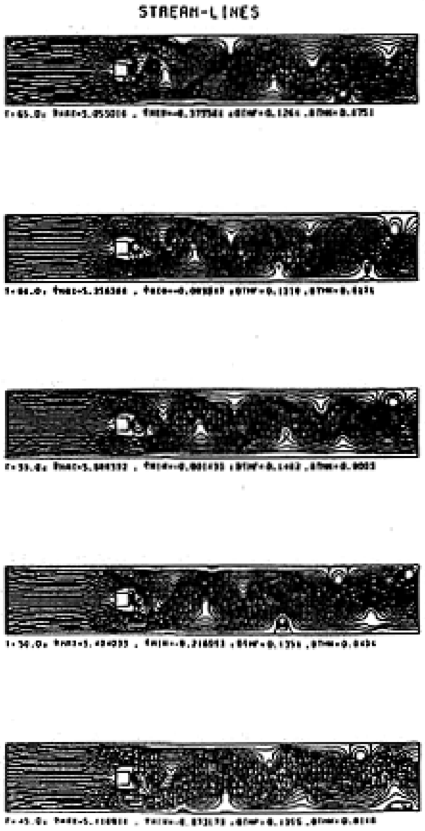


(B) Aspects of flow every t = 5 (IN = 1400)

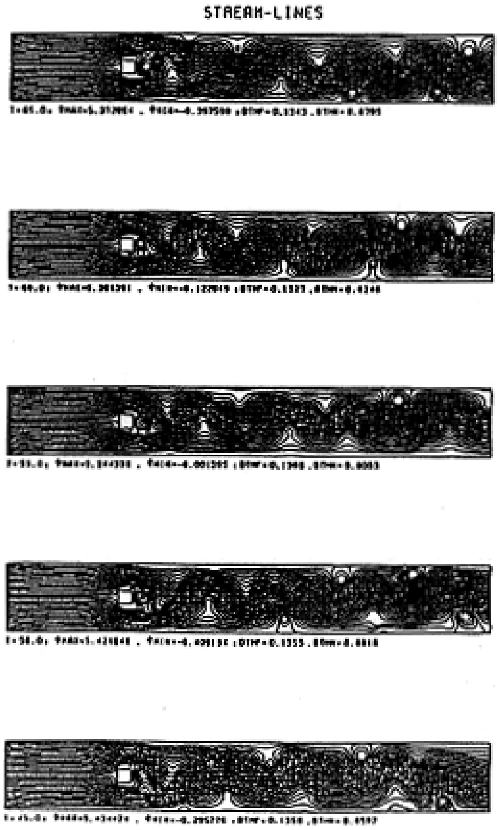


(C) Difference of flows between IN = 1200 and IN = 1400 which is drawn in piles ψ profile and ζ profile on several vertical internal points each t

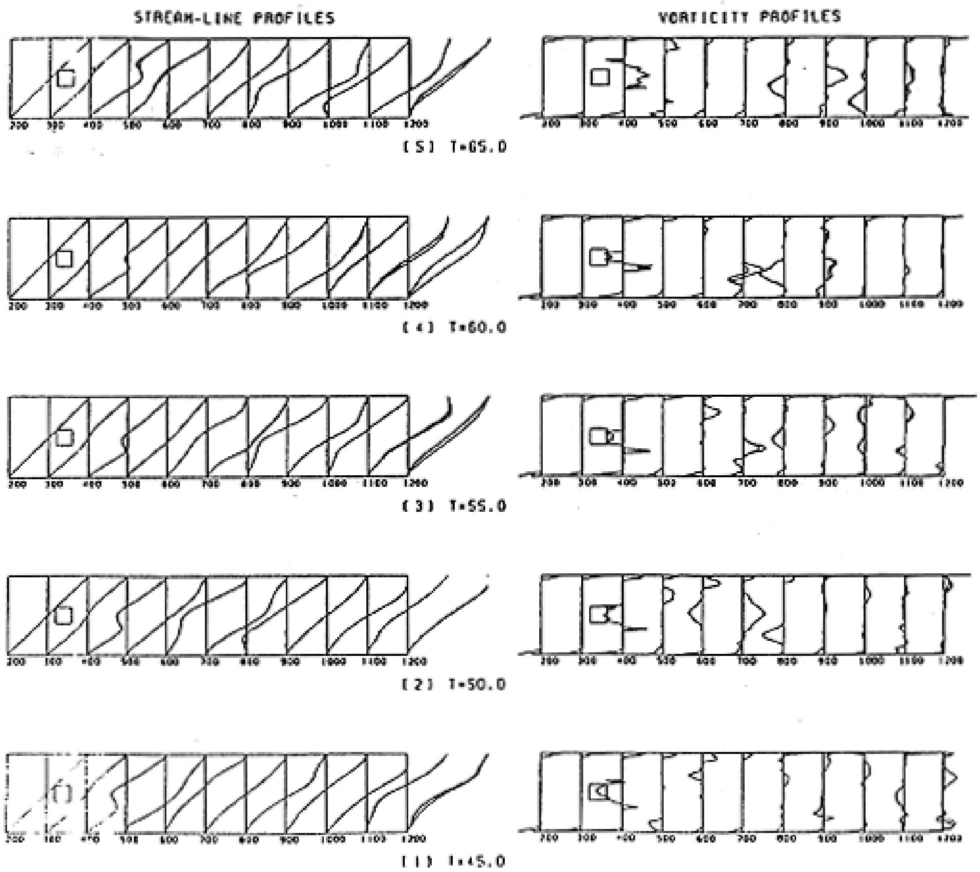
Fig. 15 Difference of flows between short and long open boundaries (Re = 1,000, OBC: no. 3)



