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**Effect of Damping on Shock Response  
Spectrum**

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# Effect of Damping on Shock Response Spectrum\*

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## ABSTRACT

This paper presents analytical expressions for the shock response spectrum of an oscillator with any amount of damping which is subjected to an arbitrary shock excitation. The residual shock spectrum is also evaluated in an analytical manner when a rectangular, triangular or half sine wave pulse is applied to the oscillator.

## 概 要

任意の衝撃力を受ける場合のショック・レスポンス・スペクトラムに対する減衰の影響について解析的に調べた。さらに、衝撃力が矩形波、三角波、正弦波の場合について、減衰のある時の応答のピーク値を解析的に求めた。

## 1. INTRODUCTION

Dynamic responses of a structure subjected to shock loadings are complex. The shock response is determined by vibration characteristics of the structure and the type and severity of loading. In practical design problems, the detailed time history of the response is not necessarily a major concern. The peak responses — induced maximum displacement, acceleration, strain and stress — are often more significant.

A shock response spectrum is a plot of the peak response of a single-degree-of-

freedom oscillator subjected to a specific shock excitation; the spectrum is a function of the natural frequency of the oscillator. The concept of the shock response spectrum was originally developed by Biot [1] in 1932 to examine the effects of earthquakes on the buildings. The shock spectrum approach has been extended and applied to a large number of engineering problems; for instance, aircraft landings [2, 3], launching and separation of sub-structures of space-vehicles [4-7], responses of an earth-moving vehicle [8], package cushioning [9], nuclear explosions [10, 11] and seismic designs [12, 13]. Figure 1 shows typical shock spectra of a half-sine pulse for different

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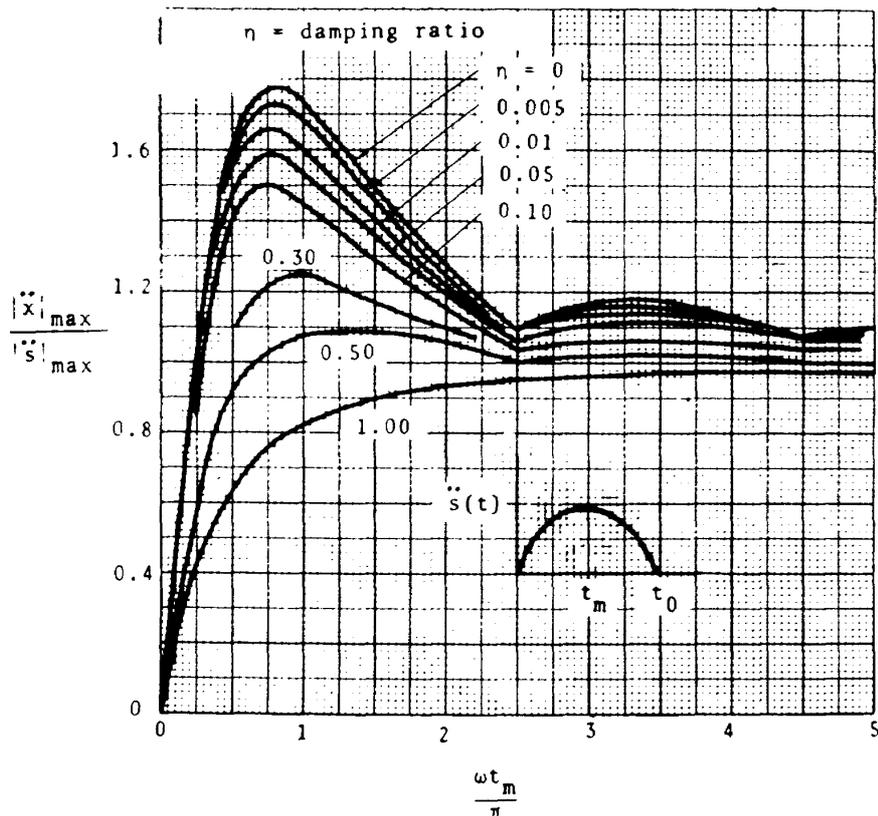


Fig. 1 Shock spectra of a half-sine pulse [9].

damping ratios [9]. The characteristics of the shock force is often not well defined at design stages. The spectrum is not as sensitive to change of the force as the time history of the response. This fact was discovered by Shappiro and Hudson [14], and confirmed by Fung [15]. From these spectra obtained for several types of the shock forces, the values for the mass, spring stiffness and damping of the system can be selected so that the response does not exceed a specified limit. If the system parameters are given, an allowable maximum amplitude of the shock force can be determined.

