

Fifth Aerodynamics Prediction Challenge (APC-V)  
2019/7/1 Tokyo

レイノルズ応力モデルを用いた  
30P30Nの空力予測

Aerodynamic prediction of 30P30N  
using a Reynolds stress model

$k$ - $\omega$  SST遷移モデルを用いた  
30P35Nのフラップ剥離予測

Flap separating prediction of 30P35N  
using a SST transition model

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Tohoku University  
○ ENDO Shunya, ARIKI Taketo, KUYA Yuichi, SAWADA Keisuke

## Contents

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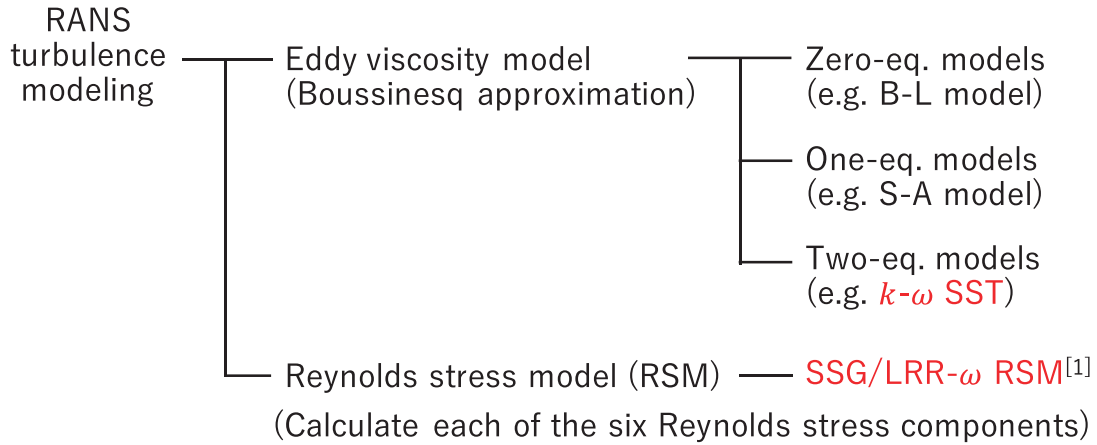
1. Overview of turbulence model
2. Computational settings
3. Results: comparison with  $k$ - $\omega$  SST
4. Concluding remarks

# Turbulence model

$$\frac{\partial(\bar{\rho}\hat{u}_i)}{\partial t} + \frac{\partial(\hat{u}_j\bar{\rho}\hat{u}_i)}{\partial x_j} = -\frac{\partial\bar{p}}{\partial x_i} + \frac{\partial\bar{\sigma}_{ij}}{\partial x_j} + \frac{\partial\bar{\tau}_{ij}}{\partial x_j}$$

$\bar{\sigma}_{ij}$ : Molecular viscous stress

$\bar{\tau}_{ij}$ : Reynolds stress



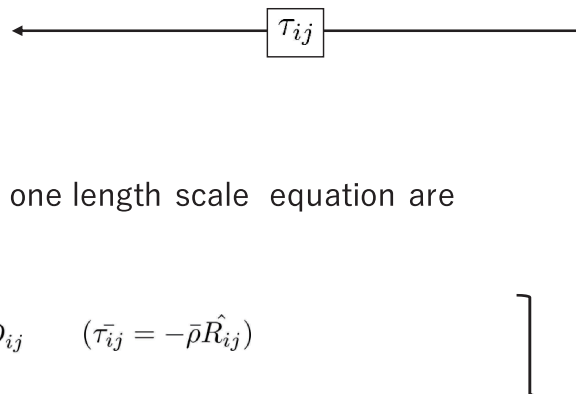
[1] Eisfeld et al., 2003

# Reynolds stress model

SSG/LRR- $\omega$  RSM: Reynolds stress model proposed by Eisfeld et al.

