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Three Shear Theory of Plasticity

Anisotropy due to Plastic Deformation

Influence of Intermediate Principal Stress on Plastic Flow

Strain Ratio Relationship in Plastic Deformation

Internal Shearing Resistances in the Three Shear Theory of Plasticity

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CONTENTS

Three Shear Theory of Plasticity*

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A new theory of plasticity was advanced. The experiments on the stress relation between plastic pure shear and plastic tension conform closely with this theory. Moreover, the hysteresis loops and the anisotropy due to plastic deformation will be accounted for by this theory. The relation between the yield point of mild steel and the plastic deformation beyond that can be also accounted for.

Introduction

The theory of plasticity should be able to account for various experimental facts of plasticity. For instance,

1. The stress relation between plastic tention and plastic pure shear.

2. Hysteresis loops A hysteresis loop is drawn when a test piece is unloaded and reloaded in the plastic region. The stress-strain relation is elastic in the early stage of reloading. But it must become plastic at a certain point, for hysteresis loop will not be drawn if it is elastic till the end. The theory should not only be able to account for this fact but also be able to predict the point where the test piece becomes plastic.

3. Anisotropy due to plastic deformation Suppose that a test piece is deformed plastically under a certain load, and unloaded, and then an another kind of load is re-applied. For example, a test piece of thin hollow cylinder is at first deformed plastically under combined tension and internal pressure, their ratio being such that the plastic strain is a pure shear exhibiting a longitudinal elongation. If the re-applied load is of the same kind, the stress-strain relation is elastic in the early stage. But if the re-applied load is the internal pressure only, it is plastic from the first.

Under the internal pressure, pure shear exhibiting a circumferential elongation will take place if the material is isotropic. Thus the re-applied loads are similar in nature though the directions are different, and yet one is elastic and the other plastic. The theory should be able to account for these facts.

4. The relation between the yielding of mild steel and the plastic deformation beyond that It is already known that the yielding of mild steel takes place under a constant shearing stress, when the stress distribution is uniform. The theory must not conflict with this fact; moreover it must be able to account for the relation between the yielding and the plastic deformation beyond that.

We had a desire to advance a new theory. To collect data for this object, we carried out various kinds of experiments.

One of them is a series of experiment on the stress relation between tension and pure shear. The experiment itself is not difficult at all, but the problem lies in selection of the material. The material for this object must be isotropic in the initial state. Test pieces are generally cut out from round bars, and the bars are rarely isotropic though they are annealed. We examined various materials, and fortunately found out a bar of brass which could be considered to be practically isotropic, and experiments were carried out with test pieces cut from this bar. The results are shown in Fig. 9.

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Many experiments concerning the hysteresis loops and anisotropy due to plastic deformation were also carried out, and they were very useful to understand plasticity. They will be explained in detail in the next paper. We will here describe only a few of them that are considered to be necessary to explain the new theory.

Hysteresis loops and anisotropy due to plastic deformation

The simplest case in plasticity is the pure shear, and this case will be considered at first. Fig. 1 shows a hysteresis loop when a thin hollow cylinder was subjected to combined tension and internal pressure, their ratio being such that the plastic strain is a pure shear exhibiting a longitudinal elongation. The material is brass; The composition is Cu 59.59%, Zn 38.36%, Pb 1.29%, Sn 0.47%, and Fe 0.29%. The outer diameter of the cylinder is 18 mm and the inner diameter 16 mm.

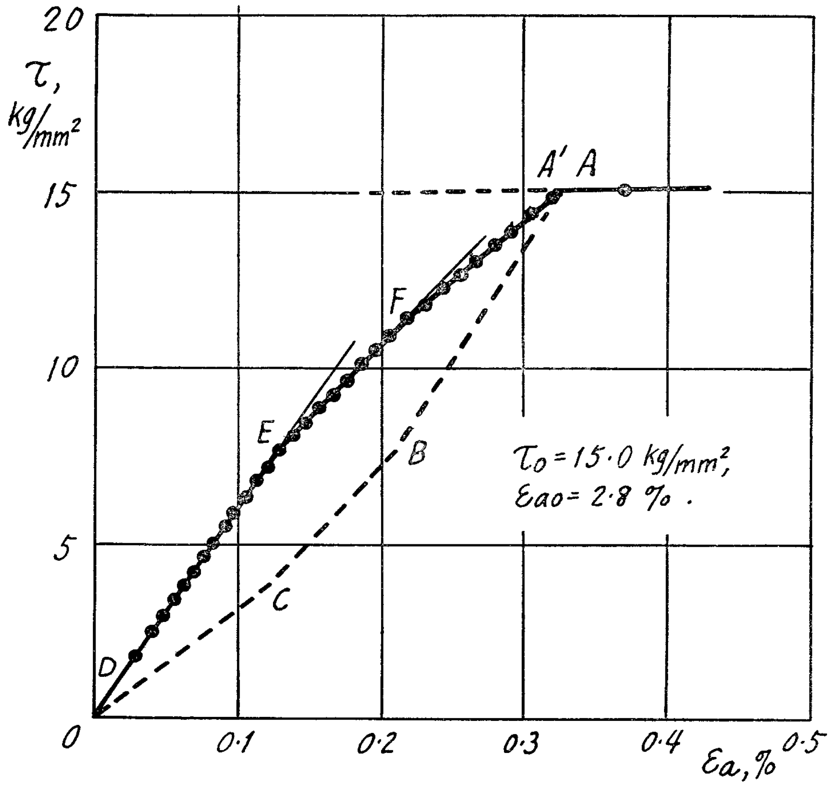

Fig. 1

The test piece was at first deformed up to point A in Fig. 1, where the shearing stress τ was τ_0 and the logitudinal strain ε_a was ε_{a0}, and then unloaded. Then a same kind of load was re-applied; $DEFA'$ in Fig. 1 is the stress-strain curve in this case. From D to E, the relation may be considered to be elastic. The curve deviates from the elastic line at E, and the shearing stress τ at E is just one half of τ_0 at A.

Let R be the stress range of elastic deformation, then. consindering the experimental result above mentioned, the following relation should hold;

$$\tau_0 = 2R .$$

The theory must be such a one that satisfies this relation.

When the re-applied load was the internal pressure only, instead of combined tension and internal pressure, the stress-strain relation became as shown in Fig. 2. In the figure, the shearing stress τ was taken in ordinate and the circumferential strain ε_t of the outer diameter in abscissa. Longitudinal strain in this case was also measured, the result is shown in Fig. 3, taking the internal pressure p in ordinate and the longitudinal strain ε_a in abscissa.

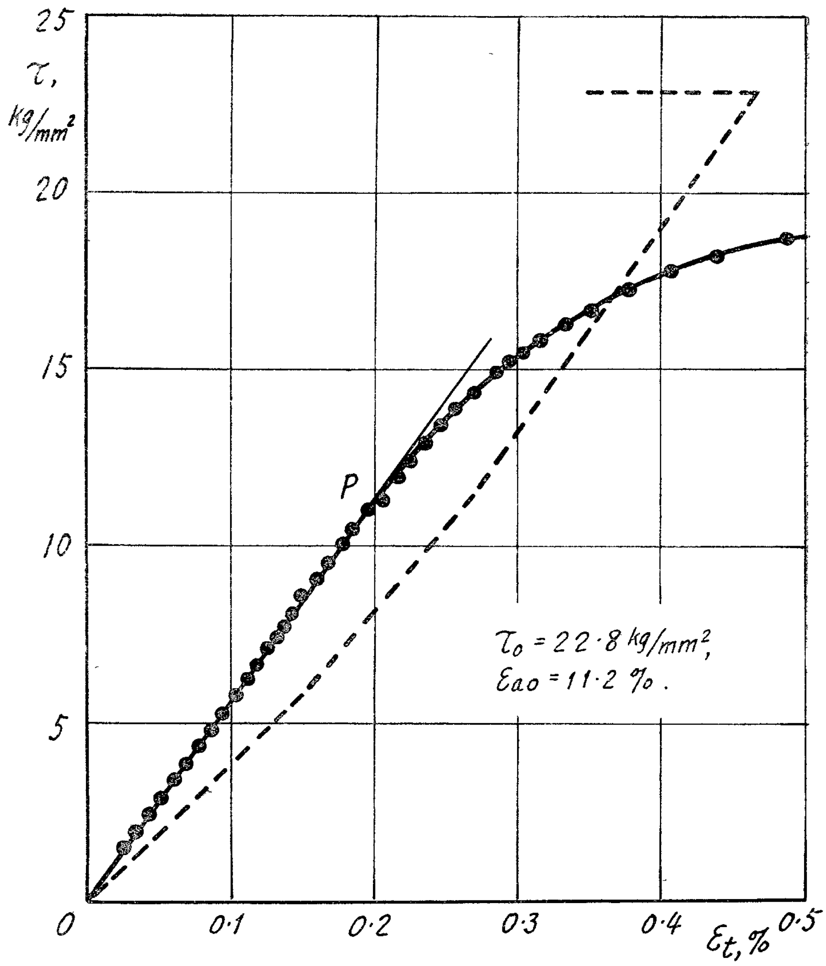
25
τ, kg/mm²
20
15
10
5
0
0·1
0·2
0·3
0·4
0·5
εt, %
P
τo = 22·8 kg/mm²,
εao = 11·2 %.

Fig. 2

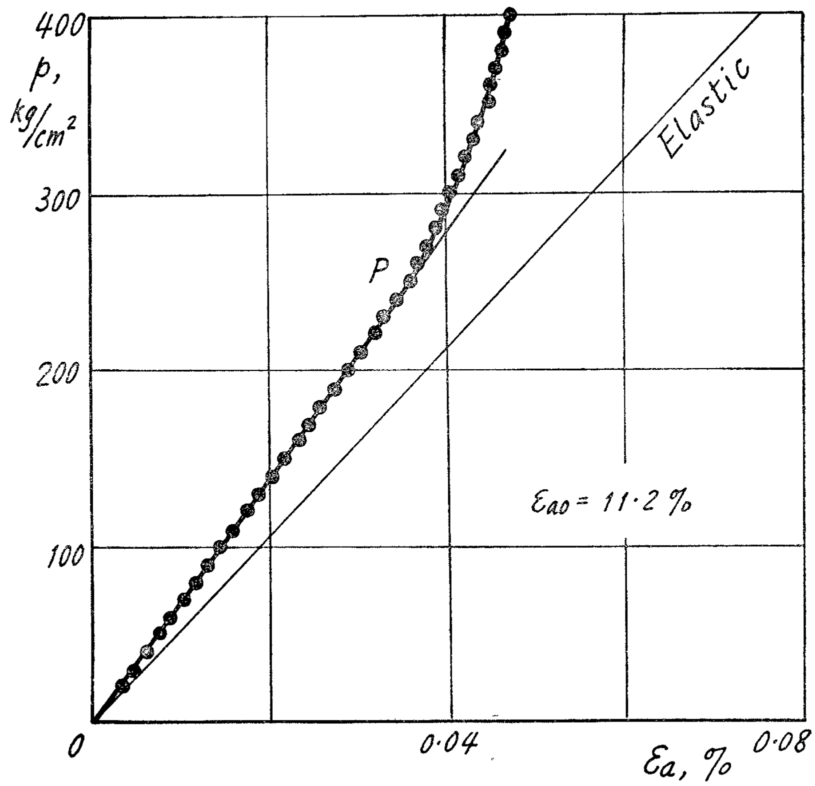
400
p, kg/cm²
300
200
100
0
0·04
0·08
εa, %
P
Elastic
εao = 11·2 %

Fig. 3

To compare with the experiment shown in Fig. 3, another case will be mentioned. A similar thin hollow cylinder was deformed plastically under the internal pressure up to $\tau=\tau_0$ and $\varepsilon_t=\varepsilon_{t0}$, and then unloaded. Then the same internal pressure was re-applied, and the longitudinal strain was measured. The result is shown in Fig. 4.

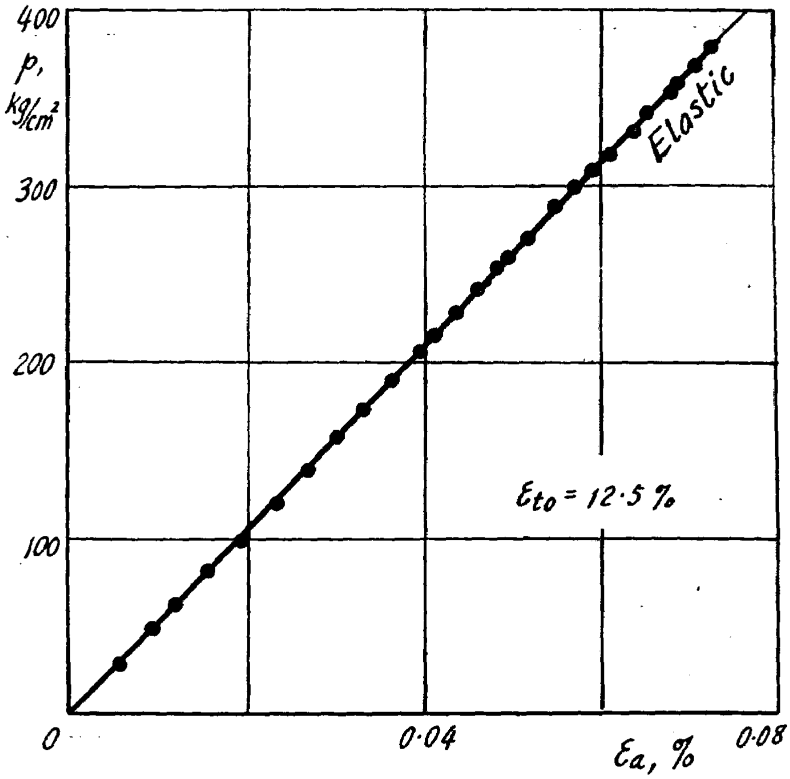

Fig. 4

The stress under internal pressure can be divided two stresses. One is a hydrostatic tension, and the other a shearing stress. Under the hydrostatic tension, elastic strain in all directions will take place. The elastic lines in Figs. 3 and 4 show the elastic strain due to this hydrostatic tension; they were drawn from the bulk modulus of that material and the dimensions of test pieces at that instant.

The measured points in Fig. 4 lie accurately on the elastic line, while the measured curve in Fig. 3 deviates from the elastic line and shows clearly the plastic longitudinal contration.

Now, take z-axis parallel to the axis of the cylinder, y-axis in the circumferential direction and x-axis in the direction of radius of the cylinder. The directions of principal stresses coincide witn these axes.

Let $\sigma_x, \sigma_y, \sigma_z$ be the principal stresses,

τ_1, τ_2, τ_3 be the principal shearing stresses, or

$$\left\{\begin{aligned} \tau_1&=\frac{1}{2}(\sigma_y-\sigma_z), \\ \tau_2&=\frac{1}{2}(\sigma_z-\sigma_x), \\ \tau_3&=\frac{1}{2}(\sigma_x-\sigma_y). \end{aligned}\right.$$

$\varepsilon_x, \varepsilon_y, \varepsilon_z$ be the principal strains,

$\gamma_1, \gamma_2, \gamma_3$ be the shear strains such as

$$\left\{\begin{aligned} \gamma_1&=\varepsilon_y-\varepsilon_z, \\ \gamma_2&=\varepsilon_z-\varepsilon_x, \\ \gamma_3&=\varepsilon_x-\varepsilon_y, \quad \text{and} \end{aligned}\right.$$

I, II, III be the directions of shears; the directions of shear strains γ_1, γ_2, and γ_3 are to be called the directions I, II, and III.

Excepting the hydrostatic tension, the state of stress under internal pressure is a shearing stress in the direction III. If the properties of the directions I and II are the same, neither longitudinal elongation nor contraction will take place under this shearing stress. Thus, in this case, the longitudinal elongation under internal pressure will be only the elastic one due to hydrostatic tension.

The stress-strain relation in the case of internal-pressure~internal-pressure is similar to that of pure-shear~pure-shear, and is elastic up to $\tau = \frac{1}{2}\tau_0$. The properties in the directions I and II are the same in this region, both being elastic. Thus the longitudinal strain in this case is only the elastic one due to the hydrostatic tension, and this agrees well with the experiment shown in Fig. 4.