The spectra for some simple shocks have been evaluated analytically [16, 17]. Few analytical examinations of the peak response to an arbitrary shock were, however, made particularly for damped systems. Analog and digital computers were

used to calculate the spectra. Charts of the shock response spectra of a linear, single-degree-of-freedom system are available in Refs. 18–21 and 26. Review papers on the shock spectrum approaches have been written by the present author on a three year basis [22, 23].

As mentioned above, few shock spectra for the damped system has analytically been evaluated because their response characteristics is very much complicated. However, it is a conservative design to calculate the peak response without the damping being taken into account, since the damping reduces the response of the structure. In addition, the effect of damping in most engineering structures in considered to be relatively small. However, for instance, airplanes are equipped with oil dampers with large damping in the landing gears. Many instruments are

mounted on shock absorbers. Some indicators for dynamic measurements are designed to damp critically. The systems which will encounter severe shock loadings are installed with absorbers with large damping. Hence, it is often very important to evaluate the effect of damping on the shock response spectrum. In this paper we will first present analytical expressions for the peak response of an oscillator with any amount of damping, and evaluate the peak response of the oscillator subjected to typical forces of simple patterns.

## 2. LINEAR SYSTEMS

The equation of motion of a single-degree-of-freedom system with spring,  $k$ , subjected to a force shock,  $af(t)$ , (Fig. 2) is written as

$$\ddot{x} + 2\eta\omega\dot{x} + \omega^2x = \frac{a}{m}f(t) \quad (1.1)$$

where  $\eta$ ,  $\omega$  and  $m$  are, respectively, the damping ratio, natural frequency and mass. The equation of motion for a ground shock is

$$\ddot{y} + 2\eta\omega\dot{y} + \omega^2y = -\ddot{s} = -af(t) \quad (1.2)$$

For the force shock,  $x$  is the displacement of the mass. For the ground shock,  $y$  and  $s$  are, respectively, the relative and ground displacements. The absolute displacement,  $x$ , of the mass is

$$x = y + s \quad (2)$$

The amplitude of the shock loading is determined by the parameter  $a$ , because

$$\begin{aligned} f(t) &\neq 0 \text{ for } 0 < t < t_0 \\ f(t) &= 0 \text{ for } t \leq 0 \text{ and } t \geq t_0 \end{aligned} \quad (3)$$

and  $\max\{f(t)\} = f(t_m) = 1$   
 $0 < t < t_0$

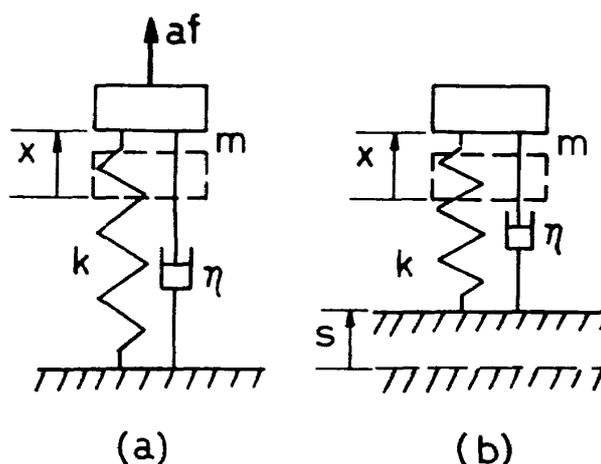


Fig. 2 Linear system models;  
 (a) force shock,  
 (b) ground shock.

In these equations  $t_0$  is a duration and  $t_m$  is a rise time. If  $f(t)$  rises monotonically to a peak value and falls back to zero, it is called a simple pulse [24]. The impact force of a typical airplane landing is known as the simple pulse [2].