(A) Aspects of flow every t = 5 (IN = 1200)

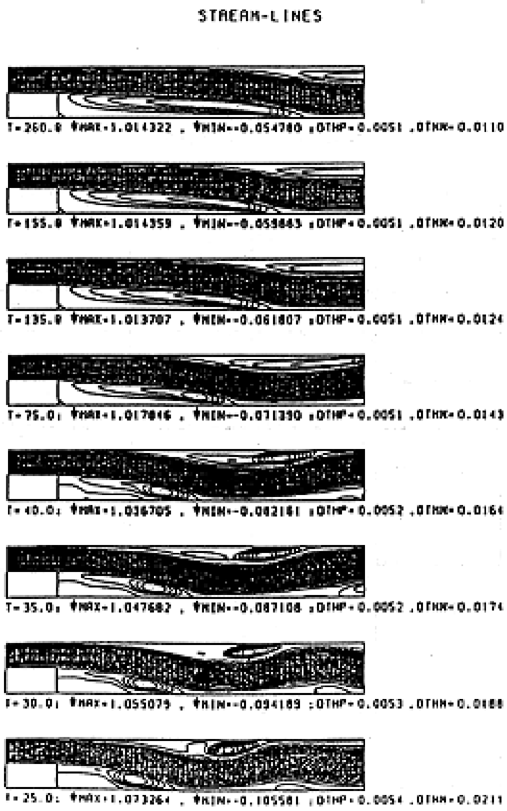


(B) Aspects of flow every t = 5 (IN = 1400)

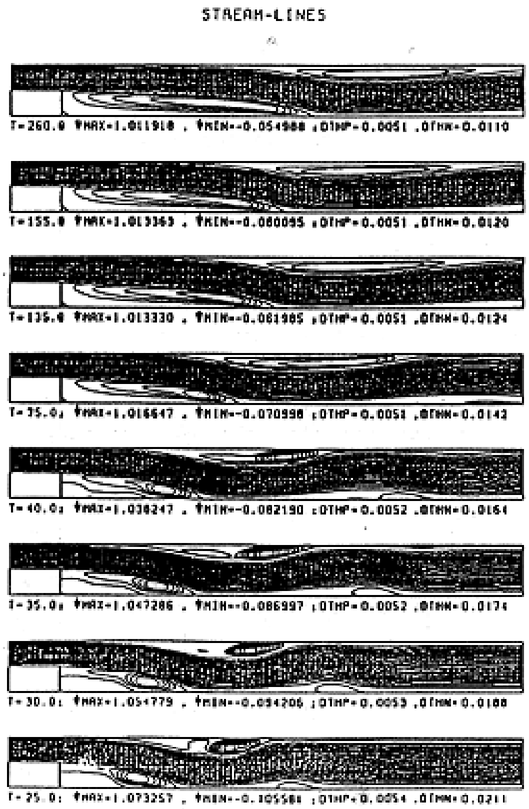


(C) Difference of flows between IN = 1200 and IN = 1400 which is drawn in piles ψ profile and ζ profile on several vertical internal points each t