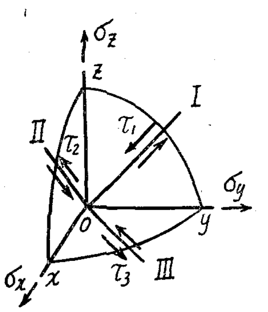

Fig. 5

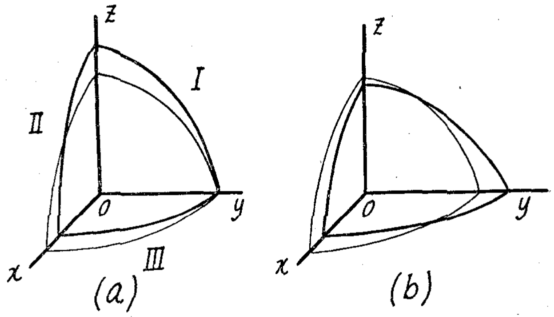

Fig. 6

Next we will consider the case of longitudinal-pure-shear~internal-pressure. The deformation of a spherical element in the case of longitudinal pure shear will be as shown in Fig. 6 (a), and then it will become as shown in Fig. 6 (b) under the internal pressure. Comparing the shears in Fig. 6 (a) and (b), we will notice that the directions of shears are equal in II and III, but are different in I. So it may be considered that the strains in directions II and III are elastic and that in direction I is plastic from the beginning of reloading. In such a case, plastic longitudinal contraction will take place under the shearing stress in direction III. Or the longitudinal strain under internal pressure will deviate from the elastic line. This agrees well with the experiment shown in Fig. 3.

Though plastic from the beginning, the stress-strain relation is linear at first as shown in Fig. 2, and it deviates from linear relation at P, this point corresponds to P in Fig. 3. The shearing stress τ_3 at P is just one half of τ_0, or it is equal to R.

It may be considered from the above explanation that in plasticity there exists not only the case where the strain in directions I, II, and III are all plastic, but also the case where those in one or two directions are plastic and the rest still elastic. This idea seems to be able to account for various experimental evidences on plasticity.

In the case of pure-shear~pure-shear, shown in Fig. 1, the stress-strain relation becomes plastic at E, where

$$\tau_2 = \frac{1}{2}\tau_0 = R.$$

It may be considered that at this point the state in II becomes plastic. In the case of pure-shear~internal-pressure, shown in Fig. 2, the stress-strain curve deviates from linear relation at P, where

$$\tau_3 = \frac{1}{2}\tau_0 = R.$$

In this case, the state in I is plastic from the beginning, and it may be considered that the state in III becomes plastic at P. Comparing the stresses, it may be concluded that the strees ranges of elastic deformations are equal in the three directions.

Theory of plasticity

A new theory of plasticity was advanced, taking into account the experimental results above mentioned.

1. Plastic deformation is to be divided into shears in three directions, and each shear is to be considered individually.
2. When the deformation increases monotonously, and when the directions of principal strains coincide with those of principal stresses, the resistances to the strain increase in the three directions R_1, R_2, and R_3 are equal in magnitude, or

$$R_1=R_2=R_3=R\,. \tag{1}$$

3. When the direction of shear strain is reversed, the reversed strain is elastic, and the stress range of this elastic strain is R concerning to each direction.

Let S_1, S_2, and S_3 be shearing stress such as shown in Fig. 7. When the material is subjected to S_1, S_2, and S_3 simultaneously, the relations between τ_1, τ_2, τ_3 and S_1, S_2, S_3 are

$$\left.\begin{aligned}\tau_1&=S_1-\frac{1}{2}S_2-\frac{1}{2}S_3\,,\\ \tau_2&=-\frac{1}{2}S_1+S_2-\frac{1}{2}S_3\,,\\ \tau_3&=-\frac{1}{2}S_1-\frac{1}{2}S_2+S_3\,.\end{aligned}\right\} \tag{2}$$

τ is a shearing stress working on a plane, and S is a shearing stress expressed by normal stresses. τ_1 consists of S_1 and components of S_2 and S_3 as seen in Eq. (2). On the other hand, R_1, R_2, and R_3 are the resistances to the strain increase in each direction, and they must correspond to shearing stresses. But as they are considered individually, they cannot correspond to τ_1, τ_2, and τ_3; they must correspond to S_1, S_2, and S_3.

4. R_1, R_2, and R_3 correspond to the shearing stresses S_1, S_2, and S_3.

Those expressed in 1, 2, 3, and 4 are the fundamental concept of the authors' new theory of plasticity.

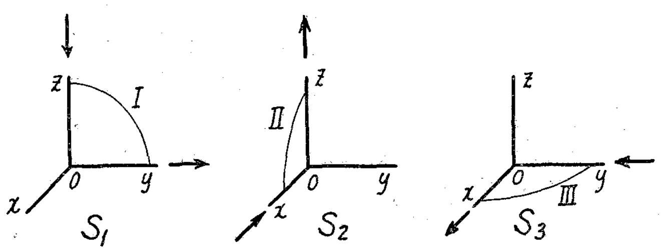

Fig. 7

This is a shearing stress theory. Concerning to one direction, $S=R$ when the shear strain increases monotonously. If the direction of strain increase is reversed and the strain begins to decrease, this strain decrease is elastic until $S=0$, as the elastic stress range is R. If the strain decrease continues still more, this decrease is plastic.

In actual cases, the shears in three directions must be considered individually and simultaneously. This is the characteristic of this theory, and in this sense this theory may be called "Three Shear Theory" of plasticity.

Hysteresis loops and anisotropy due to plastic deformation will be accounted for by this theory, but these problems will be explained in detail in the next paper.

It will be explained in this paper, how the stress relation between tension and pure shear can be accounted for, and why the yielding of mild steel takes place at constant shearing strees though it is the first step of plasticity.

Pure shear

Suppose that a thin hollow cylinder is subjected to combined tension and internal pressure, their ratio being such that the plastic strain is a pure shear exhibiting a longitudinal elongation. In this case, the deformation of a spherical element in the material will be as shown in Fig. 6 (a). The working shearing stress is that in the direction II, and under this stress the strain increases take place not only in the direction II but also in the directions I and III. Thus the following equations must hold.

$$\left.\begin{aligned} S_1&=-R_1=-R\,,\\ S_2&=R_2=R\,,\\ S_3&=-R_3=-R\,.\end{aligned}\right\}\qquad(3)$$

This is the state of stress of plastic pure shear. Expressed in τ_1, τ_2, and τ_3.

$$\left.\begin{aligned} \tau_1&=-R_1-\frac{1}{2}R_2+\frac{1}{2}R_3=-R\,,\\ \tau_2&=\frac{1}{2}R_1+R_2+\frac{1}{2}R_3=2R\,,\\ \tau_3&=\frac{1}{2}R_1-\frac{1}{2}R_2-R_3=-R\,.\end{aligned}\right\}\qquad(4)$$

τ_2 is the greatest in this case, and this value $\tau_{\max}$ is

$$\tau_{\max}=2R\,. \qquad (4)'$$

Tension

When longitudinal elongation takes place under tension, increases of shear strains take place in the directions I and II. So in this case, the values of S_1 and S_2 are

$$\left.\begin{aligned} S_1&=-R_1\,,\\ S_2&=R_2\,.\end{aligned}\right\}\qquad(5)$$

The question arises about the values of S_3. In tension it is considered that fine slips in the directions I and II take place altenately. The direction of shear in III when a slip takes place in I and that when a slip takes place in II are opposite, or the sign of shear in III is positive for the slip in I and negative for the slip in II. So S_3 must be also positive for the slip in I and negative for the slip in II. Moreover, as τ_1 and τ_2 are equal in magnitude, the value of S_3 must also be equal in magnitude for both slips.

Now assume that a small slip takes place at first in direction I. Then the shear strain γ_3 in direction III will increase a little, and R_3 will move from OA to CB as shown in Fig. 8. In this state the resistance to the slip in direction II is smaller than that in I, as the resistance in III is R_3 for the slip in I and that for the slip in II is not $-R_3$ but is zero. And thus the slip in II will take place, and the shear strain γ_3 will begin to decrease, and R_3 will move from CB to DF along the plastic line CD as shown in Fig. 8. At DF where $DE=EF$, the resistances for slips I and II are equal, the resistance in III for I being EF and that for II ED. In this state it is possible that slips in I and II take place alternatly, or the tensile elongation is possible. S_3 in this state is

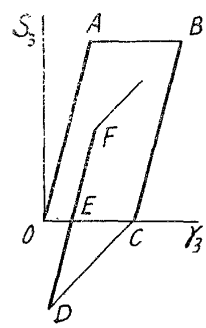

Fig. 8

$$S_3=\pm\frac{1}{2}R_3\,. \qquad (5)'$$

The state of stress in tension is given by Eqs. (5) and (5)'

Expressed in τ_1, τ_2, and τ_3, the state of stress is

$$\left.\begin{aligned}\tau_1&=-R_1-\frac{1}{2}R_2-\frac{1}{4}R_3=-\frac{7}{4}R,\\ \tau_2&=\frac{1}{2}R_1+R_2+\frac{1}{4}R_3=\frac{7}{4}R,\\ \tau_3&=\frac{1}{2}R_1-\frac{1}{2}R_2\pm\frac{1}{2}R_3=\pm\frac{1}{2}R.\end{aligned}\right\} \tag{6}$$

Concerning to one point in the material, slips take place alternately in directions I and II. Concerning to one moment, the slip in I is taking place in some parts in the material and that in II in other parts. Thus the internal stress exists in the direction III, but macroscopically τ_3 is 0.

The greatest shearing stress τ_{max} in order to continue the tensile deformation is

$$\tau_{max}=\frac{7}{4}R. \tag{6$'$}$$

Relation between pure shear and tension

When the deformation increases, the value of R increases also by work-hardening. It is considered that the work-hardening depends on the amount of total slip, or R is a function of the amount of total slip.

In the case of pure shear, the relation between the amount of slip s_{sh} and the longitudinal strain ε_{sh} is

$$s_{sh}=2\log(1+\varepsilon_{sh}), \tag{7}$$

and, in the case of tension, the relation between the amount of slip s_{tn} and the longitudinal strain ε_{tn} is also

$$s_{tn}=2\log(1+\varepsilon_{tn}). \tag{8}$$

Or, the amounts of slip are equal when

$$\varepsilon_{sh}=\varepsilon_{tn}. \tag{9}$$

Consequently we have

$$R_{sh}=R_{tn} \quad \text{at} \quad \varepsilon_{sh}=\varepsilon_{tn}, \tag{10}$$

where R_{sh} is the value of R in pure shear, and R_{tn} is the value of R in tension.
Or, from Eqs. (4)′ and (6)′,

$$\frac{\tau_{tn}}{\tau_{sh}}=\frac{7}{8} \quad \text{at} \quad \varepsilon_{sh}=\varepsilon_{tn}, \tag{11}$$

where τ_{sh} is τ_{max} in pure shear, and τ_{tn} is τ_{max} in tension.
This equation is the stress relation between plastic tension and plastic pure shear by the authors' theory.

Now this relation will be compared with experimental results which are shown in Fig. 9. The test pieces used were hollow cylinders, the outer diameter being 18 mm and the inner diameter 16 mm. The material was brass; the composition is Cu 59.59%, Zn 38.36%, Sn 0.47%, and Fe 0.29%. The material to be used in the experiments must be isotropic in the virgin state, but such a material is very rare. Fortunately we found out a bar of brass which was considerd to be practically isotropic after examining various kinds of materials, and test pieces were cut out from that bar. However, brass of the same composition is not always isotropic;

only that particular bar was isotropic. The examination and selection of the material was already reported[1] by one of the authors, so only the results of experiments will be mentioned here.

In Fig. 9, $P-A$ is the stress-strain relation under combined tension and internal pressure, their ratio being such that the plastic strain is a pure shear exhibiting a longitudinal elongation. The shearing stress τ_2 is taken in ordinate and the longitudinal elongation ε_a in abscissa. $P-T$ is the stress-strain relation under internal pressure; the plastic strain in this case is a pure shear exhibiting a circumferential elongation. The shearing stress τ_3 is taken in ordinate and the circumferential strain of mean diameter in abscissa. The measured points of $P-A$ and $P-T$ lie closely on one curve, though the directions of slips are different, and this shows the isotropy of the material in the virgin state.

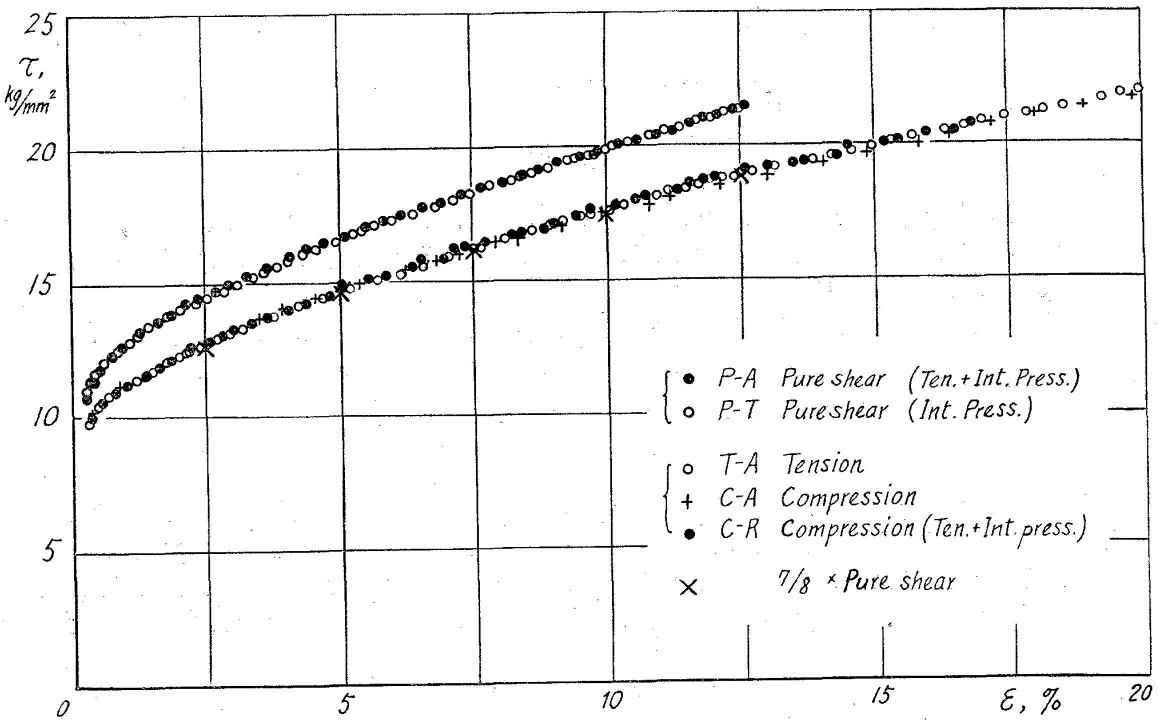

Fig. 9

$T-A$ is the stress-strain relation under tension, the longitudinal strain ε_a being taken in abscissa. $C-A$ is that under compression; this is the only case in which a solid test piece was used. In abscissa is taken such a strain that corresponds to the tensile strain ε_a under tension, calculated from the expansion of the diameter. $C-R$ is the stress-strain relation under combined tension and internal pressure, their ratio being such that the longitudinal stress is always equal to the circumferential stress. This is plastically a compression in the direction of thickness of the cylinder. In abscissa is also taken such a strain that corresponds to ε_a, calculated from longitudinal and circumferential elongations. The measured points in the three cases $T-A$, $C-A$, and $C-R$ lie closely on one curve.

These experimental results are considered to be very reliable, as the stress-strain curves of pure shears conform very closely, and those under tension and compressions also conform closely, notwithstanding the difference of slip directions.

In this figure, points of 7/8 of the values in ordinate of the stress-strain curves of pure shears are also marked; these points lie closely on the curves of tension and compressions through all the plastic region.

The yielding of mild steel and the plastic deformation beyond that

The yielding of mild steel in the case of pure shear will be considered at first. As the stress-strain relation is unstable at the yield point, the point considered here is of course not the upper yield point but the so-called lower one, and the yielding takes place as follows:— The strain of the test piece does not increase uniformly, but the strain of the yielding part passes from the elastic one to a certain finite plastic strain, and such parts come out in succession under a constant shearing stress, as the average strain of the test piece increases. The point A in Fig. 10 denotes the yield point, and P the equilibrium point in the plastic state under the constant shering stress; the strain of the yielding part passes from A to P.

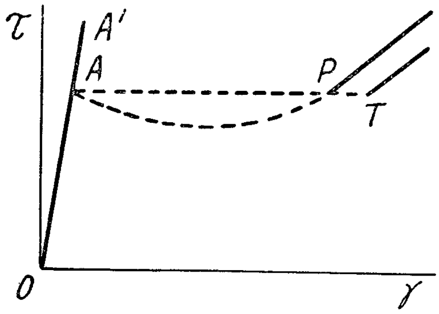

Fig. 10

Next, the yielding under tension will be considered. It was explained that, in the plastic region, fine slips take place alternately in the directions I and II, but at the yield point the manner of slip is different, as the stress-strain relation is unstable at this point. If a slip starts either in the direction I or II, only that slip will continue until the point P is reached.

At the yield point the stress-strain relation is unstable, and in both cases, pure shear and tension, the yielding spreads under this stress, or

$$\tau_y = \text{const}, \tag{12}$$

where τ_y is the shearing stress at the yield point when the stress distribution is uniform.