Governing eq.

$$\frac{\partial(\bar{\rho}\hat{u}_i)}{\partial t} + \frac{\partial(\hat{u}_j\bar{\rho}\hat{u}_i)}{\partial x_j} = -\frac{\partial\bar{p}}{\partial x_i} + \frac{\partial\bar{\sigma}_{ij}}{\partial x_j} + \frac{\partial\bar{\tau}_{ij}}{\partial x_j}$$



Turbulence model

The six Reynolds stress equations and one length scale equation are given by:

$$\frac{\partial\bar{\rho}\hat{R}_{ij}}{\partial t} + \frac{\partial(\bar{\rho}\hat{u}_l\hat{R}_{ij})}{\partial x_l} = \bar{\rho}P_{ij} + \bar{\rho}\Pi_{ij} - \bar{\rho}\epsilon_{ij} + \bar{\rho}D_{ij} \quad (\bar{\tau}_{ij} = -\bar{\rho}\hat{R}_{ij})$$

$$\frac{\partial\bar{\rho}\omega}{\partial t} + \frac{\partial(\bar{\rho}\hat{u}_l\omega)}{\partial x_l} = \frac{\alpha_\omega\omega}{\hat{k}} \frac{\bar{\rho}P_{ll}}{2} - \beta_\omega\omega^2 + \frac{\partial}{\partial x_l} \left[ \left( \bar{\mu} + \sigma_\omega \frac{\bar{\rho}\hat{k}}{\omega} \right) \frac{\partial\omega}{\partial x_l} \right] + \sigma_d \frac{\bar{\rho}}{\omega} \max \left( \frac{\partial\hat{k}}{\partial x_j} \frac{\partial\omega}{\partial x_j}, 0 \right)$$

$P_{ij}$ : production     $\Pi_{ij}$ : pressure-strain     $\epsilon_{ij}$ : dissipation     $D_{ij}$ : diffusion

## Case 1-2: Aerodynamic prediction of 30P30N airfoil

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Numerical method	
Governing eqs.	Compressible N-S eqs.
Discretization	Cell-centered finite volume
Inviscid flux	SLAU with 3rd-order U-MUSCL
Viscous flux	2nd-order
Time integration	LU-SGS dual-time stepping
Turbulence model	SSG/LRR- $\omega$ RSM, $k$ - $\omega$ SST model

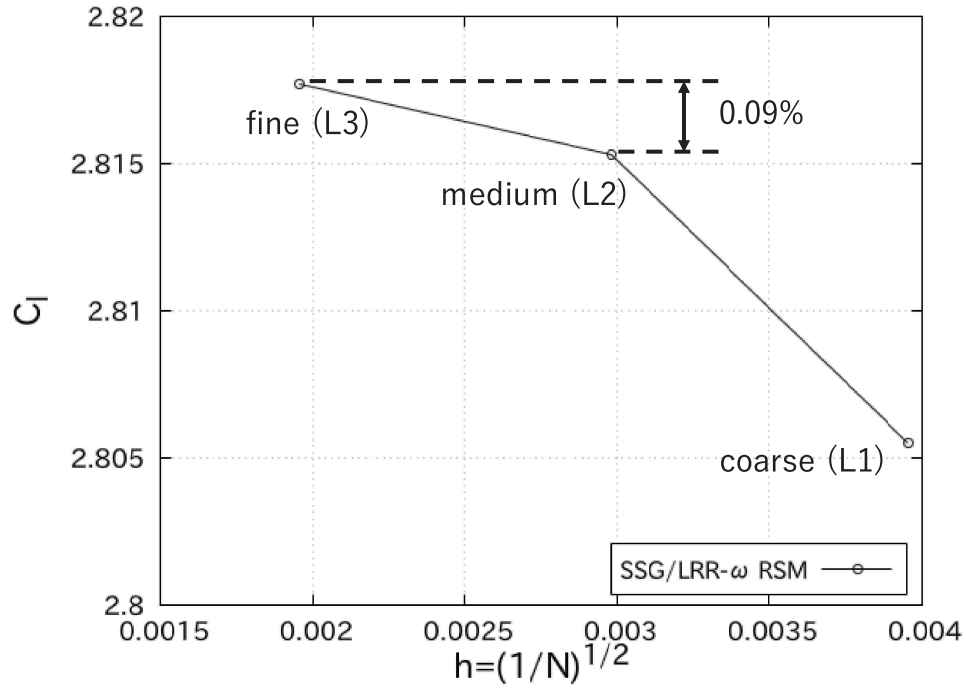
Flow condition	
$M_\infty$	0.17
$Re_c$	$1.71 \times 10^6$
$T_\infty$	295.56 K
$\alpha$	0 ~ 24 deg

