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TR-90T

The Shape of Mechanical Hysteresis Loop, Its Deformation due to Stress Repetition and Resulting Increase in Flow Stress

Part 1. Experiment

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Part 2. Theory for Torsion

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CHŌFU, TOKYO, JAPAN

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The Shape of Mechanical Hysteresis Loop, Its Deformation due to Stress Repetition and Resulting Increase in Flow Stress*

(Part 1. Experiment)

By Fujio NAKANISHI**, Yasuo SATO*** and Fumio NAGAI***

Summarys

Experimental results of the hysteresis loops during repeated torsion, pure shear and tension are presented in this paper.

The following observations are made:—

1. The stress-strain relations in reloading and unloading are symmetric.
2. The hysteresis loop is composed of several sections. For instance, the reloading or unloading curve in torsion has three straight sections and shows bends at stresses of one third and two thirds of the flow stress, Figs. 1 and 2.
3. The beginning section of reloading curve is elastic while the slopes of the other lines vary as the stress is repeated thus resulting in a change in the shape of the loop.
4. The stress repetition produces an increase in the stress for the commencement of plastic flow, but this effect vanishes as the plastic flow is continued, see Figs. 4 and 11.

1. Introduction

A new theory of plasticity has been advanced by two of the authors. This theory accounts well for the experimental results not only on the plastic stress relation between tension and pure shear[1] but also on the strain ratio relationship in the plastic deformation[2] which has not yet been explained by other theories. If this theory is developed to some extent, it may be possible to explain the shape of hysteresis loop in the stress-strain relation and its deformation due to the repetition of cyclic loading. It is well known that the stress of plastic flow rises a few percents during the repetition of cyclic loading. The reason of the rise in the stress of plastic flow observed after repetition may be explained and the magnitude of the stress may also be predicted.

For this purpose, it seems necessary at present to accumulate accurate experimental findings. In this paper, the experimental work on the hysteresis loop in elasto-plastic stress-strain relation is reported. Emphasis is put on i) shapes of the hysteresis loops under torsion, pure shear and tension, ii) deformation of the hysteresis loops and rise of plastic flow stress caused by repeating the stress cycles.

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Selection of Material

In the experimental study of plasticity, it is important to obtain materials sufficiently isotropic in the virgin state, though such a material is very rare. Previously we examined bars of several materials with respect to their isotropy and found out a bar of brass which could be considered practically isotropic in the virgin state[3]. Using that brass, experiments were carried out on the relation between simple shear and pure shear[4], on the stress relation between tension and pure shear in plastic range[1], and on the anisotropy due to plastic deformation[5]. The components of that brass were Cu 59.59%, Zn 38.36%, Pb 1.29%, Sn 0.47% and Fe 0.29%. Thereafter, an isotropic material like that brass bar has not been obtained. However, bars of brass, of which components are the same to the previous one, are not so anisotropic as that of mild steel or aluminium, and their plastic stress relation between tension and pure shear agreed well with that of the previous brass bar. Therefore, we have always used the bars of brass of the same components in the investigation of the plasticity; for example, in the experiments on the influence of intermediate principal stress on plastic flow[6], and on the strain ratio relationship in plastic deformation[2]. Also in the present experiments on the hysteresis loops, the brass of the same components were used.

Methods of Experiments

The specimens were machined from bars of brass into the form of solid cylinder of 12 mm in diameter for tension test, and hollow cylinder of 18 mm in outer diameter and 16 mm in inner diameter for torsion and pure shear tests.

Hysteresis loop was obtained by removing and reapplying load between the shearing stresses τ_0 and 0 after the specimen was deformed up to the plastic state, $\tau=\tau_0$. The testing machines used in those experiments were of ordinary self-balancing type. With the intention to obtain the hysteresis loop in which no effect of strain speed is contained, the experiments were carried out in the following way. The specimen was deformed a little under moderate speed and then the operation of the machine was ceased for a while. During this period, the strain of specimen still continued to increase while the load decreased, since in this type of machine the decrease in load is caused by the increase in the strain of specimen. Eventually the deformation of specimen stops and the equilibrium condition between load and strain is attained. The values of load and strain under this condition were taken as the point on the loop. Then, the specimen was deformed a little again and the previous procedures were repeated successively to make up the complete hysteresis loop. In the present experiments, 1~2 minutes were needed to take the data at one point and it took 1.5~2 hours to make up the complete hysteresis loop. It may be considered that the hysteresis loop obtained in this way is free from the effect of the strain speed.

Torsion

Fig. 1 shows the hysteresis loop in torsion. This loop was obtained by unloading and reloading the hollow cylinder specimen with twisting moment between stresses τ_0 and 0, after the virgin specimen was deformed under torsion up to the plastic state A, where stress $\tau=\tau_0$ and strain $\gamma=\gamma_0$. In this figure, the twisting moment M is taken

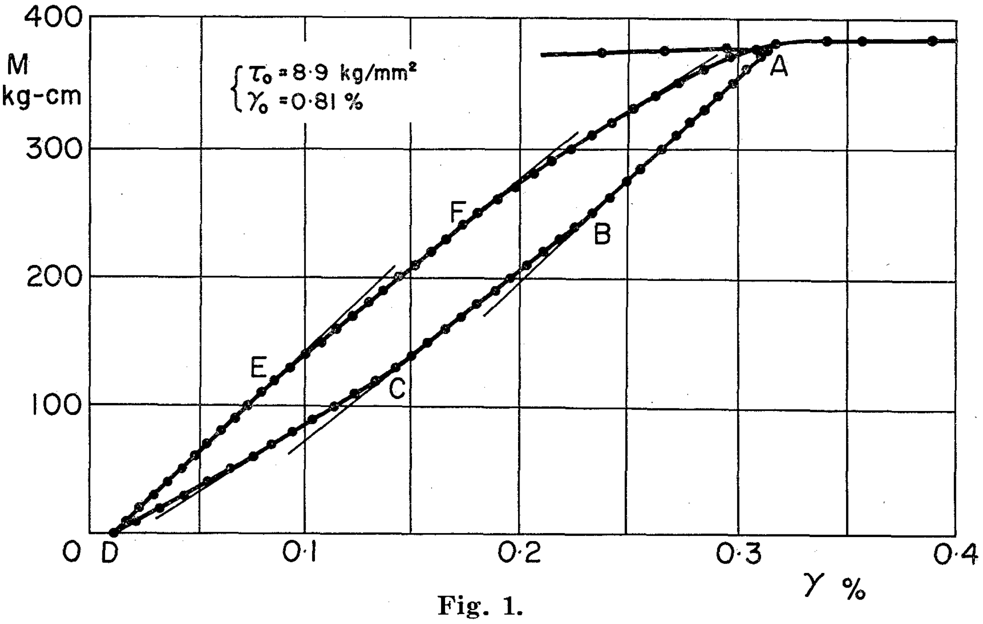

Fig. 1.

in ordinate and the strain γ in abscissa. The M-γ relations of the loop bend at points B, C, E and F. Hence, the curves under unloading and reloading consist of 3 portions respectively and as shown in Fig. 1 the shapes of the loop have point symmetry with respect to its centre. The magnitudes of stresses at point C and E are $1/3\cdot\tau_0$, and B and F are $2/3\cdot\tau_0$. The 1st portion shown by AB or DE is a straight line and the strain in this portion is elastic. The 2nd portion shown by BC or EF is also a straight line. The 3rd portion looks like a straight line in its beginning part but turns into a curve in its end.

The hysteresis loops shown in Fig. 2 are obtained after repeating the cyclic unloading and reloading, where n denotes the number of repetition. From this experiment, it is ascertained that the main features of the hysteresis loop are not changed by repeating the cycle; the symmetry in the shape of the loop is preserved, either of the unloading and the reloading curves bends at the stresses $1/3\cdot\tau_0$ and $2/3\cdot\tau_0$ and hence is still composed of 3 portions, and the 1st portion is the elastic line and the 2nd portion is a straight line. However, one exception is found in the 3rd portion; the curved part in this portion vanishes as the number of repetition is increased and eventually the portion becomes completely straight.

In order to study the deformation of the hysteresis loop due to repetition, the slopes of the straight lines in the 2nd and the 3rd portions are plotted in Fig. 3. In this figure, λ_2 and λ_3 are taken in ordinate and the number of repetition, n in abscissa,

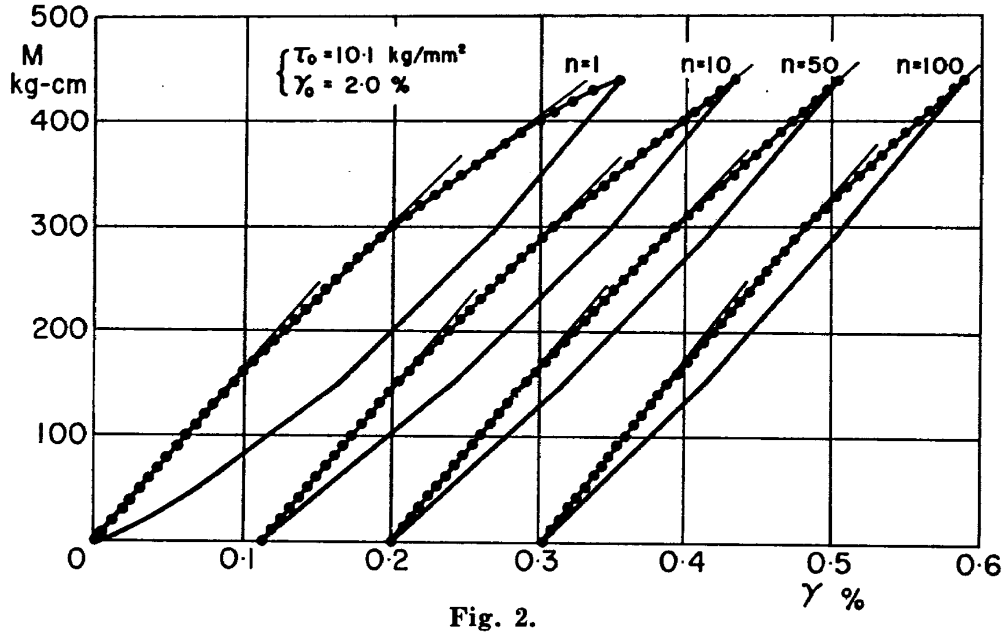

Fig. 2.