The manners of yielding are the same in both cases until the strain increase to the point P in Fig. 10. This is the equilibrium point in case of pure shear, but under tension this is not the equilibrium point. Suppose that the slip in the direction I took place at first under

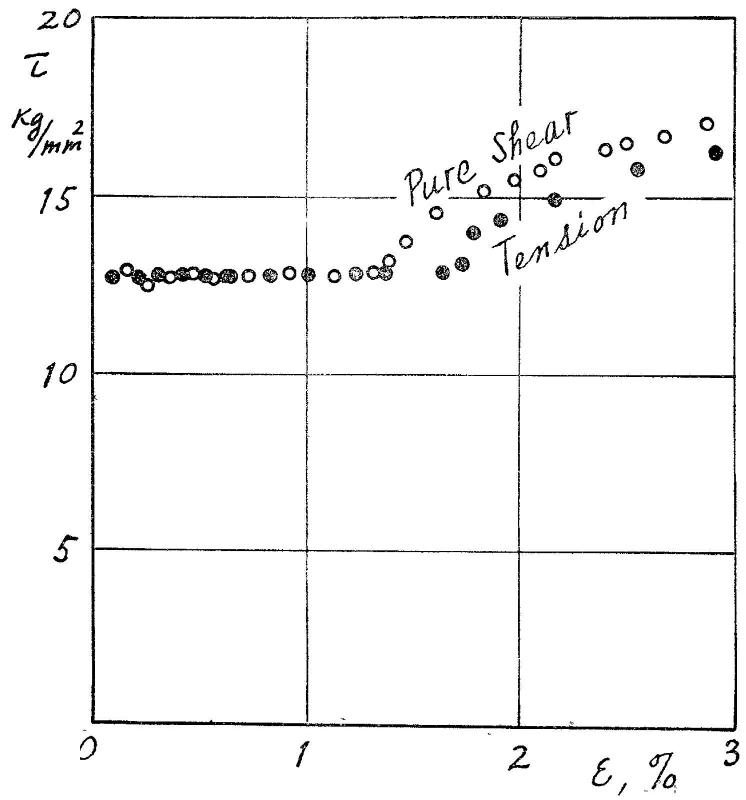

Fig. 11

tension, and the strain reached the point P. In this state, the stress and the resistance are in equilibrium in direction I, but they are not in direction II. The resistance in II is smaller than the stress, and the slip in this direction will start, and the phenomenon explained concerning to the plastic tension and shown in Fig. 8 will take place at this point. After that, the strain will increase more, and at the point T in Fig. 10 the equibrium will be attained in both directions I and II.

It was explained that the plastic stress under tension is expressed by Eq. (6)′. It is because the phenomenon shown in Fig. 8 takes place under tension. But when the material yields under tension, this phenomenon takes place at the point P in Fig. 10, and the yielding is another phenomenon prior to this. Thus the condition of yielding expressed by Eq. (12) is a different problem from this theory of plasticity, and of course there is no contradiction between them.

Fig. 11 shows the experimental results on the yielding and the plastic deformation beyond that, in the cases of pure shear and tension. The shearing stress τ is taken in ordinate and the longitudinal strain ε in abscissa. Material is a mild steel of 0.39% carbon content. Test pieces are hollow cylinders of outer diameter 18 mm and inner diameter 16 mm. The stress ratio between the plastic pure shear and plastic tension does not satisfy Eq. (11) exactly, as this material is considered to be not isotropic enough. And yet from this figure we will be able to understand what was explained above.

Conclusions

A new theory of plasticity was advanced. The experiments on the stress relation between plastic pure shear and plastic tension conformed closely with this theory. Moreover, hysteresis loops and anisotropy due to plastic deformation will also be accounted for by this theory.

As for the yielding of mild steel, it is already known that the yielding spreads under a constant shearing stress, independent of the kind of load, when the stress distribution is uniform. It was shown that this fact did not conflict with the theory, though the yielding was the first step of plastic deformation.

We wish to express our thanks to Mr. F. Nagai for his assistance in carrying out the experiments and computations.

Reference

(1) Y. Sato: *Bulletin of JSME*, Vol. 2, No. 5 (1959), p. 107.

Anisotropy Due to Plastic Deformation*

By Fujio NAKANISHI and Yasuo SATO

A new theory of plasticity was advanced to account for various experimental facts, and it was explained in the previous paper(1) that the experiments on the stress relation between tension and pure shear agreed very well with this theory, and that the relation between the yielding of mild steel and the plastic deformation after that could also be accounted for. In this paper, the theory will be compared with the experiments on the anisotropy due to plastic deformation.

Authors' Theory of Plasticity

Let

$\sigma_x, \sigma_y, \sigma_z$ be the principal stresses,

$\varepsilon_x, \varepsilon_y, \varepsilon_z$ be the principal strains,

τ_1, τ_2, τ_3 be the principal shearing stresses, or

$$\left\{\begin{aligned} \tau_1 &= \frac{1}{2}(\sigma_y - \sigma_z), \\ \tau_2 &= \frac{1}{2}(\sigma_z - \sigma_x), \\ \tau_3 &= \frac{1}{2}(\sigma_x - \sigma_y), \end{aligned}\right.$$

$\gamma_1, \gamma_2, \gamma_3$ be the shearing strains such as

$$\left\{\begin{aligned} \gamma_1 &= \varepsilon_y - \varepsilon_z, \\ \gamma_2 &= \varepsilon_z - \varepsilon_x, \\ \gamma_3 &= \varepsilon_x - \varepsilon_y, \end{aligned}\right.$$

I, II, III be the directions of shears; the directions of shear strains γ_1, γ_2, and γ_3 are to be called the directions I, II, and III, and

S_1, S_2, S_3 be the shearing stresses such as shown in Fig. 1.

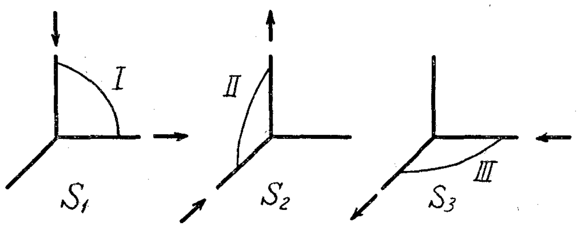

Fig. 1

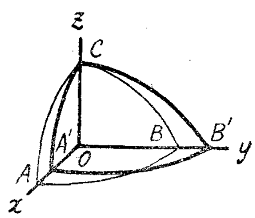

Fig. 2

The relations between τ_1, τ_2, τ_3 and S_1, S_2, S_3 are

$$\left.\begin{aligned} \tau_1 &= S_1 - \frac{1}{2}S_2 - \frac{1}{2}S_3, \\ \tau_2 &= -\frac{1}{2}S_1 + S_2 - \frac{1}{2}S_3, \\ \tau_3 &= -\frac{1}{2}S_1 - \frac{1}{2}S_2 + S_3. \end{aligned}\right\} \quad (1)$$

The authors' theory is as follows:—

1. The three directions I, II, and III are to be considered individually.

   As they are considered individually, the resistances to the strain increase must also be considered individually; let they be R_1, R_2, and R_3. For instance, when the slip takes place in the negative direction III under the shearing stress τ_3, the deformation of an elemental sphere in the material will be as shown in Fig. 2, and in this case the resistaece R_3 in direction III will have of course the direct effect, but R_1 and R_2 will also have some effect as there are strains in directions I and II. Considering the strain energy, or considering the components of R_1 and R_2 in direction III, the following relation must hold during the slip,

$$\tau_3=-\frac{1}{2}R_1-\frac{1}{2}R_2-R_3.$$

   R_1, R_2, and R_3 are resistances to the increase of shear strain in each direction, and must correspond to shearing stresses, but, as seen in the above relation, they do not correspond directly to τ_1, τ_2, and τ_3. R_1, R_2, and R_3 must correspond to the shearing stresses S_1, S_2, and S_3; this fact can be seen by comparing the above relation with equations (1).

2. When the directions of principal strains coincide with those of principal stresses, and when the shear strain increases monotonously, the magnitudes of R_1, R_2, and R_3 are equal, or

$$R_1=R_2=R_3=R.$$

3. When the direction of shear strain is reversed concerning to each direction, the stress range of elastic deformation is R.

Those explained in 1, 2, and 3 are the authors' theory of plasticity; its characteristic is to consider the three directions of shear individually. Concerning to each direction, it is a shearing stress theory; the shearing stress in this case is not τ but S.

Anisotropy due to Plastic Deformation

Now a thin hollow cylinder deformed plastically under the internal pressure will be considered. In this case, the strain is plastically a pure shear, the elongation taking place in the circumferential direction.

The deformation of the part ABC of an elemental sphere in the material will be as shown in Fig. 2, taking x-axis in the direction of radius of the cylinder, y-axis in the direction of circumference, and z-axis parallel to the axis of the cylinder. The shear strain in the direction III is negative, and those in I and II are positive, so the state of stress must be as follows;

$$\left.\begin{aligned} S_1&=R, \\ S_2&=R, \\ S_3&=-R, \end{aligned}\right\} \qquad (2)$$

or, in τ_1, τ_2, and τ_3, this state is

$$\left.\begin{aligned} \tau_1&=R, \\ \tau_2&=R, \\ \tau_3&=-2R. \end{aligned}\right\} \qquad (2)'$$

The state of stress (2) can be expressed diagrammatically as shown in Fig. 3.

Now suppose that the test piece is unloaded after it is deformed until $\tau_3=\tau_0$ and $\gamma_3=\gamma_0$, and then a certain load is re-applied. The stress range of elastic deformation in this case will be

$$\left.\begin{array}{llr} S_1=0\sim R, & \text{in the direction} & \text{I}, \\ S_2=0\sim R, & \quad\text{〃} & \text{II}, \\ S_3=0\sim -R, & \quad\text{〃} & \text{III}. \end{array}\right\} \tag{3}$$

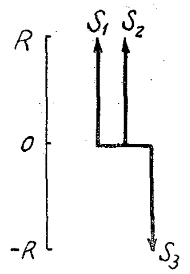

Fig. 3

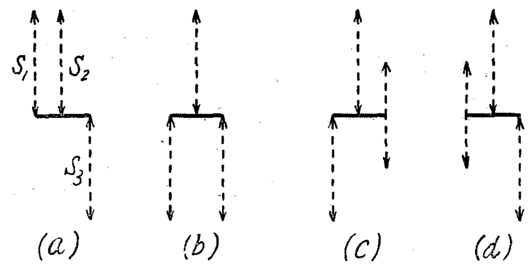

Fig. 4

These ranges are shown diagrammatically in Fig. 4(a).

The elastic stress ranges differ as to the direction as shown above; this is the anisotropy due to plastic deformation by the authors' theory.

The elastic stress ranges in other cases can be obtained similarly. When a thin hollow cylinder is subjected to combined tension and internal pressure, where the ratio of tension and internal pressure is such that the strain is plastically a pure shear, the elongation taking place in the direction of axis, the state of stress must be as follows[1]:

$$\left.\begin{array}{l} S_1=-R, \\ S_2=R, \\ S_3=-R, \end{array}\right\} \tag{4}$$

$$\left.\begin{array}{l} \tau_1=-R, \\ \tau_2=2R, \\ \tau_3=-R. \end{array}\right\} \tag{4$'$}$$

When unloaded, the elastic stress ranges will be as shown in Fig. 4(b).

The state of stress in tension is[1]

$$\left.\begin{array}{l} S_1=-R, \\ S_2=R, \\ S_3=\pm\dfrac{1}{2}R, \end{array}\right\} \tag{5}$$

where positive sign of S_3 corresponds to the slip in the direction I, and negative sign to that in II.

$$\left.\begin{array}{l} \tau_1=-\dfrac{7}{4}R, \\ \tau_2=\dfrac{7}{4}R, \\ \tau_3=0\pm\dfrac{1}{2}R. \end{array}\right\} \tag{5$'$}$$

When unloaded, the elastic stress ranges will be as shown in Fig. 4(c). In this case, $\tau_3=0$ macroscopically, but there remains the internal stress in direction III.

When a thin hollow cylinder is subjected to combined tension and internal pressure in such a ratio that the axial stress is always equal to the circumferential stress, this is plastically a compression in the direction of thickness of the cylinder, and the state of stress is

$$\left.\begin{aligned} S_1 &= \pm\frac{1}{2}R, \\ S_2 &= R, \\ S_3 &= -R, \end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned} \tau_1 &= 0\pm\frac{1}{2}R, \\ \tau_2 &= \frac{7}{4}R, \\ \tau_3 &= -\frac{7}{4}R. \end{aligned}\right\} \tag{6$'$}$$

When unloaded, the elastic stress ranges will be as shown in Fig. 4(d). In this case, the internal stress remains in direction I.

Axial Elongation under Internal Pressure

When a certain load is applied to the test pieces in the anisotropic states as shown above, the deformation in the first stage will be elastic if all the directions of strains in I, II, and III are the same as those of the elastic stress ranges, but the deformation will be plastic from the first if even one of the direction of strains differs from those of elastic stress ranges.

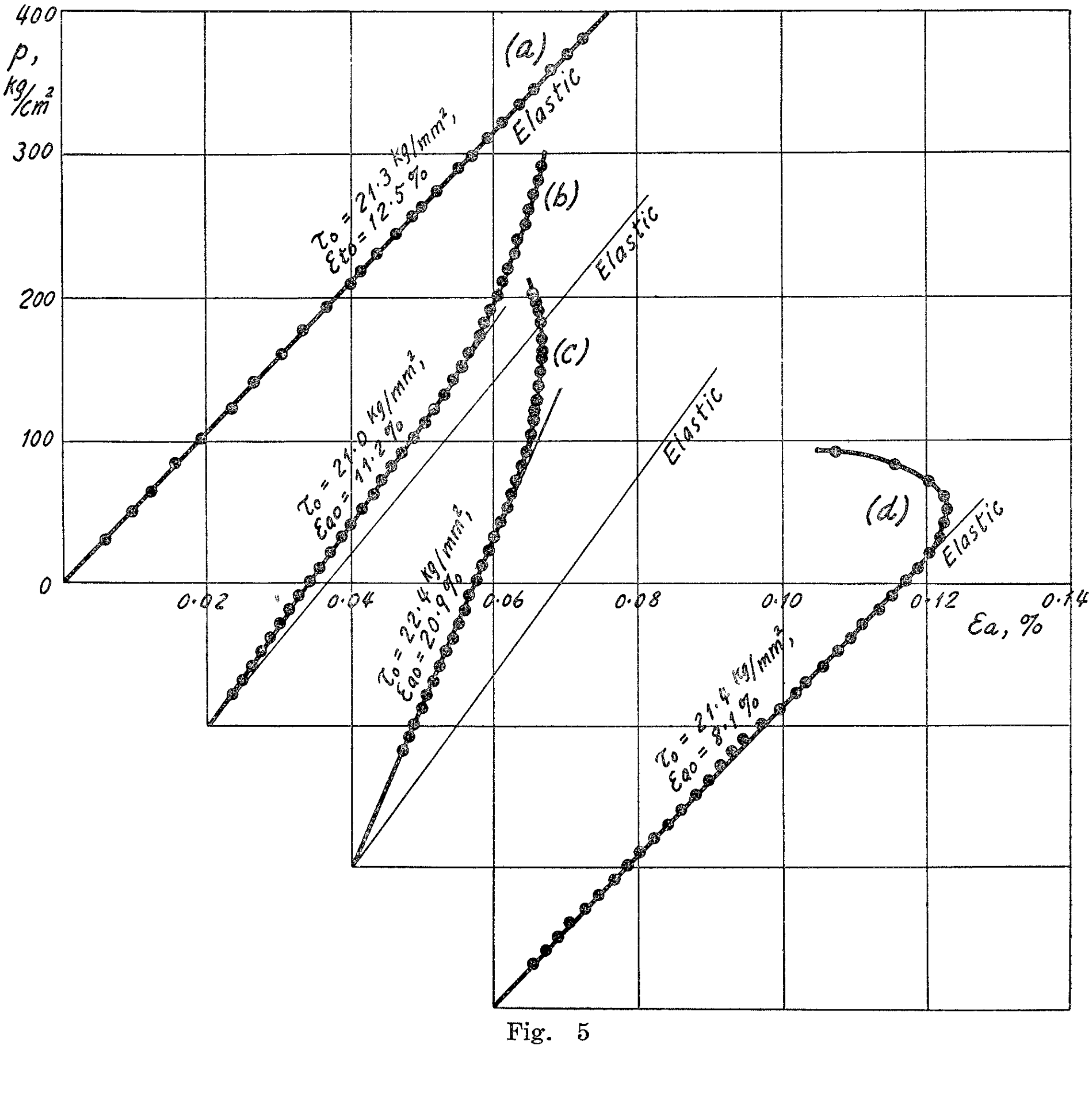

Fig. 5

Experiments were carried out. The test pieces used were hollow cylinders, the initial outer diameter being 18 mm and the inner diameter 16 mm. The material was brass; the composition is Cu 59.59%, Zn 38.36%, Pb 1.29%, Sn 0.47%, and Fe 0.29%.
The pre-applied loads were (a) internal pressure, (b) pure shear, (c) tension, and (d) compression, and the re-applied was internal pressure. The axial and circumferential elongations were measured. The results are shown in Figs. 5 and 6.