## Contents

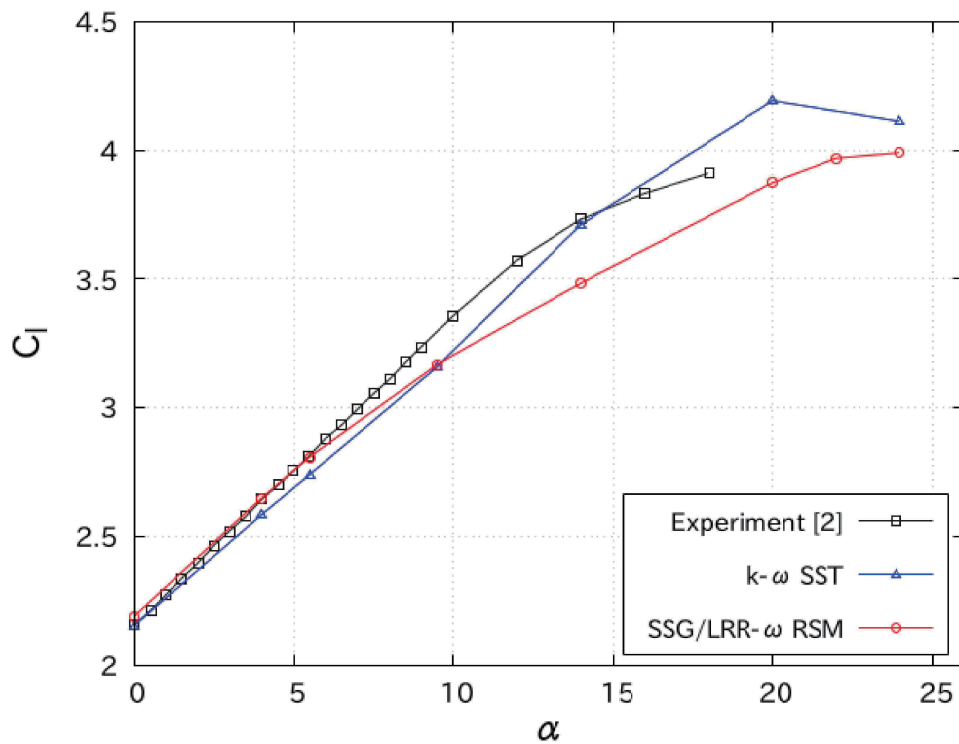
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1. Overview of Turbulence model
2. Computational settings
3. Results: comparison with  $k$ - $\omega$  SST model
  - Grid convergence study
  - Lift coefficient  $C_l$
  - $\alpha=5.5$  deg
  - $\alpha=14$  deg
4. Concluding remarks

