The Analysis of the Structure of Whole Lunar Transfer Orbit by Hybrid Rocket Kick Motor for Micro Deep Space Probe

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Abstract

We are designing lunar transfer orbits which micro probe with hybrid rocket kick motor can fly from geo transfer orbit. Kick motor acceleration performance is less than 1 km/s, and we adopt piggy back system, so we cannot decide the launch day and the position of perigee arbitrarily. In order to meet these conditions, we are analyzing relationships between launch day and acceleration considering various geo transfer conditions.

超小型探査機用ハイブリッドロケットキックモータによる大域的な月遷移軌道とその解構造の理解

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摘要

著者が所属する北海道大学大学院工学院宇宙環境システム工学研究室は小型のハイブリッドロケットキックモータを開発している．JAXA と提携して開発する超小型探査機に搭載し，地球遷移軌道から月遷移軌道に遷移させる際にキックモータを使用することでキックモータの実証を計画している．キックモータの要求性能(増速量)が 1 km/s 以下であること，他の親機と一緒に打ち上げてもらうピギーバック方式を適用する予定であるため，打上日が親機の都合に依存するということ，それにより地球静止軌道における形状が一意に定まらないという課題が生じる．本稿ではこれらの要求を満たすために，地球静止軌道に関する様々な条件を設定して，各条件における打上日と月遷移軌道に遷移する際に必要な増速量の関係を解析すし比較検討することによって有効な条件は何かを議論する．

1 Introduction

Confirming the status of space exploration at national scale, NASA, JAXA, and ESA are planning sample return missions for Mars and asteroids. Japan has achieved the world-leading result of asteroid sample return by Hayabusa, but deep space exploration missions aiming for beyond the moon have been conducted only once or twice in 10 years. There are still few opportunities to experience.

One way to increase the opportunity to experience deep space exploration projects in a limited budget is to install a kick motor on a micro spacecraft. For example, as shown in Fig. 1, if a ride from the Low Earth Orbit (LEO) to Geo Transfer Orbit (GTO) to Geostationary Earth Orbit (GEO) is carried out and the speed is increased at perigee by a kick motor, it can reach the Venus at a speed of 1.06 km/s, and Mars at a speed of 1.15 km/s. Another possible method is to reach the moon at a small speed increase (0.7 km/s) and obtain the speed increment to the target using lunar swing-by. The payload launch to deep space beyond the moon, which was carried out after 2000, was the lunar orbital satellite "Kaguya" in 2007, Venus probe "Akatsuki" in 2010, and the asteroid explorer "Hayabusa 2" in 2014. However, the total number of payload launch increases to 18 times, including beyond the GTO. Although not all of them can be shared, it is possible to secure opportunities at once every two years.

To achieve the thrust and specific thrust required for kick motors, chemical rockets are necessary, and safe and low-cost rockets are desirable. Hybrid rockets are the most suitable rockets, because they can provide safety management compared to solid rockets and liquid rockets. Hokkaido University Laboratory of Space Systems, to which the author belongs, conducts ground combustion experiments and develops hybrid rocket kick motors.

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In our laboratory, to demonstrate hybrid rocket kick motors in the space, we plan the agenda "Kick motor research and development base for micro spacecraft" with JAXA.

The implementation plan as FY30 is roughly divided into the following five elements[1].

(1) Development of kick motor
- Development of simple high-air combustion test facility
- Construction of fuel reverse speed formula
- Development of 6U class (~500N class) motors

(2) Development of attitude control system
Study of control law and start of response characteristic acquisition experiment

(3) Thermal design
Examination by simplified nodal analysis under wider orbit conditions

(4) Examination of orbit
Examination of the Lunar Transfer Orbit (LTO)

(5) Survey of demand and mission requirements
Clarify design requirements for staged development targets and kick motors when targeting the Moon and Mars

The authors are examining the trajectory corresponding to the above element (4).