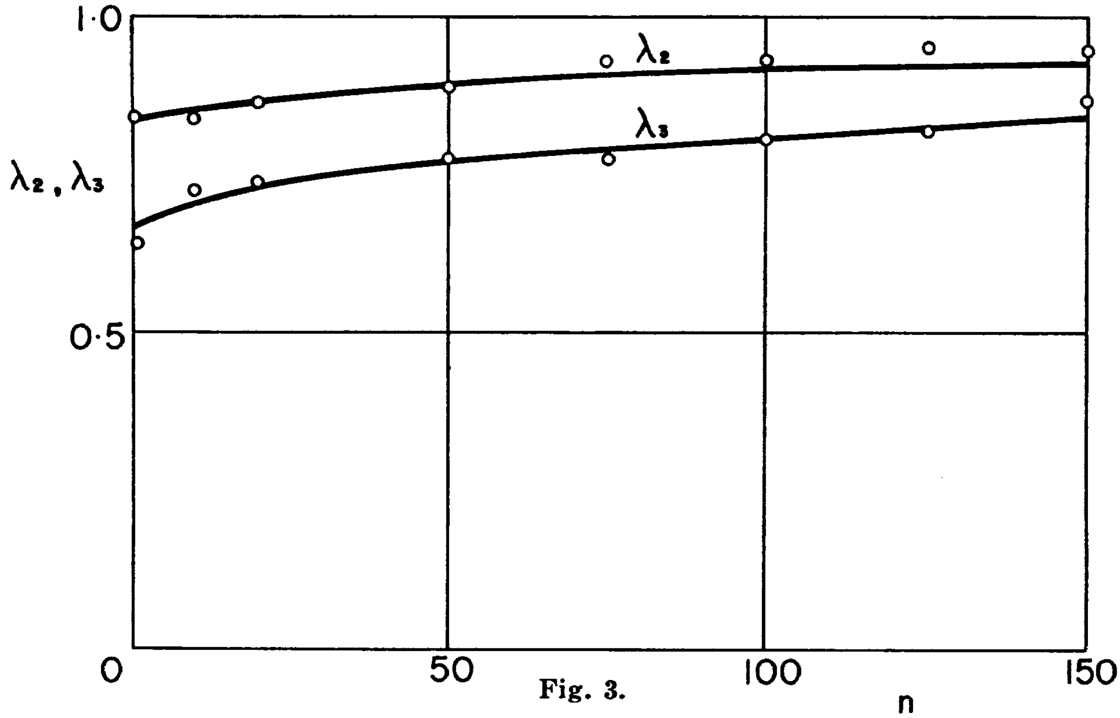

Fig. 3.

where λ_2 and λ_3 are the ratios of slopes of two straight lines in the 2nd and the 3rd portions to those of elastic lines. As the number of repetition is increased, at first the values of λ_2 and λ_3 increase but they approach asymptotically to certain constant value respectively. The deformation rate of the loops shown in Fig. 2 and the changing rates of λ_2 and λ_3 in Fig. 3, indicate that not so many times of repetition are needed to saturate the deformation of loop, practically $n=100$ is enough.

Fig. 4 shows the M-γ relation when the twisting moment is applied beyond the stress τ_0 after 150 cycles of unloading and reloading. In this case the plastic flow does not take place at the point A but does at P, and the ratio of τ_p, the stress at P, to τ_0, the stress at A, is

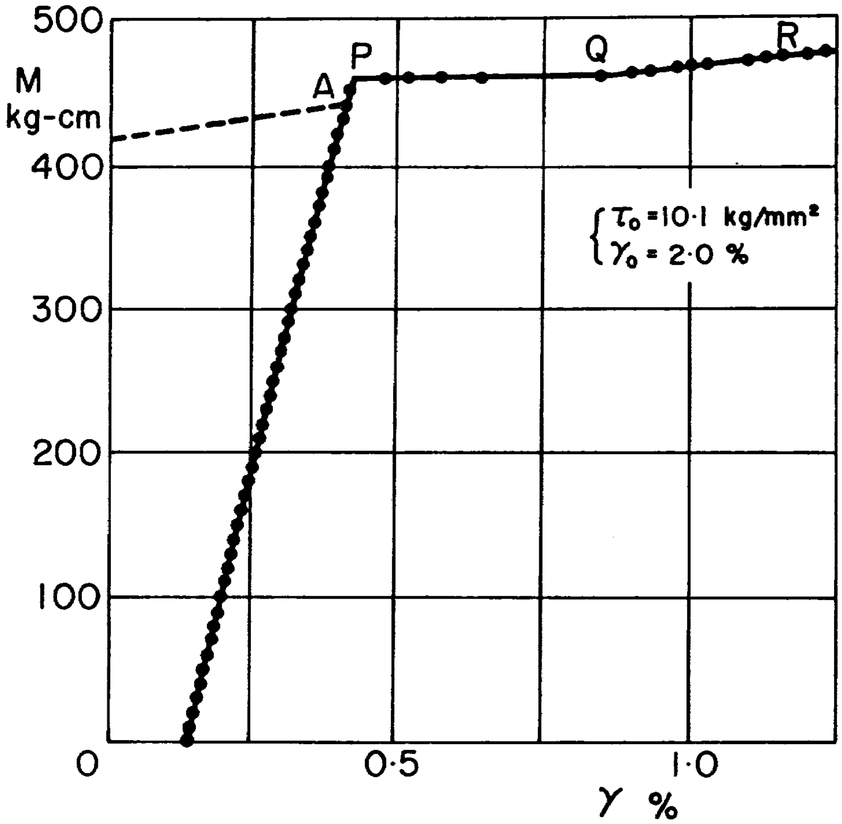

Fig. 4.

$$\frac{\tau_p}{\tau_0}=1.041$$

As to the M-γ relation shown in Fig. 4, it is also observed that the strain increases from point P to Q under constant stress, and seems that the plastic curve after point Q lies on the extension of the dotted line which is the plastic curve of the virgin specimen. From what mentioned above, it may be said that once plastic flow occurs after repetition it shifts to the plastic curve of the virgin specimen without accompanied by any work-hardening.

Besides above, a number of experiments on the shapes of hysteresis loop under torsion were carried out, and the results indicate that most hysteresis loops bend at the stresses $1/3 \cdot \tau_0$ and $2/3 \cdot \tau_0$ as pointed out in this section, but some of them bend at the stresses $1/4 \cdot \tau_0$, $2/4 \cdot \tau_0$ and $3/4 \cdot \tau_0$.

Pure Shear

The state of stress of thin-walled hollow cylinder which is closed at both ends and subjected to internal pressure can be divided into a hydrostatic tension and a shear. So the deformation of the cylinder consists of an elastic expansion due to hydrostatic tension and the pure shear, which caused the increase in diameter at the cost of the wall thickness keeping the axial length of the cylinder constant plastically. The elastic expansion can be known from the bulk modulus of material or by measuring the axial elastic strain of the cylinder. Subtracting the elastic expansion from the total deformation, the deformation due to pure shear can be separated, and thus the hysteresis loop due to pure shear is obtained.

Fig. 5 shows the hysteresis loops of hollow cylinder under internal pressure. They were obtained by cyclic unloading and reloading with internal pressure, after

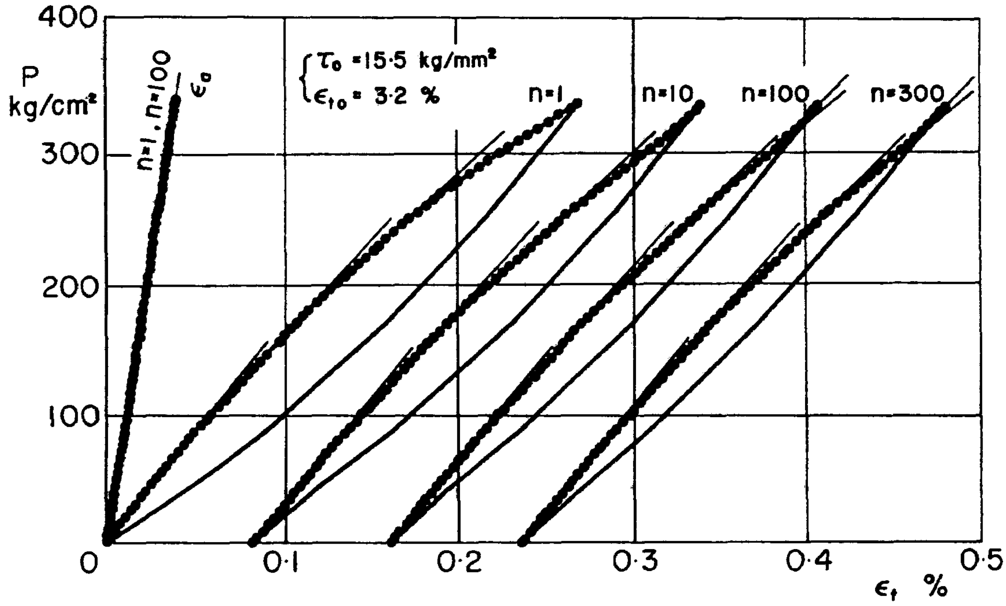

Fig. 5.