The stress under internal pressure can be divided into two stresses. One is uniform tension, and elastic tensile strain in all directions will take place under this stress. The other is a shearing stress in direction III, and the strain as shown in Fig. 2 will take place under this stress. The strains in directions I and II are positive, and that in III negative. Comparing these signs with the elastic stress ranges in Fig. 4, it will be easily understood that (a) and (d) are elastic, and (b) and (c) are plastic in direction I.

If the deformations in the both directions in I and II are elastic as in (a) and (d), neither axial elongation nor contraction will take place under the shearing stress; thus the axial elongation in this case will be only the elastic one due to uniform tension.

If it is elastic in II and plastic in I as in cases of (b) and (c), axial contraction will take place under the shearing stress. Or the axial elongation under internal pressure will deviate from the elastic line of uniform tention.

The experimental results on the axial elongation under internal pressure are shown in Fig. 5. The elastic lines in the figure were drawn from the bulk modulus of that material and the dimensions of test pieces at that instant.

Fig. 5(a) shows the axial elongation in the case of internal-pressure~internal-pressure. The test piece was at first deformed under internal pressure up to $\tau=\tau_0$ and the strain of outer diameter $\varepsilon_t=\varepsilon_{t0}$, and unloaded, and then it was subjected to internal pressure again. The measured points lie accurately on the elastic line.

Fig. 5(b) shows the axial elongation in pure-shear~internal pressure. The test piece was at first deformed under combined tension and internal pressure so as the plastic strain

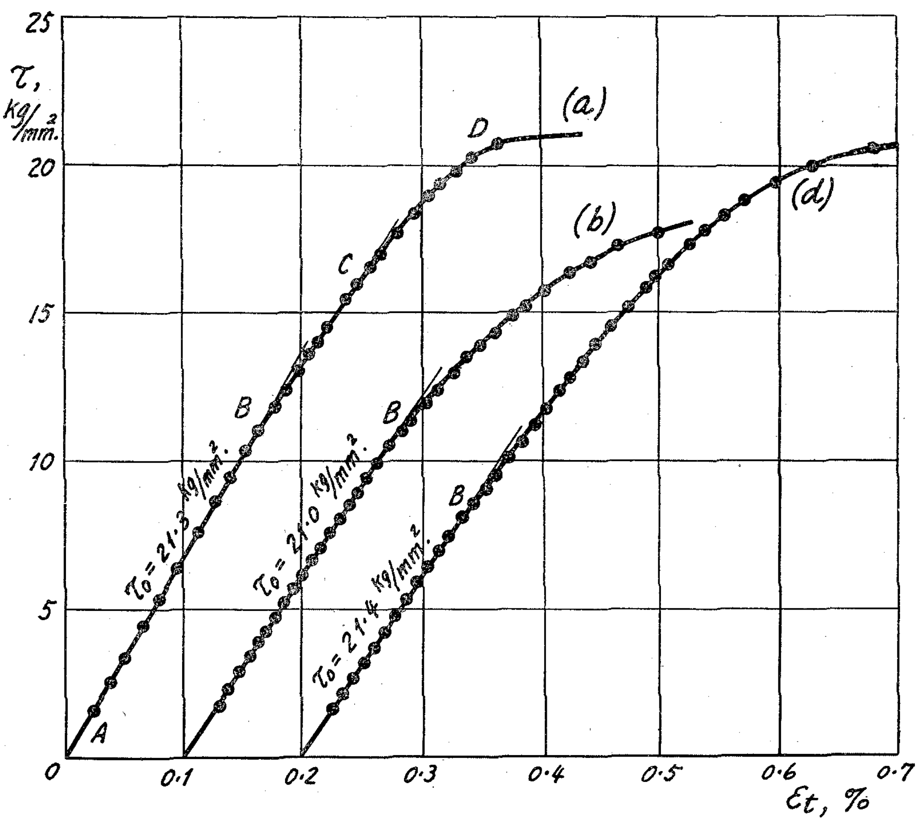

Fig. 6(a), (b), (d).

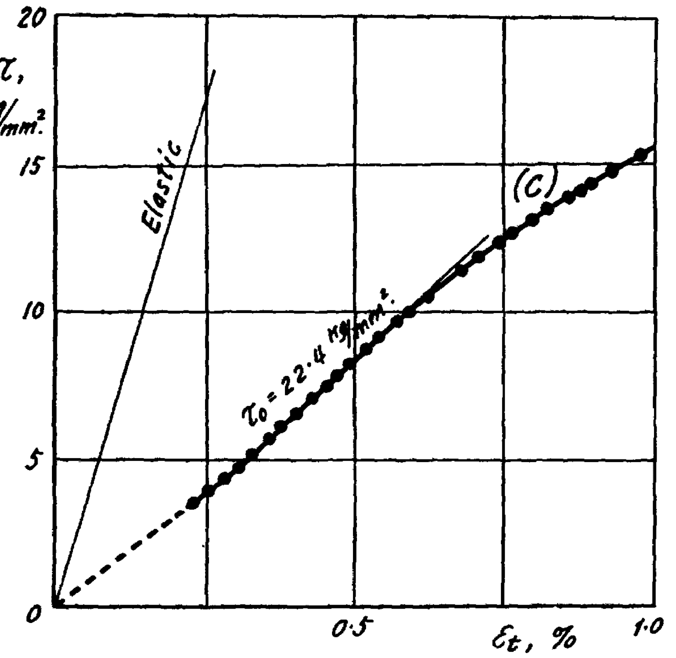

Fig. 6(c).

might be a pure shear, and unloaded after $\tau=\tau_0$ and axial strain $\varepsilon_a=\varepsilon_{a0}$; and then it was subjected to internal pressure. The measured points deviate from the elastic line, and show the plastic axial contraction clearly.

Fig. 5(c) shows the axial elongation in tension~internal-pressure. The experimental results in this case also show the plastic axial contraction.

Fig. 5(d) shows the axial elongation in radial-compression~internal-pressure. The measured points lie accurately on the elastic line.

Stress-Strain Relations

(a) Internal-Pressure~Internal-Pressure

The stress-strain relation is shown in Fig. 6(a). It is at first linear, and there are abrupt changes of slope at B and C. The stresses at B, C, and D, obtained from the figure, are

$$\left.\begin{aligned} \tau_3&=\frac{1}{2}\tau_0, && \text{at } B,\\ \tau_3&=\frac{3}{4}\tau_0, && \text{at } C,\\ \tau_3&=\tau_0, && \text{at } D. \end{aligned}\right\} \qquad (7)$$

By the authors' theory, the elastic stress range of the unloaded test piece is as shown in Fig. 7(a), and the re-applied load, internal pressure, is such a load which finally will become as shown in Fig. 7(d) concerning to the shearing stress.

In the first stage the re-applied load is S_3 itself concerning to the shearing stress, so at first the test piece will be subjected to S_3 until the state of stress shown in Fig. 7(b) is reached, where

$$\left.\begin{aligned} S_1&=0,\\ S_2&=0,\\ S_3&=-R, \end{aligned}\right\} \qquad (8)$$

$$\left.\begin{aligned}\tau_1&=\frac{1}{2}R,\\ \tau_2&=\frac{1}{2}R,\\ \tau_3&=-R=\frac{1}{2}\tau_0.\end{aligned}\right\} \qquad (8)'$$

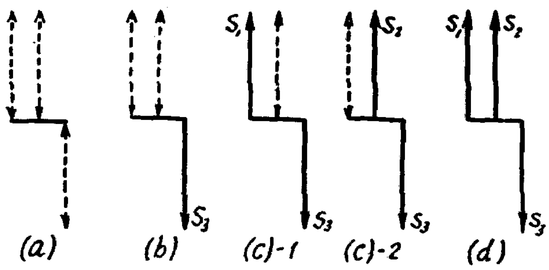

Fig. 7

The deformation in the direction III will become plastic at this point, and this corresponds to the point B in Fig. 6(a).

With an increase of pressure, as S_3 can increase no more, the test piece must be subjected to S_1 or S_2, and the state shown in Fig. 7(c)-1 or (c)-2 will be reached. These states are

$$\left.\begin{aligned}S_1&=R,\\ S_2&=0,\\ S_3&=-R,\end{aligned}\right\} \qquad (9)$$

$$\left.\begin{aligned}\tau_1&=\frac{3}{2}R,\\ \tau_2&=0,\\ \tau_3&=-\frac{3}{2}R=\frac{3}{4}\tau_0.\end{aligned}\right\} \qquad (9)'$$

$$\left.\begin{aligned}S_1&=0,\\ S_2&=R,\\ S_3&=-R,\end{aligned}\right\} \qquad (10)$$

$$\left.\begin{aligned}\tau_1&=0,\\ \tau_2&=\frac{3}{2}R,\\ \tau_3&=-\frac{3}{2}R=\frac{3}{4}\tau_0.\end{aligned}\right\} \qquad (10)'$$

The deformation in I or II becomes plastic at this point, and this corresponds to the point C in Fig. 6(a).

The state in Fig. 7(c)-1 or (c)-2 cannot be a pure shear by itself, and perhaps these states coexist mixedly, and the shearing stresses will be

$$\left.\begin{aligned}\tau_1&=\frac{3}{4}R\pm\frac{3}{4}R,\\ \tau_2&=\frac{3}{4}R\mp\frac{3}{4}R,\\ \tau_3&=-\frac{3}{2}R.\end{aligned}\right\} \qquad (11)$$

There exist internal stresses, but macroscopically this state is a pure shear.

The state of stress becomes finally as shown in Fig. 7(d). This is equal to that before unloading, and at this point the material is plastic in all three directions, and plastic flow takes place hereafter.

(b) **Pure Shear～Internal Pressure**

This is the case where a test piece in the state shown in Fig. 4(b) is subjected to internal pressure, or to the shearing stress which will finally become as shown in Fig. 3.

At first the test piece will be subjected to S_3 as before, though the deformation in I is plastic from the first, and that in III will become also plastic at the following state,

$$\left.\begin{aligned} S_1&=0,\\ S_2&=0,\\ S_3&=-R,\end{aligned}\right\} \tag{12}$$

$$\left.\begin{aligned} \tau_1&=\frac{1}{2}R,\\ \tau_2&=\frac{1}{2}R,\\ \tau_3&=-R=\frac{1}{2}\tau_0.\end{aligned}\right\} \tag{12$'$}$$

The stress-strain relation, shown in Fig. 6(b), is linear at first, and begins to deviate at B where the stress τ_3 is

$$\tau_3=\frac{1}{2}\tau_0.$$

This corresponds to (12)$'$.

After this point the test piece will be subjected to S_1 and S_2, or the transition of elastic stress range must take place in direction I. The gentle slope after B in Fig. 6(b) is considered to correspond to this transition.

(c) **Tension～Internal Pressure**

This is the case where the test piece in the state shown in Fig. 4(c), the internal stress of $\pm\frac{1}{2}R$ remaining in direction III, is subjected to internal pressure.

The stress-strain relation is shown in Fig. 6(c). The special features of the curve are that the slope is very gentle compared with other cases, and moreover that the curve is concave upwards in the early stage. Though not clear in Fig. 6(c), it was ascertained from other experiments that the change of slope took place at $\tau=\frac{2}{7}\tau_0$.

The test piece will be subjected to S_3 at first, and the transition of elastic stress range in III will take place, holding the internal stress of $\pm\frac{1}{2}R$ as it is. This transition will end at $S_3=-\frac{1}{2}R$, or taking into account of the internal stress, at

$$S_3=-\frac{1}{2}R\pm\frac{1}{2}R,$$

or

$$\tau_3=-\frac{2}{7}\tau_0\pm\frac{1}{2}R.$$

It seems that the slope of the curve during this transition is very gentle, and perhaps this is the cause that the curve is concave upwards. The same phenomenon occurs when internal pressure is applied leaving the tensile load as it is instead of unloading; this problem will be discussed in detail in the next paper on the influence of intermediate principal stress on plastic flow.

(d) Compression~Internal Pressure

This is the case where the test piece in the state shown in Fig. 4(d) is subjected to internal pressure. The stress-strain relation is shown in Fig. 6(d).

The test piece will be subjected to S_3 at first, and the deformation in III will become plastic at the following state,

$$\left.\begin{array}{l} S_1=0, \\ S_2=0, \\ S_3=-R, \end{array}\right\} \tag{13}$$

$$\tau_3=-R=\frac{4}{7}\tau_0. \tag{13)'$$

As the internal stress remains in direction I, taking into account of this stress, (13)′ will be

$$\tau_3=-R\pm\frac{1}{4}R=\frac{3}{7}\tau_0\sim\frac{5}{7}\tau_0. \qquad (13)''$$

In Fig. 6(d) the curve begins to deviate from a linear relation at B, where the stress is $3/7\,\tau_0$.

The pressure being increased more, the test piece will be subjected to S_1 and S_2 mixedly, and the direction I will become plastic at the following state of stress.

$$\left.\begin{array}{l} S_1=\frac{1}{2}R, \\ S_2=0, \\ S_3=-R, \end{array}\right\} \text{ or } \left.\begin{array}{l} S_1=0, \\ S_2=\frac{1}{2}R, \\ S_3=-R, \end{array}\right\} \tag{14}$$

$$\tau_3=-\frac{5}{4}R=\frac{5}{7}\tau_0. \qquad (14)'$$

This point is not clear in Fig. 6(d), but is corresponds to the point, in Fig. 5(d), from where the curve begins to deviate from the linear relation.

Conclusion

It was explained in the authors' theory how the anisotropy due to plastic deformation was to be considered. Experiments agreed very well with the theory.

Reference

(1) F. NAKANISHI and Y. SATO: Trans. of Japan Soc. Mech. Eng., 24-147 (1958).

Influence of Intermediate Principal Stress on Plastic Flow*

By Fujio NAKANISHI and Yasuo SATO

Let σ_1, σ_2, and $\sigma_3 (\sigma_1 > \sigma_2 > \sigma_3)$ be the principal stresses, and $\sigma_3 = 0$. Taking σ_1 and σ_2 as the axes, the criterion of plastic flow is not expressed by a smooth curve by the authors' theory, but a curve composed of two straight lines which intersect at $\sigma_1 = 4\sigma_2$ in the range between tension and pure shear.

Experiments were carried out. The results show that there is an abrupt bent as expected according to the authors' theory.

A new theory of plasticity was advanced by the authors, and it was explained in the previous paper[(1)(2)] that not only the stress relation between plastic tension and plastic pure shear, but also the hysteresis loops, the anisotropy due to plastic deformation, and the relation between the yield point of mild steel and the plastic deformation beyond that point can be accounted for by this theory. Moreover, the strain ratio relation in plastic deformation will also be accounted for.

In this paper the investigation on the criterion of plastic flow in the range between tension and pure shear will be reported.

The theory of plasticity

Let, $\sigma_x, \sigma_y, \sigma_z$ be the principal stresses,

τ_1, τ_2, τ_3 be the principal shearing stresses, or

$$\left\{ \begin{aligned} \tau_1 &= \frac{1}{2}(\sigma_y - \sigma_z), \\ \tau_2 &= \frac{1}{2}(\sigma_z - \sigma_x), \\ \tau_3 &= \frac{1}{2}(\sigma_x - \sigma_y). \end{aligned} \right.$$

S_1, S_2, S_3 be the shearing stresses such as shown in Fig. 1; or

$$\left. \begin{aligned} \tau_1 &= S_1 - \frac{1}{2}S_2 - \frac{1}{2}S_3, \\ \tau_2 &= -\frac{1}{2}S_1 + S_2 - \frac{1}{2}S_3, \\ \tau_3 &= -\frac{1}{2}S_1 - \frac{1}{2}S_2 + S_3. \end{aligned} \right\} \quad (1)$$

$\varepsilon_x, \varepsilon_y, \varepsilon_z$ be the principal strains,

$\gamma_1, \gamma_2, \gamma_3$ be the shear strains, or

$$\left\{ \begin{aligned} \gamma_1 &= \varepsilon_y - \varepsilon_z, \\ \gamma_2 &= \varepsilon_z - \varepsilon_x, \\ \gamma_3 &= \varepsilon_x - \varepsilon_y. \end{aligned} \right.$$

I, II, III be the directions of shears; the directions of shear strains γ_1, γ_2, and γ_3 are here to be called the directions I, II, and III, then, the authors' theory of plasticity is as follows: —

* Received 8th July, 1964.