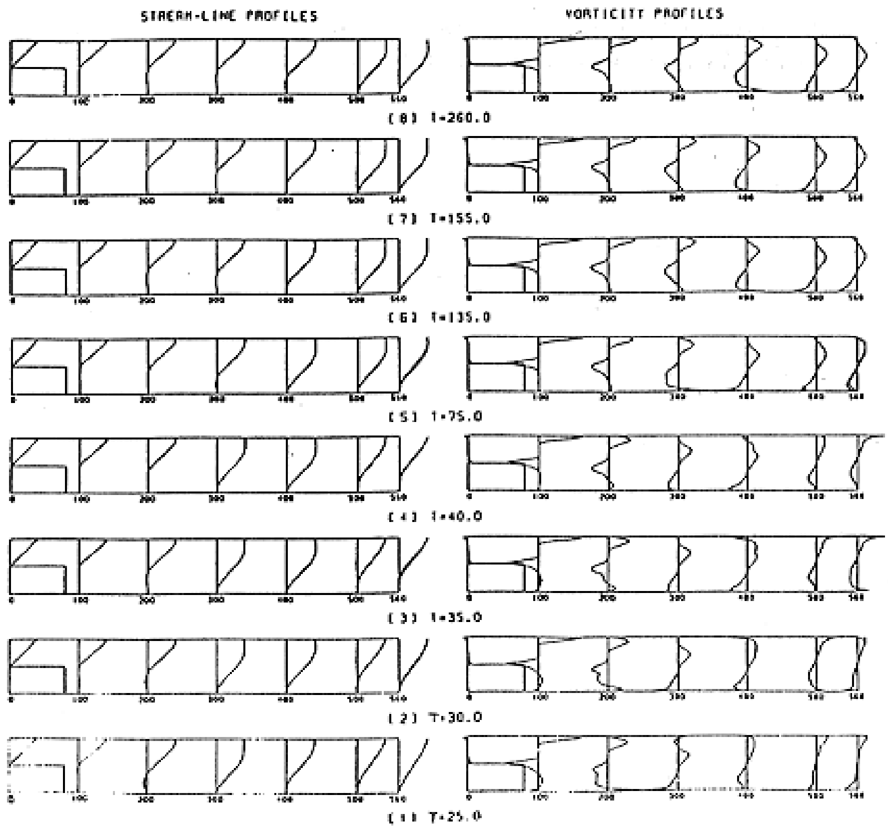
Fig. 16 Difference of flows between short and long open boundaries (Re = 1,000, OBC: no. 4)



(A) Aspects of flow every t = 5 (IN = 560)



(B) Aspects of flow every t = 5 (IN = 800)



(C) Difference of flows between IN = 560 and IN = 800 which is drawn in piles ψ profile and ζ profile on several vertical internal points each t

Fig. 17 Difference of flows between short and long open boundaries (Re = 800, OBC: no. 1)