The solution of Eq. (1.1) for an initial condition  $x(0) = \dot{x}(0) = 0$  is described by Duhamel's integral:

$$\frac{kx(t)}{a} = \int_0^t f(\xi)h(t-\xi)d\xi \quad (4.1)$$

$$= \int_0^t f(t-\xi)h(\xi)d\xi \quad (4.2)$$

$$= X(t) \quad (4.3)$$

where  $h(t)$  is an impulsive response, and

$$h(t) = \frac{\omega}{\beta} e^{-\eta\omega t} \sin(\beta\omega t) \quad (5)$$

$$\beta = (1 - \eta^2)^{1/2} \quad (6)$$

$X(t)$  is a ratio of the displacement to the maximum static deflection. The time,  $t_p$ , at which the response reaches the maximum or minimum peak is given by

$$\left. \frac{dX(t)}{dt} \right|_{t=t_p} = 0 \quad (7)$$

The nondimensional peak displacement  $X(t_p)$  is a function of the natural frequency,  $S_X(\omega)$ ; that is,  $\max\{X(t)\} = X(t_p) = S_X(\omega)$ . A plot of  $S_X(\omega)$  against a frequency parameter,  $\omega t_m/\pi$  or  $\omega t_0/\pi$ , is called the displacement spectrum, amplification spectrum, or shock response spectrum. The velocity and acceleration spectra are defined as the peak values of  $\dot{X}(t)$  and  $\ddot{X}(t)$ , respectively. The peak response, which occurs before or after  $t=t_0$ , is called, respectively, the primary or residual shock spectrum.

### 3. SHOCK SPECTRUM OF DAMPED SYSTEMS SUBJECTED TO AN ARBITRARY FORCE

The general expressions for the response to an arbitrary excitation,  $f(t)$ , is given by Eqs. (4). The characteristics of the response depends on whether  $\eta < 1$  or not.

#### 3.1) $\eta < 1$ , i.e., Damping less than critical.

(i) Peak response reaches before the end of the force, i.e.,  $t_p < t_0$  with  $0(t_0) \lesssim 1$ .

Differentiating Eq. (4.1) with respect to time, we have

$$\dot{X}(t) = \frac{\omega^2}{\beta} \int_0^t f(\xi) e^{-\eta\omega(t-\xi)} \times [\beta \cos\{\beta\omega(t-\xi)\} - \eta \sin\{\beta\omega(t-\xi)\}] d\xi$$

If  $\omega \ll 1$ , then

$$\dot{X}(t) = \omega^2 \int_0^t f(\xi) \{1 - \eta\omega(t-\xi)\}^2 d\xi \quad (8)$$

The time,  $t_p$ , is a root of Eq. (8) when  $\dot{X}(t_p) = 0$ .

It is noted that this cannot occur if

$$\int_0^t f(\xi) d\xi > 0 \quad \text{for } t \leq t_0$$

(ii) Peak response reaches after the end of the force, i.e.,  $t_p > t_0$  with  $0(t_0) \lesssim 1$ .

Eq. (4.1) may be written as

$$X(t) = \frac{\omega}{\beta} e^{-\eta\omega t} A \sin(\beta\omega t + \theta_1) \quad (9)$$

where

$$A = \left[ \left\{ \int_0^{t_0} f(\xi) e^{\eta\omega\xi} \cos(\beta\omega\xi) d\xi \right\}^2 + \left\{ \int_0^{t_0} f(\xi) e^{\eta\omega\xi} \sin(\beta\omega\xi) d\xi \right\}^2 \right]^{1/2} \quad (10.1)$$

$$\tan \theta_1 = \frac{-\int_0^{t_0} f(\xi) e^{\eta\omega\xi} \sin(\beta\omega\xi) d\xi}{\int_0^{t_0} f(\xi) e^{\eta\omega\xi} \cos(\beta\omega\xi) d\xi} \quad (10.2)$$

Differentiating Eq. (9) with respect to  $t$ , we obtain

$$\dot{X}(t) = \frac{\omega^2}{\beta} A e^{-\eta\omega t} \cos(\beta\omega t + \theta_1 + \theta_1^*) \quad (11)$$

where

$$\tan \theta_1^* = \frac{\eta}{\beta} \quad \text{for } 0 < \theta_1^* < \frac{\pi}{2} \quad (12)$$