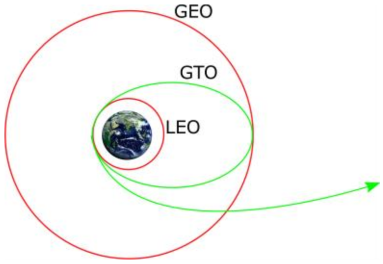

Fig. 1: Acceleration from GTO

2 Design policy

2.1 Premise conditions for this analysis

There are two premise conditions for this mission. These conditions are treated as conditions that cannot be changed thereafter.

① **The performance of the kick motor is 1 km/s or less**

Since the kick motor of this mission is assumed to be a small-scale thing used for a micro spacecraft, the increase speed performance is very small, 1 km/s or less. Therefore, there is a possibility that orbits which could be realized with past spacecraft cannot be used.

② **Apply a piggyback system**.

The piggyback system is a method of launching the spacecraft together with the main satellite using the launch capability of the rocket. It is less expensive than launching alone, and it has the advantage of increasing launch opportunities because it can be launched even in missions different from the main satellite. This mission will ride on the main satellite up to the GTO and aim for the moon from the GTO. The orbit from the GTO to the moon is expressed as Lunar Transfer Orbit (LTO).

Moreover, in the piggyback system, the opportunity for launching the spacecraft depends on the mission of the main satellite, so there is a weak point that the orbital launch time cannot be selected freely for the micro spacecraft. Therefore, it is necessary to design a trajectory that allows the spacecraft to be inserted into the moon even under various conditions.

Based on the above conditions, the goal of this study is to analyze the amount of acceleration required to meet the moon and determine whether the required performance of the kick motor is satisfied. In this study, the following three conditions were assumed for analysis.

Ⅰ. Case of accelerating in the tangential direction from perigee
Ⅱ. Case of restrictions on the orbit injection point are removed
Ⅲ. Case of the restriction on the speed direction is removed (limited in-plane direction)

2.2 GTO initial conditions

Table 1 shows the initial conditions of the GTO in equatorial plane coordinate system. The equatorial plane is the orbital plane for the earth's equator, the lunar orbital plane is the orbital plane of the moon, and the ecliptic plane is the orbital plane of the sun. Their positional relationship is as shown in Fig. 2. The red ellipse represents the equator plane, the orange ellipse represents the ecliptic plane, and the pink ellipse represents the lunar orbital plane.

Table 1: Initial conditions of GTO

Perigee of GTO r_p[km]	300
Semi-major axis a_E [km]	2.442×10^4
Eccentricity e_E [-]	0.7265
Inclination i_E [deg]	30
Argument of periapsis ω_E [deg]	0 or 180
Longitude of ascending node Ω_E [deg]	0~360

Table 2 shows the initial conditions for the orbit of the moon in the equatorial plane coordinate system.

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Table 2: Initial conditions of Lunar orbit

Revolution radius from the earth center [km]	384400(Approximate with circular orbit)
The angle between the equator and lunar orbital plane i_M (Equatorial plane coordinate system)[deg][2]	28.54
Longitude of ascending node on lunar orbital plane Ω_M(Equatorial plane coordinate system)[deg][2]	125.08
Argument of periapsis on lunar orbital plane ω_M (Equatorial plane coordinate system)[deg][2]	318.15

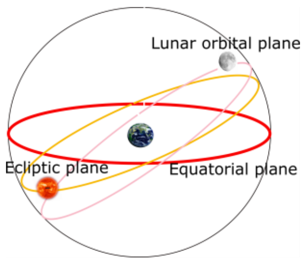

Fig. 2: The relationship of each orbital plane

3 Analysis method and results

3.1 Case of accelerating in the tangential direction from perigee

Give ΔV in the tangential direction at the perigee of GTO. Using the longitude of ascending node Ω as a parameter, we investigate the relationship with ΔV. Ignore the phase of the moon and spacecraft.

In the equatorial plane coordinate system, the equinox point direction is set as the x-axis, the rotation direction of the earth (angular momentum direction) is set as the z-axis, and the position orthogonal to these two axes is set as the y-axis.

Set the perigee direction unit vector ($\mathbf{P_E}$), on the equatorial plane reference, the speed direction unit vector **(**$\mathbf{Q_E}$**),** and the out-of-plane direction unit vector ($\mathbf{R_E}$) when accelerating in the tangential direction of the perigee as shown in Fig. 3 below.