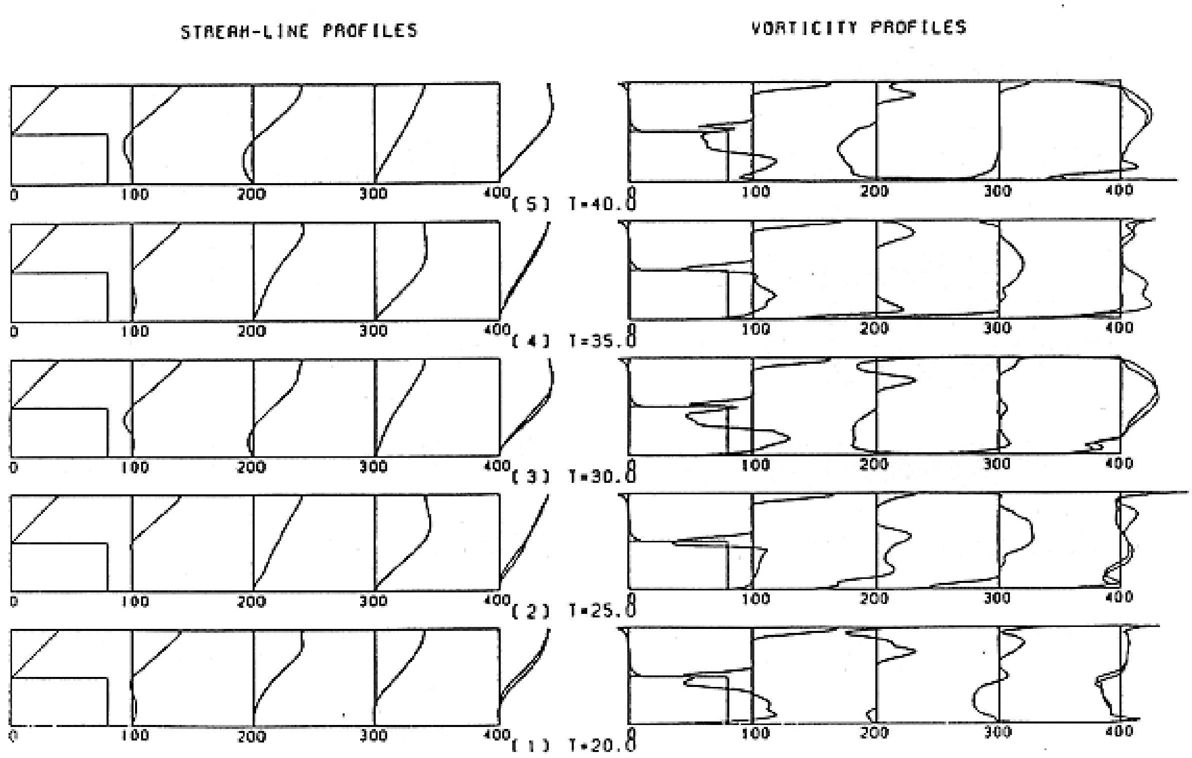
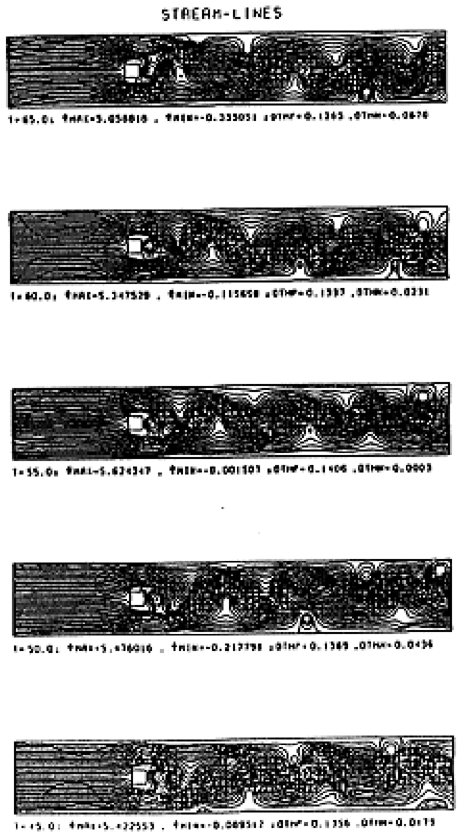
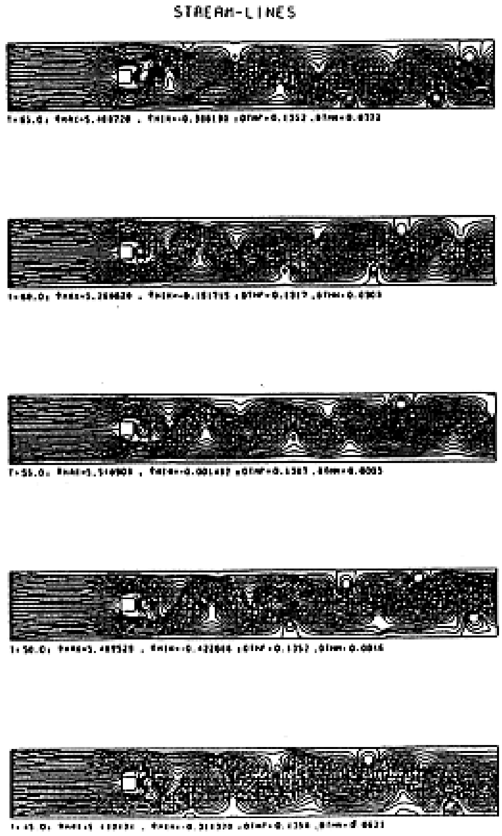