From Eq. (11), we have an expression for  $t_p$ :

$$\omega t_p = \left\{ \frac{n}{2} \pi - (\theta_1 + \theta_1^*) \right\} / \beta > 0 \quad (13)$$

for  $n = 1, 3, \dots$

Simplified results can be obtained when

$\omega \ll 1$ . If  $\omega \rightarrow 0$ , then  $A$  tends to  $|\int_0^{t_0} f(\xi) d\xi|$ , while  $\theta_1$  tends to 0 or  $\pi$  according to the sign of  $\int_0^{t_0} f(\xi) d\xi$ . Hence, from Eq. (13),

$$\omega t_p = (\frac{\pi}{2} - \theta_1^*) / \beta \quad \text{for } n = 1 \text{ or } 3 \quad (14)$$

Substituting Eq. (14) into Eq. (9) yields

$$|X(t)|_{\max} = \omega \exp\left\{-\left(\frac{\pi}{2} - \theta_1^*\right)\eta/\beta\right\} \times \left| \int_0^{t_0} f(\xi) d\xi \right| \quad (15)$$

From Eq. (15), we have

$$\frac{d}{d\omega} |X(t)|_{\max} = \exp\left\{-\left(\frac{\pi}{2} - \theta_1^*\right)\eta/\beta\right\} \times \left| \int_0^{t_0} f(\xi) d\xi \right| \quad (16)$$

If  $\eta \ll 1$ , Eq. (15) reduces to

$$|X(t)|_{\max} = \omega \exp\{-\eta\pi/2\} \left| \int_0^{t_0} f(\xi) d\xi \right| \quad (17)$$

If  $\eta$  tends to 1, then Eq. (15) becomes

$$|X(t)|_{\max} = \omega e^{-1} \left| \int_0^{t_0} f(\xi) d\xi \right| \quad (18)$$

Eq. (17) is equivalent to Eq. (7) of Ref. 24.

### 3.2) $\eta > 1$ , i.e., Damping greater than critical.

A response to an arbitrary excitation,  $f(t)$ , may be written as

$$X(t) = (\omega/\beta) \int_0^t f(\xi) \exp\{-\eta\omega(t-\xi)\} \times \sinh\{\gamma\omega(t-\xi)\} d\xi \quad (19)$$

where

$$\gamma = (\eta^2 - 1)^{1/2} \quad (20)$$

(i) In case  $t_p < t_0$ ,  $0(t_p) \lesssim$  and  $\omega \ll 1$ , we have the same expression as Eq. (8).

(ii) In case  $t_p > t_0$ :

We have

$$X(t) = (\omega/\gamma) B e^{-\eta\omega t} \sinh(\gamma\omega t - \theta_2) \quad (21)$$

where

$$B = \left[ \left\{ \int_0^{t_0} f(\xi) e^{\eta\omega\xi} \cosh(\gamma\omega\xi) d\xi \right\}^2 - \left\{ \int_0^{t_0} f(\xi) e^{\eta\omega\xi} \sinh(\gamma\omega\xi) d\xi \right\}^2 \right]^{1/2} \quad (22.1)$$

$$\tanh \theta_2 = \frac{\int_0^{t_0} f(\xi) e^{\eta\omega\xi} \sinh(\gamma\omega\xi) d\xi}{\int_0^{t_0} f(\xi) e^{\eta\omega\xi} \cosh(\gamma\omega\xi) d\xi} \quad (22.2)$$

Differentiation of Eq. (21) with respect to  $t$  yields

$$\dot{X}(t) = -(\omega^2/\gamma) B e^{-\eta\omega t} \times \sinh(\gamma\omega t - \theta_2 - \theta_2^*) \quad (23)$$

where

$$\tanh \theta_2^* = \gamma/\eta \quad (24)$$

From Eq. (23), we have

$$\omega t_p = (\theta_2 + \theta_2^*)/\gamma \quad (25)$$

Substituting Eq. (25) into Eq. (21) yields, for arbitrary  $\omega$ ,

$$|X(t)|_{\max} = \omega B \exp\{-\eta(\theta_2 + \theta_2^*)/\gamma\} \quad (26)$$

If  $\omega \ll 1$ ,  $\theta_2$  tends to zero, and

$$B \rightarrow \left| \int_0^{t_0} f(\xi) d\xi \right|$$

Hence,

$$|X(t)|_{\max} = \omega \left\{ \exp(-\eta\theta_2^*/\gamma) \right\} \times \left| \int_0^{t_0} f(\xi) d\xi \right| \quad (27)$$

As  $\eta$  approaches 1, Eq. (27) reduces to Eq. (18).