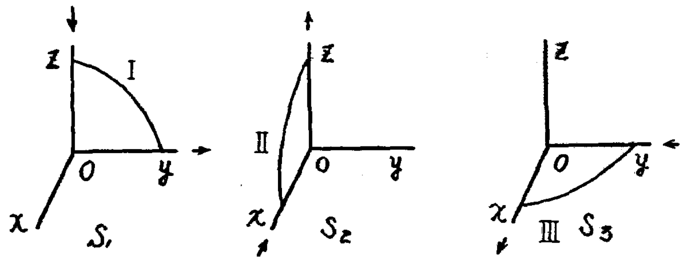

Fig. 1. Shearing stresses S_1, S_2, and S_3

1. The three shears in directions I, II, and III are to be considered individually.

Then the shearing resistances in the three directions must also be considered individually, and let them be R_1, R_2, and R_3. Now consider a deformation of an elemental sphere shown in Fig. 2; this is a pure shear in the direction II. It is generally thought that such a deformation is formed by a slip in II under the shearing stress τ_2, and in such a case τ_2 must be

$$\tau_2=\frac{1}{2}R_1+R_2+\frac{1}{2}R_3\,,$$

R_1, R_2, and R_3 are the resistances to the increase of shear, and they must correspond to the shearing stresses, but as seen in the above equation they do not correspond directly to τ_1, τ_2, and τ_3, but to S_1, S_2, and S_3.

2. When the strains are increasing monotonously, the magnitudes of R_1, R_2, and R_3 are equal, or

$$R_1=R_2=R_3=R\,,$$

provided that the directions of principal strains coincide with those of principal stresses.

3. When the direction of strain is reversed concerning each direction, the change is elastic, and the stress range of this elastic change is R.

Those explained in 1, 2, and 3 are the essence of the authors' theory of plasticity which is characterized by individual treatment of the shears in the three directions and so far as each direction is concerned, it represents a shearing stress theory.

Now the states of stress in thin hollow cylinders subjected to combined tension and internal pressure will be considered according to this theory, taking z-axis parallel to the axis of the cylinder, y-axis to the circumferential direction, and x-axis to the radial direction.

In the case of pure shear shown in Fig. 2, the strain II is of positive sign and those in I and III are negative sign, so the state of stress is[1]

$$\left.\begin{aligned} S_1&=-R\,,\\ S_2&=R\,,\\ S_3&=-R\,. \end{aligned}\right\} \tag{2}$$

Expressed in τ_i', (2) becomes

$$\left.\begin{aligned} \tau_1&=-R\,,\\ \tau_2&=2R\,,\\ \tau_3&=-R\,. \end{aligned}\right\} \tag{2'}$$

In the case of tension, the deformation will be as shown in Fig. 3, and the state of stress is[1]

$$\left.\begin{aligned} S_1&=-R\,,\\ S_2&=R\,,\\ S_3&=\pm\frac{1}{2}R\,. \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} \tau_1 &= -\frac{7}{4}R\,, \\ \tau_2 &= \frac{7}{4}R\,, \\ \tau_3 &= 0 \pm \frac{1}{2}R\,. \end{aligned}\right\} \qquad (3)'$$

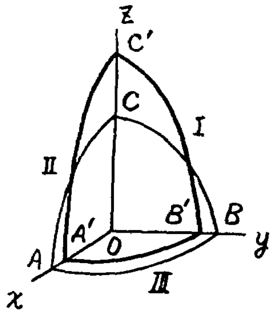

Fig. 2. Pure shear

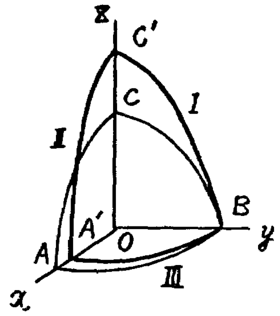

Fig. 3. Deformation under tension

It is considered that small slips in I and II take place alternately in tension. When a slip takes place in I the strain induced in III is of positive sign, but, when the slip is in II, it is of negative sign, and thus the shears of positive and negative sign are alternated in III. τ_1 and τ_2 must be equal in magnitude in tension, and as the stress range of elastic change is R, the value of S_3 should be $+R/2$ for the slip in I, and $-R/2$ for the slip in II. τ_3 is macroscopically zero, but there exists an internal stress of $\pm R/2$ in III.

Influence of intermediate principal stress

Let,

$$\left.\begin{aligned} \sigma_1 &= \sigma_z - \sigma_x\,, \\ \sigma_2 &= \sigma_y - \sigma_x\,, \\ \sigma_3 &= 0\,. \end{aligned}\right\} \qquad (4)$$

Considering that the hydrostatic compression or tension has no effect on plastic deformation, σ_1, σ_2, and σ_3 can be taken as principal stresses without losing generality.

In tension $\sigma_2 = 0$, and in pure shear $\sigma_2 = \sigma_1/2$. Here the states of stress in the plastic flow in the range between tension and pure shear, or in the range $0 < \sigma_2 < \sigma_1/2$, will be investigated. And if the states in this range are known, we can get easily the states in all the range.

The state of stress of plastic tension expressed by (3), will be diagrammatically shown as in Fig. 4 (a), and that of pure shear expressed by (2) will be shown as in Fig. 4 (e). Comparing these two, the difference lies in the value of S_3 in III. The value of S_3 varies as the intermediate principal stress σ_2 increases, and reaches $-R$ as shown in Fig. 4 (e) at $\sigma_2 = \sigma_1/2$. Thus the states of stress in the range between tension and pure shear should be as shown in Fig. 4 (b), (c), and (d).

From Fig. 4 (a) to (c), the elastic stress range of $\pm R/2$ moves as σ_2 increases, and the state of stress in this range can be given by

$$\left.\begin{aligned} S_1 &= -R\,, \\ S_2 &= R\,, \\ S_3 &= -\alpha R \pm \frac{1}{2}R\,, \end{aligned}\right\} \qquad (5)$$

where $$0<\alpha<\frac{1}{2}.$$

Or,

$$\left.\begin{aligned}\tau_1&=-\frac{7}{4}R+\frac{1}{2}\alpha R,\\ \tau_2&=\frac{7}{4}R+\frac{1}{2}\alpha R,\\ \tau_3&=-\alpha R\pm\frac{1}{2}R.\end{aligned}\right\} \quad (5)'$$

$\pm R/2$ in τ_3 is an internal stress, and when the stress induced by the external load is considered, this term can be neglected. On the other hand,

$$\left.\begin{aligned}\tau_1&=-\frac{1}{2}(\sigma_1-\sigma_2),\\ \tau_2&=\frac{1}{2}\sigma_2,\\ \tau_3&=-\frac{1}{2}\sigma_2.\end{aligned}\right\} \quad (6)$$

From Eqs. (5)′ and (6), we have

$$\left.\begin{aligned}\sigma_1&=\frac{7}{2}R+\alpha R,\\ \sigma_2&=2\alpha R.\end{aligned}\right\} \quad (7)$$

Or,

$$\sigma_1=\frac{7}{2}R+\frac{1}{2}\sigma_2. \quad (8)$$

This is the relation between σ_1 and σ_2 in the plastic flow. The straight line TQ in Fig. 5 shows this relation; the point T corresponds to tension or the stress state in Fig. 4 (a), and the point Q to the state in Fig. 4 (c).

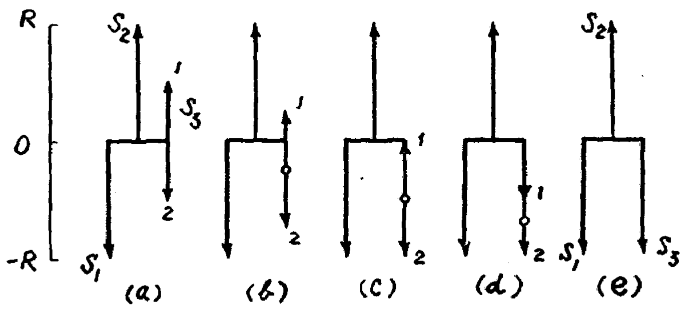

Fig. 4. Diagrammatic expressions of stress states in the range between tension and pure shear

In Fig. 4 (c) the value of S_3 for the slip in II is $-R$, and S_3 cannot decrease further. Meanwhile, the value of S_3 for the slip in I will continue to decrease, and in the state in Fig. 4 (e), or in pure shear, S_3 should become $-R$ for both slips. Thus, the states of stress between Fig. 4 (c) and (e) are

$$\left.\begin{aligned}S_1&=-R,\\ S_2&=R,\\ S_3&=-\alpha R\pm(1-\alpha)R,\end{aligned}\right\} \quad (9)$$

where $$\frac{1}{2}<\alpha<1\,.$$

Or,

$$\left.\begin{aligned}\tau_1&=-2R+\alpha R\,,\\ \tau_2&=2R\,,\\ \tau_3&=-\alpha R\pm(1-\alpha)R\,.\end{aligned}\right\}\qquad(9)'$$

From Eqs. (9)′ and (6), we have

$$\left.\begin{aligned}\sigma_1&=4R\,,\\ \sigma_2&=2\alpha R\,.\end{aligned}\right\}\qquad(10)$$

The horizontal line QP in Fig. 5 shows this relation.

Thus, the criterion of plastic flow is not expressed by a smooth curve by the authors' theory, but by a curve composed of two straight lines in the range between tension and pure shear.

By putting

$$\sigma_2/\sigma_1=n\,,$$

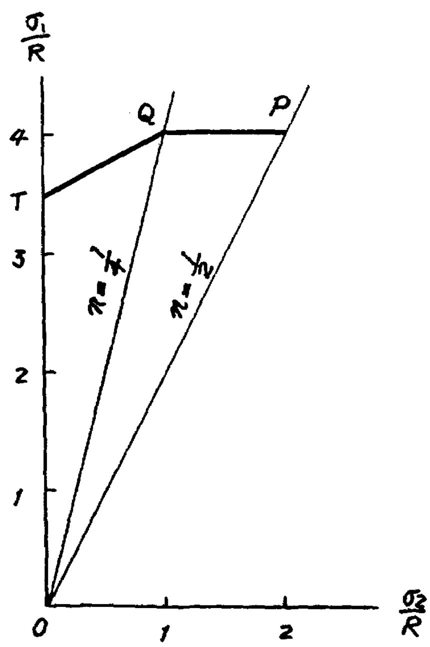

Fig. 5. Criterion of plastic flow

$n=0$ in tension, and in the range $0<n<1/4$ the criterion is expressed by a straight line of inclination equal to 1/2, and in the range $1/4<n<1/2$ it is expressed by a horizontal line, and there is an abrupt bent at $n=1/4$.

Experiment 1

It was already ascertained that the ratio of plastic stresses under tension and under pure shear was 7/8, comparing the states of stress where the values of R are equal(1). To investigate the relations in the range between tension and pure shear, experiments were carried out for various values of n.

Test pieces were hollow cylinders of 18 mm outer diameter and 16 mm inner diameter. The material was brass; the compositions were Cu 59.59%, Zn 38.36%, Pb 1.29%, Sn 0.47%, and Fe 0.29%. The experiments were carried out by applying various ratios of combined tension and internal pressure. The results are shown in Fig. 6; σ_1 is taken in the ordinate and longitudinal strain ε_z in the abscissa.

In Fig. 5 the stresses are expressed in the ratios to R. R increases with strain, and is considered to be a function of the amount of total slips, and so, in the range between tension and pure shear, the values of R will be equal when the values of ε_z are equal.

σ_1 in tension is equal to $7R/2$. Taking the ratios of σ_1 of various curves in Fig. 6 to that of tension at equal longitudinal strain, the values of σ_1 become as shown in Fig. 7. Comparing with Fig. 5, the theory and experiments are found to conform will each other very closely.

Test pieces were cut out from the same bar, but the material could not be considered to be perfectly uniform. And this experiment, though it conforms with theory very closely, will not be sufficient as the evidence of the existence of an abrupt bent at $n=1/4$. So we carried out another experiment with one test piece.

Experiment 2

A test piece of hollow cylinder was at first deformed under tension, and then an internal pressure was applied step by step to examine whether there was an abrupt bent at $n=1/4$ or not. The material and dimensions of the test piece were the same as before.

Fig. 8 shows the results of such an experiment.

In this case the testing machine used was of self-balance type. At first tension was

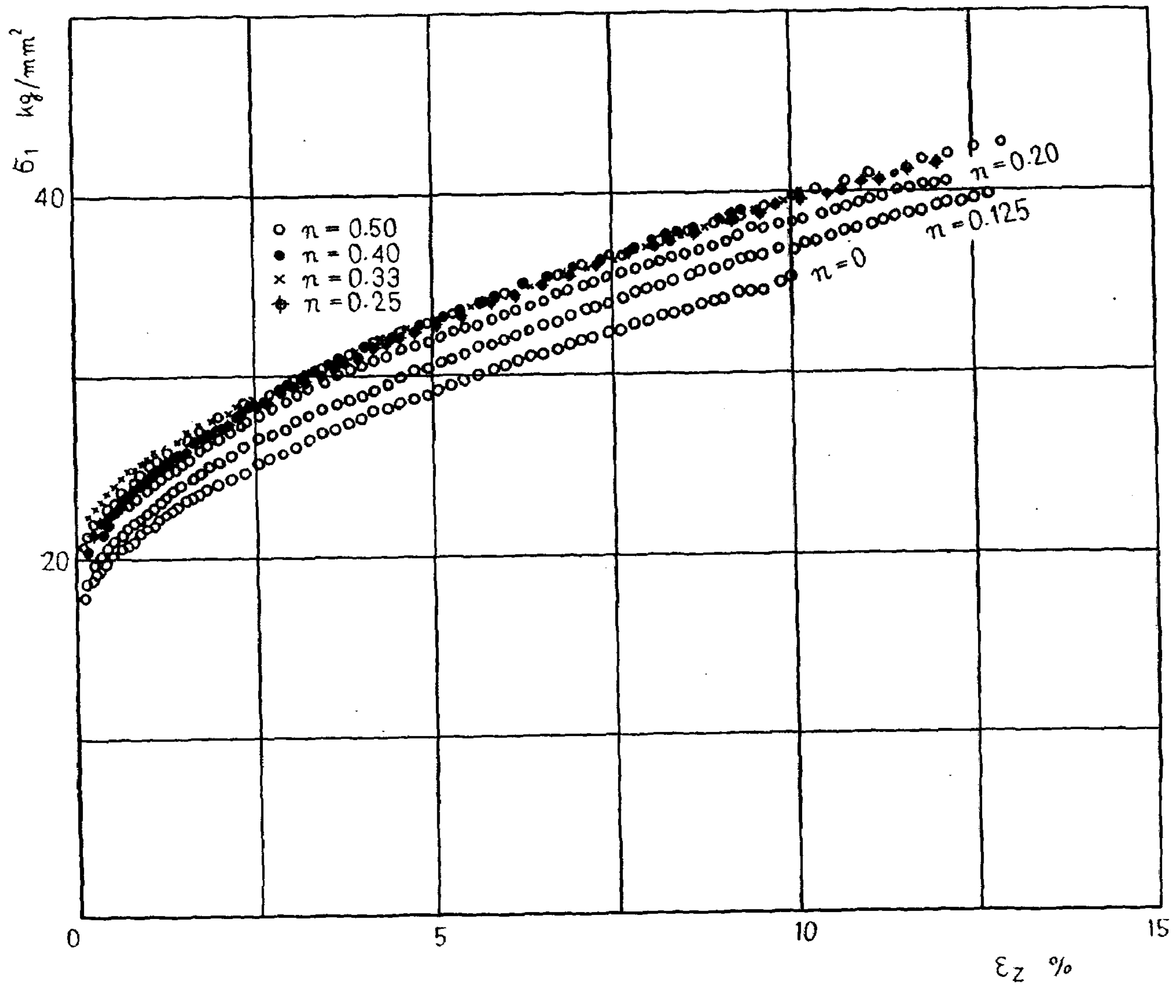

Fig. 6. Stress-strain relations for various values of n

applied until ε_z became ε_{z0}, and after that an internal pressure was applied, the testing machine being left as it was. It can be seen in the figure that there is an abrupt change at $n=1/4$, but the shape of the curve is quite different from that in Fig. 5. This is perhaps due to the fact that the method of measurement was not suitable to the purpose of this experiment. The reasons are:

1. The state of stress under internal pressure can be considered to be composed of a hydrostatic tension and a shear of negative sign in III. Elastic extension in all the direction will be induced by the hydrostatic tension, and plastic longitudinal strain will be induced by the shear in III, as the change in II is plastic and that in I is elastic. Thus the test piece elongates a little under the internal pressure, and as the testing machine is of self-balance type the tensile load decreases. Thus, what we measured in this experiment was the