## Grid convergence study ( $\alpha=5.5$ deg)

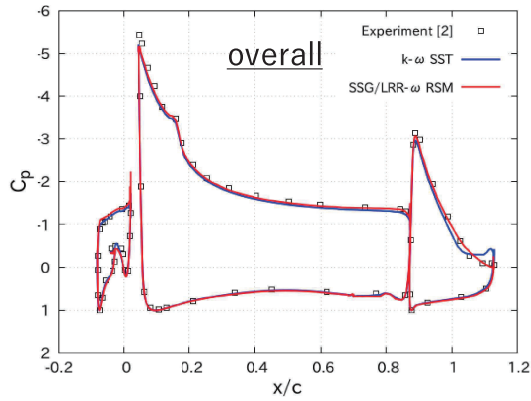


## Lift coefficient $C_l$



[2] Murayama et al., 2014

## Pressure coefficient $C_p$ ( $\alpha=5.5$ deg)

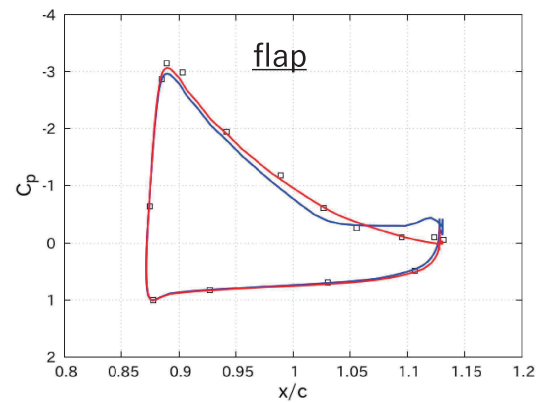
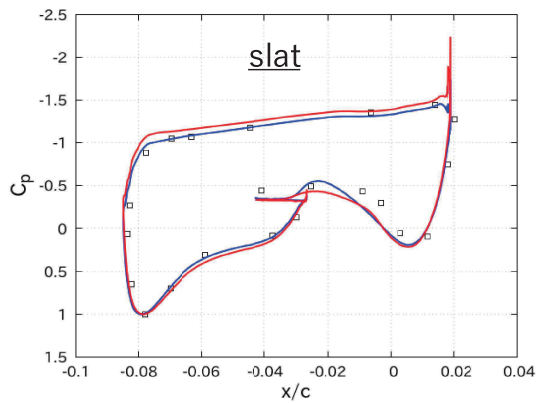