## 4. FORMULAE FOR PEAK RESPONSE TO SHOCK PULSES OF SIMPLE PATTERNS

Next, we will present analytical expressions for calculating the residual shock spectrum of a damped system when the shock pulses applied are of such forms as rectangular, triangular and half-sine waves.

These pulses which are zero except for the first cycle or so, can easily be constructed by the addition or subtraction of semi-infinite periodic or step functions,  $F(t)$  for  $t > 0$ . For instance, if  $F(t)$  is a step function with rise time  $t_m$ , i.e.,  $F(t) = 1$  for  $t \geq t_m$ , then the function defined by

$$f_0(t) = F(t) - F(t + t_m) \quad (28)$$

vanishes for  $t \geq 2t_m$ . The dimensionless response  $X_0(t)$  to the excitation given by Eq. (28) is written from Eqs. (4) as

$$X_0(t) = \int_0^t F(\xi) h(t-\xi) d\xi - \int_{t_m}^t F(\xi - t_m) h(t-\xi) d\xi \quad (29.1)$$

$$= \int_0^t F(t-\xi) h(\xi) d\xi - \int_0^{t-t_m} F(t-t_m-\xi) h(\xi) d\xi \quad (29.2)$$

$$= X(t) - X(t-t_m) \quad (29.3)$$

### 4.1) Shock spectrum of single and double rectangular wave pulses

For the single pulse case,  $F(t)$  in Eq. (28) is represented by an ordinary unit step function. Let us introduce an auxi-

liary parameter,  $\alpha$ , and

$$\alpha = \begin{cases} 1 & \text{for single rectangular pulse} \\ 2 & \text{for double rectangular pulse} \end{cases} \quad (30)$$

Then, the function for single and double rectangular excitations of durations  $t_0$  and  $2t_0$ , respectively, can be given (Fig. 3) by

$$f_\alpha(t) = \{1(t) - 1(t - t_0)\} - (\alpha - 1) \times \{1(t - t_0) - 1(t - 2t_0)\} \quad (31)$$

where  $1(t)$  denotes the unit step function. Taking into account the response to the step function  $1(t)$  which is, for instance, described at p. 106 of Ref. 25, the response to  $f_\alpha$  by Eq. (31) is written as

$$X_\alpha(t) = (e^{-\eta\omega t}/\beta) A^\alpha \sin(\beta\omega t + \theta_1 + \alpha\theta_2) \quad (32)$$

where

$$\sin \phi_1 = -\beta, \quad \cos \phi_1 = -\eta \quad (33.1)$$

$$A = \{1 - 2 \exp(\eta\omega t_0) \cos(\beta\omega t_0) + \exp(2\eta\omega t_0)\}^{1/2} \quad (33.2)$$

$$\tan \phi_2 = \frac{\exp(\eta\omega t_0) \sin(\beta\omega t_0)}{\{1 - \exp(\eta\omega t_0) \cos(\beta\omega t_0)\}} \quad (33.3)$$

Differentiation of Eq. (32) with respect to  $t$  yields

$$\dot{X}_\alpha(t) = (\omega/\beta) A^\alpha e^{-\eta\omega t} \sin(\beta\omega t + \alpha\phi_2) \quad (34)$$

Therefore, we have

$$\omega t_p = (n\pi - \alpha\phi_2)/\beta \quad (35)$$

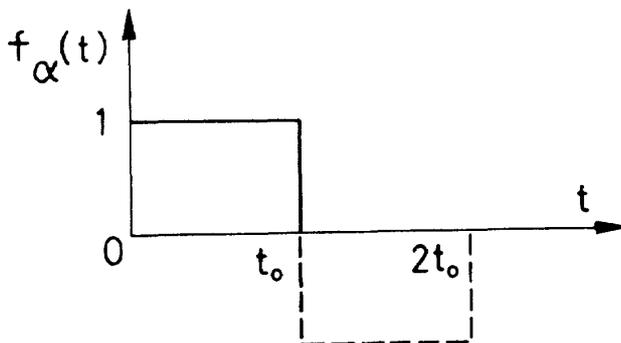


Fig. 3 Single and double rectangular pulses.