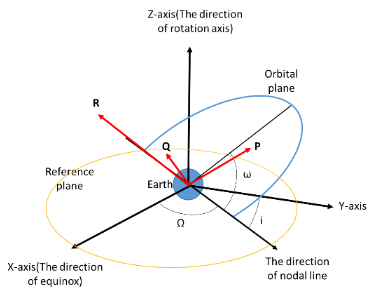

Fig 3: Unit vectors in equatorial plane

$\mathbf{P_E}$, $\mathbf{Q_E}$, and $\mathbf{R_E}$ in the orbital plane coordinate system can be obtained by rotating the equatorial plane coordinate system in the order of Ω_E (around the z axis)→i_E(around the x axis)→ω_E(around the z axis).

$$\begin{bmatrix}\mathbf{P_E}\\ \mathbf{Q_E}\\ \mathbf{R_E}\end{bmatrix} = \begin{bmatrix}\cos\omega_E & \sin\omega_E & 0\\ -\sin\omega_E & \cos\omega_E & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & \cos i_E & \sin i_E\\ 0 & -\sin i_E & \cos i_E\end{bmatrix}\begin{bmatrix}\cos\Omega_E & \sin\Omega_E & 0\\ -\sin\Omega_E & \cos\Omega_E & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

$$\mathbf{P_E} = \begin{bmatrix}\cos\omega_E\cos\Omega_E - \sin\omega_E\sin\Omega_E\cos i_E\\ \cos\omega_E\sin\Omega_E + \sin\omega_E\cos\Omega_E\cos i_E\\ \sin\omega_E\sin i_E\end{bmatrix} \quad (1)$$

$$\mathbf{Q_E} = \begin{bmatrix}-\cos\Omega_E\sin\omega_E - \sin\Omega_E\cos\omega_E\cos i_E\\ -\sin\Omega_E\sin\omega_E + \cos\Omega_E\cos\omega_E\cos i_E\\ \cos\omega_E\sin i_E\end{bmatrix} \quad (2)$$

$$\mathbf{R_E} = \begin{bmatrix}\sin\Omega_E\sin i_E\\ -\cos\Omega_E\sin i_E\\ \cos i_E\end{bmatrix} \quad (3)$$

Converts the coordinate axis in the equatorial plane reference to the coordinate axis in the lunar orbital plane reference.

The lunar orbital plane coordinate system is set so that the x-axis is the perilune direction, the z-axis is the lunar revolution direction (angular momentum direction), the y-axis is perpendicular to the x-axis, and the z-axis with the earth center as the origin.

The rotation matrix is shown as follows from the positional relationship.

$$\mathbf{A}(\text{Rotation by }\Omega_M) = \begin{bmatrix}\cos 125.08^\circ & \sin 125.08^\circ & 0\\ -\sin 125.08^\circ & \cos 125.08^\circ & 0\\ 0 & 0 & 1\end{bmatrix} \quad (4)$$

$$\mathbf{B}(\text{Rotation by }i_M) = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos 28.54^\circ & -\sin 28.54^\circ\\ 0 & \sin 28.54^\circ & \cos 28.54^\circ\end{bmatrix} \quad (5)$$

$$\mathbf{C}(\text{Rotation by }\omega_M) = \begin{bmatrix}\cos 318.15^\circ & \sin 318.15^\circ & 0\\ -\sin 318.15^\circ & \cos 318.15^\circ & 0\\ 0 & 0 & 1\end{bmatrix} \quad (6)$$

Set the unit vector after coordinate transformation as follows.

$$\mathbf{P_M}(\text{Perilune direction unit vector in lunar orbital plane reference}) = \mathbf{CBAP}$$

$$\mathbf{Q_M}(\text{Velocity direction unit vector in lunar orbital plane reference}) = \mathbf{CBAQ}$$

$$\mathbf{R_M}\left(\text{Angular momentum direction unit vector in lunar orbital plane reference}\right) = \mathbf{P_M} \times \mathbf{Q_M}$$