Lift coefficient:  $C_l$

Experiment: 2.81

$k-\omega$  SST: 2.74

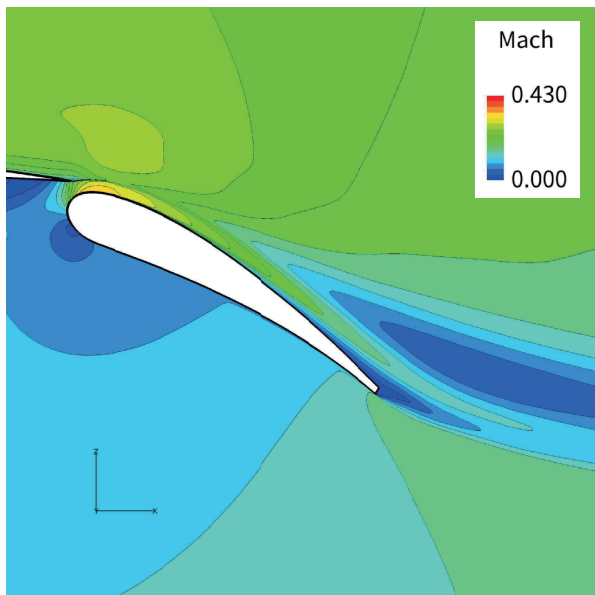
SSG/LRR- $\omega$  RSM: 2.80



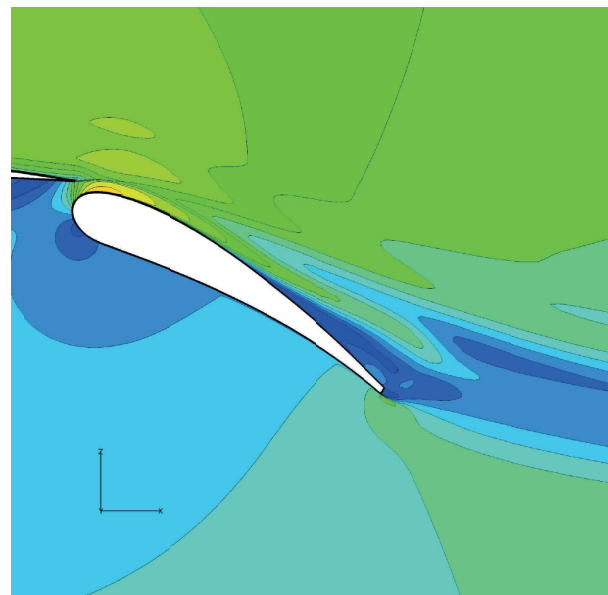
[2] Murayama et al., 2014

## Mach number ( $\alpha=5.5$ deg)

SSG/LRR- $\omega$  RSM

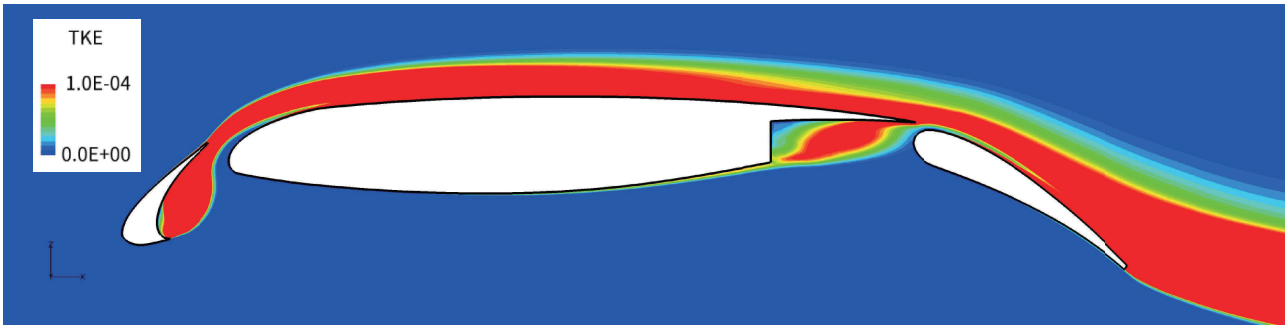


$k-\omega$  SST

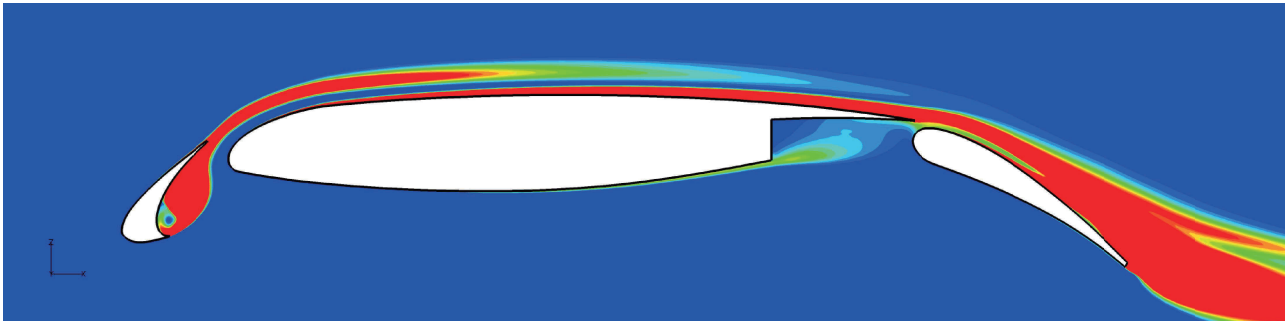


## Turbulence kinetic energy $k$ ( $\alpha=5.5$ deg)

SSG/LRR- $\omega$  RSM