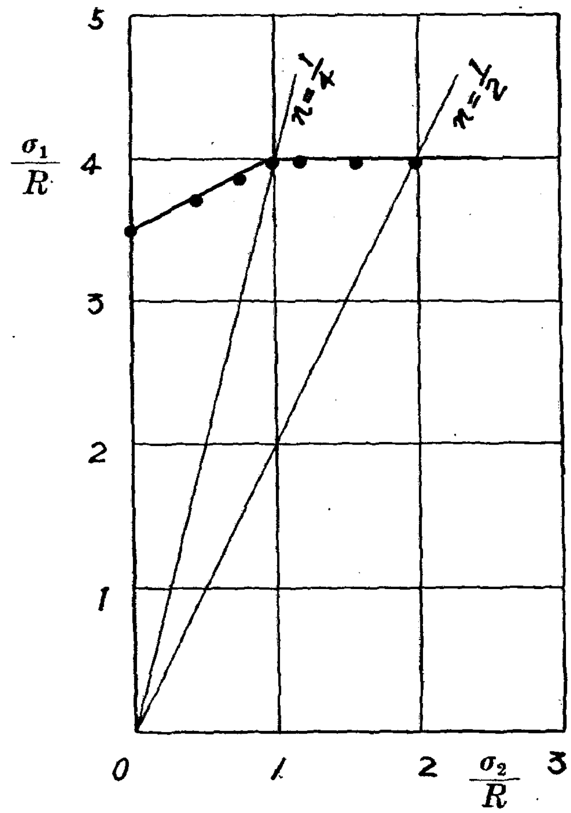

Fig. 7. Experimental results on the criterion of plastic flow

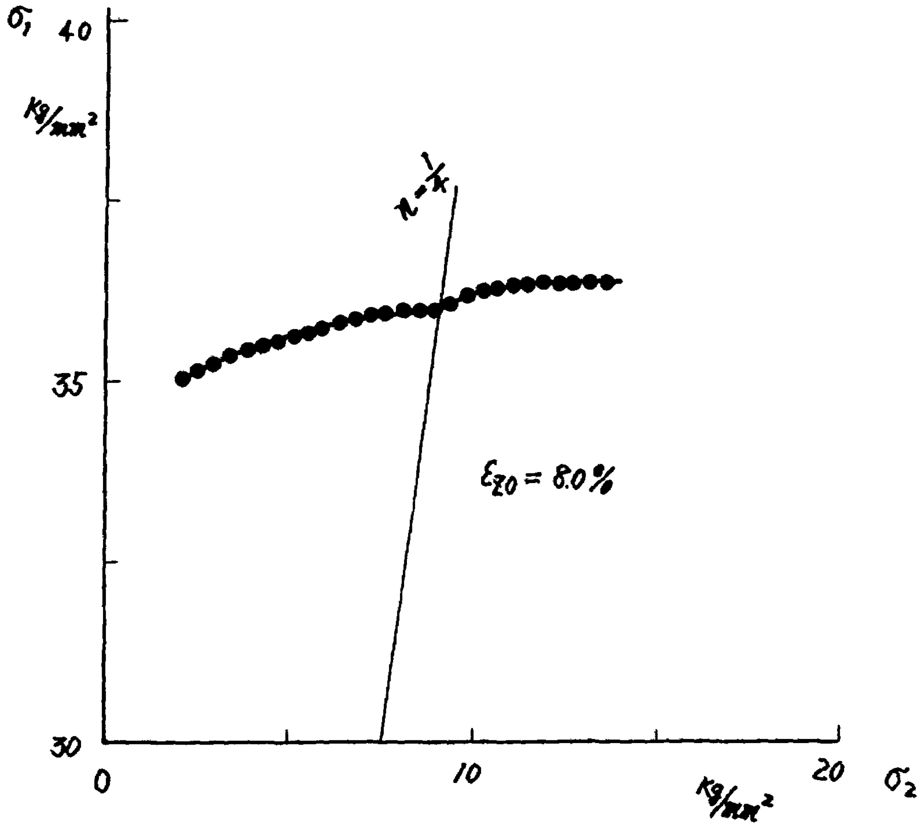

Fig. 8. Stress change due to internal pressure

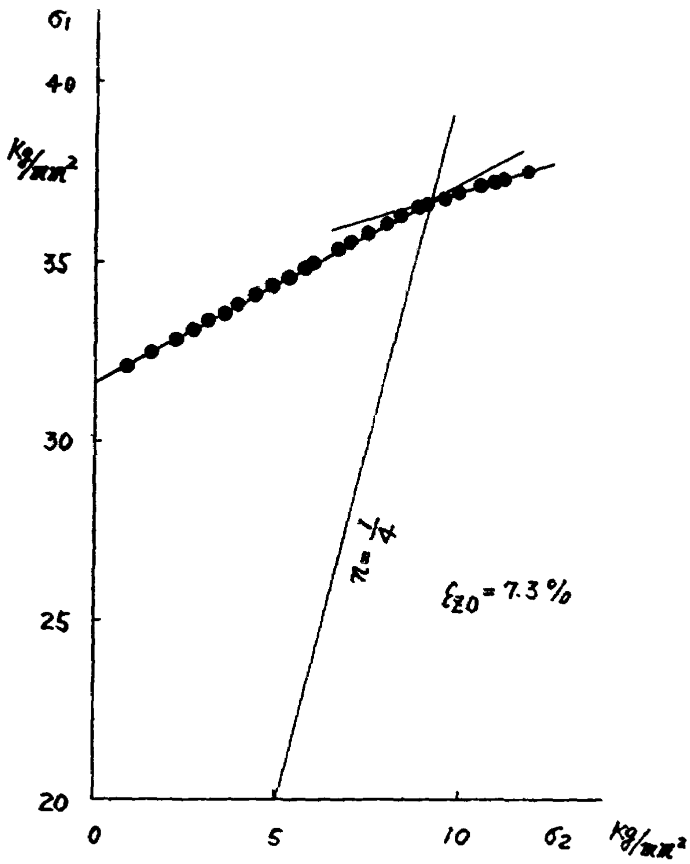

Fig. 9. Experimental results on the criterion of plastic flow

decrease of the tensile load due to the internal pressure.

2. As explained above, the change in II under the internal pressure is plastic, but that in I is elastic because the direction of shear is reversed in I, and the absolute value of strain in I decreases elastically. But, what we want to measure is the stresses in the state of plastic flow where the changes are plastic in all three directions.

To obtain the state of plastic flow where the change in I should be also plastic, it is necessary, after the application of internal pressure, to increase the tensile load until the beginning of plastic flow under tension.

Another experiment was carried out by this revised method. At first the internal pressure was increased a little, and then, the internal pressure being held constant, the tensile load was increased until the beginning of longitudinal plastic flow, and after the load settled, the reading of the tensile load was taken. This procedure was repeated.

Figs. 9 and 10 show the results obtained in such a way. In the experiment shown in Fig. 9, the test piece was at first deformed under tension until the strain ε_z became ε_{z0}, and then the internal pressure and tension were applied in such a way as mentioned above. During the measurement of the points shown in the figure the longitudinal strain increased about 2.1%; this strain is composed of the elongation due to the internal pressure, that due to the increase of tensile load and that due to the longitudinal plastic flow for measurements. Similarly in the case shown in Fig. 10 the increase of the longitudinal strain was 2.4%.

The results show clearly that there is an abrupt bent at $n=1/4$.

In Figs. 9 and 10, the ratio of σ_1 at $n=0$ to that at $n=1/4$ is not 7/8, and the values of σ_1 in the range $1/4<n<1/2$ are not constant. But this is quite natural as the value of R varies in these experiments. It is in terms of σ_1/R that the ratio at $n=0$ to that at $n=1/4$ should be 7/8, or constant in the range $1/4<n<1/2$. The purpose of these experiments is to examine whether there is an abrupt bent at $n=1/4$ or not.

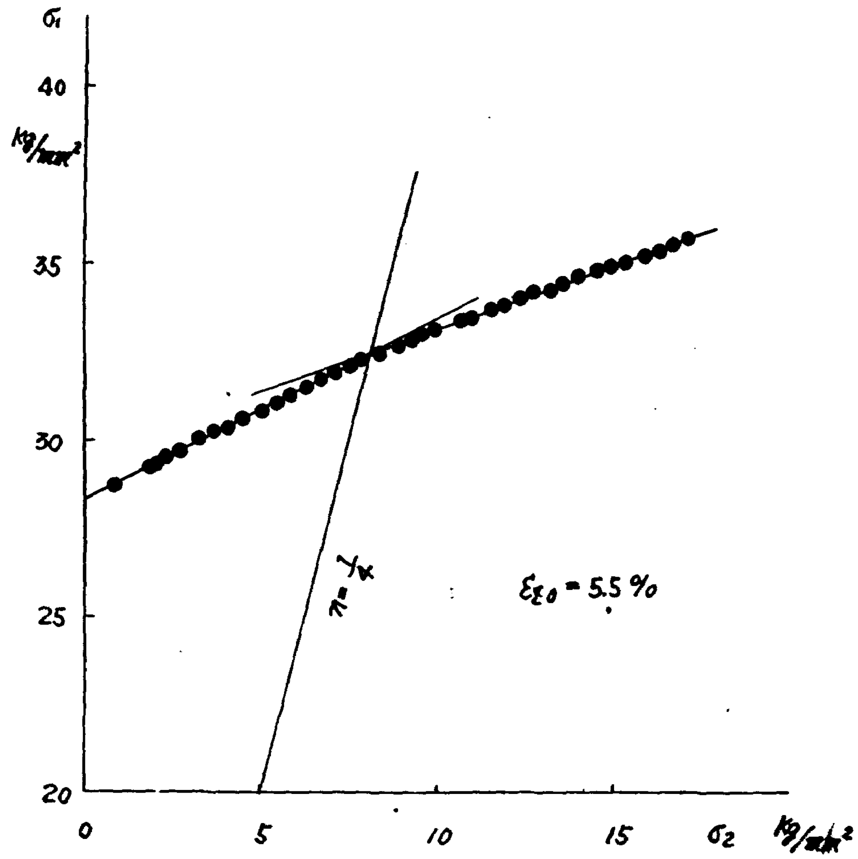

Fig. 10. Experimental results on the criterion of plastic flow

Conclusions

The criterion of plastic flow by the authors' theory is not expressed by a smooth curve, but, in the range between tension and pure shear, it is composed of two straight lines intersecting at $n=1/4$. Experiments show that there is an abrupt bent as expected according to the authors' theory.

As the criterion in the range between tension and pure shear is known, we can get easily the criterion in the whole range. It is shown in Fig. 11.

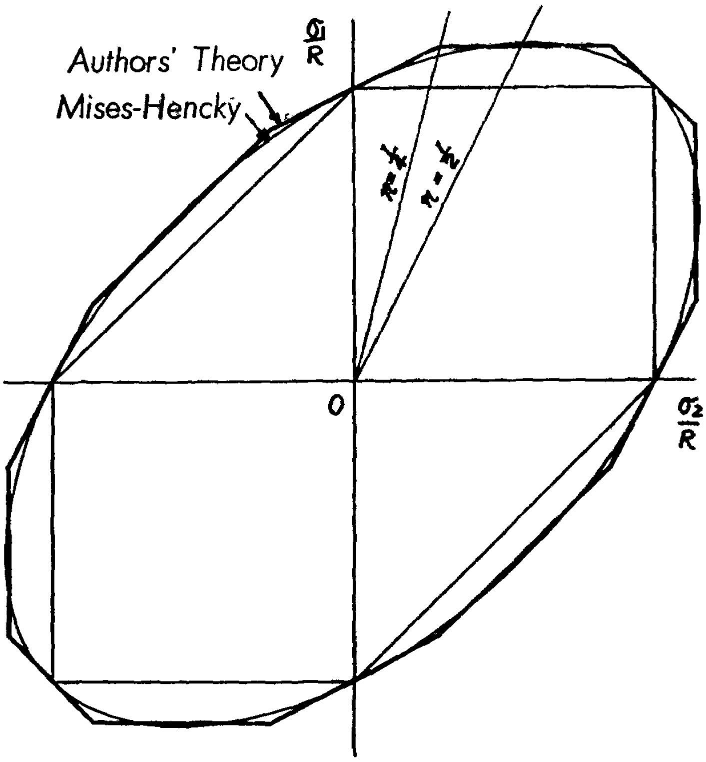

Fig. 11. Criterion of plastic flow

Mises-Hencky's theory is generally adopted as the theory of plasticity. It is rather used for the convenience of computation than for the strict correctness. Mises-Hencky's theory is also shown in Fig. 11, its point of tension being superposed upon that of the authors'. Authors consider of course that our theory is correct. Comparing with it, Mises-Hencky's theory is a very good approximation as shown in the figure, and it can be applied to the plastic flow without much error if we pay attention to the condition of application.

We wish to express our thanks to Mr. F. Nagai for his assistance in carrying out the experiments.

References

(1) F. Nakanishi and Y. Sato: *Bulletin of JSME*, Vol. 2, No. 6 (1959), p. 257.

(2) F. Nakanishi and Y. Sato: *Proc. 8th Japan Nat. Congr. Appl. Mech., 1958.* (1959) . 249.

Strain Ratio Relationship in Plastic Deformation

By Fujio NAKANISHI and Yasuo SATO

In plastic deformation, it is generally assumed that the shear strains γ_1, γ_3, and γ_2 are proportional to the shearing stresses τ_1, τ_2, and τ_3. But the experiments by Lode and also the experiments by Taylor and Quinney show that this assumption is not correct. The way of deformation such as shown by experiments, however, is not yet accounted for.

A new theory on plasticity has been advanced by the authors, and it was already explained that, not only the relation between plastic stresses under various states of stress, but also the forms of hysteresis curves, anisotropy due to plastic deformation, and the relation between the yielding of mild steel and the deformation beyond that could be well accounted for by this theory.

As for strain ratio relationship this theory also conforms very closely with the experiments.

Introduction

In what a shape a metal will be plastically deformed under a certain state of stress is one of the fundamental problems in plasticity, but it is not yet made clear. The plastic deformations under tension and under pure shear are known, but those under the intermediate states of stress between them are not known yet.

It has been generally assumed that the deformation of a metal is similar to that of a viscous mass. Now, let

x, y, z be the directions of principal stresses,

$\sigma_x, \sigma_y, \sigma_z$ be the principal stresses,

τ_1, τ_2, τ_3 be the principal shearing stresses, or

$$\left.\begin{aligned} &\tau_1=\frac{1}{2}(\sigma_y-\sigma_z), \quad \tau_2=\frac{1}{2}(\sigma_z-\sigma_x), \\ &\tau_3=\frac{1}{2}(\sigma_x-\sigma_y), \end{aligned}\right\}$$

and

$\varepsilon_x, \varepsilon_y, \varepsilon_z$ be the principal strains, or

$$\left.\begin{aligned} \gamma_1&=\varepsilon_y-\varepsilon_z, \\ \gamma_2&=\varepsilon_z-\varepsilon_x, \\ \gamma_3&=\varepsilon_x-\varepsilon_y, \end{aligned}\right\}$$

then, according to the above assumption, the platic strains will be proportional to the shearing stresses, or

$$\frac{\gamma_1}{\tau_1}=\frac{\gamma_2}{\tau_2}=\frac{\gamma_3}{\tau_3}. \tag{1}$$

Following the expression by Lode, we put

$$\left.\begin{aligned} \mu&=\frac{2\sigma_y-\sigma_z-\sigma_x}{\sigma_z-\sigma_x}=\frac{\tau_1-\tau_3}{\tau_2}, \\ \nu&=\frac{2\varepsilon_y-\varepsilon_z-\varepsilon_x}{\varepsilon_z-\varepsilon_x}=\frac{\gamma_1-\gamma_3}{\gamma_2}, \end{aligned}\right\} \tag{2}$$

* Received 8th July, 1964

μ denotes the state of stress and ν the strain ratio in plastic deformation. When the material is deformed under a pure shear, in which elongation takes place in the direction of z and contraction in x, μ and ν are both 0, and under tension in z-axis they are both -1. Under the intermediate states of stress between them, as long as Eq. (1) holds, the relation between μ and ν must be as follows:

$$\nu = \mu \qquad (1)'$$

Taking μ in abscissa and ν in ordinate, Eq. (1)′ is a straight line connecting the points of tension and pure shear as shown in Fig. 3.

On the relation between μ and ν, there are already experiments by W. Lode(1) and also by G. I. Taylor and H. Quinney(2), and these experimental data deviate clearly from the straight line $\nu=\mu$.

A new theory of plasticity, the "Three Shear Theory of Plasticity", has been advanced by the authors(3). By this theory, various experimental facts, for instance, the stress relation between plastic tension and plastic pure shear(3), the influence of the intermediate principal stress on plastic flow(4), and the anisotropy due to plastic deformation(5), can be accounted for.