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Calculate the angular momentum using the following formula. b_E is the GTO semi-minor axis, and T_E is the GTO period.

$$h_1 = \frac{2\pi a_E b_E}{T_E} \tag{7}$$

The position vector and velocity vector at the orbit injection point on the lunar orbital plane reference are shown as follows. μ_E is the gravity constant of the earth.

$$\boldsymbol{r_1} = r_p \mathbf{P_M} \tag{8}$$

$$\boldsymbol{v_1} = \frac{\mu_E}{h_1}(1 + e_E)\mathbf{Q_M} \tag{9}$$

Now that we have defined the position vector and velocity vector, we calculated the GTO longitude of ascending node Ω_{EM}, argument of periapsis ω_{EM}, and inclination i_{EM} on the basis of the lunar orbital plane [3]. In the lunar orbital plane coordinate system, the unit vectors in the x-axis, y-axis, and z-axis directions are $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$ respectively.

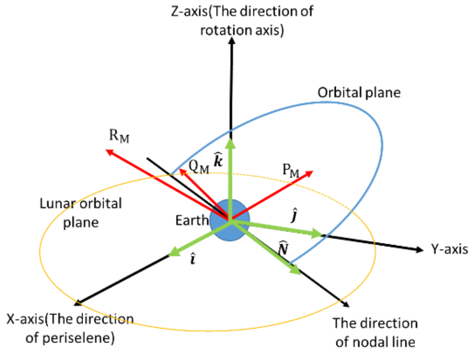

Fig. 4: Unit vectors in moon's path plane

The inclination i_{EM} can be obtained from the relationship between $\mathbf{R_M}$ and $\hat{\boldsymbol{k}}$ by the following formula.

$$i_{EM} = \cos^{-1}(\mathbf{R_M} \cdot \hat{\mathbf{k}}) \tag{10}$$

Here, the value of i_{EM} is obtained in the range of $0^\circ \leq i_{EM} \leq 180^\circ$ but this is determined solely because it matches the principal value of $\cos^{-1}$.

Next, longitude of ascending node Ω_{EM} is the unit vector in the direction of intersection using,

$$\hat{\mathbf{N}} = \frac{\hat{\mathbf{k}} \times (\boldsymbol{r_1} \times \boldsymbol{v_1})}{|\hat{\mathbf{k}} \times (\boldsymbol{r_1} \times \boldsymbol{v_1})|} \tag{11}$$

$$\Omega_{EM} = \cos^{-1}(\hat{\mathbf{N}} \cdot \hat{\mathbf{i}}) \tag{12}$$

can be obtained. In that case, if $\hat{\mathbf{N}} \cdot \hat{\mathbf{j}} \geq 0$, Ω_{EM} can be obtained as the main value of $\cos^{-1}$, that is, $0^\circ \leq \Omega_{EM} \leq 180^\circ$, but $\hat{\mathbf{N}} \cdot \hat{\mathbf{j}} < 0$, the value of Ω_{EM} must be in the range of $180^\circ < \Omega_{EM} < 360^\circ$, so can be obtained as follows.

$$\Omega_{EM} = 360^\circ - \cos^{-1}(\hat{\mathbf{N}} \cdot \hat{\mathbf{i}}) \tag{13}$$

Finally, argument of periapsis ω_{EM} can be obtained by the following formula.

$$\omega_{EM} = \cos^{-1}(\hat{\mathbf{N}} \cdot \mathbf{P_M}) \tag{14}$$

In this case, if $\mathbf{P_M} \cdot \hat{\mathbf{k}} \geq 0$, ω_{EM} can be obtained as the main value of $\cos^{-1}$, that is, a value in the range of $0^\circ \leq \omega_{EM} \leq 180^\circ$, but $\mathbf{P_M} \cdot \hat{\mathbf{k}} < 0$, the value of Ω_{EM} must be in the range of $180^\circ < \omega_{EM} < 360^\circ$ so can be obtained as follows.

$$\omega_{EM} = 360^\circ - \cos^{-1}(\hat{\mathbf{N}} \cdot \mathbf{P_M}) \tag{15}$$

Since GTO and the lunar transfer orbit are on the same plane, Ω, ω, and i in these two orbits coincide, so if you know Ω_{EM}, ω_{EM}, and i_{EM} of GTO after coordinate transformation, you can adapt to the lunar transfer orbit. As a result, if the azimuth element in the transfer orbit is set to Ω_{TM}, ω_{TM}, i_{TM},

$$\Omega_{EM} = \Omega_{TM}, \quad \omega_{EM} = \omega_{TM}, \quad i_{EM} = i_{TM}$$

are completed.