loading with internal pressure up to the plastic stage where the shearing stress attatins to τ_0. In this figure, ε_t is the circumferential strain at the outer diameter, ε_a is the axial strain which shows the one due to elastic expansion of material, and n is the number of repetition. The p-ε_t curves under the unloading and the reloading processes bend at the stresses $1/4\cdot\tau_0$, $2/4\cdot\tau_0$ and $3/4\cdot\tau_0$. The curves consist of 4 portions and as shown in Fig. 5 the shape of loop are symmetric with respect to its centre. The 1st portion is elastic. The 2nd and 3rd portions are straight lines. However, the 4th portion is a curve at the beginning of repetition but it divides into two straight lines with a gentle and a sharp slope as the repetition number is increased.

The hysteresis loops due to pure shear are obtained from the loops shown in Fig. 5 by subtracting the elastic expansion, and the variations of the slopes of the 2nd and 3rd portions are examined. They are shown against n in Fig. 6, where λ_2 and λ_3 denote the ratios of the slopes of the 2nd and the 3rd portions to that of the elastic line, and n is the number of repetition. The values of λ_2 decreases rapidly within the small number of repetition and remains constant after that.

The experiments on the rise of the stress of plastic flow observed after the repetition were carried out in the same way as in the case of torsion and τ_p is compared with τ_0, where τ_p and τ_0 are again the stresses of plastic flow after and before the repetition. The ratio of τ_p to τ_0 after 300 loading cycles was

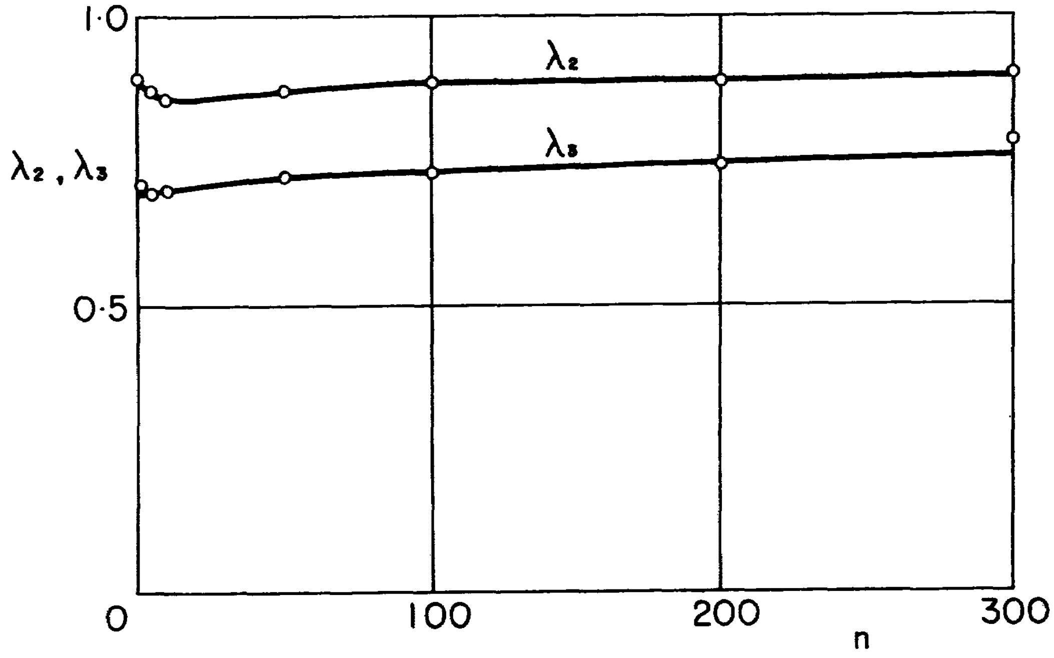

Fig. 6.

$$\frac{\tau_p}{\tau_0} = 1.033,$$

while in another experiments,

$$\frac{\tau_p}{\tau_0} = 1.037.$$

The curve of plastic flow after repetition is similar to that in the case of torsion shown in Fig. 4; the deformation of specimen continues under constant stree τ_p until the stress-strain curve crosses that of virgin specimen.

Many experiments on the hysteresis loop under pure shear were also carried out. Most hysteresis loops bend at the stresses $1/4 \cdot \tau_0$, $2/4 \cdot \tau_0$ and $3/4 \cdot \tau_0$ as shown in Fig. 5, but in some cases the bends appear at $1/3 \cdot \tau_0$ and $2/3 \cdot \tau_0$ instead of $1/4 \cdot \tau_0$, $2/4 \tau_0$ and

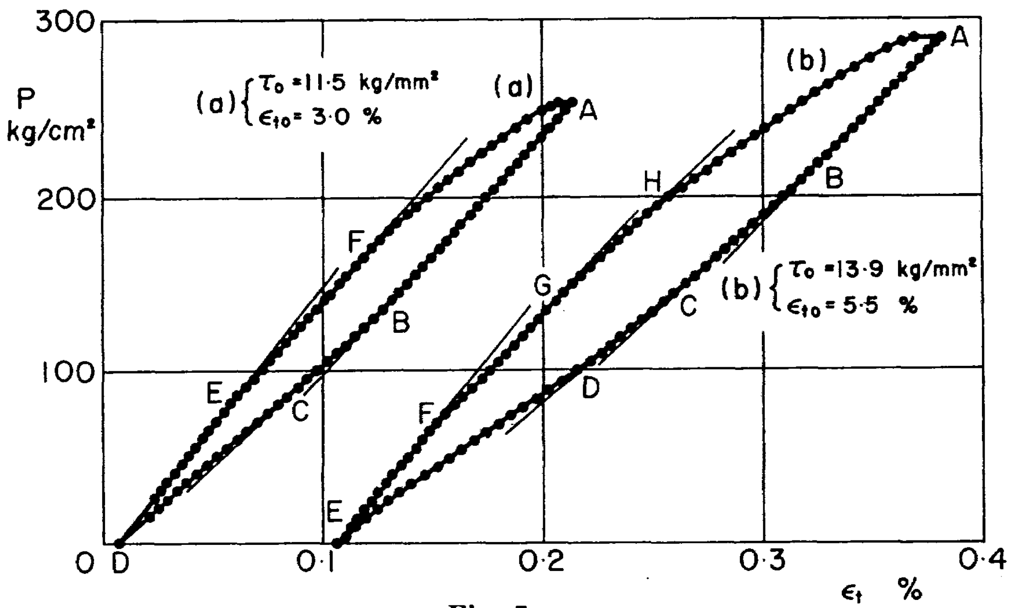

Fig. 7.

$3/4\tau_0$. Figs. 7 (a) and (b) show the examples of such cases. The hysteresis loop shown in (a) bends at point B and C in unloading process and at E and F in reloading one. The stresses at B and C are $2/4\cdot\tau_0$ and $1/3\cdot\tau_0$ and at E and F are $1/3\cdot\tau_0$ and $2/3\cdot\tau_0$. The hysteresis loop shown in (b) also bends at the stresses $3/4\cdot\tau_0$, $2/4\cdot\tau_0$ and $1/3\cdot\tau_0$ as indicated by B, C and D and at the stresses $1/4\cdot\tau_0$, $2/4\cdot\tau_0$ and $2/3\cdot\tau_0$ as indicated by F, G and H.

Tension

Fig. 8 shows the hysteresis loop in tension between stresses τ_0 and 0. This loop

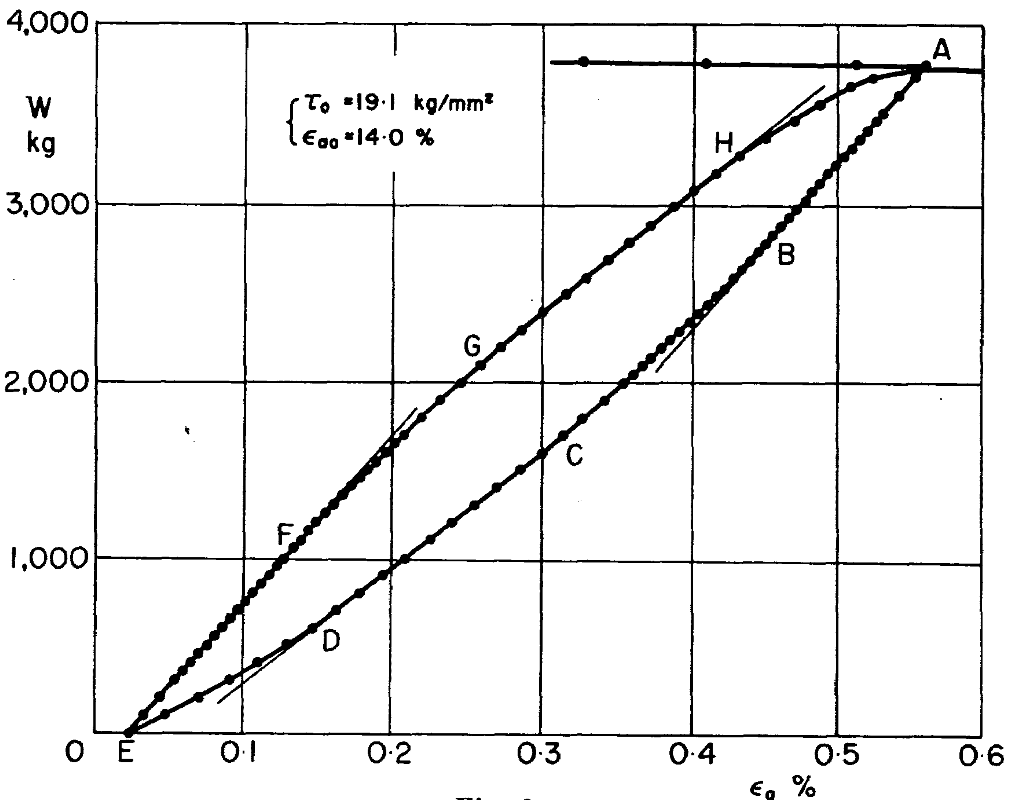

Fig. 8.