In this paper, it will be explained that the strain ratio relationship can be also accounted for by the authors' theory.

Experiments

The authors also carried out experiments on the strain ratio relationship. As mentioned above, there are already experiments on this problem, but in these experiments, a certain load was applied to the test piece which had been already deformed under another kind of load. The authors' intention was to apply the same kind of load from the first. In Taylor-Quinney's experiments, the shear is simple shear under torsion. The relation between μ and ν may be similar whether the shear is simple shear or pure shear, but pure shear is more convenient for comparison with the authors' theory, and experiments were carried out under a combined stress of tension and pure shear.

Test pieces were hollow cylinders of 18 mm outer diameter and 16 mm inner diameter. The material was brass, its composition being Cu 59.59%, Zn 38.26%, Pb 1.29%, Sn 0.47%,

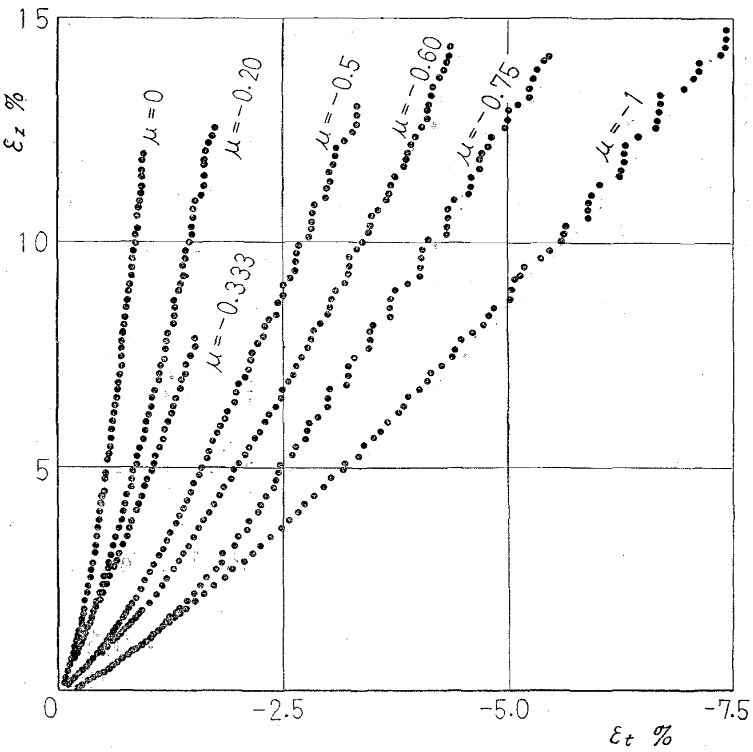

Fig. 1. Strain relations for various values of μ

and Fe 0.29%. The experiments were carried out by applying combined tension and internal pressure of various ratios, the ratios being such that the values of μ were always kept constant during the deformation.

Now, the deformation of an elemental part in the material will be considered, with z-axis taken paralled to the axis of the cylinder, y-axis in the circumferential direction, and x-axis in the radial direction. The measured quantities in the experiments were the axial strain ε_z and the circumferential strain of the outer diameter ε_t. The results are shown in Fig. 1.

The material used here cannot be called isotropic, because in the initial stage ε_t is large compared with ε_z as seen in Fig. 1. In the previous experiments[3][6] on the stress relation between plastic tension and plastic pure shear, we used a material which could be considered practically isotropic; we examined various materials and at last found out a bar of brass which could be considered to be practically isotropic, and test pieces cut out from that bar were used. After that, however, such an isotropic material could not be found out. The brass used in the present experiments is of the same composition as the previous one, but this was not isotropic.

So, the following computations were tried on the experimental relation between ε_t and ε_z of tension.

1. At first elastic strain, though very small compared with the plastic one, was subtracted, and the relation between the plastic axial strain $\varepsilon_z{}^p$ and the plastic strain of outer diameter $\varepsilon_t{}^p$ was obtained.

2. The plastic strain of the inner diameter was calculated assuming that the volume was constant during the deformation.

3. The mean values of the plastic strains of outer and inner diameter can be considered as the plastic circumferential strain $\varepsilon_y{}^p$, because in this case the wall of the cylinder is thin.

In the form of logarithmic strains, these calculated strains were plotted as shown in Fig. 2. If the material is isotropic, the slope of the curve should be 2:1. As seen in the figure, the slope of the curve is not equal to 2:1 in the early stage, but beyond the axial strain ε_z=5% the curve coincides very well with the straight line of slope 2:1. This fact shows that the material was not isotropic in the initial state, but it becomes gradually isotropic as the plastic deformation proceeds. The expression that the material becomes isotropic, however, will be liable to cause misunderstanding. The correct expression may be as follows. Beyond a certain point Q in the figure, the curve coincides with the curve of an ideal test piece of the dimension expressed by point P. So, if we measure the strain taking the point P or Q as the origin, the strain may be equal to that obtained with the test piece of isotropic material.

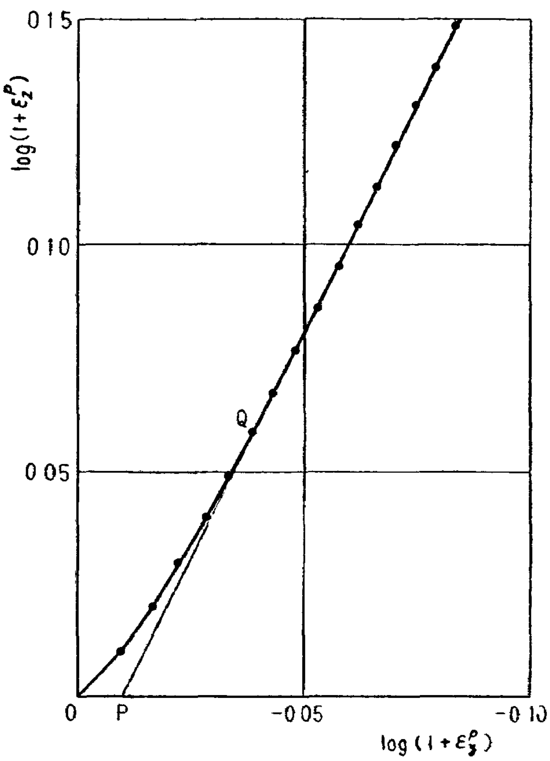

Fig. 2. Plastic strain relation for tension

Similar computations were carried out also on the other experimental results, and the values of ν were calculated respectively taking the point ε_z=6% as the origin. The values of ν thus calculated were plotted in Fig. 3. As seen in the figure, the data deviate clearly from the straight line $\nu=\mu$. And these experimental results conform closely with those of Lode and also with those of Taylor-Quinney, though the method of experiments is quite different.

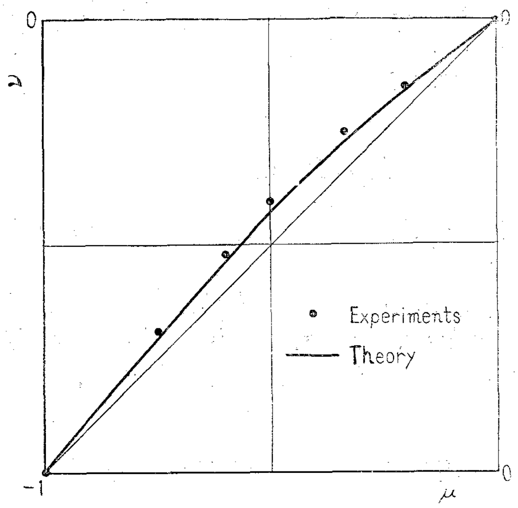

Fig. 3. Strain ratio relationship

Theory of plasticity

Let S_1, S_2, S_3 be the shearing stresses as shown in Fig. 4, and
I, II, III be the directions of shears; the directions of shear strains γ_1, γ_2, and γ_3 are to be called the directions, I, II, and III.

The authors' theory of plasticity[(3)] is as follows;

(i) The shears in the three directions I, II, and III are to be considered individually.

Then the resistances to the increase of the shear strains must also be considered independently concerning each direction. Now consider the deformation of an elemental sphere; Fig. 5 shows a pure shear in the direction II. It is generally thought that such a deformation takes place by the slip in II under the shearing stress τ_2, and in this case the resistance R_2 will of course have the direct effect. Moreover R_1 and R_3 will have also some effect, as there are strains both in I and III. Considering the strain energy or the components of R_1 and R_3 in II, the following relation must hold during the slip,

$$\tau_2=\frac{1}{2}R_1+R_2+\frac{1}{2}R_3, \tag{3}$$

R_1, R_2, and R_3 are the resistances to the increase of the shear strains, and they should correspond to the shearing stresses; as seen in the above equation they do not correspond directly to τ_1, τ_2, and τ_3, but the correspond to S_1, S_2, and S_3. The relations between τ_1, τ_2, τ_3 and S_1, S_2, S_3 are

$$\left.\begin{aligned} \tau_1&=S_1-\frac{1}{2}S_2-\frac{1}{2}S_3, \\ \tau_2&=-\frac{1}{2}S_1+S_2-\frac{1}{2}S_3, \\ \tau_3&=-\frac{1}{2}S_1-\frac{1}{2}S_2+S_3. \end{aligned}\right\} \tag{4}$$

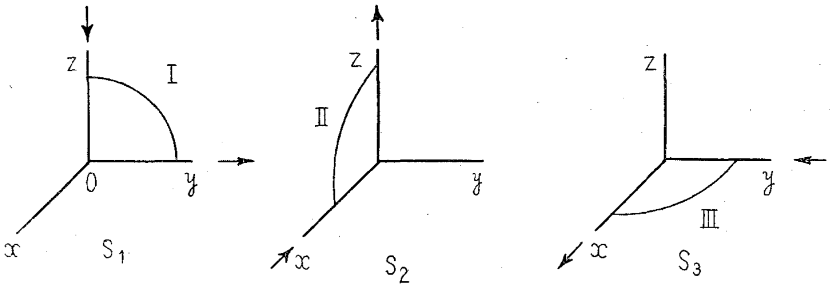

Fig. 4. Shearing stresses S_1, S_2, and S_3

Comparing Eqs. (3) and (4) taking into account the sign of the strain in each direction in Fig. 5, it will be seen clearly that R_s' corresponds directly to S_s'.

(ii) When the strains are increasing monotonously, the magnitudes of R_1, R_2, and R_3 are equal, or

$$R_1=R_2=R_3=R. \tag{5}$$

(iii) When the direction of the change of strain is reversed concerning each direction, the reversed change is elastic and the stress range of this elastic change is R.

Explained in (i), (ii), and (iii) is the authors' theory of plasticity; its characteristic is to consider three directions of shear individually. It is a shearing stress theory concerning each direction, but the shearing stress in this case is not τ but S.

Strain ratio relationship

1. Pure shear

The above theory will be applied to a pure shear where $\mu=0$. In this case the deformation will be as shown in Fig. 5; a part of an elemental sphere ABC in the material is deformed to $A'BC'$, and the sign of the strain in II is positive and those in I and III are negative. So the state of stress should be

$$\left.\begin{aligned} S_1&=-R,\\ S_2&=R,\\ S_3&=-R, \end{aligned}\right\} \tag{6}$$

or expressed by τ',

$$\left.\begin{aligned} \tau_1&=-R,\\ \tau_2&=2R,\\ \tau_3&=-R. \end{aligned}\right\} \tag{6$'$}$$

The state of stress is generally expressed in terms of σ_s' or τ_s', but by the authors' theory it is expressed in the form of Eq. (6), and Eq. (6)′ is obtained by putting Eq. (6) into Eq. (4).

The magnitudes of the three resistances and stresses are equal in Eq. (6). Under such a state of a state of stress, it is quite natural to consider that the increments of shears, ds_1, ds_2, and ds_3 in I, II, and III are all the same in magnitudes, or

$$\left.\begin{aligned} ds_1&=-ds,\\ ds_2&=ds,\\ ds_3&=-ds. \end{aligned}\right\} \tag{7}$$

By these increments of shears, or by the slips they may be called, it is clear that the elemental sphere is deformed as shown in Fig. 5.

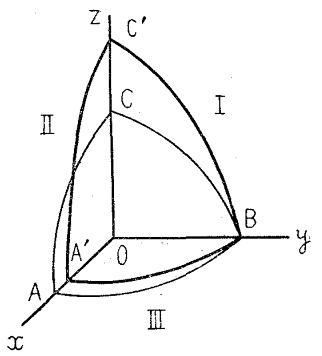

Fig. 5. Pure shear

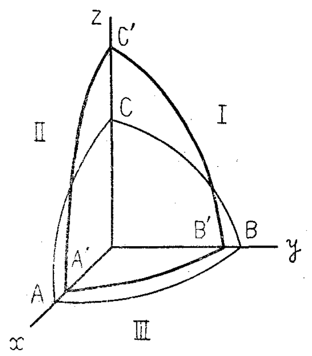

Fig. 6. Tension

2. Tension

In tension the deformation becomes as shown in Fig. 6, and the state of stress[3] is

$$\left.\begin{aligned} &S_1=-R, \qquad S_2=R, \\ &S_3=\begin{cases} +\dfrac{1}{2}R, & \text{for the slip in I}, \\ -\dfrac{1}{2}R, & \text{for the slip in II}. \end{cases} \end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned} \tau_1&=-\frac{7}{4}R, \\ \tau_2&=\frac{7}{4}R, \\ \tau_3&=\pm\frac{1}{2}R. \end{aligned}\right\} \tag{8$'$}$$

It is considered that in tension small slips in I and II take place alternately. When a slip takes place in I, the strain induced in III is positive, but when a slip takes place in II, it is negative. So the positive and negative deformations will be repeated alternately in III. As the positive and negative strains will be equal in magnitude of tension, and as the elastic stress range is R, the shearing stress S_3 must be as shown in Eq. (8).

Concerning a certain point in the material, small slips in I and II take place alternately. Concerning a certain moment, the slips in I take place in some parts in the material and those in II in other parts. Thus the stress in III is 0 macroscopically, but there exist internal stresses of $\pm R/2$.

The increments of strain or the slip in this case will be

$$\left.\begin{aligned} ds_1&=-ds, \\ ds_2&=ds, \\ ds_3&=\pm\frac{1}{2}ds. \end{aligned}\right\} \tag{9}$$

The macroscopic strain in III is 0, and the increments of strain, we measure or the macroscopic increments can be expressed by

$$\left.\begin{aligned} ds_1&=-ds, \\ ds_2&=ds, \\ ds_3&=0. \end{aligned}\right\} \tag{9$'$}$$

3. The intermediate state between tension and pure shear

Fig. 7 shows diagramatically the state of stress in this range. Fig. 7 (e) and (a) show respectively the state of stress of pure shear expressed by Eq. (6) and that of tension expressed by Eq. (8). Comparing these two, the difference lies only in III. Or S_1 and S_3 remain as they are, and only the value of S_3 changes as the state of stress changes from tension to pure shear. Taking into consideration that the elastic stress range is R, the state of stress[4] in this range should be as shown in Fig. 7 (b), (c), and (d).