Substituting Eq. (35) into Eq. (32) gives

$$X_\alpha(t)|_{\max} = \exp(-\eta\omega t_p) A^\alpha \times \sin(\phi_1 + n\pi)/\beta = (-1)^{n+1} A^\alpha \exp\{-\eta(n\pi - \alpha\phi_2)/\beta\} \quad (36)$$

### 4.2) Shock spectrum of single and double triangular wave pulses

In order to make use of Eq. (28), it is necessary to construct a step function,  $F_1(t)$ , with initial slope  $1/t_m$  and rise time  $t_m$ , from a linearly increasing function  $F_2(t) = t/t_m$  (Figs. 4). Then, substitution of  $F(t) = F_1(t)$  into Eq. (28) yields a triangular pulse of duration of  $2t_m$ .

Let

$$\alpha = \begin{cases} 1 & \text{for single pulse} \\ 2 & \text{for double pulse} \end{cases} \quad (37)$$

For the single pulse case, the response is written as

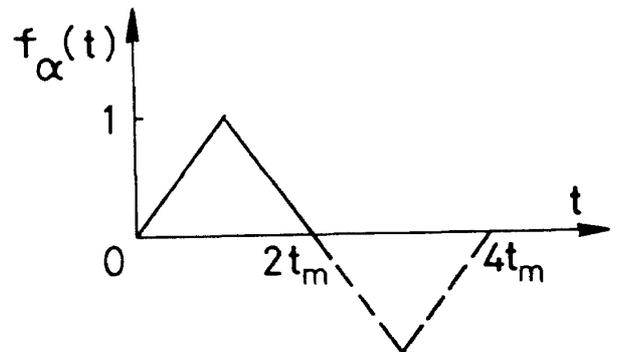


Fig. 4a Single and double triangular pulses

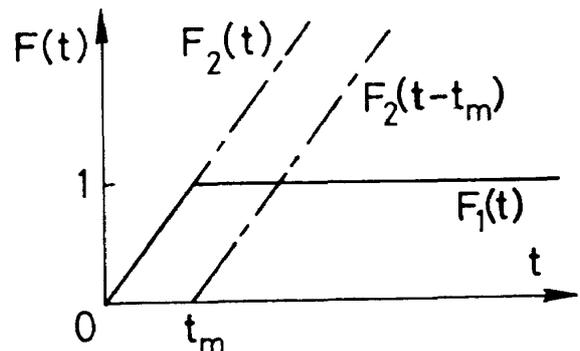


Fig. 4b  $F_1(t)$  and  $F_2(t)$ .

$$\begin{aligned}
 X_1 = X_\alpha |_{\alpha=1} &= \frac{1}{t_m} \left[ \int_0^t (t-\xi) h(\xi) d\xi \right. \\
 &\quad \left. - 2 \int_0^{t-t_m} (t-t_m-\xi) h(\xi) d\xi \right. \\
 &\quad \left. + \int_0^{t-2t_m} (t-2t_m-\xi) h(\xi) d\xi \right] \quad (38)
 \end{aligned}$$

Differentiation of Eq. (38) yields

$$\begin{aligned}
 \dot{X}_1(t) &= \frac{1}{t_m} \left[ \int_{t-t_m}^t h(\xi) d\xi \right. \\
 &\quad \left. - \int_{t-2t_m}^{t-t_m} h(\xi) d\xi \right] \quad (39)
 \end{aligned}$$

Similarly, the response for the double pulse is given as

$$X_2(t) = X_1(t) - X_1(t-2t_m) \quad (40)$$

From  $\dot{X}_\alpha = 0$ , we obtain

$$\omega t_p = \frac{1}{2i\beta} \ln\left(\frac{B}{A}\right) \quad (41)$$

where

$$\begin{aligned}
 A &= [1 - (\alpha-1) \exp\{2(\eta-i\beta)\omega t_m\}] \\
 &\quad \times [1 - \exp\{(\eta-i\beta)\omega t_m\}]^2 / (\eta-i\beta) \quad (42.1)
 \end{aligned}$$

$$\begin{aligned}
 B &= [1 - (\alpha-1) \exp\{2(\eta+i\beta)\omega t_m\}] \\
 &\quad \times [1 - \exp\{(\eta+i\beta)\omega t_m\}]^2 / (\eta+i\beta) \quad (42.2)
 \end{aligned}$$

Substitution of Eq. (41) into Eq. (38) or (40) gives the peak value of the response to the single or double pulse, respectively.