was obtained by unloading and reloading with tensile load W after the specimen was deformed under tension up to the plastic state, where the stress $\tau=\tau_0$ and the strain $\varepsilon_a=\varepsilon_{a0}$. In this figure, tensile load W is taken in ordinate and the strain ε_a in abscissa. The W-ε_a relations in unloading and reloading processes are symmetric with respect to the centre of loop; they bend at points B, C and D and points F, G and H respectively and consist of 4 portions. The 1st portion is elastic, the 2nd is not a straight line but it curves a little, the 3rd is a straight line and the 4th is a curve. As the 2nd portions BC and FG are not straight lines, the stresses corresponding to points B, C, F and G cannot be defined so clear. However, it may be said approximately that the stresses at point B, C and D are $5/7\cdot\tau_0$, $3/7\cdot\tau_0$ and $1/7\cdot\tau_0$, and F, G and H are $2/7\cdot\tau_0$, $4/7\cdot\tau_0$ and $6/7\cdot\tau_0$ respectively.

Fig. 9 shows the experimental results on the deformation of the hysteresis loops due to the repetition of the cyclic loading. Those loops were obtained by unloading

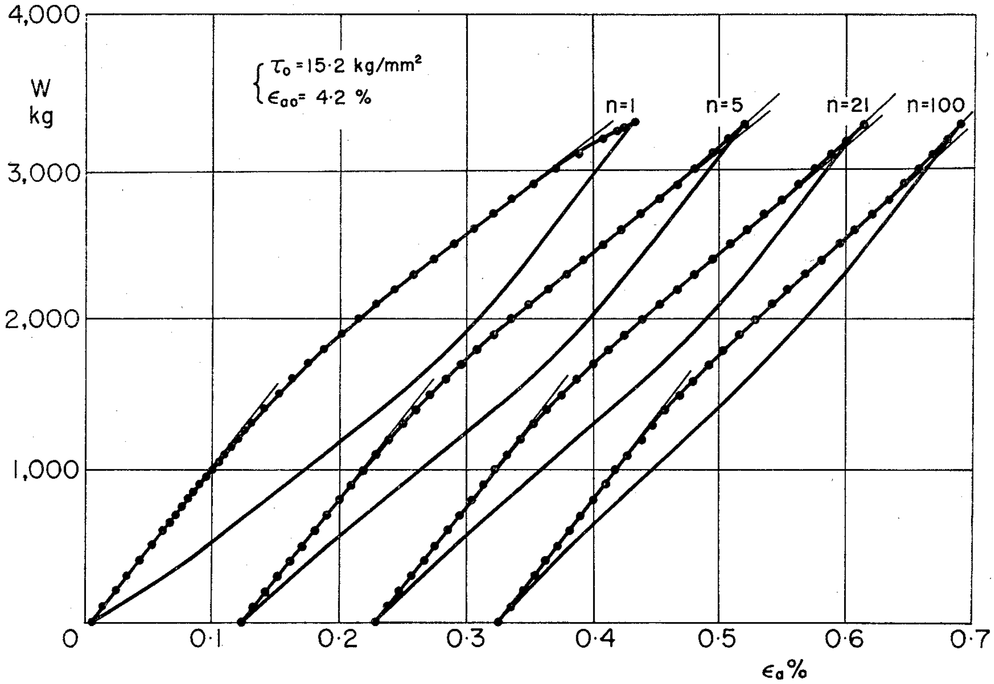

Fig. 9.

and reloading with the tensile load between τ_0 and 0, after the specimen was deformed plastically in tension up to the stress τ_0. Even if the repetition is continued, the W-ε_a relation of loop under reloading bends still at the stresses of $2/7\cdot\tau_0$, $4/7\cdot\tau_0$ and $6/7\cdot\tau_0$, and that under unloading bends at the stresses of $5/7\cdot\tau_0$, $3/7\cdot\tau_0$ and $1/7\cdot\tau_0$ as shown by solid line. They still consist of 4 portions and keep the symmetry; the 1st portion is elastic, the 2nd curves a little, and the 3rd is a straight line. However, the 4th portion turns rapidly into a straight line of sharp slope from a curve.

The deformation under tension can be divided into elastic volume expansion due to hydrostatic tension and the deformation due to shear. The elastic expansion can be calculated from bulk modulus. Subtracting the strain due to elastic expansion from the tensile strain, the deformation due to shear is separated. The hysteresis loops due to shear were separated from the hysteresis loops shown in Fig. 9, and the variation of the slope of the 3rd portion is shown in Fig. 10 against the number of repetition n, where λ_3 is the ratio of the slope of the 3rd portion to that of elastic one. The value of λ_3 seems to remain constant throughout the whole range of n.

Fig. 11 shows the stress of plastic flow after unloading and reloading were repeated up to $n=100$. In this figure, the stress τ_0 at point A is the stress of plastic flow before repetition. The plastic flow after repetition does not take place at point A, but does at point P where the stress is τ_p. The relation between τ_p and τ_0 is

$$\frac{\tau_p}{\tau_0}=1.038$$

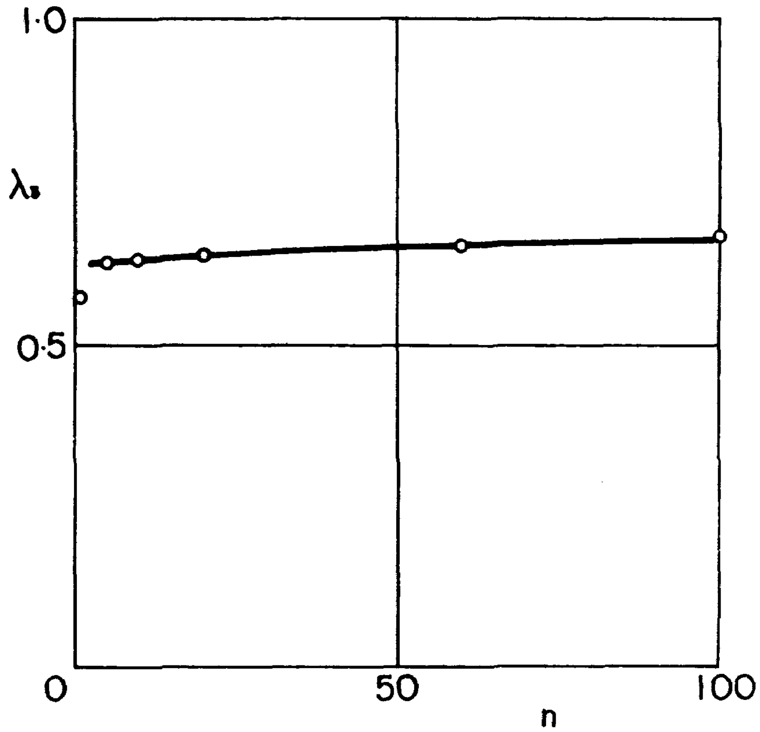
1·0
λ₃
0·5
0
50
100
n

Fig. 10.

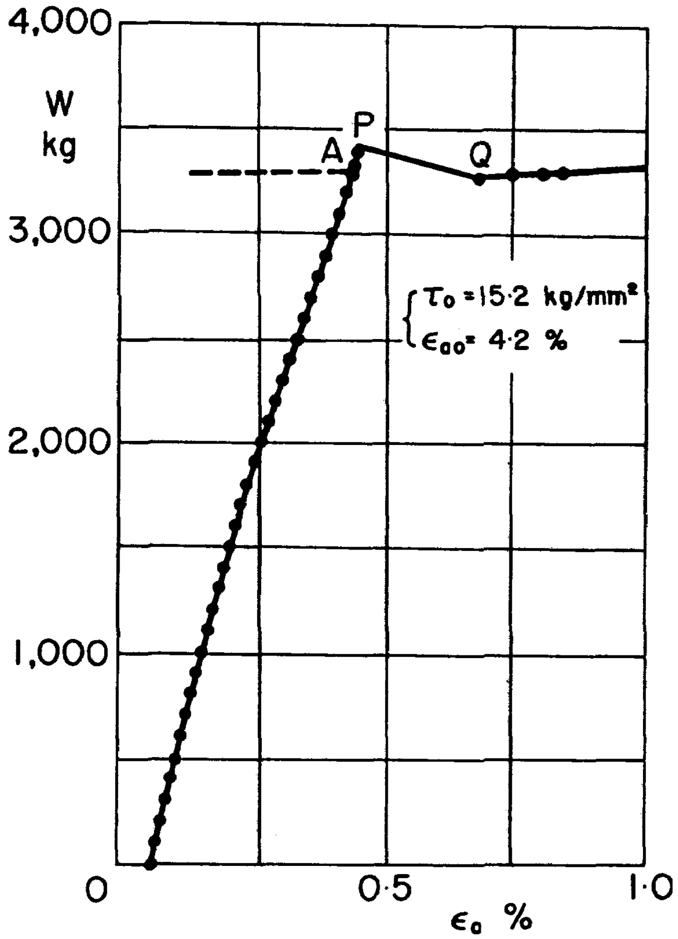
4,000
W
kg
3,000
2,000
1,000
0
0·5
1·0
ε₀ %
P
A
Q
τ₀ =15·2 kg/mm²
ε₀₀= 4·2 %

Fig. 11.