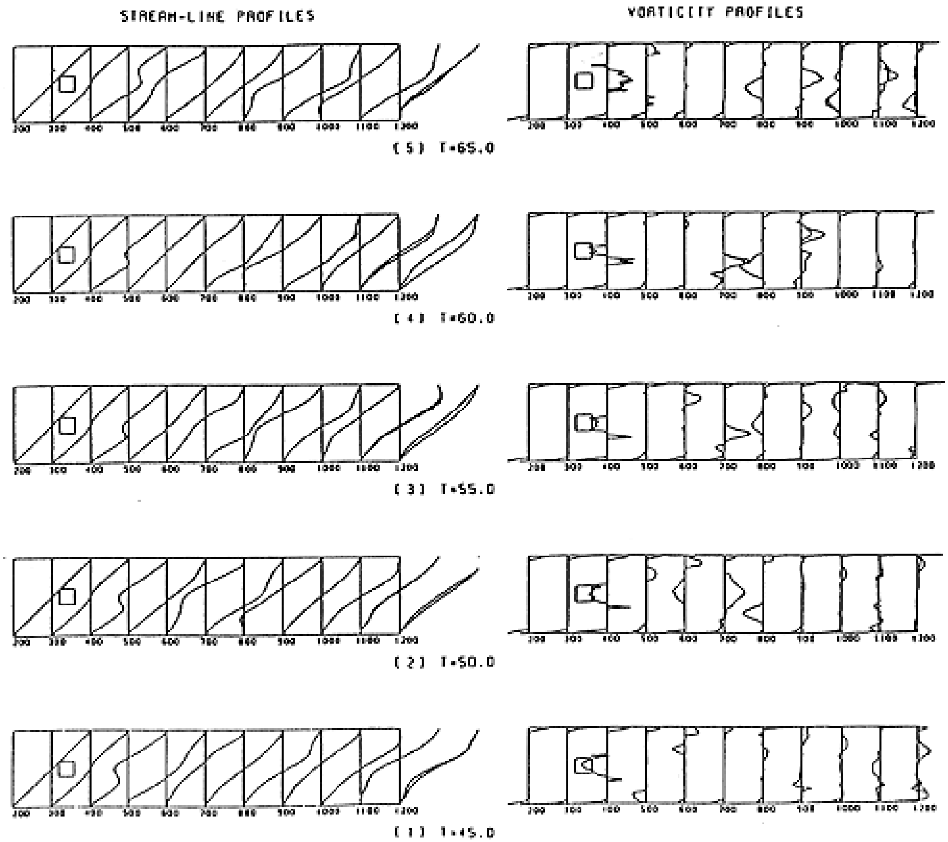
Fig. 18 Difference of flows between three OBCs which is drawn in piles ψ profile and ζ profile by each OBC on several vertical internal points within x = 400 (Re = 1,000, IN = 800)



(A) Aspects of flow every t = 5 (IN = 1200)

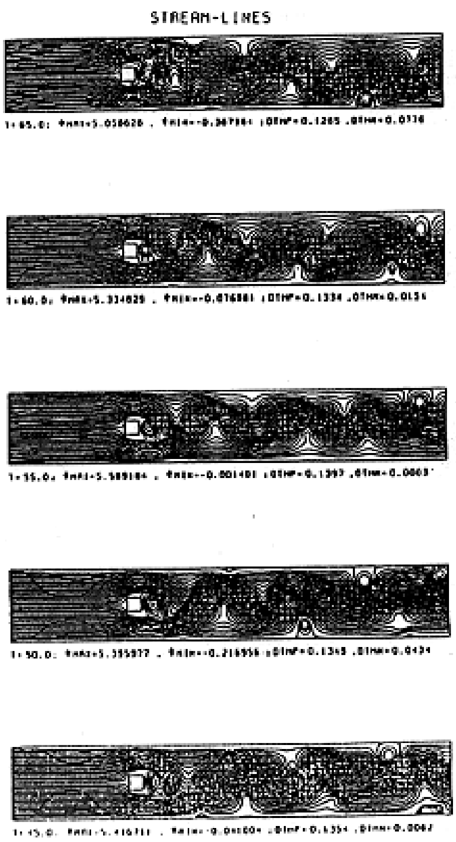


(B) Aspects of flow every t = 5 (IN = 1400)