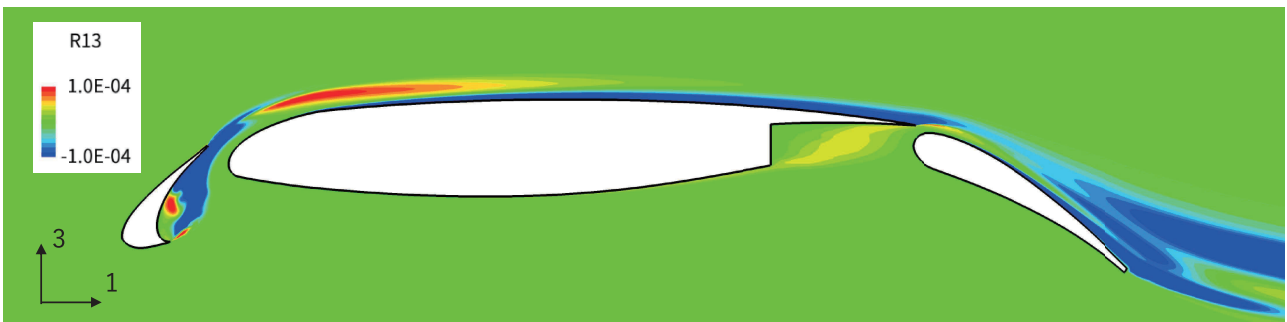


$k$ - $\omega$  SST

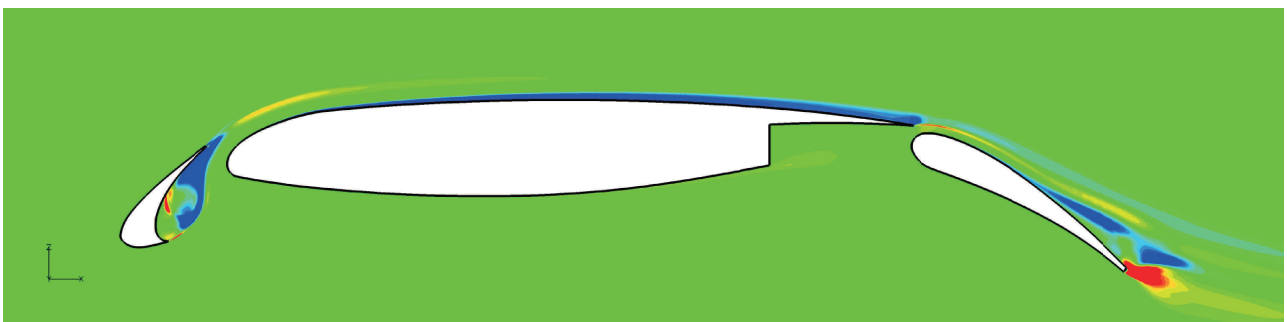


## Reynolds stress $R_{13}$ ( $\alpha=5.5$ deg)

SSG/LRR- $\omega$  RSM



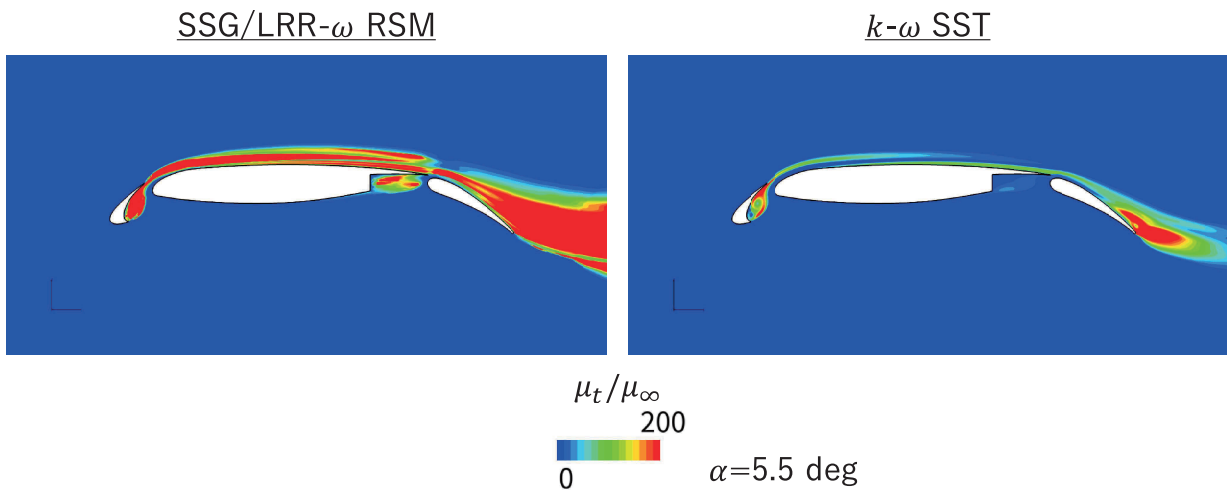
$k$ - $\omega$  SST



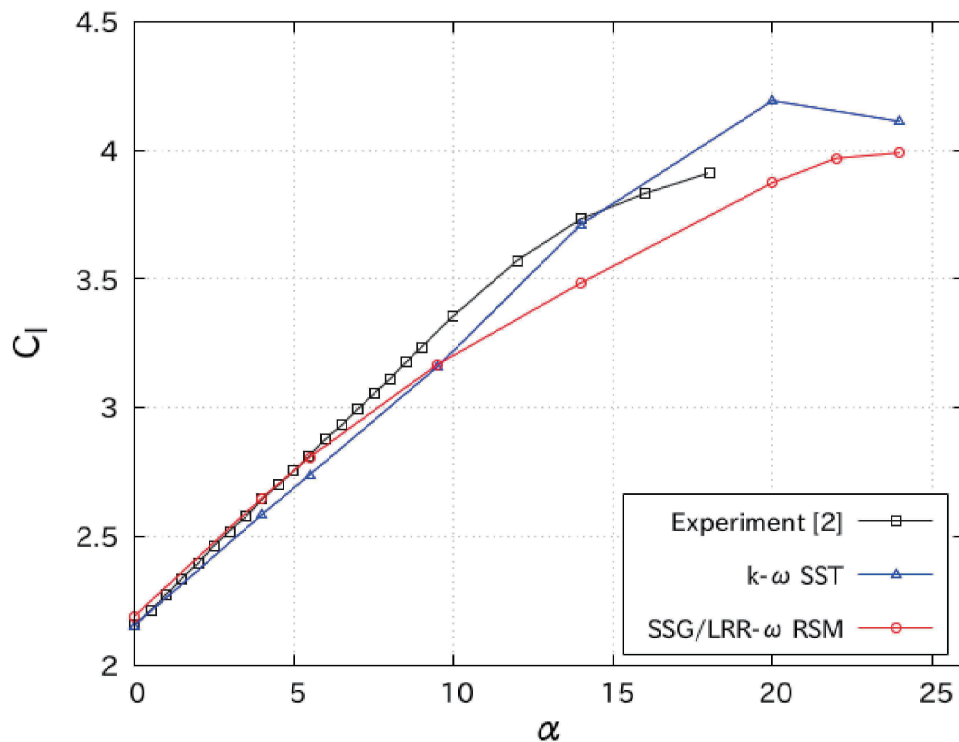
## Estimated $\mu_t$ in RSM

$$\tau_{ij} = -\rho R_{ij} = \mu_t \left( 2S_{ij} - \frac{2}{3} \frac{\partial u_l}{\partial