In this case, in contrast with the case of torsion and pure shear, the curve of plastic flow falls rapidly from point P to Q which also lies on the extension of the plastic curve of virgin specimen.

Conclusions

Various informations were obtained from the experiments on the hysteresis loops under torsion, pure shear, and tension, and they are summarized as follows:

1. The stress-strain relation under unloading has point symmetry to that under reloading with respect to the centre of the hysteresis loop.

2. Hysteresis loop is not smooth curve but there exists sharp bends. For instance, the loop under torsion consists of 3 portions and has bends at the stresses $1/3 \cdot \tau_0$ and $2/3 \cdot \tau_0$.

3. The 1st portion of the loop is an elastic line and its slope is constant, while the slopes of the other portions vary during the repetition of the cyclic loading. Especially, the last portion turns into a straight line from a curve. Those are the reason for the deformation of the hysteresis loop during repetition of cyclic loading.

4. After repetition, the stress of plastic flow rises a few percents comparing with the one before repetition, and the curve of plastic flow beyond the stress τ_0 shifts to the plastic curve of the virgin specimen after some amounts of plastic deformation without developing any work-hardening.

It has been generally considered that the hysteresis loop is a smooth curve, but its fallacy is demonstrated from our experiments. This discrepancy seems due to the effect of the strain speed; the ordinary smooth curve is obtained when the sharp bends are blurred by the strain speed effect.

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The Shape of Mechanical Hysteresis Loop, Its Deformation due to Stress Repetition and Resulting Increase in Flow Stress

(Part 2. Theory for Torsion)

By Fujio NAKANISHI**

Summarys

The observation that the beginning straight section of the hysteresis loop has the slope of G, the shear modulus, and the slopes of the second and the third sections are 0.84 G and 0.64 G is found to be in accordance with the result from the previous theory of plasticity proposed by the author, if the present strain is considered to be a combination of pure shear and rotation. The slopes tend to 0.94 G and 0.84 G respectively as the stress is repeated; this is expected from the theory by supposing that the strain changes into simple shear. The rise in the stress for the commencement of plastic flow due to stress repetition is also quantitatively predicted from the theory by allowing for the change in the type of strain.

Introduction

Concerning the plastic flow, the theory named "Three Shear Theory of Plasticity" which agrees well with the experiments, has been advanced[1),2),3)] previously. Developing this theory to some extent, it is expected to account for the elasto-plastic problems such as the hysteresis loop and Bauschinger effect. The experimental data on the hysteresis loops under various kinds of load were reported previously in Part 1. In Part 2, the behaviours of hysteresis loop in torsion are discussed and interpretation for them are given by the theory.

Theory of Plasticity

Let,

$x,\ y,\ z$: be the directions of principal stresses,

$\sigma_x,\ \sigma_y,\ \sigma_z$: be the principal tensile stresses,

$\tau_1,\ \tau_2,\ \tau_3$: be the principal shearing stresses, where the following relations hold,

$$\tau_1 = \frac{1}{2}(\sigma_y - \sigma_z),$$

$$\tau_2 = \frac{1}{2}(\sigma_z - \sigma_x),$$

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$$\tau_3=\frac{1}{2}(\sigma_x-\sigma_y).$$

ε_x, ε_y, ε_z: be the principal tensile strains,

γ_1, γ_2, γ_3: be the shearing strains given by

$$\gamma_1=\varepsilon_y-\varepsilon_z,$$
$$\gamma_2=\varepsilon_z-\varepsilon_x,$$
$$\gamma_3=\varepsilon_x-\varepsilon_y.$$

G: be the modulus of rigidity,

S_1, S_2, S_3: be the shearing stresses such as shown in Fig. 1,

I, II, III: be the directions of shears; the directions of shear strains γ_1, γ_2 and γ_3 are called as the directions I, II and III,

s: be the slip, and

R: be the resistance to the increase of shear strain.

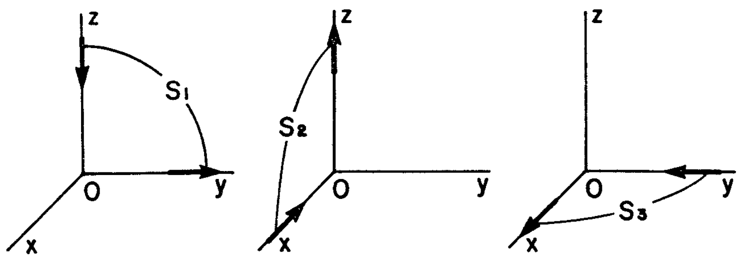

Fig. 1.

The authors' theory is as follows:

(i) The shears in three directions I, II and III are to be considered independently. This means that the resistance to the increase of strain must also be considered independently; the resistances in three directions, R_1, R_2 and R_3, should correspond to the shear stresses S_1, S_2 and S_3. The relation between τ_1, τ_2, τ_3 and s_1, s_2, s_3 are

$$\left.\begin{aligned}\tau_1&=S_1-\frac{1}{2}S_2-\frac{1}{2}S_3,\\ \tau_2&=-\frac{1}{2}S_1+S_2-\frac{1}{2}S_3,\\ \tau_3&=-\frac{1}{2}S_1-\frac{1}{2}S_2+S_3.\end{aligned}\right\}\quad(1)$$

(ii) In the plastic flow under pure shear, the magnitudes of R_1, R_2 and R_3 are equal, or

$$R_1=R_2=R_3=R. \quad (2)$$

(iii) The increments of slips ds_1, ds_2 and ds_3 are proportional to the shearing stresses S_1, S_2 and S_3 in each direction.

(iv) When direction of the change of strain is reversed in each shear direction, the reversed change is elastic until the change of S reaches R and after that it turns into plastic.

What are described in (i), (ii), (iii) and (iv) are the essence of the authors' theory of plasticity which is characterized by the independent treatment of the shear in each direction. So, this theory is the shearing stress theory concerning each direction, and the shearing stress in this case is not τ but S.

Now consider the plastic flow due to pure shear in direction II. The state of stress in this case can be expressed by the above theory as

$$\left.\begin{aligned} S_1 &= -R, \\ S_2 &= \ \ R, \\ S_3 &= -R. \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} \tau_1 &= -R, \\ \tau_2 &= \ 2R, \\ \tau_3 &= -R. \end{aligned}\right\} \tag{3$'$}$$

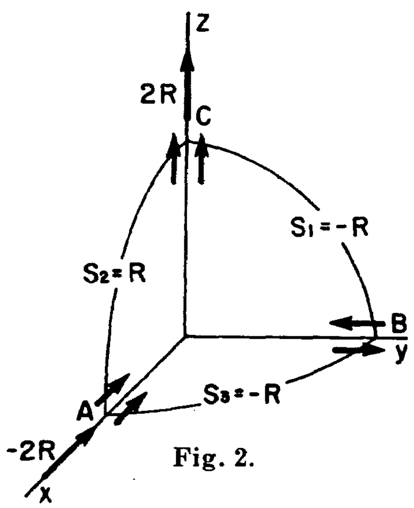

Fig. 2.

The equilibrium state of stress under pure shear is illustrated in Fig. 2 by taking an elemental sphere in the material. In this figure, ABC shows one eighth of the sphere and the plastic deformation of material can be expressed by the displacements of points A, B and C. It can be also seen that the shearing stress $\tau_2=2R$ is supported by the resistances in the three directions as shown in Fig. 2.

In the above explanations, only the plastic deformation is considered neglecting the elastic deformation. However, it is necessary to consider both deformation simultaneously in the actual elasto-plastic problems like hysteresis loop.

Yield Point (1)

Now, yield point will be defined as the point where the stress-strain relation shifts from one curve to another, though such definition is quite different from the yield point of mild steel.

