Direct Numerical Simulation of By-Pass Transition in the Plane Poiseuille Flow

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Two kinds of by-pass transitions in the plane Poiseuille flow are directly simulated on a parallel computer, one of which being triggered by a large initial disturbance existing in the flow and another by a suction/blowing from the bottom wall. Threshold amplitude of the disturbance which triggers the transition is determined as a function of Reynolds number. Two oblique waves induced by a suction/blowing excite a large streamwise mode which triggers the transition of the flow.

Key Words: Plane Poiseuille flow, By-pass transition, Initial disturbance, Suction/blowing

1. Introduction

Laminar-turbulent transition of the plane Poiseuille flow has been traditionally analyzed with the linear stability equation. As a result, it is found that the Tollmien-Schlichting (TS) waves become to be unstable and trigger the transition when the Reynolds number is greater than its critical value, 5772[1)]. Another type of the linear instability, however, has been recognized recently to be important for the transition of the flow[2)]. It has been known that the transition becomes to be dominant in the flow of which disturbances are considerably high and is triggered by amplified streamwise vortices (SV) instead of the TS wave. Thus, this type of transition is called by-pass transition.

In this study, two kinds of the by-pass transition are investigated with the direct numerical simulations: one of which is triggered by a initial disturbance existing in the flow, and another by a stationary suction/blowing from the bottom wall, which is expected to simulate the surface roughness. The simulations are conducted on a parallel computer in the NAL.

2. Explanation of DNS

Coordinate system $\{x,y,z\}$ of the physical space is taken for x in the streamwise direction, y in the spanwise and z in the direction normal to the wall. The flow is described as $U+\mathbf{u}$, where $U\left(=1-z^2\right)$ is the basic flow and $\mathbf{u}(u,v,w)$ the disturbance velocity.

The basic equation for $\mathbf{u}$ is derived from the Navier-Stokes equation

$$\frac{\partial \mathbf{u}}{\partial t}=-U\frac{\partial \mathbf{u}}{\partial x}-w\frac{\partial U}{\partial z}-\omega\times\mathbf{u}-\nabla P+\frac{1}{R}\nabla^2\mathbf{u} \quad (1)$$

where $\omega=\nabla\times\mathbf{u}$ and R is the Reynolds number defined by $U(0)$ and a half width between two walls.

The incompressible condition is

$$\nabla\cdot\mathbf{u}=0 \quad (2)$$

The velocity $\mathbf{u}$ is expanded by the Fourier series for x, y directions, while for z direction, it is calculated on the Chebyshev collocation points z_j

$$\mathbf{u}\left(x,y,z_j\right)=\sum_{k_x,k_y}\mathbf{u}(k_x,k_y,z_j)\exp\left(ik_x x+ik_y y\right) \quad (3)$$

Then, Eq. (1) is calculated by the Fourier-Chebyshev spectral method for $\mathbf{u}(k_x,k_y,z_j)$.

Energy norm for the mode (k_y, k_x) per unit mass is defined by

$$E\left(k_x,k_y\right)=\frac{1}{4}\int_{-1}^{1}|\mathbf{u}(k_x,k_y,z)|^2\,dz \quad (4)$$

The initial disturbance in the flow is given by

$$\mathbf{u}\left(k_x,k_y,z_j,0\right)=\varepsilon\,\mathbf{q}\left(k_x,k_y,z_j\right), \quad (5)$$

where ε is an amplitude parameter assigned and $\mathbf{q}$ is a random function which satisfied the solenoidal condition.

3. Transition induced by the initial disturbance

Time evolution of dominant Fourier modes of a disturbance in the transitional flow is shown in Fig. 1, in which two-dimensional modes ($0\le k_x\le 5$, $k_y=0$) are described by solid lines and three-dimensional ones ($0\le k_x\le 5$, $k_y\ne 0$) by dashed lines. Initial values of these modes are assigned as 10^{-7}. The vortical structures in the flow obtained at $t=50$ are shown in Fig. 2. We can observe many fine streamwise vortices in the flow.

From the simulations for the transition of the flow with various initial disturbances, we obtain the threshold amplitude of the initial disturbance which triggers the SV type transition. Figure 3 shows the amplitudes as a function of the Reynolds number. The symbol ○ shows the amplitude with the SV type transition, while ● shows the amplitude without the transition. The solid line shows the inclination of $R^{-1.7}$, which agrees with the result obtained by Lundbladh et al [3].