Next, the equation of motion of the elliptical orbit, the expression representing the perigee and the far apogee, is expressed as follows.

$$\mathrm{r} = \frac{a(1 - e^2)}{1 + e\cos f} \tag{16}$$

$$r_p = a(1 - e) \tag{17}$$

$$r_a = a(1 + e) \tag{18}$$

Where f is the true anomaly. The geocentric distance at the ascending intersection is set to 384400 km, and the value is equal to the revolution radius of the moon, so f $=\omega_{TM}$ can be substituted.

Since r_p(perigee), r(distance from the earth to the moon 384400 km), and f are known this time, unknowns are to determine the semi-major axis a, the eccentricity e, and the apogee r_a.

By expanding the equations (16) to (18), the eccentricity e_{TM}, the semi-major axis a_{TM}, and the apogee r_{aTM} in the lunar transfer orbit are obtained. Since the semi-major axis was found, the equation for calculating the velocity vector in the tangential direction of the perigee in the lunar transfer trajectory was used.

Case of the elliptical orbit 0<e_{TM}<1,

$$\boldsymbol{v_2} = \frac{\mu_E}{h_2}(1 + e_{TM})\mathbf{Q_M} \tag{19}$$

Case of the hyperbolic trajectory (e>1),

$$\boldsymbol{v_2} = \sqrt{\mu_E\left(\frac{2}{r_p} + \frac{1}{a_{TM}}\right)}\,\mathbf{Q_M} \tag{20}$$

Case of the parabolic trajectory (e=1),

$$\boldsymbol{v_2} = \sqrt{\frac{2\mu_E}{r_p}}\,\mathbf{Q_M} \tag{21}$$

From this, the difference $(\boldsymbol{v_2} - \boldsymbol{v_1})$ between the GTO and the velocity vector of the lunar transfer orbit is obtained, and the acceleration amount ΔV can be obtained by taking the norm of the difference.

The series of calculations was changed in the range of

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Ω_E= 0°~360° on the equatorial plane reference. The resulting graph is graphed by taking the acceleration amount ΔV on the vertical axis and the ascending intersection longitude Ω_E of the GTO in the equatorial plane coordinate system on the horizontal axis.

(i) $\omega_E = 0°$

It will intersect at the descending intersection, not at the ascending intersection.

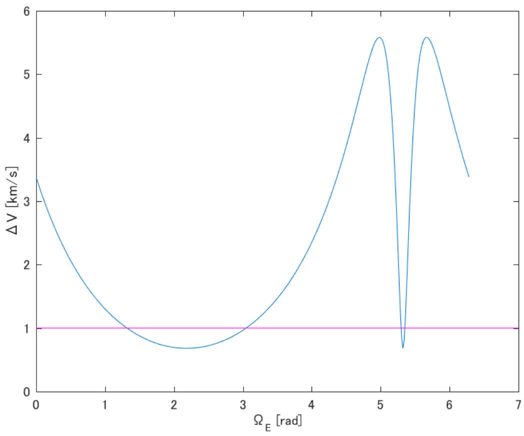

Fig. 5: The relationship between ΔV and Ω_E (ω_E=0°)

(ii) $\omega_E = 180°$

This is an intersection at the ascending intersection and not at the descending intersection. In addition, the acceleration peak was slightly larger than when ω_0=0° .