Fig. 3 shows the hysteresis loops of hollow cylinder of brass in torsion. They are obtained successively by unloading and reloading the specimen with twisting moment between stresses τ_0 and 0, after the specimen was deformed in torsion up to the plastic state, where the stress $\tau=\tau_0$ and the strain $\gamma=\gamma_0$. The hysteresis loop shown in (a) is the one for the small number of repetition of loading cycle, and (b) shows the hysteresis loop after a large number of repetition. They are examples of hysteresis loop for brass, but the same loop will be obtained by any metal as far as it has a certain elastic range

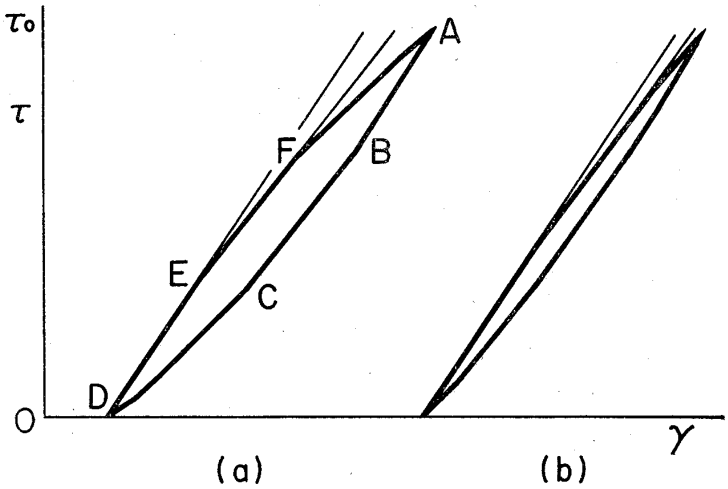

Fig. 3.

in its stress-strain relation.

As mentioned in Part 1, the specimen was deformed with moderate speed in the mere repetition of loop, but in the cycle in which the measurement was made it was deformed very slowly and every datum point was read after the equilibrium condition in stress-strain relation is established. Consequently, it may be said that on effect of strain speed is contained in the measured hysteresis loop. The characteristic of the τ-γ relation in unloading process shown in Fig. 3 (a) is as follows:

(i) AB is a straight elastic line,

(ii) BC and CD are also straight lines,

(iii) Yielding takes place at point B, where the stress is lower than τ_0 by $1/3\cdot\tau_0$, and

(iv) Yielding takes place again at point C, where the stress is also lower than that of point B by $1/3\cdot\tau_0$.

The τ-γ relations under reloading and unloading has point symmetry with respect to the centre of the loop.

According to the experiments on the hysteresis loops in torsion, the yield point appears mostly with the stress difference $|\varDelta\tau|=1/3\cdot\tau_0$ as shown in Fig. 3, but sometimes appears also with $|\varDelta\tau|=1/4\cdot\tau_0$. On the contrary, in the loops under pure shear, the yield point appears rather with the stress difference $|\varDelta\tau|=1/4\cdot\tau_0$ than with $|\varDelta\tau|=1/3\cdot\tau_0$. From these experiments, it may be concluded that there are two types in the yielding of the hysteresis loops in torsion and pure shear; it takes place with the stress difference $|\varDelta\tau|=1/3\cdot\tau_0$ in one type and with $|\varDelta\tau_0|=1/4\cdot\tau_0$ in the other.

If the condition of the yield point is determined by the magnitudes of $|\varDelta\tau|$, it should be determined uniquely and the two types will not be expected to appear. According to the authors' theory, however, the condition of the yield point is not determined by $|\varDelta\tau|$ but by $|\varDelta S|$ and the existence of several types will be expected in the yielding.

As it is considered in the theory that the increment of slip ds takes place in proportion to S, the state of S will be known inversely from the type of slip. Taking the state $\tau=\tau_0$

as the origin of the load, the unloading from $\tau=\tau_0$ means to increase the stress in the reversed direction of the previous plastic flow. Consequently, it will be ascertained that the slip under unloading is similar to that under the plastic flow, though their signs are reversed. Then, the slip under plastic flow will be discussed in order to know the slip under unloading.

Slip under Plastic Flow

As it is considered that the increment of slip ds takes place in proportion to S, the slip under plastic flow due to pure shear is as follows:

$$\left.\begin{aligned} ds_1&=-ds,\\ ds_2&=\ \ ds,\\ ds_3&=-ds. \end{aligned}\right\} \tag{4}$$

The existence of two types in the yielding reveals that there also exist two types of the slip under plastic flow. Therefore, the following types will be considered as to the slip shown in Eq. (4).

1. The slips in three directions, ds_1, ds_2 and ds_3 take place simultaneously.
2. The slips do not take place simultaneously but they take place one after another in a certain order in an elemental part of material.

The slips in 1 and 2 may be called as the 1st and the 2nd type of slip respectively.

In the 2nd type of slip, the slip ds_2 is possible to take place independently, but the slip ds_1 does not take place independently. If it is assumed that the slip $ds_1=-ds$ takes place, the plastic strain $d\gamma_2=1/2\cdot ds$ and $d\gamma_3=1/2\cdot ds$ should be induced in the directions II and III respectively. It is possible to induce $d\gamma_2=1/2\cdot ds$ in II being subjected to stress $S_2=R$, but impossible to induce $d\gamma_3=1/2\cdot ds$ in III being subjected to $S_3=-R$. So the slip ds_1 must be accompanied at least by the slip $-1/2\cdot ds$ in III in order to make $d\gamma_3=0$. As to the slip in the direction III, it is similar to the slip in I. In the case of the 2nd type, consequently, the slip shown by Eq. (4) can be divided into three kinds of slips,

$$\left\{\begin{aligned} ds_1&=0,\\ ds_2&=ds,\\ ds_3&=0. \end{aligned}\right. \qquad \left\{\begin{aligned} ds_1&=-\frac{2}{3}ds,\\ ds_2&=\ \ 0,\\ ds_3&=-\frac{1}{3}ds. \end{aligned}\right. \qquad \left\{\begin{aligned} ds_1&=-\frac{1}{3}ds,\\ ds_2&=\ \ 0,\\ ds_3&=-\frac{2}{3}ds. \end{aligned}\right. \tag{5}$$

Eq. (5) shows that shear in II, tension in the direction of z-axis and compression in the direction of x-axis respectively, and they take place successively in a certain order. If an elemental part is considered, these slips take place alternately in a certain order. On the other hand, if it is considered at one moment, one of these slips takes place in some parts, while the other two take place in the other parts. Consequently, it implies that the three kinds of slips coexist in the material.

Yield Point (2)

The type of slip after the yield point is passed in unloading process may be similar to that under plastic flow which was discussed in the previous section, though their signs are reversed. Consequently, the state of stress S in unloading process will be known from the type of slip, since ds takes place in proportion to S. Taking the state of $\tau=\tau_0$ as the origin of slip, the state of the shearing stress under unloading will be expressed as follows:

In the 1st type of slip, $S_1=-S_2=S_3$.

In the 2nd type of slip,

$$\begin{array}{ll} S_2, & \text{for } ds_2=-ds, \\ S_1,\ S_3,\ \text{where } \frac{1}{2}S_1=S_3, & \text{for } ds_1=ds, \\ S_1,\ S_3,\ \text{where } S_1=\frac{1}{2}S_3, & \text{for } ds_3=ds. \end{array}$$

In the former case, material is subjected to $S_1, -S_2$ and S_3 simultaneously. In the latter case, three parts, which are subjected to S_2 only and subjected to combined S_1 and S_3 with the ratios of $1/2\cdot S_1=S_3$ and $S_1=1/2\cdot S_3$, will coexist in the material.

Next problem is to predict the yield point, when material is subjected to S as shown above. $|\Delta S|=R$ has been considered as the condition of the yielding in the authors' theory, and this condition is satisfied when the value of S is changed in one direction. When the value of S is changed in two or three directions at a time as in the above cases, the condition of yielding can not be given by $|\Delta S|=R$ but it may be given generally by

$$\sum|\Delta S|=R \tag{6}$$

Expressing the condition of yielding by Eq. (6), the yield point is given as follows:

In the 1st type of slip,

$$\begin{cases} \Delta S_1=\frac{1}{3}R, \quad \Delta S_2=-\frac{1}{3}R, \quad \Delta S_3=\frac{1}{3}R, \\ \Delta\tau_2=-\frac{2}{3}R. \end{cases} \tag{7}$$

In the 2nd type of slip,

$$\begin{cases} \Delta S_2=-R, \\ \Delta\tau_2=-R. \end{cases}$$

$$\begin{cases} \Delta S_1=\frac{2}{3}R, \quad \Delta S_3=\frac{1}{3}R, \\ \Delta\tau_2=-\frac{1}{2}R. \end{cases} \tag{8}$$

$$\begin{cases} \Delta S_1=\frac{1}{3}R, \quad \Delta S_3=\frac{2}{3}R, \\ \Delta\tau_2=-\frac{1}{2}R. \end{cases}$$

As $\tau_0=2R$, the yield point is predicted to appear at $|\Delta\tau|=1/3\cdot\tau_0$ in the 1st type and at $|\Delta\tau|=1/4\cdot\tau_0$ in the 2nd type of slip. This prediction agrees just with the experiments.

In this paper, the yield point corresponding to the 1st type will be discussed, because the yield point mostly appears at $|\Delta\tau|=1/3\cdot\tau_0$ in the hysteresis loop under torsion. The yield point corresponding to the 2nd type will be discussed concerning with the hysteresis loop under pure shear on another occasion.

Shape of Hysteresis Loop

In the hysteresis loop shown in Fig. 3, λ_2 and λ_3 are taken as the ratios of the slope of the 2nd stage EF to that of the 1st one DE and that of the 3rd one FA to the 1st one DE, and their experimental data are plotted in Fig. 4, where n is the number of the repetition of the cyclic loading. The initial value of λ_2 is 0.84 and that of λ_3 is 0.64. Fig. 5 shows the relation between the stress τ and the slope of the stress τ versus the strain γ, $d\tau/d\gamma$, in the initial hysteresis loop, for the unloading process from the state $\tau_0=2R$. According to the experimental data under unloading from $\tau_0=2R$, the 1st stage is elastic until $\Delta\tau=-2/3\cdot R$, the slope in this stage being G, and that of the 2nd stage is 0.84 G. They are shown by lines AB and B'C in Fig. 5.