### 4.3) Shock spectrum of single and double half-sine wave pulses

Let the excitation be, (Fig. 5),

$$f_\alpha(t) = \sin(pt) \quad \text{for } 0 < t < \alpha t_m \quad (43)$$

where

$$p = \pi / (2t_m) \quad (44)$$

and

$$\alpha = \begin{cases} 2 & \text{for single half-sine} \\ 4 & \text{for double half-sine} \end{cases} \quad (45)$$

It is obvious that the response is written as

$$X_\alpha(t) = \int_0^{\alpha t_m} \sin\{p(t-\xi)\} h(\xi) d\xi \quad (46)$$

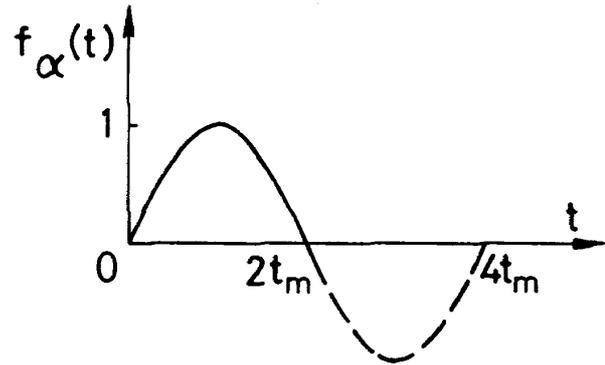


Fig. 5 Single and double half-sine pulses.

From  $\dot{X}_\alpha(t) = 0$ , we have

$$t_p = \frac{1}{2ip} \ln\left(\frac{B}{A}\right) \quad (47)$$

where

$$\begin{aligned}
 A &= \frac{1 - \exp[-\{\eta\omega + i(p+\beta\omega)\}\alpha t_m]}{\eta\omega + i(p+\beta\omega)} \\
 &\quad - \frac{1 - \exp[-\{\eta\omega + i(p-\beta\omega)\}\alpha t_m]}{\eta\omega + i(p-\beta\omega)} \quad (48.1)
 \end{aligned}$$

$$\begin{aligned}
 B &= \frac{1 - \exp[-\{\eta\omega - i(p+\beta\omega)\}\alpha t_m]}{\eta\omega - i(p+\beta\omega)} \\
 &\quad - \frac{1 - \exp[-\{\eta\omega - i(p-\beta\omega)\}\alpha t_m]}{\eta\omega - i(p-\beta\omega)} \quad (48.2)
 \end{aligned}$$

Substituting Eq. (47) into Eq. (46), we have

$$X_\alpha(t)|_{\max} = X_\alpha(t_p) \quad (49)$$

## 5. CONCLUDING REMARKS

The analytical expressions of the shock response spectrum for an arbitrary shock excitation are obtained with damping being taken into account. The spectrum is also analytically evaluated when the applied forces are of single or double rectangular, triangular and half-sine waves.