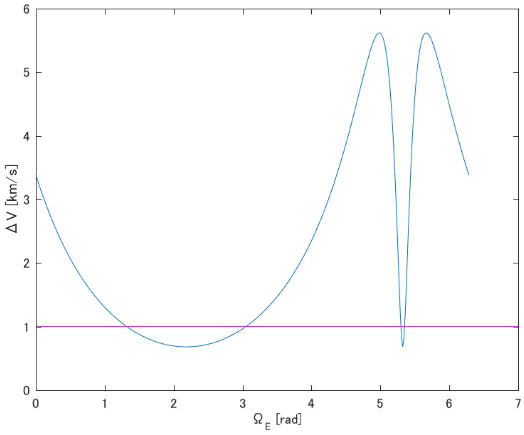

Fig. 6: The relationship between ΔV and Ω_E (ω_E=180°)

The minimum value is $\Delta V = 0.68\ \mathrm{km/s}$, when $\Omega_E = 125°, 305°$, and the range that satisfies the performance of the kick motor is $\Omega_E = 76°\sim174°,\ 305°$.

Depending on the value of Ω_E, it may be necessary to increase the speed exceeding the kick motor performance (1 km/s or less). Can be understood visually.

Therefore, by expanding the condition of increasing the tangential direction at the perigee, the condition that the required acceleration amount decreases as a whole, the range of the longitude of ascending node Ω_E that can be achieved with the acceleration amount of 1 km/s or less is expanded. We will examine the conditions to be performed.

3.2 Case of restrictions on the orbit injection point are removed

When accelerating at a point other than the perigee, the acceleration point becomes the orbit insertion point in the new transfer orbit, and the direction of the semi-major axis changes. This time, since ΔV is given in the tangential direction, the semi-major axis of the transfer orbit changes on the GTO orbital plane, and it is possible to contact the lunar orbit even at the longitude of ascending node that did not satisfy the condition when ΔV was given at the near point.

Here, if the distance from the center of the earth to the acceleration point is r_1, it can be calculated by the following formula.

θ represents true anomaly. In this analysis, θ was varied in the range of $-90°\sim90°$.

$$r_1 = \frac{a_E(1 - {e_E}^2)}{1 + e_E\cos(\omega_E + \theta)} \tag{22}$$

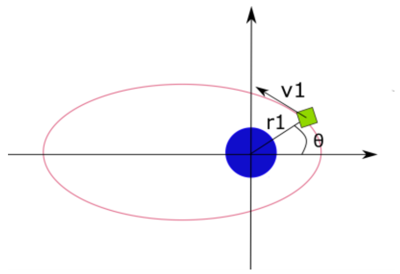

Fig. 7: The definition of the true Anomaly

The position vector and velocity vector at the orbit injection point on the lunar orbital plane reference are shown as follows [4].

$$\boldsymbol{r_1} = r_1 \begin{bmatrix} \cos\sigma\cos\Omega_{EM} - \sin\sigma\sin\Omega_{EM}\cos i_{EM} \\ \cos\sigma\sin\Omega_{EM} + \sin\sigma\cos\Omega_{EM}\cos i_{EM} \\ \sin\sigma\sin i_E \end{bmatrix} \tag{23}$$

$$\boldsymbol{v_1} = -\frac{\mu_E}{h_1}A \tag{24}$$

$$\begin{aligned} A &= [\cos\Omega_{EM}(\sin\sigma + e_{EM}\sin\omega_{EM}) \\ &+ \sin\Omega_{EM}(\cos\sigma + e_{EM}\cos\omega_{EM})\cos i_{EM}]i_x \\ &+ [\sin\Omega_{EM}(\sin\sigma + e_{EM}\sin\omega_{EM}) \\ &- \cos\Omega_{EM}(\cos\sigma + e_{EM}\cos\omega_{EM})\cos i_{EM}]i_y \\ &+ [-(\cos\sigma + \mathrm{e}\cos\omega_{EM})\sin i_{EM}]i_z \end{aligned}$$

$$\sigma = \omega_{EM} + \theta \tag{25}$$

Next, the velocity vector $\boldsymbol{v_2}$ after the injection is obtained at the orbit injection point.