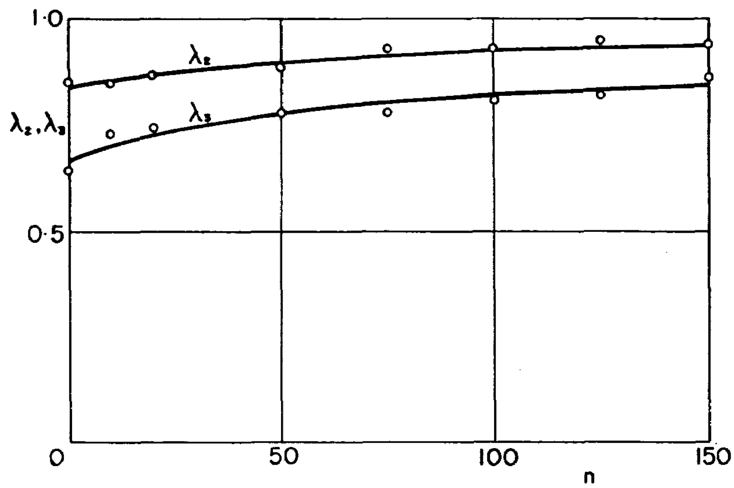

Fig. 4.

If the point A is taken as the origin of the load, unloading from the point A means to increase the load in the opposite direction. In the case of unloading, it may be considered that some kinds of mutual interference is induced among crystals of material and the slip will be difficult to take place under such condition. This is the reason for the existence of an elastic range in the hysteresis loop. As load is decreased, the mutual interference will be intensified gradually, but it will not be intensified unlimitedly. From the behaviour of the slope $d\tau/d\gamma$ which falls from AB to B'C at the yield point B, it will be supposed that the mutual interference breaks and its effect vanishes at the yield point.

Here, the slope will be discussed on the hysteresis loop in which no effect of the mutual

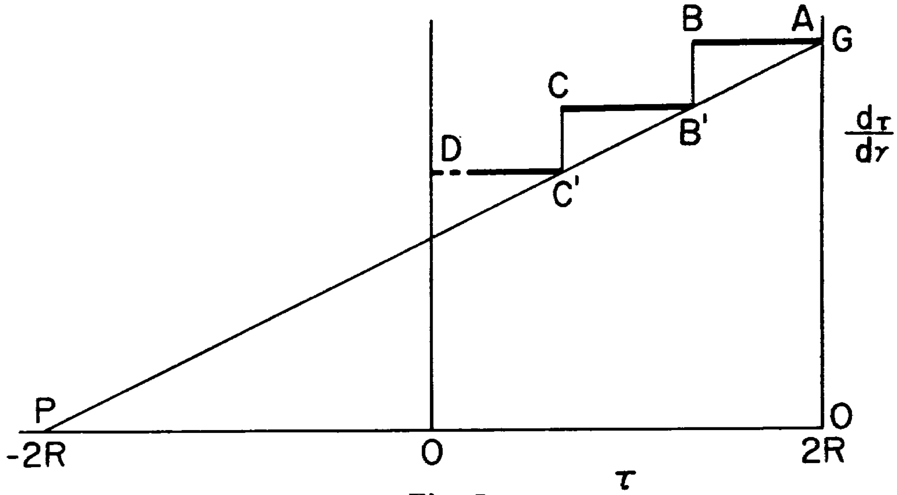

Fig. 5.

interference among the crystals is contained. In this loop, the τ-γ relation is elastic and the slope is equal to G at the initial state of the unloading process from the state $\tau_0=2R$.

When τ changed by $4R$ from $2R$ to $-2R$, the plastic flow takes place in the reversed direction and the slope of the τ-γ relation becomes zero, if the work-hardening due to plastic flow under unloading is neglected. In the stress range of $-2R<\tau<2R$, the simplest relation between the slope $d\tau/d\gamma$ and the variation of the stress $\Delta\tau$ is linear, then

$$\frac{d\tau}{d\gamma}=G\left(1-\frac{|\Delta\tau|}{4R}\right). \tag{9}$$

Eq. (9) is shown by the straight line AP in Fig. 5, and the point B' lies just on this line. The experimental behaviour on the slope of the hysteresis loop seems to be able to be interpreted by putting the following assumption.

(i) Eq. (9) gives the $d\tau/d\gamma-\Delta\gamma$ relation on the hysteresis loop in which no effect of the mutual interference among the crystals is contained.

(ii) The slope under unloading is not on the line AP but on AB owing to the effect of the mutual interference among crystals.

(iii) On account of the vanishing of the mutual interferece at the yield point B, the slope falls suddenly from B to B' which lies on the line AP.

After passing the point B', the mutual interference among the crystals will arise newly as the load is removed, and the slope of loop also does not change along the line AP. For this case, however, the intensity of the mutual interference and the $d\tau/d\gamma-\Delta\tau$ relation are not yet clearly known. According to the experiment, the slope of the 2nd stage is constant until the stress reaches next yield point C as shown by B'C in Fig. 5. The slope B'C is

$$\frac{d\tau}{d\gamma}=\frac{5}{6}G. \tag{10}$$

Assuming that the slope falls from point C to C' which lies on the line AP at the yield point C and assuming that the slope C'D is constant, the slope of the 3rd stage is

$$\frac{d\tau}{d\gamma}=\frac{2}{3}G. \tag{11}$$

Though the data of λ_2 and λ_3 are scattered a little as shown in Fig. 4, the initial values $\lambda_2=0.84$ and $\lambda_3=0.64$ agree well with Eqs. (10) and (11) respectively.

Deformation of Hysteresis Loop due to Repetition

The deformation under torsion is a simple shear. In this paper, however, considering that simple shear consists of pure shear and rotation, the strain due to pure shear has been treated. If the amounts of the slip and rotation are extremely small, the strain of simple shear may be identical with that of pure shear. However, the strain is not so small in actual cases, and it seems that the above treatment of simple shear does not hold always. Therefore, if simple shear is replaced by pure shear, some incompatibility will arise and some extra strains will be required to dissolve the incompatibility. According to the experiments, the 1st stage in the initial hysteresis loop is not elastic exactly, though it was called conventionally elastic; the slope of the 1st stage in the initial hysteresis loop is a little more gentle than that of elastic line of virgin specimen, and it is ascertatined that a small plastic strain, or an extra strain, exists in the initial hysteresis loop. From what mentioned here, it may be said that the strain in the initial hysteresis loop is caused by pure shear plus rotation, and that the incompatibility mentioned above is dissolved by the appearance of the extra strains. Moreover, it seems that the occurrence of pure shear plus rotation is easier than that of simple shear in the initial hysteresis loop under torsion, since the strain of pure shear plus rotation is somewhat larger than that of simple shear for the same load.

By experiments, it is also shown that the small plastic strain, or the extra strain, vanishes rapidly by repeating the hysteresis loop and the slope of the 1st stage approaches that of the elastic line of the virgin specimen. From this fact, it will be said that pure shear plus rotation is difficult to take place, when hysteresis loop is repeated, and the strain of pure shear plus rotation turns into that of simple shear with the increasing number of repetition. Here the deformation of hysteresis loop due to repetition will be discussed on the basis of the above considerations.

When the directions of principal stresses and strains coincide as in the case of pure shear, the shears in three directions are considered independently in the authors' theory. In general case, however, the shear in each direction must be divided into two shears, and, hence, six components of shear should be considered independently. For instance, when material is subjected to shear in the direction II, the slip in II should be divided into the slips in the directions II′ and II″ as shown in Fig. 6. In the case of pure shear, the slips in II′ and II″ are always equal and the sum of them is called as the slip in the direction II. On the other hand, the slips in II′ and II″ should be considered individually in the case of

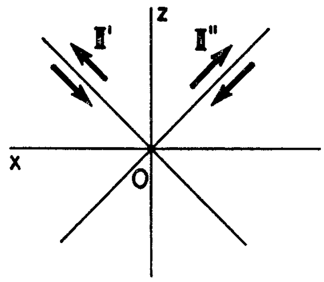

Fig. 6.

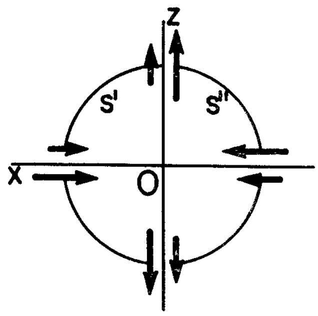

Fig. 7.

simple shear, since the slip takes place only in the one direction and the other remains unchanged.

Now the shearing stress will be considered for the case when material is subjected to shear in the direction II. The shearing stress S in II consists of S_2 and the components of S_1 and S_2, and in this case S should be also considered being divided into S' and S'' in II' and II'' as shown in Fig. 7. Then

$$S=S_2-\frac{1}{2}S_1-\frac{1}{2}S_3,$$
$$S=\tau_2=S'+S''.$$

In simple shear, the nature in II' will differ from that in II'' on account of the difference of the slips in II' and II''. This difference will be accounted for by S' and S'', though it can not be explained by the stress τ and strain γ since the values of τ and γ in II' and II'' are always eqnal.