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## REFERENCES

1. Biot, M.A., "A Mechanical Analyzer for the Prediction of Earthquake Stresses," *Bull. Seismol. Soc. Amer.*, Vol. 31, pp. 151-171 (1941).
2. Fung, Y.C., "The Analysis of Dynamic Stresses in Aircraft Structures during Landing as Nonstationary Random Processes," *Trans. ASME, J. Appl. Mech.*, Vol. 22, No. 4, pp. 449-457.
3. Leal, O.N., Bisplinghoff, R.L., and Pian, T.H.H., "Studies of Transient Stresses in an Airplane Model Wing during Drop Test," *Proc. Soc. Exptl. Stress Anal.*, Vol. 6, No. 1, pp. 115-122 (1948)
4. Bamford, R. and Trubert, M., "A Shock Spectra and Impedance Method to Determine a Bound for Spacecraft Structural Loads," *AIAA Paper No. 75-811* (1975).
5. Bucciarelli, L.L. and Askinazi, J., "Pyrotechnic Shock Synthesis Using Nonstationary Broad Band Noise," *Trans. ASME, J. Appl. Mech.*, Vol. 40, No. 2, pp. 429-432 (1973).
6. Trubert, M. and Salama, M., "A Generalized Modal Shock Spectra Method for Spacecraft Loads Analysis," *AIAA Paper No. 79-0745* (1979).
7. Crimi, P., "Analysis of Structural Shock Transmission," *J. Spacecraft*, Vol. 15, No. 2, pp. 79-84 (1978).
8. Kemper, J.D. and Ayre, R.S., "Optimum Damping and Stiffness in a Nonlinear Four-Degree-of-Freedom System Subjected to Shock Load," *Trans. ASME, J. Appl. Mech.*, Vol. 38, No. 1, pp. 135-142 (1971).
9. Mindlin, R.D., "Dynamics of Package Cushioning," *Bell System Tech. J.*, Vol. 24, pp. 353-461 (1945).
10. Barton, M.V. (ed.), "Shock and Structural Response," *Colloq. Shock and Structural Response*, ASME (1960).
11. Galbraith, F.W. and Schreiner, R.N., "Shock Spectra from Subsurface Nuclear Explosion," *ASCE, J. Engr. Mech. Div.*, EM5, pp. 1015-1027 (1968).
12. Housner, G.W., "Behavior of Structures during Earthquakes," *ASCE, J. Engr. Mech. Div.*, Paper No. 2220, EM4 (1959).
13. Wang, P.C. and Yun, C.B., "Site-Dependent Critical Design Spectra," *Earthquake Engineering and Structural Dynamics*, Vol. 7, 569-578 (1979).
14. Shapiro, H. and Hudson, D.E., "The Measurement of Acceleration Pulses with the Multi-Frequency Reed Gage," *Trans. ASME, J. Appl. Mech.*, Vol. 20, No. 3., pp. 422-426 (1953).
15. Fung, Y.C. and Barton, M.V., "Some Shock Spectra Characteristics and Uses," *Trans. ASME, J. Appl. Mech.*, Vol. 25, No. 3, pp. 365-372 (1958).
16. Frankland, J.M., "Effects of Impact on Simple Elastic Structures," *Proc.*

- Soc. Exptl. Stress Anal., Vol. 6, No. 2, pp. 7–27 (1949).
17. Jacobsen, L.S. and Ayre, R.S., *Engineering Vibrations*, McGraw-Hill (1958).
  18. Ayre, R.S., "Transient Response to Step and Pulse Function," Chap. 8, *Shock and Vibration Handbook*, (Harris, C.M. and Crede, C.E., eds.) McGraw-Hill (1961).
  19. Barton, M.V. Chobotov, V., and Fung, Y.C., "A Collection of Information on Shock Spectrum of a Linear System," Tech. Rep. No. EM 11-9, Space Tech. Lab., Calif. (1961).
  20. Fung, Y.C. "Transient Oscillations," Unpublished manuscript.
  21. Rubin, S., "Concepts in Shock Data Analysis," Chap. 23, *Shock and Vibration Handbook*, (Harris, C.M. and Crede, C.E., eds.), McGraw-Hill (1961).
  22. Matsuzaki, Y., "A Review of Shock Response Spectrum," *The Shock and Vibration Digest*, Vol. 9, No. 3, pp. 3–12 (1977).
  23. Matsuzaki, Y., "Shock Response Spectrum and Maximax Response," *The Shock and Vibration Digest*, Vol. 12, No. 3, pp. 11–15 (1980).
  24. Fung, Y.C., "Some General Properties of the Dynamic Amplification Spectra," *J. Aeronautical Sciences*, Vol. 24, No. 7, pp. 547–548 (1957).
  25. Thomson, W.T., *Vibration Theory and Applications*, Prentice Hall (1965).
  26. Kawakami, M., "On the form of impulse and the vibration (in Japanese)," *Bullutin of Western Society of Ship Building*, No. 17, pp. 65–75 (1959).

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