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When the velocity is changed, the following equation holds for the velocity vector before and after the injection.

$$\boldsymbol{v_2} = (1+\alpha)\boldsymbol{v_1} \quad (\alpha\text{: coefficient}) \tag{26}$$

Here, $\alpha|\boldsymbol{v_1}| = \Delta V$(acceleration amount), so if ΔV is set as a parameter, α can be obtained, and the velocity vector $\boldsymbol{v_2}$ after acceleration can be obtained. This time, ΔV was varied in the range of 0 km/s~2 km/s. Here, the angular momentum vector $\boldsymbol{h_2}$ is obtained by the following equation.

$$\boldsymbol{h_2} = \boldsymbol{r_1} \times \boldsymbol{v_2} \tag{27}$$

The position vector and velocity vector at the trajectory entry point in the transition trajectory are obtained, and using the method of keplerian from Cartesian, we can be obtained Ω_{TM}, ω_{TM}, i_{TM}, e_{TM}, and a_{TM}.

Next, the geocentric distance at the ascending point (or descending point) is obtained by the orbital motion equation (16). This calculation obtains the condition that the geocentric distance r_{TM} becomes the moon's revolution radius 384400km. The ascending intersection longitude was fixed, and True Anomaly θ was moved in the range of $-90°\sim90°$, and the speed increase ΔV was moved in the range of 0 km/s〜2 km/s.

3.3 Case of the restriction on the speed direction is removed (limited in-plane direction)

The position vector $\boldsymbol{r_1}$ is obtained from equation (8). If the velocity after acceleration is set to $\boldsymbol{v_2}$, it can be defined by the following formula from Fig. 8.

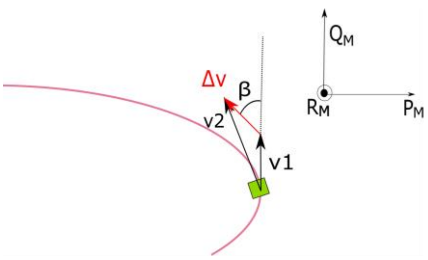

Fig. 8: The definition of the velocity vector

$$\boldsymbol{v_2} = \boldsymbol{v_1} + \Delta V cos\beta \times \mathbf{Q_M} - \Delta V sin\beta \times \mathbf{P_M} \tag{28}$$

Since the position vector and velocity vector at the orbit entry point were obtained, the orbital elements were obtained as in 3.2.

The analysis was performed by moving the ΔV direction $\Delta\beta$ in the range of $-90°\sim90°$and the speed increase ΔV in the range of 0 km/s〜2 km/s.

3.4 Result

Table 3 below summarizes the degree of freedom and the number of days allowed for launch under each condition.

Table 3: The result

Condition	Degree of freedom[-]	Launch possible days [day]
3.1	1	102
3.2	2	128
3.3	2	104

The number of days of Condition 3.2 that can be launched increased by 25.5% compared to Condition 3.1. Condition 3.3 also increased slightly from condition 3.1. The launch date could not be covered all the year under either condition.

4 Summary

As a result of analysis under the condition of increasing the speed in the tangential direction at the perigee, there was a limit on the value of Ω_E at which the lunar transfer orbit could be realized in the performance range of the kick motor (~1 km/s). As a result of analyzing the effect of increasing the degree of freedom by loosening the conditions, it was found that the selectable Ω_E increased by 25.5% depending on the condition of increasing the speed in the tangential direction near the perigee. It was also found that the increase in Ω_E by increasing the in-plane direction other than the tangential direction is not expected.

In the future, we would like to analyze how much Ω_E that can be achieved with the acceleration capability of 1 km/s or less, which is within the performance range of the kick motor, by increasing the degree of freedom further when accelerating in the out-of-plane direction at a perigee.

5 References

[1] JAXA 大学共同利用連携拠点「超小型深宇宙探査機用キックモータ研究開発拠点」
平成 30 年度研究打ち合わせ(第 2 回)　発表資料

[2] H.Hussmann，F.Sohl，J.Oberst: *Solar System*，pp. 171，Springer Materials，(2009)

[3] 半揚稔雄: *ミッション解析と軌道設計の基礎*，pp.41-46，現代数学社，(2014)

[4] RICHARD H. BATTIN: *An Introduction to the Mathematics and Methods of Astrodynamics*, pp. 125, AIAA, (1987)

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