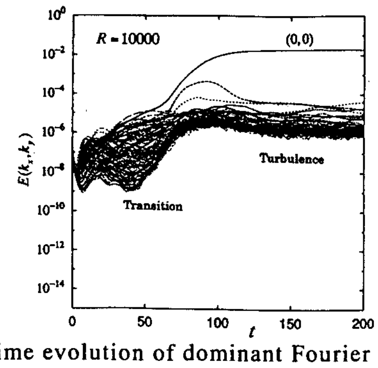

Fig. 1 Time evolution of dominant Fourier modes of a disturbance induced in the SV type transition for $R = 10000$

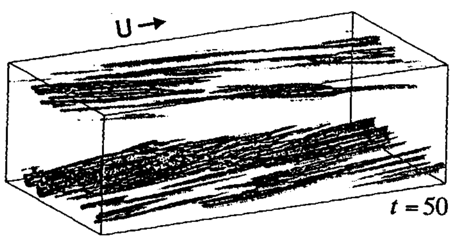

Fig. 2 Vortical structures of the disturbance flow obtained at $t = 50$ in Fig. 1

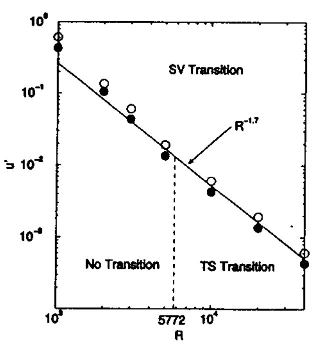

Fig. 3 Amplitudes of the initial disturbances which triggers the SV type transition versus Reynolds numbers

4. Transition induced by the suction/blowing

To simulate the effect of wall roughness on the transition, we apply a stationary suction and blowing from the bottom wall

$$w_s(x, y, -1) = A_s \cos(k_x x + k_y y) + c.c. \quad (6)$$

Figure 4 shows the time evolution of dominant Fourier modes of the disturbance induced by a suction/blowing whose wave number being (2,4) and amplitude A_s being 4×10^{-3}. In this case, a mode with the same wave number to the suction/blowing and $(0,0)$ are amplified quickly. Then, these modes excite various streamwise components in the disturbance, for instance $(0,10)$, and eventually trigger the transition of the flow at $t \approx 600$.

On the other hand, Fig. 5 shows the transition induced by a suction/blowing with two wave number components $(4,2)$ and $(-4,2)$. In this case, two oblique modes with the same wave numbers to the suction/blowing are amplified in the disturbance at once, and then, from their interaction, a streamwise mode with the wave number $(0,4)$ is amplified and soon after, $(0,8)$. Finally, the amplified streamwise modes and their higher harmonics trigger the by-pass transition[4].

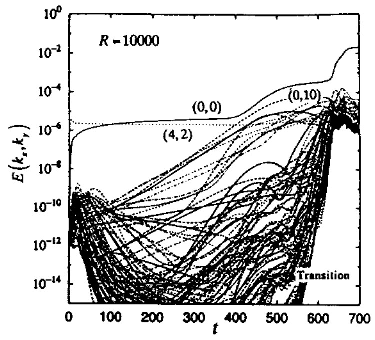

Fig. 4 Time evolution of dominant Fourier modes of a disturbance induced by a suction/blowing with a wave number component $(4,2)$

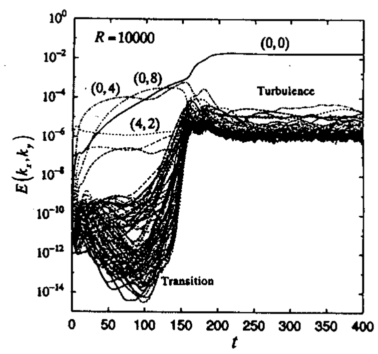

Fig. 5 Time evolution of dominant modes of a disturbance induced by a suction/blowing with two wave number components of $(4,2)$ and $(-4,2)$

5. Conclusion

Two kinds of by-pass transition are investigated with the direct numerical simulation. One of which is triggered by the large disturbance existing in the flow. The threshold amplitude of the disturbance which triggers the transition is determined as a function of the Reynolds number. Another type of the transition is induced by the suction/blowing from the wall. The suction/blowing with two oblique wave components excites a large streamwise mode which triggers the transition of the flow.

References

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