From Fig. 7 (a) to (c), the elastic stress range of $\pm R/2$ moves downwards and the state of stress in this range can be given by

$$\left.\begin{aligned} S_1&=-R, \\ S_2&=R, \\ S_3&=-\alpha R\pm\frac{1}{2}R. \end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned} \tau_1 &= -\frac{7}{4}R+\frac{1}{2}\alpha R, \\ \tau_2 &= \frac{7}{4}R+\frac{1}{2}\alpha R, \\ \tau_3 &= -\alpha R \pm \frac{1}{2}R, \end{aligned}\right\} \tag{10$'$}$$

where

$$0<\alpha<\frac{1}{2}.$$

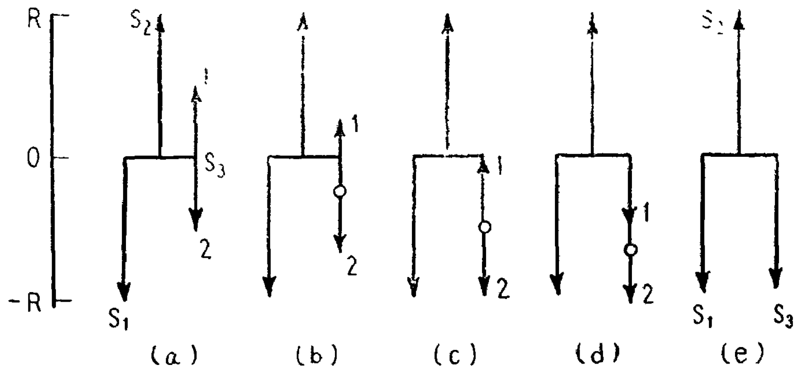

Fig. 7. Diagramatic expressions of states of stress in the range between tension and pure shear; (a) tension, (e) pure shear

Under the state of stress expressed by Eq. (10)′, the positive and negative deformations coexist in III; one is induced by the slip under $-\alpha R+R/2$, and the other is that under $-\alpha R-R/2$. And the macroscopic increments of strain will be expressed by

$$\left.\begin{aligned} ds_1 &= -ds, \\ ds_2 &= ds, \\ ds_3 &= -\alpha ds. \end{aligned}\right\} \tag{11}$$

The relation between s_i' and γ_i' is

$$\left.\begin{aligned} \gamma_1 &= s_1-\frac{1}{2}s_2-\frac{1}{2}s_3, \\ \gamma_2 &= -\frac{1}{2}s_1+s_2-\frac{1}{2}s_3, \\ \gamma_3 &= -\frac{1}{2}s_1-\frac{1}{2}s_2+s_3. \end{aligned}\right\} \tag{12}$$

From Eqs. (11) and (2), we have

$$\nu=-\frac{3(1-\alpha)}{3+\alpha}. \tag{13}$$

$\pm R/2$ in τ_3 expressed by Eq. (10)′ is an internal stress, and this term can be neglected when the stress induced by external load is considered. Then we have

$$\mu=-\frac{7-6\alpha}{7+2\alpha}. \tag{14}$$

The relation between ν and μ is

$$\nu=\frac{3+27\mu}{25+\mu}, \tag{15}$$

where

$$-1<\mu<-\frac{1}{2}. \tag{15}$$

In the range from Fig. 7 (c) to (e), the state of stress is

$$\left.\begin{aligned} S_1&=-R\,, \\ S_2&=R\,, \\ S_2&=-\alpha R\pm(1-\alpha)R\,. \end{aligned}\right\} \tag{16}$$

$$\left.\begin{aligned} \tau_1&=-2R+\alpha R\,, \\ \tau_2&=2R\,, \\ \tau_3&=-\alpha R\pm(1-\alpha)R\,, \end{aligned}\right\} \tag{16$'$}$$

where

$$\frac{1}{2}<\alpha<1\,.$$

The macroscopic strains will be expressed, as in the case of Eq. (11), by

$$\left.\begin{aligned} ds_1&=-ds\,, \\ ds_2&=ds\,, \\ ds_3&=-\alpha ds\,. \end{aligned}\right\} \tag{11$'$}$$

The value of ν, as in the case of Eq. (13), is

$$\nu=-\frac{3(1-\alpha)}{3+\alpha}. \tag{13$'$}$$

The value of μ can be obtained, as in the former case, neglecting the internal stress $+(1-\alpha)R$ in τ_3, or

$$\mu=-(1-\alpha)\,. \tag{17}$$

The relation between ν and μ is

$$\nu=\frac{3\mu}{4+\mu}\,, \tag{18}$$

where $$-\frac{1}{2}<\mu<0\,.$$

The relations between ν and μ expressed by Eqs. (15) and (18) are shown in Fig. 3. The theoretical relation conforms very closely with the experiments as seen in the figure.

Conclusions

It is generally assumed that the strain ratio relationship is expressed by $\nu=\mu$, but it is already known that this assumption does not agree with experiments. By the authors' theory it can be accounted for that the strain ratio relationship cannot be expressed by $\nu=\mu$, and moreover the theory agrees very well with experiments.

Assumption that there exists an internal stress expressed by Eq. (10) or Eq. (16), except the case of pure shear is a characteristic of the authors' theory. So when the material is of such a property that it cannot support the internal stress, it is beyond the limits of applicability of the theory. In such a case, it is the same whether we consider ds_i is proportional to S_i or $d\gamma_i$ is proportional to τ_i, all the same the relation $\nu=\mu$ will hold. In Taylor-Quinney's experiments, the results of the amorphous heated glass lie on the line $\nu=\mu$; it is quite natural. The results of cadmium and lead lie near the line $\nu=\mu$; these materials have the property to creep even at the room temperature and it is considered that they cannot support the internal stress sufficiently. The curves for metals such as aluminium, copper, nickel and mild steel conform well with the theory.

The authors would like to express their thanks to Mr. F. Nagai for his assistance in carrying out the experiments.

References

(1) W. Lode: *Mitt. u. Forschungarb.*, Ht. 303 (1928); A. Nadai: *Theory of Flow and Fracture of Solid*, (1950), McGraw Hill.

(2) G. I. Taylor and H. Quinney: *Trans. Roy. Soc. London*, Sec. A, Vol. 230, (1931).

(3) F. Nakanishi and Y. Sato: *Bulletin of JSME*, Vol. 2, No. 6, (1959), p. 257.

(4) F. Nakanishi and Y. Sato: *Bulletin of JSME*, Vol. 3, No. 9 (1960), p. 54.

(5) F. Nakanishi and Y. Sato: *Proc. 8th Japan Nat. Congr. Appl. Mech. 1958*, (1959), p. 249.

(6) Y. Sato: *Bulletin of JSME*, Vol. 2, No. 5, (1959), p. 107.

Internal Shearing Resistances in the Three Shear Theory of Plasticity*

By Fujio NAKANISHI

Introduction

A new theory of plasticity, the Three Shear Theory, has been advanced by the author(1). This theory conforms very closely with experiments on the stress-relation between plastic tension and plastic pure shear(1), the influence of intermediate principal stress on the plastic flow(3), hysteresis loops(1)(3), and anisotropy due to plastic deformation(2). It is already known that the yielding of mild steel takes place under a constant shearing stress when the stress distribution is uniform, and this fact can be also accounted for by this theory.

Let

$\sigma_x, \sigma_y, \sigma_z$ be the principal stresses,

τ_1, τ_2, τ_3 be the principal shearing stresses, or

$$\begin{cases} \tau_1=\frac{1}{2}(\sigma_y-\sigma_z), \\ \tau_2=\frac{1}{2}(\sigma_z-\sigma_x), \\ \tau_3=\frac{1}{2}(\sigma_x-\sigma_y). \end{cases}$$

$$\begin{cases} \gamma_1=\varepsilon_y-\varepsilon_z, \\ \gamma_2=\varepsilon_z-\varepsilon_x, \\ \gamma_3=\varepsilon_x-\varepsilon_y. \end{cases}$$

I, II, III be the directions of shears; the directions of shear strains γ_1, γ_2, and γ_3 are to be called the directions I, II, and III,

S_1, S_2, S_3 be the shearing stresses such as shown in Fig. 1,

R_1, R_2, R_3 be the shearing resistances in directions I, II, and III, and

s_1, s_2, s_3 be the slips in directions I, II, and III,

then, the theory is as follows:—

(i) The shears in directions I, II, and III are to be considered individually.

(ii) The shearing resistances R_1, R_2, and R_3 are equal in magnitude, or

$$R_1=R_2=R_3=R, \tag{1}$$

provided that the directions of principal strains coincide with those of the principal stresses.

(iii) When the direction of shear is reversed, the change is elastic, and the stress range of this elastic change is R concerning to each direction.

(iv) R_1, R_2, and R_3 correspond to shearing stresses S_1, S_2, and S_3.

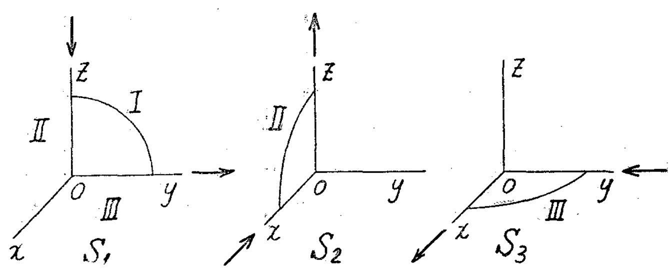

Fig. 1. Shearing Stresses, S_1, S_2, and S_3

The characteristic of this theory is that the shears in the three directions are to be considered individually, and it is a shearing stress theory concerning to each direction; the shearing stress in this case is not τ but S.

(i) was deduced from the results of experiments on the anisotropy due to plastic deformation, especially by comparing the axial elongations under internal pressure of thin hollow cylinders which had been previously subjected to various kinds of stresses.

(ii) can be also deduced from the results of experiments on the anisotropy due to plastic deformation. But, here, apart from experiments, we will study how the work hardening in each direction should be induced by slips, and how R_1, R_2, and R_3 should become equal.

Plastic Deformation and Work Hardening

1. Deformation due to Slips in One Direction

When a slip occurs in the direction I, elongation will take place in the direction of y-axis, and contraction in z-axis, and a part ABC of the elemental sphere in the material will become $AB'C'$ as shown in Fig. 2. This is a pure shear in the direction I.

The author considers that slips in the three directions generally take place in plastic deformation, but in the case shown in Fig. 2 the deformation can be formed by a slip in one direction. Assuming that this deformation was formed by a slip in one direction, the work hardening in this case will be considered.

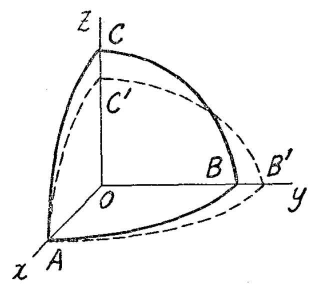

Fig. 2. Pure shear in the direction I

It will be very simple if the work hardening in each direction is a function of the strain in that direction, but this is not the case.

Here, it is assumed that a slip in a certain direction for instance $ds_1=ds$ induces the work hardening in the same direction $dR_1=dR$. It is not known, in this case, whether the work hardenings dR_2 and dR_3 may take place or not. Moreover, though they are assumed to take place, their amounts are not known. So we put the work hardenings in the three directions in this case as follows:

$$\left.\begin{aligned} dR_1&=dR\,, \\ dR_2&=\alpha dR\,, \\ dR_3&=\alpha dR\,. \end{aligned}\right\} \qquad (2)$$

2. Deformation due to Slips in Two Directions

Here we will consider a case where a slip $ds_2=-ds$ in the direction II and another slip $ds_3=-ds$ in the direction III take place. The deformation due to the slip $ds_2=-ds$ is shown in Fig. 3(a), and that due to $ds_3=-ds$ in Fig. 3(b). The combined deformation becomes as shown in Fig. 3(c), and this is the same as that shown in Fig. 2. Or, as for the deformation, the alternate slips $ds_2=-ds$ in II and $ds_3=-ds$ in III are equal to the slip $ds_1=ds$ in the direction I.

When the slips in directions II and III take place alternately, the strain γ_1 in the direction I increases monotonously, but γ_2 and γ_3 do not. Fig. 4 shows the relations between R_2, R_3 and γ_2, γ_3 when slips take place alternately in II and III.

Now suppose that internal resistances and strains are R_{20}, γ_{20} in II and R_{30}, γ_{30} in III. When the slip $ds_2=-ds$ takes place in II, the resistance R_2 shifts from R_{20} to R_{21}. In this case, as shown in Fig. 3(a), $d\gamma_3$ is equal to $-\frac{1}{2}d\gamma_2$, and so R_3 shifts from R_{30} to R_{31}. Similarly, by the slip $ds_3=-ds$, R_3 shifs from R_{31} to R_{32}, and at the same time R_2 shifs from R_{21} to R_{22}.

The change from R_{30} to R_{31} is elastic as the direction of strain in III is reversed at γ_{30}.

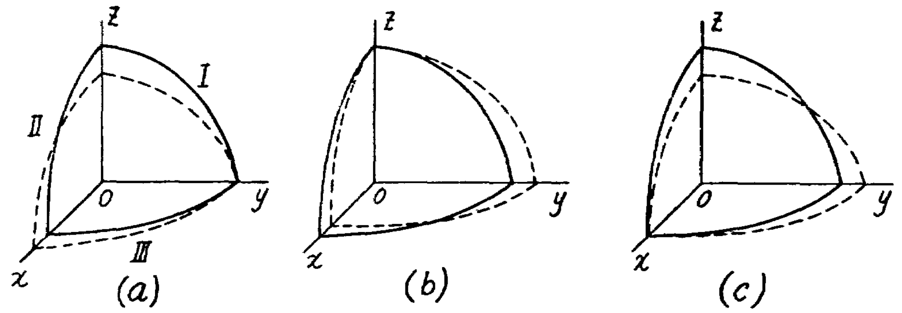

Fig. 3. (a) Deformation due to $ds_2=-ds$,
(b) Deformation due to $ds_3=-ds$,
(c) Combination of (a) and (b).

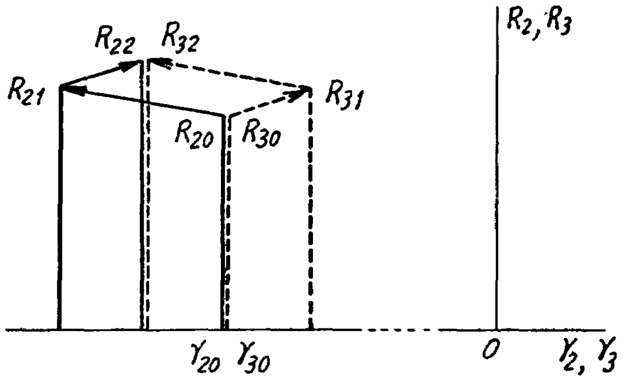

Fig. 4. R_2, R_3 and γ_2, γ_3 when slip take place alternately in II and III

Next, the fore-half of the change from R_{31} to R_{32} in III is also elastic as the direction of strain is once more reversed, and only the change beyond γ_{30} or the remaining half is plastic. Thus only one half of the slip $ds_3=-ds$ in the direction III is plastic. Similarly, a half of the slip $ds_2=-ds$ in the direction II is elastic and the remaining half is plastic.

Work hardening is induced only by plastic changes, and the plastic components ds_2^p and ds_3^p of the slips $ds_2=-ds$ and $ds_3=-ds$ are

$$\left\{\begin{aligned} ds_2^p&=-\frac{1}{2}ds, \\ ds_3^p&=-\frac{1}{2}ds. \end{aligned}\right.$$

The work hardenings due to these plastic components of slips should be similar to (2), or

$$\left.\begin{aligned} dR_1&=\frac{1}{2}\alpha dR+\frac{1}{2}\alpha dR=\alpha dR, \\ dR_2&=\frac{1}{2}dR+\frac{1}{2}\alpha dR=\frac{1}{2}(1+\alpha)dR, \\ dR_3&=\frac{1}{2}\alpha dR+\frac{1}{2}dR=\frac{1}{2}(1+\alpha)dR. \end{aligned}\right\} \tag{3}$$

3. Work Hardening in the Three Directions

Comparing the above two cases, the increments of strains are the same, and so the work hardenings are also to be considered to be the same. The strains in the three directions increase monotonously in one case, and not monotonously in the other case. But the parts of the changes in which they are not monotonous are elastic, and concerning to the plastic changes the strains increase monotonously. The work hardening is induced only by plastic changes, and is not affected by elastic ones, so the work hardenings in the above two cases should be considered to be the same.

Then, comparing Eqs. (2) and (3), we have

$$\alpha = 1 . \tag{4}$$

And Eq. (2) becomes

$$dR_1 = dR_2 = dR_3 = dR . \tag{5}$$

This means that a slip in one direction must have the same effects on the work hardenings in the three directions.

As the Eq. (5) holds for slips in any direction, the following equation generally holds when the plastic deformation increases monotonously.

$$R_1 = R_2 = R_3 = R . \tag{1}$$

Work Hardening and Amount of Plastic Slips

As the Eq. (5) holds for slips in any direction, the Eq. (1) holds generally, and at the same time the value of R should be a function of the amount of plastic slips.

The author's theory was compared with experiments in the previous paper[1], assuming that the value of R was a function of the amount of slips. Now it was shown that the assumption was right.

The author considers that the slips in the three direction generally take place in plastic deformation, and these slips should be taken into account when the strain-ratio-relations are discussed. But, when the amount of slips only is to be considered as in the case of work hardening, it is not necessary to take into account the slips in the three directions; it may be considered that the deformation is formed by slips in one direction in the pure shear, and generally by slips in two directions.

Conclusions

The conclusions are as follows:—

1. The internal resistances in the three directions are equal in magnitude.
2. The internal resistance is a function of the amount of slips.

Eq. (1) was deduced from the fact that the deformation shown in Fig. 2 is the same as that in Fig. 3(c). This fact only holds when the directions of principal strains coincide with those of principal stresses, or in the case of pure shear, so (1) is the relation in such a case. Another relation may hold in the case of simple shear like torsion.

References

(1) F. Nakanishi and Y. Sato: Trans. of Japan Soc. Mech. Eng., 24-147, (1958).
(2) F. Nakanishi and Y. Sato: 8th Japan Nat. Congr. App. Mech., (1958).
(3) F, Nakanishi and Y. Sato: Trans. of Japan Soc. Mech. Eng., 25-155, (1959).

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Anisotropy due to Plastic Deformation
Influence of Intermediate Principal Stress on Plastic Flow
Strain Ratio Relationship in Plastic Deformation
Internal Shearing Resistances in the Three Shear Theory of Plasticity

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