x_l} \delta_{ij} \right) - \frac{2}{3} \rho k \delta_{ij} \quad \text{in } k-\omega \text{ SST}$$

$$\rightarrow \mu_t = \frac{\rho}{2} \sqrt{\frac{\left( R_{ij} - \frac{2}{3} k \delta_{ij} \right)^2}{\left( S_{ij} - \frac{1}{3} \frac{\partial u_l}{\partial x_l} \delta_{ij} \right)^2}}$$

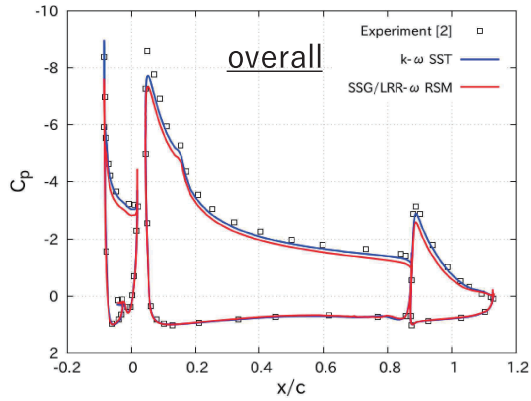


## Lift coefficient $C_l$

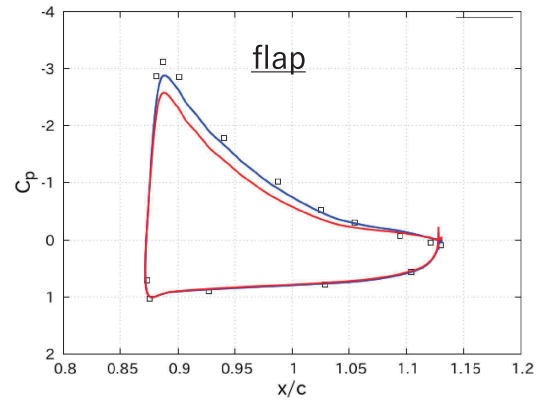
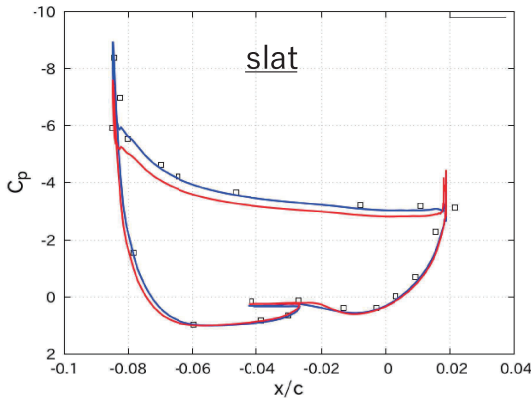