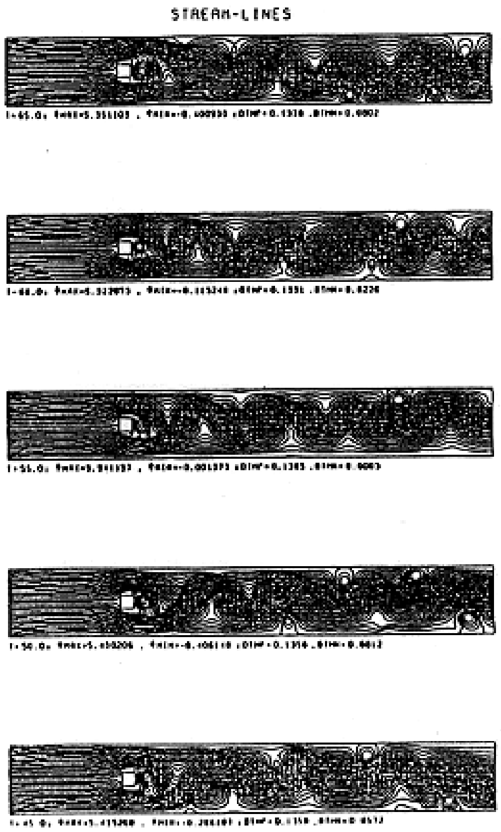


(C) Difference of flows between IN = 1200 and IN = 1400 which is drawn in piles ψ profile and ζ profile on several vertical internal points each t

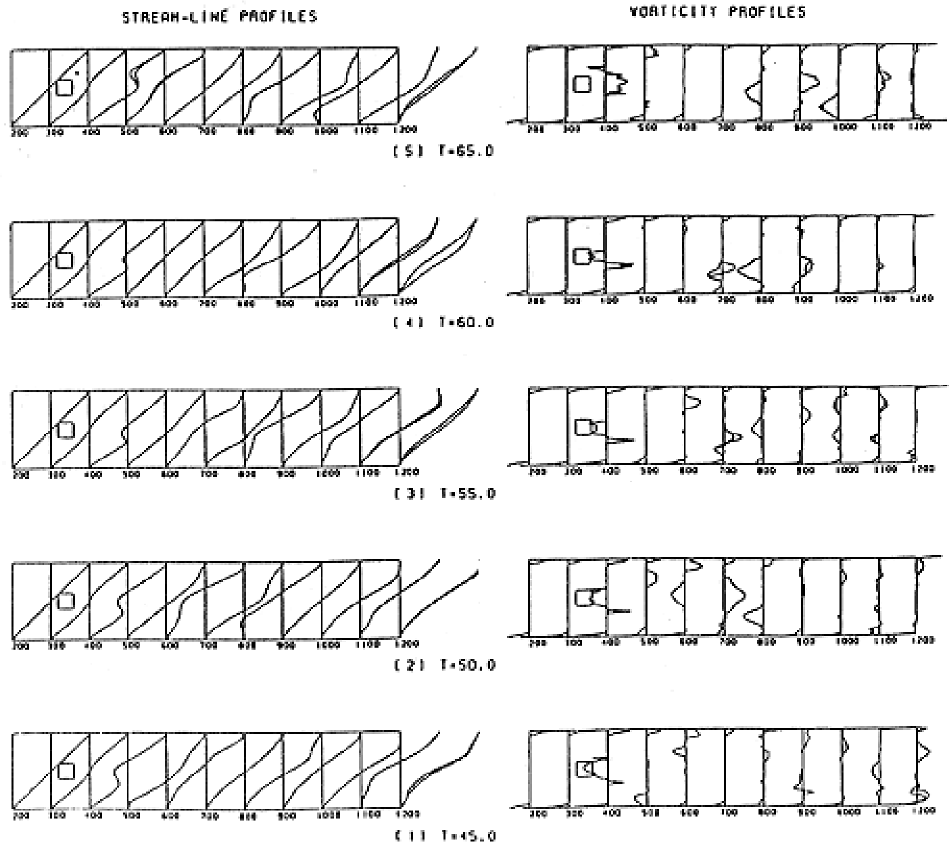
Fig. 19 Difference of flows between short and long open boundaries (Re = 1,000, OBC: no. 4, c = 0.7)



(A) Aspects of flow every t = 5 (IN = 1200)



(B) Aspects of flow every t = 5 (IN = 1400)



(C) Difference of flows between IN = 1200 and IN = 1400 which is drawn in piles ψ profile and ζ profile on several vertical internal points each t

Fig. 20 Difference of flows between short and long open boundaries (Re = 1,000, OBC: no. 4, c = 1.3)