In the 1st stage, it is elastic and the slip does not take place. Consequently, S' and S'' are equal either in pure shear or in simple shear, then

$$S'=S'',$$
$$\frac{d\tau}{d\gamma}=G,\quad \frac{dS'}{d\gamma}=\frac{1}{2}G,\quad \frac{dS''}{d\gamma}=\frac{1}{2}G, \tag{12}$$

and the stresses at the yield point are

$$\Delta S=-\frac{2}{3}R,\quad \Delta S'=-\frac{1}{3}R,\quad \Delta S''=-\frac{1}{3}R. \tag{13}$$

In the 2nd stage, S' and S'' are also equal in pure shear,

$$S'=S''$$
$$\frac{d\tau}{d\gamma}=\frac{5}{6}G,\quad \frac{dS'}{d\gamma}=\frac{5}{12}G,\quad \frac{dS''}{d\gamma}=\frac{5}{12}G. \tag{14}$$

On the other hand, S' and S'' are not equal in simple shear, because the slip takes place in the one direction and the other remains unchanged. Now assuming that the slip takes place in II′ and does not in II″, $dS'/d\gamma$ and $dS''/d\gamma$ are given by Eqs. (14) and (12) respectively and become as follows:

$$\frac{dS'}{d\gamma}=\frac{5}{12}G,\quad \frac{dS''}{d\gamma}=\frac{1}{2}G, \tag{15}$$

or

$$\frac{d\tau}{d\gamma}=\frac{dS}{d\gamma}=\frac{11}{12}G. \tag{16}$$

The λ_2-n relation in Fig. 4 shows that the value of λ_2 increases from 0.84 to 0.94 gradually as the number of repetition n is increased, and the value of $d\tau/d\gamma$ expressed by Eq. (16) agrees well with the data of λ_2 for the large number of n.

It can be considered that the allotment of the stress is proportional to the rigidity shown by Eq. (15), that is

$$\varDelta S=-\frac{2}{3}R,\quad \varDelta S'=-\frac{10}{33}R,\quad \varDelta S''=-\frac{12}{33}R. \tag{17}$$

In the 3rd stage, S' and S'' are also equal in pure shear, then

$$\frac{d\tau}{d\gamma}=\frac{2}{3}G,\quad \frac{dS'}{d\gamma}=\frac{1}{3}G,\quad \frac{dS''}{d\gamma}=\frac{1}{3}G. \tag{18}$$

In the case of simple shear, the slope in the direction II″ still remains constant and given by Eq. (12), but the slope in II′ changes on account of the change of S' in the 2nd stage. Dividing Eq. (9) into $ds'/d\gamma$ and $dS''/d\gamma$,

$$\frac{dS'}{d\gamma}=\frac{1}{2}G\left(1-\frac{|\varDelta S'|}{2R}\right). \tag{19}$$

Therefore,

$$\frac{dS'}{d\gamma}=\frac{15}{44}G,\quad \frac{dS''}{d\gamma}=\frac{1}{2}G. \tag{20}$$

$$\frac{d\tau}{d\gamma}=\frac{37}{44}G=0.841G. \tag{21}$$

Though the experimental data on λ_3 are scattered a little as shown in Fig. 4, the value of λ_3 increases from 0.64 to 0.82~0.84 gradually with the increase in the number of repetition. The value of $d\tau/d\gamma$ expressed by Eq. (21) agrees well with the experimental data of λ_3 for the large number of n.

Rise of Stress of Plastic Flow after Repetition

It is well known by experiment that the stress of plastic flow after repetition rises by a few percents comparing with τ_0 on account of the work-hardening, where τ_0 is the stress of plastic flow before repetition. From the behavior of plastic curve after repetition, it may be recognized that the work-hardening due to repetition is quite different from that in plastic flow. The work-hardening due to plastic flow depends on R in the authors' theory. However, it seems that the work-hardening due to repetition does not depend on R, because in the experiments the stresses at yield points in the hysteresis loop do not change but remain constant, even when the number of repetition is increased.

As mentioned in the previous section, the difference between S' and S'' appears as the number of repetition is increased, though they are equal in the initial state of repetition. By this difference, the rise of the flow stress during the repetition may be accounted for and its magnitude can be also predicted.

In the initial state of repetition, the strain in the hysteresis loop is pure shear and the value of S' and S'' are equal. The amplitudes of the stresses, a' and a'', are the sum of $|\Delta S'|$ and $|\Delta S''|$ in the 1st, the 2nd and the 3rd stages, or

$$\left.\begin{aligned} a' &= \sum|\Delta S'| = R\,, \\ a'' &= \sum|\Delta S''| = R. \end{aligned}\right\} \tag{22}$$

During the repetition, the strain in the hysteresis loop turns into simple shear, and S' and S'' in this state become as follows:

$$\left.\begin{aligned} a' &= \sum|\Delta S'| = 0.9066\,R, \\ a'' &= \sum|\Delta S''| = 1.0934\,R. \end{aligned}\right\} \tag{23}$$

Fig. 8 (a) shows the amplitudes of S' and S'' in the initial state. As the number of repetition is increased, the amplitude of S' becomes small and that of S'' large. Considering the fact that the stress-strain relations of hysteresis loop under unloading and reloading are symmetric and that the stresses at the yield points are independent of the number of repetition, it may be ascertained that the amplitudes of S' and S'' take the location shown in Fig. 8 (b). At the state of $\tau=\tau_0$, the value of S'' exceeds R and it is possible to induce the plastic deformation in II$''$, while S' has not yet reached R. Consequently, the plastic flow does not take place in a complete form in the direction II. It is the necessary condition for the plastic flow in II that both S' and S'' reach R. In the case of Fig. 8(a), the amplitudes of S' and S'' reach R at the same time when the state of $\tau=\tau_0$ is attained, therefore, it is possible to induce the plastic flow in II at this point. In order to induce the plastic flow in II in a complete form in the case (b), it is necessary to increase the load until S' reaches R. This is the reason for rise of plastic flow stress after the repetition.

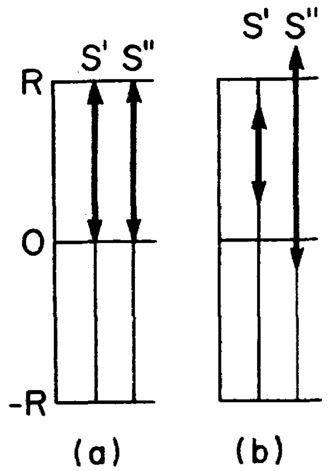

Fig. 8.

When the load is increased in the case (b), the 3rd yield point is expected to appear at $\tau=\tau_0$, and $dS'/d\gamma$ after the yield point can be calculated by Eq. (19) as

$$\frac{dS'}{d\gamma}=0.5467\frac{1}{2}G. \tag{24}$$

Concerning S'' in II'', the slip will not occur up to $\tau=\tau_0$ on account of the influence of repetition, but beyond that point yielding will occur at $\tau=\tau_0$ on account of no influence of repetition. $dS''/d\gamma$ after the yield point can be also given by the similar relation to Eq. (19), and it is

$$\frac{dS''}{d\gamma}=0.4533\frac{1}{2}G. \tag{24)'$$

When S' reaches R from the state of $\tau=\tau_0$, the increments of S' and S'' are

$$\begin{cases} \Delta S'=0.0467R, \\ \Delta S''=\Delta S'\dfrac{dS''}{dS'}=0.0387R. \end{cases} \tag{25}$$

$$\Delta\tau=0.0854R. \tag{26}$$

Expressing the stress of the plastic flow afterre petition by τ_p, the relation between τ_p and τ_0 is

$$\frac{\tau_p}{\tau_0}=1.043. \tag{27}$$

According to the experiment, the ratio of τ_p to τ_0 is

$$\frac{\tau_p}{\tau_0}=1.041.$$

This experimental relation is obtained after the hysteresis loop was repeated up to $n=150$. During this period, some hysteresis loop was observed and their slopes are shown in Fig. 4. As to τ_p/τ_0, the result of the computation agrees well with that of the experiment.

Conclusions

By the authors' theory, the behaviours of the hysteresis loop under torsion can be accounted for, not only on the shape of the hysteresis loop but also its deformation due to repetition and on the rise of stress of the plastic flow after repetition. The slip of the 1st type was assumed in the computation on the rise of the plastic flow stress. However, according to the experiment on Bauschinger effect, the 1st type of slip turns clearly into the 2nd type at the state just a little before τ becomes zero, when twisting moment is removed from $\tau=\tau_0$ to $\tau=-\tau_0$. In the hysteresis loop shown in Fig. 3(a), CD and FA curve a little in their ends, though they are regarded as straight lines, and the appearance of the curves in CD and FA portions is considered as the effect of the transformation of the slip from the 1st to the 2nd type. Concerning the amplitudes of S' and S'', it may be considered that Eq. (23) remains valid in the range of $\tau\leqq\tau_0$, because the curves in CD and FA vanish and they become almost straight as the number of the repetition is increased. However, the computation on the rise of the plastic flow stress after repetition should be carried out with respect to the slip of the 2nd type in the range of $\tau > \tau_0$. If the computation is carried out by considering the effect of the slip in the 2nd type, the value of τ_p/τ_0 becomes smaller than that of Eq. (27), but their difference is very small. Concerning the investigations on Bauschinger effect, it will be reported on another occasion.

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