[2] Murayama et al., 2014

# Pressure coefficient $C_p$ ( $\alpha=14$ deg)



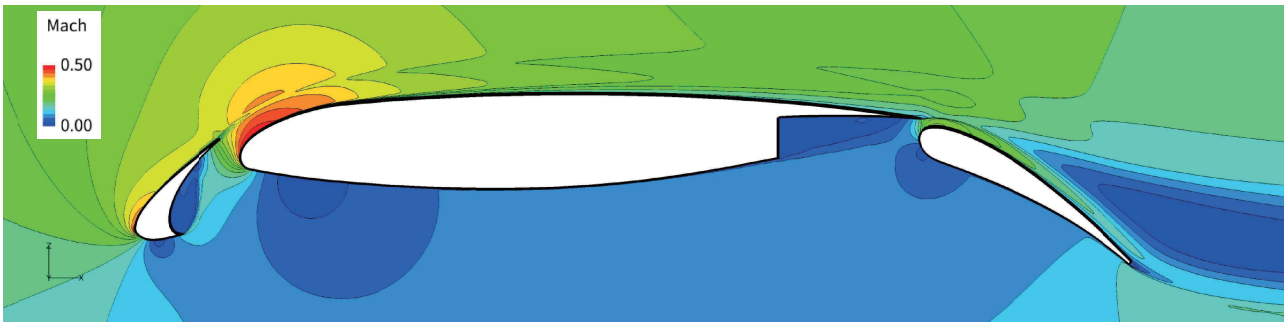
Lift coefficient:  $C_l$   
 Experiment: 3.73  
 $k-\omega$  SST: 3.71  
 SSG/LRR- $\omega$  RSM: 3.48



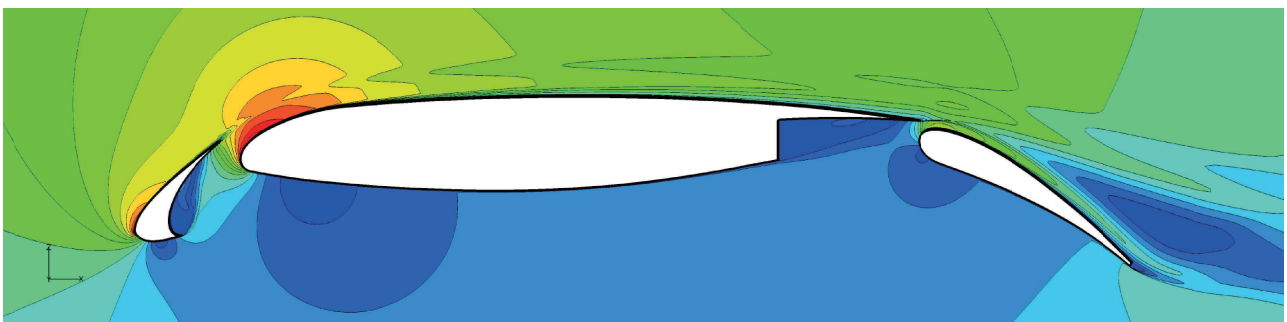
[2] Murayama et al., 2014

# Mach number ( $\alpha=14$ deg)

SSG/LRR- $\omega$  RSM



$k-\omega$  SST





## Concluding remarks

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We analyzed 30P30N airfoil by RSM

- Pressure coefficient  $C_p$  obtained by the RSM is in good agreement with the experimental result ( $\alpha=5.5$  deg), in particular around the flap
- The  $k-\omega$  SST clearly shows separation around the flap trailing edge, where the RSM predicts relatively smaller separation
- At  $\alpha=14$  deg , lift coefficient  $C_l$  is different, but no significant change is found in the flow field